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Aggregate Price Stickiness*

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We accomplish two tasks to characterize aggregate price stickiness in this paper. First, we endogenize β, the fraction of the firms keeping their price unchanged following a money supply shock in the near-rationality model (Akerlof and Yellen, 1985) by introducing a distribution of price-adjustment barriers among the firms into the near-rationality model. Second, as β can be considered an indicator of aggregate price stickiness by its definition, the endogenized β enables us to characterize aggregate price stickiness by studying its properties. We show that: (1) $\lim_{\varepsilon\to 0}\beta(\varepsilon)=1$; (2) $\left.\frac{d\beta}{d\varepsilon}\right|_{\varepsilon=0}=0$; and (3) the possibility of multiple equilibrium values of β, where ε is the fraction change in money supply.

Key Words: Small menu costs; Near-rationality; Aggregate price stickiness; Strategic complementarity; Coordination failure.
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1. INTRODUCTION

Although price stickiness is central to Keynesian models, in most such models it has no solid microeconomic foundation. Thus construction of microeconomic foundations for price stickiness is a priority for new Keynesian economists.

To meet this challenge, new Keynesian economists have put forward two parallel ideas: small menu costs (Mankiw, 1985) and near-rationality (Akerlof and Yellen, 1985). By menu costs, they mean the costs for changing prices. This is called menu costs because it could be viewed as the price of printing a new menu. These costs might include such items as "printing

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new catalogs, informing salesmen of the new price and any other costs associated with price adjustment" (Mankiw, 1985). In the small menu costs model, a firm will not adjust its price following a money supply change if the individual profit increment from adjusting its price is less than its menu costs. By near-rationality, they mean "nonmaximizing behavior in which the gains from maximizing rather than nonmaximizing are small in a well-defined sense" (Akerlof and Yellen, 1985). In the near-rationality model, the monopolistically competitive economy implies that an individual firm's profit loss resulting from keeping its price unchanged following a money supply change is only in second order of ε, where ε is the fraction change in the money supply. Thus, if a firm does not change its price following the money supply shock, its behavior is suboptimal, but still near-rational because the firm's profit loss is only in second order of ε. Blanchard and Kiyotaki (1987) and Ball and Romer (1991) expand on the paper by Akerlof and Yellen. The major difference is that they derive their results from basic optimization assumptions so that explicit welfare calculations are allowed. For a survey of this literature, see Rotemberg (1987) and Blanchard (1987).

The papers in this literature have two common features. First, they show that a second-order "small" price-adjustment barrier[1] (either menu costs or near-rationality) for an individual firm to adjust its price can cause changes in money supply to have first-order "large" effect on real economic variables, either on social welfare (Mankiw, 1985) or on employment (Akerlof and Yellen, 1985). Second, the parameter β, which is the fraction of the firms that keep their prices unchanged following a money supply shock, is exogenous. In the initial equilibrium of their models, each firm sets its price to maximize profit. Then, they introduce a money supply shock into their models. Following the money supply shock, they assume that β fraction of the firms keep their price unchanged while the remaining $(1 - \beta)$ fraction of the firms change their price to maximize profit. They either assume a general β between zero and one (Akerlof and Yellen, 1985) or assume β is equal to one (Mankiw, 1985, Blanchard and Kiyotaki, 1987, Ball and Romer, 1991) in their models. In a word, β is exogenous in their models[2].

[1]By the definition of small menu costs and near-rationality, we can see that they are equivalent routes to the same place. Therefore, for convenience of exposition, we give them a unified terminology in this paper: price-adjustment barrier.

[2]Ball and Romer (1991) endogenized the parameter β in the same static partial equilibrium setting as ours, but in a limited way. They did not study the properties of β as we have done. The three properties regarding the parameter β are the key contribution of this paper. The recent literature of state-dependent pricing (Dotsey et al., 1999; Bakhshi et al., 2004) endogenized a similar parameter in a dynamic general equilibrium setting. They did not focus on characterizing the behavior of aggregate price stickiness, either.

Although an exogenous β is fine for their purposes, β should be an endogenous variable. In addition, β can be considered an indicator of aggregate price stickiness by its definition. Therefore, provided that we could endogenize β, we can go one step further to explicitly characterize the behavior of aggregate price stickiness by studying the properties of β, which is the focus and the key contribution of this paper.

We are interested in characterizing aggregate price stickiness in this paper. In order to do so, we accomplish two tasks. First, we endogenize β, the fraction of the firms that keep their original optimal price unchanged following a money supply shock in the near-rationality model (Akerlof and Yellen, 1985). We approach this task by introducing a distribution of price-adjustment barriers among the firms into the near-rationality model. Second, on the basis of an endogenized β, we characterize the behavior of aggregate price stickiness by studying the properties of β.

We get three results regarding the properties of β. First, we show that $\lim_{\varepsilon \to 0} \beta(\varepsilon) = 1$. It says that when there is a money supply shock but turns out to be very small, β approaches one. Second, we show that $\left.\frac{d\beta}{d\varepsilon}\right|_{\varepsilon=0} = 0$. As $\left.\frac{d\beta}{d\varepsilon}\right|_{\varepsilon=0} = 0$, then by Taylor's expansion, when ε is very small (close to zero), $\beta(\varepsilon) - \beta(0) = \beta(\varepsilon) - 1 \propto \varepsilon^2$. $\beta(\varepsilon) - \beta(0) = \beta(\varepsilon) - 1 \propto \varepsilon^2$ says that when money supply shock is small, almost all of the firms will keep their original price unchanged while only a fraction that is only in second order of the money supply shock will change their price. In other words, prices are not only sticky, but price stickiness is very significant for small money supply shocks. Intuitively, only a small fraction of firms will have price-adjustment barrier so small that it pays them to change their price in response to small shocks. Third, we cannot exclude the possibility of multiple equilibrium values of β because the profit loss for a firm resulting from keeping its price unchanged rather than adjusting its price decreases as β increases. In other words, the higher the fraction of the firms that do not change their prices following a money supply shock, the less incentive for an individual firm to change its price. This is exactly the concept of strategic complementarity (Cooper and John, 1988). By strategic complementarity, they mean that the optimal strategy of a decision-maker depends positively on the strategies of the other decision-makers. In a word, due to strategic complementarity, we cannot exclude the possibility of multiple equilibria. The third result is consistent with the analysis by Blanchard and Kiyotaki (1987) and Ball and Romer (1991). This result is important because multiple equilibria further implies the possibility of coordination failure among firms. Thus, models with price stickiness and models with coordination failure are not completely competing paradigms to explain economic fluctuations, but can be compatible with each other.

The remainder of the paper consists of three sections. The first section presents the model in detail. The second section illustrates our work using an example. The third section concludes.

2. MODEL

We are interested in studying aggregate price stickiness in a monopolistically competitive economy. To do so, we first endogenize β, the fraction of the firms that keep their original optimal price unchanged following a money supply shock in the near-rationality model, by introducing a distribution of price-adjustment barrier among the firms into the near-rationality model. Second, as β can be considered an indicator of aggregate price stickiness, the endogenized β paves the way for us to characterize aggregate price stickiness by studying the properties of β.

This section is organized as follows. First, we will review the near-rationality paper by Akerlof and Yellen (1985). All the assumptions, except an exogenous β, made by Akerlof and Yellen will be kept intact in our model. Then, we introduce a new assumption into the near-rationality model: a distribution of price-adjustment barriers among the firms, which is also common knowledge among all the firms. The purpose of introducing this new assumption is to endogenize β. The first two subsections present the environment of our model economy. The third subsection analyzes the individual pricing decision following a money supply shock in our model economy. Based on the analysis of the individual pricing decision, the fourth subsection presents the equilibrium equations of the model. We will derive the equilibrium value of β in this subsection. Finally, on the basis of an endogenized β, we characterize aggregate price stickiness by studying the properties of β.

2.1. A Review of the Near-Rationality Paper by Akerlof and Yellen (1985)

In response to Lucas's challenge that it is necessary to produce models of monetary nonneutrality that meets the criterion of "no $500 bills lying on the sidewalk", Akerlof and Yellen are led to explore near-rationality as a microeconomic foundation for price stickiness. In their model, if an individual firm does not change its price following a money supply shock, the firm will incur a profit loss. However, the loss is small in a well-defined sense: the loss is only in second order of the money supply shock. In this sense, the firm's behavior is suboptimal, but still near-rational from its individual standpoint. But, the near-rational behavior viewed from individual standpoint can cause the money supply shock to have first-order effect on real economic variables such as employment and output.

Akerlof and Yellen assume a monopolistically competitive economy with a fixed number of identical firms in their model. Each firm's sales depend on the level of real aggregate demand and the firm's own price relative to the aggregate price level. They assume that the productivity of workers depends on the real wage they receive (efficiency wage hypothesis). The efficiency wage hypothesis implies that firms will be induced to set wages above the market-clearing level. In the initial equilibrium, each firm sets its price and wage to maximize profit, under the assumption that a change in its own price has no effect on the prices charged by rivals or on the aggregate price level. That is, each firm is assumed to be a Bertrand maximizer. Then, Akerlof and Yellen introduce a money supply shock into their model and assume that the money supply changes by a fraction ε. Following the money supply shock, they assume a fraction β of the firms do not change their prices, while the remaining $(1-\beta)$ of the firms change their prices to maximize profit. If a firm does not change its price following the money supply shock, it will incur a profit loss that is a function of both and . We denote this loss function as $L(\varepsilon,\beta)$. $L(\varepsilon,\beta)$ depends on both ε and β because $L(\varepsilon,\beta)$ depends the firm's sales and the firm's sales depend on both ε and β. However, the above assumptions, especially the monopolistic competition and efficiency wage hypothesis, imply that $L(\varepsilon,\beta)$ is only second-order with respect to ε, i.e., $\left.\frac{\partial L(\varepsilon,\beta)}{\partial\varepsilon}\right|_{\varepsilon=0}=0$. In this sense, if an individual firm does not change its price following the money supply shock, its behavior is still near-rational. However, Akerlof and Yellen further show that the elasticity of total employment with respect to ε is positive, i.e., $\left.\frac{d(N/N_0)}{d\varepsilon}\right|_{\varepsilon=0}>0$, where N and N_0 are the total employment and the initial employment respectively. In other words, the money supply shock has first-order effect on employment that is a real economic variable. In summary, they show that a second-order "small" price-adjustment barrier for an individual firm to change its price can cause money supply shocks to have first-order "large" effect on real economic variables such as employment and output.

2.2. A New Assumption and the Setup of Our Model

All the assumptions in the near-rationality model by Akerlof and Yellen will be kept intact in our model except one: the exogenous β. Akerlof and Yellen take β as exogenously given in their model. This is fine for their purpose. However, β should be an endogenous variable. Our tasks are first to endogenize β and then to study its properties.

In order to endogenize β, we introduce a new assumption into the near-rationality model: we assume that there is a distribution of price-adjustment barriers among the firms, which is also common knowledge among all the firms. In more detail, we assume that each firm has a price-adjustment barrier c_i which is greater than zero, where i is the firm index. The price-adjustment barriers for all the firms ($\{c_i\}$) follow a certain distribution. We assume F, the cumulative distribution function (CDF) of the price-adjustment barriers, is first-order differentiable and strictly increasing. We also assume that this distribution is common knowledge among all the firms. Because each firm's price-adjustment barrier is greater than zero, we have $F(0) = 0$. Because F is first order differentiable and strictly increasing, we have $F' > 0$, $F'(0_+) > 0$, and ${F^{-1}}'(0_+) > 0$, where F' is the first order derivative of F and ${F^{-1}}'$ is the first order derivative of F^{-1}, the inverse function of F.

2.3. Individual Pricing Decision Following a Money Supply Shock

Subsections 2.1 and 2.2 have essentially set up the environment of our model economy. In this subsection, we shall present the individual pricing decision process of each firm following a money supply shock in our model economy and its aggregate outcome.

Let's consider one specific firm, firm i. When the manager of this firm sets his price following a money supply shock, he will have rational expectations of the distribution of other firms' price-setting behavior: a fraction β of the firms will keep their original optimal price unchanged while $(1-\beta)$ of the firms will change their price and charge the new optimal price. Why does he form such a rational expectation of the distribution of other firms' price-setting behavior? A reasonable explanation is: If $L > c_i$, then firm i will charge the new optimal price; otherwise, firm i will keep its price unchanged, i.e., charge the original optimal price. The key point here is that $\{c_i\}$ follows a certain distribution, which is common knowledge among all the firms. When all the firms follow the above behavior, the consistent outcome (equilibrium) with the expectation is that a fraction β of the firms will keep their original optimal price unchanged while $(1-\beta)$ of the firms will change their price and charge the new optimal price.

2.4. Equilibrium Equations of the Model

Based on the above analysis of pricing decision process following a money supply shock, we can write down the equilibrium equations and derive the equilibrium value of β.

First, given loss L, the fraction of the firms whose price-adjustment barrier is less than L is $F(L)$. If a firm's price-adjustment barrier is less than L, the firm will change its price and charge the new optimal price. Therefore, $F(L)$ is the fraction of the firms that change their price and charge the new optimal price. $(1-\beta)$ is also the fraction of the firms that change their price and charge the new optimal price by the definition of β. Thus we have equation (1):

$$1-\beta = F(L). \tag{1}$$

Second, L is a function of ε and β. Intuitively, L depends on ε because an individual firm's sales depend on aggregate demand and aggregate demand depends on ε. Similarly, L depends on β because an individual firm's sales depend on aggregate price level and aggregate price level depends on β. Therefore, we have equation (2):

$$L = L(\varepsilon, \beta). \tag{2}$$

Finally, regarding the properties of the loss function, we have equation (3), (4), (5), and (6), which result from the nature of the economy, i.e., the monopolistically competitive economy. Equation (3) and (4) have been shown by Akerlof and Yellen. Especially, equation (4) is one of the key findings by Akerlof and Yellen, which says that the loss to an individual firm resulting from keeping its price unchanged following a money supply shock is only in second order of ε, the parameter describing the money supply shock. Equation (5) and (6) have been derived by Ball and Romer (1991). Especially, equation (6) says that the loss for an individual firm resulting from keeping its price unchanged following the money supply shock is decreasing in the fraction of the firms that keep their price unchanged. In other words, the higher the fraction of the firms that keep their price unchanged following the money supply shock, the less incentive for an individual firm to change its price. Thus, this is exactly the concept of strategic complementarity (Cooper and John, 1988) because by strategic complementarity, they mean that the optimal strategy of a decision-maker depends positively on the strategies of the other decision-makers.

$$\lim_{\varepsilon\to 0} L(\varepsilon,\beta) = 0, \tag{3}$$

$$\left.\frac{\partial L(\varepsilon,\beta)}{\partial \varepsilon}\right|_{\varepsilon=0} = 0, \tag{4}$$

$$\left.\frac{\partial L(\varepsilon,\beta)}{\partial \beta}\right|_{\varepsilon=0} = 0, \tag{5}$$

$$\left.\frac{\partial L(\varepsilon,\beta)}{\partial \beta}\right|_{\varepsilon>0} < 0. \tag{6}$$

Thus, we have completed the model. In summary, equation (1) and (2) together describe the interaction between L and β. Equation (3), (4), (5) and (6) prescribe the properties of the loss function L that are implied by the nature of the monopolistically competitive economy.

Simply by plugging equation (2) into equation (1), we obtain equation (7).

$$1-\beta = F(L(\varepsilon,\beta)) \text{ or } F^{-1}(1-\beta) = L(\varepsilon,\beta). \tag{7}$$

Equation (7) gives us the equilibrium value of β which has been endogenized by introducing a distribution of price-adjustment barrier among the firms into the near-rationality model.

Ball and Romer (1991) also used the same equilibrium concept in which an endogenous β was derived given the distribution of menu costs. This paper resurrects that notion of equilibrium. However, it uses that equilibrium notion in a different way, by characterizing the behavior of aggregate price stickiness via studying the properties of β. In the next subsection, we will accomplish our second task: characterize aggregate price stickiness by studying the properties of β, which is the focus and key contribution of this paper.

2.5. Properties of β

So far, we have obtained the equilibrium value of β. As β can be considered an indicator of aggregate price stickiness, the endogenized β paves the way for us to characterize aggregate price stickiness by studying the properties of β. Our analysis reaches three results.

PROPOSITION 1. $\lim_{\varepsilon\to 0}\beta(\varepsilon) = 1$.

Proof. By equation (3), we have $\lim_{\varepsilon\to 0} L(\varepsilon,\beta) = 0$. Because $F(0) = 0$ and F is continuous implied by the first order differentiable assumption, we have $\lim_{x\to 0} F(x) = 0$. Taken together, we have $\lim_{\varepsilon\to 0} F(L(\varepsilon,\beta)) = 0$. By equation (1), we have $\lim_{\varepsilon\to 0}(1-\beta(\varepsilon)) = \lim_{\varepsilon\to 0} F(L(\varepsilon,\beta))$. Thus, we have $\lim_{\varepsilon\to 0}(1-\beta(\varepsilon)) = 0$ which implies that $\lim_{\varepsilon\to 0}\beta(\varepsilon) = 1$. ▮

Interpretation: $\lim_{\varepsilon\to 0}\beta(\varepsilon) = 1$ says that when there is a money supply shock but it turns out to be very small, β approaches one.

PROPOSITION 2. $\left.\dfrac{d\beta}{d\varepsilon}\right|_{\varepsilon=0} = 0$.

Proof. Take derivative with respect to ε on both sides of equation (7) and by simple algebra manipulation, we get equation (8):

$$\frac{d\beta}{d\varepsilon} = -L_1/(L_2 + F^{-1'}(1-\beta)), \tag{8}$$

where L_1 is the partial derivative of the loss function L with respect to ε, L_2 is the partial derivative of the loss function L with respect to β, and $F^{-1'}(1-\beta)$ is the derivative of function F^{-1} with respect to $(1-\beta)$.

Take limits on both sides of equation (8), we have equation (9):

$$\lim_{\varepsilon \to 0} \frac{d\beta}{d\varepsilon} = \lim_{\varepsilon \to 0} (-L_1/(L_2 + F^{-1'}(1-\beta(\varepsilon)))). \tag{9}$$

We have $\left.\frac{\partial L(\varepsilon, \beta)}{\partial \varepsilon}\right|_{\varepsilon=0} = 0$ by equation (4) and have $\lim_{\varepsilon \to 0} \beta(\varepsilon) = 1$ by proposition 1, therefore we have

$$\lim_{\varepsilon \to 0} L_1(\varepsilon, \beta) = \lim_{\varepsilon \to 0} L_1(\varepsilon, \beta(\varepsilon)) = \left.\frac{\partial L(\varepsilon, \beta)}{\partial \varepsilon}\right|_{\varepsilon=0, \beta=1} = 0$$

by the continuity of L_1. By equation (5), we have $\left.\frac{\partial L(\varepsilon, \beta)}{\partial \beta}\right|_{\varepsilon=0} = 0$, therefore we have $\lim_{\varepsilon \to 0} L_2(\varepsilon, \beta) = 0$ by the continuity of L_2. Because $F^{-1'}(0_+) > 0$ by assumption and $\lim_{\varepsilon \to 0} \beta(\varepsilon) = 1$ by proposition 1, we have $\lim_{\varepsilon \to 0} F^{-1'}(1 - \beta(\varepsilon)) > 0$. Hence, the left hand side of equation (9) is equal to zero. Thus, the right hand side of equation (9) is equal to zero, i.e., $\left.\frac{d\beta}{d\varepsilon}\right|_{\varepsilon=0} = 0$. ▮

Interpretation: When ε is close to zero, because $\left.\frac{d\beta}{d\varepsilon}\right|_{\varepsilon=0} = 0$, then by Taylor's expansion, we have $\beta(\varepsilon) - \beta(0) = \beta(\varepsilon) - 1 \propto \varepsilon^2$. Thus, the change of β is only in second order of ε. It says that when the money supply shock is small (close to zero), almost all of the firms will keep their original optimal price unchanged while only a fraction that is in second order of the money supply shock will change their price and charge the new optimal price. In other words, the fraction declines very slowly for small positive shocks. In this sense, prices are not only sticky, but seem to be very sticky for small positive money supply shocks. Intuitively, only a small fraction of firms will have price-adjustment barrier so small that it pays them to change their price in response to small shocks.

Proposition 3. *The possibility of multiple equilibrium values of β.*

Proof. Because $\left.\frac{\partial L(\varepsilon, \beta)}{\partial \beta}\right|_{\varepsilon>0} < 0$ by equation (6) and $F' > 0$ by assumption, we have $F(L(\varepsilon, \beta))$ is decreasing in β. It is clear that $(1-\beta)$ is also decreasing in β. Therefore, we cannot exclude the possibility of multiple equilibrium values of β because $1-\beta = F(L(\varepsilon,\beta))$ by equation (7). By equation (7), it can also be seen that whether we have unique equilibrium or multiple equilibria depends on the size of ε and the shape of F. ▌

Interpretation: Intuitively, due to strategic complementarity implied by equation (6), we cannot exclude the possibility of multiple equilibria. This result is consistent with the analysis by Blanchard and Kiyotaki (1987) and Ball and Romer (1991). This result is important because the possibility of multiple equilibria further implies the possibility of coordination failure among the firms. Thus, this result implies that we cannot simply say that models with price stickiness and models with coordination failure are completely competing paradigms to explain economic fluctuations. Instead, property 3 shows that these two types of models can be compatible with each other.

3. EXAMPLE

This section will illustrate the use of the previous properties in an example adapted from Mankiw's model of monopolistic competition with barrier for price adjustment (Mankiw, 1985).

Let's consider a monopolist with a constant cost curve and a linear demand curve. Suppose the constant cost is 0 and demand is $q = m - p + \overline{p}$ where m is the money supply, p is the price of the product of the individual firm and $\overline{p}$ is the aggregate price level. In the initial equilibrium, each firm is setting its price to maximize its own profit, taking the aggregate price level as given. Each individual firm's own price has negligible effect on the aggregate price level. Now, we introduce a money supply shock. Following the money supply shock, the new demand curve is $q = m(1+\varepsilon) - p + \overline{p}$. We assume the price-adjustment barriers follow a uniform distribution $u[0, A]$, which is a common knowledge among all the firms.

Following the money supply shock, if a fraction β of the firms keep their price unchanged and if the monopolist decides to keep his original optimal price unchanged as well rather than charge the new optimal price, he will lose:

$$L = m^2(1-x)^2,$$

where $x = p_m/m$ and p_m is the new optimal price. L is the loss function and x satisfies equation (10):

$$(1+\varepsilon) - 2x + x^{(1-\beta)} = 0. \tag{10}$$

The derivation of the loss function follows the same procedures as those in Akerlof and Yellen (1985). The details of how to derive the loss function are available in Appendix 1.

Therefore, if $\beta = 1$, then $x = 1 + \varepsilon/2$ by equation (10) and we get the minimum loss $L_{\min} = 0.25m^2\varepsilon^2$. If $\beta = 0$, then $x = 1 + \varepsilon$ by equation (10) and we get the maximum loss $L_{\max} = m^2\varepsilon^2$. Thus, if $\beta \in [0, 1]$, then $x = 1 + k(\beta)\varepsilon$ and we get the general loss function

$$L(\varepsilon, \beta) = m^2(1 - x)^2 = m^2k(\beta)^2\varepsilon^2, \tag{11}$$

where $k(beta) \in [0.5, 1]$ and $dk/d\beta < 0$, i.e., k is strictly decreasing in β and for each $\beta \in [0, 1]$, there is a unique x that satisfies equation (10). The uniqueness of x for each β can also be shown graphically by drawing the intersection of function $f(x) = 2x - (1 + \varepsilon)$ and function $g(x) = x^{(1-\beta)}$. $dk/d\beta < 0$ implies that $L(\varepsilon, \beta) = m^2(1 - x)^2 = m^2k(\beta)^2\varepsilon^2$ is decreasing in β, which is "strategic complementarity".

Because $1 - \beta = F(L)$ and the price-adjustment barriers follow the uniform distribution $u[0, A]$, i.e., $F(y) = y/A$ for $\forall y \in [0, A]$, β must satisfy:

$$\beta = 1 - m^2k(\beta)^2\varepsilon^2/A, \tag{12}$$

when ε approaches 0, then by equation (11) and (12), we can see that L approaches 0 and β approaches 1 respectively. This illustrates property 1 in the context of this example.

By equation (11), we get $\partial L/\partial\varepsilon = 2m^2k(\beta)^2\varepsilon$. So, if $\varepsilon = 0$, then $\partial L/\partial\varepsilon = 0$. Then follow the same proof as that for property 2, we get $\left.\frac{d\beta}{d\varepsilon}\right|_{\varepsilon=0} = 0$, thus also illustrating property 2 in the context of this example.

Because $dk/d\beta < 0$, we have $F(L(\varepsilon, \beta)) = m^2k(\beta)^2\varepsilon^2/A$ is decreasing in β. $(1 - \beta)$ is also decreasing in β. Therefore, we cannot exclude the possibility of multiple equilibrium values of β by equation (12). This illustrates proposition 3 in the context of this example.

4. CONCLUSION

To build a microeconomic foundation for price stickiness, new Keynesian economists have proposed two parallel ideas: small menu costs and near-rationality. It has been well understood that these ideas were very similar. For convenience of exposition, we give these two concepts a unified terminology: price-adjustment barrier. The striking result of this literature is that they show that second-order "small" price-adjustment barrier for an individual firm to adjust its price can cause money supply shocks to have first-order "large" effect on real economic variables such as social welfare and employment and output.

The papers in this literature assume that β, the fraction of the firms that keep their price unchanged following a money supply shock is exogenous. This assumption does not hurt their research purposes at all. However, the fraction should be an endogenous variable. In addition, the fraction can be considered an indicator of aggregate price stickiness. Therefore, an endogenized β makes it possible for us to characterize aggregate price stickiness by studying the properties of β, which is the focus and key contribution of this paper.

In this paper, we accomplish two tasks. First, we endogenize β in the near-rationality model by Akerlof and Yellen (1985). We accomplish this task by introducing a distribution of price-adjustment barriers among the firms into the near-rationality model. Second, we characterize aggregate price stickiness by studying the properties of β. We show that: (1) $\lim_{\varepsilon \to 0} \beta(\varepsilon) = 1$; (2) $\left.\frac{d\beta}{d\varepsilon}\right|_{\varepsilon=0} = 0$; and (3) we cannot exclude the possibility of multiple equilibrium values of β.

The example adapted Mankiw's model of monopolistic competition with price-adjustment barriers for changing prices illustrates these results.

APPENDIX A

In the initial equilibrium, each firm is taking the aggregate price as given and setting its price to maximize its own profit. Essentially, they are solving the following maximization problem:. $\max_{\{p\}} p(m - p + \overline{p})$. The first order condition for this optimization problem is:

$$m - 2p + \overline{p} = 0. \tag{A.1}$$

As each firm is charging the same price, we have

$$\overline{p} = p. \tag{A.2}$$

By equation (A.1) and (A.2), we have $\overline{p} = p = m$

Now, we introduce a money supply shock and money supply increases from m to $m(1 + \varepsilon)$. Following this change in money supply, we assume β fraction of the firms keep their original optimal price unchanged, i.e., their price is still m. However, the remaining $(1 - \beta)$ fraction of the firms change their price and charge the new optimal price. Essentially, they take the new aggregate price as given and solve the following maximization problem: $\max_{\{p_m\}} p_m(m(1+\varepsilon) - p_m + \overline{p}_{new})$. The first order condition for this optimization problem is:

$$m(1+\varepsilon) - 2p_m + \overline{p}_{new} = 0. \tag{A.3}$$

As β fraction of the firms charge m and the remaining $(1-\beta)$ fraction of the firms charge p_m, by the definition of aggregate price we have:

$$\overline{p}_{new} = m^{\beta} p_m^{1-\beta}. \tag{A.4}$$

Plug equation (A.4) into equation (A.3), we have:

$$m(1+\varepsilon) - 2p_m + m^{\beta} p_m^{1-\beta} = 0. \tag{A.5}$$

If we define $x = p_m/m$, we can rewrite equation (A.5) as:

$$(1+\varepsilon) - 2x + x^{(1-\beta)} = 0. \tag{A.6}$$

Now, we can write down the loss function

$$L(\varepsilon,\beta) = p_m[m(1+\varepsilon) - p_m + \overline{p}_{new}] - m[m(1+\varepsilon) - m + \overline{p}_{new}]. \tag{A.7}$$

Plug equation (A.4) into equation (A.7) and take advantage of equation (A.5), we have:

$$L(\varepsilon,\beta) = p_m p_m - m(2p_m - m) = (m - p_m)^2. \tag{A.8}$$

As $x = p_m/m$, we can rewrite equation (A.8) as $L(\varepsilon,\beta) = m^2(1-x)^2$, where x satisfies equation (A.6).

Thus we have derived the loss function in the example.

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