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Could Risk Management Be Harmful to Firms?

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Based on a theoretical model, this paper shows that risk management policies shielding firms from marketwide risk exposures could be harmful to the firms. Specifically, if a firm's operation is delegated to a manager and subject to moral hazard problems, risk exposures could align the manager's interests with the firm owner's so that they alleviate the moral hazard problems and raise the firm's value. As a result, the risk management policies could reduce the firm's value to the owner.

Key Words: Risk Management; Dynamic Contracts; Moral Hazard.
JEL Classification Numbers: G32, D81, D82.

1. INTRODUCTION

The 2008 financial crisis has been deemed as a wake up call for stricter risk management policies in the financial industry. After this crisis, many regulatory authorities and non-government associations mandated or advocated various risk management enhancements. For example, as part of the Dodd-Frank Act, the Volcker Rule, which went into effect in 2012, restricts banks in the United States from speculative investments such as hedge funds and private equity funds. In 2011, the Basel Committee on Banking Supervision, an international bank risk management association, proposed the Third Basel Accord, which requires higher standards of capital reservation and leverage for compliance than previous accords. Such risk management policies could prevent reckless behavior that exposes financial companies to marketwide turbulence, which could be the result of a financial crisis, unexpected exchange rate shocks,[1] or a natural disaster. The majority of investors, policy makers, and researchers believe that risk budgeting—that is minimizing a firm's risk exposure given a target expected return on the firm's assets—should always be beneficial to firm

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[1]See Gong et al. (2009) for example.

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owners and the financial market. However, based on a theoretical model, this paper shows that risk management policies that shield firms from marketwide risk exposure could be harmful to a firm if the problem of moral hazard arises. Intuitively, when the operation of a firm is delegated to a manager, the manager could increase his private benefits at the cost of the firm if such misbehavior cannot be directly detected. In this situation, if the firm is exposed to marketwide risks beyond the manager's control, uncertainty about the firm's future would also imply uncertainty about the manager's future. This uncertainty could then create a precautionary savings effect[2] that aligns the manager's interests with the owner's, thereby restricting the manager from engaging in misbehavior and raising the firm's value. In this way, a risk management policy that prevents such risk exposure could end up hurting the firm.

Specifically, in the model, a firm owner delegates the operation of her firm to a manager over a long time horizon. The firm generates cash flows using its capital. Capital accumulation is based on the firm's investment funds and has two key features. First, investment is operated by the manager, who could secretly exploit the funds to enhance his own benefits at the cost of the firm owner. For example, a fund manager could use investment funds to inflate the prices of the securities he plans to purge from his personal account. Or an asset manager could refrain from exerting reasonable effort to maximize the expected return on the assets he manages. Obviously, such misbehavior would reduce the firm's value, so incentive compensation packages are needed to induce managers to behave appropriately. Second, the firm could choose whether to implement a risk management policy that shields the firm's capital marketwide shocks that arrive randomly. The protection from this policy would help the firm to avoid potential capital loss upon a marketwide shock. Such a protection is costly, however, and lowers the rate of capital growth in the short run. For example, purchasing insurance or hedging derivatives uses up a firm's resources, and refraining from selling credit defaults swaps (CDSs) or buying speculative securities reduces the firm's current profitability. In expectation, the avoidance of the potential losses upon the marketwide shocks is balanced by the decrease in the capital rate of return, so the risk management policy only reduces the volatility of capital accumulation without affecting its expected return. Overall, no implementation cost is associated with this risk management policy. I show that even if the risk management policy that only reduces marketwide risk exposure is costless, it could still decrease the firm's value to the owner.

[2]Precautionary savings refer to the additional savings that people reserve for future uncertainty.

To understand the key intuition here, notice that, over time, the manager's compensation under the delegation contract is positively correlated with the firm's capital. As a result, the marketwide risk exposure of the firm's capital implies that the manager is faced with a similar level of risk exposure. Since the manager is risk averse, greater uncertainty about his compensation due to his risk exposure lowers his expected utility level in the future. If his intertemporal preference exhibits a strong income effect, a precautionary savings effect arises: the prospect of a "poorer" future makes the manager more cautious about the firm's prospective growth and causes him to abstain from engaging in current misbehavior in order to improve the firm's long-run profitability.[3] Consequently, marketwide risk exposure alleviates the moral hazard problem and makes the delegation contracts more efficient. Risk management policies that prevent such exposure could be strictly unfavorable to the owner because of the moral hazard problem.

This paper is based on the literature on dynamic contract design with information frictions and includes DeMarzo and Sannikov (2006), Sannikov (2008), DeMarzo et al. (2012), Williams (2011), and Zhu (2013). Different from the existing papers in the literature, this paper studies how risk management policies that prevent marketwide risk exposure affect the firm's value under the optimal contract. This paper is also related to the literature on the dynamic principal-agent model with endogenous risk taking, which includes Ou-Yang (2003), DeMarzo et al. (2014), Feng and Westerfield (2016), and Leung (2014). These papers study the manager's hidden risk exposure decision, which is part of the moral hazard problem. In contrast, this paper considers exogenous risk management policies that are mandated by the government or arises from compliance with industrial risk management standards. Another related paper is by Fedele and Mantovani (2014), who study how public financial institutions can mitigate a credit crunch problem based on a moral hazard model.

The rest of the paper is organized as follows. Section 2 introduces the model, Section 3 illustrates the key intuition of this paper by demonstrating a contract with a closed-form solution, Section 4 shows how risk management affects the firm's value, and Section 5 concludes.

2. THE MODEL

2.1. Capital accumulation, marketwide risks, and risk management

Assume that a risk-neutral owner delegates the operation of her firm to a risk-averse manager over the time horizon $[0, \infty)$. At each instant in time t, the firm has total assets or capital K_t and generates an operating profit

[3] See Leland (1968) and Obstfeld (1994) for more on the precautionary savings effect.

AK_t with $A > 0$ being a productivity parameter. The firm accumulates capital through investments that are funded by the firm but operated by the manager. The law of motion for K_t follows:[4]

$$dK_t = K_t \left[(i_t - \delta)\, dt + \sigma dB_t + \lambda l dt - l dN_t \right] \text{ for all } t \geq 0. \tag{1}$$

Specifically, i_t is the investment-to-capital ratio chosen by the manager, and $\delta \geq 0$ is the capital depreciation rate. Capital accumulation is subject to two types of shocks. The first type consists of firm-level shocks, which are characterized by a standard Brownian motion, $\{B_t\}$, and a volatility rate, $\sigma > 0$. I assume that firm-level shocks cannot be distinguished from the manager's investment policy by the owner, so they create a moral hazard problem in the model (to be introduced shortly). The second type consists of marketwide shocks, which arrive infrequently and are characterized by a Poisson counting process, $\{N_t\}$, with a fixed arriving rate $\lambda > 0$.[5] A marketwide shock could be a financial crisis that triggers a downturn in the economy, a sudden increase in crude oil prices, or a sharp nationwide decrease in housing prices. Such a shock is publicly observable but beyond the firm's control.

As indicated by the last term in parentheses on the right-hand side of equation (1), the firm may lose a fraction of its capital upon a marketwide shock, depending on whether it has adopted a risk management policy from the beginning. The policy protects the firm capital from the shocks so that the capital loss $l = 0$. Without the policy protection, $l = \bar{l} \in (0, 1)$. So we use $l \in \{0, \bar{l}\}$ to indicate the firm's choice of the risk management policy. According to the term next to the last one in parentheses on the right-hand side of (1), if the firm chooses not to adopt the risk management policy ($l = \bar{l}$), there is an additional rate of capital growth $\lambda \bar{l} > 0$. This additional return can be interpreted as the cost of risk protection, which could be the firm's resources used to purchase insurance and hedging derivatives, or the short-run gains in capital that could be earned by purchasing risky high-yield securities prohibited by the risk management policy. In balance, risk protection is costly in the short run but eliminates the firm's exposure to market-level losses.

Recall that the arriving rate of the market wide shock is λ, and the rate of capital loss upon a shock is l. Therefore, the short-term gain is balanced by the potential losses upon shocks in expectation. Technically, the last two terms in parentheses on the right-hand side of (1) form a compensated

[4]See Wen (2007) for a discrete-time version of the capital accumulation technology.
[5]If a shock hits at time t, $dN_t = 1$. Otherwise, $dN_t = 0$.

jump martingale and the expected stochastic integral

$$E_t \left\{ \int_t^{t+\Delta} K_s \left[\lambda l ds + l dN_s \right] \right\} = 0 \text{ for all } t \geq 0 \text{ and } \Delta > 0.$$

Therefore, in this model, implementing the risk management policy only reduces the volatility of capital accumulation over time without affecting the expected rate of return. This paper shows that such a policy could strictly reduce the expected firm value to the owner.

2.2. Resource constraint, preferences, and moral hazard

At each instant t, the generated operating profit, AK_t, is divided among the owner's dividend payment, D_t, the manager's compensation, C_t, and the capital investment fund, $i_t K_t$. So, the resource constraint of the firm is

$$D_t + C_t + i_t K_t = AK_t \text{ for all } t \geq 0. \tag{2}$$

In addition, I assume that the firm owner cannot issue additional equity and the manager is protected by limited liability so that

$$D_t \geq 0 \text{ and } C_t \geq 0 \text{ for all } t \geq 0.$$

Delegation of the firm's operation is according to a contract that specifies the division of the operating profit at each instant in time, based on the past performance of the manager. Although the capital accumulation is publicly observable, the owner cannot distinguish the manager's actual investments from the firm-level shocks, dB_t. Namely, when a low rate of capital growth is realized over a period of time, the owner cannot tell whether it is because the manager did not make appropriate investment decisions or because the firm capital received negative firm-level shocks. Therefore, the manager could divert part of the investment funds, $i_t K_t$, to raise his consumption C_t without being detected, so the owner needs to design the contract to induce the manager to make appropriate investments.

Because of the manager's hidden-diversion ability, without loss of generality, we assume that, under a contract, the owner makes a transfer payment P_t to the manager at each instant t and the manager allocates it between his consumption and the capital investment. Namely, $P_t = C_t + i_t K_t$. Given the initial risk management policy set at $t = 0$, a contract is denoted $(\{P_t\})$, with P_t depending on the history of the contract. Under the contract, the manager chooses investment to maximize his expected utility,

$$E_0 \left[\beta \int_0^\infty e^{-\beta t} \frac{C_t^{1-\gamma}}{1-\gamma} dt \right] = E_0 \left[\beta \int_0^\infty e^{-\beta t} \frac{(P_t - i_t K_t)^{1-\gamma}}{1-\gamma} dt \right], \tag{3}$$

subject to (1), with $\beta > 0$ being his discount rate and $\gamma > 0$ being the relative risk aversion coefficient. Notice that the background probability distribution behind the time-zero expectation, E_0, depends on the manager's investments and the initial risk management policy. The owner designs the optimal contract to maximize the net present value of her dividends,

$$E_0\left[\int_0^{\infty} e^{-\beta t} D_t dt\right] = E_0\left[\int_0^{\infty} e^{-\beta t}\left(AK_t - P_t\right) dt\right].$$

I assume that the firm owner has full bargaining power and chooses the initial expected utility that is promised to the manager, W_0, to maximize the initial value of her firm under the contract. The optimal choice of W_0 will be explained later.

3. KEY INTUITION AND THE CONTRACT WITH A TIME-INVARIANT FRACTION OF PAYMENT

This section considers a type of the contract under which the owner constantly delivers a fixed fraction, $p \in [0, 1]$, of the firm's operating profit to the manager. Namely, $P_t = pAK_t$ for all $t \geq 0$, and the transfer payment under his control for consumption and investment is proportional to the firm's capital. So, to some extent the manager has an incentive to restrict his consumption at each instant in time to make investments. Although this type of contract is not optimal, it shares some common features with the optimal contract, and its simplicity allows us to clearly understand the key intuition of the model.

Fixing the initial risk management policy, l, and the fraction p, I solve the manager's optimal investment policy $\{i_t\}$, which maximizes his expected utility defined by (3), using dynamic programming. Let K_t be the state variable and $W(K)$ be the value function of the manager. The law of motion for K_t implies that $W(K)$ satisfies the following Hamilton-Jacobi-Bellman (HJB hereafter) equation:

$$\begin{aligned} 0 = \max_{i} \quad & \beta\frac{\left[(pA - i)\, K\right]^{1-\gamma}}{1-\gamma} - \beta W(K) + W'(K)K(i - \delta + \lambda l) \\ & + \frac{1}{2}W''(K)K^2\sigma^2 + \lambda\left[W\left((1-l)K\right) - W(K)\right]. \end{aligned} \tag{4}$$

The last term on the right-hand side of (3) corresponds to the impact of the market-level shocks on the manager's utility under the contract. Notice that only a fraction $1-l$ of the capital stock remains after a shock. Since the manager's utility is homogeneous of degree $1-\gamma$ in K, it is straightforward to show that

$$W(K) = \underline{w}K^{1-\gamma} \tag{5}$$

for some scalar $\underline{w}$. In fact, $\underline{w} = W(K)/K^{1-\gamma}$ is the manager's utility relative to the firm's capital, which can be interpreted as the manager's stake in the firm. Under this type of contract, the manager's stake is time invariant. With the form of the value function in (5), the HJB equation (3) can be written as

$$0 = \max_{i} \quad \frac{\beta\left(pA - i\right)^{1-\gamma}}{1-\gamma} - \beta\underline{w} + (1-\gamma)\,\underline{w}\left((i-\delta) - \frac{1}{2}\gamma\sigma^2\right) + (1-\gamma)\underline{w}\lambda\left(l + \frac{(1-l)^{1-\gamma} - 1}{1-\gamma}\right). \quad (6)$$

The objective function on the right hand side of (6) implies the manager's optimal investment-to-capital ratio:

$$\hat{\imath} = \frac{1}{\gamma}\left(pA - \beta - (1-\gamma)\,\delta - \frac{1}{2}\gamma\,(1-\gamma)\,\sigma^2 + (1-\gamma)\lambda\left(l + \frac{(1-l)^{1-\gamma} - 1}{1-\gamma}\right)\right), \quad (7)$$

and then $\underline{w} = \frac{\beta}{1-\gamma}\left(pA - \hat{\imath}\right)^{-\gamma}$. The investment-to-capital ratio is time invariant, depending only on the model parameters, the payment fraction p, and the initial risk management policy l. On the other hand, if the initial capital stock is K, the firm owner's expected payoff under the contract is

$$E_0\left[\int_0^\infty e^{-\beta t}(1-p)AK_t dt\right] \text{ with } K_0 = K.$$

Since the investment-to-capital ratio is constant and K_t follows (1), the expected payoff is homogeneous of degree one in K and equal to $\underline{v}K$ with

$$\underline{v} = \frac{(1-p)A}{\beta + \delta - \hat{\imath}}. \quad (8)$$

In fact, $\underline{v}$ is the value of one unit of capital to the owner under the contract, which is time invariant as well.

To see how the initial risk management policy, l, affects the value of capital to the owner, notice that she is risk-neutral, and implementing the policy only reduces the volatility without changing the expected rate of capital growth. Therefore, as shown by (8),[6] the policy does not directly affect the capital value, but does affect it indirectly through its effect on the manager's investment. Clearly, $\underline{v}$ increases with $\hat{\imath}$, and $\hat{\imath}$ depends on l through the last term in parentheses on the right-hand side of (7), which is equal to zero if the firm chooses to impose risk management. To see the dependence of the owner's expected payoff under the contract on the risk management policy, I show the following result.

[6]To guarantee that the owner's expected payoff is finite, I assume that $\hat{\imath}$ is smaller than $\beta + \delta$.

LEMMA 1. *$\bar{l} + \frac{(1-\bar{l})^{1-\gamma}-1}{1-\gamma} < 0$ for all $\bar{l} \in (0,1)$.*

Proof. See Appendix A.1. ▮

According to Lemma 1, equation (7) implies the following proposition.

PROPOSITION 1. *If $\gamma > 1$, for any $p \in [0,1]$, under the contract that delivers a fraction p of the operating profits to the manager, the firm owner's expected payoff is lower with risk management.*

Obviously, according to (7), when the manager's relative risk aversion coefficient is greater than one, not imposing the risk management policy from the beginning induces the manager to choose a higher level for the investment-to-capital ratio and a lower level of consumption. The higher level of investment strictly raises the firm's value, in favor of the firm's owner. To understand the intuition, notice that, under the contract with the total transfer payment to the manager being proportional to the operating profit, the manager's future benefit is proportional to the firm's future capital stock. When making the decision on how to allocate the transfer payment, pAK_t, between investment and consumption, the key trade-off the manager is subject to is between his current utility and future benefits, which is similar to the trade-off a household is subject to when making decisions on consumption and saving with income uncertainty—a topic studied in the macroeconomics literature. The level of investment is determined by the intertemporal income and substitution effects. When $\gamma > 1$, the intertemporal elasticity of substitution, which is $1/\gamma$, is smaller than one so that the income effect dominates. Without risk management, the firm is exposed to marketwide shocks that are beyond the manager's control. Since the manager is risk averse, greater risk exposure lowers his future utility level and the dominating intertemporal income effect makes him more cautious, makes him abstain from engaging in misbehavior, and increases the level of investment to raise his future benefits. In fact, this phenomenon resemblances the precautionary savings effect studied in the macroeconomics literature. As a result, under the contract, exposing the firm to marketwide risks would align the manager's interests with the owner's. Therefore, even risk management policies that only reduce risk exposure without affecting expected capital accumulation would have a strictly negative effect on the firm owner's benefits.

The following proposition shows that, if the firm were owned by the manager himself, he would be better off by implementing the risk management by choosing $l = 0$. Therefore, moral hazard in firm delegation is a key factor that makes marketwide risk exposure strictly desirable to the owner.

PROPOSITION 2. *Under the contract paying a time-invariant fraction of the operating profits to the manager, risk management improves the manager's expected utility.*

Proof. According to the definition of $\underline{w}$, $(1-\gamma)\,\underline{w} > 0$. Therefore, Lemma 1 and the objective function on the right-hand side of (6) imply that the manager could yield a higher utility level if $l = 0$ as $\underline{w}$ decreases with $\hat{\imath}$. So we have the desired result. ∎

Under the optimal contract, to be discussed in the next section, instead of being time invariant, the fraction of the operating profit, p_t, delivered to the manager is optimally chosen at each instant according to the past performance of the manager. In general, to provide incentives, the fraction increases with the past growth of the firm capital. The resource constraint (2) implies that p_t is bounded in $[0,1]$ for all $t \geq 0$. Therefore, the marketwide risk exposures of firm capital implies a level of risk exposure to the manager's benefits as under the contract discussed in the current section. Therefore, through the same mechanism, the initial risk management policy could still imply a lower level of investment and reduce the firm owner's expected payoff under the optimal contract.

Notice that the scalars $\underline{w}$ and $\underline{v}$ defined above depend on the fraction p and the risk management policy choice l. To facilitate the discussion of the optimal contract in the next section, I write them as functions of p and l: $\underline{w}\,(p,l)$ and $\underline{v}\,(p,l)$.

4. EFFECT OF RISK MANAGEMENT ON THE OPTIMAL CONTRACT

This section studies how the initial choice of the risk management policy affects the firm owner's expected payoff under the optimal contract. Given the initial risk management policy, l, define the manager's continuation utility under the contract, $(\{P_t\})$, as

$$W_t = E_t\left[\beta\int_0^\infty e^{-\beta(s-t)}\frac{(P_s - i_s K_s)^{1-\gamma}}{1-\gamma}ds\right] \text{ for all } t \geq 0,$$

which is the expected utility that the manager will experience from time t on.

We use dynamic programming to characterize the optimal contract with W_t being a state variable. The Martingale Representation Theorem implies the following law of motion for W_t:[7]

$$\begin{aligned} dW_t &= \beta \left(W_t - \frac{(P_t - i_t K_t)^{1-\gamma}}{1-\gamma} \right) dt + g_t (1-\gamma) W_t \sigma dB_t \\ &+ j_t (1-\gamma) W_t (\lambda dt - dN_t), \end{aligned} \tag{9}$$

with $\{g_t\}$ and $\{j_t\}$ being two squared integrable processes that respectively indicate the sensitivity of W_t with respect to the firm-level and marketwide shocks. According to the last term on the right-hand side of (8), when a marketwide shock hits, W_t is instantaneously adjusted by $j_t (1-\gamma) W_t$ under the optimal contract. In fact, g_t and j_t are optimally chosen as part of the contract design problem.

Let $V(K_t, W_t)$ be the value function of the owner under the optimal contract with K_t and W_t being the state variables. Because of the homogeneity of the contract design problem, $V(K_t, W_t) = v\left(\frac{W_t}{K_t^{1-\gamma}}\right) K_t$ for some function $v(w)$, which we call the normalized value function. Here, $w_t = \frac{W_t}{K_t^{1-\gamma}}$ is the normalized continuation utility of the manager, the ratio of the manager's future utility to the size of the firm. A higher level of w_t indicates that a greater fraction of the future operating profits will be consumed by the manager, so that w_t is interpreted as the manager's stake in the firm. Furthermore, $v(w)$ is the value of one unit of firm capital to the owner given the manager's stake w. As discussed in Section 3, w_t and $v(w_t)$ are time invariant under the contract delivering a fixed fraction of the operating profits to the manager. However, under the optimal contract, equation (8) implies that w_t follows

$$dw_t = (1-\gamma) w_t \left[\begin{array}{c} \left(\frac{\beta}{1-\gamma} \left(1 - \frac{(p_t A - i_t)^{1-\gamma}}{(1-\gamma) w_t} \right) + \left(1 - \frac{1}{2}\gamma\right) \sigma^2 - (1-\gamma) g \sigma^2 + \lambda j_t \right) dt \\ - (i_t - \delta + \lambda l) dt + (g_t - 1) \sigma dB_t + \frac{1-(1-\gamma) j_t - (1-l)^{1-\gamma}}{(1-\gamma)(1-l)^{1-\gamma}} dN_t \end{array} \right]. \tag{10}$$

Let $p_t = \frac{P_t}{AK_t}$ be the fraction of the operating profits that are delivered to the manager at time t. Since the owner's dividend payment and the manager's compensation are nonnegative, given the initial risk management policy l, w_t is bounded in the interval $[\underline{w}(0,l), \underline{w}(1,l)]$ for all $t \geq 0$. The lower bound can only be achieved by constantly paying the agent zero transfer, $p_t = 0$ for all $t \geq 0$. The upper bound can only be achieved by paying all the firm's operating profits to the manager, $p_t = 1$ and $P_t = AK_t$ for all $t \geq 0$. Clearly, the value of one unit of capital on the left and right boundaries is $\underline{v}(0,l)$ and $\underline{v}(1,l) = 0$, respectively. Accordingly, I normalize the manager's consumption level with respect to capital K: $dc = \frac{dC}{K}$.

To characterize the optimal contract, we show the following incentive compatibility condition which determines the manager's investment-to-capital ratio, i_t.

[7]See Sannikov (2008) for technical details.

LEMMA 2. *Under a contract, the agent chooses i_t such that*

$$g_t\,(1-\gamma)\,w_t = \beta\,(p_t A - i_t)^{-\gamma} \text{ for all } t \geq 0. \tag{11}$$

Proof. See Appendix A.2. ∎

Since g_t indicates the sensitivity of the manager's future benefits to capital accumulation, the left-hand side of (11) is the marginal benefit of additional investment. Additional investment entails a cost on current consumption given the amount of the delivered transfer payment, and the marginal cost of investment is on right-hand side. Essentially, equation (11) is the first-order condition for the manager's optimal investment level. Obviously, if $\gamma > 1$, the marketwide risk exposure without risk management would imply a higher level of $(1-\gamma)\,w_t$ because $(1-\gamma) < 0$ and greater uncertainty about his future benefits implies a lower level of w_t. Therefore, given the same fraction of transferred operating profit, p_t, and sensitivity g_t, greater risk exposure would induce the manager to invest more and consume less. As a result, the risk management policy preventing the risk exposure could weaken the incentive provisions of the contract.

Equation (11) determines the sensitivity g_t as a function of the transfer payment ratio, p_t, and the normalized consumption, c_t, which I denote $g(p,i)$. Based on our analysis, we have the following proposition, which characterizes the normalized value function of the owner under the optimal contract, $v(w)$.

PROPOSITION 3. *Given the risk management policy, l, the normalized value function $v(w)$ satisfies the following HJB differential equation over $[\underline{w}\,(0,l)\,,\underline{w}\,(1,l)]$:*

$$\begin{aligned} 0 &= \max_{p,i,j} A(1-p) + (i-\delta-\beta+\lambda l)\,v(w) \\ &+ (1-\gamma)\,wv'(w)\left[\begin{array}{c}\frac{\beta}{1-\gamma}\left(1-\frac{(Ap-i)^{1-\gamma}}{(1-\gamma)w}+\frac{\lambda}{\beta}(1-\gamma)j\right)-(i-\delta+\lambda l) \\ -\frac{1}{2}\gamma\sigma^2+\gamma g(p,i)\sigma^2\end{array}\right] \\ &+ \frac{1}{2}\,(1-\gamma)^2\,w^2 v''(w)\sigma^2\,(g(p,i)-1)^2+\lambda\left[v\left(\frac{1-(1-\gamma)j}{(1-l)^{1-\gamma}}w\right)-v(w)\right] \end{aligned} \tag{12}$$

and the boundary conditions

$$v\,(\underline{w}\,(0,l)) = \underline{v}\,(0,l) \text{ and } v\,(\underline{w}\,(1,l)) = 0. \tag{13}$$

Under the optimal contract, the agent's normalized utility at $t=0$ is

$$w_0 = \arg\max_{\tilde{w}\in[\underline{w}(0,l),\underline{w}(1,l)]} v\,(\tilde{w}) \tag{14}$$

and evolves according to (10) *until it is absorbed by the left or right boundary. The policies* $p(u_t)$, $i(u_t)$, *and* $j(u_t)$ *are the maximizers of the objective function on the right-hand side of* (12).

Proof. See Appendix A.3. ▮

As mentioned earlier, the owner has full bargaining power when offering the delegation contract, so she always chooses the manager's initial expected utility $W_0 = w_0 K_0$ to maximize the firm's value according to (14). Because of the complexity of the model, analytically characterizing the effect of the risk management policy on the firm's value to the owner under the optimal contract is difficult. I therefore demonstrate it numerically based on the benchmark parameter values listed in Table 1.

TABLE 1.

Benchmark Parameter Values

Parameters	Descriptions	Values
A	Productivity of capital	0.11
δ	Depreciation rate of capital	0.05
σ	Volatility of firm-level shocks to capital	0.10
λ	Arriving rate of marketwide shocks to capital	0.05
$\bar{l}$	Potential fraction of capital loss upon a marketwide shock	0.30
β	Discount rate of the manager	0.05
r	Interest rate	0.05

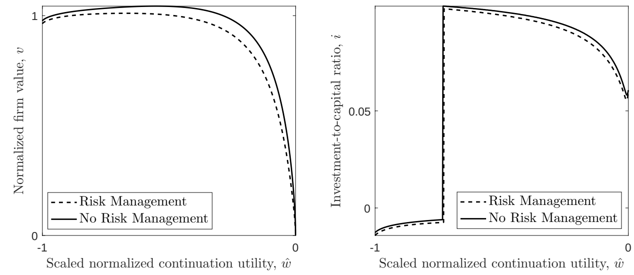
Notes - This table lists the parameter values in the benchmark numerical example.

To facilitate the interpretation of the comparison between the optimal contract with and without risk management, I define the manager's scaled normalized continuation utility as

$$\hat{w} \equiv -\frac{w - \underline{w}(1, l)}{\underline{w}(0, l)} \text{ for } l = 0, \bar{l}.$$

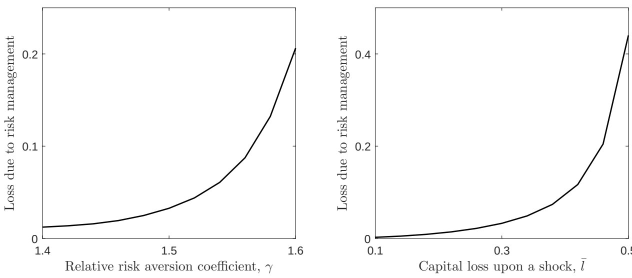
Obviously, $\hat{w}$ is a linear monotonic transformation of w with a range of $[-1, 0]$, which is independent of the initial risk management policy choice, l. In Figure 1, I plot the normalized value, v, and the investment-to-capital ratio, i, as functions of the scaled normalized continuation utility under the contract with and without risk management. As we see in the left panel, without risk management, the value of one unit of capital is greater because the manager is induced to choose higher levels of investment as interpreted in the right panel. So risk management preventing risk exposure lowers the firm's value to the owner, as what happens under the contract discussed in Section 3.

FIG. 1. Value Functions and Optimal Policies with Different Levels of Risk Taking.



Notes - In the left panel, I plot the normalized firm value, v, as a function of the scaled normalized continuation utility of the manager in the case of risk management (dashed curve) and the case of no risk management (solid curve). In the right panel, I plot the investment-to-capital ratio as a function of the scaled normalized continuation utility of the manager in the case of risk management (dashed curve) and the case of no risk management (solid curve).

FIG. 2. Dependencies of loss of risk management on the manager's risk aversion and the potential capital loss upon a marketwide risk shock.



Notes - I define the loss arising from risk management as the difference between the initial normalized firm value, $v(w_0)$, under the contract with and without risk management. The left panel shows how the loss depends on the manager's relative risk aversion coefficient, γ. The right panel shows how the loss depends on the potential capital loss upon a marketwide risk shock, $\bar{l}$. The benchmark parameter values are listed in Table 1.

In Figure 2, based on the benchmark parameter values listed in Table 1, I demonstrate two comparative static analyses to show how the loss arising from the firm's risk management policy depends on the manager's relative risk aversion coefficient, γ, and the potential loss of capital upon a marketwide risk shock. The loss of firm value is measured by the difference between the initial normalized firm values, $v(w_0)$, under the contract with and without risk management. As shown in the left panel, the loss from risk management increases with γ. Intuitively, the greater the risk aversion coefficient, the smaller the intertemporal elasticity of substitution, the more significant the precautionary savings effect is, and the more favorable the marketwide risk exposure is to the owner. As seen in the right panel, the loss arising from risk management goes up if the potential fraction of capital loss upon a shock increases. Clearly, a higher potential loss implies greater uncertainty of the firm and the manager in the future, which reinforces the precautionary savings effect and further prevents the manager from engaging misbehavior that could lower the firm's future profitability.

5. CONCLUSION

In this paper, I propose a theoretical model to show that risk management policies that shield firms from marketwide risk exposure could reduce firm's value to owners when a moral hazard problem in firm delegation arises. The level of risk exposure of a firm implies that the manager's prospective benefits and uncertainties associated with his future compensations are subject to a similar level of risk exposure under a delegation contract. If the manager's intertemporal preference exhibits a dominating income effect, greater uncertainty with respect to future benefits makes him more cautious and makes him abstain from engaging in misbehavior that may raise his current utility but hurt the firm's long-run profitability. As a result, risk exposure prevented by risk management policies could alleviate the moral hazard problem, enhance incentive provisions, and raise firms' value.

APPENDIX

A.1. PROOF OF LEMMA 1

Let's consider three cases. In case 1, $\gamma = 1$, since

$$\lim_{\gamma \to 1} \bar{l} + \frac{\left(1-\bar{l}\right)^{1-\gamma} - 1}{1-\gamma} = \bar{l} + \ln\left(1-\bar{l}\right)$$

which is negative for all $\bar{l} \in (0,1)$.

In case 2, $\gamma < 1$. Notice that

$$\bar{l} + \frac{\left(1-\bar{l}\right)^{1-\gamma} - 1}{1-\gamma} = \frac{-\gamma}{1-\gamma} + \left(-\left(1-\bar{l}\right) + \frac{\left(1-\bar{l}\right)^{1-\gamma}}{1-\gamma}\right),$$

which is negative.

In case 3, $\gamma > 1$, since $\bar{l} + \frac{(1-\bar{l})^{1-\gamma}-1}{1-\gamma} = \frac{-\gamma}{1-\gamma} + \left(-\left(1-\bar{l}\right) + \frac{(1-\bar{l})^{1-\gamma}}{1-\gamma}\right)$. Let us consider the terms in the second parentheses on the right-hand side. Since

$$\frac{d}{dx}\left(-(1-x) + \frac{(1-x)^{1-\gamma}}{1-\gamma}\right) = 1 - \frac{1}{(1-x)^{\gamma}} < 0 \text{ for all } x \in (0,1).$$

Furthermore, when $x = 0$, $-(1-x) + \frac{(1-x)^{1-\gamma}}{1-\gamma} = -1 - \frac{1}{1-\gamma}$. Therefore

$$\frac{-\gamma}{1-\gamma} + \left(-\left(1-\bar{l}\right) + \frac{\left(1-\bar{l}\right)^{1-\gamma}}{1-\gamma}\right) < 0 \text{ for all } \bar{l} \in (0,1).$$

A.2. PROOF OF LEMMA 2

To show (11), it is equivalent to prove that, under $(\{P_t\})$,

$$i_t \in \arg\max_i \beta \frac{\left((p_t A_t - i)\, K_t\right)^{1-\gamma}}{1-\gamma} + g_t\,(1-\gamma)\, W_t i_t \text{ for all } t \geq 0. \quad \text{(A.1)}$$

Suppose there is an alternative investment policy $\{i'_t\}$ that does not satisfy (A.1). For any $t \geq 0$, define

$$\Phi_t = \beta \int_0^t e^{-\beta s} \frac{\left((p_t A - i'_t)\, K_s\right)^{1-\gamma}}{1-\gamma} + e^{-\beta t} W_t.$$

According to its definition, Φ_t is the conditional expected utility of the manager based on the time-t history if he adopts policy i' up to time t and then switches to policy i. Obviously, $\Phi_0 = W_0$, the expected utility of the manager if he chooses i from the beginning. According to (8),

$$\begin{aligned} d\Phi_t &= e^{-\beta t}\left\{\beta \frac{\left((p_t A - i'_t)\, K_s\right)^{1-\gamma}}{1-\gamma} dt - \beta W_t + dW_t\right\} \\ &= e^{-\beta t}\left\{\begin{array}{c} \beta\left[\frac{\left((p_t A - i'_t) K_s\right)^{1-\gamma}}{1-\gamma} - \frac{((p_t A - i_t) K_s)^{1-\gamma}}{1-\gamma}\right] dt \\ + g_t\,(1-\gamma)\, W_t \sigma dB_t + j_t\,(1-\gamma)\, W_t\,(\lambda dt - dN_t) \end{array}\right\}. \end{aligned}$$

Notice that $\{B_t\}$ is a standard Brownian motion under the measure generated by policy i. Let $\{B'_t\}$ be a standard Brownian motion under the measure generated by policy i'. The transfer between the two measures is according to Girsanov's theorem. According to (1),

$$dB_t = dB_t' + \frac{i_t' - i_t}{\sigma} \text{ for all } t \geq 0$$

and then

$$d\Phi_t = e^{-\beta t} \left\{ \begin{array}{c} \left[\beta \left(\frac{((p_t A - i_t') K_s)^{1-\gamma}}{1-\gamma} - \frac{((p_t A - i_t) K_s)^{1-\gamma}}{1-\gamma} \right) + g_t (1-\gamma) W_t (i_t' - i_t) \right] dt \\ + g_t (1-\gamma) W_t \sigma dB_t' + j_t (1-\gamma) W_t (\lambda dt - dN_t) \end{array} \right\}. \tag{A.2}$$

Clearly, according to the drift term on the right hand side of (A.2), $\{\Phi_t\}$ is a super martingale if and only if (A.1) is satisfied. In addition, $\{\Phi_t\}$ is a super martingale if and only if

$$E'[\Phi_t] \geq \Phi_0 = W_0.$$

Here, E' is the expectation operator based on policy i'. Consequently, policy i should be adopted at the beginning of the contract if and only if (A.1) is satisfied, and we have the desired result.

A.3. PROOF OF PROPOSITION 3

The boundary conditions are obvious. I show the HJB equation of $v(w)$, (12). According to (1) and (8), $V(K,W)$ satisfies the following HJB differential equation.

$$\begin{aligned} 0 = \max_{p,i,j} \quad & (1-p)AK - \beta V(K,W) + V_K(K,W) K (i - \delta + \lambda l) \\ & + V_W(K,W) \beta \left(W - \frac{((Ap - i) K)^{1-\gamma}}{1-\gamma} - \frac{\lambda}{\beta} j (1-\gamma) W \right) \\ & + \frac{1}{2} V_{KK}(K,W) \sigma^2 K^2 + \frac{1}{2} V_{WW}(K,W) g^2(p,i)(1-\gamma)^2 W^2 \\ & + K_{KW}(K,W) K g(p,i)(1-\gamma) W \\ & + \lambda \left[V((1-l)K, (1 + j(1-\gamma))W) - V(K,W) \right]. \end{aligned} \tag{A.3}$$

According to the normalization, $V_K(K,W) = v(w) - (1-\gamma) w v'(w)$, $V_{KK}(K,W) = \frac{1}{K}(1-\gamma) w (-\gamma v'(w) + (1-\gamma) w v''(w))$, $V_{KW}(K,W) = \frac{1}{K^{1-\gamma}} (\gamma v'(w) - (1-\gamma) w v''(w))$, $V_W(K,W) = K^{\gamma} v'(w)$, and $V_{WW} = K^{2\gamma-1} v''(w)$. Then (A.3) implies (12).

REFERENCES

DeMarzo, P., M. Fishman, Z. He, and N. Wang, 2012. Dynamic agency and q theory of investment. *Journal of Finance* **67**, 2295-2340.

DeMarzo, P., D. Livdan, and A. Tchistyi, 2014. Risking Other People's Money: Gambling, Limited Liability, and Optimal Incentives. Working Paper, Stanford University.

DeMarzo, P. and Y. Sannikov, 2006. Optimal security design and dynamic capital structure in a continuous-time agency model. *Journal of Finance* **61**,2681-2724.

Fedele, A. and A. Mantovani, 2014. Credit availability in the crisis: Which role for public financial institutions? *Annals of Economics and Finance* **15-1**,161-183.

Feng, F. and M. Westerfield, 2016. Dynamic asset allocation with hidden volatility. Working paper, University of Washington.

Gong, Q., J. Shen, and J. Lin, 2009. Firm liquidation and economic crisis under unexpected exchange rate shock. *Annals of Economics and Finance* **10-1**, 1-14.

Leland, H., 1968. Saving and uncertainty: The precautionary demand for saving. *Quarterly Journal of Economics* **2**, 465-473.

Leung, R., 2014. Continuous-time principal-agent problem with drift and stochastic volatility control: With applications to delegated portfolio management. Working paper, Cheung Kong Graduate School of Business.

Obstfeld, M., 1994. Risk-taking, global diversification, and growth. *American Economic Review* **84**, 1310-1329.

Ou-Yang, H., 2003. Optimal contracts in a continuous-time delegated portfolio management problem. *Review of Financial Studies* **16**, 173-208.

Sannikov, Y., 2008. A continuous-time version of the principal-agent problem. *Review of Economic Studies* **75**, 957-984.

Wen, Y., 2007. Proproduct and inventory behavior of capital. *Annals of Economics and Finance* **8-1**, 95-112.

Williams, N., 2011. Persistent private information. *Econometrica* **79**,1233-1275.

Zhu, J., 2013. Optimal contracts with shirking. *Review of Economic Studies* **80**, 812-839.