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Health, Taxes, and Growth

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This paper studies capital accumulation and consumption in the traditional Ramsey model under an exogenous growth framework. The model has three important features: (1) treating health as a simple function of consumption, which enable the study of health and growth in an aggregate macroeconomic model; (2) the existence of multiple equilibria of capital stock, health, and consumption, which is more consistent with the real world situation - rich countries may end up with high capital, better health, and higher consumption than poor countries; (3) the fundamental proposition of a consumption tax instead of capital taxation from the traditional growth model does not hold anymore in our model. As long as consumption goods contribute to health formation, the issue of a consumption tax versus an income (or capital) tax should be re-examined.

Key Words: Health; Capital accumulation; Taxation.
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1. INTRODUCTION

The relationship between health and economic growth has long attracted researchers as well as practitioners from many disciplines including economics, sociology, physiology, etc. There have already been a large number of evidences indicating that health is positively related to economic growth, but it is still not clear how health interacts with economic growth. Moreover, existing studies are largely limited to examining the empirical relationship between health, measured by health expenditure, intensity of health care, and life expectancy, and economic growth. Recently there have been some theoretical studies analyzing this causality relationship between health and economic growth (Ehrlich and Lui, 1991; Barro, 1996; Zon and Muysken, 2001, 2003; Morand, 2004; etc.). However, since the interaction mechanism between health and growth is quite complicated, these theoretical studies only analyzing a portion of the whole interaction mechanism between health and growth. To our knowledge, no study has investigated how income and nutrition improvement influence on growth, which is another important channel through which health affects economic growth as indicated by Fogel (1994a, 1994b, 2002) who argued that the combined effort of the increases in the dietary energy available for work, and of the increased human efficiency in transforming dietary energy into work output, appears to account for about 50 percent of the British economic growth since 1790 (Fogel, 1994a, p.388). In this paper, following Fogel's research, we intend to explore the interaction between health and growth through the channel that increases in income and nutrition improve the health capital accumulation and hence raise the labor productivity. Furthermore, we also want to study when consumption affects health capital and hence labor productivity, whether the accumulation of health capital will lead to endogenous economic growth or it is just a by-product of economic growth.

Using an extended Ramsey (1928) model, we assume that consumption not only increases agents' utility but also improves agents' health. Under this assumption, we study the relationship among consumption, health capital and physical capital accumulation, and discuss the effect of health on economic growth. We find that health capital is not the motivation but the by-product of economic growth, which is consistent with Boumol (1967) and Zon and Muysken (2001, 2003). We also find that health capital accumulation is able to magnify economic growth driven by exogenous technology, which is consistent with Fogel's results (Fogel, 1994a, 1994b, 2002). Moreover, in the case of a special product function, we also find the existence of multiple equilibria of capital stock, health, and consumption, which is highly relevant to the real world situation that rich countries may end up with higher capital accumulation, better health, and higher consumption than the poor countries. This result helps to understand the polarization between the developing countries and the developed countries in the real world. Finally, we also reconsider the effects of consumption tax and capital tax. We find that the fundamental proposition of a consumption tax instead of a capital tax contributes to growth in the traditional growth models does not hold anymore: Once the consumption goods contribute to health formation, the issue of a consumption tax versus an income (or capital) tax should be re-examined. It is necessary to point out that the consumption here denotes the categories of commodity which are able to benefit health improvement.

There are increasing theoretical and empirical investigations on the effect of health on economic growth. The empirical studies can be divided into three categories (Jamison, et al., 2004). The first category comprises the historical case studies that may be more or less quantitative (Fogel, 1994a, 1994b, 2002; Strauss and Thomas, 1998; Sohn, 2000). As stated above, these studies all concluded that nutritional improvement is the main force that enhances health human capital improvement and hence economic growth in the long term. The second category is characterized by many "micro" studies which involve either household surveys that include one or more measures of health status along with other extensive information, or the assessment of the impact of specific diseases. Strauss and Thomas (1998) provided a major review (extensively updated by Thomas and Frankenberg, 2002), and Savedoff and Schultz (2000) surveyed methods used in the household studies and summarized findings of recent analyses from five Latin American countries. Recent studies include Liu et al (2008) on China and Laxminarayan (2004) on Vietnam. This literature confirms that health is positively associated with productivity on the micro level, which is consistent with our assumption that health human capital constitutes a type of production factor. The third category focuses on the relationship between health and economic growth from a macroeconomic perspective. These studies mainly rely on cross-national data to assess the impact of health at the national level, measured in life expectancy, adult survival rates, adult mortality rates or other indexes, on income growth rates and most confirmed that health is positively related to growth (Hicks, 1979; Wheeler, 1980; Barro, 1996; Sachs & Warner, 1997; Bloom and Williamson, 1998; Arora, 2001; Bloom et al., 2004; McDonald and Roberts, 2006; Lorentzen, et al., 2008). On the microeconomic and macroeconomic contribution of health to economic growth and development, Shurcke, et al. (2006) reviewed recent evidence.

The theoretical studies on the relationship between health and growth did not appear until about 20 years ago. Early theoretical studies on this issue mainly focused on the provision of health services from a microeconomic demand perspective and did not analyze the effect of health in the form of human capital promotes economic growth (Grossman, 1972;

Muurinen, 1982; Forster, 1989; Ehrilch and Lui, 1991; Johansson & Lofgren, 1995; Mertzer, 1997). Barro (1996) is the first study to propose a theoretical framework to analyze the macroeconomic effects of health as one of the most important components of human capital on economic growth. In a three-sector neoclassical growth model considering simultaneous both health and education human capitals, Barro analyzed the effects of health human capital on education and physical capital and the interaction between these three forms of capitals, and further discussed the effects of public policy of health services as a publicly subsidized private good and as a public good. Muysken, et al. (1999) also investigated the growth implications of endogenous health on steady-state growth and the transitional dynamics in a standard neo-classical growth framework.

Extending the Lucas (1988) endogenous growth model to include health investment and take into account that health services can provide utility, Zon and Muysken (2001, 2003) discussed the macroeconomic effects of health investment on economic growth. Compared to Barro (1996), besides the effect of health on labor productivity, Zon and Muysken (2001, 2003) considered three other channels through which health influences economic growth: 1) better health helps the accumulation of education human capital; 2) health services increase an agent's utility; and 3) health improvement increases longevity and hence leads to an aging population. While the first two effects of health on labor productivity and on education human capital accumulation tend to facilitate economic growth, the last two effects suggest that health investment may exceed the optimal level at which the marginal contribution of health investment to growth equals the marginal cost. This may crowd out resources which could have been used for physical capital investment. Therefore, in such a situation, health investment may impede the progress of economic growth. By introducing the effects of skill-driven technological change (henceforth SDTC) into the Zon and Muysken (2001, 2003) framework, Hosoya (2002, 2003) further investigated the relationships among economic growth, average health level, labor allocation, and longevity of the population in an endogenous growth model that integrates SDTC and human capital accumulation through formal schooling with health human capital accumulation. In addition, through integrating the accumulation of human capital, innovation in medical technology, health and longevity into a four-sector (education, consumption goods, R&D sector devoted to health research, and health goods) endogenous growth model with "keeping up with the Jones" preferences and an altruism utility function, Sanso and Asia (2006) also studied the bidirectional interaction between health and economic growth. They concluded that health, by influencing longevity, may become a source of endogenous growth.

In order to explain the real-world situation that rich countries may end up with higher capital, better health, and higher consumption than poor countries, the existence of multiple steady states and the poverty trap are also important issues in the literature on the relationship between health and economic development. Chakraborty (2004) and Bunzel and Qiao (2005) introduced endogenous mortality risk into a two-period overlapping generations model to study the effect of health (measured in mortality) on economic growth and confirmed the existence of multiple steady states. Hemmi, et al. (2007) studied the interaction between decisions on financing after-retirement health shocks and precautionary saving motives, and demonstrated that, at low levels of income, individuals choose not to save to finance the cost of after-retirement health shocks. However, once individuals become sufficiently rich, they do choose to save to finance the cost of these shocks. Therefore, this change in the individual saving behavior may also give rise to multiple steady state equilibria and result in the poverty trap.

Compared with the above literature, this paper has two important contributions to the existing literature: first, we analyze the effects of health improvement derived from increasing consumption and nutrition intake on the long-run economic growth, which have been ignored by all the previous studies; second, we build on the existing literature and discuss the effects of fiscal policies on the long-run capital stock and consumption level with health capital stock included as a variable.

This paper is organized as follows. Section 2 presents a theoretical model with health generated by consumption. Section 3 develops the accumulation of physical capital and health capital in an exogenous growth model. Multiple equilibria have been found in this framework. Section 4 studies the effects of the income tax and the consumption tax on the long-run consumption level and capital stock. Section 5 concludes with a discussion on the implications of these results and the future research.

2. BASIC MODEL

Consider an intertemporal model with the representative agent choosing his consumption path, c, and his capital accumulation path, k, to maximize his discounted utility, namely

$$\max \int_0^\infty u(c)e^{-\beta t}dt \tag{1}$$

subject to

$$\dot{k} = y - c - \delta k \tag{2}$$

where a dot over a variable denotes the derivative of the variable with respect to time, y denotes the agent's income, and $k(0) = k_0$ the initial capital stock. The discount rate β $(0 < \beta < 1)$ is a given constant. The instantaneous utility function is defined as $u(c)$. It is assumed that the marginal utility of consumption is positive, but diminishing, i.e. $u'(.) > 0$ and $u''(.) < 0$.

One of the main channels through which health influences the economic growth lies in the production function in which an increase in health can improve the labor productivity. In this paper, the production function is assumed as follows:

$$y = f(k, hl), \tag{3}$$

where l and h denote labor supply and health capital respectively. Compared with the normal neoclassical production function, the uniqueness of the above production function lies in the health capital entering into the product function. In fact, the existing literature points out several channels through which better health will raise the productivity and output. Most directly, healthier workers have more energy and robustness and are able to work harder and for a longer time. People with healthier body are less likely to be caught by disease and have lower chance to be absent from work. The fact that labor productivity is positively associated with health has been confirmed both in empirical micro- and macro-economic researches, especially in low-income settings (Strauss and Thomas, 1998; Bloom, et al, 2004; etc.). In addition, there are some indirect channels through which health influences productivity. For instance, improvement in health raises the incentive to acquire more schooling, since investment in schooling can be amortized over a longer working life. Healthier students also have lower absenteeism and higher cognitive function, and thus receive a better education for a given level of schooling (Howitt, 2005; Kalemli-Ozcan, et al., 2000; Weil, 2007; etc.). All these factors lead to healthier people with higher productivity. Therefore, it is very rational and natural for the health variable to enter the production function, just as Barro (1996), Issa (2003), Hosoya (2002, 2003), Muysken, et al. (1999), Zon and Muysken (2001, 2003), Weil (2007) did. Furthermore, just as what Fogel (2002, p.24) observed, the contribution of nutrition and health to economic growth may be thought of as labor-enhancing technological changes. In Zon and Muysken (2001, p. xiii), they also considered the contribution of health to production ability as Harrod-neutral technical change. In addition, we assume that

$$f_h > 0, \quad f_k > 0, \quad f_{hh} < 0, \quad f_{kk} < 0, \quad f_{kk} f_{hh} > f_{hk}^2 \tag{4}$$

which implies that the marginal productivity of physical capital and health capital are positive but diminishing, and the production function is convex in h and k.

The second main aspect of the interaction mechanism between health and economic growth in our paper lies in the effect of income on the health through consumption and nutrition improvement. As most economists observed, there are at least three main ways to improve an individual's health. First, sufficient nutrition is indispensable to keep a healthy body. Fogel (1994a, 1994b, 2002) and Strauss and Thomas (1998) indicated that, measured in life expectation or in height, an increase in nutrition is the main factor to improve the population's health in the long run in many countries, including Britain, France, United states, Vietnam and others. For the case of the underdeveloped periods of developed countries or the presently low- and middle-income countries, the main approach to improve health is still to increase nutrition and calorie intakes which are mainly embodied in food consumption. The second approach to improve health is health investment (Grossman, 1972; Strauss and Thomas, 1998; Zon and Muysken, 2001, 2003). By Grossman (1972), the health investment includes the own time of consumers and market goods such as medical care, diet, exercise, recreation, housing, which are obvious included in total consumption. Moreover, the health investment may also include an individual's medical cure activities when he/she is caught by some diseases or infections, in that these actions can shorten ill health time and/or avoid incidental death caused by illness (Zon and Muysken, 2003). The third way of health improvement may be related to an individual's knowledge on health protection and life behavior. Since the goal of this paper is to study the relationship between the health and the long term growth, we mainly focus on health derived from improvement in nutrition and consumption. In the long term, just as Fogel (1994a, 1994b, 2002) and Strauss and Thomas (1998) indicated, income and hence total consumption is the main force that promote health improvement. To this end, we assume that health is determined mainly by an agent's consumption, and people with more consumption will be much healthier, though other factors are also crucial factors to determine the health level. Therefore, we assume that the health generation function is given below[1]

$$h = h(c) \tag{5}$$

[1]Note that in equation (5), health is considered as a flow variable rather than a stock variable and hence no depreciation is allowed as well. However, even if in the case that health is a stock variable and there exists health capital depreciation, the general conclusion of the paper is not affected.

We assume that the marginal health productivity of consumption is nonnegative and non-increasing:

$$h'(c) \geq 0, \quad h''(c) \leq 0 \tag{6}$$

The assumption of nondecreasing $h(c)$ implies that, with the increase of consumption, the health capital h will at least not decrease. Alternatively, we can assume that $h(c)$ is not a monotonic function. For example, there exists a consumption level, $\bar{c} > 0$, such that $h(c)$ increases when consumption is less than $\bar{c}$; and the function $h(c)$ is kept constant when consumption is larger than $\bar{c}$. That is to say, we have $h'(c) \geq 0$, when $c < \bar{c}$; and $h'(c) \leq 0$, otherwise. We will discuss this kind of health generation function in section 3.3.

In order to solve the consumer's optimization problem, we define the Hamiltonian associated with the optimization problem

$$H = u(c) + \lambda[f(k, h(c)) - c - \delta k] \tag{7}$$

where λ is the co-state variable representing the marginal utility of physical capital investment measured in utility. By the Pontryagin's Principle, we obtain the first-order conditions

$$\lambda = u'(c) + \lambda f_h(k, h(c))h'(c) \tag{8}$$
$$\dot{\lambda} = \lambda]\beta + \delta - f_k(k, h(c))] \tag{9}$$

and the transversality condition $\lim_{t\to\infty} \lambda k e^{-\beta t} = 0$.

PROPOSITION 1. *Under the above assumptions on the utility function, production function and health generation function, if and only if a pair of real number, $(c(t), k(t))$, satisfies*

$$1 > f_h(k, h(c))h'(c) \tag{10}$$

then the pair $(c(t), k(t))$ satisfying equations (6), (8), (9) and the transversality condition which maximizes the objective function arrives.

Proof. (See appendix A) ▮

Equation (8) indicates that the marginal value of physical capital investment equals the marginal value of consumption, which is the sum of the marginal utility of consumption and the marginal contribution of consumption to production. From equation (8), we can express λ as a function of consumption and capital stock, $\lambda(c, k)$.

$$\lambda = \frac{u'(c)}{1 - f_h(k, h(c))h'(c)} \tag{11}$$

In equation (11), $f_h(k, h(c))h'(c)$ denotes the increase in production brought by increasing the unit consumption through increasing health capital and hence improving productivity, and $1 - f_h(k, h(c))h'(c)$ denotes the cost of increasing the unit consumption measured in consumption goods. Hence, the right side of equation (11) represents the marginal value of increasing the unit consumption or the marginal cost of increasing the unit investment measured in utility. The left side of (11) represents the marginal value of investment. Therefore, equation (11) implies that the agent divides his/her income between investment and consumption subject to that the marginal value of investment equals the marginal cost. Compared with the standard Ramsey model, the uniqueness of this consumption optimal condition is that there is an additional term $f_h(k, h(c))h'(c)$ in the denominator of the right side in equation (11). If consumption has no effect on health, i.e., $h'(c) = 0$, then equation (11) is the same as in the standard Ramsey model.

From equation (11), we know why the condition of $1 > f_h(k, h(c))h'(c)$ should be satisfied if an agent's investment is optimal. Given any positive investment, as we can see from equation (11), if $1 \leq f_h(k, h(c))h'(c)$, then the marginal value of investment measured in utility will be negative or zero. Since the marginal utility of consumption, $u'(c)$, is definitely positive, a decrease in investment or/and an increase in consumption always increases the utility. Therefore, if $1 \leq f_h(k, h(c))h'(c)$, the agent who maximizes his/her lifetime utility will keep increasing his/her consumption and decreasing his/her investment till the marginal value of investment becomes positive and equals the marginal cost of investment.

Differentiating equation (11) with respect to c and k respectively, we are able to obtain the following short-run effects of consumption and capital stock on the marginal value of capital:

$$\lambda_c = \frac{u_{cc}[1 - f_h(k, h(c))h'(c)] + u_c[f_{hh}(k, h(c))(h'(c))^2 + f_h(k, h(c))h''(c)]}{[1 - f_h(k, h(c))h'(c)]^2} < 0 \tag{12}$$

$$\lambda_k = \frac{u_c f_{hk}(k, h(c))h'(c)}{[1 - f_h(k, h(c))h'(c)]^2} > 0 \tag{13}$$

From equations (12) and (13), it is clear that when consumption increases, the marginal value of investment will decrease, which is the same as the standard Ramsey model. The difference between our model and the standard Ramsey model is that the marginal value of investment decrease more in our model than in the standard Ramsey model, which results from the decreasing marginal health productivity of consumption $(u_c f_{hh}(k, h(c))(h'(c))^2)$ and the decreasing marginal productivity of health $(u_c f_h(k, h(c))h''(c))$. However, when capital stock increases, the marginal value of investment will increase, which is constant in the standard Ramsey model. The reason for this result is that in a standard Ramsey model, the marginal cost of investment, $u'(c)$, has no correlation with the capital stock which results in the marginal value of the optimal investment, which also equals to $u'(c)$, has no correlation with the capital stock. However, in our model, the marginal cost of investment, $u'(c)/[1 - f_h(k, h(c))h'(c)]$, is determined not only by consumption but also by capital stock. Therefore, when capital stock increases, the marginal productivity of capital will also increase, and hence the decrease in production brought by increasing the unit consumption will decrease. Consequently, with capital stock increasing, the marginal value of the optimal consumption or/and the marginal cost of the optimal investment will increase, which results in the increasing marginal value of the optimal investment, λ.

By equations (5), (8), (9) and (11), we derive the dynamic equation of consumption as follows

$$\dot{c} = -\frac{\lambda}{\lambda_c}[f_k(k, h(c)) - \delta - \beta] - \frac{\lambda_k}{\lambda_c}[f(k, h(c)) - c - \delta k] \tag{14}$$

Equations (2) and (14) determine the accumulation paths for capital stock and consumption. In the following sections, we analyze the dynamic behavior of the consumption, capital accumulation, and hence health accumulation.

3. DYNAMICS OF CAPITAL ACCUMULATION AND CONSUMPTION

By equations (2) and (14), the consumption and the capital stock approach the steady-state value when $\dot{c} = \dot{k} = 0$. It can be characterized as

$$f(k, h(c)) - c - \delta k = 0 \tag{15}$$

$$f_k(k, h(c)) - \delta - \beta = 0 \tag{16}$$

Under the assumption of the neoclassical production function, the existence of a steady state is obvious. But we cannot guarantee its uniqueness. We will give examples for the existence of unique steady state and multiple steady states. In Appendix B, we study the stability of the steady state. The saddle-point stability requires that

$$\Lambda \equiv \beta h'(c) f_{kh}(k, h(c)) + [1 - f_h(k, h(c))h'(c)] f_{kk}(k, h(c)) < 0 \tag{17}$$

In generally, we cannot determine the stability and the uniqueness of the steady state, we will present some examples to analyze it.

3.1. Unique Steady State

Consider the following special forms of the utility function, the output production function and the health generation function

$$u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}, \quad f(k,h) = Ak^{\alpha}h^{1-\alpha}, \quad h(c) = \xi c + \eta c^{\theta} \tag{18}$$

where A denotes technology level, α the elasticity of capital with respect to output, σ the intertemporal substitute elasticity. θ, ξ and η are parameters in the health generation function. All these parameters are assumed to be positive constants. By equations (8), (15) and (16), the steady state satisfies

$$c^{-\sigma} + \lambda A(1-\alpha)k^{\alpha}(\xi c + \eta c^{\theta})^{-\alpha}(\xi + \eta\theta c^{\theta-1}) = \lambda \tag{19}$$

$$Ak^{\alpha-1}(\xi c + \eta c^{\theta})^{1-\alpha} - c/k - \delta = 0 \tag{20}$$

$$A\alpha k^{\alpha-1}(\xi c + \eta c^{\theta})^{1-\alpha} - \beta - \delta = 0 \tag{21}$$

Therefore, if $\frac{\alpha}{\eta(\beta+\delta-\alpha\delta)}\left(\frac{\beta+\delta}{A\alpha}\right)^{\frac{1}{1-\alpha}} > \frac{\xi}{\eta}$, then equations (19), (20), and (21) determine the unique steady state:

$$c^* = \left[\frac{\alpha}{\eta(\beta+\delta-\alpha\delta)}\left(\frac{\beta+\delta}{A\alpha}\right)^{\frac{1}{1-\alpha}} - \frac{\xi}{\eta}\right]^{\frac{1}{\theta-1}} \tag{22}$$

$$k^* = c^*\alpha/(\beta+\delta-\delta\alpha) \tag{23}$$

It is easy to verify that the saddle-point stable condition of equation (17) is satisfied when β and/or ξ are small enough to ensure a unique steady state which is saddle-point stable.

If we set the parameters as: $\delta = 0.1$, $\alpha = 0.7$, $\xi = 0.01$, $\eta = 0.5$, $\theta = 0.5$, $\beta = 0.05$, $\sigma = 0.5$, and $A = 1$, then the associated capital stock $k = 2.4316$, the consumption level $c = 0.27709$, the health capital $h = 0.54148$, the output $y = 1.54953$, and $\Lambda = -1.4 \times 10^{-5}$. As a result, the steady state is saddle-point stable. We also present in Table 1 the simulation results of the corresponding equilibrium values of the variables of this economy when we assume different values of A ranging from 1 to 1.5 while assuming other parameters unchanged.[2]

Based on these simulation results, we have the following findings on the effects of health on economic growth. First, the above results indicate

[2]Our simulation results indicate that when A is greater than 1.5, we need much less ξ or β to guarantee the existence and stability of the steady state. This is because when A is too large, the condition of the existence and stability of steady state, $[\alpha/(\eta\beta+\eta\delta-\eta\alpha\delta)][(\beta+\delta)/(A\alpha)]^{\frac{1}{1-\alpha}} > \xi/\eta$, can not be satisfied.

TABLE 1.

Simulation results of unique steady state

A	c	k	h	y	$c+\delta k$	Λ	g_y	g_A
1	145.01	1268.85	7.47	271.90	271.90	$-1.4E-05$		
1.01	157.50	1378.17	7.85	292.40	292.40	$-1.3E-05$	0.075	0.01
1.03	185.75	1625.28	8.67	348.27	348.27	$-1.1E-05$	0.28	0.03
1.07	258.16	2258.94	10.62	465.96	465.96	$-7.5E-06$	0.59	0.07
1.10	330.62	2892.95	12.40	619.92	619.92	$-5.7E-06$	1.28	0.10
1.50	22468.59	196600.2	299.63	42128.61	42128.61	$-2.9E-08$	153.94	0.50

Note: g_y denotes the rate of technology progress, g_A the output growth rate and other parameters are the same as defined in the previous part of this paper.

that the economy has a steady state and there is no persistent economic growth in this economy. Hence, even if a rise in consumption and nutrition can improve health capital and hence improve labor productivity, the health capital improvement is not able to induce persistent economic growth. Therefore, the improvement in health capital derived from increment in consumption and nutrition is not the motivation but the by-product of economic growth, which is consistent with what was stated in Boumol (1967) and Zon and Muysken (2001, 2003). Second, from Table 1 we can see that, when there is 1 percent increment of technology level from 1 to 1.01, the output increases by 7.5 percent. In contrast, when technology level increases by 50 percent from 1 to 1.5, the production output increases by about 154 times. Therefore, we find that while improvement in health capital can not introduce persistent economic growth, it is able to enlarge the economic growth driven by exogenous technology, which is consists with Fogel's result. This conclusion is also correct when there are multiple steady states in the economy as what would be discussed in the following section.

3.2. The Existence of Multiple Steady States

If we change the production function to

$$f(k,h) = Ak^{\alpha}h^{1-\alpha} + Bk^{\omega_3} + Dh^{\omega_4} \tag{24}$$

where $\alpha, \omega_3, \omega_4, A, B$ and D are positive constants. The utility function and the health generation function are still the same as equation (18).

Under the specified functions, we will discuss the existence of steady state and the stability of them. For simplicity, we discuss these using numerical solutions.

Case 1: Set the parameters as: $\theta = 0.5$, $\alpha = 0.5$, $\omega_3 = 0.7$, $\omega_4 = 0.1$, $A = 0.5$, $B = 0.5$, $\delta = 0.15$, $\xi = 0.4$, $\eta = 0.1$, $\beta = 0.1$, and $D = 0.3$. In this case, we get the steady states values as:

$$
\begin{aligned}
k_1^* &= 0.00475, & c_1^* &= 0.27499, & h_1^* &= 0.16244 \\
k_2^* &= 0.18408, & c_2^* &= 0.49376, & h_2^* &= 0.26777 \\
k_3^* &= 8.33639, & c_3^* &= 2.59078, & h_3^* &= 1.19727
\end{aligned}
$$

and the corresponding Hamiltonian multipliers are $\lambda_1^* = 2.11286$, $\lambda_2^* = 1.66239$, and $\lambda_3^* = 0.88170$ respectively.

We can prove that k_1^* and k_3^* are the saddle-point stable steady states while the second steady state capital stock k_2^* is a critical steady state. If the initial capital stock is less than k_2^*, the capital stock, consumption, and health will converge to the first steady state. In contrast, if the initial capital stock is larger than k_2^*, the capital stock, the consumption, and the health will rise to the third steady state.

Case 2: We consider the situation with low marginal productivity of consumption and we select the parameters as: $\theta = 0.5$, $\alpha = 0.5$, $\omega_3 = 0.7$, $\omega_4 = 0.1$, $A = 0.5$, $B = 0.5$, $\delta = 0.15$, $\xi = 0.1$, $\eta = 0.1$, $\beta = 0.1$, and $D = 0.3$. Now, we can get the steady states as:

$$
\begin{aligned}
k_1^* &= 0.00038, & c_1^* &= 0.23519, & h_1^* &= 0.07202 \\
k_2^* &= 0.64704, & c_2^* &= 0.65349, & h_2^* &= 0.14619 \\
k_3^* &= 4.67104, & c_3^* &= 1.44939, & h_3^* &= 0.26533
\end{aligned}
$$

and the corresponding Hamiltonian multipliers are $\lambda_1^* = 2.21422$, $\lambda_2^* = 1.39389$, and $\lambda_3^* = 0.99177$ respectively.

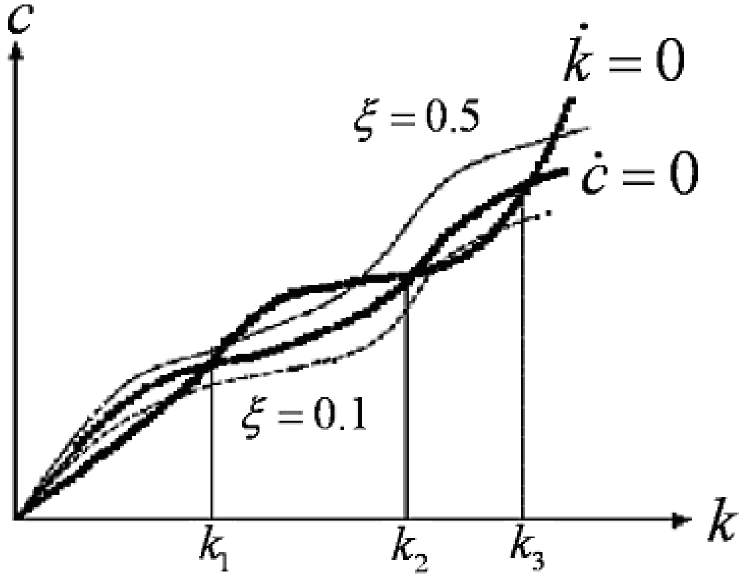
Case 3: We consider the situation with high marginal productivity of consumption and we select the parameters as: $\theta = 0.5$, $\alpha = 0.5$, $\omega_3 = 0.7$, $\omega_4 = 0.1$, $A = 0.5$, $B = 0.5$, $\delta = 0.15$, $\xi = 0.5$, $\eta = 0.1$, $\beta = 0.1$, and $D = 0.3$, we can get the steady states as:

$$
\begin{aligned}
k_1^* &= 0.01137, & c_1^* &= 0.29984, & h_1^* &= 0.20468 \\
k_2^* &= 0.09943, & c_2^* &= 0.42903, & h_2^* &= 0.28002 \\
k_3^* &= 9.99119, & c_3^* &= 3.08705, & h_3^* &= 1.71923
\end{aligned}
$$

and the corresponding Hamiltonian multipliers are $\lambda_1^* = 2.04917$, $\lambda_2^* = 1.77569$, and $\lambda_3^* = 0.84564$ respectively. The results from this situation are very much similar to those of the previous example.

We present the multiple steady states in Figure 1. The solid curves are $\dot{c} = 0$, and $\dot{k} = 0$ when $\xi = 0.4$. These two curves suggest that there are three steady states. In particular, the steady states k_1 and k_3 are saddle-point stable, while k_2 is unstable. With the increment of ξ (say $\xi = 0.5$), the curve $\dot{c} = 0$ will shift up. If ξ decreases, for example to $\xi = 0.1$, the

FIG. 1. The Existence of Multiple Steady States



curve $\dot{c} = 0$ will shift down. For further details on the dynamics, please check Figure 1.

3.3. A Nonmonotonic Health Generation Function

In the previous sections, we analyze the dynamics of the consumption accumulation path and the capital stock accumulation path under the assumption of a monotonic health generation function. In this subsection, we analyze these dynamics under the assumption of a nonmonotonic health generation function. Suppose we define the health generation function as

$$h(c) = \xi c + \eta c^{\theta} \tag{25}$$

where ξ and $\theta > 1$ are positive constants, $\eta < 0$ is a negative constant. The production function and utility function are specified the same as in equations (24) and (18), respectively:

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma} \tag{26}$$

$$f(k, h) = Ak^{\alpha}h^{1-\alpha} + Bk^{\omega_3} + Dh^{\omega_4} \tag{27}$$

where $\sigma, \alpha, \omega_3, \omega_4$, and A are positive constants.

For the selected parameters: $\theta = 2, \alpha = 0.5$, $\omega_3 = 0.6, \omega_4 = 0.1$, $A = 0.5, B = 0.5$, $\delta = 0.15, \xi = 0.5$, $\eta = 0.1, \beta = 0.1$, and $D = 0.3$, we obtain the critical consumption level $\bar{c} = 2.5$, so we have $h'(c) > 0$ when $c \leq 2.5$; and $h'(c) \leq 0$ otherwise. We get the steady states:

$$\begin{aligned} k_1^* &= 0.000004, & c_1^* &= 0.24946, & h_1^* &= 0.11851 \\ k_2^* &= 0.30234, & c_2^* &= 0.60283, & h_2^* &= 0.26508 \\ k_3^* &= 4.57422, & c_3^* &= 1.63457, & h_3^* &= 0.55010 \end{aligned}$$

and the corresponding Hamiltonian multipliers are $\lambda_1^* = 2.20676$, $\lambda_2^* = 1.49573$, $\lambda_3^* = 0.90285$, respectively.

We can prove that k_1^* and k_3^* are the saddle-point stable steady states capital stocks. The second steady state capital stock k_2^* is a critical steady state. If the initial capital stock is less than k_2^*, the capital stock, the consumption, and the health will converge to the first steady state. In contrast, if the initial capital stock is larger than k_2^*, the capital stock, the consumption, and the health will rise to the third steady state.

4. POLICY ANALYSIS

Introducing government tax to the above model, the budget constraint of the agent can be rewritten as

$$\dot{k} = (1 - \tau_y) f(k, h(c)) - (1 + \tau_c)c - \delta k \tag{2'}$$

where τ_y and τ_c are the income tax rate and the consumption tax rate, respectively.

The first-order conditions (8) and (9) can be rewritten as

$$u_c + \lambda(1 - \tau_y) f_h(k, h(c)) h'(c) = \lambda(1 + \tau_c) \tag{8'}$$

$$\dot{\lambda} = \beta\lambda - \lambda[(1 - \tau_y) f_k(k, h(c)) - \delta] \tag{9'}$$

with the same transversality condition as defined in the previous section.

From equation (8'), we can also express the marginal value of physical capital investment as a function of consumption and capital stock

$$\lambda = u_c / D \tag{28}$$

where $D = 1 + \tau_c - (1 - \tau_y) f_h(k, h(c)) h'(c)$.

The steady state is characterized by the following two equations:

$$(1 - \tau_y) f(k, h(c)) - (1 + \tau_c)c - \delta k = 0 \tag{29}$$

$$(1 - \tau_y) f_k(k, h(c)) - \delta - \beta = 0 \tag{30}$$

Suppose the steady state exists and is saddle-point stable. We then focus on the effects of the income tax and consumption tax on the steady-state consumption and capital stock. Taking total differentiate with respect to τ_c and τ_y on equations (29) and (30), we get

$$\begin{aligned}&\begin{pmatrix} \beta & (1-\tau_y)f_h(k,h(c))h'(c)-1-\tau_c \\ (1-\tau_y)f_{kk}(k,h(c)) & (1-\tau_y)f_{kh}(k,h(c))h'(c) \end{pmatrix}\begin{pmatrix} dk \\ dc \end{pmatrix} \\ =\ &\begin{pmatrix} c \\ 0 \end{pmatrix} d\tau_c + \begin{pmatrix} f \\ -f_k(k,h(c)) \end{pmatrix} d\tau_y \end{aligned} \tag{31}$$

The effects of consumption tax on the steady-state capital stock and consumption can be derived as

$$\begin{aligned} \frac{dk}{d\tau_c} &= -\frac{c(1-\tau_y)f_{kh}(k,h(c))h'(c)}{\Delta} < 0, \\ \frac{dc}{d\tau_c} &= \frac{c(1-\tau_y)f_{kk}(k,h(c))}{\Delta} < 0 \end{aligned} \tag{32}$$

where

$$\begin{aligned} \Delta &= (1-\tau_y)f_{kh}(k,h(c))h'(c)\beta \\ &- (1-\tau_y)f_{kk}(k,h(c))((1-\tau_y)f_h(k,h(c))h'(c)-1-\tau_c) \end{aligned} \tag{33}$$

which is negative from the saddle-point stability condition (14).From equation (32), we find that with the increase of the consumption tax rate, the steady-state capital stock and consumption will decrease.

Similarly, for the effects of the income tax on the steady-state capital stock and consumption, from equation (31), we have

$$\begin{aligned} \frac{dk}{d\tau_y} &= -\frac{1}{\Delta}f(1-\tau_y)f_{kh}(k,h(c))h'(c) \\ &+ f_k(k,h(c))((1-\tau_y)f_h(k,h(c))h'(c)-1-\tau_c) \end{aligned} \tag{34}$$

$$\frac{dc}{d\tau_y} = -\frac{1}{\Delta}\beta f_k(k,h(c)) + f(1-\tau_y)f_{kk}(k,h(c)) \tag{35}$$

which show that the effects of the income tax rate on the steady-state capital stock and consumption is ambiguous.

In this paper, we identify the negative effects of consumption tax on the long-run consumption level and capital stock. In other words, we find that with the increase in the consumption tax rate, the long-run capital stock and consumption level will decrease. The reason is that with the increment of the consumption tax rate, the cost of consumption will be increasing, which in turn decreases the long-run consumption level. However, with the decrease of the consumption, the health generation will decrease and the output and investment will also decrease which results in decrease in the capital stock. These causality relationships found in this study are significantly different from what we find in the existing literature. In the traditional literature, such as Rebelo (1991), the consumption tax, which decreases the long-run consumption level, has no effects on the long-run capital stock.

Furthermore, we present the ambiguous effects of the income tax rate on the steady-state capital stock and the consumption, which are different from the negative effects of the income tax rate on the capital stock in the existing literature. The reason behind this discrepancy between our results and those of the existing literature is that as the income tax rate increases, the return on the capital stock will decrease which in turn decreases the steady state capital stock and increases the consumption level. With the increase of the consumption level, the health generation will increase, and the marginal productivity of the capital stock will also increase which leads to the increment of the returns on the capital stock, which will lead to a higher steady state capital stock. The overall effect of the income tax rate on the capital stock will depend on the interaction of the above two effects. Thus, we derive ambiguous effects of the income tax rate on the steady-state capital stock as well as on the long-run consumption level.

5. CONCLUSIONS

In this paper, we first presented a theoretical framework to discuss the consumption path and the capital accumulation path by introducing health as a sector of output production. We assume that health can be generated by private consumption. Based on the simulation results of Case 1 in Section 3.1 we found that, when the improvement in health capital is induced by a rise in consumption, this consumption and nutrition driven health capital is not the motivation but the by-product of economic growth, which is consistent with the conclusion in Boumol (1967) and Zon and Muysken (2001, 2003) concluded. However, we also found that the resulting health capital is able to expand the economic growth driven by exogenous technology, which is consistent with the result of Fogel (1994a, 1994b, and 2002).

Secondly, under the assumption of a nonmonotonic health generation function, we could not derive the uniqueness of steady state, like Kurz (1968). In the given numerical examples, we derived three steady states under some given parameter specifications. The existence of multiple steady states can be used to explain the economic growth puzzle posed by Lucas (1993): Why would two countries like South Korea and the Philippines, whose wealth and endowment levels were quite close not so long ago, differ so drastically in their recent growth experience.

Lastly, we discussed the effects of consumption tax and income tax on long-run capital stock and consumption. The results obtained from the theoretical framework in our study were different from those found by Rebelo (1991). We found negative effects of the consumption tax rate on the long-run capital stock and the consumption while the effects of the income tax rate on the long-run capital stock and the consumption level were ambiguous.

The model has three important features: (1) treating health as a simple function of consumption, which enable the study of the relationship between health and long-term economic growth in an aggregate macroeconomic model; (2) the existence of multiple equilibria of capital stock, health, and consumption, which is more consistent with the real world situation — rich countries may end up with higher capital stock, better health, and higher consumption level than poor countries; (3) the fundamental proposition of a consumption tax instead of capital taxation based on the traditional growth model does not hold anymore in our model. As long as consumption goods contribute to health formation, the issue of a consumption tax versus an income (or capital) tax should be re-examined.

For future theoretical studies, research should focus on the monetary policy implications based on the framework proposed in our study. We will also extend our model to a two-sector framework to consider simultanously the physical capital and the human capital. Furthermore, it is interesting to analyze the effects of government expenditures on the long-run capital stock and the consumption with the consideration of health. For future empirical studies, we are interested in studying whether the gap between rich and poor countries is widening, which is suggested by the multiple equilibria framework proposed in our study.

APPENDIX A

The proof of Proposition 1:

1) We proof equation (10) is a necessary condition.

By the Hamilton function, we have

$$\frac{\partial H}{\partial c} = u'(c) + \lambda[f_h(k, h(c))h'(c) - 1] \tag{A.1}$$

$$\frac{\partial H}{\partial k} = \lambda[f_k(k, h(c)) - \delta] \tag{A.2}$$

and

$$\frac{\partial^2 H}{\partial c^2} = u''(c) + \lambda[f_h(k,h(c))h''(c) + f_{hh}(k,h(c))(h'(c))^2] \quad \text{(A.3)}$$

$$\frac{\partial^2 H}{\partial c \partial k} = \lambda f_{ck}(k,h(c))h'(c) \quad \text{(A.4)}$$

$$\frac{\partial^2 H}{\partial k^2} = \lambda f_{kk}(k,h(c)) \quad \text{(A.5)}$$

If the objective function arrives at maximum when (c,k) satisfies the first-order conditions, then the Hamilton function must be nonpositive. Therefore, $\frac{\partial^2 H}{\partial c^2}$ and $\frac{\partial^2 H}{\partial k^2}$ must be nonpositive and the determinant of Hessian second-order matrix must be nonnegative. By assumption (3), $f_{kk} < 0$, in order that $\frac{\partial^2 H}{\partial k^2} \leq 0$, there must be $\lambda \geq 0$, which result in $f_h(k,h(c))h'(c) < 1$.

2) We prove equation (10) is a sufficient condition. First, when $f_h(k,h(c))h'(c) < 1$, then $\lambda > 0$. It is obviously that $\frac{\partial^2 H}{\partial c^2}$ and $\frac{\partial^2 H}{\partial k^2}$ are positive. Second, the determinant of the Haiseir matrix is

$$\begin{aligned} \begin{vmatrix} H_{cc} & H_{ck} \\ H_{kc} & H_{kk} \end{vmatrix} &= \lambda f_{kk}\{u'' + \lambda[h'' f_h + {h'}^2 f_{hh}]\} - (\lambda h' f_{hk})^2 \\ &= \lambda f_{kk} u'' + \lambda^2 [f_{kk} f_h h'' + {h'}^2 (f_{kk} f_{hh} - f_{hk}^2)] \end{aligned}$$

By the assumption on the utility function, production function and health generation function, the determinant of Hessian matrix must be positive. And hence, (c,k) satisfying equation (6), (8), (9) and transversality condition maximizes the objective function arrives.

APPENDIX B

The condition of saddle-point stability

We linearize system (2') and (11) around the steady state

$$\begin{pmatrix} \frac{dk}{dt} \\ \frac{dc}{dt} \end{pmatrix} = J \begin{pmatrix} k - k^* \\ c - c^* \end{pmatrix} \quad \text{(B.1)}$$

where

$$J = \begin{pmatrix} \beta & 10 f_h(k,h(c))h'(c) \\ -\frac{\lambda}{\lambda_c} f_{kk} - \frac{\lambda_k}{\lambda_c}\beta & 0 \end{pmatrix} \quad \text{(B.2)}$$

is the coefficient matrix associated with the above linear system. The eigenvalues μ_1 and μ_2 of the matrix J satisfy

$$\mu_1 + \mu_2 = \beta \quad \text{(B.3)}$$

$$\mu_1\mu_2 = -\frac{\lambda}{\lambda_c}\{\beta h'(c)f_{hk}(k,h(c)) - [h'(c)f_h(k,h(c)) - 1]f_{kk}(k,h(c))\} \quad \text{(B.4)}$$

Thus, the saddle-point stability requires that

$$\beta h'(c)f_{hk}(k,h(c)) - [h'(c)f_h(k,h(c)) - 1]f_{kk}(k,h(c)) < 0. \quad \text{(B.5)}$$

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