

On Collusion and Industry Size

Marc Escrihuela-Villar*

Universitat de les Illes Balears
E-mail: marc.escrihuela@uib.es

and

Jorge Guillén†

ESAN
E-mail: jguillen@esan.edu.pe

In this paper we investigate the connection between the number of competitors and the sustainability of collusion within the context of a infinitely repeated symmetric Cournot model where only a subset of firms cooperate. We show that, in our model, an increase in the number of cartel firms may increase collusion likelihood by diminishing the negative effects for collusion of the existence of a competitive fringe. Also, we show that an increase in the number of fringe firms makes collusion harder to sustain.

Key Words: Collusion; Sustainability; Fringe.

JEL Classification Numbers: L11, L13, L41, D43.

1. INTRODUCTION

It is a standard view to link the likelihood of collusion to several structural indexes such as the number of firms in the industry. For instance, it is commonly believed that as the number of suppliers increases attaining a collusive agreement becomes more difficult. There are a variety of reasons to expect that cartel success is negatively related to the number of firms in the industry like that a large number of firms creates coordination problems or increases the likelihood that there exists a firm willing to cheat. This seems to be particularly relevant as agreements are illegal and there-

* Corresponding author. Mailing address: Departamento de Economía Aplicada. Edificio Jovellanos Ctra. Valldemossa km 7.5 07122 Palma de Mallorca Balears - España.

† Mailing address: Escuela de Administración de Negocios para Graduados (ESAN). Alonso de Molina 1652, Monterrico, Surco, Lima - Perú

fore have to be kept secret or are indeed completely tacit. In this respect, the tacit collusion literature has shown that the critical threshold for the discount factor above which collusion is sustainable increases (and consequently collusion becomes less likely) as the number of firms increases. The intuition is that, with more firms, each firm gets a lower share of the pie from colluding, thus increasing the gains from cheating as well as reducing the attractiveness of long-term collusion (see for example Osborne (1976) and Vives (1999)). In accordance with this structural view, a concentrated oligopoly has been often held as a necessary condition whenever a collusive agreement is absent.¹ The empirical results, however, are ambivalent in this regard since a large number of firms have also been involved in cartel cases from the mid-80s (see for instance Levenstein and Suslow (2006) and Motta (2009)). On the other hand, it can also be observed that, in practice, many collusive agreements do not involve all firms in the industry.² Despite this empirical evidence, the Industrial Organization analysis of tacit collusion in quantity-setting supergames has usually focused on the symmetric subgame perfect Nash equilibrium —henceforth, SPNE— that maximizes industry profits (see for example the seminal paper by Friedman (1971) or Rothschild (1999)).³ Consequently, our central purpose in this paper is to determine which is the effect on cartel sustainability of a variation on the number of cartel and fringe firms when collusion involves only a subset of firms.

To that extent, we develop a multi-period oligopoly model with homogeneous, quantity-setting firms, an exogenous subset of which are assumed to collude. We assume that the remaining (fringe) firms choose their output levels non-cooperatively. We use SPNE as solution concept. It is well known that this repeated game setting exhibits multiple SPNE collusive agreements, thus to select among those equilibria, we adopt the particular criterion of restricting strategies to grim “trigger strategies”. Our main results are (i) that an increase in the number of cartel firms might make collusion easier to sustain by alleviating the harm that the existence a fringe represents for the agreement and (ii) an increase in the number of fringe firms makes generally collusion more difficult to sustain.

¹An example would be the UK tractors decision (1992) where the European Commission explicitly took account of the high concentration in the market of agricultural tractors in the UK (see European Commission, UK Agricultural Tractor Registration Exchange, 1992).

²A significant example is the citric acid industry where three North-American and five European firms were fined for fixing prices and allocating sales in the worldwide market. Their joint market share was around 60 percent (see Levenstein and Suslow (2006)).

³Among the few exceptions are Escrihuela-Villar (2008) that analyzes the price effects of horizontal mergers, Escrihuela-Villar (2009) that considers how the sequence of play between the cartel and the fringe affects cartel stability or Bos and Harrington (2010) that endogenize the composition of a cartel with heterogeneous production capacities.

Using our model, we argue then that collusion likelihood may either increase or decrease due to a variation in the industry size. As a consequence, the interpretation that a smaller industry size fosters collusion, as traditionally used from an antitrust viewpoint, can be misleading since it should be also considered whether there exists or not a competitive fringe. In this sense, the theoretical possibility that a rising number of firms can make collusion easier to sustain is an application of the “topsy-turvy” principle of supergame theory that states that any underlying market condition that makes competitive behavior possible and credible can, by lowering (punishment profits), actually promote collusion. From a theoretical perspective, our analysis can also be related to the strand of the literature that considers the sustainability of collusion when the number of firms grows. For instance, with capacity constraints, Brock and Scheinkman (1985) find that since the cutoff of the discount factor depends non-monotonically on per firm capacity to produce, for a low number of firms, the ability to collude increases with an additional firm, but eventually, it falls again. Harrington (1991) and MacLeod (1987) obtain that joint profit maximum can always be sustained in equilibrium as long as the height of the entry barriers is positive and with free entry respectively. However, differently from these papers and according to the empirical evidence, we assumed that not all firms participate in the agreement.

The rest of the paper is structured as follows. In section 2, we present the model. In section 3, we analyze the effect of a variation on the number of cartel and fringe firms on cartel sustainability. Section 4 tests the robustness of our results using an optimal punishment—the stick-and-carrot strategies proposed by Abreu (1986,1988)—and establishes that the main results continue to hold. We conclude in section 5. All proofs are grouped together in the appendix.

2. THE MODEL

We consider an industry with $N > 2$ firms, indexed by $i = 1, \dots, N$. Firms simultaneously produces a quantity of a homogeneous product where costs are assumed to be linear and normalized to zero. The industry inverse demand is given by the piecewise linear function $p(Q) = \max(0, a - Q)$ where $Q = \sum_{i=1}^N q_i$ is the industry output, p is the output price and $a > 0$. We assume that $K \in [2, N)$ firms, indexed by $k = 1, \dots, K$ —henceforth, cartel firms— behave cooperatively so as to maximize their joint profits. The remaining $(N - K)$ firms constitute the fringe and choose their output in a non-cooperative way. We assume that only one cooperative group is

formed and we take K as exogenous.⁴ This assumption is based on the fact that cartels often involve an agreement between firms which can easily coordinate with each other (e.g. because they are based in the same country or have the same business culture). The fringe consists of foreign firms or new entrants that could not coordinate their behavior with the cartel firms even if they wish so.⁵ We also assume that firms compete repeatedly over an infinite horizon with complete information (i.e. each of the firms either fringe or cartel observes the whole history of actions) and discount the future using a discount factor $\delta \in (0, 1)$. Time is discrete and dates are denoted by $t = 1, 2, \dots$. In this framework, a pure strategy for firm k is an infinite sequence of functions $\{S_k^t\}_{t=1}^\infty$ with $S_k^t : \sum^{t-1} \rightarrow \mathcal{Q}$ where \sum^{t-1} is the set of all possible histories of actions (output choices) of all cartel firms up to $t - 1$, with typical element σ_j^τ , $j = 1, \dots, K$, $\tau = 1, \dots, t - 1$, and \mathcal{Q} is the set of output choices available to each cartel firm. Following Friedman (1971), we restrict our attention to the case where each cartel firm is only allowed to follow grim trigger strategies. In words, these strategies are such that cartel firms adhere to the collusive agreement until there is a defection, in which case they revert forever to the static N -firm Nash equilibrium. Since firms are symmetric, each cartel firm produces the same amount of output that we denote by q . The output corresponding to noncooperative behavior is denoted by q_n . Since we restrict attention to trigger strategies, $\{S_k^t\}_{t=1}^\infty$ can be specified as follows. At $t = 1$, $S_k^1 = q$, while at $t = 2, 3, \dots$

$$S_k^t(\sigma_j^\tau) = \begin{cases} q & \text{if } \sigma_j^\tau = q \text{ for all } j = 1, \dots, K \text{ and } \tau = 1, \dots, t - 1 \\ q_n & \text{otherwise} \end{cases} \quad (1)$$

Regarding fringe firms, their optimal response consists of maximizing their current period's payoff. We denote the output produced by each fringe firm by q_f . We denote by $\Pi^c(N, K)$ and $\Pi^f(N, K)$ the profit function of a cartel firm and that of a fringe firm respectively. As shown by Friedman (1971), cartel firms colluding in each period can be sustained as a SPNE of the repeated game with the strategy profile (1) if and only if for given values of N, K and δ , the following condition is satisfied

$$\frac{\Pi^c(N, K)}{1 - \delta} \geq \Pi^d(N, K) + \frac{\delta \Pi(N)}{1 - \delta} \quad (2)$$

where $\Pi^d(N, K)$ denotes the profits attained by an optimal deviation from the collusive output, and $\Pi(N)$ denotes the Nash equilibrium profits. If δ

⁴It is a well-known result that if we endogeneize cartel formation using the concept of cartel stability by d'Aspremont, et al. (1983) only cartels containing just over half the firms in the industry are stable (see for instance Shaffer (1995) and Donsimoni et al. (1986)).

⁵For instance, in the citric acid industry the fringe included a variety of minor companies based in Eastern Europe, Russia and China (see Levenstein and Suslow (2006)).

exceeds a certain critical level, (2) is not a binding constraint. For given values of N and K , we denote by $\delta_K(N, K)$ the minimum δ required for the condition (2) to be satisfied. Then, a cartel of K firms is said to be sustainable if $\delta \geq \delta_K(N, K)$ and $\delta_K(N, K) \in (0, 1)$. It can be verified that the above mentioned critical level of the discount factor is $\delta_K(N, K) = \frac{(1-K)(1+N)^2}{4K^3+(1+N)^2+3K(1+N)^2-4K^2(3+2N)}$.

3. COLLUSION AND INDUSTRY SIZE

In this section, we analyze the effect of a change in the industry size on the critical discount factor above which full collusion is sustainable. The existing literature (see for instance Shapiro (1989) or Vives (1999)) concentrates on analyzing the effect of a change in N on the minimal threshold for collusion which is valid whenever it is assumed that all firms participate in the collusive agreement. However, this analysis is insufficient when the agreement involves only a part of the industry. Consequently, since we assumed that we have cartel and fringe firms, we can extend the analysis and use our model to study the effect of a variation of the number of cartel firms with either constant number of fringe firms or with constant number of total firms and a variation of the number of fringe firms. In other words, in this section we consider three different cases: the effect on $\delta_K(N, K)$ of (i) a variation of N with $(N - K)$ fixed, (ii) a variation of K with N fixed and (iii) a variation of N with K fixed. In the first case a convenient change of variable is $N - K \equiv F$ (where F represents the number of fringe firms) to calculate $\frac{\partial \delta_K(N, F)}{\partial N}$. We obtain the following result.

PROPOSITION 1. *For all $\delta_K(N, F) \in (0, 1)$, $\frac{\partial \delta_K(N, F)}{\partial N}$ is negative when N is small enough and positive when N is large enough.*

When the number of fringe firms is fixed, an increase in the number of cartel firms makes collusion easier to sustain when N is relatively small. The reverse is true when N is large enough. In other words, whenever cartel firms face a fringe an increase in the number of cartel firms helps collusion when the cartel is relatively small. It is easy to see that this results holds because, since $\delta_K(N, K)$ can be rewritten like $\delta_K(N, K) = \frac{1 - \frac{\Pi^c(N, F)}{\Pi^d(N, F)}}{1 - \frac{\Pi(N)}{\Pi^d(N, F)}}$, a variation of N has two different effects. First, $\frac{\Pi^c(N, F)}{\Pi^d(N, F)}$ decreases when N increases because deviation profits increase more than profits from being in the cartel of K firms. This would increase $\delta_K(N, K)$. Second, as N increases, $\frac{\Pi(N)}{\Pi^d(N, F)}$ also decreases. This second effect would decrease $\delta_K(N, K)$, representing that the punishment becomes more severe. When

N is small enough, the second effect dominates the first one. Roughly speaking, the intuition behind Proposition 1 is as follows. Cartel firms cut production in order to rise price while fringe firms produce more than cartel firms. Consequently, the larger the number of cartel firms cutting production, the less harmful is for the agreement the existence of the fringe and the cartel is, therefore, easier to sustain. This is true only up to a point. When the number of cartel firms is relatively large, the free riding effect of fringe firms is proportionally less important and an increase in the number of cartel firms makes collusion more difficult to sustain. In the second case, we also analyze an increase in cartel size but for a given total number of firms in the industry.

PROPOSITION 2. *For all $\delta_K(N, K) \in (0, 1)$, $\frac{\partial \delta_K^S(N, K)}{\partial K}$ is negative.*

When the number of cartel firms increases collusion is easier to sustain. The intuition behind is that a variation of K has two different effects. First, $\frac{\Pi^c(N, K)}{\Pi^d(N, K)}$ decreases with K since deviation profits increase more than cartel profits. Second, $\frac{\Pi(N)}{\Pi^d(N, K)}$ also decreases with K given that $\Pi(N)$ does not depend on K and deviation profits increase with K . Then, the result comes from the fact that the second effect dominates the first one. Intuitively, for given values of N , as K increases cartel firms face a harsher punishment in the event that deviation from the collusive agreement occurs, and thus collusion is easier to sustain. Regarding the third case, $\frac{\partial \delta_K(N, K)}{\partial N}$ evaluates a variation on the number of fringe firms for a given cartel size. We can establish the following result.

PROPOSITION 3. *For all $\delta_K(N, K) \in (0, 1)$, $\frac{\partial \delta_K(N, K)}{\partial N}$ is positive.*

When the number of cartel firms is fixed, an increase in the number of fringe firms makes collusion more difficult to sustain when decision between the cartel and the fringe is simultaneous. Intuitively, an increase in the number of firms that produce beyond the production cut agreement hinders cartel sustainability.

4. EXTENSIONS

To test the robustness of our results it is natural to consider a set of strategies that are less grim than the trigger strategies. We consider here the two-phase output path (with a “stick-and-carrot” pattern) presented by Abreu (1986, 1988). As in section 2, the strategy space consists of a sequence of decisions rules, describing each player’s action as a function of the past history of the play. Then, a pure strategy for firm k is an

infinite sequence of functions $\{S_k^t\}_{t=1}^\infty$ with $S_k^t : \Sigma^{t-1} \rightarrow \mathcal{Q}$ where Σ^{t-1} is the set of all possible histories of actions of all firms up to $t-1$, with typical element σ_j^τ , and \mathcal{Q} is the set of output choices available to each firm. We assume that if a deviation from the collusive agreement occurs, then all firms expand their output for one period so as to drive price below cost and return to the most collusive sustainable output in the remaining periods, provided that every player went along with the first phase of the punishment. Let q and q^p denote the output produced by each firm in a collusive and in a punishment phase respectively. $\{S_k^t\}_{t=1}^\infty$ can be specified as follows. At $t=1$, $S_k^1 = q$, while at $t=2, 3, \dots$

$$S_k^t(\sigma_j^\tau) = \begin{cases} q & \text{if } \sigma_j^\tau = q \text{ for all } j = 1, \dots, K, \tau = 2, \dots, t-1 \\ q & \text{if } \sigma_j^\tau = q^p \text{ for all } j = 1, \dots, K, \tau = t-1 \\ q^p & \text{otherwise.} \end{cases} \quad (3)$$

Under the conditions specified in Abreu (1986), each cartel firm producing the quantity q in each period can be sustained as a SPNE of the repeated game with the strategy profile (3). As in section 2, let $\delta_K(N, K)$ denote the minimum discount factor required for a cartel of K firms maximizing their joint profits at equilibrium and $F \equiv N - K$.

PROPOSITION 4. *For all $\delta_K(N, K) \in (0, 1)$ $\frac{\partial \delta_K(N, F)}{\partial N}$ is negative, $\frac{\partial \delta_K(N, K)}{\partial N}$ is positive and $\frac{\partial \delta_K(N, F)}{\partial K}$ is negative.*

In words, an increase in the number of cartel firms makes collusion easier to sustain both with a fixed number of fringe firms and with a fixed number of total firms. Conversely, an increase in the number of fringe firms makes collusion more difficult to sustain. Thus, we have studied a more severe punishment and established that, with simultaneous decision, the results of section 2 continue to hold.⁶

5. CONCLUDING COMMENTS

Two stylized facts about collusion are that (i) some of the best known examples of cartels involve only a part of the industry and (ii) often cartels involve a large number of firms. However, the relationship between cartel sustainability and the industry size when not all firms collude is a problem that we believe has not been extensively considered. Consequently, we develop in this paper a theoretical framework to study the effects of a

⁶The only exception is that with Abreu's penal code, an increase in the cartel size with a fixed number of fringe firms helps cartel sustainability regardless of the number of firms in the industry. We recall, however, that we have in this section focused on the case where the number of firms (total and cartel firms) is relatively small.

variation in the number of firms on cartel sustainability. Against the standard view that suggests a negative link between the likelihood of collusive behavior and the number of firms in the industry, we show that the critical discount factor above which joint profit maximization is sustained may decrease with the number of cartel firms when not all firms participate in the agreement. The main intuition is that an increase in the number of cartel firms minimizes the negative impact of the fringe in cartel sustainability. We conclude, then, that the existence of a competitive fringe could be a possible explanation for the collusive agreements involving many firms.

Finally, the limited context of the present model is acknowledged: to analyze real-world cases of cartels, firms' capacities, cost asymmetries or a wider range of demand functions should also be considered. We believe that those are subjects for future research.

APPENDIX

Proof of Proposition 1. With the corresponding change of variable

$$\delta_K(N, F) = \frac{(1+N)^2(-1-F+N)}{(N-2F-1)(1-2F^2+F(-5+N)+N(6+N))}.$$

Thus,

$$\begin{aligned} & \frac{\partial \delta_K(N, F)}{\partial N} \\ = & \frac{4(1+N)(-2F^4 + 4F^3(-2+N) + (-1+N)^3 - 2F^2(5 + (-7+N)N) - 2F(3 + N(-5+4N)))}{(-1-2F+N)^2(1-2F^2+F(-5+N)+N(6+N))^2}. \end{aligned}$$

It can be easily proved that $\delta_K(N, F) \in (0, 1)$ when $N \geq 1 + F(3 + F)$.

Then, we only have to check that $\frac{\partial \delta_K(N, F)}{\partial N} < 0$ when

$N < f(F) \equiv \frac{1}{3}(3+8F+2F^2 + \frac{(2F(1+F)(9+2F(4+F)))}{h}) + h$ and $\frac{\partial \delta_K(N, F)}{\partial N} > 0$ when $N > f(F)$

where $h \equiv (3\sqrt{3}\sqrt{F^2(1+F)^3(2+F)^2(27+F(27+8F))} + F(1+F)^2(54 + F(81 + 44F + 8F^2)))^{\frac{1}{3}}$ and $f(F) > 1 + F(3 + F) \forall F > 0$. ■

Proof of Proposition 2. It is immediate to verify that

$$\frac{\partial \delta_K(N, K)}{\partial N} = -\frac{((8(-1+K)K^2(-2+K-N)(1+N))}{((1-2K+N)^2(1+N+K(5-2K+3N)))^2} > 0 \forall N > K. \quad \blacksquare$$

Proof of Proposition 3. The result follows from the fact that

$\frac{\partial \delta_K(N, K)}{\partial K} = -\frac{4(1+N)^2(1-2K(3+(-3+K)K)+2N+2(-2+K)KN+N^2)}{((1-2K+N)^2(1+N+K(5-2K+3N)))^2}$. The numerator would be negative only whenever

$$N < -1 - (-2 + K)K + \sqrt{K(2 + (-2 + K)K^2)} < K.$$

■

Proof of Proposition 4. The conditions specified in Abreu (1986) under which all cartel firms producing q constitutes a SPNE are the following $\Pi_p^c(N, K) + \frac{\delta}{1-\delta}\Pi^c(N, K) = 0$ where $\Pi_p^c(N, K)$ denote the profits obtained

by a cartel firm when producing q^p , and $\frac{1}{1-\delta}\Pi^c(N, K) \geq \Pi^d(N, K)$. We denote by δ_a the minimum discount factor needed for the first inequality to be satisfied. Equivalently, we denote by δ_b the minimum discount factor needed for the second inequality to be satisfied. Then, it can be verified that

$\delta_a = \frac{4K(-1+K-N)}{-1+K(-2+3K-4N)}$, and $\delta_b = \frac{(1-2K+N)^2}{(1+N)^2}$. The minimum discount factor required for a cartel of size K maximizing their joint profits at equilibrium (δ_K) will be given by the envelope from above of δ_a and δ_b , that is $\delta_K = \max\{\delta_a, \delta_b\}$. Consequently, the cutoff of the discount factor is $\delta_K(N, K) = \frac{4K(-1+K-N)}{-1+K(-2+3K-4N)}$ for $N < 12$ while for $N \geq 12$, $\delta_K(N, K) = \frac{4K(-1+K-N)}{-1+K(-2+3K-4N)}$ if $K < \frac{7+5N+\sqrt{25+N(46+25N)}}{12}$ and $\delta_K(N, K) = \frac{(1-2K+N)^2}{(1+N)^2}$ if $K \geq \frac{7+5N+\sqrt{25+N(46+25N)}}{12}$. We denote $f(N) \equiv \frac{7+5N+\sqrt{25+N(46+25N)}}{12}$. Then, since $\frac{f(N)}{N} \in (0, \frac{5}{6})$ and for instance $\frac{f(N)}{N} \leq 0.85 \forall N < 58$, we concentrate, for simplicity, on the case where $\delta_K(N, K) = \frac{4K(-1+K-N)}{-1+K(-2+3K-4N)}$.⁷ Then, since $N - K \equiv F$, $\frac{\partial \delta_K(N, F)}{\partial N} = -\frac{4(-1+(N-F)^2(1+F))}{(1+N)^2+2(-1+N)F-3F^2} < 0$. Also, $\frac{\partial \delta_K(N, K)}{\partial N} = \frac{4(-1+K)^2K}{(1+K(2-3K+4N))^2} > 0$ and $\frac{\partial \delta_K(N, K)}{\partial K} = -\frac{4(-1+K)(1+K(-1+N)+N)}{(1+K(2-3K+4N))^2} < 0$. ■

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⁷The empirical evidence on partial cartels suggests that collusion may involve a large number of firms. However, it seems plausible to consider the case where the number of firms is below 60 and cartel market share below 85%. For instance in Levenstein and Suslow (2006) the characteristics of collusion in each of the U.S. price-fixing samples specify that the mean number of firms ranges across the samples from 7.25 to 29.1 with cartel market shares between 50 and 75%.

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