

## Trade, Sectorial Reallocation, and Growth\*

Pengfei Wang and Danyang Xie†

This paper introduces sectorial heterogeneity in TFPs in a growth model to generate new insights on trade, sectorial reallocation, and economic growth. The rate of overall economic growth in this model is a simple average of sectorial growth in a closed economy, but will depend on trade parameters in an open economy as openness to trade shifts resources toward fast-growing sectors. We find that the overall growth rate is unambiguously higher as the number of trading partners increases. These conclusions survive even after trade cost is introduced. Nevertheless, trade share and growth rate may not move in the same direction as trade liberalization is pursued or as the number of trading partners is increased. This finding may explain why the existing empirical evidence concerning this relationship between growth and trade share remains inconclusive.

*Key Words:* Heterogeneous Sectors; International Trade; Growth.

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## 1. INTRODUCTION

It comes as no surprise that productivity growth is dramatically different across sectors. Table 1, extracted from Jorgenson and Gollop (1992), highlights this difference in the case of the U.S.A. for the post-war period between 1947-85. Using OECD's intersectoral database, Bernard and Jones (1996) document TFP differences both across sectors and across countries (see their Table 1). As a first step, we ignore TFP differences across countries in this paper. Rather, we focus on TFP differences across sectors and investigate how these differences affect the aggregate growth. Furthermore, we examine the interaction between international trade and growth.

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† Wang: Department of Economics, HKUST. E-mail: pfwang@ust.hk; Xie: Department of Economics, HKUST; Center for Economic Development Research and Economics and Management School, Wuhan University. E-mail: danyang.xie@gmail.com.

Clearly, we need to go beyond “one sector” growth models (Solow 1956, for example) to incorporate differences in sector-level productivity growth.

**TABLE 1.**

Average Annual TFP Growth across Sectors (in %)

Agriculture	1.58
Manufacturing	0.72
Transportation	0.96
Communications	2.04
Utilities	0.87
Trade	0.90
Fire	0.24
Other Services	-0.13

To gain more insight into the impact of heterogeneity, we develop a model that is highly tractable. This is achieved by making two technical assumptions. First, we adopt a special case of the production function used in Acemoglu, Antras and Helpman (2007), which greatly simplifies the aggregation. Second, we assume that sectorial growth rates are driven by a spectrum of exogenous TFP growth. We show that, in this case, the growth rate of the aggregate output is a simple average of the sectorial productivity growth rates. When the model is extended to an open economy, the simple average is replaced by a weighted average with the weights depending on trade parameters. Hence, our model could be viewed as a semi-endogenous growth model in the spirit of Jones (1995) to distinguish it from the endogenous growth models of Romer (1986, 1990), Lucas (1988), Grossman and Helpman (1991) and Aghion and Howitt (1992).

The main findings of this paper are as follows. First, we identify a *resource reallocation effect*: only the relatively more productive sectors engage in international trade and the resources are directed more toward these sectors than in the closed economy case. We show that this is an equilibrium outcome and is consistent with the assumption underlying the Harrod-Balassa-Samuelson hypothesis that the average technological progress in tradable sectors is faster than that in nontradable ones. Our result, obtained in a growth model, complements the insights obtained from the stationary trade models of Eaton and Kortum (2002), Bernard, Eaton, Jensen and Kortum (2003), Melitz (2003), Helpman, Melitz and Yeaple (2004), Ghironi and Melitz (2005), Bernard, Redding and Schott (2007) and Melitz and Ottaviano (2008). The common theme of this New Trade Theory is that trade liberalization leads to reallocations of resources among firms: the least productive firms are forced to exit and the more productive firms enter the export sector and benefit from a larger international mar-

ket. The New Trade Theory has been confirmed in a number of empirical studies.<sup>1</sup> For example, Bernard and Jensen (1995, 1999), Aw, Chung, and Roberts (2000), Eaton, Kortum, and Kramarz (2004), among others, have documented that differences in firm productivity are strongly correlated with a firm's decision to engage in international transactions (such as exporting, importing intermediate goods from foreign suppliers, or investing in foreign subsidiaries).

Second, our approach has the advantage that the resource reallocation produces a growth effect rather than a one-time level effect<sup>2</sup>. As a result, gains from trade can be very large compared to those from the stationary trade models of the New Trade Theory. Arkolakis, Costinot and Rodriguez-Clare (2012) show that the total gains from trade can be identical for a large class of New Trade models regardless of their micro-level implication. Their estimate for the United States suggests that the gains from trade are very small, ranging from 0.7 percent to 1.4 percent. In a similar spirit, Fan, Lai and Qi (2013) show that the gains from the reduction of trade costs can also be represented by the same formula for a large class of New Trade models. Again, their estimate finds that the global welfare gains from a worldwide reduction in international shipping time in the last 50 years are not large, ranging from 2.98% to 8.81%. Since trade improves growth in our model, there could be a large gain due to compounding. Our example illustrates that despite only a small change in the growth rate, the present value gains more than 20%.

We show that two types of trade policies can enhance growth. First, we show that growth is a non-increasing function of the fixed costs of trade. As the trade cost declines, the tradable sectors expand. Since the tradables on average grow faster than the nontradables, overall growth increases. We then show that the growth rate is unambiguously higher as the number of trading partners increases. More trading partners will intensify the competition and the resources will be reallocated to the sectors with higher productivity growth, leading to higher overall growth. This is in line with empirical evidence on the positive relationship between growth and various measures of openness. In particular, Sala-i-Martin (1997) shows that, in a cross-country study, the number of years an economy has been open is robustly linked to the growth rate. Frankel and Romer (1999), using countries' geographic characteristics as instrumental variables, find evidence suggesting that trade has a quantitatively large and robust positive ef-

<sup>1</sup>See Melitz (2007) for a survey on the New Trade Theory.

<sup>2</sup>There is a large literature that studies trade and growth in R&D-based models of growth. Trade affects growth by changing the benefits and costs of R&D. These theoretical analyses find that the effect of trade on growth is ambiguous (see e.g. Grossman and Helpman 1993). Baldwin and Robert-Nicoud (2008) add firm-heterogeneity to that literature and find that the growth effect of trade is, again, ambiguous.

fect on income. Alcalá and Ciccone (2004) introduce a concept of “real openness” and show its significant and statistically robust positive effect on productivity.<sup>3</sup> The charts in Lucas (2007) also suggest that openness (classified based on the five-test approach in Sachs and Werner, 1995) is positively linked to growth.

Our third result is that when heterogeneity is allowed in trade cost, trade composition matters for growth. Although the growth rate in the open economy is always higher than that in the closed economy, there is no monotonic relationship between the trade-to-GDP share and the growth rate. This finding may explain why the existing empirical evidence concerning this relationship is not conclusive (see Rodríguez and Rodrik (2000) for a very influential skeptical review of the cross-country evidence on trade and economic growth).

We should emphasize that it is not difficult to setup a version of the existing R&D-based models of economic growth in which trade can enhance growth. Since this type of models typically feature scale effects in innovation, trade can naturally raise growth rate (see e.g., Rivera-Batiz and Rommer (1991) and Grossman and Helpman (1991b)). Undoubtedly, innovation is a crucial determinant of economic growth. Nevertheless, viewing it as the only engine of growth is somewhat unsatisfactory in explaining the data. For example, Jones (1995) criticized that the typical “scale effects” assumed in the existing growth models is inconsistent with time-series evidence from industrial countries. Jones (1995b) pointed out that many R&D-based endogenous growth model predict a counterfactual “growth effects” of permanent changes in government policy. As summarized by Temple (1999), the empirical research spurred by the new growth theory typically and somewhat ironically lend support to the exogenous growth theory. As a result, researchers have searched for different mechanisms for economic growth. For instance, Jones (1995) builds a semi-endogenous growth model in which the growth rate, in the absence of “scale effects”, depends on a technological parameter and the exogenous rate of population growth. Lucas and Moll (2014), Perla and Tonetti (2014), and Benhabib, Perla and Tonetti (2014, 2015) have shown that diffusion of ideas across people/firms can generate endogenous growth. Perla and Tonetti (2014) and Waugh (2014) extend this type of models to the open economies and study how trade affect the diffusion of ideas and hence growth. Our paper adds to the literature by pointing out the resource reallocation across sectors induced by trade can be a new growth engine. To highlight this new growth engine, we intentionally start with a closed economy endowed with

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<sup>3</sup>Alcalá and Ciccone (2004) define “real openness” as imports plus exports in exchange-rate U.S. dollars relative to GDP in purchasing power parity U.S. dollars in an attempt to eliminate distortions due to cross-country differences in the relative price of nontradable goods.

exogenous TFP growth rates in multiple sectors and then isolate the endogenous contribution to aggregate growth from trade. Other recent studies also investigate trade in multi-sector settings (see, e.g., Allen, Arkolakis and Takahashi (2014), Caliendo and Parro (2014), Levchenko and Zhang (2013), Giovanni, Levchenko and Zhang (2014)), but these studies focus on trade and structural change and have found larger welfare gains of trade liberalization than in the one-sector models, and leave out the growth effect of trade.

The remainder of this paper is organized as follows. In Section 2, we set up a closed economy model incorporating heterogeneous productivity growth across sectors. We show that the overall growth rate is a simple average of the productivity growth across sectors. In Section 3, we extend the model to an open economy and characterize the endogenous trade patterns at the equilibrium. In Section 4, we examine the interaction between trade openness, trade composition and growth. Section 5 concludes the paper and discusses directions for future research.

## 2. A CLOSED ECONOMY MODEL

We focus on modelling the heterogeneity of productivity growth across sectors. For tractability, we abstract away from capital and make no attempt to model investment and savings decisions.

The production of the closed economy consists of two layers. The upper layer produces a final good in a competitive market by combining a continuum of sectoral goods  $i \in [0, 1]$  from the lower layer.

### 2.1. Final Good Production

The final good is produced competitively. We assume that the production function of the final good is given by

$$Y = \exp \left( \int_0^1 \log(Y(i)) di \right). \quad (1)$$

This production function implies equal cost-shares for different sectoral inputs, and hence guarantees balanced growth in all sectors. Normalizing the price of the final good to unity, the first-order condition for the profit maximization on the part of the final goods producers yields the following inverse demand curve for sector  $i$ :

$$P_t(i)Y_t(i) = Y_t. \quad (2)$$

## 2.2. Sectorial Goods Production

In sector  $i$ , a single monopoly has the technology

$$Y_t(i) = A_t(i)N_t(i), \quad (3)$$

and the technology in sector  $i$  grows at  $g_i$ , namely

$$A_t(i) = A_0 \exp(g_i t). \quad (4)$$

We assume that  $g_i$  is drawn independently from a common distribution with cumulative distribution function of  $F$  on the interval of  $[g_{\min}, g_{\max}]$  across sectors. We also assume that  $\bar{g} = \int_{g_{\min}}^{g_{\max}} g dF(g)$ . As the main focus is on the economic growth, we assume that  $A_t(i) = A_0$  for all  $i$  without loss of generality. Finally we assume that each sector has a potential entrant, who has an inferior technology in production. The entrant can produce according to

$$Y'_t(i) = \frac{A_t(i)}{\mu} N'_t(i), \quad (5)$$

where  $\mu > 1$ . We assume these two firms engage in Bertrand competition. It follows that the optimal price set by the monopoly firm is

$$P_t(i) = \mu \frac{w_t}{A_t(i)} = \mu \frac{w_t}{A_0 \exp(g_i t)}. \quad (6)$$

And its profit is

$$\Pi_t(i) = P_t(i)Y_t(i) - w_t N_t(i) = P_t(i)Y_t(i) - w_t \frac{Y_t(i)}{A_0 \exp(g_i t)} = \frac{\mu - 1}{\mu} P_t(i)Y_t(i). \quad (7)$$

And by equation (2), we have

$$\Pi_t(i) = \frac{\mu - 1}{\mu} Y_t. \quad (8)$$

## 2.3. The Equilibrium

In equilibrium, the labor market must clear. By (2) and (6) we have

$$N_t(i) = \frac{Y_t(i)}{A_0 \exp(g_i t)} = \frac{Y_t(j)}{A_0 \exp(g_j t)} = N_t(j), \quad (9)$$

for any  $i$  and  $j$ . Let  $\bar{N}$  denote the total labor in the economy:  $\bar{N} = \int_0^1 N_t(i) di$ . The result above implies

$$N_t(i) = \bar{N}, \text{ for any } i. \quad (10)$$

Finally, the final good output in period  $t$  is

$$\log Y_t = \int_0^1 g_i t di + \log \bar{N} + \log A_0, \quad (11)$$

or

$$Y_t = A_0 \bar{N} \exp \int_0^1 g_i t di = A_0 \bar{N} \exp \left[ t \int_{g_{\min}}^{g_{\max}} g f(g) dg \right]. \quad (12)$$

The output growth rate is given by

$$\frac{\dot{Y}_t}{Y_t} = \int_{g_{\min}}^{g_{\max}} g f(g) dg \equiv \bar{g}, \quad (13)$$

which is a simple average of the growth rates of all sectors.

### 3. AN OPEN ECONOMY MODEL

We begin with a symmetric two-country model and then generalize our equilibrium characterization to a symmetric  $m + 1$ -country case.

We denote the two countries as home,  $H$ , and foreign,  $F$ . Each country has a continuum of intermediate goods sectors. We assume the final goods producer must produce with domestically produced intermediate goods, but can choose whether to use foreign intermediate goods. If he chooses to use foreign intermediate goods, he is free to choose the type of intermediate goods to use. Suppose a final good producer chooses a set of foreign intermediate goods denoted by  $I_H$ . His production function becomes

$$Y_H = \Omega_H \exp \left\{ \frac{1}{\Omega_H} \left[ \int_0^1 \log Y_H^H(i) di + \int_{i \in I_H \subset [1,2]} \log Y_H^F(i) di \right] \right\}, \quad (14)$$

where  $Y_H^H(i)$  denotes the domestic intermediate goods,  $i \in [0, 1]$ , and  $Y_H^F(i)$  denotes the foreign intermediate goods,  $i \in I_H \subset [1, 2]$ ;  $\Omega_H$  is the total measure of intermediate goods used in the production. We use the notation  $|I_H| = \int_{i \in I_H \subset [1,2]} di$  to denote the measure of imported intermediate goods, so  $\Omega_H = 1 + |I_H|$ . In our notation, whenever the subscript and the superscript appear at the same time, the subscript indicates where intermediate goods are used and the superscript indicates where the intermediate goods

are produced. Notice if none of the foreign goods is used, the production is simply given by (1). Given the choice of  $I_H$ , the production function given in (14) is a special case of the CES-type used in Acemoglu, Antras and Helpman (2007)<sup>4</sup>. The unit elasticity of substitution in this case is known to be necessary for a balanced growth in an economy with heterogenous sectorial TFP growth. Note that this type of production function exhibits constant returns to scale allowing us to focus on the characterization of the unit cost.

Denote by  $P_H(i)$  and  $P_F(i)$  the prices of home intermediate goods and foreign intermediate goods, respectively. Then for a given set  $I_H$ , the unit cost of production in the home country can be obtained by solving the following cost minimization problem:

$$c(I_H) = \min_{\{Y_H^H(i), Y_H^F(i)\}} \int_0^1 P_H(i) Y_H^H(i) di + \int_{i \in I_H \subset [1,2]} P_F(i) Y_H^F(i) di, \quad (15)$$

with the constraint

$$\Omega_H \exp \left\{ \frac{1}{\Omega_H} \left[ \int_0^1 \log Y_H^H(i) di + \int_{i \in I_H \subset [1,2]} \log Y_H^F(i) di \right] \right\} \geq 1. \quad (16)$$

The above problem yields

$$c(I_H) = \exp \left\{ \frac{1}{\Omega_H} \left[ \int_0^1 \log P_H(i) di + \int_{i \in I_H \subset [1,2]} \log P_F(i) di \right] \right\}. \quad (17)$$

Given the unit cost above, the firm's profit maximization problem is to choose output quantity  $Y_H$  and the set of foreign intermediate goods  $I_H$  as follows:

$$\max_{Y_H, I_H} Y_H - c(I_H) Y_H \quad (18)$$

Notice that regardless of the level of production  $Y_H$ , the optimal set  $I_H$  is given by  $I_H^* = \arg \min c(I_H)$ , and its analytical form will be derived in the next section when we characterize the equilibrium. The demand for home

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<sup>4</sup>The treatment of domestic and foreign inputs in our production technology is asymmetric: the final good producers use all types of domestic intermediate goods but only a chosen set of foreign intermediate goods. This assumption, though convenient and consistent with the well-known home bias in international trade, is not crucial for our analysis. We relax this assumption in the Appendix and show that the result is robust.



goods and that for foreign goods are given by

$$Y_H^H(i) = \frac{c(I_H^*)}{1 + |I_H^*|} \frac{1}{P_H(i)} Y_H,$$

$$Y_H^F(i) = \begin{cases} \frac{c(I_H^*)}{1 + |I_H^*|} \frac{1}{P_F(i)} Y_H, & \text{if } i \in I_H^* \\ 0 & \text{otherwise} \end{cases},$$

respectively.

Similarly, given the choice of imported intermediate goods  $I_F$  from home country, the final good production function in the foreign country is given by

$$Y_F = \Omega_F \exp \left\{ \frac{1}{\Omega_F} \left[ \int_1^2 \log Y_F^F(i) di + \int_{i \in I_F \subset [0,1]} \log Y_F^H(i) di \right] \right\}, \quad (19)$$

where  $\Omega_F = 1 + |I_F|$ .

In the absence of trade costs, the law of one price must hold for any goods. As in the closed economy, Bertrand competition leads to

$$P_{H,t}(i) = \mu \frac{w_{Ht}}{A_0 \exp(g_it)}, P_{F,t}(i) = \mu \frac{w_{Ft}}{A_0 \exp(g_it)}. \quad (20)$$

To gain a better understanding of the production function and the firm's optimal choice of  $I_H$ , we consider several examples.

**Example 1**  $P_H(i) = 1$  for any  $i \in [0, 1]$  and  $P_F(i) = P_F > 1$  for any  $i \in [1, 2]$ . For any  $I_H \subset [1, 2]$ , the unit cost (in log) is  $\log(c(I_H)) = \frac{|I_H|}{1 + |I_H|} \log P_F \geq 0$ . It attains the minimum when  $|I_H| = 0$ . Therefore the optimal choice  $I_H^*$  is the empty set  $\emptyset$ , i.e. the final goods firm will not use any foreign imported goods. The demand for each type of intermediate goods is then given by  $Y_H^H(i) = \frac{1}{P_H(i)} Y_H = Y_H$  for any  $i \in [0, 1]$  and  $Y_H^F(i) = 0$  for any  $i \in [1, 2]$ .

**Example 2**  $P_H(i) = 1$  for any  $i \in [0, 1]$  and  $P_F(i) = P_F < 1$  for any  $i \in [1, 2]$ . For any  $I_H \subset [1, 2]$ , the unit cost (in log) is  $\log(c(I_H)) = \frac{|I_H|}{1 + |I_H|} \log P_F \leq 0$ . The cost attains its minimum when  $|I_H| = 1$ . Therefore the optimal choice  $I_H$  is  $[1, 2]$ , i.e. the final goods firm will use the entire set of foreign intermediate goods. The demand for each type intermediate goods is then given by  $Y_H^H(i) = \frac{1}{2} Y_H$  for any  $i \in [0, 1]$  and  $Y_H^F(i) = \frac{1}{2P_F} Y_H$  for any  $i \in [1, 2]$ .

**Example 3**  $P_H(i) = 1$  and  $\log P_F(i) \equiv p_F$  is a random variable, with a non-degenerate cumulative distribution function  $S$  and  $\int p_F dS(p_F) = 0$ . In this case  $\log c(\emptyset) = \log c([1, 2]) = 0$ . It is easy to see that a firm can achieve a lower unit cost by choosing  $I_H = \{i \mid \log P_F(i) < 0\}$ . The cost is then given

by  $\log c(I_H) = \frac{1}{1+S(0)} \int_{p_F < 0} p_F dS(p_F) < 0$ . It follows that the final goods firms will only use a subset of foreign intermediate goods. It is intuitive to conjecture that the final goods firms will use the foreign intermediate goods only if they are relatively cheap. So given a distribution  $S$ , there must exist a shreshold price  $P_F^*$  such that  $i \in I_H^*$  if and only if  $P_F(i) \leq P_F^*$ . We now formally prove this conjecture. First notice for any  $I_H$ , we can construct another set of intermediate goods  $\tilde{I}_H = \{i | \log P_F(i) \leq p_F^*\}$ , where  $p_F^* = S^{-1}(|I_H|)$ . Notice also that  $\int_{-\infty}^{p_F^*} dS(x) = \int_{i \in I_H} di = |I_H|$  while  $\int_{i \in \tilde{I}_H} \log P_F(i) di = \int_{-\infty}^{p_F^*} x dS(x) \leq \int_{i \in I_H} \log P_F(i) di$  by construction, where the inequality will hold strictly if  $\tilde{I}_H$  and  $I_H$  differ with positive measure. So without loss of generality, the optimal set  $I_H^*$  will take the form  $I_H^* = \{i | \log P_F(i) \leq p_F^*\}$ , with  $p_F^*$  to be endogenously determined. To find  $I_H^*$  is then equivalent to solving  $\min \frac{1}{1+S(p_F)} \int_{-\infty}^{p_F} x dS(x)$ , which yields  $p_F^* = \frac{1}{1+S(p_F^*)} \int_{-\infty}^{p_F^*} x dS(x)$ . The right-hand side is the unit cost of production, which in turn is the average price of intermediate goods used in production. The left-hand side is the price of the most expensive intermediate goods imported and used in production. The optimal choice of the type of foreign intermediate goods used in production should achieve the lowest unit production cost. A firm needs to pay  $p_F^*$  to add one additional type of intermediate goods to production, but it would reduce its cost by  $\frac{1}{1+S(p_F^*)} \int_{-\infty}^{p_F^*} x dS(x)$ . The optimal set of intermediate goods should lead to equal gains and costs. Namely  $p_F^* = \frac{1}{1+S(p_F^*)} \int_{-\infty}^{p_F^*} x dS(x)$ . To give a more concrete characterization, we assume  $p_F$  is uniformly distributed across  $[-a, a]$ . In this case we can obtain  $p_F^* = a(\sqrt{8} - 3) < 0$ .

### 3.1. Equilibrium

Since the two countries are symmetric, we have

$$w_{Ht} = w_{Ft} \equiv w_t, \quad Y_{Ht} = Y_{Ft} \equiv Y_t \quad (21)$$

In this case, given the set  $I_{Ht}^*$ , the unit cost for final goods producers in home country becomes

$$c_t^* = c_t(I_{Ht}^*) = \mu w_t \exp \left\{ -\frac{t}{\Omega_H} \left[ \int_0^1 g_i di + \int_{i \in I_{Ht}^* \cap [1,2]} g_i di \right] \right\} \quad (22)$$

Given the expression of  $c_t^*$ , it immediately follows, as in Example 3, that the selection of the optimal set  $I_{Ht}^*$  is equivalent to choosing a shreshold growth rate  $g_t^*$  such that for all  $g_i \geq g_t^*$ , we have  $i \in I_{Ht}^*$ . The cost minimization is

then equivalent to finding the cutoff  $g_t^*$ ,

$$g_t^* = \arg \max_g \frac{\int_{g_{\min}}^{g_{\max}} x f(x) dx + \int_g^{g_{\max}} x f(x) dx}{1 + \int_g^{g_{\max}} f(x) dx}. \quad (23)$$

The first-order condition then implies

$$g_t^* = g^* = \frac{\int_{g_{\min}}^{g_{\max}} x f(x) dx + \int_{g^*}^{g_{\max}} x f(x) dx}{1 + \int_{g^*}^{g_{\max}} f(x) dx}. \quad (24)$$

Hence  $I_{Ht}^* = I^* = \{i | g_i \geq g^*\}$ . And by symmetry  $I_F^* = I^*$ .

We now prove the existence and the uniqueness of this cutoff growth rate,  $g^*$ . To that end, let us define an auxiliary function:

$$\Phi(g) = g + g \int_g^{g_{\max}} f(x) dx - \int_{g_{\min}}^{g_{\max}} x f(x) dx - \int_g^{g_{\max}} x f(x) dx, \quad (25)$$

Notice that  $\Phi(g_{\min}) = 2g_{\min} - 2\bar{g} < 0$ , and  $\Phi(g_{\max}) = g_{\max} - \bar{g} > 0$ . So by the Intermediate Value Theorem, there exists a value  $g^*$  such that  $\Phi(g^*) = 0$ . Finally

$$\Phi'(g) = 1 + \int_g^{g_{\max}} f(x) dx > 0, \quad (26)$$

so  $g^*$  is unique by monotonicity. We then have  $|I^*| = \int_{g^*}^{g_{\max}} f(x) dx = 1 - F(g^*)$ .

Perfect competition among final good producers implies

$$c_t^* = 1. \quad (27)$$

Then the total demand for each type of intermediate goods is

$$P_{H,t}(i) Y_{Ht}^H(i) = \frac{1}{2 - F(g^*)} Y_t, \quad (28)$$

and  $P_{F,t}(i) Y_{Ht}^F(i) = \begin{cases} \frac{1}{2 - F(g^*)} Y_t & \text{if } g_i \geq g^* \\ 0 & \text{otherwise} \end{cases}$ .

By symmetry, we have

$$P_{F,t}(i) Y_{Ft}^F(i) = \frac{1}{2 - F(g^*)} Y_t, \quad (29)$$

and  $P_{H,t}(i) Y_{Ft}^H(i) = \begin{cases} \frac{1}{2 - F(g^*)} Y_t & \text{if } g_i \geq g^* \\ 0 & \text{otherwise} \end{cases}$ .

The market clearing condition for each type of intermediate goods is given by,

$$Y_{Ht}(i) = Y_{Ht}^H(i) + Y_{Ht}^F(i),$$

and by symmetry we have,

$$Y_{Ht}(i) = \left\{ \begin{array}{ll} \frac{1}{2-F(g^*)} \frac{1}{P_{Ht}(i)} Y_t & \text{if } g_i < g^* \\ \frac{1}{2-F(g^*)} \frac{2}{P_{Ht}(i)} Y_t & \text{if } g_i \geq g^* \end{array} \right\}. \quad (30)$$

Finally, equation (20) yields the total labor used in sector  $i$ ,

$$N_t(i) = \left\{ \begin{array}{ll} 2n_t & \text{if } g_i \geq g^* \\ n_t & \text{otherwise} \end{array} \right\}, \quad (31)$$

where  $n_t$  is determined by the aggregate labor market clearing condition:

$$n_t + n_t \int_{g^*}^{g^{\max}} f(x) dx = \bar{N},$$

which yields  $n = \frac{1}{2-F(g^*)} \bar{N}$ . Finally the aggregate output can be written as

$$Y_t = A_0 n_t (2 - F(g^*)) \exp \left\{ \frac{\bar{g} + \int_{g^*}^{g^{\max}} x f(x) dx}{1 + \int_{g^*}^{g^{\max}} f(x) dx} t \right\} = A_0 \bar{N} \exp(g^* t). \quad (32)$$

The growth rate under the open economy is then given by

$$\frac{\dot{Y}_t}{Y_t} = g^* > \bar{g}. \quad (33)$$

Several remarks are in order. First, even without any trade costs, some goods will be nontradable at equilibrium. In our model any intermediate goods sector with  $g_i < g^*$  will be nontradable. Because of the low technological progress in these sectors, their prices will be relatively too high and hence they will not find any demand from final goods producers abroad. Second, the phenomenon that tradable sectors' productivity grows faster than nontradable sector's productivity is an equilibrium outcome. It hence provides a micro-foundation for the well-known Balassa-Samuelson hypothesis.

To gain a better understanding of why openness to trade can increase growth, we take a closer look at its effect on different sectors. Since  $0 < F(g^*) < 1$ , the labor in sector  $i$  with  $g_i \geq g^*$  is  $N_t(i) = \frac{2\bar{N}}{2-F(g^*)} > \bar{N}$ , while the labor in sector  $i$  with  $g_i < g^*$  is  $N_t(i) = \frac{\bar{N}}{2-F(g^*)} < \bar{N}$ . Thus, compared

with the closed economy, the labor is shifted towards the tradable sectors. We now show that this resource reallocation is the key to achieving higher growth in the open economy. To see this, note that

$$\begin{aligned} \int_0^1 \frac{N_t(i)}{N} g_i di &= \int_{g_{\min}}^{g^*} \frac{n}{N} g f(g) dg + \int_{g^*}^{g_{\max}} \frac{2n}{N} g f(g) dg \\ &= \frac{\int_{g_{\min}}^{g^*} n g f(g) dg + \int_{g^*}^{g_{\max}} 2n g f(g) dg}{n(2 - F(g^*))} \\ &= \frac{\bar{g} + \int_{g^*}^{g_{\max}} g f(g) dg}{1 + \int_{g^*}^{g_{\max}} f(g) dg} \\ &= g^*, \end{aligned}$$

namely,  $g^*$  is a weighted average of the TFP growth rates. As resource reallocation in the open economy raises the weights on the high TFP growth,  $g^*$  is naturally higher than  $\bar{g}$  in the closed economy, which is a simple average. Thus, we have shown:

**PROPOSITION 1.** *Trade has a growth effect:  $g^* > \bar{g}$ . The effect comes from the reallocation of labor from sectors with low TFP growth to sectors with high TFP growth.*

#### 4. TRADE POLICY AND GROWTH

In this section, we discuss trade policy and growth. In particular, we ask whether a reduction in trade cost would have a growth effect. For that purpose, we depart from the free trade case above by assuming that in each period a firm needs to pay  $\phi > 0$  units of labor in order to export its goods to the foreign market<sup>5</sup>.

Similar to the free trade case, there exists a threshold  $g^*(\phi)$  such that a firm will export if and only if  $g_i \geq g^*(\phi)$ . Clearly,  $g^*(\phi) \geq g^*$ , since an intermediate good that is nontradable under free trade will certainly remain so under costly trade.

Notice that given the cutoff  $g^*(\phi)$ , the demand for each type of intermediate goods is then given by

$$P_{Ht}(i) Y_{Ht}^H(i) = \begin{cases} \frac{1}{1 + \int_{g^*(\phi)}^{g_{\max}} f(g) dg} Y_t & \text{if } g_i < g^*(\phi) \\ \frac{2}{1 + \int_{g^*(\phi)}^{g_{\max}} f(g) dg} Y_t & \text{if } g_i \geq g^*(\phi) \end{cases}, \quad (34)$$

<sup>5</sup>An iceberg cost in our model does not have an impact on long-term growth. We hence omit its analysis.

where  $P_{H_t}(i)$  is given by equation (20) as before. We can then write the total labor used in each sector as

$$N_t(i) = \begin{cases} 2n_t + \phi & \text{if } g_i \geq g^*(\phi) \\ n_t & \text{otherwise} \end{cases}, \quad (35)$$

where  $n_t$  remains to be determined by the labor market equilibrium condition. The constant markup between its production cost and price implies that the profit for the firm in sector  $i$  is

$$\Pi_t(i) = \begin{cases} 2n_t(\mu - 1)w_t - \phi w_t & \text{if } g_i \geq g^*(\phi) \\ n_t(\mu - 1)w_t & \text{otherwise} \end{cases}. \quad (36)$$

The labor market clearing condition is

$$n_t + (n_t + \phi) \int_{g^*(\phi)}^{g_{\max}} f(g)dg = \bar{N}. \quad (37)$$

In terms of the fixed cost, there are three cases: prohibitive, negligible, and moderate. In the prohibitive case, the fixed cost is too high, namely

$$2n_t(\mu - 1)w_t - \phi w_t < n_t(\mu - 1)w_t, \quad (38)$$

so that even the most productive sectors will not export. So we have

$$n_t = \bar{N}, \quad (39)$$

and from (38),

$$\phi > (\mu - 1)\bar{N} \equiv \phi_{\max}.$$

In the negligible case,  $g^*(\phi) = g^*$ , we have

$$n(\mu - 1) \geq \phi, \quad (40)$$

where  $n$  can be solved from

$$n + (n + \phi) \int_{g^*}^{g_{\max}} f(g)dg = \bar{N}. \quad (41)$$

This requires

$$\frac{\bar{N} - \phi \int_{g^*}^{g_{\max}} f(g)dg}{1 + \int_{g^*}^{g_{\max}} f(g)dg} (\mu - 1) \geq \phi, \quad (42)$$

or

$$\phi \leq \frac{\bar{N}(\mu - 1)}{1 + \mu \int_{g^*}^{g_{\max}} f(g)dg} \equiv \phi_{\min}. \quad (43)$$

In this case, the aggregate output will be

$$Y_t = A_0 \left[ \bar{N} - \phi \int_{g^*}^{g_{\max}} f(x)dx \right] \exp(g^*t). \quad (44)$$

If  $\phi < \phi_{\min}$ , a further reduction in the trade cost will not affect the growth rate but will have a level effect on aggregate output.

Finally in the moderate case,  $\phi_{\min} < \phi < \phi_{\max}$ , we have  $g^*(\phi) > g^*$ . In this case, although the final goods firm would like to use those foreign indeterminate goods  $i \in [1, 2]$  where  $g_i$  falls between  $g^*$  and  $g^*(\phi)$ , the foreign producers of these intermediate goods will not find it profitable to export them given the fixed cost. The cutoff  $g^*(\phi)$  and  $n$  are jointly determined by

$$n + (n + \phi) \int_{g^*(\phi)}^{g_{\max}} f(g)dg = \bar{N} \quad (45)$$

$$n(\mu - 1) = \phi \quad (46)$$

Or simply,

$$\frac{\phi}{\mu - 1} + \frac{\mu\phi}{\mu - 1} \int_{g^*(\phi)}^{g_{\max}} f(g)dg = \bar{N} \quad (47)$$

It is easy to see that  $\frac{\partial g^*(\phi)}{\partial \phi} > 0$ . The aggregate output in this case is given by

$$Y_t = A_0 \left[ \bar{N} - \phi \int_{g^*(\phi)}^{g_{\max}} f(g)dg \right] \exp(\hat{g}(\phi)t), \quad (48)$$

where  $\hat{g}(\phi)$  is the economic growth rate in the presence of trade cost, and is given by,

$$\hat{g}(\phi) = \frac{\bar{g} + \int_{g^*(\phi)}^{g_{\max}} xf(x)dx}{1 + \int_{g^*(\phi)}^{g_{\max}} f(x)dx} < \frac{\bar{g} + \int_{g^*}^{g_{\max}} xf(x)dx}{1 + \int_{g^*}^{g_{\max}} f(x)dx} = g^*. \quad (49)$$

The inequality above follows from the definition of  $g^*$  (see equation (23)).

We now show that when the fixed cost is moderate, trade cost has both level and growth effects on output. First, the total trade cost is

$$\Psi(\phi) = \phi \int_{g^*(\phi)}^{g_{\max}} f(g)dg. \quad (50)$$

Notice that (47) can be written as  $\frac{\phi}{\mu-1} + \frac{\mu}{\mu-1}\Psi(\phi) = \bar{N}$ . We have  $\frac{\partial\Psi}{\partial\phi} < 0$ . Hence a reduction in trade cost will increase the total trade costs as the range of tradables expands. Therefore, a reduction in trade cost will generate a negative level effect on output. We now turn our attention to the effect of  $\phi$  on the growth rate. We first show that  $\frac{\partial\hat{g}}{\partial\phi} < 0$ .

$$\frac{\partial\hat{g}}{\partial\phi} = -\frac{f(g^*(\phi))}{[1 + \int_{g^*(\phi)}^{g_{\max}} f(g)dg]^2} \Phi(g^*(\phi)) \frac{\partial g^*(\phi)}{\partial\phi} < 0 \quad (51)$$

where we used the fact that  $\Phi'(g) > 0$  and  $\Phi(g^*) = 0$ , which implies  $\Phi(g^*(\phi)) > 0$  (note that  $g^*(\phi) > g^*$ ). In other words, a high trade cost will reduce the growth rate for the moderate case of  $\phi$ ,  $\phi_{\min} < \phi < \phi_{\max}$ .

The discussions above can be summarized into a proposition.

**PROPOSITION 2.** *In the moderate case, as trade cost reduces, the economic growth rate rises:  $\frac{\partial\hat{g}}{\partial\phi} > 0$ .*

Note that Proposition 2 does not depend on the asymmetric treatment of domestic and foreign intermediate inputs assumed in production function (14), see the Appendix for the robustness check.

To illustrate the three cases: the prohibitive, the negligible, and the moderate, we examine a simple numerical example below.

**Example 4** We assume that the growth rate of each sector follows a power distribution with  $F(g) = (g/g_{\max})^\gamma$ . The mean growth rate in the close economy is given by  $\bar{g} = \int_0^{g_{\max}} g dF(g) = \frac{\gamma}{\gamma+1} g_{\max}$ . We set  $g_{\max} = 2\%$  and  $\gamma = 1$  so that the average growth rate in the closed economy is given by  $\bar{g} = 1\%$ , similar to that in Table 1. Without loss of generality, we normalize  $\bar{N} = 1$ . We set  $\mu = 1.1$  so that the markup is 10%, matching the parameter value in the standard New Keynesian monopolistic competition model. The growth in the open economy with free trade is given by

$$g^* = \frac{0.01 + \frac{1}{0.04}((0.02)^2 - g^{*2})}{2 - g^*/0.02}, \quad (52)$$

which yields

$$g^* = 1.17\%.$$

Under free trade, the range of tradables is given by  $1 - F(g^*) = 0.415$ , namely, the tradables account for 41.5% of the intermediate goods sector. Suppose that the interest rate  $r = 2\%$ . Despite only a small increase in the growth rate, the present value of total output however will jump from  $1/(r - \bar{g})$  to  $1/(r - g^*)$ , an increase of 20.48%, by opening up to trade.



Now let us look at the three cases when a trade cost is present. It is straightforward to obtain  $\phi_{\min} = 0.0687$  and  $\phi_{\max} = 0.1$ . The threshold growth rate of TFPs for tradable sectors is,

$$g^*(\phi) = \left\{ \begin{array}{ll} 1.17\% & \text{if } \phi \leq 0.0687 \\ \frac{2.1\phi - 0.1}{1.1\phi} \times 2\% & \text{if } 0.0687 < \phi < 0.1 \\ 2\% & \text{if } \phi \geq 0.1 \end{array} \right\}. \quad (53)$$

and the corresponding growth rate of the output is,

$$\hat{g}(\phi) = \left\{ \begin{array}{ll} 1.17\% & \text{if } \phi \leq 0.0687 \\ \frac{2 - \left(\frac{2.1\phi - 0.1}{1.1\phi}\right)^2}{2 - \frac{2.1\phi - 0.1}{1.1\phi}}\% & \text{if } 0.0687 < \phi < 0.1 \\ 1\% & \text{if } \phi \geq 0.1 \end{array} \right\}. \quad (54)$$

#### 4.1. M+1-Country Model

We now extend our model to the  $M + 1$  symmetrical countries. The home country produces a continuum of intermediate goods indexed by  $i \in [0, 1]$ . The set of intermediate goods produced by country  $m = 1, 2, \dots, M$  is  $[m, m + 1]$ , respectively.

**Free Trade** We first look at the equilibrium in the case of free trade. As discussed in the two-country model, due to symmetry, each country will produce the same amount of final output and have the same wage under free trade. The price of intermediate goods  $i$  is then given by

$$P_t(i) = \mu \frac{w_t}{A_0 \exp(g_i t)}. \quad (55)$$

As in the two-country model, there exists a cutoff  $g^*$  and sector  $i$  will export if and only if  $g_i > g^*$ . The unit cost of production is given by

$$c_t^* = \mu w_t \exp \left\{ -\frac{t}{\Omega} \left[ \int_0^1 g_i di + M \int_{g_i > g^*} g_i di \right] \right\}, \quad (56)$$

where  $\Omega = 1 + M \int_{g^*}^{g_{\max}} f(g) dg$ . Again, cutoff  $g^*$  yields the lowest unit production cost for the final goods firm, namely,  $g^*$  is determined by

$$g^* = \arg \max_g \frac{\bar{g} + M \int_g^{g_{\max}} x f(x) dx}{1 + M \int_g^{g_{\max}} f(x) dx}. \quad (57)$$

Then the first-order condition is,

$$g^* = \frac{\bar{g} + M \int_{g^*}^{g_{\max}} g f(g) dg}{1 + M \int_{g^*}^{g_{\max}} f(g) dg}. \quad (58)$$

Again, we define

$$\Phi(g, M) = g + Mg \int_g^{g_{\max}} f(x)dx - \bar{g} - M \int_g^{g_{\max}} xf(x)dx. \quad (59)$$

For any  $M$ , we have  $\Phi(g_{\min}, M) = g_{\min}(M + 1) - (M + 1)\bar{g} < 0$  and  $\Phi(g_{\max}, M) = g_{\max} - \bar{g} > 0$ . So there exists a solution  $g^*$  such that  $\Phi(g^*, M) = 0$ . Again given  $\Phi'_g(g, M) = 1 + M \int_g^{g_{\max}} f(x)dx > 0$ , the solution is unique. Notice that for any given  $g$ , we have  $\Phi'_M(g, M) = g \int_g^{g_{\max}} f(x)dx - \int_g^{g_{\max}} xf(x)dx < 0$ . This implies that  $\frac{\partial g^*}{\partial M} = -\frac{\Phi'_M(g^*, M)}{\Phi'_g(g^*, M)} > 0$ , namely, as the number of trading partners increases, the growth rate in each country will increase under free trade. The intuition is as follows. More trading partners will intensify the competition in trade, which will increase the cutoff  $g^*$ . As the tradable sectors now are more concentrated in sectors with high growth rate, the average growth increases.

**Trade with Fixed cost** Now we consider the effect of fixed cost. There exists a unique threshold TFP growth rate,  $g^*(\phi)$ , such that the firm in sector  $i$  will choose to export if and only if  $g_i \geq g^*(\phi)$ . As in the two-country model,  $g^*(\phi) \geq g^*$ . We can write the labor demand in each sector as

$$N_t(i) = \begin{cases} (1 + M)n_t + M\phi & \text{if } g_i \geq g^*(\phi) \\ n_t & \text{otherwise} \end{cases}, \quad (60)$$

and the profit function for each firm as

$$\Pi_t(i) = \begin{cases} (1 + M)n_t(\mu - 1)w_t - M\phi w_t & \text{if } g_i \geq g^*(\phi) \\ n_t(\mu - 1)w_t & \text{otherwise} \end{cases}. \quad (61)$$

And the labor market clearing condition implies

$$n + M(n + \phi) \int_{g^*(\phi)}^{g_{\max}} f(x)dx = \bar{N}. \quad (62)$$

Given other parameter values, the relationship between  $\phi$  and the unique threshold  $g^*(\phi)$  is given by

$$g^*(\phi) = \begin{cases} g_{\max} & \text{if } \phi \geq \phi_{\max} \\ F^{-1}\left(1 - \frac{(\mu-1)\bar{N}-\phi}{M\mu}\right) & \text{if } \phi_{\min} < \phi < \phi_{\max} \\ g^* & \text{if } \phi \leq \phi_{\min} \end{cases}. \quad (63)$$

Here  $\phi_{\max} = (\mu - 1)\bar{N}$ , and  $\phi_{\min} = \frac{(\mu-1)\bar{N}}{1+M\mu \int_{g^*}^{g_{\max}} f(x)dx}$  where  $g^*$  is the threshold TFP growth rate without trade cost defined in equation (58).

Notice that  $g^* < g^*(\phi) < g_{\max}$  if  $\phi_{\min} < \phi < \phi_{\max}$ . Equation (63) states that when the trade cost is prohibitive large, countries do not trade with each other. When the trade cost drops below the upper threshold level,  $\phi_{\max}$ , the range of tradable sectors gradually increases. When the trade cost drops further below the lower threshold level,  $\phi_{\min}$ , the range of tradable sectors stays at the equilibrium range reached in the free trade case.

The discussions on the prohibitive and negligible cases are as in the two-country case. For the moderate case, exporting yields zero profits for the firms, namely,

$$n(\mu - 1) = \phi. \quad (64)$$

The labor market equilibrium condition is,

$$n + M(n + \phi) \int_{g^*(\phi)}^{g_{\max}} f(x) dx = \bar{N}. \quad (65)$$

Combining these two equations gives

$$\frac{\phi}{\mu - 1} + M \frac{\mu\phi}{\mu - 1} \int_{g^*(\phi)}^{\infty} f(x) dx = \bar{N}. \quad (66)$$

Re-arranging terms yields  $1 - F(g^*(\phi)) = \frac{(\mu-1)\bar{N}-\phi}{M\mu\phi}$ , or the second line in equation (63).

Given  $g^*(\phi)$ , the aggregate output is,

$$Y = A_0 [1 - M\phi \int_{g^*(\phi)}^{g_{\max}} f(x) dx] \exp(\hat{g}(\phi)t), \quad (67)$$

where  $\hat{g}(\phi)$  is the output growth rate given by,

$$\hat{g}(\phi) = \frac{\bar{g} + M \int_{g^*(\phi)}^{g_{\max}} x f(x) dx}{1 + M \int_{g^*(\phi)}^{g_{\max}} f(x) dx}. \quad (68)$$

Similar to the two-country model, we can show  $\frac{\partial \hat{g}}{\partial \phi} > 0$  for  $\phi_{\min} < \phi < \phi_{\max}$ , namely, the growth rate will increase when the trade cost decreases. The proof is similar to the two-country model, so we omit it for conciseness.

We now study the impact of an increase in the number of trading partners on the output growth rate. Differentiating (68) yields

$$\begin{aligned} \frac{\partial \hat{g}(\phi)}{\partial M} &= \frac{\int_{g^*(\phi)}^{g_{\max}} x f(x) dx - M g^*(\phi) f(g^*(\phi)) \frac{\partial g^*(\phi)}{\partial M}}{1 + M \int_{g^*(\phi)}^{g_{\max}} f(x) dx} \\ &= \frac{\int_{g^*(\phi)}^{g_{\max}} g f(g) dg - g^*(\phi) \int_{g^*(\phi)}^{g_{\max}} f(x) dx}{1 + M \int_{\bar{g}}^{\infty} f(x) dx} > 0, \end{aligned} \quad (69)$$

where we have used  $\frac{\partial g^*(\phi)}{\partial M} = \frac{1}{Mf(g^*(\phi))} \int_{g^*(\phi)}^{g_{\max}} f(x)dx$  from equation (66). In other words, similar to the case without trade costs, the growth rate of output will increase with the number of trading partners.

**PROPOSITION 3.** *As the number of trading partners increases, overall growth is enhanced. This result holds with or without trade cost.*

**Example 5** We now extend Example 4 to the case of multiple countries. We first compute the growth rate in the open economy with free trade as

$$g^* = \frac{0.01 + \frac{M}{0.04}((0.02)^2 - g^{*2})}{1 + M\left(\frac{0.02 - g^*}{0.02}\right)}, \quad (70)$$

or  $g^* = 0.02\left(1 - \frac{1}{1 + \sqrt{M+1}}\right)$ . The total range of imported intermediate goods is  $M[1 - F(g^*)] = \sqrt{M+1} - 1$ . The trade share as measured by the ratio of the value of total imported intermediate goods to output in each country is given by  $1 - 1/\sqrt{M+1}$ . It is easy to see that both the output growth rate and trade share increase monotonically with the number of trading partners. In this example, trade increases growth via two channels. First, there is the labor reallocation effect. When the number of trading partners increases, competition intensifies and labor shifts toward the high productivity-growth sectors. There is also a *total product-variety effect*, which is less straightforward than we might otherwise think. As the number of trading partners increases, each country exports a *narrower* range of goods, but the total variety of exports from all countries expands, leading to higher overall growth.

As for the effect of trade cost in this example, we have

$$\phi_{\min} = \frac{\mu - 1}{1 + \mu \frac{M}{1 + \sqrt{M+1}}}, \quad \phi_{\max} = \mu - 1.$$

We focus on the case with  $\phi_{\min} < \phi < g_{\max}$ . The cutoff  $g^*(\phi)$  is then given by

$$g^*(\phi) = \left[1 - \frac{\mu - 1 - \phi}{M\mu\phi}\right] g_{\max},$$

and the growth rate is given by

$$\hat{g}(\phi) = 0.01 \left[1 + \frac{(\mu - 1 - \phi)\left(1 - \frac{\mu - 1 - \phi}{M\mu\phi}\right)}{(\mu - 1)(\phi + 1)}\right].$$

It is evident that  $\hat{g}(\phi)$  is increasing in  $M$ .

## 5. TRADE SHARE AND GROWTH

It is shown above that a decrease in trade cost and an increase in the number of trading partners would boost growth rates unambiguously. Will the trade share, measured as Trade/GDP, move in the same direction? Empirical evidence seems to be inconclusive (see, e.g., Rodriguez and Rodrik (2001)). Our results below may reconcile the conflicting empirical findings.

First, consider a reduction in trade cost, due to either trade liberalization or an improvement in transportation and/or communication technologies. If trade cost is homogeneous across sectors, the range of tradables will widen, leading to higher growth and a higher Trade/GDP ratio. However, if trade cost is heterogeneous, and the reduction in trade cost is not uniform across sectors, growth and the Trade/GDP ratio may move in opposite directions. This can be seen from the following example. Let the growth rates for sectors  $i \in [0, 1]$  take only three values:  $g_1$ ,  $g_2$  and  $g_3$  and let  $g_1 < g_2 < g_3$ , with the corresponding fractions  $\theta_1, \theta_2$ , and  $\theta_3$ , respectively. We assume  $\theta_2 > \theta_3$ ,  $g_2 > \theta_1 g_1 + g_2 \theta_2 + g_3 \theta_3$ , and  $\theta_3 g_3 > \theta_2 g_2$ . Suppose the trading costs are 0,  $\phi_2$  and  $\phi_3$  accordingly. Suppose also that initially  $\phi_2 > (\mu - 1)$  and  $\phi_3 > (\mu - 1)$ . Hence it is easy to prove that there is no trade initially. We now consider trade liberalization.

Case 1: Suppose trade liberalization reduces  $\phi_2$  to zero but  $\phi_3$  remains the same. Notice that in this case, the final goods producer will choose  $\alpha$  fraction of foreign type-2 intermediate goods to solve

$$\max_{\alpha \leq \theta_2} \frac{\bar{g} + \alpha g_2}{1 + \alpha}. \quad (71)$$

Given that  $g_2 > \bar{g}$ , we have  $\alpha = \theta_2$ , hence all foreign type-2 intermediate goods will be used. In this case the overall economic growth rate is  $g_{c1} = \frac{\bar{g} + \theta_2 g_2}{1 + \theta_2}$ , and the Trade/GDP ratio is  $\frac{\theta_2}{1 + \theta_2}$ .

Case 2: Suppose trade liberalization reduces  $\phi_3$  to zero but  $\phi_2$  remains the same. The growth rate is  $g_{c2} = \frac{\bar{g} + \theta_3 g_3}{1 + \theta_3}$  and the Trade/GDP ratio is given by  $\frac{\theta_3}{1 + \theta_3}$ . Under these parameter values the growth rate in case 2 is greater than the growth rate in Case 1, i.e.

$$g_{c2} = \frac{\bar{g} + \theta_3 g_3}{1 + \theta_3} > g_{c1} = \frac{\bar{g} + \theta_2 g_2}{1 + \theta_2}, \quad (72)$$

but the trade share in Case 2 is lower than that in Case 1. This example hence illustrates that trade does not yield a simple monotonic relationship between growth and commonly used proxy of openness such as the trade share.

## 6. CONCLUDING REMARKS

As far as we know, this is the first attempt to discuss the growth effect in a trade model with heterogeneity in productivity growth across sectors. We show that although the growth rate in each sector is exogenous, the overall growth rate is endogenous, depending on trade parameters in an open economy. We show that as trade cost declines and the number of trading partners rises, the resources will shift to sectors with higher productivity growth, leading to a higher overall growth. Nevertheless, trade openness, measured as Trade/GDP, may not always increase. The model could also be used to analyze the effect of fiscal, industrial, and tariff policies on growth, which we leave for future study.

Another area for future study is to introduce capital into the model. In this richer setting, one would be able to discuss investment and savings decisions and intertemporal trade-offs. Introducing capital would also add a richer dynamics to the reallocation of labor across sectors, generating implication beyond the balanced growth as in Kongsamut, Rebelo and Xie (2001), Ngai and Pissarides (2007), and Acemoglu and Guerrieri (2008).

Finally, the most difficult exercise would be to allow for asymmetric countries. Such a framework would be useful for discussing trade and FDI patterns as well as convergence and may generate insights on different industry policies from the perspectives of developed and developing economies.

## APPENDIX: SYMMETRIC PRODUCTION FUNCTION

We have assumed an asymmetry between domestically produced intermediates and their foreign counterparts. We now relax this assumption and show that our main results regarding the effect of trade on growth remain robust. For simplicity we focus on the two-country model. The production function is changed to:

$$Y_H = \Omega_H \exp \left\{ \frac{1}{\Omega_H} \left[ \int_{i \in I_C [0,1]} \log Y_H^H(i) di + \int_{i \in I_H \subset [1,2]} \log Y_H^F(i) di \right] \right\} \quad (A1)$$

and  $\Omega_H = |I| + |I_H| = 1$ . Namely, we now assume that the production in each country requires a measure of unity of intermediate goods, either produced by domestic intermediate goods producers or foreign intermediate goods producers. By construction, the growth rate in the closed economy is still given by equation (13). Similar to the model in Section 3, only sectors with high TFP growth rate will engage in international trade. Since the total measure of intermediate goods in production is now fixed to unity, some sectors with low productivity will be driven out by competition. The

labor used in each sector is given by,

$$N_t(i) = \begin{cases} 0 & \text{if } g_i < g^{**} \\ n & \text{if } g^{**} \leq g_i < g^* \\ 2n + \phi & \text{if } g_i \geq g^* \end{cases}. \quad (\text{A2})$$

The above equation says that sectors with  $g_i < g^{**}$  will be out of production due to competition from foreign intermediate goods; sectors with  $g^{**} \leq g_i < g^*$  will only serve the domestic market; and sectors with  $g_i \geq g^*$  will serve both markets. By symmetry, the domestic final goods producers will use  $1 - F(g^*)$  measure of foreign intermediate goods. Namely,  $|I| = 1 - F(g^{**})$ , and  $|I_H| = 1 - F(g^*)$ . Since the total measure of intermediate goods sectors is unity, we must have  $1 - F(g^*) + 1 - F(g^{**}) = 1$ , or

$$F(g^{**}) = 1 - F(g^*), \quad (\text{A3})$$

For an interior solution, we must reach the point at which the firms are indifferent to exporting, so  $n(\mu - 1) = \phi$ , as before. Finally the labor market clearing requires  $[1 - F(g^{**})]n + (2n + \phi)[1 - F(g^*)] = \bar{N} = 1$ . These two conditions imply

$$1 - F(g^*) = \frac{\mu - 1 - \phi}{\mu\phi}. \quad (\text{A4})$$

Since  $g^{**} \leq g^*$ , we have  $1 - F(g^*) \leq F(g^*)$ . Or  $F(g^*) \geq \frac{1}{2}$ . The interior solution requires  $\frac{2\mu-2}{2+\mu} \leq \phi \leq \mu - 1$ . It is easy to see that that  $\frac{dg^*}{d\phi} > 0$ . Equation (A3) then implies  $\frac{dg^{**}}{d\phi} = -\frac{f(g^*)}{f(g^{**})} \frac{dg^*}{d\phi}$ . The aggregate output is given by

$$Y = A \left[ 1 - \phi \int_{g^*}^{g^{\max}} f(x) dx \right] \exp(\hat{g}t), \quad (\text{A5})$$

where  $\hat{g}$  is the output growth rate given by

$$\hat{g} = \int_{g^*}^{g^{\max}} x f(x) dx + \int_{g^{**}}^{g^{\max}} x f(x) dx, \quad (\text{A6})$$

which implies  $\frac{d\hat{g}}{d\phi} = -g^* f(g^*) \frac{dg^*}{d\phi} - g^{**} f(g^{**}) \frac{dg^{**}}{d\phi} = (g^{**} - g^*) \frac{dg^*}{d\phi} < 0$ . In other words, a reduction in fixed cost of trade will lead to higher economic growth as in Section 4.

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