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Cournot and Bertrand Competition in a Differentiated Duopoly with Endogenous Technology Adoption *

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We study firms' endogenous technology adoption in a two-stage differentiated duopoly game. Firms choose technologies from a continuous technology set in stage one, and engage in either Cournot or Bertrand competition in stage two. If the technology set is sufficiently convex and the degree of product differentiation is sufficiently high, we find that (i) Bertrand competition leads to more interior technology choices than Cournot competition, and (ii) Cournot competition induces a greater incentive to innovate for both firms. Furthermore, welfare analysis shows that Bertrand competition always yields higher consumer surplus and social welfare than Cournot competition although the marginal cost of production is higher.

Key Words: Technology adoption; Differentiated duopoly; Cournot competition; Bertrand competition.
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1. INTRODUCTION

In a technology-driven economy, remarkable developments in technology innovation provide firms with incentives to adopt cost-reducing technologies so as to gain a competitive advantage. In the past few decades, a growing literature has emerged to study firms' endogenous technology adoption considering the tradeoff between fixed and marginal costs. In a two-stage oligopoly game, we turn our attention to the comparison of Cournot and Bertrand framework to see which mode of competition produces a greater incentive to innovate.

Our paper is based on Mills and Smith (1996), which proposes a two-stage duopoly game in a homogenous good industry to study why firms adopt different technologies with identical opportunity sets. In their model, firms simultaneously choose technologies in stage one and outputs in stage two. The available technology set is continuous and identical for both firms, with each technology corresponding to a constant marginal cost and associated fixed cost. If the technology set is insufficiently convex, the authors show that firms choose only extreme technologies in equilibrium, and asymmetric technology choices arise under certain conditions on the technology set and demand.

In this paper, we extend the Mills and Smith (1996) model to study technology adoption in a differentiated duopoly under both Cournot and Bertrand competition. Focusing on the comparison between Bertrand and Cournot equilibria, we aim to address the questions of which mode of competition leads to more interior technology adoption and which one provides greater incentive to make cost-reducing investment. If the technology set is sufficiently convex and the degree of product differentiation is sufficiently high, we find that (i) Bertrand Competition leads to more interior technology choices than Cournot competition, and (ii) Cournot competition provides both firms with more incentives to invest on technology. Furthermore, we conduct welfare analysis under both modes of competition and show that (i) equilibrium prices are lower and quantities are larger under Bertrand competition than Cournot competition, and (ii) Bertrand competition always yields higher social welfare than Cournot competition which is consistent with the standard welfare ranking.

A number of papers on strategic technology adoption have followed up on the work of Mills and Smith (1996). Elberfeld (2003) extends the original Cournot duopoly setting to an Cournot oligopoly. They show that asymmetric technology choices can arise as in the duopoly case but the welfare implications are not robust. Elberfeld and Nti (2004) introduce uncertainty into the Cournot oligopoly model to analyze the adoption of a new technology. Pal (2010) examines firms' choice of technology adoption in a differentiated duopoly considering both Cournot and Bertrand competition in stage two. More recently, Zhang et al (2014) consider the effect of technology spillover on technology choice based on the model of Elberfeld and Nti (2004). In all of these papers, firms simultaneously make a choice between two alternative production technologies, which represents the two extreme technologies in the technology set originally introduced by Mills and Smith (1996). Our paper, in contrast, studies how firms choose from a continuous technology set rather than two alternatives choices. The comparison between Cournot and Bertrand equilibria is further conducted to see which mode generates more interior technology adoption and which one provides greater incentive to invest on technology innovation. We believe our analysis will yield some interesting results to enrich the current literature.

This paper is a companion to the study in Wang and Zeng (2014), which focuses on the explanation of heterogenous firms in a Cournot duopoly. Although both papers study technology adoption following Mills and Smith (1996), they have different problem scenarios. Wang and Zeng (2014) consider a two-stage Cournot duopoly game in a homogenous good industry and introduce sequential moves in stage one. Their focus is which mode of moves, simultaneous moves or sequential moves, leads to more asymmetric technology choices. In this paper, we consider a two-stage duopoly game in a differentiated good industry and introduce Bertrand competition in stage two. We mainly study which mode of competition provides greater incentive to adopt cost-reducing technology and generates higher social welfare.

Our paper is also related to another strand of literature which study Cournot and Bertrand competition in a differentiated duopoly. Singh and Vives (1984) find that if the goods are substitutes, firms always make larger profits under Cournot competition than under Bertrand competition. By enlarging the parameter space originally considered by Singh and Vives (1984) to allow for a wider range of cost asymmetry, Zanchettin (2006) finds that Singh and Vives's ranking of profits between the two modes of competition fails to hold. Wang (2008) revisits this price and quantity competition by Zanchettin (2006), and concludes that the possibility of (partial) reversal in profit relationships between two modes of competition will not alter the celebrated result by Singh and Vives (1984) that firms always choose a quantity contract in a two-stage game.

The next section presents our model setup and analysis of both Cournot and Bertrand equilibria. Section 3 provides a comparison of the two competitions. Section 4 concludes the paper. All proofs are in the appendix.

2. THE MODEL

Consider a differentiated goods market with two firms. The demand functions are given by $q_i = \frac{1}{1-\gamma^2}[(1-\gamma)a - p_i + \gamma p_j]$, where $i, j = 1, 2, i \neq j$,[1] q_i (p_i) is the quantity (price) of firm i and γ ($0 < \gamma < 1$) is the product differentiation parameter. The corresponding inverse demand functions are $p_i = a - q_i - \gamma q_j$.

Firms play the following two-stage game:

• In stage 1: firms simultaneously select a technology from a continuous set characterized by constant marginal costs $c_i \in [c_\beta, c_\alpha]$ and fixed costs $F(c_i)$, where $c_\alpha < a$, $F(c_i)$ is non-negative and twice differentiable on $[c_\beta, c_\alpha]$ with $F' < 0, F'' > 0$.
• In stage 2: firms play Cournot (Bertrand) competition, choosing outputs (prices) given the technologies chosen in stage 1.

For simplicity and easy tractability, we assume a quadratic fixed cost function given by $F(c) = tc^2 - \lambda c + k$, where $t > 0$, $\lambda > 2c_\alpha$, $k \geq \lambda c_\alpha - tc_\alpha^2$. With this specification, $F \geq 0$, $F' < 0$ and $F'' = 2t > 0$ for all $c \in [c_\beta, c_\alpha]$.

The game is solved by backward induction. We first take a look at Cournot competition. In the second stage, each firm chooses its output to achieve profit maximization, given both firms' technology choices in stage 1 and the rate of output of the other firm:

$$\max_{q_i}(a - q_i - \gamma q_j - c_i)q_i - F(c_i).$$

By solving firm i's first-order condition to the maximization problem, we obtain that

$$q_i(c_i, c_j) = \frac{2(a - c_i) - \gamma(a - c_j)}{4 - \gamma^2}. \tag{1}$$

Therefore, we can rewrite firm i's profit as

$$\pi_i(c_i) = \big(a - q_i(c_i, c_j) - \gamma q_j(c_i, c_j) - c_i\big)q_i(c_i, c_j) - F(c_i), \tag{2}$$

which represents the expected profit for firm i in stage 1 given firm j's technology choice c_j. The second-order condition for an interior maximum to $\pi_i(c_i)$ is that

$$F''(c_i) > 2(\frac{2}{4 - \gamma^2})^2 \equiv g^C(\gamma), \quad \forall c_i \in (c_\beta, c_\alpha). \tag{3}$$

[1]In other places where i and/or j appear, we also have $i, j = 1, 2, i \neq j$ unless otherwise specified.

Under Bertrand competition, each firm decides on its price to maximize profit in the second stage, given technology choices in stage 1 and the rate of price of the other firm:

$$\max_{p_i}(p_i - c_i)\frac{(1-\gamma)a - p_i + \gamma p_j}{1-\gamma^2} - F(c_i).$$

The first-order condition to the above profit maximizing problem yields that

$$p_i(c_i, c_j) = \frac{(2+\gamma)(1-\gamma)a + 2c_i + \gamma c_j}{4-\gamma^2}. \tag{4}$$

We thus obtain firm i's expected profit in stage 1 as

$$\pi_i(c_i) = \big(p_i(c_i, c_j) - c_i\big)\frac{(1-\gamma)a - p_i(c_i, c_j) + \gamma p_j(c_i, c_j)}{1-\gamma^2} - F(c_i). \tag{5}$$

The second-order condition for an interior maximum to $\pi_i(c_i)$ is that

$$F''(c_i) > \frac{2}{1-\gamma^2}(\frac{2-\gamma^2}{4-\gamma^2})^2 \equiv g^B(\gamma), \quad \forall c_i \in (c_\beta, c_\alpha). \tag{6}$$

The terms $g^B(\gamma)$ and $g^C(\gamma)$ satisfy the following relationship for all $\gamma \in (0,1)$ (see derivation in the Appendix):

$$g^B(\gamma) > g^C(\gamma) > 1/2.$$

The above analysis implies that if the continuous technology set is insufficiently convex (i.e., $F''(c_i) < g^C(\gamma), \forall c_i \in (c_\beta, c_\alpha)$), then both firms' stage-one profit functions are convex under either Cournot or Bertrand competition, leading them to choose only the corners of the technology set (i.e., $c_i \in \{c_\beta, c_\alpha\}$). This situation corresponds to the two-choice technology adoption model in previous research such as Elberfeld (2003) and Elberfeld and Nti (2004).

In a closely related paper, Pal (2010) investigates technology adoption in a differentiated duopoly under two modes of competition (Cournot and Bertrand) assuming that firms choose between two alternative production technologies. Pal's model corresponds to the case of $t \leq 1/4$ in our model in which $F''(c_i) \leq 1/2$. That is, Pal (2010) focuses on the case that the technology set is insufficiently convex.

In this paper, we shall focus on the interesting but still unexplored case in which the technology set is sufficiently convex. More specifically, we focus on the situation in which $t > 1/4$ and the degree of product differentiation is sufficiently high such that $0 < \gamma < \gamma^*$, where $g^B(\gamma^*) = F''(c_i) = 2t$. With this specification, $F''(c_i) > g^B(\gamma) > g^C(\gamma), \forall c_i \in (c_\beta, c_\alpha)$.[2] As a result, both firms' stage-one profit functions are concave under either Cournot or Bertrand, and both will choose from a continuous technology set rather than the two extreme technologies as in previous literature.

Without loss of generality, we assume $t = 1$ and focus on $0 < \gamma < \gamma^* = 0.93$.[3] Hence, the fixed cost function becomes $F(c) = c^2 - \lambda c + k$. As λ rises, fixed cost falls for all c. The cost parameter λ in our model plays a similar role as r in Pal (2010), and many of our results below are provided in terms of the level of λ.

2.1. Equilibrium under Cournot Competition

Under Cournot competition, in stage 1 firms choose technologies to maximize expected profits in (2). The first-order condition for firm i is

$$\frac{2}{4-\gamma^2}\frac{2[\gamma(a-c_j)-2(a-c_i)]}{4-\gamma^2} = F'(c_i) = 2c_i - \lambda,$$

which yields the best response functions

$$c_i = \frac{4\gamma(a-c_j) - 8a + (4-\gamma^2)^2\lambda}{2(4-\gamma^2)^2 - 8}. \tag{7}$$

Solving for equilibrium we have

$$c_1^* = c_2^* = \frac{(2+\gamma)^2(2-\gamma)\lambda - 4a}{2(2+\gamma)^2(2-\gamma) - 4} \equiv c_c^*. \tag{8}$$

Straightforward calculations imply that $c_\beta < c_c^* < c_\alpha$ if and only if $\underline{\lambda}^C < \lambda < \overline{\lambda}^C$, where

$$\underline{\lambda}^C = 2c_\beta + \frac{4(a-c_\beta)}{(2+\gamma)^2(2-\gamma)}, \tag{9}$$

$$\overline{\lambda}^C = 2c_\alpha + \frac{4(a-c_\alpha)}{(2+\gamma)^2(2-\gamma)}. \tag{10}$$

The next proposition summarizes the above results.

PROPOSITION 1. *Under Cournot competition, in equilibrium the firms choose (c_β, c_β) if $\lambda \le \underline{\lambda}^C$; (c_α, c_α) if $\lambda \ge \overline{\lambda}^C$; and (c_c^*, c_c^*) if $\underline{\lambda}^C < \lambda < \overline{\lambda}^C$.*

[2]Our analysis can be easily extended to the case where $g^C(\gamma) < F''(c_i) < g^B(\gamma)$. In this case the equilibrium outcomes under Cournot competition in general admit interior solutions (as in the case that $F''(c_i) > g^B(\gamma)$) while the equilibrium outcomes under Bertrand competition always lead to extreme solutions (as in the case that $F''(c_i) < g^C(\gamma)$).

[3]The threshold value $\gamma^* = 0.93$ is obtained from $g^B(\gamma^*) = 2$.

We may interpret choosing c_β as making full investment in new technology, choosing c_α as making no investment in new technology, and choosing c_c^* as making intermediate investment. Notice that both firms always adopt identical technology. In equilibrium with technology (c^*, c^*), we have $q_i^C(c^*, c^*) = p_i^C(c^*, c^*) - c^* = \frac{a-c^*}{2+\gamma}$, and $\pi_i^C(c^*, c^*) = \frac{(a-c^*)^2}{(2+\gamma)^2} - F(c^*)$, $i = 1, 2$.[4]

2.2. Equilibrium under Bertrand Competition

Under Bertrand competition, in stage 1 firms choose technologies to maximize expected profits in (5). The first-order condition for firm i is

$$\frac{\gamma^2 - 2}{(1-\gamma^2)(4-\gamma^2)} \left(\frac{2(1-\gamma)a}{2-\gamma} + \frac{2\gamma c_j + 2(\gamma^2 - 2)c_i}{4-\gamma^2} \right) = 2c_i - \lambda,$$

which yields the best response function

$$c_i = \frac{-2\gamma(2-\gamma^2)c_j + (1-\gamma^2)(4-\gamma^2)^2\lambda - 2(2-\gamma^2)(2-\gamma-\gamma^2)a}{2(1-\gamma^2)(4-\gamma^2)^2 - 2(2-\gamma^2)^2}. \quad (11)$$

Solving for equilibrium we have[5]

$$c_1^* = c_2^* = \frac{2(\gamma^2 - 2)a + \lambda(2+\gamma-\gamma^2)(4-\gamma^2)}{2(\gamma^2-2) + 2(2+\gamma-\gamma^2)(4-\gamma^2)} \equiv c_b^*. \quad (12)$$

Straightforward calculations imply that $c_\beta < c_b^* < c_\alpha$ if and only if $\underline{\lambda}^B < \lambda < \overline{\lambda}^B$, where

$$\underline{\lambda}^B = 2c_\beta + \frac{2(2-\gamma^2)(a-c_\beta)}{(2+\gamma-\gamma^2)(4-\gamma^2)}, \quad (13)$$

$$\overline{\lambda}^B = 2c_\alpha + \frac{2(2-\gamma^2)(a-c_\alpha)}{(2+\gamma-\gamma^2)(4-\gamma^2)}. \quad (14)$$

The next proposition summarizes the above results.

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[4]We show how firms use innovation as a strategic device to compete in a Cournot duopoly game. In different settings, some economists examine firms' competition from other aspects, such as capacity investment, inventory investment and strategic outsourcing. See in Ohnishi(2009, 2011) and Chen (2010).

[5]Note that asymmetric equilibria may occur when $\underline{\lambda}^B < \lambda < \overline{\lambda}^B$. If the best response equations (11) have slopes less than -1 (or equivalently, $\gamma < 0.8658$), in addition to the interior Nash equilibrium, there can be two asymmetric Nash equilibria which are located on the boundary of $[c_\beta, c_\alpha] \times [c_\beta, c_\alpha]$. In the following analysis, we ignore the two asymmetric equilibria since they are unstable.

PROPOSITION 2. *Under Bertrand competition, in equilibrium the firms choose* (c_β, c_β) *if* $\lambda \leq \underline{\lambda}^B$*;* (c_α, c_α) *if* $\lambda \geq \overline{\lambda}^B$*; and* (c_b^*, c_b^*) *if* $\underline{\lambda}^B < \lambda < \overline{\lambda}^B$.

In equilibrium with technology (c^*, c^*), we have $q_i^B(c^*, c^*) = \frac{p_i^B(c^*,c^*)-c^*}{1-\gamma^2} = \frac{a-c^*}{(2-\gamma)(1+\gamma)}$, and $\pi_i^B(c^*, c^*) = \frac{(a-c^*)^2(1-\gamma)}{(2-\gamma)^2(1+\gamma)} - F(c^*)$, $i = 1, 2$.

3. COMPARISON OF COURNOT AND BERTRAND EQUILIBRIA

In this section we compare Cournot and Bertrand equilibria in terms of firms' technology adoption decisions and social welfare.

3.1. Cournot vs. Bertrand in Technology Adoption

To see how firms' technology adoption decisions differ under alternative modes of product market competition, we obtain the relationship between the critical values given in (9), (10), (13) and (14), in the following lemma.

LEMMA 1. *Denote* $H(\gamma) = \frac{\gamma^3}{(6+4\gamma-2\gamma^2-\gamma^3)(2-\gamma)(1+\gamma)}$.[6] *We have that*

(i) $\underline{\lambda}^B < \underline{\lambda}^C \leq \bar{\lambda}^B < \bar{\lambda}^C$ *if* $c_\alpha - c_\beta \geq H(\gamma)(a - c_\alpha)$*;*
(ii) $\underline{\lambda}^B < \bar{\lambda}^B < \underline{\lambda}^C < \bar{\lambda}^C$ *if* $c_\alpha - c_\beta < H(\gamma)(a - c_\alpha)$.

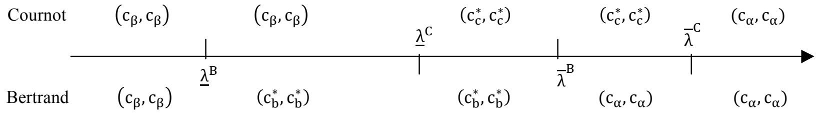
Figure 1 depicts the pure-strategy Nash equilibrium choices of firms regarding technology adoption corresponding to different levels of λ under alternative modes of competition. The two patterns correspond to the two cases in Lemma 1.

Clearly, Cournot competition and Bertrand competition lead to different choices of technology for $\underline{\lambda}^B < \lambda < \bar{\lambda}^C$, while both firms choose (c_β, c_β) for $\lambda \leq \underline{\lambda}^B$ and (c_α, c_α) for $\lambda \geq \bar{\lambda}^C$. Secondly, the range for both firms to make full investment in new technology is wider under Cournot competition than under Bertrand competition. On the contrary, the range for no firm to adopt the best (i.e., lowest marginal cost) technology is narrower under Cournot competition than under Bertrand competition. Furthermore, Cournot competition leads to more full investment while Bertrand competition leads to more zero investment. Firms may either make intermediate investment in certain regions depending on the value of the adoption cost parameter λ.

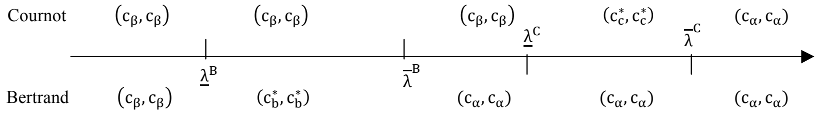
A natural question then arises regarding which competition mode leads to more interior solutions and which one in general invests more to reduce marginal cost.

[6] Note that $H(\gamma)$ is an increasing function of γ.

FIG. 1. Technology adoption under Cournot and Bertrand



(a) Pattern 1: $c_\alpha - c_\beta \geq H(\gamma)(a - c_\alpha)$



(b) Pattern 2: $c_\alpha - c_\beta < H(\gamma)(a - c_\alpha)$

LEMMA 2. *Regarding interior solutions, we have that*

(i) $\bar{\lambda}^B - \underline{\lambda}^B > \bar{\lambda}^C - \underline{\lambda}^C$*;*
(ii) $c_b^* - c_c^* > 0$*, and* $\partial(c_b^* - c_c^*)/\partial\lambda < 0$*.*

Lemma 2(i) implies that Bertrand Competition leads to more interior technology solutions than Cournot competition. When both firms choose an interior technology, the marginal cost is lower under Cournot competition and the difference in marginal costs decreases with the cost of technology adoption. Combining the results in Lemma 2(ii) and Figure 1 yields at any given λ, both firms are (weakly) more willing to invest to reduce the marginal cost under Cournot competition than under Bertrand competition.

The results in Lemma 1 and Lemma 2 give us the following Proposition.

PROPOSITION 3. *Firms choose symmetric technology under both Cournot and Bertrand competition. Although Bertrand competition leads to more interior technology solutions than Cournot competition, firms invest more on technology to reduce marginal costs under Cournot competition.*

Our results are based on the assumption that the technology set is sufficiently convex so that both firms choose from a continuous technology set rather than the two extreme technologies in the set. However, if the technology set is insufficiently convex, then both firms choose from the two extreme technologies and conclusions similar to those in Pal (2010) apply.

3.2. Cournot vs. Bertrand in Welfare

Next, we move on to the ranking of Cournot and Bertrand competition in terms of social welfare. We start with the equilibrium prices and quantities. Based on the results in Section 2.1 and Section 2.2, we directly compare equilibrium prices and quantities under two modes of competition in each region in Figure 1 and then obtain the following two lemmas.

LEMMA 3. *The equilibrium prices of both firms are higher under Cournot competition than under Bertrand competition.*

LEMMA 4. *The equilibrium quantities of both firms are larger under Bertrand competition than under Cournot competition.*

Implied by the above two lemmas, although firms invest more on technology to reduce marginal costs of production under Cournot competition than Bertrand competition (Proposition 3), firms still charge lower prices and produce more under Bertrand competition than under Cournot competition. It also follows from Lemma 3 that consumer surplus is higher under Bertrand competition since the equilibrium prices of both firms are lower under Bertrand competition.

We next turn to the ranking of firms' profits.

LEMMA 5. *Both firm's profits are higher under Cournot competition than under Bertrand competition.*

Lemma 5 implies that higher technology adoption under Cournot competition generates higher industry profit.

Based on the above results, a switch from Cournot competition to Bertrand competition increases consumer surplus but decreases firms' profits. As a result, the welfare ranking of the Cournot and Bertrand equilibria is not so clear. Given the equilibrium strategy (c^*, c^*), we obtain equilibrium consumer surplus ($CS = \frac{1}{2}(q_1^2 + 2\gamma q_1 q_2 + q_2^2)$ under both modes of competition as

$$CS^C(c^*, c^*) = (1+\gamma)(\frac{a-c^*}{2+\gamma})^2, \quad CS^B(c^*, c^*) = \frac{(a-c^*)^2}{(2-\gamma)^2(1+\gamma)}.$$

Thus, we are able to obtain social welfare under both modes, which is the sum of consumer surplus and industry profits.

$$W^C(c^*, c^*) = (3+\gamma)(\frac{a-c^*}{2+\gamma})^2 - 2F(c^*), \tag{15}$$

$$W^B(c^*, c^*) = (3 - 2\gamma)\frac{(a - c^*)^2}{(2 - \gamma)^2(1 + \gamma)} - 2F(c^*). \tag{16}$$

PROPOSITION 4. *In a two-stage duopoly game where firms simultaneously decide the optimal technology adoption from a continuous technology set in stage one and compete in either Cournot or Bertrand competition in stage two, Social welfare is higher under Bertrand competition than under Cournot competition.*

Although a switch from Bertrand competition to Cournot competition increases firms' profits, the gain in firms' profits is smaller than the loss in consumer surplus. Thus, as for social welfare, Bertrand competition yields more. That is, the standard welfare ranking holds. Recall that we focus on the case that the technology set is sufficiently convex and the product differentiation is relatively high. If the technology set is insufficiently convex, firms choose two extreme technologies and Cournot competition may lead to higher social welfare than Bertrand competition in some situations (see Pal (2010)).

COROLLARY 1. *If the technology set is insufficiently convex, for any given $\gamma \in (0, 1)$, Cournot competition may lead to higher social welfare than Bertrand competition when Cournot competition leads to higher technology adoption than Bertrand competition.*

Implied by Proposition 4 and Corollary 1, the convexity of technology set and product differentiation are very critical to the social welfare ranking of the two modes of competition. The traditional efficiency result holds for a sufficiently convex technology set (large t) and high product differentiation (small γ). Otherwise, the opposite results may hold in some situations.

4. CONCLUSION

In a horizontally differentiated industry, firms have the motivation to employ a cost-reducing technology to gain a competitive advantage. Economists have investigated from different aspects to see how technology adoption occurs in a duopoly as a result of strategic choices. Among them, some researchers focused attention on the comparison of Cournot and Bertrand competition to see which one provides a greater incentive to innovate.

In this paper we revisited Cournot and Bertrand competition in a differentiated duopoly with endogenous technology adoption. We assume that the technology set is a continuous interval as opposed to the two choice model in the literature to investigate firms' incentives to adopt new technologies under Cournot and Bertrand competition. With this enlarged parameter region, we find that the convexity of technology set and the degree of product differentiation play a very important role in technology adoption and efficiency comparison of the two modes of competition. Specifically, with our attention focused on the case with a sufficiently convex technology set and sufficiently high product differentiation, we show that Bertrand competition leads to more interior solutions but Cournot competition provides more incentives to invest in technology. Furthermore, the standard ranking of welfare continues to hold in this dynamic environment.

APPENDIX: PROOFS

Derivation for $g^B(\gamma) > g^C(\gamma)$:

It is obvious from (3) that $g^C(\gamma)$ is increasing in γ and therefore $g^C(\gamma) > g^C(0) = 1/2$. Furthermore, based on (3) and (6), we have

$$g^B(\gamma) - g^C(\gamma) = \frac{2}{(4-\gamma^2)^2}\left(\frac{(2-\gamma^2)^2}{1-\gamma^2} - 4\right) = \frac{2}{(4-\gamma^2)^2}\frac{\gamma^4}{1-\gamma^2} > 0.$$

Proof of Proposition 1:

Since their profit functions are concave, both firms choose interior technology when $c_\beta < c_c^* < c_\alpha$. That is, $c_\beta < \frac{(2+\gamma)^2(2-\gamma)\lambda - 4a}{2(2+\gamma)^2(2-\gamma)-4} < c_\alpha$, which reduces to

$$2c_\beta + \frac{4(a-c_\beta)}{(2+\gamma)^2(2-\gamma)} < \lambda < 2c_\alpha + \frac{4(a-c_\alpha)}{(2+\gamma)^2(2-\gamma)}.$$

Since $4\gamma < 2(4-\gamma^2)^2 - 8$, the slope of the best response equation (7), $\frac{-4\gamma}{2(4-\gamma^2)^2-8} > -1$. As a result, the interior solution obtained in this case is unique.

Furthermore, each firm's best response curve is always equal to c_β if $\lambda \leq 2c_\beta + \frac{4(a-c_\beta)}{(2+\gamma)^2(2-\gamma)}$ and c_α if $\lambda \geq 2c_\alpha + \frac{4(a-c_\alpha)}{(2+\gamma)^2(2-\gamma)}$. Thus we have the results.

Proof of Proposition 2:

Since their profit functions are concave, both firms choose interior technology when $c_\beta < c_b^* < c_\alpha$. That is, $c_\beta < \frac{2(\gamma^2-2)a+\lambda(2+\gamma-\gamma^2)(4-\gamma^2)}{2(\gamma^2-2)+2(2+\gamma-\gamma^2)(4-\gamma^2)} < c_\alpha$, which reduces to

$$2c_\beta + \frac{2(2-\gamma^2)(a-c_\beta)}{(2+\gamma-\gamma^2)(4-\gamma^2)} < \lambda < 2c_\alpha + \frac{2(2-\gamma^2)(a-c_\alpha)}{(2+\gamma-\gamma^2)(4-\gamma^2)}.$$

Furthermore, each firm's best response curve is always equal to c_β if $\lambda \le 2c_\beta + \frac{2(2-\gamma^2)(a-c_\beta)}{(2+\gamma-\gamma^2)(4-\gamma^2)}$ and c_α if $\lambda \ge 2c_\alpha + \frac{2(2-\gamma^2)(a-c_\alpha)}{(2+\gamma-\gamma^2)(4-\gamma^2)}$. Thus we have the results.

Proof of Lemma 1:

We first show that $\bar{\lambda}^C > \bar{\lambda}^B$ and $\underline{\lambda}^C > \underline{\lambda}^B$. From (10) and (14), we obtain that

$$\begin{aligned}
\bar{\lambda}^C - \bar{\lambda}^B &= 2c_\alpha + \frac{4(a-c_\alpha)}{(2+\gamma)^2(2-\gamma)} - 2c_\alpha - \frac{2(2-\gamma^2)(a-c_\alpha)}{(2+\gamma-\gamma^2)(4-\gamma^2)} \\
&= \frac{2(a-c_\alpha)}{4-\gamma^2}\left(\frac{2}{2+\gamma} - \frac{2-\gamma^2}{(1+\gamma)(2-\gamma)}\right) \\
&= \frac{2(a-c_\alpha)}{4-\gamma^2}\frac{\gamma^3}{(4-\gamma^2)(1+\gamma)} \\
&> 0.
\end{aligned}$$

Similarly, we have

$$\underline{\lambda}^C = 2c_\beta + \frac{4(a-c_\beta)}{(2+\gamma)^2(2-\gamma)} > \underline{\lambda}^B = 2c_\beta + \frac{2(2-\gamma^2)(a-c_\beta)}{(2+\gamma-\gamma^2)(4-\gamma^2)}.$$

Next, we focus on the relationship between $\bar{\lambda}^B$ and $\underline{\lambda}^C$. We have that

$$\begin{aligned}
\bar{\lambda}^B - \underline{\lambda}^C &= 2c_\alpha + \frac{2(2-\gamma^2)(a-c_\alpha)}{(2+\gamma-\gamma^2)(4-\gamma^2)} - 2c_\beta - \frac{4(a-c_\beta)}{(2+\gamma)^2(2-\gamma)} \\
&= 2(c_\alpha - c_\beta) + \frac{2}{4-\gamma^2}\left(\frac{(2-\gamma^2)(a-c_\alpha)}{2+\gamma-\gamma^2} - \frac{2(a-c_\beta)}{2+\gamma}\right) \\
&= 2(c_\alpha - c_\beta) + \frac{2}{4-\gamma^2}\left(\frac{(2-\gamma^2)(a-c_\alpha)}{2+\gamma-\gamma^2} - \frac{2(a-c_\alpha)}{2+\gamma} - \frac{2(c_\alpha-c_\beta)}{2+\gamma}\right) \\
&= (2 - \frac{4}{(4-\gamma^2)(2+\gamma)})(c_\alpha - c_\beta) - \frac{2\gamma^3}{(4-\gamma^2)^2(1+\gamma)}(a-c_\alpha) \\
&= \frac{2(6+4\gamma-2\gamma^2-\gamma^3)}{(4-\gamma^2)(2+\gamma)}(c_\alpha - c_\beta) - \frac{2\gamma^3}{(4-\gamma^2)^2(1+\gamma)}(a-c_\alpha).
\end{aligned}$$

Hence, $\bar{\lambda}^B \ge \underline{\lambda}^C$ if $c_\alpha - c_\beta \ge \frac{\gamma^3(a-c_\alpha)}{(6+4\gamma-2\gamma^2-\gamma^3)(2-\gamma)(1+\gamma)}$, and $\bar{\lambda}^B < \underline{\lambda}^C$ if $c_\alpha - c_\beta < \frac{\gamma^3(a-c_\alpha)}{(6+4\gamma-2\gamma^2-\gamma^3)(2-\gamma)(1+\gamma)}$.

Proof of Lemma 2:

We first look at part (i). From (9) and (10), we have that

$$\overline{\lambda}^C - \underline{\lambda}^C = 2(c_\alpha - c_\beta)(1 - \frac{2}{(2+\gamma)^2(2-\gamma)}).$$

From (13) and (14), we have that

$$\overline{\lambda}^B - \underline{\lambda}^B = 2(c_\alpha - c_\beta)(1 - \frac{2-\gamma^2}{(2+\gamma-\gamma^2)(4-\gamma^2)}).$$

As a result,

$$\begin{aligned}
&\overline{\lambda}^B - \underline{\lambda}^B - (\overline{\lambda}^C - \underline{\lambda}^C) \\
&= 2(c_\alpha - c_\beta)\left(\frac{2}{(2+\gamma)^2(2-\gamma)} - \frac{2-\gamma^2}{(2+\gamma-\gamma^2)(4-\gamma^2)}\right) \\
&= 2(c_\alpha - c_\beta)\frac{2(2-\gamma)(1+\gamma) - (2-\gamma^2)(2+\gamma)}{(2+\gamma)^2(2-\gamma)^2(1+\gamma)} \\
&= 2(c_\alpha - c_\beta)\frac{\gamma^3}{(2+\gamma)^2(2-\gamma)^2(1+\gamma)} \\
&> 0.
\end{aligned}$$

Thus we have part (i).

To prove part (ii), we rewrite c_c^* and c_b^* as follows:

$$c_c^* = \frac{\lambda(2+\gamma)^2(2-\gamma) - 4a}{2(2+\gamma)^2(2-\gamma) - 4} = a + \frac{(\lambda - 2a)(2+\gamma)^2(2-\gamma)}{2(2+\gamma)^2(2-\gamma) - 4},$$

$$c_b^* = \frac{2(\gamma^2-2)a + \lambda(2+\gamma-\gamma^2)(4-\gamma^2)}{2(\gamma^2-2) + 2(2+\gamma-\gamma^2)(4-\gamma^2)} = a + \frac{(\lambda-2a)(2+\gamma-\gamma^2)(4-\gamma^2)}{2(\gamma^2-2) + 2(2+\gamma-\gamma^2)(4-\gamma^2)}.$$

As a result,

$$\begin{aligned}
&c_b^* - c_c^* \\
&= \frac{(\lambda-2a)(4-\gamma^2)}{2}\left(\frac{(2-\gamma)(1+\gamma)}{(\gamma^2-2) + (2+\gamma-\gamma^2)(4-\gamma^2)} - \frac{2+\gamma}{(2+\gamma)^2(2-\gamma) - 2}\right) \\
&= \frac{(\lambda-2a)(4-\gamma^2)}{2}\frac{-\gamma^3}{\big((\gamma^2-2) + (2+\gamma-\gamma^2)(4-\gamma^2)\big)\big((2+\gamma)^2(2-\gamma) - 2\big)} \\
&= (2a - \lambda)\frac{\gamma^3(4-\gamma^2)}{2\big((\gamma^2-2) + (2+\gamma-\gamma^2)(4-\gamma^2)\big)\big((2+\gamma)^2(2-\gamma) - 2\big)}.
\end{aligned}$$

It is obvious that the fraction (the second term) in the last expression has a positive sign because $0 < \gamma < 1$. Therefore we have $\partial(c_b^* - c_c^*)/\partial\lambda < 0$.

Next we look at the first term in the last expression. Since $\underline{\lambda}^B < \lambda < \bar{\lambda}^C$, we have

$$\begin{aligned}2a - \lambda > 2a - \bar{\lambda}^C &= 2a - 2c_\alpha - \frac{4(a - c_\alpha)}{(2+\gamma)^2(2-\gamma)} \\ &= 2(a - c_\alpha)(1 - \frac{2}{(2+\gamma)^2(2-\gamma)}) \\ &> 0.\end{aligned}$$

Thus we have $c_b^* - c_c^* > 0$. That is, $c_b^* > c_c^*$.

Proof of Lemma 3:

Under Cournot competition, $p_i^C(c^*, c^*) = \frac{a-c^*}{2+\gamma} + c^*$, and under Bertrand competition $p_i^B(c^*, c^*) = \frac{(a-c^*)(1-\gamma)}{2-\gamma} + c^*$. When technology adoption is identical under both Cournot and Bertrand competition, it follows obviously that $p_i^C > p_i^B$. Next, we show that $p_i^C > p_i^B$ holds when technology adoptions are different under Cournot and Bertrand competition.

(i). When firms choose (c_β, c_β) under Cournot competition and (c_b^*, c_b^*) under Bertrand competition, we have $\lambda < \underline{\lambda}^C$. After simplification, we obtain that

$$\begin{aligned}p_i^C(c_\beta, c_\beta) - p_i^B(c_b^*, c_b^*) &= \frac{a - c_\beta}{2+\gamma} + c_\beta - (\frac{(a - c_b^*)(1-\gamma)}{2-\gamma} + c_b^*) \\ &= \frac{(1+\gamma)\left(2a(2+3\gamma^2-\gamma^4) + 2c_\beta(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4) + (\gamma-2)(2+\gamma)^2\lambda\right)}{2(2+\gamma)(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)},\end{aligned}$$

which is a decreasing function of λ. Thus, $p_i^C(c_\beta, c_\beta) - p_i^B(c_b^*, c_b^*)$ is greater than

$$\frac{(1+\gamma)\left(2a(2+3\gamma^2-\gamma^4) + 2c_\beta(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4) + (\gamma-2)(2+\gamma)^2\underline{\lambda}^C\right)}{2(2+\gamma)(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)},$$

which can be reduced to

$$\frac{2(1+\gamma)(a - c_\beta)\gamma^2(3-\gamma^2)}{2(2+\gamma)(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)}.$$

Therefore, we have $p_i^C(c_\beta, c_\beta) - p_i^B(c_b^*, c_b^*) > 0$.

(ii). When firms choose (c_c^*, c_c^*) under Cournot competition and (c_b^*, c_b^*) under Bertrand competition, we have $\underline{\lambda}^C < \lambda < \bar{\lambda}^B$. After simplification, we obtain that

$$p_i^C(c_c^*, c_c^*) - p_i^B(c_b^*, c_b^*) = -\frac{\gamma^2(12+12\gamma-7\gamma^2-7\gamma^3+\gamma^4+\gamma^5))(2a-\lambda)}{2(-6-4\gamma+2\gamma^2+\gamma^3)(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)},$$

which is a decreasing function of λ. Thus,

$$p_i^C(c_c^*, c_c^*) - p_i^B(c_b^*, c_b^*) > -\frac{\gamma^2(12+12\gamma-7\gamma^2-7\gamma^3+\gamma^4+\gamma^5))(2a-\bar{\lambda}^B)}{2(-6-4\gamma+2\gamma^2+\gamma^3)(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)}$$
$$= \frac{(a-c_\alpha)\gamma^2(3-\gamma^2)}{12+2\gamma-8\gamma^2+\gamma^4} > 0.$$

(iii). Similarly, after simplification we obtain that

$$p_i^C(c_c^*, c_c^*) - p_i^B(c_\alpha, c_\alpha) = \frac{2a(\gamma^2-2)+(-4-4\gamma+\gamma^2+\gamma^3)\lambda}{2(-6-4\gamma+2\gamma^2+\gamma^3)} - c_\alpha - \frac{(a-c_\alpha)(1-\gamma)}{2-\gamma},$$

which increases in λ. Thus,

$$\begin{aligned} & p_i^C(c_c^*, c_c^*) - p_i^B(c_\alpha, c_\alpha) \\ > & \frac{2a(\gamma^2-2)+(-4-4\gamma+\gamma^2+\gamma^3)\bar{\lambda}^B}{2(-6-4\gamma+2\gamma^2+\gamma^3)} - c_\alpha - \frac{(a-c_\alpha)(1-\gamma)}{2-\gamma} \\ = & \frac{(a-c_\alpha)\gamma^2(3-\gamma^2)}{12+2\gamma-8\gamma^2+\gamma^4} > 0. \end{aligned}$$

(iv). We next show that $p_i^C(c_\beta, c_\beta) > p_i^B(c_\alpha, c_\alpha)$ when pattern 2 arises. Note that for pattern 2 we have $c_\alpha - c_\beta < H(\gamma)(a-c_\alpha)$, which can be rewritten as $(a-c_\beta) < (1+H(\gamma))(a-c_\alpha)$. We therefore have

$$\begin{aligned} p_i^C(c_\beta, c_\beta) - p_i^B(c_\alpha, c_\alpha) &= \frac{a-c_\alpha}{2-\gamma} - \frac{(1+\gamma)(a-c_\beta)}{2+\gamma} \\ &> \frac{a-c_\beta}{(2-\gamma)(1+H(\gamma))} - \frac{(1+\gamma)(a-c_\beta)}{2+\gamma} \\ &= \frac{\gamma^2(1+\gamma)(3-\gamma^2)}{(2+\gamma)(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)} > 0. \end{aligned}$$

Proof of Lemma 4:

When technology adoption is identical under both Cournot and Bertrand competition, it follows obviously that $q_i^B > q_i^C$. The following analysis is similar to the proof for Lemma 3.

(i). After simplification, we obtain that

$$\begin{aligned} & q_i^B(c_\beta, c_\beta) - q_i^C(c_b^*, c_b^*) \\ = & \frac{a(4+6\gamma^2-2\gamma^4)+2c_\beta(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)+(\gamma^2-4)\lambda}{2(2+\gamma)(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)}, \end{aligned}$$

which decreases in λ. Thus, $q_i^B(c_\beta, c_\beta) - q_i^C(c_b^*, c_b^*)$ is greater than

$$\frac{a(4+6\gamma^2-2\gamma^4)+2c_\beta(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)+(\gamma^2-4)\underline{\lambda}^C}{2(2+\gamma)(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)},$$

which can be reduced to

$$\frac{(a-c_\beta)\gamma^2(3-\gamma^2)}{2(2+\gamma)(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)}.$$

Therefore, we have $q_i^B(c_\beta, c_\beta) - q_i^C(c_b^*, c_b^*) > 0$.

(ii). After simplification, we obtain that

$$q_i^B(c_b^*, c_b^*) - q_i^C(c_c^*, c_c^*) = -\frac{\gamma^2(12-7\gamma^2+\gamma^4))(2a-\lambda)}{2(-6-4\gamma+2\gamma^2+\gamma^3)(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)},$$

which is a decreasing function of λ. Thus,

$$\begin{aligned} q_i^B(c_b^*, c_b^*) - q_i^C(c_c^*, c_c^*) &> -\frac{\gamma^2(12-7\gamma^2+\gamma^4))(2a-\bar{\lambda}^B)}{2(-6-4\gamma+2\gamma^2+\gamma^3)(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)} \\ &= \frac{(a-c_\alpha)\gamma^2(3-\gamma^2)}{(2-\gamma)(1+\gamma)(6+4\gamma-2\gamma^2-\gamma^3)} > 0. \end{aligned}$$

(iii). Similarly, after simplification we obtain that

$$q_i^B(c_\alpha, c_\alpha) - q_i^C(c_c^*, c_c^*) = \frac{a-c_\alpha}{(2-\gamma)(1+\gamma)} - \frac{(\gamma^2-4)(2a-\lambda)}{2(-6-4\gamma+2\gamma^2+\gamma^3)},$$

which increases in λ. Thus,

$$\begin{aligned} q_i^B(c_\alpha, c_\alpha) - q_i^C(c_c^*, c_c^*) &> \frac{a-c_\alpha}{(2-\gamma)(1+\gamma)} - \frac{(\gamma^2-4)(2a-\bar{\lambda}^B)}{2(-6-4\gamma+2\gamma^2+\gamma^3)} \\ &= \frac{(a-c_\alpha)\gamma^2(3-\gamma^2)}{(2-\gamma)(1+\gamma)(6+4\gamma-2\gamma^2-\gamma^3)} > 0. \end{aligned}$$

(iv). After simplification, we obtain that

$$q_i^B(c_\alpha, c_\alpha) - q_i^C(c_\beta, c_\beta) = \frac{a-c_\alpha}{(2-\gamma)(1+\gamma)} - \frac{a-c_\beta}{2+\gamma}.$$

Following part (iv) in the proof for Lemma 3, we immediately obtain that $q_i^B(c_\alpha, c_\alpha) > q_i^C(c_\beta, c_\beta)$.

Proof of Lemma 5:

First, when technology adoption is identical under both Cournot and Bertrand competition, for example (c_β, c_β), we have that

$$\pi_i^C(c_\beta, c_\beta) - \pi_i^B(c_\beta, c_\beta) = (a-c_\beta)^2 \frac{2\gamma^3}{(2+\gamma)^2(2-\gamma)^2(1+\gamma)} > 0.$$

Similarly, $\pi_i^C(c_\alpha, c_\alpha) - \pi_i^B(c_\alpha, c_\alpha) > 0$. Next we look at the situations when technology adoptions are different under the two modes of competition.

(i). In pattern 1, if $\underline{\lambda}^B < \lambda < \underline{\lambda}^C$, and (c_b^*, c_b^*) under Bertrand competition. we have $\pi_i^C(c_\beta, c_\beta) - \pi_i^B(c_b^*, c_b^*) = \frac{(a-c_\beta)^2}{(2+\gamma)^2} - \frac{(a-c_b^*)^2(1-\gamma)}{(2-\gamma)^2(1+\gamma)} - (\lambda - c_b^* - c_\beta)(c_b^* - c_\beta)$, where c_b^* is given in (12) as a function of λ. As a result, we write

$$\pi_i^C(c_\beta, c_\beta) - \pi_i^B(c_b^*, c_b^*) \triangleq F_1(\lambda).$$

Next we show that $F_1(\lambda)$ decreases in λ.

To do that, we denote $\frac{\partial F_1(\lambda)}{\partial \lambda} \triangleq G_1(\lambda)$. Differentiating $F_1(\lambda)$ with regard to λ yields $G_1(\lambda) = \frac{1}{4}\Big(\frac{2(-1+\gamma)(1+\gamma)(-4+\gamma^2)^2(-2a+\lambda)}{(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)^2}$
$+ \frac{\left((-4-4\gamma+4\gamma^2+\gamma^3-\gamma^4)\left(2a(-2+\gamma^2)-2c_\beta(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)+(-2+\gamma)^2(2+3\gamma+\gamma^2)\lambda\right)\right)}{(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)^2}$
$+ \frac{(-2+\gamma)^2(2+3\gamma+\gamma^2)\left(2a(-2+\gamma^2)+2c_\beta(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)+(-4-4\gamma+4\gamma^2+\gamma^3-\gamma^4)\lambda\right)}{(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)^2}\Big)$

Similarly, differentiating $G_1(\lambda)$ with regard to λ yields that

$$\begin{aligned}\frac{\partial G_1(\lambda)}{\partial \lambda} =& \frac{1}{4}\Big(\frac{2(\gamma^2-1)(-4+\gamma^2)^2}{(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)^2} \\ &+ \frac{2(-2+\gamma)^2(2+3\gamma+\gamma^2)(-4-4\gamma+4\gamma^2+\gamma^3-\gamma^4)}{(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)^2}\Big) < 0.\end{aligned}$$

Thus, $G_1(\lambda)$ is decreasing in λ. That is, for all $\underline{\lambda}^B < \lambda < \underline{\lambda}^C$,

$$\frac{\partial F_1(\lambda)}{\partial \lambda} \triangleq G_1(\lambda) < G_1(\underline{\lambda}^B) = \frac{(-a+c_\beta)\gamma}{6+4\gamma-5\gamma^2-\gamma^3+\gamma^4} < 0,$$

which implies that $F_1(\lambda)$ is also decreasing in λ. As a result, for all $\underline{\lambda}^B < \lambda < \underline{\lambda}^C$,

$$F_1(\lambda) > F_1(\underline{\lambda}^C) = \frac{\gamma^3(-72-84\gamma+60\gamma^2+73\gamma^3-18\gamma^4-21\gamma^5+2\gamma^6+2\gamma^7)}{(-2+\gamma)(2+\gamma)^3(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)^2} > 0.$$

That is, $\pi_i^C(c_\beta, c_\beta) > \pi_i^B(c_b^*, c_b^*)$.

Similarly, for $\underline{\lambda}^B < \lambda < \bar{\lambda}^B$ in pattern 2, we also have $G_1(\lambda) < G_1(\underline{\lambda}^B) < 0$. Thus, $F_1(\lambda)$ is decreasing in λ. Furthermore, since $\underline{\lambda}^C > \bar{\lambda}^B$, we have $F_1(\lambda) > F_1(\bar{\lambda}^B) > F_1(\underline{\lambda}^C) > 0$. That is, $\pi_i^C(c_\beta, c_\beta) > \pi_i^B(c_b^*, c_b^*)$ continues to hold.

(ii). In pattern 1, if $\underline{\lambda}^C < \lambda < \bar{\lambda}^B$, we have

$$\begin{aligned}
&\pi_i^C(c_c^*, c_c^*)) - \pi_i^B(c_b^*, c_b^*)\\
&= \frac{(a-c_c^*)^2}{(2+\gamma)^2} - \frac{(a-c_b^*)^2(1-\gamma)}{(2-\gamma)^2(1+\gamma)} - (\lambda - c_b^* - c_c^*)(c_b^* - c_c^*)\\
&= \frac{\gamma^3(288+336\gamma-312\gamma^2-376\gamma^3+132\gamma^4+157\gamma^5-26\gamma^6-29\gamma^7+2\gamma^8+2\gamma^9)(\lambda-2a)^2}{4(-6-4\gamma+2\gamma^2+\gamma^3)^2(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)^2}\\
&> 0.
\end{aligned}$$

(iii). In pattern 1, if $\bar{\lambda}^B < \lambda < \bar{\lambda}^C$, we have $\pi_i^C(c_c^*, c_c^*)) - \pi_i^B(c_\alpha, c_\alpha) = \frac{(a-c_c^*)^2}{(2+\gamma)^2} - \frac{(a-c_\alpha)^2(1-\gamma)}{(2-\gamma)^2(1+\gamma)} - (\lambda - c_\alpha - c_c^*)(c_\alpha - c_c^*)$, where c_c^* is given in (8) as a function of λ. As a result, we write

$$\pi_i^C(c_c^*, c_c^*)) - \pi_i^B(c_\alpha, c_\alpha) \triangleq F_2(\lambda).$$

Next we show that $F_2(\lambda)$ increases in λ.

To do that, we denote $\frac{\partial F_2(\lambda)}{\partial \lambda} \triangleq G_2(\lambda)$. Differentiating $F_2(\lambda)$ with regard to λ yields $G_2(\lambda) = \frac{(-2+\gamma)^2(2+\gamma)^2(-2a+\lambda)}{2(-6-4\gamma+2\gamma^2+\gamma^3)^2} - \big(1 + \frac{(-2+\gamma)(2+\gamma)^2}{-4-2(-2+\gamma)(2+\gamma)^2}\big)\big(c_\alpha - \frac{-4a-(-2+\gamma)(2+\gamma)^2\lambda}{-4-2(-2+\gamma)(2+\gamma)^2}\big) - \frac{(-2+\gamma)(2+\gamma)^2\big(-c_\alpha+\lambda-\frac{-4a-(-2+\gamma)(2+\gamma)^2\lambda}{-4-2(-2+\gamma)(2+\gamma)^2}\big)}{-4-2(-2+\gamma)(2+\gamma)^2}$. We then have that

$$\frac{\partial G_2(\lambda)}{\partial \lambda} = \frac{(-2+\gamma)^2(2+\gamma)^2}{2(-6-4\gamma+2\gamma^2+\gamma^3)^2} - \frac{2(-2+\gamma)(2+\gamma)^2\big(1+\frac{(-2+\gamma)(2+\gamma)^2}{-4-2(-2+\gamma)(2+\gamma)^2}\big)}{-4-2(-2+\gamma)(2+\gamma)^2} > 0.$$

Thus, for all $\bar{\lambda}^B < \lambda < \bar{\lambda}^C$,

$$\frac{\partial F_2(\lambda)}{\partial \lambda} \triangleq G_2(\lambda) > G_2(\bar{\lambda}^B) = \frac{-(a-c_\alpha)\gamma(12+14\gamma-12\gamma^2-11\gamma^3+3\gamma^4+2\gamma^5)}{(-2+\gamma)(1+\gamma)(-6-4\gamma+2\gamma^2+\gamma^3)^2} > 0,$$

which implies that $F_2(\lambda)$ is also increasing in λ.
Thus, for all $\bar{\lambda}^B < \lambda < \bar{\lambda}^C$,

$$F_2(\lambda) > F_2(\bar{\lambda}^B) = \frac{\gamma^3(-72-84\gamma+60\gamma^2+73\gamma^3-18\gamma^4-21\gamma^5+2\gamma^6+2\gamma^7)}{(-2+\gamma)^3(1+\gamma)^2(2+\gamma)(-6-4\gamma+2\gamma^2+\gamma^3)^2} > 0$$

That is, $\pi_i^C(c_c^*, c_c^*) > \pi_i^B(c_\alpha, c_\alpha)$.

Similarly, for $\underline{\lambda}^C < \lambda < \bar{\lambda}^C$ in pattern 2, we also have $G_2(\lambda) > G_2(\underline{\lambda}^C) > G_2(\bar{\lambda}^B) > 0$. Thus, $F_2(\lambda)$ is increasing in λ. As a result, $F_2(\lambda) > F_2(\underline{\lambda}^C) > F_2(\bar{\lambda}^B) > 0$. That is, $\pi_i^C(c_c^*, c_c^*)) > \pi_i^B(c_\alpha, c_\alpha)$ continues to hold.

(iv). In pattern 2, $\bar{\lambda}^B < \lambda < \underline{\lambda}^C$, we have

$$\begin{aligned}
&\pi_i^C(c_\beta, c_\beta) - \pi_i^B(c_\alpha, c_\alpha)\\
&= \frac{(a-c_\beta)^2}{(2+\gamma)^2} - \frac{(a-c_\alpha)^2(1-\gamma)}{(2-\gamma)^2(1+\gamma)} - (\lambda - c_\alpha - c_\beta)(c_\alpha - c_\beta)\\
&> \frac{(a-c_\beta)^2}{(2+\gamma)^2} - \frac{(a-c_\alpha)^2(1-\gamma)}{(2-\gamma)^2(1+\gamma)} - (\underline{\lambda}^C - c_\alpha - c_\beta)(c_\alpha - c_\beta)\\
&= (a-c_\alpha)^2\Big(\frac{-3+\gamma^2}{-4+\gamma^2}\frac{(a-c_\beta)^2}{(a-c_\alpha)^2} + \Big(\frac{4}{(2+\gamma)^2(2-\gamma)} - 2\Big)\frac{a-c_\beta}{a-c_\alpha} + \frac{3+\gamma-3\gamma^2+\gamma^3}{(2-\gamma)^2(1+\gamma)}\Big).
\end{aligned}$$

Notice that the last equation is a quadratic expression of $\frac{a-c_\beta}{a-c_\alpha}$. Since $\frac{-3+\gamma^2}{-4+\gamma^2} > 0$ and $(\frac{4}{(2+\gamma)^2(2-\gamma)} - 2)^2 - 4(\frac{-3+\gamma^2}{-4+\gamma^2})(\frac{3+\gamma-3\gamma^2+\gamma^3}{(2-\gamma)^2(1+\gamma)}) < 0$, we therefore have that

$$\frac{-3+\gamma^2}{-4+\gamma^2}\frac{(a-c_\beta)^2}{(a-c_\alpha)^2} + \Big(\frac{4}{(2+\gamma)^2(2-\gamma)} - 2\Big)\frac{a-c_\beta}{a-c_\alpha} + \frac{3+\gamma-3\gamma^2+\gamma^3}{(2-\gamma)^2(1+\gamma)} > 0.$$

As a result, $\pi_i^C(c_\beta, c_\beta) > \pi_i^B(c_\alpha, c_\alpha)$.

Proof of Proposition 4:

First, when technology adoption is identical under both Cournot and Bertrand competition, for example (c_β, c_β), we have that

$$W^C(c_\beta, c_\beta) - W^B(c_\beta, c_\beta) = (a-c_\beta)^2\frac{\gamma^2(\gamma^2+2\gamma-4)}{(2+\gamma)^2(2-\gamma)^2(1+\gamma)} < 0.$$

Similarly, $W^C(c_\alpha, c_\alpha) - W^B(c_\alpha, c_\alpha) < 0$. Therefore, Bertrand competition generates more social welfare than Cournot competition.

Denote the marginal cost under Cournot competition by c^c and that under Bertrand competition by c^b, where $c^c < c^b$. Next, we show that when firms choose different technologies under Cournot and Bertrand competition, social welfare is lower under Cournot competition. Note that the welfare functions (15) and (16) consist of two terms, where the second term is the fixed cost of technology adoption. We know that $F(c_i)$ varies inversely with c_i, so $F(c^c) > F(c^b)$. As a result, if the first term in (15) is smaller than the first term in (16), we obviously obtain that Bertrand competition leads to higher social welfare. In the following, we show that

$$(3+\gamma)\Big(\frac{a-c^c}{2+\gamma}\Big)^2 < (3-2\gamma)\frac{(a-c^b)^2}{(2-\gamma)^2(1+\gamma)}$$

always holds in the region $\underline{\lambda}^B < \lambda < \bar{\lambda}^C$.

(i). In pattern 1, $\underline{\lambda}^B < \lambda < \underline{\lambda}^C$, firms choose (c_β, c_β) under Cournot competition and (c_b^*, c_b^*) under Bertrand competition. We have

$$(3-2\gamma)\frac{(a-c_b^*)^2}{(2-\gamma)^2(1+\gamma)} = \frac{(1+\gamma)(3-2\gamma)(4-\gamma^2)^2(2a-\lambda)^2}{4(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)},$$

which is a decreasing function of λ. Thus,

$$\begin{aligned}
&(3+\gamma)(\frac{a-c_\beta}{2+\gamma})^2 - (3-2\gamma)\frac{(a-c_b^*)^2}{(2-\gamma)^2(1+\gamma)} \\
&< (3+\gamma)(\frac{a-c_\beta}{2+\gamma})^2 - \frac{(1+\gamma)(3-2\gamma)(4-\gamma^2)^2(2a-\underline{\lambda}^C)^2}{4(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)} \\
&= \frac{(a-c_\beta)^2\gamma^2(-36-12\gamma+59\gamma^2+19\gamma^3-24\gamma^4-8\gamma^5+3\gamma^6+\gamma^7)}{(2+\gamma)^2(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)^2} \\
&< 0.
\end{aligned}$$

(ii). In pattern 1, $\underline{\lambda}^C < \lambda < \bar{\lambda}^B$, firms choose (c_c^*, c_c^*) under Cournot competition and (c_b^*, c_b^*) under Bertrand competition. We have

$$\begin{aligned}
&(3+\gamma)(\frac{a-c_c^*}{2+\gamma})^2 - (3-2\gamma)\frac{(a-c_b^*)^2}{(2-\gamma)^2(1+\gamma)} \\
&= \frac{(2a-\lambda)^2(4-\gamma^2)^2\gamma^2(-36-12\gamma+59\gamma^2+19\gamma^3-24\gamma^4-8\gamma^5+3\gamma^6+\gamma^7)}{4(-6-4\gamma+2\gamma^2+\gamma^3)^2(6+4\gamma-5\gamma^2-\gamma^3+\gamma^4)^2} \\
&< 0.
\end{aligned}$$

(iii). In pattern 1, $\bar{\lambda}^B < \lambda < \bar{\lambda}^C$, firms choose (c_c^*, c_c^*) under Cournot competition and (c_α, c_α) under Bertrand competition. We have $(3+\gamma)(\frac{a-c_c^*}{2+\gamma})^2 = \frac{(4-\gamma^2)^2(3+\gamma)(2a-\lambda)^2}{4(-6-4\gamma+2\gamma^2+\gamma^3)}$, which is a decreasing function of λ. Thus,

$$\begin{aligned}
&(3+\gamma)(\frac{a-c_c^*}{2+\gamma})^2 - (3-2\gamma)\frac{(a-c_\alpha)^2}{(2-\gamma)^2(1+\gamma)} \\
&< \frac{(4-\gamma^2)^2(3+\gamma)(2a-\bar{\lambda}^B)^2}{4(-6-4\gamma+2\gamma^2+\gamma^3)} - (3-2\gamma)\frac{(a-c_\alpha)^2}{(2-\gamma)^2(1+\gamma)} \\
&= \frac{(a-c_\alpha)^2\gamma^2(-36-12\gamma+59\gamma^2+19\gamma^3-24\gamma^4-8\gamma^5+3\gamma^6+\gamma^7)}{(2-\gamma)^2(1+\gamma)^2(-6-4\gamma+2\gamma^2+\gamma^3)^2} \\
&< 0.
\end{aligned}$$

(iv). In pattern 2, $\underline{\lambda}^B < \lambda < \bar{\lambda}^B$, firms choose (c_β, c_β) under Cournot competition and (c_b^*, c_b^*) under Bertrand competition. Since $\bar{\lambda}^B < \underline{\lambda}^C$, it follows straightforwardly from above part (i) that $(3+\gamma)(\frac{a-c_\beta}{2+\gamma})^2 - (3-2\gamma)\frac{(a-c_b^*)^2}{(2-\gamma)^2(1+\gamma)} < 0$.

(v). In pattern 2, $\bar{\lambda}^B < \lambda < \underline{\lambda}^C$, firms choose (c_β, c_β) under Cournot competition and (c_α, c_α) under Bertrand competition. Note that for pattern 2 we have $c_\alpha - c_\beta < H(\gamma)(a-c_\alpha)$, which can be rewritten as $(a-c_\beta) < (1+H(\gamma))(a-c_\alpha)$. We therefore have

$$
\begin{aligned}
&(3+\gamma)(\frac{a-c_\beta}{2+\gamma})^2 - (3-2\gamma)\frac{(a-c_\alpha)^2}{(2-\gamma)^2(1+\gamma)} \\
&< (3+\gamma)\frac{(1+H(\gamma))^2(a-c_\alpha)^2}{(2+\gamma)^2} - (3-2\gamma)\frac{(a-c_\alpha)^2}{(2-\gamma)^2(1+\gamma)} \\
&= \frac{(a-c_\alpha)^2\gamma^2(-36-12\gamma+59\gamma^2+19\gamma^3-24\gamma^4-8\gamma^5+3\gamma^6+\gamma^7)}{(2-\gamma)^2(1+\gamma)^2(-6-4\gamma+2\gamma^2+\gamma^3)^2} \\
&< 0.
\end{aligned}
$$

(vi). In pattern 2, $\underline{\lambda}^C < \lambda < \bar{\lambda}^C$, firms choose (c_c^*, c_c^*) under Cournot competition and (c_α, c_α) under Bertrand competition. Since $\bar{\lambda}^B < \underline{\lambda}^C$, it follows straightforwardly from above part (iii) that $(3+\gamma)(\frac{a-c_c^*}{2+\gamma})^2 - (3-2\gamma)\frac{(a-c_\alpha)^2}{(2-\gamma)^2(1+\gamma)} < 0$.

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