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Private and Public Health Expenditures in an Endogenous Growth Model with Inflation Targeting

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This paper develops a monetary endogenous growth overlapping generations model characterized by endogenous longevity and an inflation targeting monetary authority, and analyzes the growth dynamics that emerges from this framework. Besides the endogenous longevity which depends on the complimentarity of private and public health expenditures, the growth process is endogenized by allowing for a productive role of government expenditure on infrastructure. Following the huge existing literature, money is introduced by assuming that banks are obligated to hold a fraction of the deposits as cash reserve requirements. Given this framework, we show that multiple equilibria emerges, with the low-growth (high-growth) equilibrium being unstable (stable) and locally determinate (locally indeterminate). In addition, we show that, under certain conditions, endogenous fluctuations and even chaos could emerge around the high-growth equilibrium.

Key Words: Indeterminacy; Inflation targeting; Longevity; Multiple equilibria; Overlapping generations; Public health.
JEL Classification Numbers: C62, D84, E10, E31, E32, E52, J10, O10, O40.

1. INTRODUCTION

There exits a new, yet burgeoning, literature that incorporates the role of mortality in dynamic general equilibrium models. Most noteworthy among them are Blackburn and Cipriani (2002), Kalemli-Ozcan (2003, 2008), Chakraborty (2004), Bunzel and Qiao (2005), Cervellati and Sunde (2005), Hashimoto and Tabata (2005), Agénor (2006, 2009), Aísa and

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Pueyo (2006), Finlay (2006), Hazan and Zoabi (2006), Sarkar (2007, 2008), Bhattacharya and Qiao (2007), Tang and Zhang (2007), Castelló-Climent and Doménech (2008), Osang and Sarkar (2008) and Gupta and Ziramba (forthcoming c). While some of these studies endogenize the probability of survival by assuming the mortality rate to depend on public health capital, other studies emphasize the role of per-capita income or consumption expenditure or even private spending on health in affecting longevity. To the best of our knowledge, Bhattacharya and Qiao (2007) and Agénor (2009) are the only two studies which takes into account the simultaneous role of private expenditure (in form of resources or time) and public health expenditures in affecting the probability of survival.

On one hand, Bhattacharya and Qiao (2007) endogenized longevity into a standard Solow (1956)-type overlapping generations growth model by assuming that a young agent may increase the length of old age by incurring investments in health. In addition, the private health investments are assumed to be more 'productive' if accompanied by complementary tax-financed public health programs. The paper shows that the existence of the public input in the private longevity function exposes the economy to aggregate endogenous fluctuations and even chaos, which is otherwise impossible in its absence. On the other hand, Agénor (2009) studied growth dynamics and health outcomes in a three-period overlapping generations model with public capital. Reproductive agents are assumed to face a non-zero probability of death in both childhood and adulthood. Besides working, adults allocate time to their own health and child rearing, with health status in adulthood depending upon health in childhood. The paper showed that with partial persistence in health, pure stagnation might occur, while, with full persistence, a stagnating equilibrium with low growth and high fertility might result due to poor access to public capital. In addition, multiple growth regimes might emerge, given threshold effects in health status. From a policy perspective, the analysis highlighted that a reallocation of public spending toward health or infrastructure may shift the economy from a low-growth equilibrium to a high-growth and low-fertility steady-state.

Against this backdrop, our paper develops a monetary endogenous growth version of the model designed by Bhattacharya and Qiao (2007) and analyzes the growth dynamics that emerge from this framework. Besides the endogenous longevity which depends on the complimentarity of private and public health expenditures, as in Bhattacharya and Qiao (2007), the growth process is endogenized along the lines of Barro (1990)-type production structure. Following the huge existing literature of introducing money in general equilibrium models,[1] money is incorporated by assuming that banks, otherwise operating in a perfectly competitive environment, are obligated to hold a fraction of the deposits as cash reserve requirements. Finally, growth dynamics are introduced in the overlapping generations framework by assuming an inflation targeting monetary authority, instead of the standard assumption of a monetary authority following a money-growth rule (see subsection 2.4 and Kudoh (2004a, b) for further details). The motivation behind extending the paper of Bhattacharya and Qiao (2007) emanates from the basic drawback of a Solow (1956)-type growth model, which is, essentially its lack of ability to explain the non-zero growth rate of the standard of living in steady-state observed in the data. In this sense, our framework is more closer to reality, and as we show below helps us obtain more richer set of dynamics over and above those yielded by the work of Bhattacharya and Qiao (2007). The remainder of the paper is organized as follows: Section 2 lays out the economic environment and Section 3 defines the equilibrium. Sections 4 and 5, respectively, discuss the growth dynamics and concludes.

2. ECONOMIC ENVIRONMENT

Time is divided into discrete segments, and is indexed by $t = 1, 2, \ldots$. There are four theaters of economic activities: (i) each (possible) two-period lived overlapping generations household (consumer/worker) is endowed with one unit of labor when young, but the agent retires when old. Thus, at each date t, there are two coexisting generations of young and old. N people are born at each time point $t \geq 1$. At $t = 1$, there exist N people in the economy, called the initial old, who live for only one period. The young earns their wage income by inelastically supplying one unit of the labor endowment.[2] A fraction of the wage income is invested in their own health when young, with such expenditure being determined endogenously to maximize utility, and the rest is deposited into banks for future consumption; (ii) the banks simply convert one period deposit contracts into loans, after meeting the cash reserve requirements. No resources are assumed to be spent in running the banks[3]; (ii) each infinitely-lived producer is endowed with a production technology to manufacture a single final good using the inelastically supplied labor, physical (financed by credit facili-

[1]See Gupta (2005, 2008, forthcoming) and Gupta and Ziramba (2008, 2009, forthcoming a, b, d) and references cited there in for further details.

[2]The decision to rule out any labor-leisure choice is for the sake of simplicity. Our basic results continue to remain the same irrespective of whether we have the consumer working a fraction of the available time endowment. See footnote 7 for further details.

[3]This is a simplifying assumption and has no bearing on our final results. See footnote 5 for further details.

tated by the financial intermediaries) and public capital, and; (iv) there is an infinitely-lived government which meets its expenditure on the provision of infrastructure and health (the latter being complementary to the private health expenditure in the survival function) via inflation tax. Note that taxes have been ignored not only for the sake of simplicity, but more importantly because inclusion or non-inclusion of income taxes in our model does not affect the main conclusions of the paper. However, public debt has not been considered due to technical reasons outlined in subsection 2.4. There is a continuum of each type of economic agents with unit mass.

The sequence of events can be outlined as follows: When young, a household works and receives wages, spends a part of the income on health expenses and deposits the rest into banks. A bank, after meeting the reserve requirement, provides a loan to a goods producer, which subsequently manufactures the final good and returns the loan with interests. Finally, the banks pay back the deposits with interest to households at the end of the first period, and the latter consumes in the second period.

2.1. Consumers

Let c_{t+1} denote the consumption of an old representative agent born in period t. The corresponding utility function is given as $u(c_{t+1} = E_t u(c_{t+1})$, where $u : \mathcal{R}_+ \rightarrow \mathcal{R}_+$ is twice-continuously differentiable, strictly increasing and strictly concave in old-age consumption. In our case, we use $u(c) = c^{(1-\sigma)}/(1-\sigma)$, with $0 < \sigma < 1$. All young agents survive to the second period, but a specific agent is alive only for $0 < \theta \leq 1$ fraction of the old-age. Thus, $(1+\theta)$ is a measure of longevity in the model. Agents can influence their longevity by spending ϕ fraction of their wage income. As in Bhattacharya and Qiao (2007), we assume that θ is strictly increasing and strictly concave in ϕ, and is represented by a constant-elasticity functional form of the following type for the longevity production function:

$$\theta(\phi; \eta) = b\eta\phi^{b\eta} \tag{1}$$

where $b(> 0)$ is a scale parameter and $0 < \eta < 1$ is the endogenous public input (ratio of real government expenditure on health to real wage) in private longevity. In all our discussion, we assume that $0 < \theta \leq 1$. Strict concavity of θ requires $b\eta \in (0, 1)$. Let g_t denote the total real government expenditure on health and infrastructure, with $0 \leq \lambda \leq 1$ $(0 \leq (1-\lambda) \leq 1)$ denoting the fraction of g_t devoted to health (infrastructure). We assume that η is a function of $(\lambda g_t/w_t)$, i.e., $\eta = \eta(\lambda g_t/w_t)$. Specifically, we assume that $\eta = (1 + \lambda g_t/w_t)$, such that $\eta \geq 1$ and $\eta(0) = 1$. Given that $\eta \geq 1$, the condition for strict concavity of $\theta(\phi)$ boils down to $b \in (0, 1/\eta)$. The longevity production function in (1) suggests that if there does not exist private efforts to improve health, then a better public health system cannot be of much help in raising longevity and vice versa, thus highlighting the complementary nature of private and public components to health systems.[4]

Formally the representative young agent's problem (born in period $t \geq 1$) can be described as follows:

$$\max_{\phi_t} \theta(\phi_t; \eta_t) \frac{(c_{t+1})^{(1-\sigma)}}{1-\sigma} \tag{2}$$

subject to

$$D_t \leq (1-\phi_t) p_t w_t n_t \tag{3}$$

$$p_{t+1} c_{t+1} \leq (1 + i_{D_{t+1}}) D_t \tag{4}$$

where equation (3) is the feasibility (first-period budget) constraint and equation (4) denotes the old-age budget constraint for the consumer. $n_t(= 1)$ is the inelastic labor-supply; w_t is the real wage at t; D_t is the size of bank deposits in nominal terms; $i_{D_{t+1}}$ is the nominal interest rate received on the deposits at $t+1$, and; $p_t(p_{t+1})$ is the price-level in period $t(t+1)$. Young agents spend a fraction of their wage income ϕ in health to achieve a longer lifespan in old-age. There is, however, a trade off between their investment in health with contemporaneous investment in physical capital via their savings (deposits). Return on savings are made available to the old agents right at the very beginning of the second-period of their life, which, in turn, allows us to abstract away from the complications that might arise with the fate of savings of those agents who die before receiving any return.

Assuming that $0 < b\eta < \sigma$, $\theta(\phi_t)u(\phi_t)$ is strictly concave in ϕ, and the agent's problem has a unique interior solution, yields:

$$\phi_t = \frac{b(1+\lambda \frac{g_t}{w_t})}{1-\sigma+b(1+\lambda \frac{g_t}{w_t})} \tag{5}$$

$$d_t = \frac{(1-\sigma)}{1-\sigma+b(1+\lambda \frac{g_t}{w_t})} \tag{6}$$

where $d_t(= D_t/p_t)$ measures real deposits. Note, *ceteris paribus*, private investment in health increases and optimal deposits falls as public expenditure on health increases.

2.2. Financial Intermediaries

At the start of each period the financial intermediaries accept deposits and make their portfolio decision (that is, loans and cash reserves choices) with a goal of maximizing profits. At the end of the period they receive

[4]See Bhattacharya and Qiao (2007) for further discussion on this issue.

their interest income from the loans made and meets the interest obligations on the deposits. Note that the intermediaries are constrained by legal requirements on the choice of their portfolio (that is, reserve requirements), as well as by feasibility. Given such a structure, the intermediaries obtain the optimal choice for L_t by solving the following problem:

$$\max_{L,D} \pi_b = i_{L_t} L_t - i_{D_t} D_t \tag{7}$$

$$s.t. \; : \; \gamma_t D_t + L_t D_t \tag{8}$$

where π_b is the profit function for the financial intermediary, and $M_t \geq \gamma_t D_t$ defines the legal reserve requirement. M_t is the cash reserves held by the bank; L_t is the loans; i_{L_t} is the interest rate on loans, and; γ_t is the reserve requirement ratio. The reserve requirement ratio is the ratio of required reserves (which must be held in form of currency) to deposits.

To gain some economic intuition of the role of reserve requirements, let us consider the solution of the problem for a typical intermediary. Free entry, drives profits to zero and we have:

$$i_{L_t}(1 - \gamma_t) - i_{D_t} = 0 \tag{9}$$

Simplifying, in equilibrium, the following condition must hold:

$$i_{L_t} = \frac{i_{D_t}}{1 - \gamma_t} \tag{10}$$

Thus, reserve requirements tend to induce a wedge between the interest rate on savings and lending rates for the financial intermediary.[5]

2.3. Firms

All firms are identical and produce a single final good using a Barro (1990)-type production technology, such that:

$$y_t = A k_t^{\alpha} (n_t((1 - \lambda) \times g_t))^{(1-\alpha)} \tag{11}$$

where $A > 0; 0 < \alpha(1 - \alpha) < 1$, is the elasticity of output with respect to capital (labor or publicly-provided infrastructure), with k_t, n_t and $[(1 - \lambda) \times g_t]$ respectively denoting capital, labor, and government infrastructure expenditure inputs at time t. At time t the final good can

[5]For the sake of simplicity, we have assumed that no resources are required to run a bank. But now suppose that the profit function of the bank is redefined as follows: $\max_{L,D} \pi_b = i_{L_t} L_t - i_{D_t} D_t - c D_t$, where c captures the fraction of the deposits spent as resource cost. Given the same constraint as above, profit maximization would imply: $i_{L_t} = \frac{i_{D_t}}{1-\gamma_t - c}$. In essence, if we redefine γ_{1t} as $(\gamma_t + c)$, our results are unaffected.

either be consumed or stored. We assume that producers are able to convert bank loans L_t into fixed capital formation such that $p_t i_{kt} = L_t$, where i_t denotes the investment in physical capital. In each of the respective technologies the production transformation schedule is linear so that the same technology applies to both capital formation and the production of the consumption good and hence both investment and consumption good sell for the same price p. We follow Diamond and Yellin (1990) and Chen *et al.* (2008) in assuming that the goods producer is a residual claimer, that is, the producer uses up the unsold consumption good in a way which is consistent with lifetime value maximization of the firms. This assumption regarding ownership avoids the "unnecessary" Arrow-Debreu redistribution from firms to consumers and simultaneously retains the general equilibrium structure of the models.

The representative firm at any point of time t maximizes the discounted stream of profit flows subject to the capital evolution and loan constraint. Formally, the problem of the firm can be outlined as follows

$$\max_{k_{t+1},n_t} \sum_{i=0}^{\infty} \rho^i [p_t A k_t^{\alpha}(n_t((1-\lambda)\times g_t))^{(1-\alpha)} - p_t w_t n_t - (1+i_{L_t})L_t] \quad (12)$$

$$k_{t+1} \leq (1-\delta)k_t + i_{k_t} \quad (13)$$

$$p_t i_{k_t} = L_t \quad (14)$$

where ρ is the firm owners (constant) discount factor, and δ_k is the (constant) rate of capital depreciation. The firm solves the above problem to determine the demand for labor and investment.

The firm's problem can be written in the following recursive formulation:

$$\begin{aligned} V(k_t) &= \max_{n,k}[p_t A k_t^{\alpha}(n_t((1-\lambda)\times g_t))^{(1-\alpha)} \\ &\quad -p_t w_t n_t - p_t(1+i_{L_t})(k_{t+1}-(1-\delta_k)k_t)] + \rho V(k_{t+1}) \quad (15) \end{aligned}$$

The upshot of the above dynamic programming problem are the following efficiency conditions:

$$\begin{aligned} &k_{t+1} : (1+i_{L_t}) \quad (16) \\ &= \rho\left(\frac{p_{t+1}}{p_t}\right)\left[\alpha A\left(\frac{n_{t+1}(1-\lambda)g_{t+1}}{k_{t+1}}\right)^{(1-\alpha)} + (1+i_{L_{t+1}})(1-\delta_k)\right] \end{aligned}$$

$$n_t : A(1-\alpha)\left(\frac{n_t(1-\lambda)g_t}{k_t}\right)^{(1-\alpha)} k_t = w_t \quad (17)$$

Equation (16) provides the condition for the optimal investment decision of the firm. The firm compares the cost of increasing investment in the current period with the future stream of benefit generated from the extra capital invested in the current period. Assuming δ_k to be unity without any loss of generality, simplifies equation (19) to $(1 + i_{L_t}) = \rho(\frac{p_{t+1}}{p_t})[\alpha A(\frac{n_{t+1}(1-\lambda)g_{t+1}}{k_{t+1}})^{(1-\alpha)}]$. And equation (17) simply states that the firm hires labor up to the point where the marginal product of labor equates the real wage.

2.4. Government

As discussed above we have an infinitely-lived government (monetary authority). The monetary authority finances its expenditure on health and infrastructure: $p_t[\lambda g_t + (1-\lambda)g_t]$ through inflation tax (seigniorage). Formally the government budget constraint can be written as follows:

$$p_t g_t = (M_t - M_{t-1}). \tag{18}$$

The monetary authority targets the inflation rate. Namely, we assume that $\pi_t = \hat{\pi}$, for all t. Note that, with $(1+i_{L_t}) = \rho\hat{\pi}[\alpha A(\frac{n_{t+1}(1-\lambda)g_{t+1}}{k_{t+1}})^{(1-\alpha)}]$, $n_{t+1} = 1$, and $\frac{g_{t+1}}{k_{t+1}}$ constant in steady-state, targeting inflation also implies targeting the nominal interest rate on loans (and hence deposits). Given this policy rule for the rate of inflation, the nominal quantity of money adjusts endogenously to satisfy the demand for money. Using $M_t = \gamma_t D_t$, the government budget constraint in real terms can be rewritten as

$$g_t = \gamma_t d_t \left(1 - \frac{1}{[\Omega_t \times \hat{\pi}]}\right) \tag{19}$$

where Ω_t is the gross growth rate at time t and $\hat{\pi}$ is the gross inflation target. Notable exceptions from the government budget constraint are taxes and government bonds. As suggested earlier, though taxes have been ignored for simplicity, bonds are not included for the following technical reason: In a world of no uncertainty, incorporating government bonds in either the consumer or the bank problem would imply plausible multiplicity of optimal allocations of deposits or loans and government bonds, since the arbitrage condition would imply a relative price of one between deposits or loans and government debt.[6]

[6] One way to incorporate government bonds is to have the financial intermediaries hold government bonds as part of obligatory reserve requirements. Or alternatively, assume that there exists a fixed ratio of government bonds to money.

3. EQUILIBRIUM

A valid perfect-foresight competitive equilibrium for this economy is a sequence of prices $\{p_t, i_{D_t}, i_{L_t}\}_{t=0}^{\infty}$, allocations $\{c_{t+1}, n_t, i_{k_t}\}_{t=0}^{\infty}$, stocks of financial assets $\{m_t, d_t\}_{t=0}^{\infty}$, and policy variables $\{\gamma_t, g_t\}_{t=0}^{\infty}$ such that:

• Taking $g_t, \gamma_t, p_t, p_{t+1}, i_{D_{t+1}}$ and w_t, the consumer optimally chooses ϕ_t such that (5) and (6) holds;
• Banks maximize profits, taking, i_{L_t}, i_{D_t}, and γ_t as given and such that (10) holds;
• The real allocations solve the firm's date–t profit maximization problem, given prices and policy variables, such that (16) and (17) holds;

The money market equilibrium conditions: $m_t = \gamma_t d_t$ is satisfied for all $t \geq 0$;

• The loanable funds market equilibrium condition: $p_t i_{k_t} = (1 - \gamma_t) D_t$ where the total supply of loans $L_t = (1 - \gamma_t) D_t$ is satisfied for all $t \geq 0$;
• The goods market equilibrium condition require: $c_t + i_{k_t} + g_t = (1 - \phi_t) A k_t^{\alpha} (n_t((1 - \lambda) \times g_t))^{(1-\alpha)}$ is satisfied for all $t \geq 0$;
• The labor market equilibrium condition: $(n_t)^d = 1$ for all $t \geq 0$;
• The government budget constraint (equation (18)) is balanced on a period-by-period basis;
• d_t, i_{d_t}, i_{L_t}, and p_t must be positive at all dates.

4. GROWTH DYNAMICS

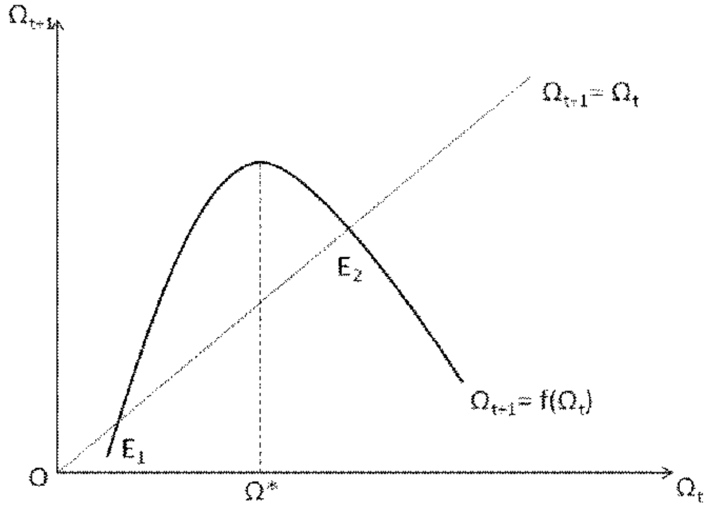
In this section, we analyze the growth dynamics obtained from this model. Using equations (6), (13), (14), (17) and (19) yields the following relationship between Ω_{t+1} and Ω_t, i.e., $\Omega_{t+1} = f(\Omega_t)$:

$$\Omega_{t+1} = \frac{2A(1-\alpha)(1-\gamma)(1-\sigma)\hat{\pi}\Omega_t \left(2^{-1/\alpha} \left(\frac{A(\alpha-1)(\lambda-1)(-b\hat{\pi}\Omega_t + \sigma\hat{\pi}\Omega_t - \hat{\pi}\Omega_t + \sqrt{\hat{\pi}\Omega_t((b-\sigma+1)^2 \hat{\pi}\Omega_t - 4b\gamma\lambda(\sigma-1)(\hat{\pi}\Omega_t - 1))})}{b\lambda\hat{\pi}\Omega_t} \right)^{\frac{1}{\alpha}} \right)^{1-\alpha}}{b\hat{\pi}\Omega_t - \sigma\hat{\pi}\Omega_t + \hat{\pi}\Omega_t + \sqrt{\hat{\pi}\Omega_t((b-\sigma+1)^2\hat{\pi}\Omega_t - 4b\gamma\lambda(\sigma-1)(\hat{\pi}\Omega_t - 1))}} \tag{20}$$

The function $f(\Omega)$ satisfies (a) $f'(\Omega) > 0$ for $\Omega < \Omega^* \left[= \frac{\gamma(\sigma-1)}{(\gamma(\sigma-1) - \frac{(\alpha-1)\alpha(b-\sigma+1)^2}{b(1-2\alpha)^2\lambda})\hat{\pi}} \right]$; (b) $f'(\Omega) = 0$ for $\Omega = \Omega^*$; (c) $f'(\Omega) < 0$ for $\Omega > \Omega^*$; (d) $\lim_{\Omega \to 0} f'(\Omega) = \infty$; (e) $\lim_{\Omega \to \infty} f'(ga) = -\infty$; and (f) $\lim_{\Omega \to \infty} f(\Omega) = B$ where

$$B = \frac{2A(1-\alpha)(1-\gamma)(1-\sigma) \left(2^{-1/\alpha} \left(- \frac{A(\alpha-1)(\lambda-1)(b-\sigma+\sqrt{(b-\sigma+1)^2 - 4b\gamma\lambda(\sigma-1)}+1)}{b\lambda} \right)^{\frac{1}{\alpha}} \right)^{1-\alpha}}{b - \sigma - \sqrt{(b-\sigma+1)^2 - 4b\gamma\lambda(\sigma-1)} + 1}.$$

FIG. 1. Multiple Equilibria with Endogenous Fluctuations (and Possible Chaos)

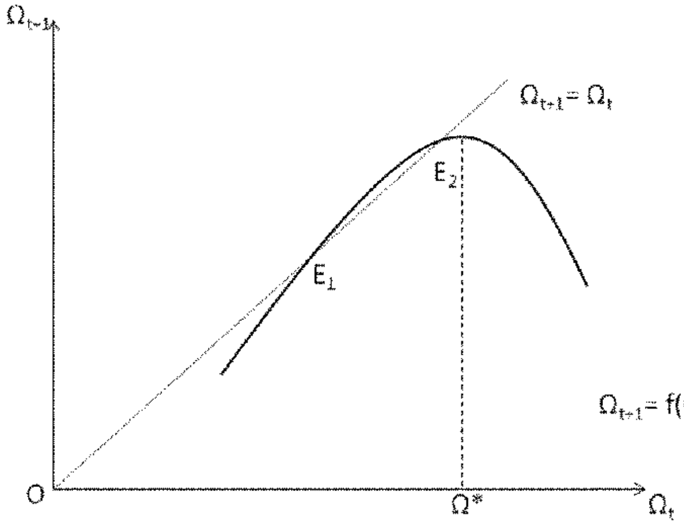


Depending upon the values of $A, \alpha, \gamma, \sigma, \lambda$ and b and given the properties of $f(\Omega)$ we can have two different types of balanced growth paths, yielding two distinct forms of the high-growth equilibrium equilibria, as depicted in Figures 1 and 2.

Intuitively, the inverted u-shaped nature of the f locus is easy to understand given the existence of a positive effect and a negative effect of an increase in Ω_t. As Ω_t increases, higher seigniorage revenue is created which, in turn, raises the ratio of real government expenditure to real wage. This translates into higher growth rate via the higher available resources for productive public infrastructure relative to real wage, given $(1-\lambda)$. However, higher value of the ratio of real government expenditure to real wage also implies higher availability of resources for public expenditure on health relative to real wage, given λ. Due to the complementary nature of private and public investment in health in the longevity production function, private investment in health increases at the cost of ratio of deposits to real wage and hence loans and investment, causing a negative effect on growth.[7] The positive effect dominates at lower levels of Ω_t resulting in the

[7]Note, if the consumer was only working a fraction of the total labor-time endowment, a higher value of Ω_t would increase the ratio of real government expenditure to real wage, and, hence, ϕ_t, which, in turn, would raise the survival probability and the labor time supplied via an income effect to compensate for the loss in labor income due to the higher private investment in health. A higher value of the labor time supplied would

FIG. 2. Multiple Equilibria without Endogenous Fluctuations



f locus to be positively-sloped, but after Ω^* the negative effect dominates to cause the f locus to become negatively-sloped.[8] Note that high (low) values of A or low (high) values of $\alpha, \gamma, \sigma, \lambda$ and b moves the function up (down) to yield situations plotted in Figures 1 and 2 respectively. Clearly then, the existence of the equilibria depends critically on the values of the above set of parameters, which, in turn, needs to be such that the function intersects the 45 degree line. Specifically speaking, we can draw the following set of conclusions:

- The low-growth (high-growth) equilibrium is unstable (stable) and locally determinate (locally indeterminate). The low-growth (high-growth)

then add to the already existing direct positive effect on Ω_{t+1}, implying that the positive and negative effects would cancel out now at a relatively higher value of Ω_t relative to what we have under the case when $n_t = 1$.

[8]Recall that, $c_{t+1} = (1 + r_{D_{t+1}})d_t$, where r_{D_t} is the net real interest rate on deposits. Using equation (10), we can show that $1+r_{D_t} = (1-\gamma_t)(1+r_{L_t}) + \frac{\gamma_t}{1+\pi_t}$. Moreover, given that $(1 + r_{L_t}) = \rho[\alpha A \left(\frac{n_{t+1}(1-\lambda)g_{t+1}}{k_{t+1}}\right)^{(1-\alpha)}]$, a higher value of $\frac{g_{t+1}}{k_{t+1}}$ would translate into a higher value of $(1 + r_{D_t})$ and a lower value of d_t, the latter as shown in equation (6). Given this, the relationship between second-period consumption, and, hence, utility, and the ratio of real government expenditure to capital (or real wage) would be captured by an inverted u-shaped curve. Note, the expected utility function will also have the same shape, but in addition to the positive effect on utility via the rise in the real interest rate on deposits following a rise in the ratio of real government expenditure to capital, there will be another positive effect via the increased probability of survival.

equilibrium is unstable (stable) under perfect foresight because the f locus intersects the 45 degree line from below (above). Further, although k_t is a state variable and cannot jump, $\Omega \equiv k_t/k_{t-1}$ is not a state-variable and, hence, can jump. This implies that there is infinitely many rational expectations paths to the high-growth and stable equilibrium from any initial given value k_1. Thus, the stable equilibrium in this economy suffers from the problem of indeterminacy;

• As portrayed in Figure 1, the high-growth and stable equilibrium can also be characterized by endogenous fluctuations, given that the slope of the f locus is negative at point E_2. Let us now consider the intuition behind the endogenous fluctuations. Consider the region around E_2 where the negative effect (discussed above) dominates, and suppose that the economy starts off at a relatively low level of Ω_t. Then in that period, the size of seigniorage revenue is low and, hence, the public investment in longevity (as well as infrastructure) will be relatively low. As a result, because of the complementary nature of the private and public inputs in longevity, the private investment in longevity by the young will be relatively low, implying that the young-age deposits will be higher, causing next period's growth to be higher. This, however, is not guaranteed as indicated in Figure 2, where the slope of the f locus is still positive. Recall that the position of the f locus depends on the values of the structural parameters of the model $(A, \alpha, \gamma, \sigma, \lambda$ and $b)$;

• As in Bhattacharya and Qiao (2007), our framework too can yield chaotic behavior of growth rate around the high-growth equilibrium, provided the following set of conditions are satisfied: As outlined in Mitra (2001), the map f is required to be a continuous function from X to X, where the state space X is an interval on the non-negative part of the real line, with (X, f) defining the dynamical system. Further, the map f must be unimodal with a maximum at Ω^* with $f(\Omega^*) > \Omega^*$ and the high-growth equilibrium at E_2 must ensure that the steady-state level of growth rate corresponding to the high-growth equilibrium must exceed Ω^* (see Mitra (2001) and Bhattacharya and Qiao (2007) for further details). Formally, if (X, f) is a dynamical system, and if f satisfies $f^2(\Omega^*) < \Omega^*$ and $f^3(\Omega^*) < \Omega^H$ (the steady-state gross growth rate at the high-growth equilibrium), then (X, f) exhibits topological chaos (Mitra, 2001). Understandably, we can only have chaos under the situation depicted in Figure 1, once the above set of conditions have been satisfied. Purely from an economic perspective, chaotic behavior implies the possibility of dramatic reversals in life expectancy, which is in line with experiences of many countries over the last three decades (see Bhattacharya and Qiao (2007) for further details).

5. CONCLUSION

There exits a new, but growing, literature that incorporates the role of mortality in dynamic general equilibrium models. While some of these studies endogenize the probability of survival by assuming the mortality rate to depend on public health capital, other studies emphasize the role of per-capita income or consumption expenditure or even private spending on health in affecting longevity. To the best of our knowledge, Bhattacharya and Qiao (2007) and Agénor (2009) are the only two studies which takes into account the simultaneous role of private and public health expenditures in affecting the probability of survival. Against this backdrop, our paper develops a monetary endogenous growth version of the model designed by Bhattacharya and Qiao (2007) and analyzes the growth dynamics that emerges from this framework. Besides the endogenous longevity which depends on the complimentarity of private and public health expenditures, as in Bhattacharya and Qiao (2007), the growth process is endogenized along the lines of a Barro (1990)-type production structure. Following the huge existing literature of incorporating money in general equilibrium models, money is introduced by assuming that banks, otherwise operating in a perfectly competitive environment, are obligated to hold a fraction of the deposits as cash reserve requirements. Finally, growth dynamics are introduced in the overlapping generations framework by assuming an inflation targeting monetary authority, instead of the standard assumption of a monetary authority following a money-growth rule. Given this framework, we show that multiple equilibria emerges, where the low-growth (high-growth) equilibrium is unstable (stable) and locally determinate (locally indeterminate). Furthermore, endogenous fluctuations and even chaotic behavior of the growth rate could also be observed around the high-growth equilibrium, under a certain set of conditions.

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