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Optimal Taxation under Income Uncertainty

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Optimal taxation under income uncertainty has been extensively developed in expected utility theory, but it is still open for inseparable utility function between income and effort. As an alternative of decision-making under uncertainty, prospect theory (Kahneman and Tversky (1979), Tversky and Kahneman (1992)) has been obtained empirical support, for example, Kahneman and Tversky (1979), and Camerer and Lowenstein (2003). It is beginning to explore optimal taxation in the context of prospect theory, for example, Oswald (1983), Tuomala (1990) in conventional setting without utility interdependence, and Kanbur, Pirttila, and Tuomala (2008) for separable value functions between income and effort. It is challenging in the prospect theory to treat with optimal taxation for inseparable value function between income and effort. In this paper, we model taxation under income uncertainty by sufficiently considering government's risk aversion and individuals' loss aversion. We obtain its sufficient condition for the first order approach to general value functions including inseparable value function between income and effort, hence generalizing Oswald (1983), Tuomala (1990) to optimal taxation with utility interdependence, and Kanbur, Pirttila, and Tuomala (2008) to inseparable value functions between income and effort.

Key Words: Optimal taxation; Income uncertainty; Moral hazard; Expected utility theory; Prospect theory; Risk aversion; Loss aversion.
JEL Classification Numbers: D81, H21.

1. INTRODUCTION

Optimal taxation under income uncertainty was characterized, for example, Mirrlees (1971, 1974, 1976), Atkinson and Stiglitz (1976, 1980), Golosov, Kocherlakota and Tsyvinski (2003), where the government (principal) has access to taxation, ex-ante identical individuals (agents) have heterogenous skills level unobservable to others, and the agent's utility depends randomly on effort under income uncertainty. However, the aim of taxation is to transform resources from the highly skilled to the less skilled

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in an efficient way, given that incomes, not skills, are observable. In most principal-agent analysis, including Mirrlees (1971, 1974, 1976), Atkinson and Stiglitz (1976, 1980), Golosov, Kocherlakota and Tsyvinski (2003), the expected utility theory (EUT henceforth) is extensively applied to describe agents' behavior under income uncertainty.

In the EUT, agent's utility, in which his/her preference is exhibited with regard to uncertain outcomes, is represented by a function of payoff, probability of occurrence and risk aversion. Different utility conforms to different preferences. As an alternative of decision-making under uncertainty, prospect theory (PT henceforth, Kahneman and Tversky (1979), Tversky and Kahneman (1992)) has been obtained empirical support, for example, Kahneman and Tversky (1979), and Camerer and Lowenstein (2003). Different from the EUT, individuals' utilities, when applying the PT, depend on how the outcome deviates from reference points, rather than directly on the absolute value of the outcomes. Individuals are loss-averse. In addition, PT has been applied elsewhere, for example, Maskin and Riley (1984) and Dai (2010) on efficient auction.

As an alternative to Mirrlees (1971, 1974, 1976), Atkinson and Stiglitz (1976, 1980), and Golosov, Kocherlakota and Tsyvinski (2003), we model taxation, in this paper, under income uncertainty by introducing elements of the PT in accordance with the emerging empirical consensus, that is, the individuals' utilities depend on their gains and losses, however, they are more sensitive to losses than to gains. Thus we obtain the sufficient condition for the first order approach (FOA henceforth) to general value function over income and effort.

As a keynote to the PT, we must ask what determines the reference income level for the individuals to assess losses and gains? One possibility is for the individuals to compare their ex-post outcome relative to the means of the outcome for other individuals, close to models of optimal taxation with utility interdependence (or envy). Those were explored in the conventional setting without income uncertainty by Oswald (1983) and Tuomala (1990). This paper will analyze taxation with utility interdependence under income uncertainty, hence generalizing Kanbur, Pirttila, and Tuomala (2008). If the reference points could be a past consumption level, Kanbur, Pirttila, and Tuomala (2008) and this analysis will resemble habit formation models, for example, Ljunqvist and Uhlig (2000) and Carroll, Overland and Weil (2000).

Though Kanbur, Pirttila, and Tuomala (2008) explored optimal taxation under income uncertainty when individuals behave according to the PT, they obtained sufficent condition required for the FOA to separable functions between income and effort, and found that optimal marginal tax rates on low incomes tend to be lower under the PT than under the EUT. Therefore we construct Pareto-optimization programme for nonlinear taxation, on top of which we develop sufficient condition for the FOA to general utility functions including inseparable functions between income and effort, hence generalizing Kanbur, Pirttila, and Tuomala (2008). Then we check taxation under income uncertainty when the individuals behave according to the PT, but the government applies the EUT.

This paper falls into a rapidly expanding field of behavioral public economics, whose central focus is on public policy while the individual preferences differ from social ones, for example, O'Donoghue and Rabin (2003) on optimal paternalistic taxes which the government imposes to correct individuals' behavior regarding consumption of harmful goods, Sheshinski (2003) on faulty individuals' decision making, where restricting individuals' choices leads to welfare improvements, and Kanbur, Pirttila, and Tuomala (2006) on non-welfarist optimal taxation.

In Section 2, we model taxation problem under income uncertainty when the individuals apply the PT. In Section 3, we introduce the FOA, its sufficient condition and how it is developed for inseparable value functions between income and effort. Some concluding remarks are presented in Section 4.

2. BASIC MODEL

Assuming that the individuals make decisions according to the PT, and the government according to the EUT, this section models taxation under income uncertainty. In the PT, the value functions are defined on deviations from the reference points, generally concave for gains and convex for losses, and steeper for losses than for gains (Kahneman and Tversky (1979)).

When the individual does not know what income he or she will receive for each possible level of effort as in Mirrlees (1971, 1974, 1976), the individual's gross income, z, depends randomly on effort, y. For a single individual or all ex-ante identical individuals of heterogenous skill levels unobservable to others, income differences are not coming from differences in innate skills as in Mirrlees (1971, 1974, 1976), but from any given level of effort. The government and the individuals here perceive the same distribution of income z and effort y. Let $f(z, y)$ and $F(z, y)$ be the density and distribution functions of income z given that effort y is undertaken by the individual. Similar to Kanbur, Pirttila, and Tuomala (2008), we assume $f(z, y)$ and $F(z, y)$ not only positive for $\forall z$, and $\forall y$, but also twice continuously differentiable.

Assumption 1 $f(z, y) > 0$, $\int f(z, y)dz = 1$ and $F(z, y) > 0$, for $\forall z$, and $\forall y$.

Assumption 2 $f(z, y)$ and $F(z, y)$ are twice continuously differentiable with respect to every income z and effort y, also $F_{yy}(z, y) > 0$.

Note that $F_{yy}(z, y) > 0$ is called the convexity of the distribution function condition (CDFC henceforth) in Mirrlees (1976), Rogerson (1985), Jewitt (1988), Alvi (1997).

Let $g(z, y) = \frac{f_y(z,y)}{f(z,y)}$ be likelihood ratio, we assume the likelihood ratio monotone with respect to income z.

Assumption 3 $\frac{\partial g(z,y)}{\partial z} > 0$.

Assumption 3 is called the monotone likelihood ratio condition (MLRC henceforth) in Mirrlees (1976), Rogerson (1985), Jewitt (1988), Alvi (1997).

Let $x = z - T(z)$ be the income after tax, $\bar{x}$ the exogenous reference income for the individual, and $\tilde{x} = x - \bar{x}$ the change over the income after tax from the income reference which may be formulated historically or from comparison with other individuals.

In our situation, the individuals are risk-averse with value function over income and effort, their values depend on their gains and loss, and they are more loss aversion to loss than to gains, hence the value function $e(x - \bar{x}, y)$ has following characteristics.

Assumption 4

$$\frac{\partial e}{\partial \tilde{x}} > 0, \tag{1}$$

$$\frac{\partial^2 e}{\partial \tilde{x}^2} > 0, \forall x < \bar{x}; \tag{2}$$

$$\frac{\partial^2 e}{\partial \tilde{x}^2} = 0, x = \bar{x}; \tag{3}$$

$$\frac{\partial^2 e}{\partial \tilde{x}^2} < 0, \forall x > \bar{x}. \tag{4}$$

For an additively separable value function between income and effort, the value function can be simplified $e(x - \bar{x}, y) = \tilde{e}(x - \bar{x}) - y$ without loss of generality, when units of efforts are properly chosen. Here there was no reference point for effort in our models at this stage, since reference point for effort can not change working method along the additively separable value function, but it is much complicated for general inseparable formulation, see, Alvi (1997), for example. Since the individuals could lose from working less or more than the reference effort, we here concentrate on the value function with the reference point merely for income.

For mathematical simplicity and the FOA works at least for income over the reference income in our situation including inseparable value functions, we need following assumption.

Assumption 5

$$\frac{\partial e(\tilde{x}, y)}{\partial y} < 0, \tag{5}$$

$$\frac{\partial^2 e(\tilde{x}, y)}{\partial \tilde{x} \partial y} \geq 0. \tag{6}$$

Let δ_e be degree of the individuals' loss aversion, i.e. $\delta_e = \frac{\frac{\partial^2 e(\tilde{x},y)}{\partial^2 \tilde{x}}}{\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}$. We have following characteristics in accordance with empirical facts.

Assumption 6

$$\frac{\partial \delta_e}{\partial y} > 0, \forall x < \bar{x}; \tag{7}$$

$$\frac{\partial \delta_e}{\partial y} = 0, x = \bar{x}; \tag{8}$$

$$\frac{\partial \delta_e}{\partial y} < 0, \forall x > \bar{x}. \tag{9}$$

The incentive individuals choose effort y to maximize expectation of their value functions

$$\int e(x - \bar{x}, y) f(z, y) dz. \tag{10}$$

Assumption 7

$$y \in \arg\max_t \int e(x - \bar{x}, t) f(z, t) dz. \tag{11}$$

The budget constraints run as follows for sufficiently large identical population with independent and identically distributed states of nature.

Assumption 8

$$\int (z - x) f(z, y) dz = 0. \tag{12}$$

Subject to the individuals' optimization constraints (11) and the budget constraints (12), the government maximizes expected utility

$$\int v(x - \bar{x}) f(z, y) dz. \tag{13}$$

The government's utility function has following characteristics.

Assumption 9

$$\frac{dv(\tilde{x})}{d\tilde{x}} > 0; \tag{14}$$

$$\frac{d^2 v(\tilde{x})}{d\tilde{x}^2} \leq 0. \tag{15}$$

The government has access to nonlinear taxation, the individual's utility depends randomly on effort under income uncertainty, however, the aim of taxation is to transform resources from the highly skilled to the less skilled in an efficient way, given that incomes not skills are observable. A taxation is Pareto optimal if no taxation exists which gives the government and the individuals more higher expected value.

Definition 2.1. A taxation T is said to be Pareto optimal if it solves the following program:

$$\max_T \int v(x-\bar{x})f(z,y)dz \tag{16}$$

subject to

$$\int (z-x)f(z,y)dz = 0, \tag{17}$$

and

$$y \in \arg\max_t \int e(x-\bar{x},t)f(z,t)dz. \tag{18}$$

Definition 2.2. Program (16-18) is called Pareto-optimization program.

In the following, we will define relaxed Pareto-optimization program. To clearly distinguish program (16-18) from the relaxed Pareto-optimization program, the Pareto-optimization program (16-18) is sometimes referred to as the unrelaxed Pareto-optimization program.

It is difficult to analyze the Pareto-optimization program (16-18), since the incentive compatibility constraint for the individuals, i.e.

$$y \in \arg\max_t \int e(x-\bar{x},t)f(z,t)dz \tag{19}$$

is in fact a continuum of constraints

$$\int e(x-\bar{x},y)f(z,y)dz \geq \int e(x-\bar{x},t)f(z,t)dz \tag{20}$$

for every effort t.

If we can replace (19) with the stationary effort for the incentive compatible individuals, i.e

$$\int [e_y(x-\bar{x},y)f(z,y)+e(x-\bar{x},y)f_y(z,y)]dz = 0, \tag{21}$$

where $e_y(x-\bar{x}, y)$ and $f_y(z, y)$ denote the partial derivatives of the functions $e(x-\bar{x}, y)$ and $f(z, y)$ respectively with respect to the second variable y. Thus the solution can be calculated by the FOA.

In place of (21) for (19), it enlarges the constraint set, since all the stationary points are included instead of merely global maxima. The resulting program (22-23-24) is referred to as the relaxed program, since the constraint set (19) has been relaxed.

Definition 2.3. The following program

$$\max_{T} \int v(x-\bar{x}) f(z, y) dz \tag{22}$$

subject to

$$\int (z-x) f(z, y) dz = 0, \tag{23}$$

and

$$\int [e_y(x-\bar{x}, y) f(z, y) + e(x-\bar{x}, y) f_y(z, y)] dz = 0 \tag{24}$$

is called the relaxed Pareto-optimization program.

3. EQUILIBRIUM TAX

The solutions between the unrelaxed program (16-18) and the relaxed program (22-23-24) are not always the same even in the EUT, as Mirrlees (1975, 1976, 1997), Rogerson (1985), Jewitt (1988), and Alvi (1997) pointed out. Particularly, the necessary condition to solve the relaxed program (22-23-24) may not even the one to solve the unrelaxed program (16-18).

In fact, the FOA is not necessarily valid, as Mirrlees (1975, 1997) pointed out in the conventional setting, since it might lead to a local optimum instead of a global one. Necessary and/or sufficient conditions for the FOA are explored in Mirrlees (1976), Rogerson (1985), Jewitt (1988), Alvi (1997) for optimal taxation in the EUT. For the separable utility function in the EUT, sufficient conditions are the MLRC and CDFC, but still no sufficient and necessary condition exists for the separable utility function even in the EUT. Therefore it is challenging, in the PT, to deal with this Pareto-optimization program for general value functions including inseparable value functions between income and effort.

Taking multipliers λ and α respectively for the constraints (23) and (24), the Lagrangian runs as follows

$$\mathcal{L} = \int \{v(\tilde{x})f(z,y)+\alpha[\frac{\partial e(\tilde{x},y)}{\partial y}f(z,y)+e(\tilde{x},y)\frac{\partial f(z,y)}{\partial y}]+\lambda(z-x)f(z,y)\}dz, \tag{25}$$

where the Lagrangian depends on T, y, λ, α if $\bar{x}$ is exogenous. Recalling the likelihood ratio $g(z,y) = \frac{\frac{\partial f(z,y)}{\partial y}}{f(z,y)}$, we yield the first-order condition with respect to x (point-wise optimization) as follows.

$$\frac{dv(\tilde{x})}{d\tilde{x}} + \alpha g\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}} + \alpha\frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}\partial y} = \lambda. \tag{26}$$

Dividing both sides by $\frac{dv(\tilde{x})}{d\tilde{x}}$ yields

$$1 + \alpha g\frac{\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}} + \alpha\frac{\frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}\partial y}}{\frac{dv(\tilde{x})}{d\tilde{x}}} = \frac{\lambda}{\frac{dv(\tilde{x})}{d\tilde{x}}}. \tag{27}$$

Following Jewitt (1988), Laffont and Martimort (2002) and Kanbur, Pirttila, and Tuomala (2008), the following result is obtained.

Lemma 1.

$$Cov(v(\tilde{x}), \frac{1}{\frac{dv(\tilde{x})}{d\tilde{x}}}) = \frac{\alpha}{\lambda}[Cov(v(\tilde{x}), g(z,y)\frac{\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}}) + Cov(v(\tilde{x}), \frac{\frac{\partial^2 e(\tilde{x},y)}{\partial y\partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}})], \tag{28}$$

where $Cov(A,B)$ denote covariance between A and B.

Proof. Dividing (27) by λ, multiplying it with the density function f and integrating over the support $[\underline{z}, \overline{z}]$ yields

$$\frac{1}{\lambda} + \frac{\alpha}{\lambda}\int \frac{\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}}gfdz + \frac{\alpha}{\lambda}\int \frac{\frac{\partial^2 e(\tilde{x},y)}{\partial y\partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}}fdz = \int \frac{1}{\frac{dv(\tilde{x})}{d\tilde{x}}}fdz, \tag{29}$$

given $\int f(z,y)dz = 1$. Applying (27) gives

$$\frac{1}{\frac{dv(\tilde{x})}{d\tilde{x}}} - \frac{\alpha g}{\lambda}\frac{\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}} - \frac{\alpha}{\lambda}\frac{\frac{\partial^2 e(\tilde{x},y)}{\partial y\partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}} + \frac{\alpha}{\lambda}\int \frac{\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}}gfdz + \frac{\alpha}{\lambda}\int \frac{\frac{\partial^2 e(\tilde{x},y)}{\partial y\partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}}fdz = \int \frac{1}{\frac{dv(\tilde{x})}{d\tilde{x}}}fdz. \tag{30}$$

Multiplying both sides by vf, integrating over the support $[\underline{z}, \overline{z}]$, and reorganizing yields

$$Cov(v(\tilde{x}), \frac{1}{\frac{dv(\tilde{x})}{d\tilde{x}}}) = \frac{\alpha}{\lambda}[Cov(v(\tilde{x}), g(z,y)\frac{\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}}) + Cov(v(\tilde{x}), \frac{\frac{\partial^2 e(\tilde{x},y)}{\partial y \partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}})], \tag{31}$$

▮

Corollary 1. *For the separable value functions between income and effort, i.e.* $e(x-\bar{x}, y) = \tilde{e}(x-\bar{x}) - y$,

$$Cov(v(\tilde{x}), \frac{1}{\frac{dv(\tilde{x})}{d\tilde{x}}}) = \frac{\alpha}{\lambda} Cov(v(\tilde{x}), g(z,y)\frac{\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}}). \tag{32}$$

Proof. For $e(x-\bar{x}, y) = \tilde{e}(x-\bar{x}) - y$,

$$\frac{\partial e(\tilde{x}, y)}{\partial y} = -1, \tag{33}$$

$$\frac{\partial^2 e(\tilde{x}, y)}{\partial y \partial \tilde{x}} = 0. \tag{34}$$

The result follows from the substitution of them into Lemma 1. ▮

Lemma 2. $\alpha > 0$ *holds trivially for* $x < \bar{x}$ *and also if a combination of the government's risk aversion and the individuals' loss aversion is less than* $-\frac{dv(\tilde{x})}{d\tilde{x}}$ *for* $x > \bar{x}$.

Proof. Dividing both sides of equality (26) by $\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}$, multiplying $f(z,y)$, integrating over the support $[\underline{z}, \overline{z}]$ yields

$$\lambda \int \frac{f(z,y)}{\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}} dz = \int \frac{\frac{dv(\tilde{x})}{d\tilde{x}}}{\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}} f(z,y) dz + \alpha \int \frac{\frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x} \partial y}}{\frac{\partial e(\tilde{x},y)}{\partial x}} f(z,y) dz. \tag{35}$$

Inserting λ from Lemma 1 into it, and reorganizing yields

$$\alpha[Cov(v(\tilde{x}), g(z,y)\frac{\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}}) \int \frac{f(z,y)}{\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}} dz \tag{36}$$

$$+ Cov(v(\tilde{x}), \frac{\frac{\partial^2 e(\tilde{x},y)}{\partial y \partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}}) \int \frac{f(z,y)}{\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}} dz \tag{37}$$

$$-Cov(v(\tilde{x}),\frac{1}{\frac{dv(\tilde{x})}{d\tilde{x}}})\int\frac{\frac{\partial^2 e(\tilde{x},y)}{\partial\tilde{x}\partial y}}{\frac{\partial e(\tilde{x},y)}{\partial\tilde{x}}}f(z,y)dz] \tag{38}$$

$$= Cov(v(\tilde{x}),\frac{1}{\frac{dv(\tilde{x})}{d\tilde{x}}})\int\frac{\frac{dv(\tilde{x})}{d\tilde{x}}}{\frac{\partial e(\tilde{x},y)}{\partial\tilde{x}}}f(z,y)dz. \tag{39}$$

Since $\frac{dv(\tilde{x})}{d\tilde{x}} > 0$, and $\frac{d^2v(\tilde{x})}{d\tilde{x}^2} < 0$, $v(\tilde{x})$ and $\frac{1}{\frac{dv(\tilde{x})}{d\tilde{x}}}$ co-vary consistently, i.e.

$$Cov(v(\tilde{x}),\frac{1}{\frac{dv(\tilde{x})}{d\tilde{x}}}) > 0. \tag{40}$$

Since $\frac{dv(\tilde{x})}{d\tilde{x}} > 0$, and $\frac{\partial e(\tilde{x},y)}{\partial\tilde{x}} > 0$, then $\int\frac{\frac{dv(\tilde{x})}{d\tilde{x}}}{\frac{\partial e(\tilde{x},y)}{\partial\tilde{x}}}f(z,y)dz > 0$.
For $x > \bar{x}$, $\frac{dv(\tilde{x})}{d\tilde{x}} > 0$, $\frac{\partial e(\tilde{x},y)}{\partial\tilde{x}} > 0$, $\delta_e(\tilde{x},y) < 0$, and $\delta_v(\tilde{x}) > 0$,

$$\frac{\partial^3 e(\tilde{x},y)}{\partial\tilde{x}^2\partial y} = \frac{\partial\delta_e(\tilde{x},y)}{\partial y}\frac{\partial e(\tilde{x},y)}{\partial\tilde{x}} + \delta_e(\tilde{x},y)\frac{\partial^2 e(\tilde{x},y)}{\partial\tilde{x}\partial y} \le 0, \tag{41}$$

but a combination of the government's risk aversion and the individuals' loss aversion is less than $-\frac{dv(\tilde{x})}{d\tilde{x}}$, i.e.

$$v(\tilde{x})\delta_v(\tilde{x}) + v(\tilde{x})\delta_e(\tilde{x},y) < -\frac{dv(\tilde{x})}{d\tilde{x}}. \tag{42}$$

For $x < \bar{x}$, $\frac{dv(\tilde{x})}{d\tilde{x}} > 0$, $\frac{\partial e(\tilde{x},y)}{\partial\tilde{x}} > 0$, $\delta_e(\tilde{x},y) > 0$, and $\delta_v(\tilde{x}) > 0$,

$$\frac{\partial^3 e(\tilde{x},y)}{\partial\tilde{x}^2\partial y} = \frac{\partial\delta_e(\tilde{x},y)}{\partial y}\frac{\partial e(\tilde{x},y)}{\partial\tilde{x}} + \delta_e(\tilde{x},y)\frac{\partial^2 e(\tilde{x},y)}{\partial\tilde{x}\partial y} \ge 0, \tag{43}$$

but a combination of the government's risk aversion and the individuals' loss aversion is trivially bigger than $-\frac{dv(\tilde{x})}{d\tilde{x}}$, i.e.

$$v(\tilde{x})\delta_v(\tilde{x}) + v(\tilde{x})\delta_e(\tilde{x},y) > -\frac{dv(\tilde{x})}{d\tilde{x}}. \tag{44}$$

On either case,

$$Cov(v(\tilde{x}),g(z,y)\frac{\frac{\partial e(\tilde{x},y)}{\partial\tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}})\int\frac{f(z,y)}{\frac{\partial e(\tilde{x},y)}{\partial\tilde{x}}}dz + Cov(v(\tilde{x}),\frac{\frac{\partial^2 e(\tilde{x},y)}{\partial y\partial\tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}})\int\frac{f(z,y)}{\frac{\partial e(\tilde{x},y)}{\partial\tilde{x}}}dz \tag{45}$$

$$> Cov(v(\tilde{x}),\frac{1}{\frac{dv(\tilde{x})}{d\tilde{x}}})\int\frac{\frac{\partial^2 e(\tilde{x},y)}{\partial\tilde{x}\partial y}}{\frac{\partial e(\tilde{x},y)}{\partial x}}f(z,y)dz. \tag{46}$$

Hence, $\alpha > 0$. ▮

Differentiate (26) with respect to z, and reorganize to obtain $\frac{d\tilde{x}}{dz}$ as follows.

LEMMA 3.

$$\frac{d\tilde{x}}{dz} = -\frac{\alpha \frac{\partial g(z,y)}{\partial z} \frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\alpha \frac{\partial^3 e(\tilde{x},y)}{\partial \tilde{x}^2 \partial y} + \frac{d^2 v(\tilde{x})}{d\tilde{x}^2} + \alpha g(z,y) \frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}^2}}. \tag{47}$$

Proof. Differentiating (26) with respect to z yields

$$\frac{d\tilde{x}}{dz}[\alpha \frac{\partial^3 e(\tilde{x},y)}{\partial \tilde{x}^2 \partial y} + \frac{d^2 v(\tilde{x})}{d\tilde{x}^2} + \alpha g(z,y) \frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}^2}] = -\alpha \frac{\partial g(z,y)}{\partial z} \frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}. \tag{48}$$

Reorganizing it yields

$$\frac{d\tilde{x}}{dz} = -\frac{\alpha \frac{\partial g(z,y)}{\partial z} \frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\alpha \frac{\partial^3 e(\tilde{x},y)}{\partial \tilde{x}^2 \partial y} + \frac{d^2 v(\tilde{x})}{d\tilde{x}^2} + \alpha g(z,y) \frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}^2}}. \tag{49}$$

▮

COROLLARY 2. *For the separable value functions between income and effort, i.e.* $e(x - \bar{x}, y) = \tilde{e}(x - \bar{x}) - y$*,*

$$\frac{d\tilde{x}}{dz} = -\frac{\alpha \frac{\partial g(z,y)}{\partial z} \frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\frac{d^2 v(\tilde{x})}{d\tilde{x}^2} + \alpha g(z,y) \frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}^2}}. \tag{50}$$

Proof. For $e(x - \bar{x}, y) = \tilde{e}(x - \bar{x}) - y$,

$$\frac{\partial^3 e(\tilde{x},y)}{\partial \tilde{x}^2 \partial y} = 0. \tag{51}$$

The result follows from the substitution of it into equality (47). ▮

LEMMA 4.

$$\frac{d\tilde{x}}{dz} > 0 \tag{52}$$

holds trivially for $x > \bar{x}$*, but for* $x < \bar{x}$ *if the government's risk aversion sufficiently out-weights a combination of the individuals' loss aversion and their partial derivative with respect to effort.*

Proof. Recalling Lemma 3,

$$\frac{d\tilde{x}}{dz} = -\frac{\alpha \frac{\partial g(z,y)}{\partial z} \frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\alpha \frac{\partial^3 e(\tilde{x},y)}{\partial \tilde{x}^2 \partial y} + \frac{d^2 v(\tilde{x})}{d\tilde{x}^2} + \alpha g(z,y) \frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}^2}}. \tag{53}$$

Since $\frac{d^2 v(\tilde{x})}{d\tilde{x}^2} = -\delta_v(\tilde{x}) \frac{dv(\tilde{x})}{d\tilde{x}}$, $\frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}^2} = \delta_e(\tilde{x},y) \frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}$, $\frac{\partial^3 e(\tilde{x},y)}{\partial \tilde{x}^2 \partial y} = \frac{\partial \delta_e(\tilde{x},y)}{\partial y} \frac{\partial e(\tilde{x},y)}{\partial \tilde{x}} + \delta_e(\tilde{x},y) \frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}^2}$, and equality (26) holds,

$$\frac{d\tilde{x}}{dz} = \frac{\alpha \frac{\partial g(z,y)}{\partial z} \frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\delta_v(\tilde{x}) \frac{dv(\tilde{x})}{d\tilde{x}} - \delta_e(\tilde{x},y)(\lambda - \frac{dv(\tilde{x})}{d\tilde{x}}) - \alpha \frac{\partial \delta_e(\tilde{x},y)}{\partial y} \frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}. \tag{54}$$

According to Assumption 3, 4, and 9,

$$\frac{\partial g(z,y)}{\partial z} > 0, \tag{55}$$

$$\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}} > 0, \tag{56}$$

$$\frac{dv(\tilde{x})}{d\tilde{x}} > 0. \tag{57}$$

Recalling equality (26), and $\alpha > 0$ from Lemma 2,

$$\lambda - \frac{dv(\tilde{x})}{d\tilde{x}} > 0. \tag{58}$$

For $x > \bar{x}$, $\frac{dv(\tilde{x})}{d\tilde{x}} > 0$, $\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}} > 0$, $\delta_e(\tilde{x},y) < 0$, $\frac{\partial \delta_e(\tilde{x},y)}{\partial y} < 0$, and $\delta_v(\tilde{x}) > 0$, then

$$\frac{d\tilde{x}}{dz} > 0. \tag{59}$$

For $x < \bar{x}$, $\frac{dv(\tilde{x})}{d\tilde{x}} > 0$, $\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}} > 0$, $\delta_e(\tilde{x},y) > 0$, $\frac{\partial \delta_e(\tilde{x},y)}{\partial y} > 0$, and $\delta_v(\tilde{x}) > 0$, but the government's risk aversion sufficiently out-weights a combination of the individuals' loss aversion and their partial derivative with respect to effort, i.e.

$$\delta_v(\tilde{x}) \frac{dv(\tilde{x},y)}{d\tilde{x}} - \delta_e(\tilde{x},y)(\lambda - \frac{dv(\tilde{x})}{d\tilde{x}}) - \alpha \frac{\partial \delta_e(\tilde{x},y)}{\partial y} \frac{\partial e(\tilde{x},y)}{\partial \tilde{x}} > 0, \tag{60}$$

then

$$\frac{d\tilde{x}}{dz} > 0. \tag{61}$$

∎

LEMMA 5. *(1) For $x < \bar{x}$, $\frac{\alpha}{\lambda} > 0$ holds trivially;*

(2) For $x > \bar{x}$, if the government's risk aversion sufficiently out-weights a combination of the individuals' loss aversion and their partial derivative with respect to effort, and out-weights a combination of the individuals' loss aversion and $\frac{\partial g(z,y)}{\partial \tilde{x}}$, $\frac{\alpha}{\lambda} > 0$.

Proof. Since $\frac{dv(\tilde{x})}{d\tilde{x}} > 0$, and $\frac{d^2v(\tilde{x})}{d\tilde{x}^2} < 0$, $v(\tilde{x})$ and $\frac{1}{\frac{dv(\tilde{x})}{d\tilde{x}}}$ co-vary consistently, i.e.

$$Cov(v(\tilde{x}), \frac{1}{\frac{dv(\tilde{x})}{d\tilde{x}}}) > 0. \tag{62}$$

For $x < \bar{x}$, $\frac{dv(\tilde{x})}{d\tilde{x}} > 0$, $\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}} > 0$, $\delta_e(\tilde{x}, y) > 0$, and $\delta_v(\tilde{x}) > 0$, then

$$\frac{\partial g(z,y)}{\partial \tilde{x}} + g(z,y)\delta_e(\tilde{x},y) + g(z,y)\delta_v(\tilde{x}) > 0. \tag{63}$$

Hence, $v(\tilde{x})$ and $g(z,y)\frac{\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}}$ co-vary consistently, i.e. $Cov(v(\tilde{x}), g(z,y)\frac{\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}}) > 0$ holds.

For $x > \bar{x}$, $\frac{dv(\tilde{x})}{d\tilde{x}} > 0$, $\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}} > 0$, $\delta_e(\tilde{x}, y) < 0$, and $\delta_v(\tilde{x}) > 0$, but the government's risk aversion sufficiently out-weights a combination of the individuals' loss aversion and $\frac{\partial g(z,y)}{\partial \tilde{x}}$, i.e.

$$\frac{\partial g(z,y)}{\partial \tilde{x}} + g(z,y)\delta_e(\tilde{x},y) + g(z,y)\delta_v(\tilde{x}) > 0. \tag{64}$$

Hence, $v(\tilde{x})$ and $g(z,y)\frac{\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}}$ co-vary consistently, i.e. $Cov(v(\tilde{x}), g(z,y)\frac{\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}}) > 0$ holds.

For $x < \bar{x}$, $\frac{dv(\tilde{x})}{d\tilde{x}} > 0$, $\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}} > 0$, $\delta_e(\tilde{x}, y) > 0$, $\frac{\partial \delta_e(\tilde{x},y)}{\partial y} > 0$, $\delta_v(\tilde{x}) > 0$, and $\frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}\partial y} > 0$, then

$$\frac{\partial \delta_e(\tilde{x},y)}{\partial y}\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}} + \delta_e(\tilde{x},y)\frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}\partial y} + \delta_v(\tilde{x})\frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}\partial y} > 0. \tag{65}$$

Hence, $v(\tilde{x})$ and $\frac{\frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}\partial y}}{\frac{dv(\tilde{x})}{d\tilde{x}}}$ co-vary consistently, i.e. $Cov(v(\tilde{x}), \frac{\frac{\partial^2 e(\tilde{x},y)}{\partial y\partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}}) > 0$ holds.

For $x > \bar{x}$, $\frac{dv(\tilde{x})}{d\tilde{x}} > 0$, $\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}} > 0$, $\delta_e(\tilde{x}, y) < 0$, $\frac{\partial \delta_e(\tilde{x},y)}{\partial y} < 0$, $\delta_v(\tilde{x}) > 0$, and $\frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}\partial y} > 0$, but the government's risk aversion sufficiently out-weights a combination of the individuals' loss aversion and their partial derivative with respect to effort, i.e.

$$\frac{\partial \delta_e(\tilde{x},y)}{\partial y}\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}} + \delta_e(\tilde{x},y)\frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}\partial y} + \delta_v(\tilde{x})\frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}\partial y} > 0, \tag{66}$$

so $v(\tilde{x})$ and $\frac{\frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}\partial y}}{\frac{dv(\tilde{x})}{d\tilde{x}}}$ co-vary consistently, i.e. $Cov(v(\tilde{x}), \frac{\frac{\partial^2 e(\tilde{x},y)}{\partial y\partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}}) > 0$ holds.

According to Lemma 1, $\frac{\alpha}{\lambda} > 0$ if

$$Cov(v(\tilde{x}), g(z,y)\frac{\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}}) + Cov(v(\tilde{x}), \frac{\frac{\partial^2 e(\tilde{x},y)}{\partial y\partial \tilde{x}}}{\frac{dv(\tilde{x})}{d\tilde{x}}}) > 0. \tag{67}$$

As a consequence, for $x < \bar{x}$, $\frac{\alpha}{\lambda} > 0$ holds trivially; for $x > \bar{x}$, if the government's risk aversion sufficiently out-weights a combination of the individuals' loss aversion and their partial derivative with respect to effort, and out-weights a combination of the individuals' loss aversion and $\frac{\partial g(z,y)}{\partial \tilde{x}}$, $\frac{\alpha}{\lambda} > 0$. ▮

For the validity of the FOA to the relaxed Pareto-optimization program (22-23-24), since $\tilde{x} = x - \bar{x}$ and $x = z - T(z)$, where $T(z)$ is the tax for income z, the marginal tax rate (MTR henceforth) runs as follows due to equality (47).

Lemma 6.

$$MTR = 1 + \frac{\alpha\frac{\partial g(z,y)}{\partial z}\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\alpha\frac{\partial^3 e(\tilde{x},y)}{\partial \tilde{x}^2\partial y} + \frac{d^2 v(\tilde{x})}{d\tilde{x}^2} + \alpha g(z,y)\frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}^2}}. \tag{68}$$

Proof.

$$MTR \equiv \frac{dT(x)}{dz} \tag{69}$$

$$= 1 - \frac{d\tilde{x}}{dz} \tag{70}$$

$$= 1 + \frac{\alpha\frac{\partial g(z,y)}{\partial z}\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\alpha\frac{\partial^3 e(\tilde{x},y)}{\partial \tilde{x}^2\partial y} + \frac{d^2 v(\tilde{x})}{d\tilde{x}^2} + \alpha g(z,y)\frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}^2}}. \tag{71}$$

▮

Corollary 3. *For the separable value functions between income and effort, i.e.* $e(x-\bar{x},y) = \tilde{e}(x-\bar{x}) - y$,

$$MTR = 1 + \frac{\alpha\frac{\partial g(z,y)}{\partial z}\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\frac{d^2 v(\tilde{x})}{d\tilde{x}^2} + \alpha g(z,y)\frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}^2}}. \tag{72}$$

Proof. For $e(x-\bar{x},y)=\tilde{e}(x-\bar{x})-y$,

$$\frac{\partial^3 e(\tilde{x},y)}{\partial \tilde{x}^2 \partial y}=0. \tag{73}$$

The result follows from the substitution of it into Lemma 6. ▮

LEMMA 7. *The expected value function of the government is concave in effort y.*

Proof.

$$\int v(x-\bar{x})f(z,y)dz=[v(\tilde{x})F(z,y)]_{\underline{z}}^{\overline{z}}-\int \frac{dv(\tilde{x})}{d\tilde{x}}\frac{d\tilde{x}}{dz}F(z,y)dz \tag{74}$$

$$=v(\tilde{x}(\overline{z}))-\int \frac{dv(\tilde{x})}{d\tilde{x}}\frac{d\tilde{x}}{dz}F(z,y)dz, \tag{75}$$

which is concave because of the CDFC, i.e. $F_{yy}>0$. ▮

To summarize,

THEOREM 1. *In nonlinear taxation under income uncertainty, when the government makes decision upon the EUT, and the individuals make decisions upon the PT, under Assumptions 1-9, the FOA is valid to the relaxed Pareto-optimization program (22-23-24) for following two cases:*
(1) for $x<\bar{x}$ when the government's risk aversion sufficiently out-weights a combination of the individuals' loss aversion and their partial derivative with respect to effort;
(2) for $x>\bar{x}$ when a combination of the government's risk aversion and the individuals' loss aversion is less than $-\frac{dv(\tilde{x})}{d\tilde{x}}$, the government's risk aversion sufficiently out-weights a combination of the individuals' loss aversion and their partial derivative with respect to effort, and the government's risk aversion sufficiently out-weights a combination of the individuals' loss aversion and $\frac{\partial g(z,y)}{\partial \tilde{x}}$.
Moreover, the MTR is given by

$$MTR=1+\frac{\alpha\frac{\partial g(z,y)}{\partial z}\frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\alpha\frac{\partial^3 e(\tilde{x},y)}{\partial \tilde{x}^2\partial y}+\frac{d^2 v(\tilde{x})}{d\tilde{x}^2}+\alpha g(z,y)\frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}^2}}. \tag{76}$$

Proof. For the validity of the FOA to the relaxed Pareto-optimization program (22-23-24), recalling Lemma 7, it suffices that

(1) $\alpha > 0$, hence, for all income x, the incentive constraints (22) hold;

(2) $\lambda > 0$, hence, for for all income x, the budget constraints (23) hold;

(3) Realized income $\tilde{x}$ should be increasing in income z for the individuals to exert effort y, i.e., $\frac{d\tilde{x}}{dz} > 0$.

All are guaranteed by Lemma 2, 4, and 5.

For the validity of the FOA, the MTR is given by Lemma 6, i.e.

$$MTR = 1 + \frac{\alpha \frac{\partial g(z,y)}{\partial z} \frac{\partial e(\tilde{x},y)}{\partial \tilde{x}}}{\alpha \frac{\partial^3 e(\tilde{x},y)}{\partial \tilde{x}^2 \partial y} + \frac{d^2 v(\tilde{x})}{d\tilde{x}^2} + \alpha g(z,y) \frac{\partial^2 e(\tilde{x},y)}{\partial \tilde{x}^2}}. \tag{77}$$

$\blacksquare$

It is not surprising that the sufficient condition is complicated for the validity of the FOA, because of the inclusion of interactive terms between income and effort as well as the dependence of the value functions on inseparable variables between income and effort, and the individuals' loss aversion is hard to compare with the government's risk aversion in general domain of income for the inseparable value function between income and effort. For the separable value functions between income and effort, for example, $e(x - \bar{x}, y) = \tilde{e}(x - \bar{x}) - y$, $\frac{\partial^2 e(\tilde{x},y)}{\partial y \partial \tilde{x}} = 0$, it reduces to Kanbur, Pirttila and Tuomala (2008).

For $x < \bar{x}$, the government and individuals' valuation functions are convex with respect to realized income $\tilde{x}$ according to Assumption 4. Hence the Pareto-optimization Program (22)-(23)-(24) is relatively difficult to deal with. For $x < \bar{x}$ and the separable value function $e(x-\bar{x}, y) = \tilde{e}(x-\bar{x}) - y$, the FOA is invalid for optimal nonlinear taxation, the government may turn for random schedule, see Arnott and Stiglitz (1988).

4. CONCLUSION

In this paper, we modeled taxation under income uncertainty applying the PT developed by Kahneman and Tversky (1979), and obtained its sufficient condition for the FOA to general value functions including inseparable ones between income and effort.

Our results generalizes Oswald (1983) and Tuomala (1990) since we analyzed taxation with utility interdependence under income uncertainty, while Oswald (1983) and Tuomala (1990) worked in the conventional setting without income uncertainty.

Our results also generalizes Kanbur, Pirttila and Tuomala (2008), since they only considered the separable value functions between income and effort, but we treated general value functions including both separable and inseparable ones.

Some problems are still open. It is a challenge to check how the MTR changes when risk aversion or loss aversion changes. This is because the Lagrangian multipliers may change when changing the parameters, and the value functions of the model.

It is also interesting to examine how the marginal tax rate increases or decreases with income in general. The sign of the change in the MTR remains ambiguous in general. Still no intuition has developed, to our knowledge, for the third derivative of the value functions in the PT.

Recalling people's uncertainty about their future productivity or realistic restrictions on taxes, optimal taxation was developed given only minimal restrictions on the set of possible tax instruments, or on the nature of shocks affecting people in economy, which provides a connection between dynamic optimal taxation and dynamic principal-agent theory. When people behave according to the PT, in particular, with different attitudes to loss aversion, the connection between dynamic optimal taxation and dynamic principal-agent theory is worth discussion.

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REFERENCES

Alvi, E., 1997. First Order Approach to the Principal-Agent Problems: A Generalization. *The Geneva Papers on Risk and Insurance Theory* **22**, 59-65.

Arnott, R. and J. E. Stiglitz, 1988. Randomization with asymmetric information. *RAND Journal of Economics* **19**, 344-362.

Atkinson, A. and J. E. Stiglitz, 1976. The Design of Tax Structure: Direct Versus Indirect Taxation. *Journal of Pubic Economics* **6**, 55-75.

Atkinson, A. and J. E. Stiglitz, 1980. Lectures on Pubic Economics, New York: McGraw-Hill.

Carroll, C. D., J. Overland, and D. N. Weil, 2000. Saving and growth with habit formation. *American Economic Review* **90**, 341-355.

Camerer, C. F. and G. Loewenstein, 2003. Behavioral economics: Past, Present, Future, in C. F. Camerer, G. Loewenstein and M. Rabin (eds.), Advances in Behavioral Economics, Princeton: University Press.

Dai, X., 2010. Efficient Auction for Risk Averse Buyers. *2010 International Conference on Management and Service Science (MASS 2010)*, ISBN-13:9781424453269, IEEE Computer Society.

O'Donoghue, T. and M. Rabin, 2003. Studying optimal paternalism, illustrated by a model of sin taxes. *American Economic Review* **93**, 186-191.

Mikhail Golosov, Narayana Kocherlakota and Aleh Tsyvinski, 2003. Optimal Indirect and Capital Taxation. *Review of Economic Studies* **70**, 569-587.

Holmstrom, B., 1979. Moral hazard and observability. *Bell Journal of Economics* **10**, 74-91.

Jewitt, I., 1988. Justifying the first order approach to principal-agent problems. *Econometrica* **56**, 1177-90.

Kahneman, D. and A. Tversky, 1979. Prospect theory: An analysis of decision under risk. *Econometrica* **47**, 263-281.

Ravi Kanbur, Jukka Pirttila, and Matti Tuomala, 2006. Non-Welfarist Optimal Taxation and Behavioral Public Economics. *Journal of Economic Surveys* **20**, 849-868.

Ravi Kanbur, Jukka Pirttila, and Matti Tuomala, 2008. Moral Hazard, Income Taxation, and Prospect Theory. *Scandinavian Journal of Economics* **110**, 321-337.

Laffont, J.-J. and D. Martimort, 2002. The theory of incentives. The principal-agent model, Princeton University Press.

Ljunqvist, L. and H. Uhlig, 2000. Tax policy and aggregate demand management under catching up with the Joneses. *American Economic Review* **90**, 356-366.

Maskin, E. and J. Riley, 1984. Optimal Auctions with Risk Averse Buyers. *Econometrica* **52**, 1473-1518

Mirrlees, J. A., 1971. An Exploration in the Theory of Optimum Income Taxation. *Review of Economic Studies* **38**, 175-208.

Mirrlees, J. A., 1974. Notes on welfare economics, information and uncertainty, in Balch, McFadden and Wu (Eds.). *Essays on Economic Behaviour under Uncertainty*, Amsterdam: North Holland.

Mirrlees, J. A., 1975,1999. The Theory of Moral Hazard and Unobservable Behaviour. Part I. *Review of Economic Studies* **66**, 3-22.

Mirrlees, J. A., 1976. The optimal structure of authority and incentives within an organization. *Bell Journal of Economics* **7**, 105-31.

Mirrlees, J. A., 1997. Information and incentives: The economics of carrot and sticks. *The Economic Journal* **107**, 1311-1329.

Oswald, A., 1983. Altruism, jealousy and the theory of optimal nonlinear taxation. *Journal of Public Economics* **20**, 77-87.

Rogerson, W., 1985. The First-Order Approach to Principal-Agent Problems. *Econometrica* **53**, 1357-67.

Sheshinski, E., 2003. Optimal policy to influence individual choice probabilities, working paper of Hebrew University.

Tuomala, M., 1990. Optimal income tax and redistribution, Oxford: Clarendon Press.

Tversky, A. and D. Kahneman, 1992. Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty* **5**, 297-323.