ANNALS OF ECONOMICS AND FINANCE **9-1**, 91–101 (2008)

Does the Utility Function Form Matter for Indeterminacy in a Two Sector Small Open Economy

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In his paper "Does utility curvature matter for indeterminacy", Kim (2005) analyzed the relationship among the utility function form, curvature and indeterminacy, concluding that the relationship between curvature and indeterminacy is not robust in neoclassical growth model and the indeterminacy may disappear under the utility specification as in Greenwood et.al (1998). The models he discussed are confined within one sector closed economy. Weder (2001), Meng and Velasco (2004) extend the Benhabib and Farmer (1996) and Benhabib and Nishimura (1998)'s closed economy two sector models into open economy, showing that indeterminacy can occur under small external effects, independently of the intertemporal elasticities in consumption. Meng and Velasco (2003) went further, showing the independence between the elasticity of labor supply and indeterminacy in open economy. Under nonseparable utility forms like in King, Plosser and Rebelo (1988, henceforth KPR) or Bennett-Farmer (2000) form, do we still have this property? In other words, is the independence between curvature and indeterminacy in small open economy models robust to the specification of utility functions? In this note, I tackle this issue under two different versions of nonseparable utility functions commonly used in the literature. The answer is "yes" to KPR form but "no" to Bennett-Farmer form. Endogenous time preference and consumable nontradable goods are two elements to deliver this result.

Key Words: Indeterminacy; Endogenous time preference.
JEL Classification Numbers: E32, F4.

* Chapter 4 of my PhD dissertation. Helpful comments from Jess Benhabib, Kim Jinill, Pierpaolo Benigno, Viktor Tsyrennikov and Shenghao Zhu are greatly appreciated and all the remaining errors are mine. I also thank the Editor, Heng-fu Zou for his support. For a recent extensive survey of the literature, see Benhabib and Farmer (1999).

1529-7373/2008

1. INTRODUCTION

It is well understood by now that under certain market imperfection conditions models of business cycle can be subject to indeterminacy. Indeterminacy means that from the same initial condition there exist an infinite number of equilibria, all of which converge to a unique steady state. Most early models like Benhabib-Farmer-Guo and Bennett-Farmer models in the literature are closed-economy, and focus on the empirical plausibility of the conditions for indeterminacy. Recent research demonstrates that only small market imperfections are needed to generate indeterminacy instead of early large increasing returns or external effects. One interesting issue is that indeterminacy also relies on the preference. Kim (2005) discussed the relationship between the utility curvature and indeterminacy but cannot find a generic property between them, Benhabib-Farmer-Guo's indeterminacy result even disappears under Greenwood et.al (in short GHH) utility form.

Recently Weder (2001), Meng and Velasco (2004) extend the Benhabib and Farmer (1996) and Benhabib and Nishimura (1998)'s closed economy two sector models into open economy, showing that indeterminacy can occur under small external effects, independently of the intertemporal elasticity of consumption. Meng and Velasco(2003) went further by showing the independence between the elasticity of labor supply and indeterminacy in open economy. One remaining issue in theirs work is that under nonseparable utility form like in KPR or Bennett-Farmer, do we still have this property? In other words, is the independence between curvature and indeterminacy in small open economy models robust to the form of utility functions?

In this paper, we tackle this issue further and find that the answer is "yes" to King et al form ($u^{KPR} = \frac{[C^{\theta}(1-l)^{1-\theta}]^{1-\sigma}-1}{1-\sigma}$) but "no" to Bennett-Farmer form ($u^{Bennett-Farmer1} = \frac{[C\exp(-\frac{l^{1+\chi}}{1+\chi})]^{1-\sigma}-1}{1-\sigma}$). We also derive the indeterminacy conditions under the two types of utility functions.

Meng and Velasco (2003) and Bian and Meng (2004) prove the independence under GHH and $(\frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\chi}}{1+\chi})$ forms. While the nonseparable forms are needed to deal with carefully since this kind of preference like u^{KPR} is compatible with a BGP and consistent with the high real exchange rate volatility that is observed in data (see Lucio Sarno 2001). Also this preference provides more plausible implications for the short run dynamics of several macroeconomics variables than the separable one.

We follow the literature of small open economy RBC models by incorporating into the model an endogenously determined discount rate and

[1] Their utility is slightly more general than this, but this generalization doesn't change the result too much. See Kim (2005)

allowing the nontradable goods consumable. Under such a preference specification, we show that indeterminacy can occur for technologies with arbitrarily small externalities and the difficulty of deriving the indeterminacy condition under nonseparable utility function in Meng and Velasco (2003) is overcome.

2. THE TWO-SECTOR SMALL OPEN RBC ECONOMY

2.1. The model Case 1: KPR form $u^{KPR} = \frac{[C^{\theta}(1-l)^{1-\theta}]^{1-\sigma}-1}{1-\sigma}$

Consider a small open economy inhabited by an infinite-lived representative agent who maximizes the intertemporal utility function

$$U = \int_0^{\infty} \left[\frac{[C_t^{\theta}(1-l_t)^{1-\theta}]^{1-\sigma}-1}{1-\sigma}\right] e^{-\int_0^t \rho(\bar{C}_s)ds} dt, \sigma > 0, \theta \in (0,1) \quad (1)$$

where $C_t = [\omega(C_t^T)^{-\mu} + (1-\omega)(C_t^N)^{-\mu}]^{-\frac{1}{\mu}}$ represents the isolated aggregator of consumption of traded goods C_t^T and nontraded goods C_t^N.[2] We follow this specification as in Mendoza and Uribe (1999). $\frac{1}{1+\mu}$ denotes the substitution elasicity between traded and nontraded consumptions. $\omega \in (0,1)$ is the share of traded consumption in the bundle. We assume the discount rate is of modified Uzawa type as in Schmitt-Grohe and Uribe (2003) and Campell and Cochrane (1999).[3] In particular, it is strictly positive, and is an increasing function of the economy-wide average consumption, i.e.,

$$\rho'(\overline{C}_s) > 0 \quad (2)$$

$\overline{C}_t$ is the economy wide average consumption, at the equilibrium $\overline{C}_t = C_t$.

The economy is open to full international borrowing and lending, so that the agent has access to net foreign bonds d_t, denominated in units of consumption goods, that pay an exogenously given world interest rate r.

The traded good sector produces the traded consumption good y_{1t} as numeraire. The nontraded sector goods y_{2t} can be used either for con-

[2]Meng and Velasco (2003) don't assume the nontradable goods consumable and endogenous discount rate, they cannot derive the sufficient condition under nonseparable utility functions. Mendoza and Uribe (1999) and Sen and Turnovsky (1995) relax the assumption, allowing for nontradable goods consumable.

[3]The average consumption in the discount rate captures the "jealousy" (or "admiration") effect of consumption externalities, recently emphasized among other areas in the literature on asset pricing like Campell and Cochrane (1999).

sumption C_t^N or for investment (i_t), with relative price p_t.[4] Producers use two factors (nontraded capital k_t and labor) in two sector productions. The production functions are assumed to be the same as Benhabib and Nishimura (1998), Cobb-Douglas with factor input generating externalities,[5]

$$y_{1t} = l_{1t}^{\alpha_0} k_{1t}^{\alpha_1} \overline{l_{1t}^{a_0} k_{1t}^{a_1}}, y_{2t} = l_{2t}^{\beta_0} k_{2t}^{\beta_1} \overline{l_{2t}^{b_0} k_{2t}^{b_1}} \tag{3}$$

where

$$k_{1t} + k_{2t} = k_t, l_{1t} + l_{2t} = l_t \tag{4}$$

Here l_{1t} and k_{1t} denote the labor services and capital used by the individual firm in the traded good producing sector, and l_{2t} and k_{2t} for the nontraded good producing sector. k_t, l_t are the aggregate capital stock and labor supply. The production functions satisfy the following assumption.

Assumption 1. *The technologies in Eq.(3) exhibit social constant returns to scale, and private decreasing returns to scale, that is,*

$$\begin{aligned} a_0 + \alpha_0 + \alpha_1 + a_1 &= \beta_0 + \beta_1 + b_1 + b_0 = 1 \\ a_0, \alpha_0, \alpha_1, a_1 &\geq 0,\ \beta_0, \beta_1, b_1, b_0 \geq 0 \end{aligned}$$

In the case of private decreasing returns, since firms earn positive profits, a fixed entry cost is required to deter new entrants.[6]

The rate of accumulation of bonds ($\dot{d}_t$) is subject to

$$\dot{d}_t = rd_t + y_{1t} + p_t y_{2t} - C_t^T - p_t C_t^N - pi_t \tag{5}$$

and the law of motion for the capital is

$$\dot{k}_t = i_t \tag{6}$$

Eqs. (5) and (6) can be consolidated into

$$\dot{z}_t = rz_t + y_{1t} + p_t y_{2t} - C_t^T - p_t C_t^N + k_t(\dot{p}_t - rp_t) \tag{7}$$

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[4]Sen and Turnovsky (1995) and Mendoza and Uribe (1999) analyze the two sector small open economy with one traded pure consumption good and one nontraded goods which can be used as investment and consumption.

[5]$l_{1t}^{a_0} k_{1t}^{a_1}, l_{2t}^{b_0} k_{2t}^{b_1}$ are factor input generating externalities in the two sectiors,

[6]The explanation of dynamic increasing return induced by the fixed entry cost is shown in Meng and Velasco (2004).

where the total wealth $z_t = p_t k_t + d_t$.[7]The agent is to choose (C_t^T, C_t^N), labor supply (l_t) and its allocation (l_{1t}, l_{2t}),capital allocation decisions (k_{1t}, k_{2t}), rates of investment (i_t) and d_t, maximizing equation (1), subject to equations (3), (4) and (7), given k_0 and d_0.

The Hamiltonian is

$$\begin{aligned} H &= \left\{ \frac{[C_t^\theta (1-l_t)^{1-\theta}]^{1-\sigma} - 1}{1-\sigma} \right\} e^{-\int_0^t \rho(\bar{C_s})ds} \\ &\quad + \phi_t [r z_t + l_{1t}^{\alpha_0} k_{1t}^{\alpha_1} \overline{l_{1t}^{a_0} k_{1t}^{a_1}} + p_t l_{2t}^{\beta_0} k_{2t}^{\beta_1} \overline{l_{2t}^{b_0} k_{2t}^{b_1}} - C_t^T - p_t C_t^N + k_t(\dot{p_t} - r p_t)] \\ &\quad + u_t(k_t - k_{1t} - k_{2t}) + w_t(l_t - l_{1t} - l_{2t}) \end{aligned} \tag{8}$$

where ϕ_t is costate variable, u_t, w_t are the rental prices of capital and labor. In solving the problem, the agent takes the average consumption $\bar{C_t}$ as given, at the equilibrium $\bar{C_t} = C_t$. First-order conditions are (denoting $\alpha_0 + a_0 = \alpha, \beta_0 + b_0 = \beta$).

$$\begin{aligned} & [C_t^\theta (1-l_t)^{1-\theta}]^{-\sigma} (1-l_t)^{1-\theta} \theta C_t^{\theta-1} \frac{\partial C_t}{\partial C_t^T} e^{-\int_0^t \rho(\bar{C_s})ds} \\ = \; & \phi_t, \frac{\partial C_t}{\partial C_t^T} = C_t^{1+\mu} [\omega (C_t^T)^{-(1+\mu)}] \end{aligned} \tag{9}$$

$$\begin{aligned} & [C_t^\theta (1-l_t)^{1-\theta}]^{-\sigma} (1-l_t)^{1-\theta} \theta C_t^{\theta-1} \frac{\partial C_t}{\partial C_t^N} e^{-\int_0^t \rho(\bar{C_s})ds} \\ = \; & \phi_t p_t, \frac{\partial C_t}{\partial C_t^N} = C_t^{1+\mu} [(1-\omega)(C_t^N)^{-(1+\mu)}] \end{aligned} \tag{10}$$

$$[C_t^\theta (1-l_t)^{1-\theta}]^{-\sigma} (1-\theta)(1-l_t)^{-\theta} C_t^\theta e^{-\int_0^t \rho(\bar{C_s})ds} = w_t \tag{11}$$

$$r_t \phi_t = -\dot{\phi_t} \tag{12}$$

$$u_t = \phi_t \alpha_1 l_{1t}^\alpha k_{1t}^{-\alpha} = \phi_t p_t \beta_1 l_{2t}^\beta k_{2t}^{-\beta} \tag{13}$$

$$w_t = \phi_t \alpha_0 l_{1t}^{\alpha-1} k_{1t}^{1-\alpha} = \phi_t p_t \beta_0 l_{2t}^{\beta-1} k_{2t}^{1-\beta} \tag{14}$$

$$\dot{p_t} = p_t (r - \beta_1 l_{2t}^\beta k_{2t}^{-\beta}) \tag{15}$$

[7]We can show that with this transformation, we can derive same indeterminacy result as we use the equations 5 and 6.

The market clearing condition for nontraded capital and the current account,

$$\dot{k}_t = y_{2t} - C_t^N \tag{16.1}$$

$$\dot{d}_t = rd_t + y_{1t} - C_t^T \tag{16.2}$$

In the appendix, we derive the dynamic equations system,

$$\dot{C}_t = C_t[\rho(C_t) - \beta_1 g^\beta(p_t)](\frac{-1}{\theta\sigma}) \tag{17}$$

$$\dot{p}_t = p_t[r - \beta_1 g^\beta(p_t)] \tag{18}$$

$$\dot{k}_t = \frac{\beta_1\alpha_0 g^\beta(p_t)}{\beta_1\alpha_0 - \beta_0\alpha_1}k_t - \frac{\beta_0\alpha_1[1 - \frac{1-\theta}{\theta(1-\omega)}C_t\Delta^{\frac{(1+\mu)}{\mu}}]}{(\beta_1\alpha_0 - \beta_0\alpha_1)g^{1-\beta}(p_t)} - C_t\Delta^{\frac{1}{\mu}} \tag{19}$$

$$\dot{d}_t = rd_t + y_{1t}(C_t, p_t, k_t) - C_t\Delta^{\frac{1}{\mu}}(\frac{\omega}{1-\omega}p_t)^{\frac{1}{\mu+1}} \tag{20}$$

where $\Delta = [\omega(\frac{\omega}{1-\omega}p_t)^{\frac{-\mu}{\mu+1}} + (1-\omega)]$, $g(p) = \xi p^{\frac{1}{(\alpha_0+a_0)(\beta_1+b_1)-(\alpha_1+a_1)(\beta_0+b_0)}}$, ξ is a positive parameter.

Lemma 1. *There exists a unique steady state in the above ODE system.*

Proof. Noting the block recursive differential equation system, from the second one, p^* is unique since $r = \beta_1 g^\beta(p^*)$. Given p^*, from the first equation, we can derive $r = \rho(C^*) = \beta_1 g^\beta(p^*)$. Due to the fact that $\rho(C_t)$ is a monotone function, we know that C^* is unique. Given C^* and p^*, from the third eqation, we know k^* is unique. ▌

The dynamic system consists of four differential equations (Eqs. (17)–(20)) for (C_t, p_t, k_t, d_t). This is in contrast to closed-economy models in the literature that are generally associated with a system of two differential equations. Linearizing around the unique steady state, we obtain

$$\begin{pmatrix} \dot{C}_t \\ \dot{p}_t \\ \dot{k}_t \\ \dot{d}_t \end{pmatrix} = \begin{bmatrix} -\frac{C^*\rho'(C^*)}{\theta\sigma} & j_{12} & 0 & 0 \\ 0 & j_{22} & 0 & 0 \\ j_{31} & j_{32} & j_{33} & 0 \\ j_{41} & j_{42} & j_{43} & r \end{bmatrix} \begin{pmatrix} C_t - C^* \\ p_t - p^* \\ k_t - k^* \\ d_t - d^* \end{pmatrix}$$

The four eigenvalues of the Jacobian are $-\frac{c^*\rho'(\bar{c}^*)}{\theta\sigma} < 0, r > 0$

$$j_{22} = \frac{\beta r}{-(\alpha_0 + a_0)(\beta_1 + b_1) + (\alpha_1 + a_1)(\beta_0 + b_0)} \tag{21}$$

$$j_{33} = \frac{F_1}{\beta_1\alpha_0 - \beta_0\alpha_1}, \; F_1 = \beta_1\alpha_0 g^{\beta}(p^*) \tag{22}$$

Proposition 1. *If the nontraded good sector is labor intensive from private perspective ($j_{22} < 0$) but capital intensive from the social perspective ($j_{33} < 0$), then there exits a continuum of equilibria that converge to the unique steady state.*

The reason is that nontraded capital k_t is a predetermined variable and evolves continuously, while p_t and C_t are jump variables. Indeterminacy requires both j_{22} and j_{33} to be negative which makes the dimension of indeterminacy be one in this case. Then the indeterminacy conditions are quite similar with those in Meng and Velasco (2003) i,e to small externalities, indeterminacy can occur under the factor intensity conditions given in the proposition.

It is clear from the proposition that indeterminacy can arise under arbitrarily small externalities. Moreover, the indeterminacy condition is independent of the intertemporal elasticities in consumption and labor allocation between work and leisure. The intuition for this result is straightforward. In the open economy, the curvature of the utility function does not affect the investment decision, since unlike in the closed economy the agent can always borrow from the outside world to finance his consumption. The above indeterminacy result is in contrast to the two-sector closed-economy indeterminacy result in Benhabib and Nishimura (1998), which requires the extreme assumption of linear or close-to-linear utility.

2.2. Case 2: Bennett- Farmer form

$$u^{Bennett-Farmer} = \frac{[C\exp(-\frac{l^{1+\chi}}{1+\chi})]^{1-\sigma} - 1}{1-\sigma} \qquad \chi, \sigma > 0$$

We can easily derive the dynamic equations system,

$$\dot{C_t} = \frac{C_t[\rho(C_t) - r + \dot{p_t}\frac{m'(p_t)}{m(p_t)}] - n'(p_t)\dot{p_t}C_t^{-\frac{1}{\chi}}}{-\sigma - \frac{1+\chi}{\chi}n(p_t)C_t^{-\frac{1+\chi}{\chi}}}$$

where $n(p_t) = \frac{\sigma-1}{1+\chi}[\frac{(1-\omega)}{\Delta^{\frac{1}{\mu}+1}}\beta_0 g^{\beta-1}(p_t)]^{\frac{1+\chi}{\chi}}$, $m(p_t) = \frac{p_t}{1-\omega}\Delta^{\frac{(1+\mu)}{\mu}}$

$$\dot{p_t} = p_t[r - \beta_1 g^{\beta}(p_t)]$$

$$\dot{k}_t = \frac{\beta_1\alpha_0 g^{\beta}(p_t)}{\beta_1\alpha_0 - \beta_0\alpha_1}k_t - \frac{\beta_0\alpha_1[\frac{(1-\omega)}{C_t\Delta^{\frac{1}{\mu}+1}}\beta_0 g^{\beta-1}(p_t)]^{\frac{1}{\chi}}}{(\beta_1\alpha_0 - \beta_0\alpha_1)g^{1-\beta}(p_t)} - C_t\Delta^{\frac{1}{\mu}}$$

$$\dot{d}_t = rd_t + y_{1t}(C_t, p_t, k_t) - C_t\Delta^{\frac{1}{\mu}}(\frac{\omega}{1-\omega}p_t)^{\frac{1}{\mu+1}}$$

LEMMA 2. *There exists a unique steady state in the above ODE system.*

Proof. Noting the block recursive differential equation system, from the second one, p^* is unique since $r = \beta_1 g^{\beta}(p^*)$. Given p^*, from the first equation, we can derive $r = \rho(C^*) = \beta_1 g^{\beta}(p^*)$. Due to the fact that $\rho(C_t)$ is a monotone function, we know that $C^*$is unique. Given C^* and p^*, from the third eqation, we know k^* is unique. d^* is determined from the last equation given C^*, p^* and k^*. ▮

The linearization around the steady state becomes:

$$\begin{pmatrix} \dot{C}_t \\ \dot{p}_t \\ \dot{k}_t \\ \dot{d}_t \end{pmatrix} = \begin{bmatrix} -\frac{c^*\rho'(C^*)}{\sigma + \frac{1+\chi}{\chi}n(p^*)C^{*-\frac{1+\chi}{\chi}}} & j_{12} & 0 & 0 \\ 0 & j_{22} & 0 & 0 \\ j_{31} & j_{32} & j_{33} & 0 \\ j_{41} & j_{42} & j_{43} & r \end{bmatrix} \begin{pmatrix} C_t - C^* \\ p_t - p^* \\ k_t - k^* \\ d_t - d^* \end{pmatrix}$$

The four eigenvalues of the Jacobian are $-\frac{c^*\rho'(C^*)}{\sigma + \frac{1+\chi}{\chi}n(p^*)C^{*-\frac{1+\chi}{\chi}}} < 0$ (as $\sigma \geq \frac{\eta(r)}{1+\eta(r)}$) , $r > 0$

$$j_{22} = \frac{\beta r}{-(\alpha_0 + a_0)(\beta_1 + b_1) + (\alpha_1 + a_1)(\beta_0 + b_0)}$$

$$j_{33} = \frac{F_2}{\beta_1\alpha_0 - \beta_0\alpha_1}, F_2 = \beta_1\alpha_0 g^{\beta}(p^*)$$

PROPOSITION 2.
$\eta(r) = \frac{C^{*-\frac{1+\chi}{\chi}}}{\chi}\frac{1}{1+\chi}[\frac{(1-\omega)}{\Delta(p^*)^{\frac{1}{\mu}+1}}\beta_0 g^{\beta-1}(p^*)]^{\frac{1+\chi}{\chi}} > 0$ *As* $\sigma > \frac{\eta(r)}{1+\eta(r)}$, *if the nontraded good sector is labor intensive from private perspective* ($j_{22} < 0$) *but capital intensive from the social perspective* ($j_{33} < 0$)*, then there exits a continuum of equilibria that converge to the unique steady state. As* $\sigma \in$

$[0, \frac{\eta(r)}{1+\eta(r)})$, *there is no indeterminacy even if the factor intensity reversal condition is satisfied*[8].

The results that Kim (2005) has regarding utility function change dramatically when we move to a small open economy. In this paper, I check two classes of nonseparable utility functions often used in the indeterminacy literature. Coupled with Meng and Velasco and Meng and Bian's finding, the independence between curvature and indeterminacy in open economy is robust to three kinds commonly used utility functions. The Bennett and Farmer form is exceptional since the conclusion also depends on the form of endogenous time preference[9].

Compared with the results of Weder (2001) and Meng and Velasco (2003, 2004), we can derive a closed form condition for indeterminacy under nonseparable utility function with leisure. Under Bennett and Farmer utility form, our indeterminacy still depends on the constant intertemporal elasticity of substitution σ. The surprising result that $\sigma \in [0, \frac{\eta(r)}{1+\eta(r)})$ implies determinacy may be due to the nonconcavity of the Bennett Farmer form[10]. If the time preference is constant and equal to the given world interest rate r, Jacobian has zero root and it is hard for us to derive the sufficient condition of indeterminacy even if it exits.

APPENDIX A

Under the case 1:

$$\frac{l_{1t}}{k_{1t}} = \frac{l_{2t}}{k_{2t}} \frac{\alpha_0\beta_1}{\alpha_1\beta_0} \tag{A.1}$$

$$\frac{l_{2t}}{k_{2t}} = g(p_t) = (\xi p_t)^{\frac{1}{(\alpha_0+a_0)(\beta_1+b_1)-(\alpha_1+a_1)(\beta_0+b_0)}}, \; \xi = \frac{\beta_1}{\alpha_1}(\frac{\alpha_0\beta_1}{\alpha_1\beta_0})^{-\alpha} \tag{A.2}$$

$$k_{2t} = \frac{\beta_1\alpha_0}{\beta_1\alpha_0 - \beta_0\alpha_1} k_t - \frac{\beta_0\alpha_1[1 - \frac{1-\theta}{\theta(1-\omega)} C_t \Delta^{\frac{(1+\mu)}{\mu}}]}{(\beta_1\alpha_0 - \beta_0\alpha_1) g(p_t)} \tag{A.3}$$

[8] I am thankful to Jess Benhabib to point out a mistake in the old version of this paper related to this proposition. Note that p^*, C^* are functions of r at the steady state.

[9] The lower bound of the indeterminacy region depends on the form of the endogenous time preference.

[10] Note that under the GHH form, the indeterminacy exists in open economy model as $\sigma = 0$. For the nonconcavity analysis of Bennett–Farmer utility form, see Hintermaier (2003).

$$l_t = [1 - \frac{1-\theta}{\theta(1-\omega)} C_t \Delta^{\frac{(1+\mu)}{\mu}}], \Delta = [\omega(\frac{\omega}{1-\omega} p_t)^{\frac{-\mu}{\mu+1}} + (1-\omega)] \tag{A.4}$$

$$y_{2t} = \frac{\beta_1 \alpha_0 g^{\beta}(p_t)}{\beta_1 \alpha_0 - \beta_0 \alpha_1} k_t - \frac{\beta_0 \alpha_1 [1 - \frac{1-\theta}{\theta(1-\omega)} C_t \Delta^{\frac{(1+\mu)}{\mu}}]}{(\beta_1 \alpha_0 - \beta_0 \alpha_1) g^{1-\beta}(p_t)} \tag{A.5}$$

$$C_t^N = C_t \Delta^{\frac{1}{\mu}}, C_t^T = (\frac{\omega}{1-\omega} p_t)^{\frac{1}{\mu+1}} C_t \Delta^{\frac{1}{\mu}} \tag{A.6}$$

APPENDIX B

Under case 2:

$$l_t = [\frac{(1-\omega)}{C_t \Delta^{\frac{1}{\mu}+1}} \beta_0 g^{\beta-1}(p_t)]^{\frac{1}{\chi}} \tag{B.1}$$

the equation 9 becomes:

$$\exp\{\frac{\sigma-1}{1+\chi}[\frac{(1-\omega)}{\Delta^{\frac{1}{\mu}+1}} \beta_0 g^{\beta-1}(p_t)]^{\frac{1+\chi}{\chi}} C_t^{-\frac{1+\chi}{\chi}}\} C_t^{-\sigma} = \frac{p_t}{1-\omega} \Delta^{\frac{(1+\mu)}{\mu}} e^{\int_0^t \rho(\bar{c_s}) ds} \phi_t \tag{B.2}$$

the dynamics of C_t,

$$\dot{C_t} = \frac{C_t[\rho(C_t) - r + \dot{p_t} \frac{m'(p_t)}{m(p_t)}] - n'(p) \dot{p_t} C_t^{-\frac{1}{\chi}}}{-\sigma - \frac{1+\chi}{\chi} n(p_t) C_t^{-\frac{1+\chi}{\chi}}} \tag{B.3}$$

where $n(p_t) = \frac{\sigma-1}{1+\chi}[\frac{(1-\omega)}{\Delta^{\frac{1}{\mu}+1}} \beta_0 g^{\beta-1}(p_t)]^{\frac{1+\chi}{\chi}}, m(p_t) = \frac{p_t}{1-\omega} \Delta^{\frac{(1+\mu)}{\mu}}$

$$\dot{p_t} = p_t[r - \beta_1 g^{\beta}(p_t)] \tag{B.4}$$

$$\dot{k_t} = \frac{\beta_1 \alpha_0 g^{\beta}(p_t)}{\beta_1 \alpha_0 - \beta_0 \alpha_1} k_t - \frac{\beta_0 \alpha_1 [\frac{(1-\omega)}{C_t \Delta^{\frac{1}{\mu}+1}} \beta_0 g^{\beta-1}(p_t)]^{\frac{1}{\chi}}}{(\beta_1 \alpha_0 - \beta_0 \alpha_1) g^{1-\beta}(p_t)} - C_t \Delta^{\frac{1}{\mu}} \tag{B.5}$$

$$\dot{d_t} = r d_t + y_{1t}(C_t, p_t, k_t) - C_t \Delta^{\frac{1}{\mu}} (\frac{\omega}{1-\omega} p_t)^{\frac{1}{\mu+1}} \tag{B.6}$$

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