Asymmetric Exchange Rate Pass-through and Monetary Policy in Open Economy*

Sheng Wang

Economics and Management School, Wuhan University, Wuhan, China
Institute for Advanced Study, Wuhan University, Wuhan, China
E-mail: shengwang522@whu.edu.cn

and

Rufei Guo

Department of Economics, Chinese University of Hong Kong, Hong Kong, China
E-mail: rufei.guo@link.cuhk.edu.hk

We extend the open economy model of optimal monetary policy to incorporate asymmetric exchange rate pass-through, with a focus on the effect of sticky price on monetary policy transmission and welfare analysis. Under incomplete pass-through in the home country and full pass-through in the foreign country, we find that country-specific productivity shocks have complex effects on optimal monetary policies, which also depend on the elasticity of money demand. In a world Nash equilibrium, foreign monetary policy depends on the degree of home exchange rate pass-through. Asymmetry in exchange rate pass-through leads to asymmetric welfare effects. The welfare level of the home country is higher than that of the foreign country in the Nash equilibrium. However, international cooperation can improve world welfare level.

Key Words: Optimal monetary policy; Exchange rate pass-through; Producer-currency pricing; Local-currency pricing.

JEL Classification Numbers: E52, F41, F42.

1. INTRODUCTION

The new open economy macroeconomics pioneered by the seminal contributions of Obstfeld and Rogoff (1995, 1996) has boosted up an outpour-

* This research is supported by National Natural Science funds of China “The Research on Monetary Policy and Welfare Analysis under Incomplete Exchange Rate Pass-Through” (71273200).
ing of research on open-economy dynamic general equilibrium models that incorporate imperfect competition and nominal rigidity. Under this new framework, extensive theoretic and empirical works have been devoted to the analysis of monetary policy and exchange rate policy.

The assumption on the choice of the specific currency on which prices are set is critical in the analysis of optimal monetary policy under sticky prices. Two specifications of price setting have been widely used. In the first specification, nominal prices are pre-set in the currency of the producer (denoted as producer-currency pricing or PCP), as used in Obstfeld and Rogoff (1995). Under PCP, exchange rate fluctuation causes one-for-one price movement, which implies a prominent expenditure-switching effect in response to exchange rate shocks. Hence PCP features full exchange rate pass-through. In the second specification, prices are pre-set in the currency of the consumers (denoted as local-currency pricing or LCP), the implications of which are better supported by existing empirical works (e.g., Campa and Goldberg, 2008; Ihrig, 2006). Under LCP, there is zero exchange rate pass-through and the expenditure-switching effect can be eliminated (e.g., Corsetti and Pesenti, 2001; Tille, 2001).

The pioneering research of Obstfeld and Rogoff (1995, 1996, 2000) use the PCP specification and assume that the exchange rate pass-through is complete. Betts and Devereux (2000) model incomplete exchange rate pass-through by assuming that only a fraction of firms set prices under LCP, which reduces the expenditure-switching effect of exchange rate changes on consumption. As a result, short-run exchange rate overshooting, which is ruled out in the basic model, becomes possible, for the magnitude of the exchange rate movement required to satisfy the monetary equilibrium condition has been enlarged. Devereux and Engel (2003) prove that under LCP optimal monetary policy leads to a fixed exchange rate regime, which is taken as a major challenge to the Friedman case. Duarte and Obstfeld (2008) and Obstfeld (2006) extend Devereux and Engel (2003) by introducing non-tradable goods. Both papers, with a difference on the assumptions of the means of control of the government, draw a similar conclusion that even a complete absence of the expenditure-switching effects need not nullify the need for flexible exchange rates. Tervala (2012) further examines the international welfare effects of monetary policy in the short-run and medium-run under both PCP and LCP.

Corsetti and Pesenti (2005) and Sutherland (2005) consider a general case of incomplete exchange rate pass-through, and analyze the relationship between the degree of pass-through and welfare. Engel (2011, 2014) introduce currency misalignment resulted from incomplete pass-through, and study the influence of currency misalignment on optimal monetary policy. Devereux and Yetman (2014) analyze the effects of incomplete pass-through on the choice of monetary policy in Asia’s emerging economies.
Almost all previous researches focus on optimal monetary policy under symmetric exchange rate pass-through. The assumption of symmetric exchange rate pass-through is plausible for industrial economies (Engel 2002), but not appropriate for the study of emerging economies and their interactions with industrial economies. Devereux et al. (2007) derive an optimal monetary policy in a world with a dollar standard, in which all goods prices are set in US dollars, which allows asymmetric exchange rate pass-through between US and the other country.

Reinhart et al. (2014) show that dollarization is increasingly a defining characteristic of many emerging market economies, and ascertain its impact on the effectiveness of monetary and exchange rate policy. Shi and Xu (2010) find that twin dollarization, a phenomenon where firms borrow in US dollars and also set export prices in US dollar, can reduce the welfare loss caused by the fixed exchange rate regime. Wang and Zou (2015) modify Devereux et al. (2007) by adding the large trade in intermediate goods to study the optimal policy.

The study of dollarization begins to focus on the analysis in the asymmetric conditions, but the existing studies stikk focus on the special case that one country is under PCP and the other is under LCP. We modify Devereux et al. (2007) to analyze the optimal monetary policy in a more general asymmetric condition, and demonstrate that flexible exchange rate regimes are optimal in an asymmetric two countries model. In addition, asymmetric exchange rate pass-through implies different optimal monetary policies and different welfare levels.

The paper is organized as follows. Section 2 presents the basic model. Section 3 and 4 describe the equilibriums under flexible prices and sticky prices. Section 4 solves for the optimal monetary policy and presents the welfare analysis. Section 5 gives an empirical test to a special implication of the theoretic model. Section 6 concludes.

2. THE MODEL

We extend the basic setup of Devereux and Engel (2003) and Devereux et al. (2007) in two ways. First, we assume asymmetric exchange rate pass-through in a two-country model: incomplete pass-through in the home country and full pass-through in the foreign country. Second, we introduce non-tradable goods into the aggregate consumption baskets.

There are two countries, namely home and foreign. Each country is inhabited by a unit mass of identical consumers. Firms in each country produce a continuum of tradable goods and a continuum of nontradable goods, and each producer is the monopolistic supplier of a variety. Varieties (tradable and non-tradable) are indexed by \( i \in [0, 1] \) in the home country and by \( i^* \in [0, 1] \) in the foreign country.
Each home representative agent is an atomistic yeoman producer of one differentiated tradable good $i$ and one differentiated nontradable good $i$, using its own labor. The producer of generic good $i$ maximizes

$$U_0(i) = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \ln C_t(i) + \frac{\chi}{1 - \varepsilon} \left( \frac{M_t(i)}{P_t} \right)^{1-\varepsilon} V_t - \eta L_t(i) \right] \right\}, \quad (1)$$

where $M$ is the domestic money holdings, $V$ is the shock to money demand, $L$ is the labor supply, and $\beta \in [0, 1]$. Consumers have access to a complete market of state-contingent money payoffs, receive labor income and dividends from domestic firms, and get transfers from the government.

As in Obstfeld and Rogoff (2000), overall consumption depends on consumption of tradables and nontradables,

$$C = \frac{C_H^{1-\gamma} C_N^\gamma}{\gamma^\gamma (1 - \gamma)^{1-\gamma}}.$$ 

And the consumption on tradables is a combination of home and foreign produced tradables

$$C_T = 2C_H^{1/2} C_F^{1/2},$$

where $C_H$, $C_F$ and $C_N$ are CES functions of the available varieties,

$$C_j = \left[ \int_0^1 C_j(i)^{\frac{\theta - 1}{\theta}} di \right]^{\frac{1}{\theta - 1}}, \quad \theta > 1 \quad j = H, F, N.$$

For convenience, we define price indices of each consumption bundle faced by home consumers as follows:

$$P = P_T^\gamma P_N^{1-\gamma},$$

$$P_T = P_H^{\frac{1}{2}} P_F^{\frac{1}{2}},$$

$$P_j = \left[ \int_0^1 P_j(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}, \quad j = H, F, N.$$

We assume complete markets of contingent claims on future money payments, which imply that agents can fully diversify all country-specific income risk. In particular, agents will equalize the marginal utility of one unit of the nominal asset in all states of the world. As in Backus and Smith (1993) and Devereux and Engel (2003), the optimal monetary rules are determined after the state-contingent financial markets have closed, so
the resulting ex post allocation satisfies the condition

$$\frac{C^{-1}_t}{P_t} = \frac{(C^*)^{-1}}{S_tP^*_t}. \tag{2}$$

In all dates and states, $S$ is the nominal exchange rate, $C^*$ is the foreign consumption and $P^*$ is the foreign price level measured in foreign currency. Since purchasing power parity need not hold ex post in this model, the preceding condition does not generally equalize marginal utilities of consumption of goods produced by different countries.

The key assumption in our model is asymmetric exchange rate pass-through between the home country and the foreign country. We assume incomplete exchange rate pass-through in the home country and follow the specification of Sutherland (2005). Home agents are required to sign separate price contracts for sales in home and foreign markets. The contract signed by home agents for sales to foreign consumers is assumed to set a fixed degree of indexation to unanticipated exchange rate changes. Home agent $i$ who sells to foreign consumers chooses a price $P_H(i)$ denominated in home currency. Then the actual foreign currency price charged is determined by the following formula

$$P^*_H(i) = P_H(i) \left( \frac{S}{S_E} \right)^{1 - \eta_i}, \tag{3}$$

where $S_E$ is the ex ante expected exchange rate and $\eta_i \in [0, 1]$. This structure allows a full range of degrees of pass-through. $\eta_i = 1$ implies PCP and full pass-through. $\eta_i = 0$ implies LCP and zero pass-through. We further assume complete exchange rate pass-through in the foreign country. Foreign agents set the same prices $P^*_F(i)$ in terms of foreign currency for both home and foreign consumers. Hence the optimal price of foreign goods sold in the home country denominated in home currency is $P_F(i) = S P^*_F(i)$.

There are two sectors of production in each country. The production functions for firms in the home country are given by

$$Y_H = AL_H, \quad Y_N = AL_N.$$  

The random variable $A$ represents the economy-wide country-specific productivity shock. Production functions for foreign firms are similar.

The governments equally distribute local currencies to their inhabitants. The gross growth rate of money supply is denoted by $\mu_t$, and monetary policy is described by a rule for $\mu_t$.

The model is driven by shocks to money demand (velocity) and productivity in each country. We assume that the log technology and log velocity
shocks follow random walks, with $u_t$ and $u^*_t$ denoting the innovations in log technology and $v_t$ and $v^*_t$ in log velocity. We use lower-case letters to denote natural logarithms, and write the stochastic processes as

$$a_t = a_{t-1} + u_t, \quad a^*_t = a^*_{t-1} + u^*_t,$$

$$\ln V_t = \ln V_{t-1} + v_t, \quad \ln V^*_t = \ln V^*_{t-1} + v^*_t.$$

All the innovations are independent identically distributed with zero means and normally distributed.

3. FLEXIBLE PRICE EQUILIBRIUM

When prices are flexible, each firm set the price as a constant markup, $\theta/(\theta - 1)$, over nominal marginal cost ($W/A$ in the home country and $W^*/A^*$ in the foreign country). The aggregate consumption levels are given by the first-order conditions for individual consumption and labor-consumption tradeoff:

$$C = \left( \frac{\theta - 1}{\theta \eta} \right) A^{1 - \frac{1}{\eta}} (A^*)^{\frac{1}{\eta}} \, A^{\frac{1}{\eta}}, \quad C^* = \left( \frac{\theta - 1}{\theta \eta} \right) (A^*)^{1 - \frac{1}{\eta}} A^{\frac{1}{\eta}} \, A^{\frac{1}{\eta}}. \tag{4}$$

When all goods are tradable, consumption is equalized across countries, as it depends on the same weighted combination of home and foreign productivity levels. But the situation is different when we introduce nontradable goods. The possibility of asymmetric equilibrium consumptions under flexible price implies that the optimal responses of monetary policies to country-specific shocks may differ across countries (Obstfeld, 2006; Duarte and Obstfeld, 2008). The situation will be more complex under sticky price, especially when the two countries have asymmetric exchange rate pass-through as in our paper.

4. PRESET NOMINAL PRICE EQUILIBRIUM

Under sticky price, there are two first-order conditions for the price setting problem of tradable goods in the home country. One for the price charged to home consumers and another for the price charged to foreign consumers. The solutions for optimal price settings are given in Table 1. The price charged to foreign consumers is adjusted by a factor which reflects the effects of the exchange rate on demand under incomplete pass-through. The first-order conditions for foreign price setting are different because there is full pass-through in the foreign country.

Under incomplete exchange rate pass-through, home firms choose one price for nontradable goods and two different prices for tradable goods.
TABLE 1.

<table>
<thead>
<tr>
<th>Home country</th>
<th>Foreign country</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{H,t} = \frac{\eta}{\theta-1}E_{t-1}{P_tC_t/A_t}$</td>
<td>$P_{F,t} = \frac{\eta}{\theta-1}E_{t-1}{P^<em>_Ct/A^</em>_t}$</td>
</tr>
<tr>
<td>$P^*<em>{H,t} = \frac{\eta}{\theta-1}(S_t)^{-\eta}E</em>{t-1}\left{\frac{P_tC_t}{A_t}\right}$</td>
<td>$P^<em>_F,t = S_tP^</em>_F,t$</td>
</tr>
<tr>
<td>$P_{N,t} = P_{H,t}$</td>
<td>$P^<em>_{N,t} = P^</em>_N,t$</td>
</tr>
</tbody>
</table>

Substitute the conditions of the tradeoff between labor supply and consumption into the price setting conditions, we know that home firms set the price of tradable goods charged in the home country, $P_{H,t}$, as a fixed markup over expected unit labor cost in the home country.

To maximize the profit of tradable goods sold in the foreign country, firms in the home country should set the optimal price in the foreign country, $P^*_{H,t}$, according to the consumption level of the foreign country. Thus in the presence of exogenous shocks, the law of one price does not hold for tradable goods produced by the home country. The price of home nontradable goods is determined by the home unit labor cost and is equal to $P_{H,t}$.

However, since firms in the foreign country follow the PCP pricing rules, the law of one price still holds for tradable goods produced by the foreign country. Specifically, price charged by the foreign firms to home consumers, $P^*_{F,t}$, is equal to the price charged to foreign consumers. And the other two prices charged by foreign firms to foreign consumers, $P^*_F,t$ and $P^*_N,t$, follow similar pricing rules as their home counterparts.

The expected consumption in logarithm form in the two countries can be derived from the pricing formulas and the definitions of price indices. As noted above, lower-case letters denote the natural logarithms of variables and $e_t = \log S_t$.

\begin{align*}
E_{t-1}c_t &= -\ln \frac{\theta\eta}{\theta-1} - \frac{1}{2} \sigma^2_e + \left[\left(1 - \frac{\gamma}{2}\right)a_{t-1} + \frac{\gamma}{2}a^*_{t-1}\right] - \frac{1}{2} \left[\left(1 - \frac{\gamma}{2}\right)\sigma^2_u + \frac{\gamma}{2}\sigma^2_{u^*}\right] \quad (5) \\
&+ \left[\left(1 - \frac{\gamma}{2}\right)\sigma_{cu} + \frac{\gamma}{2}\sigma_{cu^*}\right] - \frac{1}{2} \left[\left(1 - \frac{\gamma}{2}\right)\sigma^2_e + \frac{\gamma}{2}\sigma^2_e^* + \left(1 - \frac{\gamma}{2}\right)(\sigma_{cu} - \sigma_{cu^*})\right] \\
E_{t-1}c^*_t &= -\ln \frac{\theta\eta}{\theta-1} - \frac{1}{2} \sigma^2_e + \left[\frac{\gamma}{2}a_{t-1} + \left(1 - \frac{\gamma}{2}\right)a^*_{t-1}\right] - \frac{1}{2} \left[\frac{\gamma}{2}\sigma^2_u + \left(1 - \frac{\gamma}{2}\right)\sigma^2_{u^*}\right] \quad (6) \\
&+ \left[\frac{\gamma}{2}\sigma_{cu} + \left(1 - \frac{\gamma}{2}\right)\sigma_{cu^*}\right] - \frac{1}{2} \left(\eta_1\right)^2\left(\frac{\gamma}{2}\right)^2\left(1 - \frac{\gamma}{2}\right)\sigma^2_e^* + \eta_2\frac{\gamma}{2}\left(1 - \frac{\gamma}{2}\right)(\sigma_{cu} - \sigma_{cu^*})
\end{align*}

Asymmetric exchange rate pass-through results in an obvious difference in expected consumption between the two countries. Because home country agents set optimal price under incomplete exchange rate pass-through, the variability of import price of the foreign country depends on the degree of home exchange rate pass-through. Thus, expected consumption in the foreign country is affected by both the variance of exchange rate and
the degree of exchange rate pass-through. If there is zero exchange rate pass-through in the home country, all the price indices faced by foreign consumers are fixed in advance, which makes exchange rate shocks irrelevant to foreign consumers. Thus, expected consumption in the foreign country is only affected by the variance of consumption and its covariance with technology shocks, as in Duarte and Obstfeld (2008) in which both countries use LCP. In contrast, due to PCP of foreign firms, the import price of the home country can not be set in advance and is affected by the fluctuation of exchange rate. So the expected consumption in the home country can be further attributed to the variance of exchange rate and its covariance with technology shocks.

Substitute the expected consumption terms by the money demand functions of both countries, we have

\[
c_t - E_{t-1}c_t = \frac{1 + \frac{i}{1+i}}{1 + \frac{i}{1+i}} \left[ \mu_t - \frac{1}{\varepsilon} v_t - \frac{\gamma}{2} (e_t - E_{t-1}e_t) \right] \\
+ \frac{\varepsilon - 1}{\varepsilon(1+i)} \left[ (1 - \frac{\gamma}{2}) u_t + \frac{\gamma}{2} u_t^* \right], \tag{7}
\]

\[
c_t^* - E_{t-1}c_t^* = \frac{1 + \frac{i}{1+i}}{1 + \frac{i}{1+i}} \left[ (\mu_t^* - \frac{1}{\varepsilon} v_t^* + \eta (e_t - E_{t-1}e_t)) \right] \\
+ \frac{\varepsilon - 1}{\varepsilon(1+i)} \left[ \frac{\gamma}{2} u_t + (1 - \frac{\gamma}{2}) u_t^* \right], \tag{8}
\]

where \( i \) denotes the steady-state nominal interest rate.

The above two equations disclose the key difference in consumption innovations between the two countries under asymmetric exchange rate pass-through. The consumption innovation in the home country depends on monetary shocks and policies in both countries, for it is affected by the exchange rate innovations and hence by the monetary policy of the other country. In contrast, the consumption innovation in the foreign country depends only partly on home monetary shocks and home monetary policies. Specifically, when the home country features zero exchange rate pass-through, the consumption innovation in the foreign country depends only on its own monetary shock, \( v_t^* \), and monetary policy, \( \mu_t^* \). Therefore, the degree of pass-through is critical for the welfare analysis of monetary policy.

There are two reasons why productivity shocks have different effects on consumption in the home country and the foreign country. The first one is the presence of nontradable goods. The second one is asymmetric exchange rate pass-through. Asymmetric pass-through implies that exchange rate innovations have asymmetric effects on the consumption of the two countries. In equilibrium, consumption in the foreign country depends partly on home productivity shocks because of incomplete exchange rate pass-through.
Substitute the risk-sharing condition into Equations (7) and (8), we solve for the exchange rate innovations:

\[ e_t - E_{t-1}e_t = \frac{1}{\lambda} \left\{ \frac{1 + \varepsilon}{1 + i} \left[ (\mu_t - \mu_t^*) - \frac{1}{\varepsilon}(v_t - v_t^*) \right] + \frac{\varepsilon - 1}{\varepsilon(1 + i)} [(1 - \gamma)(u_t - u_t^*)] \right\}, \]

(9)

where \( \lambda = 1 + \frac{\varepsilon(\varepsilon - 1)}{1 + i} (\eta_t + 1)^2 \). If all goods are tradable, exchange rate innovations do not depend on country-specific productivity shocks.

5. OPTIMAL MONETARY POLICY

Now we derive the implications of asymmetric exchange rate pass-through on optimal monetary policies. The monetary authority in each country is assumed to commit to preannounced state-contingent monetary policy feedback rules. As discussed in Devereux and Engel (2003) and Duarte and Obstfeld (2008), the monetary authorities choose policies to maximize the nonmonetary expected utility of the country’s representative consumer, taking the other country’s monetary policy rule as given. We assume that central banks implement monetary policy by setting nominal money growth rates,

\[
\begin{align*}
\mu_t &= a_1 u_t + a_2 u_t^* + a_3 v_t + a_4 v_t^*, \\
\mu_t^* &= b_1 u_t^* + b_2 u_t + b_3 v_t^* + b_4 v_t.
\end{align*}
\]

Monetary policies are log-linear functions of productivity and velocity innovations. It should be noted that we analyze optimal policies as \( \chi \to 0 \) in the utility function \( \hat{U} \). By the labor market clearing conditions and the pricing equations, we have:

\[
\begin{align*}
E_{t-1}\hat{U}_t &= E_{t-1}c_t - \frac{\theta - 1}{\theta}, \\
E_{t-1}\hat{U}_t^* &= E_{t-1}c_t^* - \frac{\theta - 1}{\theta}.
\end{align*}
\]

(10)

(11)

The Nash equilibrium in monetary policies is defined as the set \( \{a^N, b^N\} \), where \( a = \{a_1, a_2, a_3, a_4\} \) and \( b = \{b_1, b_2, b_3, b_4\} \), which solves the following problems:

\[
\begin{align*}
\max_a E_{t-1}\hat{U}_t(a, b^N), \\
\max_b E_{t-1}\hat{U}_t^*(a^N, b).
\end{align*}
\]
In order to simplify the solutions and crystalize the intuition, we assume that the equilibrium nominal interest rate is zero. The solutions to the problems are given in Table 2.

**TABLE 2.**

<table>
<thead>
<tr>
<th>Optimal Monetary Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Home country</strong></td>
</tr>
<tr>
<td>$a_1 = 1 - \left(\frac{\varepsilon}{\varepsilon - 1}\right) \left(1 - \frac{\gamma_2}{\delta}\right)$</td>
</tr>
<tr>
<td>$a_2 = -\left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{1}{\delta}$</td>
</tr>
<tr>
<td>$a_3 = \frac{1}{\delta}$</td>
</tr>
<tr>
<td>$a_4 = 0$</td>
</tr>
</tbody>
</table>

Note: $\Phi = \frac{(1 - \frac{\gamma_2}{\delta})}{(1 - \eta_2) \left(1 - \eta_1 \frac{\gamma_2}{\delta}\right)^2 \delta^2}.$

It is interesting that the parameters of home optimal policy are all constant and are not affected by the degree of home exchange rate pass-through. This is because of the complete exchange rate pass-through to home import, and incomplete exchange rate pass-through only affect foreign import price. On the other hand, it is apparent that, the optimal policy set by the foreign country will be affected by the degree of exchange rate pass-through in the home country. When the degree of exchange rate pass-through goes up, foreign optimal monetary policy depends more on foreign productivity shocks and depends less on home productivity shocks.

It is easy to prove that in the extreme case of zero exchange rate pass-through in the home country, foreign optimal monetary policy is not affected by home monetary policy. The reason is simple from Equation (5), the expected utility of representative foreign agent depends only on his own consumption, the innovation of which is only affected by the monetary policy of his own country. In contrast, the optimal monetary policy in the home country is affected by the foreign country’s monetary policy. Due to PCP of foreign agents, the innovations of home consumption depend both on home and foreign monetary policies. Hence the optimal monetary policy set by the foreign country will have a direct influence on the expected utility of home individuals, and an indirect influence on home optimal monetary policy.

Although the asymmetric exchange rate pass-through indicates the dependence of home optimal monetary policy on foreign monetary policy, it does not change the effect of money demand shocks on optimal monetary policy. That is to say, optimal monetary policy rules respond only to own-country money demand shocks rather than other-country’s money demand shocks.
Moreover, the responses of optimal monetary policy rules to home and foreign productivity shocks are asymmetric. The reasons are the existence of non-tradable goods and the asymmetric exchange rate pass-through.

If there is no international trade, namely assuming the absence of tradable goods ($\gamma = 0$), $a_2$ and $b_2$ are equal to zero. This is quite intuitive: in absence of international trade, the optimal policy depends only on its own domestic shocks.

Because country-specific real shocks have asymmetric effect on the consumption level of the two countries, the nominal exchange rate is in general not constant. In fact, it can be seen that the conditional variance of the exchange rate under optimal monetary policies is given by

$$\text{var}_{t-1}(e_t) = \Phi^2 [\text{var}(u_t) + \text{var}(u^*_t)]. \quad (12)$$

The minimum exchange rate volatility is $(1 - \gamma^2)^2$ when there is zero exchange rate pass-through in the home country, while the maximum is just 1 when there is full exchange rate pass-through in the home country.

Under asymmetric exchange rate pass-through, the optimal monetary policies do not support the full flexible price equilibrium. Hence the welfare levels of both countries decrease in different extents, as shown in the following equations:

$$E_{t-1}U^N_t = -\frac{\theta - 1}{\theta} - \ln \frac{\theta \eta}{\theta - 1} + \left[\left(1 - \frac{\gamma}{2}\right)a_{t-1} + \frac{\gamma}{2}a^*_t\right] + \text{Loss}\left[\sigma^2_u + \sigma^2_u^*\right], \quad (13)$$

Loss $= -\frac{\gamma}{4} (\Phi - 1)^2$,

$$E_{t-1}U^N_t^* = -\frac{\theta - 1}{\theta} - \ln \frac{\theta \eta}{\theta - 1} + \left[\frac{\gamma}{2}a_{t-1} + \left(1 - \frac{\gamma}{2}\right)a^*_t\right] + \text{Loss}^*\left[\sigma^2_u + \sigma^2_u^*\right], \quad (14)$$

Loss $^* = -\frac{1}{2} \left\{ \left(\frac{\gamma}{2}\right)^2 (1 - \frac{\gamma}{2}) (\eta \Phi - 1)^2 + \left[\left(1 - \frac{\gamma}{2}\right)^2 (1 - \gamma)\right] \right\}.$

Even if the level of productivity is the same in the two countries, there is still some difference in the welfare levels of two countries. In addition to the distortion of sticky pricing, incomplete exchange rate pass-through will bring more welfare loss to the foreign country than the home country. As a result, the welfare level of the foreign country is lower than that of the home country as long as there are tradable goods. In absence of tradable goods, the optimal monetary policies can replicate flexible price equilibrium and offset all the loss.

Figure 1 illustrates the welfare loss of the two countries when there are no nontradable goods ($\gamma = 1$). The dashed line and the star line show the losses of the home country and the foreign country in Nash equilibrium.

---

1We assume the money demand elasticity is equal to 2.5 as in Chari et al. (2002) and Schmidt (2006). In fact, the money demand elasticity only affects the optimal policies but not the level of welfare loss.
It is intuitive that the foreign country will suffer heavier loss due to the incomplete pass-through of exchange rate volatility. In this case, as the degree of pass-through rises, the losses of both countries decrease. The effect is quantitatively larger in the foreign country because the degree of pass-through affects foreign optimal monetary policy directly.

As the monetary policies in the Nash equilibrium don’t support the full flexible price equilibrium, potential benefits exist in the cooperative policy game. Under cooperation, with equal weights assigning to the welfare levels of home and foreign households, the objective function is

$$\max_{a,b} \mathbb{E}_{t-1}\left[\frac{1}{2}\hat{U}_t(a, b) + \frac{1}{2}\hat{U}^*_t(a, b)\right].$$

Analogous to the analysis in Nash equilibrium, we can get the welfare losses in the cooperation equilibrium. The solid line and the dotted line in the Figure 1 show the losses of the home country and the foreign country in the cooperation equilibrium. The total welfare losses of both countries are falling gradually as the degree of exchange rate pass-through rises. Because the rise of the degree of pass-through increases the response of import prices to exchange rate volatility, monetary authorities in both countries can adjust the economies more quickly to exogenous shocks and raise the aggregate welfare levels.

**FIG. 1.** The welfare loss of optimal monetary policy

When exchange rate pass-through is small, agents in home country get better off from cooperation while foreign agents will be worse off. Equation (5) shows that foreign welfare depends almost only on foreign domestic monetary policy in this case, so foreign authority has the dominant monetary policy strategy in a world Nash equilibrium. Thus foreign agents can get close to maximum welfare level. As a result, agents in foreign
country cannot gain in an international economic cooperation. However, as exchange rate pass-through rises, foreign welfare begins to deviate from the maximum, and cooperation equilibrium can improve the welfare of the foreign country. When pass-through is large enough, home agents might be worse off in cooperation. If there is full pass-through in the home country, optimal polices in Nash equilibrium replicate the flexible price allocations and leave no potential benefits for cooperation, as in Devereux and Engel (2003).

In our model of asymmetric exchange rate pass-through, monetary policies can affect home consumption directly. And Nash equilibrium does not support the full flexible equilibrium as long as pass-through in the home country is incomplete. Thus cooperation can increase the whole world welfare. This is of great importance to the understanding of international economic cooperation and policy coordination.

6. EMPIRICAL TESTS

In this section, we test the two main implications of the theoretical model in the special case of zero exchange rate pass-through in the home country. The first implication is on the interaction of the monetary policies of the two countries. According to Section 5, the foreign monetary authority plays a dominant strategy in the Nash Equilibrium while the home monetary policy depends on the foreign monetary policy. The second implication concerns the effects of monetary policies on consumption innovations of the two countries, which can be directly observed in Equation (7) and Equation (8). By Equation (8), home monetary policy has no influence on foreign consumption innovation. And by Equations (7) and (9), foreign monetary policy has a contemporary positive effect on home consumption innovation through exchange rate. For clarity and easy reference, the two implications are summarized as follows:

**Implication 1**: Foreign monetary policy does affect home monetary policy while Home monetary policy does not affect foreign monetary policy.

**Implication 2**: Foreign monetary policy has a contemporary positive effect on home consumption innovation while Home monetary policy has no influence on foreign consumption innovation.

In this section, we employ the impulse response functions of an FAVAR model to test the two implications. Before we come to the details of the tests, a brief introduction of the data specifications and the method is necessary.

China and the U.S. are taken as the home country and the foreign country respectively for the following reasons: Firstly, since the U.S. dollar is the currency of international settlement and the world currency, U.S. is the best candidate for the PCP foreign country in the theoretical model. Secondly,
the two-country setting of the theoretical model mandates a selection of an
economy that has a substantial influence in the world, which makes China a
natural candidate. Thirdly, Kim et. al. (2013) find that more than 97% of
US import from China and almost 100% of US export to China are priced
in the US dollar. This is consistent with Gopinath and Rigobon (2008)s
finding that more than 90% of US imported goods are priced in dollars.
Thus China has a high tendency to feature zero pass-through. Fourthly,
although Japan seems like a natural substitute for China, the M2 data
of Japan is available only since the year 2004, which deprives of Japans
candidate. Consequently, since China’s M2 data is available from 1993, we
use the quarterly time series of China and the U.S. in our analysis.

The data are taken from EIU country data. All available China and
U.S. data are used and most of them are transformed into the log linear
form. Interest rates and net balances are kept in original forms but scaled
into commensurable units with other variables for the purposes of principal
component analysis in the FAVAR. Seasonal adjustments are conducted by
regressing on seasonal dummies and taking the residuals.

The application of the FAVAR model in the study of monetary policies is
pioneered by Bernanke et al. (2005) and advanced into a global context by
Boivin and Giannoni (2008), where a detailed presentation of FAVAR can
be found. In short, the FAVAR model attempts to extract information from
all available macro indicators, which can not be handled by the traditional
VAR models, by using recursive principal component analysis or other more
complicated methods.

In our FAVAR model, we use the growth rate of M2 in China and US as
their policy instruments to fit the theoretical model. As for the consump-
tion innovations, since the \( E_{t-1}C_t \) and the \( E_{t-1}C_t^* \) terms are constant
terms at period \( t \) and can therefore be absorbed into the intercepts, we can
safely use the growth rate of consumptions alone to replace consumption
innovations in the empirical analysis. In addition, in lack of the quarterly
data of consumption for China and given the theoretical model’s dismissal
of investment, we use the difference between real GDP and real net export
to stand for real consumption. In fact, using the U.S. data in which real
consumption is available, we find that the correlation of this differenced
value with real consumption is as high as 0.9950. Lastly, the first four
principal components of all the China and U.S. time series are used in the
final VAR.

We report the results of the VAR with 2 lags in Figure 2 and Figure 3.
The results are robust in VAR models with 3 lags and 4 lags. Moreover, the
SVAR models with contemporary terms of monetary policy on consumption
also yield similar results.

Figure 2 shows the orthogonal impulse response functions of the impacts
of China and U.S.’s monetary policy on each other. It can be seen that
FIG. 2. The interaction of monetary policies of China and the U.S.

The influence of U.S.’s monetary policy on China’s is much larger than the impact of China’s monetary policy on U.S.’s. Although the confidence interval is large because of the limited sample size, the response of China’s money growth on U.S.’s money growth is significantly positive at 2 lags at the 10% significant level. However, China’s money growth has no effect on that of U.S.: the responses are always near zero at all periods up to 16. Obviously, the results support the first implication.

FIG. 3. The effect of monetary policy on the other country’s consumption

If the support from Figure 2 for implication 1 is moderate, the support from Figure 3 for implication 2 is strong. As we can see in Figure 3, U.S.’s monetary policy has a significant positive effect on China’s consumption at the zero lag at the 5% significance level and the effect is significant
only at the zero lag. Whereas, China’s monetary policy does not affect U.S.’s monetary policy at all lags up to 16. Specifically, a 1% increase in U.S. money growth rate will boost up Chinas contemporary consumption growth by 0.31%. The empirical results in Figure 3 strongly support the second implication.

In conclusion, the statistical and economic significance of the two sets of OIRFs are consistent with the implications of the theoretical model.

7. CONCLUSIONS

Most previous research assumes symmetric exchange rate pass-through. Devereux and Engel (2003) make an important advancement in demonstrating how alternative price setting arrangements can alter the transmission mechanism of monetary policies in the open economy. Duarte and Obstfeld (2008) and Obstfeld (2006) develop the benchmark model by introducing nontradable goods and find out that the fixed exchange rate prescription is not optimal even if exchange rate changes have no expenditure-switching effect.

A distinct contribution of this paper lies on its focus on optimal monetary policy under asymmetric exchange rate pass-through in the open economy. In reality, exchange rate pass-through is always asymmetric across countries, especially between industrial and emerging economies, such as US and China. Consequently, consumption, optimal monetary policy, and welfare effects can be asymmetric. Our findings can be summarized as follows:

Firstly, even without the presence of nontradable goods, the asymmetric exchange rate pass-through in our paper changes the effect of home productivity on the optimal monetary policy. Compared with the case of both countries under PCP, the above effect is more complex and depends on the elasticity of money demand. Similar to the analysis in Duarte and Obstfeld (2008), we suggest that the degree of exchange rate pass-through to foreign import prices affects exchange rate volatility.

Secondly, under the special assumption of LCP in the home country and PCP in the foreign country, the consumption innovations in the home country depend on foreign monetary policy. As a result, the optimal monetary policy in the home country is affected by foreign monetary authority. Nevertheless, the optimal monetary policy of the foreign country only depends on its own consumption, which is completely identical to the monetary rule of Duarte and Obstfeld (2008).

Moreover, asymmetry exchange rate pass-through leads to asymmetry in welfare effects. The welfare level of the home country is higher than the level of the foreign country in the Nash equilibrium. While in cooperation equilibrium, the optimal monetary policies can effectively promote the total world welfare, but the welfare effects on two countries are different and
maybe opposite, which in turn depend on the degree of exchange rate pass-through in home country.

Further research should be done in the following directions. Firstly, a more comprehensive quantitative analysis of asymmetric pricing arrangement should be carried out to test the theoretical implications. Secondly, since the log consumption setting in the utility function is vital to perfect international sharing of consumption risks in tradable goods, we should try more general utility functions, which may have important implications on international policy coordination.

**APPENDIX A**

**Optimal money demand function:**
Solve the utility maximization problem of equation (1) under budget constraint, we have

\[
\chi \left( \frac{M_t}{P_t} \right)^{-\varepsilon} V_t = \frac{1}{C_t} \left[ 1 - E_t \left( \beta \frac{P_t C_t}{P_{t+1} C_{t+1}} \right) \right],
\]

which gives us

\[
1 - \chi \left( \frac{M_t}{P_t} \right)^{-\varepsilon} V_t C_t = \beta E_t \left( \frac{P_t C_t}{P_{t+1} C_{t+1}} \right).
\]

Log-linearization in the steady state gives us

\[
m_t - p_t = \frac{1}{\varepsilon} c_t - \frac{1}{i\varepsilon} [(E_t p_{t+1} - p_t) + (E_t c_{t+1} - c_t)] + \frac{1}{\varepsilon} \ln V_t + \Gamma_m, \quad (A.1)
\]

where \( \Gamma_m \) is a constant term. Similarly, foreign money demand function can be simplified as:

\[
m_t^* - p_t^* = \frac{1}{\varepsilon} c_t^* - \frac{1}{i\varepsilon} [(E_t p_{t+1}^* - p_t^*) + (E_t c_{t+1}^* - c_t^*)] + \frac{1}{\varepsilon} \ln V_t^* + \Gamma_m^*, \quad (A.2)
\]

**Sticky-Price Equilibrium**
Using the optimal prices \( P_{H,t} \) and \( P_{F,t} \) of table 1 and the definition of price indices, we have

\[
\frac{\theta \eta}{\theta - 1} \left( E_{t-1} \left\{ S_t^2 C_t / A_t \right\} \right)^{1-\frac{2}{1+2}} \left( E_{t-1} \left\{ S_t^{2-1} C_t / A_t^* \right\} \right)^{\frac{2}{2}} = 1. \quad (A.3)
\]

By the log-normal distribution of exogenous shocks, exchange rate and consumption must also be log-normal distributed. Take log of (A.3) give us equation (5) in the text.
Using $P^{*}_{H,t}$ and $P^{*}_{F,t}$ of Table 1, we have
\[
\frac{\theta_{H}}{\varrho_{H}} \left( E_{t-1} \left\{ S^{n_{t}(1-\hat{\gamma})}C_{t}^{*}/A_{t} \right\} \right)^{\hat{\gamma}} \left( E_{t-1} \left\{ S^{-n_{t}(\hat{\gamma})}C_{t}^{*}/A_{t}^{*} \right\} \right)^{1-\hat{\gamma}} = 1. 
\]  
(A.4)

It is easy to derive equation (6) from (A.4).

Asymmetric price stickiness renders the correlation of $p_{t}$ with the exchange rate at period $t$, making the derivation of home consumption innovation more complex.

Firstly, analogous to the derivation of (A.1), we can get
\[
\mu_{t} - (p_{t} - E_{t-1}p_{t}) = \frac{1}{i\varepsilon} \left( c_{t} - E_{t-1}c_{t} \right) \]  
(A.5)

\[- \frac{1}{i\varepsilon} \left[ (E_{t}p_{t+1} - E_{t-1}p_{t+1}) - (p_{t} - E_{t-1}p_{t}) + (E_{t}c_{t+1} - E_{t-1}c_{t+1}) \right] + \frac{1}{i\varepsilon}v_{t}. \]

Using the property of price stickiness and the definition of price index, we can easily get
\[
p_{t} - E_{t-1}p_{t} = \frac{\gamma}{2}(e_{t} - E_{t-1}e_{t}). \]  
(A.6)

Substitute (A.6) into (A.5), we can finally get equation (7).

Take expectation at $t-1$ period, using (A.2) and equation (6) we can get:
\[
\mu_{t}^{*} - (p_{t}^{*} - E_{t-1}p_{t}^{*}) = \frac{1}{i\varepsilon} \left( c_{t}^{*} - E_{t-1}c_{t}^{*} \right) \]  
(A.7)

\[- \frac{1}{i\varepsilon} \left[ (E_{t}p_{t+1}^{*} - E_{t-1}p_{t+1}^{*}) - (p_{t}^{*} - E_{t-1}p_{t}^{*}) + (E_{t}c_{t+1}^{*} - E_{t-1}c_{t+1}^{*}) \right] + \frac{1}{i\varepsilon}v_{t}^{*} \]

Using the property of price stickiness and the definition of price index, we can easily get
\[
p_{t}^{*} - E_{t-1}p_{t}^{*} = -\eta_{1} \frac{\gamma}{2}(e_{t} - E_{t-1}e_{t}), \]  
(A.8)

which, combined with (6) and (A.7), gives us equation (8).

Taking advantage of equation (2) and (A.8), we can get
\[
c_{t} - E_{t-1}c_{t} = (c_{t}^{*} - E_{t-1}c_{t}^{*}) + \left[ 1 - \frac{\gamma}{2}(\eta_{1} + 1) \right] (e_{t} - E_{t-1}e_{t}). \]  
(A.9)

Combined with equation (6) and (7), (A.9) gives us equation (9).

**Optimal monetary policy in the Nash Equilibrium:**
According to equation (7), (8) and (9), we can get

\[ \sigma^2_e = \left\{ \left[ \left( 1 - \frac{\gamma}{2} \right) a_1 + \left( \frac{\gamma}{2} \right) b_2 \right] + \frac{\varepsilon - 1}{\varepsilon} \left[ \left( 1 - \frac{\gamma}{2} \right) - \frac{\gamma}{2} \right] \right\} \sigma^2_u \]

\[ + \left\{ \left( 1 - \frac{\gamma}{2} \right) \left( a_3 - \frac{1}{\varepsilon} \right) + \frac{\gamma}{2} b_4 \right\} \sigma^2_v \]

\[ + \left\{ \left[ \left( 1 - \frac{\gamma}{2} \right) a_2 + \left( \frac{\gamma}{2} \right) b_1 \right] + \frac{\varepsilon - 1}{\varepsilon} \left[ \frac{\gamma}{2} + \gamma \left( 1 - \gamma \right) \right] \right\} \sigma^2_v. \]

\[ \sigma^2_{au} = \left\{ \eta_1 \frac{\gamma}{2} a_1 + \left( 1 - \eta_1 \frac{\gamma}{2} \right) b_2 + \frac{\varepsilon - 1}{\varepsilon} \left( \frac{\gamma}{2} + \eta_1 \frac{\gamma}{2} \right) \left( 1 - \gamma \right) \right\} \sigma^2_u \]

\[ + \left\{ \eta_1 \frac{\gamma}{2} \left( a_3 - \frac{1}{\varepsilon} \right) + \left( 1 - \eta_1 \frac{\gamma}{2} \right) b_4 \right\} \sigma^2_v \]

\[ + \left\{ \eta_1 \frac{\gamma}{2} a_2 + \left( 1 - \eta_1 \frac{\gamma}{2} \right) b_1 + \frac{\varepsilon - 1}{\varepsilon} \left( 1 - \eta_1 \frac{\gamma}{2} \right) \right\} \sigma^2_v. \]

\[ \sigma^2_{au} = \left\{ \left( a_1 - b_2 \right) + \frac{\varepsilon - 1}{\varepsilon} \left( 1 - \gamma \right) \right\} \sigma^2_u + \left\{ \left( a_2 - b_1 \right) + \frac{\varepsilon - 1}{\varepsilon} \right\} \sigma^2_v + \left\{ \left( a_3 - b_4 - \frac{1}{\varepsilon} \right) \right\} \sigma^2_v. \]

\[ \sigma^2_{eu} = \left\{ \left[ \left( 1 - \frac{\gamma}{2} \right) a_1 + \left( \frac{\gamma}{2} \right) b_2 \right] + \frac{\varepsilon - 1}{\varepsilon} \left[ \left( 1 - \frac{\gamma}{2} \right) - \frac{\gamma}{2} \right] \right\} \sigma^2_u, \]

\[ \sigma^2_{eu} = \left\{ \left[ \left( 1 - \frac{\gamma}{2} \right) a_2 + \left( \frac{\gamma}{2} \right) b_1 \right] + \frac{\varepsilon - 1}{\varepsilon} \left[ \frac{\gamma}{2} + \gamma \left( 1 - \gamma \right) \right] \right\} \sigma^2_v. \]

\[ \sigma^2_{eu} = \left\{ \left( a_1 - b_2 \right) + \frac{\varepsilon - 1}{\varepsilon} \left( 1 - \gamma \right) \right\} \sigma^2_u, \]

\[ \sigma^2_{eu} = \left\{ \left( a_2 - b_1 \right) + \frac{\varepsilon - 1}{\varepsilon} \left( 1 - \gamma \right) \right\} \sigma^2_u, \]

\[ \sigma^2_{eu} = \left\{ \eta_1 \frac{\gamma}{2} a_1 + \left( 1 - \eta_1 \frac{\gamma}{2} \right) b_2 + \frac{\varepsilon - 1}{\varepsilon} \left( \frac{\gamma}{2} + \eta_1 \frac{\gamma}{2} \right) \left( 1 - \gamma \right) \right\} \sigma^2_u, \]

\[ \sigma^2_{eu} = \left\{ \eta_1 \frac{\gamma}{2} a_2 + \left( 1 - \eta_1 \frac{\gamma}{2} \right) b_1 + \frac{\varepsilon - 1}{\varepsilon} \left( 1 - \eta_1 \frac{\gamma}{2} \right) \right\} \sigma^2_u. \]
Substitute the above variance and covariance in to equation (4) and (5), and solve the welfare maximization problem about (10) and (11), we have the Nash Equilibrium of optimal monetary policy in Table 2.

REFERENCES


