What can China Expect from an Increase of the Mandatory Retirement Age?

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Because of an aging population, China is expected to raise the mandatory retirement age in order to mitigate the pressure on its pension system. Using an Overlapping Generations model we analyze the economic impacts resulting from an increase of the life expectancy and an increased retirement age on the pension system. The results show that it is ambiguous if an increasing retirement age will cause an increase or decrease of the pension benefits. We show that, the higher the share of capital income, the more probable it becomes that an increase of retirement age will exacerbate China's pension problems.

Key Words: China; OLG model; PAYG pension system; Fertility; Mandatory retirement age.


1. INTRODUCTION

China like many other countries, even though has a relatively young population (median age in 2015 was 37.1 years), is facing an aging problem and it will have to struggle with it in the future. The life expectancy at birth in China was 76 in 2014 — an increase of 3 years and 6 years in comparison to 2004 and 1994, respectively.1 The average annual population growth rate decreased from an annual average of 1.23 percent during 1990-1995 to 0.52 percent during 2010-20152. The total fertility rate (children

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per female) decreased from 2.4 in 1990 to 1.6 in 2014 and it is expected that the fertility rate will stay around 1.7 during the 2050s.\(^3\) Another indicator which can be used to emphasize the change of the demographic structure in China is the old age dependency ratio\(^4\). China’s old-age dependency ratio has continuously increased since the 1970s (7 old to 100 young), in 2015 the respective value was 13 and according to the population projections of the United Nations, this indicator is expected to remain increasing for the next fifty years and may reach nearly 40 in 2050.

With such an adverse demographic trend, the pension and also the healthcare system come under financial pressure, because the ratio between contributors and potential receivers is decreasing. Therefore, the financial sustainability of the systems must be taken into question.

The current pension scheme of China for the enterprise workers consists of two parts: one part is organized like a pay-as-you-go pension system, where the contribution rate is 20% and a second part which is fully-funded, where the contribution rate is 8%.\(^5\) The contributions of the latter part go into an individual account, and when workers are retired, the balance of the account will be paid to them on a monthly basis. The contributions of the former part lead to pension benefit claim which is related to the years of employment and the employee’s salary.

The current pension system in China is confronted with some inherent conundrums, which have not been and will not be easily resolved by politics. Besides some institutional challenges,\(^6\) the most important one for the pension system is the demographic trend toward an aging population. China’s pension age is 60 for men, 50 for women who are blue collar workers, and 55 for women who are white collar workers. Thus, as the life expectancy in China is increasing but the retirement age being unchanged, the ratio of the retirees to the workers is increasing. At the moment, the pension can still cover the demands of the retirees, but in the long run, the pension system will run into financial trouble if the aging trend continues.

As potential pension crisis looms, some experts suggested that China should progressively prolong the retirement age, proposing that the mandatory retirement age should be raised by five years each for women and for men (Sin, 2005, Oksanen 2010). The government has also signaled its intention to follow this advice by setting a timeline for the move. Officials from

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\(^4\) Old age dependency ratio, is the ratio of people older than 64 to the (working) population of the age between 15-64. The usual indicator in the data is the number of elderly per 100 working-age persons.

\(^5\) In 2015 a similar pension system was introduced for the public service and public institutions.

\(^6\) For example the pension system is organized at a provincial level, so that every province has its own pension system and pension claims are hardly to be transferred from one provincial pension system to another.
the ministry of human resources and social security had recently unveiled a detailed plan for raising the mandatory retirement age: the retirement age of females should be increased by one year every three years, and the male retirement age should be deferred by one year every six years, so that the retirement age for males and females in China could be equalized in 2045. However, this is one of many other alternative proposals still under discussion to the best of our knowledge.

The resulting question is then: does an increase of the mandatory retirement age help to solve China’s pension problems?

In order to have a deep insight into this issue, this paper aims to analyze the economic effects caused by an increase of the working time. Because of the fact that the increase of mandatory retirement age is politically mostly justified by the increase of the life expectancy, we additionally investigate the economic effects generated by an increase of the life expectancy. The rest of the paper is organized as follows: in the following section, we review shortly the relevant literature. Then we introduce the model and derive the respective equilibrium values by assuming that China is best represented as a closed economy. In the fourth section, we analyze the impacts caused by an increase of the life expectancy and an increased retirement age. In the last section, we conclude.

2. LITERATURE REVIEW

Although the aging of societies can be observed since decades, only a relative small number of papers investigated theoretically the effects caused by (mandatory) retirement age on pension benefits. Further, the majority of the literature is concerned with the normative and political-economic issues of the retirement age (Hu 1979, Marchand et al. 1996, Crettez and Le Maitre 2002, Momota 2003, Conde-Ruiz and Galasso 2004, Casamatta et al 2005, Lacomba and Lagos 2006, Zhang and Zhang 2009, Gonzalez-Eiras and Niepelt 2012, Michel and Pestieau 2013). We found only three studies (Kunze 2014; Fanti 2014 and 2015) considering the positive analysis of a change of the retirement age, which are directly comparable to our paper. Using a three period (childhood, adulthood, old age) OLG model (Diamond 1965), Kunze (2014) and Fanti (2014, 2015) investigate the effects caused by an increasing retirement age, where the individual utility function only depends on the consumption in the two last periods of life and where population growth is exogenously given. However, the papers are different regarding the assumed production technology. Fanti (2015) uses an AK production function introduced by Grossman and Yangawa (1993). In contrast Fanti (2014) applies a standard Cobb-Douglas production function, while Kunze (2014) uses an approach where human capital accumulation is the driver of growth and applies this idea to a Cobb-Douglas function with
human capital. The human capital accumulation process is modeled by using an idea introduced by Azariadis and Drazen (1990) combined with a human capital technology used by Kunze (2012) and Lambrecht et al. (2005). It is worth to note that the adults work in the second and third period of life where the third period is separated into a working phase and a retirement phase. The transition from one to the other phase is determined by the retirement age. Further, an adult inherits the human capital from her parents and it can be improved through investments in this stage of life. The improved human capital increases the labor productivity in the third period of life. Hence, the adult solves a trade-off between investments in human and physical capital, where the latter investments lead to an interest payment and the former to an increased wage income in the third period of life. In our view Kunze’s model fits to analyze the idea of life-long learning.

Not surprisingly, the outcomes of the three models differ. Kunze (2014) derives an inverted U-shaped relationship between retirement age and growth. The results of Fanti (2015) are obvious because an increasing retirement age leads to a decline of savings and therefore to a decline of the growth rate of incomes and pension benefits. The reason is that the capital accumulation is the driver of growth in models with an AK production function. In the model of Fanti (2014), the results are not unique and depend on the capital income share — if the share exceeds 50%, the pension benefits will decrease. A different strand of literature (Echevarría and Iza 2000, Echevarría 2003, Echevarría and Iza 2006) uses a finite horizon Blanchard-Yaari model to investigate the relationship between life-expectancy, economic growth and pensions. Like in Kunze (2014) a trade-off between investing in human and physical capital is decisive that an older retirement age leads to higher yields of human capital investments, which finally leads to higher growth rates and pension benefits.

However, beside this theoretical literature, some authors like Zeng (2011) or Chen and Groenewold (2017) try to estimate the effects caused by an increase of the retirement age on China’s pension system. The disadvantage of these models is that a number of ad hoc assumptions have to be made and important features of the economy are ignored. For example, in both papers the capital stock, human capital stock, the fertility rate and growth rate is taken as exogenously given. However, because of these strong assumptions the demographic change is the most influential factor in these models. Therefore it is not surprising that an increase of the retirement age has positive effects.
3. THE MODEL

Our model differs from the literature in the sense that we extend the Diamond’s (1965) OLG model by including endogenous fertility decisions. For this purpose we consider Becker’s (1960) quality-quantity trade-off (Stauvermann and Kumar 2016; de la Croix and Doepke 2003, 2004) between parental investments in human capital and the number of children. We assume that parents enjoy the number of children as well as the children’s level of education. The adults supply their labor wage inelastically in the second and partly in the third period of life. Similar to Kunze (2014), the length of the working time in the third period of life depends on the mandatory retirement age. Our model can be classified as an extension and a generalization of Fanti’s (2014) model, and it is a complement to Kunze’s (2014) model.

The human capital accumulation technology is similar to the one used by de la Croix and Doepke (2003, 2004), Azariadis (1993), Azariadis and Drazen (1990), Stauvermann and Kumar (2016), among others. The human capital per capita $h_{t+1}$ is represented by

$$
h_{t+1} = \begin{cases} 
 Bh_t q_t^e, & \text{for } Bq_t^e - 1 > 0 \\
 h_t, & \text{for } Bq_t^e - 1 \leq 0 
\end{cases}
$$

where $B > 0$, $h_0 = 1$ and $e \in (0, 1)$. The parents invest $q_t \geq 0$ in the education of each child. To avoid the existence of low development traps we make the simplifying assumption that $B > 1/q_t^e$. We note that we do not take into account positive externalities generated in the human capital building process, like it is proposed by Lucas (1988). Furthermore, we interpret, like Galor and Weil (1999); de la Croix and Doepke (2003, 2004) and Stauvermann and Kumar (2016, 2017), children’s human capital as child quality in the sense of Becker (1960). As Diamond (1965), we assume three periods of life — childhood, adulthood and old age. Only adults make economic decisions. They receive a labor income of $h_t w_t$, where $h_t$ is the human capital stock resulting from education in childhood and $w_t$ is the wage rate per human capital unit. Additionally, they decide about the number of children $n_t$, where the child-rearing costs including a child tax are $eh_t w_t$ and the educational costs per child are $q_t h_t w_t$. The variable $1 > e > 0$ is exogenously given and expressed as a share of the wage income.

To consider the one-child policy of China we make the assumption, that the child rearing costs $e$ includes the fine charged by the government for getting too much children. The fine can be interpreted as a child tax.\(^7\)

\(^7\)By omitting this aspect, our results become more robust.

\(^8\)For simplicity we assume that all children are taxed in our model. The exclusion of the first child or in our context the first half of a child would increase the paperwork without gaining any additional insight.
Furthermore, adults consume $c^1_t$ units and save a part of their incomes $s_t$; and they have to contribute $\tau P h_tw_t$ to the pension system in the second period of life. The corresponding budget restriction in the second period of life is given by:

$$c^1_t = h_tw_t(1 - \tau_P - (q_t + e)n_t) - s_t. \tag{2}$$

Moreover, the seniors enjoy their third period of life only with the survival probability $\rho$. The surviving seniors work the fraction $\omega$ of the last period of life, and they enjoy their retirement by a fraction $1 - \omega$. The fraction $\omega$ determines the mandatory retirement age. The consumption expenditures of the seniors are sourced by three incomes: their wage income $\omega(1 - \tau_P)h_tw_{t+1}$ in the third period of life, their savings plus interest, and their pension benefit $P_{t+1}$. Even in third stage of life, they have to contribute $\omega\tau P h_tw_{t+1}$ to pension system. The corresponding budget restriction in the third period of life is represented by:

$$c^2_{t+1} = (1 - \tau_P)\omega h_tw_{t+1} + P_{t+1} + \frac{R_{t+1}s_t}{\rho}. \tag{3}$$

Following Ehrlich and Lui (1991) and Cipriani (2014) the probability $\rho$ to be alive in the third period is assumed to be exogenously given. Moreover, as Cipriani (2014), we assume a perfectly competitive financial market with the risk-free interest factor $\frac{R_{t+1}}{\rho}$. Because of the assumption made above, the individual capital stock is growing: $h_{t+1} > h_t$, and thus the wage income per time unit of a senior worker is lower than that of an adult worker.

To make comparisons with the related literature mentioned above possible, we extend the utility function of Michel and Pestieau (2013) by adding a term depending on the human capital and number of children. Therefore, the utility of a representative individual born in period $t - 1$ is given by:

$$U_t(c^1_t, c^2_{t+1}, n_t, q_t, \omega) = \ln(c^1_t) + \rho\chi \ln(c^2_{t+1}) + \mu \ln(n_t h_{t+1}) + \theta \ln(1 - \omega). \tag{4}$$

Further, the subjective discount factor $\chi$, the preference parameters for the quantity of educated children $\mu$ and retirement time $\theta$ fulfill: $\{\chi, \mu, \theta\} \in [0, 1]$. We normalize the length of a period to one. The government decides about the mandatory retirement age, which is determined by the fraction $\omega$.\footnote{If the fraction $\omega$ is a choice variable, then its optimal value will become zero ($\omega = 0$), when $\theta > \frac{\epsilon(1 + \mu + \rho\chi) - \mu(1 - \omega)\tau_P G_h}{\tau_P G_h}$, where $G_h$ is growth factor of human capital (see (11) below). Without loss of generality, we assume that this condition is fulfilled throughout the paper.} Hence, a representative agent maximizes her utility (4) regarding the
restrictions (1), (2) and (3). Therefore we insert (1), (2) and (3) in (4) and solve the following maximization problem:

$$\max_{\{s_t, n_t, q_t, \omega\}} U_t(s_t, n_t, q_t, \omega) = \ln(h_t w_t (1 - \tau_P - (q_t + e)n_t) - s_t)$$

$$+ \rho \ln \left( (1 - \tau_P) \omega h_t w_{t+1} + P_{t+1} + \frac{R_{t+1}s_t}{\rho} \right)$$

$$+ \mu \ln(n_t B h_t q_t^c) + \theta \ln(1 - \omega).$$  \hspace{1cm} (5)

We differentiate (5) with respect to the savings, the number of children and the investments in education. Then, we derive three first order conditions (FOCs):

$$\frac{1}{h_t w_t (1 - \tau_P - (q_t + e)n_t) - s_t} = \frac{\chi R_{t+1}}{(1 - \tau_P) \omega h_t w_{t+1} + P_{t+1} + \frac{R_{t+1}s_t}{\rho}}$$  \hspace{1cm} (6)

$$\frac{h_t w_t (q_t + e)}{h_t w_t (1 - \tau_P - (q_t + e)n_t) - s_t} = \frac{\mu}{n_t}.$$  \hspace{1cm} (7)

$$\frac{h_t w_t (1 - \tau_P - (q_t + e)n_t) - s_t}{h_t w_t n_t} = \frac{\mu e}{q_t}.$$  \hspace{1cm} (8)

A balanced budget of the PAYG pension system given in per capita terms requires:

$$P_{t+1} = \frac{\tau_P h_{t+1} w_{t+1} n_t}{\rho} + \tau_P \omega h_t w_{t+1}.$$  \hspace{1cm} (9)

The per capita pension benefit contains two parts: the contributions of the adult workers divided by the survival probability \( \left( \frac{\tau_P h_{t+1} w_{t+1} n_t}{\rho} \right) \) and the contributions of the senior workers \( (\tau_P \omega h_t w_{t+1}) \). In this simple model with identical agents, the first part of the pension benefit consists implicitly of the contributions of her children divided by the survival probability and the second part consists of her own contribution.

Now we insert (9) in (6), (7) and (8) and solve to get the optimal savings, optimal number of children and optimal investments in education. We get as a result:

$$q^* = \frac{e e}{(1 - e)} > 0.$$  \hspace{1cm} (10)

Putting (10) in (1), the human capital stock per capita in period \( t + 1 \) results and dividing this by \( h_t \) delivers the growth factor of human capital:

$$G_t^{h} = \frac{h_{t+1}}{h_t} = B \left( \frac{e e}{(1 - e)} \right)^{e}.$$  \hspace{1cm} (11)
Obviously, the growth factor of human capital is constant, and it increases in the costs of child rearing. Because of the assumption that these costs include the child tax, it is worth to note that China’s one child policy has a positive impact on the educational investments and hence the growth rate of human capital. Additionally, this formulation coincides with the empirical fact (Barro 2013) that education is one of the main drivers of economic growth.

However, this result (11) differs from the outcomes of Kunze (2012, 2014) and Lambrecht et al. (2005), who assume either that adults educate themselves or assume a bequest motive so that the returns of investments of physical and human capital must be equalized. The main difference between our model and the models of Kunze (2012, 2014), Echevarría (2003), Echevarría and Iza (2006) is caused by our assumption that parents’ investments in education are motivated by a form of parental altruism and not by an investment motive. This explains why the growth rate of human capital is neither affected by the survival probability nor by the retirement age.

From equation (11) we derive the optimal savings and optimal number of children:

\[
\begin{align*}
  n^*_t & = \frac{(1 - \varepsilon)\mu(w_t R_{t+1}(1 - \tau^P) + \rho_\omega w_{t+1})}{R_{t+1} e(1 + \rho_\chi + \mu) w_t - \mu G^h \tau^P (1 - \varepsilon) w_{t+1}}. \\
  s^*_t & = \frac{w_t h_t [\rho_\chi e R_{t+1} w_t (1 - \tau^P) - w_{t+1} (\mu G^h \tau^P (1 - \varepsilon) (1 - \tau^P) + e(1 + \mu) \rho_\omega)]}{R_{t+1} e(1 + \rho_\chi + \mu) w_t - \mu G^h \tau^P (1 - \varepsilon) w_{t+1}}.
\end{align*}
\]

We have to note that the savings are only positive, if the first summand of the numerator exceeds the second. The intuitive reason for this is that the individuals do not want to save if the pension benefits plus the wage income at old age are relatively high compared to the wage income in adulthood. To ensure that the savings are positive the following condition has to hold.

\[
R_{t+1}(1 - \tau^P) > \frac{w_{t+1} (\mu G^h \tau^P (1 - \varepsilon)(1 - \tau^P) + e(1 + \mu) \rho_\omega)}{\rho_\chi e},
\]

Inequality (14) requires a sufficiently low contribution rate \( \tau^P \), a sufficiently low retirement age \( \omega \) and a sufficiently high life expectancy \( \rho \). To avoid pathological cases in our analysis we assume for the rest of the paper that (14) holds.

To determine the resulting pension benefits we insert the growth factor (11) and optimal number of children (12) into the equation of the pension benefits (3) and we get:

\[
P^*_t = \tau^P w_{t+1} h_t \left( \frac{G^h (1 - \varepsilon) \mu(w_t R_{t+1}(1 - \tau^P) + \rho_\omega w_{t+1})}{\rho [R_{t+1} e (1 + \rho_\chi + \mu) w_t - \mu G^h \tau^P (1 - \varepsilon) w_{t+1}]} + \omega \right).
\]
On the production side of the economy we apply a usual neoclassical production function depending on the physical and the human capital:

\[ Y_t = F(K_t, H_t), \quad (16) \]

The aggregate human capital stock \( H_t \) is defined as the product of human capital units times the number of workers \( (H_t = h_tN_t) \). The variable \( Y_t \) is the aggregate output and \( K_t \) is the aggregate physical capital stock. The production function exhibits the usual diminishing marginal productivities in each input factor, fulfills the Inada conditions and is linear homogenous.

We reformulate the aggregate production function to a per human capital production function, by dividing (16) by the aggregate human capital stock:

\[ y_t = f(k_t), \quad (17) \]

where we define \( y_t = \frac{Y_t}{h_tN_t} \) as production per human capital unit and \( k_t = \frac{K_t}{h_tN_t} \) as capital intensity. Please note this definition of capital intensity is not referred as (physical) capital per capita, but as the (physical) capital per human capital unit. To get the variables in per capita terms, we have to multiply the capital intensity with individual human capital stock \( h_t \).

The depreciation rate of physical capital is assumed to be 100 per cent per period. Considering perfect competitive factor and good markets, we get for the wage rate per human capital unit and the interest factor:

\[ w_t = w(k_t) = f(k_t) - f'(k_t)k_t, \quad (18) \]
\[ R_t = f'(k_t). \quad (19) \]

To complete the model, we assume the existence of a unique equilibrium capital intensity \( k^* \). We derive the capital market clearing condition \( S_t(k_t, k_{t+1}) = K_{t+1} \), where \( S_t \) represents the aggregate savings, in terms of human capital units. For this purpose, we use the individual savings (13), the optimal number of children (12) and the growth factor of human capital (11) and get:

\[ \frac{w(k_t)h_t[\rho (\chi e f'(k_{t+1})w(k_t)(1 - \tau^P) - w(k_{t+1})(\mu G^h + \tau^P (1 - \varepsilon)(1 - \tau^P)) + (1 + \mu)\omega)]}{[f'(k_{t+1})e(1 + \rho \chi + \mu)w(k_t) - \mu G^h + \tau^P (1 - \varepsilon)w(k_{t+1})]} - k_{t+1}n^*_th_{t+1} = 0. \quad (20) \]
After further simplifications, the capital market clearing condition can be rewritten in per human capita form as:

\[ \Omega(k_t, k_{t+1}) = \frac{w(k_t)\rho\epsilon f'(k_{t+1})w(k_t)(1 - \tau_P) - w(k_{t+1})[\mu G^h \tau_P (1 - \varepsilon)(1 - \tau_P) + \varepsilon(1 + \mu)\rho\omega]}{G^h(1 - \varepsilon)\mu(w(k_t)f'(k_{t+1})(1 - \tau_P) + \rho\omega w(k_{t+1}))} - k_{t+1} = 0. \]  

As it is well known, the stability of the equilibrium is only guaranteed, if \( 0 < \frac{dk_{t+1}}{dk_t} < 1 \) holds. Hence, we apply the implicit function theorem to calculate the respective derivative and this leads to the following slope of (21):

\[ \frac{dk_{t+1}}{dk_t} = \frac{-f''(k^*)k^*[(B + w(k^*)\rho\epsilon f'(k^*)(1 - \tau_P)) - f'(k^*)k^*(1 - \tau_P)G^h(1 - \varepsilon)\mu]}{-w(k^*)f''(k^*)[w(k^*)\rho\epsilon(1 - \tau_P) + k^*A] + \Gamma + k^*f''(k^*)G^h(1 - \varepsilon)\mu[w(k^*)(1 - \tau_P) - \rho\omega k^*]} \]

where:

\[ B = w(k^*)\left(\rho\epsilon f'(k^*)(1 - \tau_P) - (\mu G^h \tau_P (1 - \varepsilon)(1 - \tau_P) + \varepsilon(1 + \mu)\rho\omega)\right) > 0, \]

\[ A = (\mu G^h \tau_P (1 - \varepsilon)(1 - \tau_P) + \varepsilon(1 + \mu)\rho\omega) > 0, \]

\[ \Gamma = G^h(1 - \varepsilon)\mu w(k^*)[f'(k^*)(1 - \tau_P) + \rho\omega] > 0. \]

We assume that the stability condition is fulfilled.

In the next step we derive the long-run equilibrium values for the number of children, per capita savings and pension benefits.

\[ n^E_{t+1} = \frac{(1 - \varepsilon)\mu(f'(k^*)(1 - \tau_P) + \rho\omega)}{f'(k^*)\varepsilon(1 + \rho\chi + \mu) - \mu G^h \tau_P (1 - \varepsilon)}. \]

\[ s^E_{t+1} = \frac{w(k^*)b_t[\rho\epsilon f'(k^*)(1 - \tau_P) - (\mu G^h \tau_P (1 - \varepsilon)(1 - \tau_P) + \varepsilon(1 + \mu)\rho\omega)]}{f'(k^*)\varepsilon(1 + \rho\chi + \mu) - \mu G^h \tau_P (1 - \varepsilon)} \]

\[ P^E_{t+1} = \tau^P w(k^*)b_t \left( G^h \frac{n^E_{t+1}}{\rho} + \omega \right) \]

\[ = \tau^P w(k^*)b_t \left( G^h(1 - \varepsilon)\mu(f'(k^*)(1 - \tau_P) + \rho\omega) \right. \]

\[ \left. \frac{1}{\rho[f'(k^*)\varepsilon(1 + \rho\chi + \mu) - \mu G^h \tau_P (1 - \varepsilon)] + \omega} \right). \]

4. INCREASING LIFE EXPECTANCY AND RETIREMENT AGE

Before we begin to analyze the economic effects caused by an increase of the retirement age, we analyze the impact of one of the two triggers, which causes the Chinese government to consider an increase of the retirement age: the increasing life expectancy. We do not take into account the second trigger, a decreasing fertility rate, because the decline was intended by the Chinese government, but the first trigger, the increasing life expectancy.
To do so, we differentiate (21) in the steady state equilibrium with respect to the life expectancy to derive its impact on the capital intensity. We get:

\[
\frac{dk^*}{d\rho} = -\frac{w(k^*)w(\chi f'(k^*)/(1 - \tau P) - \omega(1 + \mu) + \omega k^* \mu G^h(1 - \varepsilon))}{d\ln(k^* k^* / k^*)} > 0.
\] (26)

The sign of the denominator is negative because of the stability condition. When the retirement age is sufficiently low, an increasing life expectancy leads to an increase of the physical capital per human capital unit. The increasing life expectancy induces the agent to save more and hence the capital intensity increases. This outcome corresponds to the result of Cipriani (2014). Because of the fact that the human capital grows with a constant rate, the (physical) capital per capita increases if the capital intensity increases.

Differentiating (23) with respect to the life expectancy \( \rho \) we get the effect of an increasing life expectancy on the number of children as:

\[
\frac{dn_t^E}{d\rho} = \frac{\partial n_t^E}{\partial \rho} \bigg|_{dk^* = 0} + \frac{\partial n_t^E}{\partial k^*} \frac{dk^*}{d\rho} \geq 0,
\] (27)

where the direct and short-run effect is represented by

\[
\frac{\partial n_t^E}{\partial \rho} \bigg|_{dk^* = 0} = -\frac{\mu(1 - \varepsilon)[f''(k^*) e \chi f'(k^*) (1 - \tau P) - \omega(1 + \mu) + (1 - \varepsilon) \omega \mu G^h \tau P]}{(f'(k^*) e (1 + \rho \chi + \mu) - \mu G^h \tau P (1 - \varepsilon))^2} < 0.
\] (28)

and

\[
\frac{\partial n_t^E}{\partial k^*} = -\frac{\mu(1 - \varepsilon) f''(k^*) [(1 - \tau P) \mu G^h \tau P (1 - \varepsilon) + \rho \omega (1 + \rho \chi + \mu)]}{(f'(k^*) e (1 + \rho \chi + \mu) - \mu G^h \tau P (1 - \varepsilon))^2} > 0.
\] (29)

The indirect and long-run effect, represented by the second term on the RHS of (27), depends on the signs of (26) and (29), which are both positive. The overall effect on the number of children caused by an increasing life expectancy is not clear. In the short run the number of children will decline, because the adults have an increased desire to save more because of the longer retirement period. In the long run, this trend may be reversed because the capital intensity increases because of the increased savings and hence the wage incomes per capita increase accordingly and the interest factor declines. The higher incomes allow getting more children. Whether the latter increase of the fertility rate will exceed the former decrease is not clear in general.
Now we will derive the effect of the life expectancy on the pension benefits. Differentiation of (25), and using (27), we get:

\[
\frac{dP_{E,t+1}}{d\rho} = \frac{\tau}{\rho} h_t k^* \left[ -f''(k^*) \frac{dk^*}{d\rho} \left( \frac{G^h n_E^E}{\rho} + \omega \right) + f'(k^*) G^h n_E^E \left( \frac{1}{\eta_{y,k}} - 1 \right) \left( \frac{\eta_{n,\rho} - 1}{\rho^2} \right) \right] \geq 0,
\]

where \( \eta_{y,k} = f'(k^*) k^* \) and \( \eta_{n,\rho} = \frac{dn_E^E}{d\rho} \frac{\bar{\omega}}{n_E^E} \).

The sign of (30) is ambiguous. It is not clear if an increase of the life expectancy causes a decrease or an increase of the pension benefits. The positive effect represented by the first summand in the brackets is an outcome of the higher wage caused by the higher capital intensity. Given that the number of children declines, the overall effect becomes positive if the production elasticity of capital or the capital income share is sufficiently large. If the number of children rises, the pension benefits are always positively affected. The possible rise of the pension benefits through an increased life expectancy is somehow paradoxical, because intuitively it seems to be obvious that a longer life will decrease the pension benefits. This paradoxical result occurs if the second summand in the brackets is either positive, which will occur when the number of children depends positively on the life expectancy, or negative, but its absolute value is very close to zero. The latter requires a relatively high capital share. However, looking at the empirical data of China, it must be stated that the labor income share in China is less than 50% since more than 10 years (ILO, 2015 or Bai and Qian, 2010), if not since than 30 years (Qi, 2014). Thus the functional distribution of income has a strong impact on the reactions which can be expected by a change of the life expectancy and mandatory retirement age. Therefore, the argument, that an increasing life expectancy requires necessarily a pension system reform can be debatable, because in the case of China we would expect an increase of the pension benefits caused by an increasing life expectancy.

Nevertheless, in many countries which apply a PAYG pension system, it is believed in politics and economic advisory bodies that aging of the society decreases the pension benefits. Not surprisingly, one important policy recommendation is then to increase the retirement age to counteract the decline of the pension benefits. Hence, we analyze now how an increase of the retirement age affects the pension benefits.
If the government increases the retirement age, the equilibrium capital intensity is affected in the following way:

\[
\frac{dk^*}{d\omega} = w(k^*)\rho w(k^*)e(1+\mu) + k^* \mu G^h (1-\varepsilon) < 0. \tag{31}
\]

The stability condition ensures that the denominator is negative and consequently the derivative (31) is negative. The intuition is that increasing the retirement age reduces the need to save, because of the higher income as a senior induced by the extended working time. Therefore, the capital intensity will decline. However, the effect of an increased retirement age on the fertility rate is not unique:

\[
\frac{dn^E_t}{d\omega} = \frac{\partial n^E_t}{\partial \omega} \bigg|_{dk^*=0} + \frac{\partial n^E_t}{\partial k^*} \frac{dk^*}{d\omega} > 0, \tag{32}
\]

where

\[
\frac{\partial n^E_t}{\partial \omega} \bigg|_{dk^*=0} = \frac{\mu(1-\varepsilon)\rho}{f''(k^*)e(1+\rho\chi + \mu) - \mu G^h \gamma^P (1-\varepsilon)} > 0. \tag{33}
\]

On the one hand, an increased retirement age increases the number of children, but on the other hand, the declined capital intensity increases the interest factor and decreases the wage rate. As a consequence the number of children declines. Thus, in general the overall effect is ambiguous.

Finally, we investigate the effect of an increase of the retirement age on the pension benefits. Differentiating (25) with respect to the retirement age, we get:

\[
\frac{dP_{t+1}^E}{d\omega} = \tau^P h^t k^* \left[ -f''(k^*) \frac{dk^*}{d\omega} \left( \frac{G^h d\omega}{\rho} + \omega \right) + f'(k^*) n^E_t \left( \frac{1}{\eta_{y,k}} - 1 \right) \left( \frac{G^h d\omega}{\rho} + 1 \right) \right] > 0. \tag{34}
\]

The wage rate declines, and that leads directly to a decline of the pension benefits, but the latter decline of the pension benefits is counteracted by increased contributions of the longer working elderly. Additionally the number of children may increase, what also lead to increased contributions. A decisive variable is again, the capital income share \( \eta_{y,k} \). When it is sufficiently huge, the second summand in derivative (32) becomes smaller and further a high capital income share leads to reduction of the number of children. As a consequence the overall effect on the pension benefits
induced by an increased retirement age becomes negative and such a policy measure will harm all future generations. To illustrate our result, we have calibrated it with a Cobb-Douglas production function of the form \( y = A k^\alpha \). The following figure presents the pension levels for four different capital shares; \( \alpha \in \{0.45, 0.50, 0.55, 0.57\} \) and the retirement age \( \omega \) moves from zero to one.

\[ \text{FIG. 1. For the calculation we assumed:} \ A = 40, \rho = 0.5, h_t = 10, \theta = 0.5, \chi = 0.98, e = 0.09, \varepsilon = 0.8, \mu = 0.8, \tau^P = 0.2, B = 16.5 \]

Obviously, the equilibrium pension benefits are declining. Additionally, we observe the higher the capital income share, the lower is the level of the pension benefits. This is the case because the upper dotted line in graph represents the case where \( \alpha = 0.45 \) and the lower solid line represents the case with \( \alpha = 0.57 \). The lines representing the remaining two cases, are lying between them (the line with dashes and dots represents \( \alpha = 0.5 \), and the line only with dashes \( \alpha = 0.55 \)). The selected parameter values fits in our view more or less to the Chinese economy, because assuming one period consists of 25 years, then the corresponding growth rate per year is around 8.2%, the total fertility rate is around 1.3-1.4 children per female, and the yearly rate of return lies in the range between 12-14%.

5. SOME WELFARE IMPLICATIONS

However, until now we did not consider any welfare effects, and although this is beyond the scope of the paper, we should indicate some of the ob-
vious outcomes related to welfare. We know from (33) that an increase of the retirement age will decrease the equilibrium capital intensity, which leads to a fall of the wages per human capital unit. The consequence is that all future generations have not only to work longer they have to do it at a relative lower wage, because the growth rate of the economy remains unchanged. That of course makes it also impossible, that the resulting higher interest rate is able to compensate them for the undesired increase of the mandatory retirement age. In addition, the fully funded part of the Chinese pension system will also suffer because of lower wages and the resulting lower contributions. To make this point clear, we assume that \( \tau_{PG} \) is the contribution rate of the fully funded part of the pensions. Then the pension benefit generated by the fully-funded part is in the equilibrium \( P_{G}^* = h_{t} \tau_{PG} w(k^*) f'(k^*) \). Differentiating this with respect to the mandatory retirement age gives:

\[
\frac{\partial P_{G}^*}{\partial \omega} = \tau_{PG} f''(k^*) (w(k^*) - f'(k^*) k^*) \frac{dk^*}{d\omega}
\]

\[
= \tau_{PG} f''(k^*) f(k^*) (1 - 2\eta_{y,k}) \frac{dk^*}{d\omega} < 0, \tag{35}
\]

if \( \eta_{y,k} > \frac{1}{2} \).

Because of the fact that the capital income share exceeds the labor income share in China, the pension benefits from the fully-funded part of the system will decrease as a consequence of an increase of the mandatory retirement age.

6. CONCLUSIONS

Facing the undeniable fact of an aging population, China is expected to extend the retirement age in order to slow the speed of the decrease in the labor force. But does an increase of the mandatory retirement age help to solve China’s pension problems? In this paper, we use an OLG model of a closed economy including endogenous fertility and human capital accumulation. We have shown that it cannot be excluded that it is possible that increasing life expectancy can cause an increase of the pension benefits, and an increasing retirement age can cause a decrease of the pension benefits. Combined with the fact that China’s capital income share is very close or exceeds the labor income share it is not unlikely that an increasing retirement age will not solve its problems of the pension system, but it will

\[\text{According to a poll of the Canton Public Opinion Research Center only 26\% of the population back the idea of increasing the mandatory retirement age (China’s Daily, 2013).}\]

\[\text{We have ignored this part until here, because the fully-funded part is a perfect substitute to private savings.}\]
exacerbate the problems. However, a useful policy recommendation is to investigate empirically, if the increasing life expectancy of Chinese citizens is in fact positively associated with an increasing pressure on the financial situation of the pension system, and if an increasing retirement age will increase the contributions or not. Only if this is the case, then an increase of the retirement age should be taken under consideration. However, if the retirement age will be extended by one year in 2025, then a rough estimation delivers the result that ceteris paribus 12-15 million more senior employees will stay in their jobs in each year in the period 2025-2045\(^1\) or in other words the labor force will be increased by roughly 2-3\%, and if the unemployment rate should remain constant, this amount of additional jobs have to be created per each year of retirement extension. Or in 2038, roughly the labor force will be increased by roughly 33 million additional senior workers, who have to work if the retirement age is extended to 65 years. Even that this estimation is admittedly a very rough one; it illustrates the challenges policy-makers have to manage if they increase the retirement age.

An alternative policy proposal would be to increase the subsidies for education to increase the human capital growth rate and as a consequence the per capita incomes and pension benefits will increase.\(^2\) Another alternative would be, that the government takes some redistributive measures to increase the labor income share; the wages would increase and consequently pension contributions and benefits.

REFERENCES


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\(^{1}\)This rough estimation is based on the population prospects of the United Nations https://esa.un.org/unpd/wpp/ and assuming that the labor market participation rate of the specific vintage is between 60-70%.

\(^{2}\)See e.g. Stauvermann and Kumar (2017)


