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Limiting End-user Piracy — The Role of Private and Public Anti-Piracy Measure*

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We study when the original product developer makes costly investment to deter end-user piracy in a given regime of IPR protection. We find that when the consumers' tastes are sufficiently diverse and IPR protection is weak, only then it is profitable for the product developer to tolerate piracy. In all the other cases, it is profitable to deter, unless piracy is blockaded by strong IPR protection. The relationship between the optimal deterrence level from the product developer (private anti-piracy measure) and the degree of IPR protection in the economy (public anti-piracy measure) can be monotonic or non-monotonic. The private optimal deterrence level generally increases with the quality of the pirated good except when consumers' tastes are sufficiently diverse and the quality of the pirated good is sufficiently high. Public anti-piracy measure unambiguously reduces piracy.

Key Words: IPR protection; Private copyright protection; Piracy rate; Product quality; End-user piracy.
JEL Classification Numbers: D23, D43, L13, L86, O3.

1. INTRODUCTION

The problem with copyright violations or piracy of digital products in today's world is one of the major issues often discussed. Because of the rapid technological progress in all fronts and the very nature of a digital product, it became relatively easy to copy any kind of digital goods. Broadly speaking, there are two types of piracy that happen, commercial and end-user.

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The piracy is commercial when a pirate sells copies of an original digital product at a low price (compared to the price of the original product) for profits. This is mostly observed in poor and developing countries where the price of an original digital product is beyond the reach of the majority of the people. The other type of piracy that is observed, more or less in all parts of the world, is end-user piracy. In end-user piracy, a potential user (i.e. a consumer) can be a pirate as well. However, unlike commercial piracy, there is no explicit profit motive here for the pirate, but it is done mainly for personal usage or convenience.

In this particular analysis, the digital products we have in mind are mostly various software products that are useful in performing several tasks and can be upgraded or enhanced with various applications or add-ins in future (e.g. Microsoft office).[1]

Given piracy in either form reduces the revenue for the original producer of the good (i.e. the copyright holder), it is often the case where the original producer invests in various piracy deterring activities. Although the intellectual property right (IPR) laws (i.e. the public anti-piracy measure) of a country play a major role in stopping or limiting piracy, but often they are not enough to tackle the problem.[2] More often a private and public partnership is more effective in addressing the problem of piracy. To this end, Lu and Poddar (2012) analyzed a problem where the original producer makes costly investment to deter a commercial pirate in a given regime of IPR protection. Various interesting results were derived in the analysis. However, the focus of the study was only on commercial piracy. We would like to know what happens when the piracy is end-user type. End-user piracy is big in the digital world of copyright violations, and in this paper, we focus our study to that problem.

We study a scenario where the copyright holder faces numerous end-user pirates for its product, and makes costly investment to raise the cost of the piracy to the pirates in a given regime of IPR protection. In the analysis, first, we see the optimal responses of the copyright holder under various circumstances and their consequences. Later, we see how the private anti-piracy measure interacts with the public anti-piracy measure (i.e. IPR protection), and how the quality of the pirated good impacts the private deterrence effort.

In the literature, both commercial and end-user piracy are studied under different contexts. Studies by Conner and Rumelt (1991), Takeyama (1994), Slive and Bernhardt (1998), Shy and Thisse (1999), Chen and Png (2003), King and Lamp (2003), Bae and Choi (2006), Belleflamme and Pi-

[1] We are not considering digital products like music, movies, games where future enhancements are limited or not necessary.

[2] One of the problems in poorer countries is widespread corruption due to which IPR laws are not strictly enforced.

card (2007), Cremer and Pestieau (2009), Lahiri and Dey (2012), among others mainly focused on end-user piracy; whereas studies by Banerjee (2003, 2011, 2013), Lu and Poddar (2012, 2018), Kiema (2008), Jaisingh (2009), Martinez-Sanchez (2010), and more recently Lopez-Cunat and Martinez-Sanchez (2015) among others focused solely on commercial piracy.[3] Tunca and Wu (2013) considered a model where both end-user and commercial piracy co-exist, and showed that in the presence of commercial piracy, a higher number of end-user pirates actually can increase the copyright holder's profit and higher detection rate of end-user piracy can actually reduce copyright holder' profit, thus capturing the interaction between commercial and end-user piracy in the same market. Dey et. al. (2019), developed another type of model which makes a distinction between anti-piracy efforts that restrict supply of pirated goods (supply-side enforcement) and ones that penalize illegal consumption (demand-side enforcement) and discovered some fundamental differences between these two types in terms of their impacts on welfare and innovation.

In our end-user piracy model, we assume that there is one original product developer and a group of heterogeneous consumers who are also potential pirates. The original product developer makes costly investment to deter piracy in a given regime of IPR protection. The original producer's investment increases the copying or subsequent usage cost of the pirated product, thus effectively reducing the value of the pirated product. This can be done in various ways e.g., by encrypting the original program, providing no support service, making the pirated version incompatible with future updates, add-ins and new applications associated with the product. We call these specific efforts to deter piracy as private measure for anti-piracy from the copyright holder and the prevailing IPR protection policy as public measure of anti-piracy. The private deterrence effort is a choice variable of the product developer while the IPR protection level is assumed to be exogenous in the models and treated as a parameter.[4]

The original product developer is a monopolist, but faces numerous potential end-user pirates who are willing to pirate the product instead of buying it. If the pirates are successful in pirating the product, the overall quality of the pirated product is always lower than the original product.

[3]For a comprehensive survey on the recent development on the theory of digital piracy, see Belleflamme and Peitz (2012) and for empirical analysis see Waldfogel (2012).

[4]It is fairly well documented that different countries have different levels of IPR protection; it can be weak or strong. More importantly, for a country it takes a long time to adjust its IPR policy (more so if the government of that country is not very pro-active to reform IPR related policies), hence we assume it to be exogenous in our model. We do acknowledge that in many studies, IPR policy instruments, like monitoring the pirate and imposing penalty, are modeled endogenously, however in this analysis our focus is different. We will, however, do comparative static analysis on various levels of IPR measures in the analysis to see the impact of IPR.

The IPR protection policy as well as the deterrence effort of the original producer targets the end-users to limit piracy.

The main findings of our analysis are as follows. While characterizing the equilibrium outcomes, we find that when the buyers' or consumers'tastes are sufficiently diverse and IPR protection is weak, only then it is profitable for the original producer to tolerate piracy. In all the other situations piracy is deterred, unless it is blockaded by strong IPR protection. In order to find the interaction between two anti-piracy measures i.e. public protection (IPR protection) and private protection (the optimal deterrence level from copyright holder), we see that when piracy is tolerated, the optimal deterrence level from copyright holder is increasing in the public protection measure (complementary); while it is decreasing in the public protection measure (substitutes) when piracy is deterred. Thus, the relationship between the optimal deterrence level from the product developer and the degree of IPR protection can be monotonic or non-monotonic depending on whether piracy is tolerated or deterred, which in turn depends on the extent of diversification of the consumers' tastes, and the strength of the IPR policy. Naturally, this result has policy implications.

We are also interested to see how the optimal deterrence level from the copyright holder needs to be adjusted as the quality of the pirated good changes since the quality of the pirated good impacts the copyright holder's profit. Note that the intrinsic quality of the pirated good can widely vary from not very reliable to highly reliable and the copyright holder needs to adjust its response for deterrence accordingly. This intrinsic quality of the pirated good depends on the copying technology and other factors, like consumers' valuation or perception of the pirated good. We find that the optimal deterrence level chosen by the copyright holder increases as the pirated quality increases when piracy is tolerated. It also increases with the pirated quality under deterrence except when the consumers' tastes are sufficiently diverse and the pirated quality is sufficiently high.

The rate of piracy is an important metric often used to know the extent of piracy in a country.[5] Piracy watch-dog organizations, like BSA (Business Software Alliances), estimate this metric every year for various countries. To this end, we verify our natural intuition that the public anti-piracy measure must reduce the overall piracy rate.

The rest of the paper is organized as follows. In the next section, we lay out the basic framework of end-user piracy. In section 3, we do our main analysis. Important findings from the comparative static analysis are reported in section 4. Section 5 concludes with a discussion.

[5]The rate of piracy will be defined appropriately later.

2. THE MODEL OF END-USER PIRACY

Consider an original product developer (a monopolist) and a continuum of consumers. Consumers have different valuations for the product indexed by X which is uniformly distributed over the interval $[0, \theta]$ with density $1/\theta$. A high value of X means high valuation for the product (or higher willingness to pay) and a low value of X means low valuation for the product (or lower willingness to pay). Consumers have the choice to buy the original product from the monopolist or they can use a pirated version. We assume all the consumers are potential pirates. We also assume the pirated product is of lower quality than the original. The intrinsic product quality of the pirated good (compared to the original one) is captured by the parameter q, $q \in (0, 1)$.[6]

The original product developer undertakes costly investment in order to deter or limit piracy. It targets the end user pirates to stop or limit piracy as it stands to lose its potential market share because of them. It tries to make the pirated product less valuable to the end-users by increasing the cost of copying or increasing the cost of subsequent usage or restricting the overall future usage of the pirated product.[7] We summarize this overall increase in cost to the pirate by $x(x \geq 0)$. This can also be interpreted as the deterrence level from the original producer. We assume the cost of investment of the original product developer to set a deterrence level, x, is given by $c_0(x) = x^2/2$. Thus, the higher the investment made by the original product developer, the higher would be the deterrence or overall cost of piracy to the end-user pirate.

There exists a general level of exogenous IPR protection to reduce piracy in the economy which is denoted by $c(c \geq 0)$. When the level of private deterrence is x, and the level of public deterrence is c, we assume the total deterrence or the cost of copying to an end-user is $c + x$. We assumed an additive form between c and x since both the original firm's private effort (investment) and the legal protection and enforcement of copyright legislations (public protection) contribute to the deterrence of piracy.

We would like to interpret c as the degree or the strength of IPR protection. It essentially captures the strength of legal protection and enforcement to stop or limit piracy and it is beyond the control of the original firm (i.e. the copyright holder). It is generally understood that the government or the regulatory authority can influence c.[8]

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[6] Implicitly we assume the quality index of the original good is 1.

[7] For example, if the digital product is not original, there will be no support service or future upgrades; new applications will not be compatible, which reduces the overall value of the product.

[8] According to a recent study by Andres (2006) (also see Park and Ginarte (1997)), the strength of IPR protection of a country mainly consists of two categories: membership in the international copyright treaties and enforcement provisions. Going by the definition

A type-X consumer's utility function is given as:

$$U = \begin{cases} X - p & \text{if buys the original product} \\ qX - (c+x) & \text{if pirates the original product,} \\ 0 & \text{otherwise} \end{cases}$$

where p is the price of the original product.

3. THE MAIN ANALYSIS

3.1. Deriving Demand for the Original and Pirated Products

The demand for the original product and for the pirated product, D_0 and D_p, can be derived from the distribution of buyers as follows.

The marginal consumer, $\hat{X}$, who is indifferent between buying the original product and pirating is given by $\hat{X} = \frac{p-(c+x)}{1-q}$. The marginal consumer, $\hat{Y}$, who is indifferent between pirating the product and neither buying the original product nor pirating is given by $\hat{Y} = \frac{c+x}{q}$. Thus, the demand for the original product is $D_0 = \dfrac{1}{\theta}\displaystyle\int_{\hat{X}}^{\theta} dx = \dfrac{(1-q)\theta - p + (c+x)}{(1-q)\theta}$ and the demand for the pirated product is $D_p = \dfrac{1}{\theta}\displaystyle\int_{\hat{Y}}^{\hat{X}} dx = \dfrac{qp-(c+x)}{q(1-q)\theta}$. Here we have implicitly assumed $qp \geq c+x$ so that the demand for the pirate product is nonnegative. When instead $qp \leq c+x$, the developer's demand is $D_0 = \frac{\theta-p}{\theta}$.

3.2. Choice of Price and Deterrence Level by the Product Developer

Since the original producer is a monopolist and there is no strategic interaction in our model, we can either assume that the original producer chooses the deterrence level and price simultaneously or assume that it chooses the deterrence level first and then chooses price. We are going to take the latter approach.

and measure of the strength of IPR protection as discussed in Andres (2006), we can generally find a relatively high c in the developed countries where piracy is taken as a serious crime; hence it raises the cost of piracy significantly. On the contrary, in most of the developing countries, we will probably find c to be relatively low, because even if the laws are there to stop piracy, the enforcement policies against piracy may not be as strict due to corruption etc.; hence the cost of piracy would remain relatively small.

When the developer chooses p such that $qp \geq c + x$, the firm's profit maximization problem is

$$\max_{p\geq 0} \pi_0 = pD_0 - c_0(x) = p\left(\frac{(1-q)\theta - p + (c+x)}{(1-q)\theta}\right) - \frac{1}{2}x^2,$$
$$s.t. \quad qp \geq c + x^2$$

which is labeled Problem I.

When the developer chooses p such that $qp \leq c + x$, the firm's profit maximization problem is

$$\max_{p\geq 0} \pi_0 = pD_0 - c_0(x) = p\left(\frac{\theta - p}{\theta}\right) - \frac{1}{2}x^2,$$
$$s.t. \quad qp \leq c + x$$

which is labeled Problem II.

Solving Problems I and II (see Appendix) and combining the solutions, we get the original producer's optimal choice of price as a function of the deterrence level:

$$p(x) = \begin{cases} \frac{(1-q)\theta + c + x}{2} & \text{if } c + x \leq \frac{q(1-q)\theta}{2-q} \\ \frac{c+x}{q} & \text{if } \frac{q(1-q)\theta}{2-q} \leq c + x \leq \frac{q\theta}{2} \\ \frac{\theta}{2} & \text{if } c + x \geq \frac{q\theta}{2} \end{cases}. \tag{1}$$

When the total deterrence is low, the original developer tolerates end-user piracy; when the total deterrence is intermediate, it deters end-user piracy completely; and when the total deterrence is high, it acts as a monopolist when choosing its price. Note that when $c \geq \frac{q(1-q)\theta}{2-q}$, ender-user piracy will not be tolerated; and that when $c \geq \frac{q\theta}{2}$, the original developer will have no need to deter end-user piracy and will act as a monopolist: end-user piracy is blockaded.

Next we consider the original developer's choice of deterrence level. The original developer's profit function is the following (see Appendix):

$$\pi(x) = \begin{cases} \frac{((1-q)\theta + c + x)^2}{4(1-q)\theta} - \frac{1}{2}x^2 & \text{if } x \leq \frac{q(1-q)\theta}{2-q} - c \\ \frac{(c+x)(q\theta - (c+x))}{q^2\theta} - \frac{1}{2}x^2 & \text{if } \frac{q(1-q)\theta}{2-q} - c \leq x \leq \frac{q\theta}{2} - c \\ \frac{\theta}{4} - \frac{1}{2}x^2 & \text{if } x \geq \frac{q\theta}{2} - c \end{cases}. \tag{2}$$

Note that it is continuous in x. Then we obtain the original producer's marginal profit function of increasing x,

$$\pi'(x) = \begin{cases} \frac{(1-q)\theta+c+x}{2(1-q)\theta} - x & \text{if } x \le \frac{q(1-q)\theta}{2-q} - c \\ \frac{q\theta-2(c+x)}{q^2\theta} - x & \text{if } \frac{q(1-q)\theta}{2-q} - c \le x \le \frac{q\theta}{2} - c. \\ -x & \text{if } x \ge \frac{q\theta}{2} - c \end{cases} \tag{3}$$

Note that this function is also continuous in x. We can also obtain

$$\pi''(x) = \begin{cases} \frac{1}{2(1-q)\theta} - 1 & \text{if } x \le \frac{q(1-q)\theta}{2-q} - c \\ \frac{-2}{q^2\theta} - 1 & \text{if } \frac{q(1-q)\theta}{2-q} - c \le x \le \frac{q\theta}{2} - c. \\ -1 & \text{if } x \ge \frac{q\theta}{2} - c \end{cases} \tag{4}$$

It is clear that if $2(1-q)\theta \le 1$, then the profit is increasing in x when $x \le \frac{q(1-q)\theta}{2-q} - c$, and concave (increasing first and then decreasing) in x when $\frac{q(1-q)\theta}{2-q} - c \le x \le \frac{q\theta}{2} - c$, thus the maximum profit is obtained at $x^* = \frac{q\theta-2c}{2+q^2\theta}$ (such that $\frac{q\theta-2(c+x)}{q^2\theta} - x = 0$) as long as $c < \frac{q\theta}{2}$ (such that $x^* = \frac{q\theta-2c}{2+q^2\theta} > 0$); if $2(1-q)\theta > 1$, then the profit is concave in x when $x \le \frac{q(1-q)\theta}{2-q} - c$ and decreasing in x when $\frac{q(1-q)\theta}{2-q} - c \le x \le \frac{q\theta}{2} - c$, thus the maximum profit is obtained at $x^* = \frac{(1-q)\theta+c}{2(1-q)\theta-1}$ (such that $\frac{(1-q)\theta+c+x}{2(1-q)\theta} - x = 0$) as long as $c \le \frac{q(1-q)\theta-1}{2-q}$ (such that $x^* = \frac{(1-q)\theta+c}{2(1-q)\theta-1} \le \frac{q(1-q)\theta}{2-q} - c$), and otherwise $x^* = \frac{q\theta-2c}{2+q^2\theta}$. Of course, if $c \ge \frac{q\theta}{2}$, then $x^* = 0$.

Plugging the expression of the optimal deterrence level into (1) gives us the expression of the optimal price. Entry accommodation and entry deterrence equilibria are characterized as follows.

Define $\delta(q,\theta) \equiv \frac{q(1-q)\theta-1}{2-q}$.

PROPOSITION 1.

(i) When $0 \le c \le \delta(q,\theta)$, the original developer tolerates piracy, the optimal price is $p^* = \frac{(1-q)\theta((1-q)\theta+c)}{2(1-q)\theta-1}$ and the optimal deterrence level is $x^* = \frac{(1-q)\theta+c}{2(1-q)\theta-1}$. This case arises only when $\theta > \frac{1}{q(1-q)}$[9] (such that $\delta(q,\theta) > 0$).

(ii) When $c \ge q\theta/2$, the piracy is blockaded and the original developer's optimal price is the monopoly price $p^* = \frac{\theta}{2}$, and the optimal deterrence level is $x^* = 0$.

(iii) In all the other situations, the original developer deters piracy, the optimal price is $p^* = \frac{\theta(1+qc)}{2+q^2\theta}$ and the optimal deterrence level is $x^* = \frac{q\theta-2c}{2+q^2\theta}$.

[9]This also ensures $2(1-q)\theta > 1$.

The condition $\theta > \frac{1}{q(1-q)}$ (in Proposition 1(i)) can be interpreted as when the consumers' tastes are sufficiently diverse. Let's also define the following.

$$\begin{aligned} 0 \leq c \leq \delta(q,\theta) &\rightarrow \text{Low IPR protection} \\ \delta(q,\theta) \leq c \leq \frac{q\theta}{2} &\rightarrow \text{Intermediate IPR protection} \\ c \geq \frac{q\theta}{2} &\rightarrow \text{High IPR protection} \end{aligned}$$

When the consumers' tastes are sufficiently diverse (i.e. the market is diversified enough), the original developer tolerates ender-user piracy when the degree of IPR protection is low. This result is very intuitive. When the market is diversified enough, the original developer finds it very costly to completely deter end-user piracy when the IPR protection is low.

When IPR protection is high enough i.e. $c \geq q\theta/2$ then piracy is blockaded. Strong IPR completely stops ender-user piracy. This is policy induced piracy deterrence. No private deterrence measure (i.e. $x = 0$) from the original producer is necessary to remain as a pure monopolist in the market.

In all the other situations, i.e. when the consumers' tastes are not sufficiently diverse and IPR protection is low, or when the consumers' tastes are sufficiently diverse but IPR protection is intermediate, the original developer deters piracy completely.

4. COMPARATIVE STATICS

4.1. The Relationship between the Private Optimal Deterrence Level and the Public Anti-piracy Measure

To see the interaction between the private and public measures of piracy deterrence, the following comparative static analysis is useful. First we state the result.

PROPOSITION 2. *When piracy is tolerated,* $\frac{\partial x^*}{\partial c} > 0$*; and when piracy is deterred,* $\frac{\partial x^*}{\partial c} < 0$.

Proof. The proof is straightforward and follows from the respective expressions of x^*. ∎

We find that the optimal private anti-piracy measure (x^*) decreases in the public anti-piracy measure (c) when the consumers' tastes are not sufficiently diverse ($\theta \leq \frac{1}{q(1-q)}$), until it reaches zero. On the contrary, when the consumers' tastes are sufficiently diverse ($\theta > \frac{1}{q(1-q)}$), the overall relationship between x^* and c is non-monotonic: x^* first increases in c, then decreases until it reaches zero. Therefore, the relationship between the optimal deterrence level from the product developer and the degree of IPR protection in the economy can be monotonic or non-monotonic depending on the degree of the diversification of the consumers' tastes.

Economic Intuition: As the original producer increases the deterrence level x, it has two effects on its revenue: the direct effect and the indirect effect. The direct effect is how increasing x affects the original producer's demand (and thus its revenue), and the indirect effect is how increasing x affects the choice of price and thus its demand and revenue.

When the end-user piracy is tolerated, the original producer's choice of price is an interior solution for any given x. By envelope theorem, the indirect effect of increasing x on the original producer's revenue through the choice of price is zero. So we need to consider only the direct effect, which is equal to $p(x)\frac{\partial D_0}{\partial x} = p(x)\frac{1}{(1-q)\theta}$. It is clearly increasing in c since a higher c raises the piracy cost and the original producer charges a higher price (see equation (1)). Indeed, from equation (3), we know that $MR(x) = \frac{(1-q)\theta+c+x}{2(1-q)\theta}$ is increasing in c.[10] This explains why the optimal private effort is increasing in the public effort when the end-user piracy is tolerated.

On the contrary, when the end-user piracy is deterred, the original producer's choice of price is a boundary solution for any given x. Now the direct effect of increasing x on the original producer's revenue is zero ($D_0 = \frac{\theta-p}{\theta}$ is independent of x), so we need to consider only the indirect effect of increasing x on the original producer's revenue through the choice of price, which is equal to $MR(p(x))\frac{dp(x)}{dx} = MR(p(x))\frac{1}{q}$. It is clearly decreasing in c since a higher c raises the piracy cost and the original producer charges a higher price at the boundary solution (see equation (1)) and the marginal revenue of increasing price $MR(p(x))$ at the boundary solution is lower. Indeed, from (3), we know that $MR(x) = \frac{q\theta-2(c+x)}{q^2\theta}$ is decreasing in c. This explains why the optimal private effort is decreasing in the public effort when the end-user piracy is deterred.

We also believe this is an important result from the policy perspective. If we think that stopping or limiting piracy is a joint responsibility of government/public institutions (i.e. IPR laws and enforcements) and private organizations (like the innovative firm or the copyright holder), then the additive piracy deterring cost structure $(c+x)$ that we have assumed here is rather appropriate. Now given this additive structure, from the outset it is natural to expect that these two efforts are substitutes in stopping or limiting piracy. However, we show that it may not be the case always.

[10]The first term on the right-hand side of (3) is the marginal revenue function of increasing x.

Thus, to suggest a policy which aims to reduce piracy, the policy makers must take a note of this particular fact explicitly. The nature of interaction between the public and private measure under tolerance and deterrence are different. It would be an important result to verify empirically as well.

Above can be empirically tested with the following suitable hypothesis: Test the relationship between the product developer's share of R&D expenditure to deter piracy and the strength of the copyright protection law (generally proxied by IPR protection index) under piracy tolerance and deterrence situations.

4.2. The Private Optimal Deterrence Level and the Quality of the Pirated Product

In the presence of piracy, the original producer is always concerned about the quality of the pirated product as it affects its market share, revenue and profit. Therefore it is important for the original producer to respond optimally in terms of its deterrence level for a varying degree of quality levels of the pirated product. The following comparative static analysis is done for that purpose. We have the following finding.

PROPOSITION 3.

(i) When piracy is tolerated, $\frac{\partial x^*}{\partial q} > 0$*;*

(ii) When piracy is deterred, $\frac{\partial x^*}{\partial q} > 0$ *if* $q < \min\left\{\frac{2c+\sqrt{2\theta+4c^2}}{\theta}, 1\right\}$*;* $\frac{\partial x^*}{\partial q} < 0$ *if* $\frac{2c+\sqrt{2\theta+4c^2}}{\theta} < q < 1$ *(this implicitly requires* $\theta > 2 + 4c$*).*

Proof. The proof is straightforward and follows from the respective expressions of x^*. ▮

Generally speaking, when the quality of the pirated good increases it becomes a greater threat to the original product developer. Hence it wants to raise its deterrence effort to successfully fight the pirates to maintain its profit. This is true except when the consumers' tastes are much diversified (high θ), i.e. when the market is diversified enough to accommodate both the original developer and end-user pirates.

To further explain this relationship closely, we turn to the original producer's marginal revenue function of increasing x (the first term on the right-hand side of (3)). Under tolerance, the marginal revenue of increasing x is always increasing in q. Under deterrence, the marginal revenue of increasing x is increasing in q when $c + x > \frac{q\theta}{4}$, (i.e. θ is relatively low or medium) and decreasing otherwise. We are going to provide illustrative examples below to see this clearly.

Examples: In the first example, we fix $\theta = 5$ and $c = 0.1 (\theta > 2 + 4c)$. Then it is straightforward to obtain the following optimal deterrence effort as a function of q:

$$x^* = \begin{cases} 0 & \text{if } 0 < q \leq 0.04, \\ (5q-0.2)/(5q^2+2) & \text{if } 0.04 \leq q \leq 0.36823 \text{ or } 0.65177 \leq q < 1, \\ (5.1-5q)/(9-10q) & \text{if } 0.36823 \leq q \leq 0.65177. \end{cases}$$

In this example, note that when the quality of the pirated product is intermediate (medium, i.e. the third line in the above equation), the original producer tolerates piracy. As a result, under tolerance,

$$\frac{dx(q)}{dq} = \frac{d}{dq}\left(\frac{5.1-5q}{9-10q}\right) = \frac{6}{(9-10q)^2} > 0.$$

Under complete deterrence,

$$\begin{aligned} \frac{dx(q)}{dq} &= \frac{d}{dq}\left(\frac{5q-0.2}{5q^2+2}\right) \\ &= \frac{-25q^2+2q+10}{(5q^2+2)^2} \begin{cases} < 0 & \text{for } 0.67372 < q < 1, \\ > 0 & \text{for } 0.04 \leq q \leq 0.36823 \text{ or } 0.65177 \leq q < 0.67372. \end{cases} \end{aligned}$$

We see a non-monotonic relationship between the optimal deterrence effort and the quality of the pirated product: x^* is zero when $q \leq 0.04$; as q increases further, x^* increases until q reaches 0.67372 and then decreases.

In the second example, we fix $\theta = 5$ and $c = 1 (\theta < 2 + 4c)$. Then we can get the following optimal deterrence effort as a function of q:

$$x^* = \begin{cases} 0 & \text{if } 0 < q \leq 0.4, \\ (5q-2)/(5q^2+2) & \text{if } 0.4 \leq q < 1. \end{cases}$$

In this example, tolerance is never optimal for the original producer. IPR protection is high enough to deter piracy completely when q is small, while strategic complete deterrence is required from the copyright holder for larger q. And under complete deterrence, $\frac{dx(q)}{dq} = \frac{d}{dq}\left(\frac{5q-2}{5q^2+2}\right) = \frac{-25q^2+20q+10}{(5q^2+2)^2} > 0$ for $0.4 \leq q < 1$. We see a monotonic relationship between the optimal deterrence effort and the quality of the pirated product: x^* is zero when $q \leq 0.4$; as q increases further, x^* increases.

In the second example, when θ is relatively small ($\theta < 2+4c$), the optimal deterrence effort is increasing in the quality of the pirated product. On the contrary, in the first example, when θ is relatively big ($\theta > 2 + 4c$), the optimal deterrence level is increasing in the quality under tolerance; under deterrence, it is also increasing in the quality when quality is relatively low or medium and only decreasing when the quality is relatively high. Hence, we get the non-monotonicity.

4.3. The Rate of Piracy

We define the ratio of $D_p/(D_0 + D_p)$ to measure the rate of piracy. The piracy rate is an important metric often used to know the extent of piracy in a country. Piracy watch-dog organizations, like BSA (Business Software Alliances), estimate this metric every year for various countries.

4.3.1. The Rate of Piracy and the Public Anti-piracy Measure

In the following comparative static analysis, we want to find out how the piracy rate gets affected with the change in the IPR policy.

When and $\theta > \frac{1}{q(1-q)}$ and $c \leq \delta(q, \theta)$ (equivalently, $\frac{\theta+c-\sqrt{\theta^2+c^2-6\theta c-4\theta}}{2\theta} \leq q \leq \frac{\theta+c+\sqrt{\theta^2+c^2-6\theta c-4\theta}}{2\theta}$), i.e. when the original firm tolerates piracy, it is straightforward to get $\frac{D_p}{D_0+D_p} = \frac{q(1-q)\theta-(2-q)c-1}{2q(1-q)\theta-2(1-q)c-1}$.

In all the other cases, entry is either deterred completely or blockaded; thus, the rate of piracy is zero.

Simple computation yields

$$\frac{\partial}{\partial c}\left(\frac{D_p}{D_0 + D_p}\right) = \frac{q(1-2(1-q)\theta)}{(2q(1-q)\theta - 2(1-q)c - 1)^2} < 0 \qquad (5)$$

Thus, we have the following proposition.

PROPOSITION 4. *The rate of piracy decreases in the public anti-piracy measure.*

This result is very intuitive. As the public anti-piracy measure increases, piracy becomes more costly to end-users. Furthermore, as Proposition 2 says, under tolerance, the increase of the public anti-piracy measure induces a higher optimal deterrence level, which further increases the cost of piracy. Thus, the rate of piracy decreases.[11]

5. CONCLUDING DISCUSSIONS

In this paper, we study the problem of digital piracy when the copyright holder faces numerous end-user pirates for its product. The original product developer makes costly investment to raise the overall cost of the piracy (hence reduce the value of the pirated product) to the end-user pirates in

[11] Vasquez and Watt (2011) studied the impact of the anti-piracy public and private measure separately in controlling piracy and interestingly found that the public anti-piracy measure may actually increase piracy instead of reducing, but the private measure of anti-piracy always decreases piracy. However, they do not consider two policies together which will typically impact each other as we do here.

a given regime of intellectual property rights (IPR) protection which also aims to limit piracy.

We find that when the consumers' tastes are sufficiently diverse and IPR protection is weak, only then it is profitable for the copyright holder to tolerate piracy, and that the copyright holder completely deters piracy in all the other situations unless piracy is blockaded by strong IPR protection. We also find the relationship between the optimal deterrence level from the product developer and the degree of IPR protection in the economy can be monotonic or non-monotonic depending on whether piracy is tolerated or deterred.

The original good is assumed to have higher quality and the pirated good is of lower quality. But since the intrinsic quality of the pirated good can vary widely in reality, we want to see how the quality of the pirated good impacts the choice of deterrence level for the copyright holder by doing comparative static analysis. We find the optimal deterrence level chosen by the copyright holder increases as the quality of the pirated good increases when piracy is tolerated. It also increases with the quality of the pirated good under piracy deterrence except when the consumers' tastes are sufficiently diverse and the quality of the pirated good is sufficiently high. The piracy rate goes down unambiguously as the public anti-piracy measure increases.

Our model has limitations. We do not endogenize the choice of IPR protection policy, i.e. c is not chosen endogenously. An interaction between IPR protection policy i.e. the public effort (c) and the private effort (x) could be more explicitly captured if c is endogenized, say, by the government. However, in this model, if we just allow the government to choose c with an objective to maximize the overall welfare of the society then the optimal value of c goes to zero, a conclusion which is neither interesting nor realistic. We need to have a dynamic model where future innovation is valued to get an optimal c which is positive and meaningful.

We do not consider the case, where both commercial and end-users piracy co-exist in the same market. We believe that the model can be extended in this direction. One can build a unified framework where both types of piracy exist simultaneously, and where the government or IPR protection authorities play a more pro-active role in controlling piracy by monitoring and penalizing the pirate(s) and thus IPR protection policy is endogenized.

On another dimension, it would be also interesting to see as a country grows from a lower income to a higher income nation, how the composition of the piracy (between commercial and end-user) in the society and the piracy rate change endogenously as a response to IPR and private protection policies. We want to pursue all these in our future research.

APPENDIX

Problem I

Define Lagrangian $L_1(p,\lambda) = p\left(\frac{(1-q)\theta-p+(c+x)}{(1-q)\theta}\right) - \frac{1}{2}x^2 - \lambda(c+x-qp)$. Note that $p\left(\frac{(1-q)\theta-p+(c+x)}{(1-q)\theta}\right)$ is concave in p and $c+x-qp$ is convex in p (linearity is a special case of convexity). Thus the Kuhn-Tucker necessary conditions are sufficient. The sufficient and necessary conditions for the optimum are the following:

$$\frac{\partial L_1(p,x,\lambda)}{\partial p} = \frac{(1-q)\theta-2p+(c+x)}{(1-q)\theta} + \lambda q = 0, \tag{A.1}$$

$$\lambda(c+x-qp) = 0, \quad \lambda \geq 0, \quad qp \geq c+x. \tag{A.2}$$

If $\lambda = 0$, then we can solve for p from (A1) after plugging $\lambda = 0$ and get $p = \frac{(1-q)\theta+c+x}{2}$. We also need to check whether $qp \geq c+x$ is satisfied and we find that this condition is satisfied when $c \leq \frac{q(1-q)\theta}{2-q} - x$. In this case, the developer's profit is $\pi = \frac{((1-q)\theta+c+x)^2}{4(1-q)\theta} - \frac{1}{2}x^2$.

If instead $qp = c+x$, then $p = \frac{c+x}{q}$. We also need to check whether $\lambda \geq 0$ is satisfied and we find that this condition is satisfied when $c \geq \frac{q(1-q)\theta}{2-q} - x$. In this case, the developer's profit is $\pi = \frac{(c+x)(q\theta-(c+x))}{q^2\theta} - \frac{1}{2}x^2$.

Problem II

Define Lagrangian $L_2(p,\kappa) = p\left(\frac{\theta-p}{\theta}\right) - \frac{1}{2}x^2 - \kappa(qp-c-x)$. Note that $p\left(\frac{\theta-p}{\theta}\right)$ is concave in p and $qp-c-x$ is convex in p (linearity is a special case of convexity). Thus the Kuhn-Tucker necessary conditions are sufficient. The sufficient and necessary conditions for the optimum are the following:

$$\frac{\partial L_2(p,x,\kappa)}{\partial p} = \frac{\theta-2p}{\theta} - \kappa q = 0, \tag{A.3}$$

$$\kappa(qp-c-x) = 0, \quad \kappa \geq 0, \quad qp \leq c+x. \tag{A.4}$$

If $\kappa = 0$, then we can solve for p from (A.3) after plugging $\kappa = 0$ and get $p = \frac{\theta}{2}$. We also need to check whether $qp \leq c+x$ is satisfied and we find that this condition is satisfied when $x \leq \frac{q\theta}{2} - c$. In this case, the developer's profit is $\pi = \frac{\theta}{4} - \frac{1}{2}x^2$.

If instead $qp = c+x$, then $p = \frac{c+x}{q}$. We also need to check whether $\kappa \geq 0$ is satisfied and we find that this condition is satisfied when $x \geq \frac{q\theta}{2} - c$. In this case, the developer's profit is $\pi = \frac{(c+x)(q\theta-(c+x))}{q^2\theta} - \frac{1}{2}x^2$.

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