Sectoral Heterogeneity and the "Dual" Structural Change in China*

Shan Pan and Liutang Gong†

This paper develops a dynamic growth model with two industries and three sectors. This model characterizes the "dual" economic transition from the agriculture to the non-agriculture sectors and from the state to the nonstate sectors during the past thirty years in China. This model also proposes sectoral heterogeneity as the source of dual structural change, which reflects in factor income share between the agriculture and non-agriculture sectors while reflecting in technological progress between the state and nonstate sectors. In addition, there is nonbalanced sectoral growth in the Balanced Growth Path (BGP). Finally, we calibrate our model using data from 1978 to 2011 in China. Numerical simulation verifies the fitness of the model to the real data.

Key Words: "Dual" structural change; Sectoral heterogeneity; Elasticity of substitution; Nonbalanced growth.

1. INTRODUCTION

Economic growth is an important topic in macroeconomics. The economic growth theory is developed from exogenous growth to endogenous growth and then to nonbalanced growth. On the one hand, most models of economic growth are consistent with the "Kaldor facts" (Kaldor 1961), that is, the relative constancy of the growth rate, the capital-output ratio, the capital income share in output and the real interest rate in the long term. On the other hand, striking structural changes accompany the growth process, that is, the systematical reallocation of output and employment across different sectors, which leads to sectoral nonbalanced growth, such as the "Kuznets facts" (Kuznets 1973).

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† Pan: Corresponding author. Institute of Industrial Economics, Jinan University, China. Email: panshan@pku.edu.cn; Gong: Guanghua School of Management, Peking University, China. Email: ltgong@gsm.pku.edu.cn.

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China has achieved remarkable growth performance over last three decades of reform. Along with the high speed of growth, structural change has emerged. Between 1978 and 2011, employment and real output have presented significant reallocation trends from the agriculture to the non-agriculture sectors, and from the state non-agriculture to the nonstate non-agriculture sectors[1]. That is, China has experienced a "dual" structural change.

The transition from the agriculture to the non-agriculture sectors has received much attention in the literature. Some studies focus on the impact of structural change on economic growth; Young (2003) suggests that the TFP growth in China is modest, while the transfer of labor out of agriculture is the key driving force behind the impressive growth. Using micro data on Chinese manufacturing establishments, Hsieh and Klenow (2009) point out that resource misallocation can lower aggregate TFP and that China may have boosted its TFP by 2% per year by winnowing its resource distortion. Hayashi and Prescott (2008) argue that the release of labor into the non-agriculture sector played an important role in post-World War II growth acceleration in Japan, and the depressing factor of the prewar Japanese economy was attributed to the barrier that kept agriculture employment constant.

Others attempt to explore the sources of structural change. Kongsamut et al. (2001), Matsuyama (1992, 2002), Echevarria (1997), and Caselli and Coleman (2001) start from the demand side and argue that sectoral reallocation results from different income elasticity of demand. Based on Baumol (1967), Nagi and Pissarides (2007) highlight the difference in the elasticity of substitution across final goods and TFP, a low elasticity of substitution leading to shifts in employment shares to sectors with low TFP growth. Meanwhile, Acemoglu and Guerrieri (2008) emphasize the role of factor proportions, with capital deepening increasing the relative output of the more capital-intensive sector but simultaneously inducing a reallocation factor away from that sector. Zou and Liu (2010) propose a dynamic economic transition model with endogenous supply of skilled labor, the constraint of skilled labor affect both the economys turning point from traditional to modern growth and the subsequent growth path. Wang and Xie (2018) also emphasize the role of heterogeneity, they introduces sectorial heterogeneity in TFPs in a growth model to generate new insights on trade, sectorial reallocation and economic growth.

Hence, most studies of structural change simply concentrate on the process of transition from the agriculture to the non-agriculture sectors, while ignoring the reallocation facts from the state to the nonstate sectors, with the exception of two papers: Brandt and Zhu (2010) and Song et al. (2011). Brandt and Zhu creatively break down the non-agriculture sector into state

[1] For the remainder of the paper, we simplify these as state sector and non-state sector.

and nonstate components and quantify the source of China's growth based on a three-sector model. These authors find that the rising TFP in the nonstate sector and the labor reallocation from the state to the nonstate sectors are the key drivers of growth. Song et al. (2011) constructs a two-sector model containing state-owned firms and private firms, where firms are heterogeneous in productivity and access to financial markets. Their finding is that financial frictions and reallocations of resources across firms are the focal points of economic transition in China.

Although Brandt and Zhu (2010) distinguish between the state and nonstate sectors within the non-agriculture sector and mention the "dual" reallocation process, their model cannot intuitively account for the dynamic transitions of employment and output across sectors. Song et al. (2011), however, simply focus their study on the non-agricultural sector. Thus, both studies fail to clearly describe the "dual" structural change from the agriculture to the non-agriculture sectors and from the state to the nonstate sectors in China. Since these two transition processes coexist and are correlated to each other, we try to discuss China's distinctive "dual" structural change in a unified framework using a three-sector dynamic model that includes the agriculture sector, the state sector and the nonstate sector.

We show that the source of China's "dual" structural change is the elasticity of substitution and sectoral heterogeneity. Although our main idea incorporates points presented in Acemoglu and Guerrieri (2008) and Ngai and Pissarides (2007), we expand on those findings. (1) We extend the two-sector model in Acemoglu and Guerrieri (2008) to a two-industry, three-sector model and introduce double-CES functions, which allow us to simultaneously analyze the resource reallocation from the agriculture to the non-agriculture sector and from the state to the nonstate sector. (2) Following Ngai and Pissarides (2007), we adopt the assumption of the separate elasticity of substitutions across sectors. We then not only consider the heterogeneous factor in sectoral TFP growth rate, which is emphasized in Ngai and Pissarides (2007), but also allow for the difference in factor income share between the agriculture and non-agriculture sectors. (3) Our results incorporate their main findings, that is, both factor proportions and TFP matter.

Based on Chinese macro data from 1978 to 2011, the elasticity of substitution between the agriculture and non-agriculture sectors and between the state and nonstate sectors are both larger than 1; thus, factors will be reallocated toward the more rapidly growing sector. The heterogeneity between the agriculture and non-agriculture sectors reflects in factor income share and sectoral technological progress, wherein the agriculture sector has a higher TFP growth but a lower capital income share than the non-agriculture sector. These two heterogeneities have a contradictory influence on sectoral growth, but since the effect of factor income share is dominant, labor in China will flow from the agriculture to the non-agriculture sectors. Whereas the heterogeneity within the non-agriculture sector reflects sectoral TFP growth, the TFP growth in the nonstate sector is much larger than that in the state sector, which induces labor reallocation from the state to the nonstate sectors. With the movements of labor across sectors, sectoral real output also shows similar reallocation patterns. Therefore, both labor and output in China present "dual" structural change. In addition, factors will eventually flow into the nonstate sector, and there is nonbalanced sectoral growth in the Balanced Growth Path (BGP) while still remaining consistent with "Kaldor" facts.

The rest of the paper is organized as follows. Section 2 describes the facts of "dual" structural change in China. Section 3 constructs a three-sector dynamic growth model and characterizes the dual sectoral reallocation process and the Balanced Growth Path. Section 4 calibrates the model and provides simulation results. Sector 5 is the conclusion. The appendix contains the proof of some primary results

2. THE "DUAL" STRUCTURAL CHANGE IN CHINA

Li Yining (1997, 2013) has proposed that the Chinese economy in "dual" transition is the greatest background event in China; this "dual" transition is the transition from a traditional agricultural economy to a modern non-agricultural economy, which is represented by industrialization, and the transition from a planned economy to a market-oriented economy, which is represented by marketization. In such a "dual" development transition and institutional transition process, China has experienced rapid growth and a "dual" structural change, from the agriculture to the non-agriculture sectors and from the state to the nonstate sectors[2].

Figure 1 shows the transition trends of employment in China. Between 1978 and 2011, the employment share of the agriculture sector declined from 71% to 35%, and the state sector's share of non-agriculture employment declined from 69% to 15%. Therefore, there are significant "dual" labor reallocation trends from the agriculture to the non-agriculture sectors and from the state to the nonstate sectors.

Figures 2 and 3 present the transition trends of real output ratio in China. As shown above, there has been a sharp decline in the real output ratio of the agriculture sector relative to the non-agriculture sector and of the state sector relative to the nonstate sector. Therefore, the real output has also experienced similar transition trends with employment, that is,

[2]State sector includes state-owned units and urban collective units; data source: the National Bureau of Statistics in China (NBS) and the China Compendium of Statistics.

FIG. 1. Sectoral employment share change in China

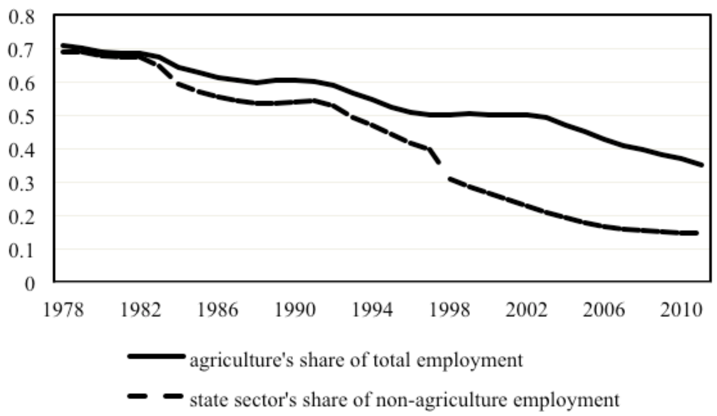
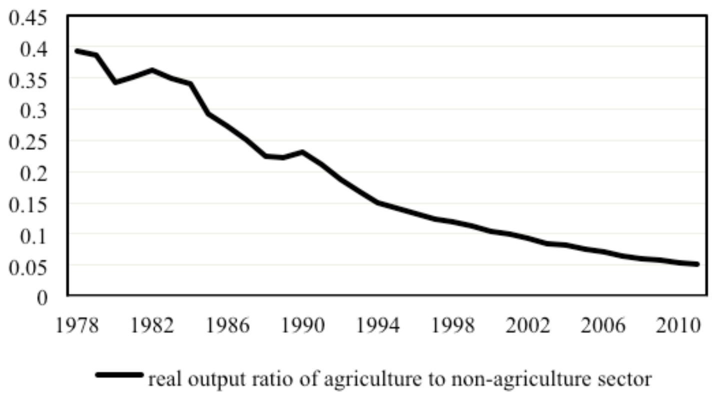


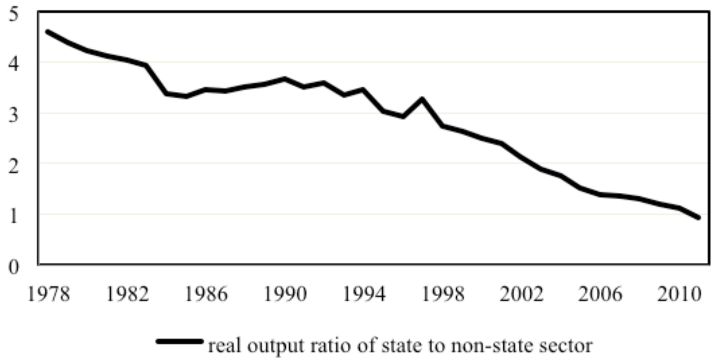
FIG. 2. The trends of real output ratio of agriculture to non-agriculture in China



from the agriculture to the non-agriculture sectors and from the state to the nonstate sectors.

This “dual” transition in China is unprecedented. After World War II, those newly independent developing countries, which had never before adopted a planned economic system, only experienced developmental transition from a traditional agricultural society to an industrial society. However, China faced a different situation; on one hand, to remove a planned system and replace it with a market-oriented economy, and on the other hand, to transition from an agricultural society to a modern industrial society. As seen from the figure above, the reallocation trends of employment

FIG. 3. The trends of real output ratio of state to nonstate in China



and output indicate that China has been gradually realizing its target of "dual" structural change.

3. THE MODEL

3.1. Setup of the Model

The representative household has standard constant relative risk aversion (CRRA) preferences

$$\int_0^\infty \exp[-(\rho - n)t]\frac{c(t)^{1-\theta} - 1}{1-\theta}dt \tag{1}$$

where $c(t)$ is consumption of the representative household of time t, $\rho > 0$ is the rate of time preferences and $\theta > 0$ is the inverse of the intertemporal elasticity of substitution.

There is a final goods sector Y, which comprises two intermediate goods sectors, the agriculture sector Y_a and the non-agriculture sector Y_{na}. The non-agriculture sector Y_{na} comprises two subsectors, the state sector Y_s and the nonstate sector Y_{ns}. Both variables take the constant elasticity of substitution (CES) form:

$$Y(t) = \left[\gamma Y_a(t)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)Y_{na}(t)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2}$$

$$Y_{na}(t) = \left[\varphi Y_s(t)^{\frac{\eta-1}{\eta}} + (1-\varphi)Y_{ns}(t)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{3}$$

where $\gamma, \varphi \in (0,1)$ denote the weight of the agriculture sector in the final output and the weight of the state sector in the non-agriculture sector, respectively. The final output can be seen as the double CES functions of intermediate sectors Y_a, Y_s and Y_{ns}. The elasticity of substitution is ε between the agriculture and non-agriculture sectors and η between the state and nonstate sectors. If the elasticity of substitution is larger than 1, these two sectors are substitutable. If the elasticity of substitution is smaller than 1, they are complementary. If the elasticity of substitution is equal to 1, then the production function reduces to Cobb-Douglas form. Such double CES forms allow inter-sectors and intra-sectors to have different elasticity of substitutions.

Three intermediate goods sectors are produced with Cobb-Douglas technologies using capital and labor

$$\begin{aligned} Y_a(t) &= M_a(t)L_a(t)^{\alpha}K_a(t)^{1-\alpha} \\ Y_s(t) &= M_s(t)L_s(t)^{\beta}K_s(t)^{1-\beta} \\ Y_{ns}(t) &= M_{ns}(t)L_{ns}(t)^{\beta}K_{ns}(t)^{1-\beta} \end{aligned} \tag{4}$$

where $L_i(t), K_i(t)$ are the levels of labor and capital used in each sector, $\alpha, \beta \in (0.1)$ are the labor income share of the agriculture sector and non-agriculture sector.[3] $M_i(t)$ denotes the TFP of sector i; here technological progress is exogenous and TFP growth rate is m_i

$$\frac{\dot{M}_i}{M_i} = m_i \quad i \in \{a, s, ns\} \tag{5}$$

Based on Chinese data, we make the following assumptions:

Assumption 1. $m_s < m_{ns} < m_a$, $\alpha > \beta$.

According to growth accounting for China in Brandt and Zhu (2010), TFP growth of the agriculture sector is the highest, while that in the state sector is the lowest. Since the agriculture sector is labor-intensive and the non-agriculture sector is capital-intensive, the labor income share of the agriculture sector is larger than that of the non-agriculture sector.

Labor is supplied inelastically and is equal to population $L(t)$ at each time t, which grows at the exponential rate n, so that

$$L(t) = \exp(nt)L(0). \tag{6}$$

Capital and labor markets are all competitive and market clearing requires

$$L_a(t) + L_s(t) + L_{ns}(t) = L(t) \tag{7}$$

$$K_a(t) + K_s(t) + K_{ns}(t) = K(t) \tag{8}$$

[3]For simplicity, we do not allow for the difference in factor income share between state and non-state sectors.

where $L(t)$ and $K(t)$ denote the aggregate labor and capital at each time.

3.2. The Optimal Allocation

Since all markets are complete and competitive, according to the second welfare theorem, the competitive equilibrium is equivalent to the social planner's problem, which is solved by maximizing the utility of the representative household.

$$\max_{\{L_i(t),K_i(t),K(t),c(t)\}} \int_0^\infty \exp[-(\rho-n)t]\frac{c(t)^{1-\theta}-1}{1-\theta}dt \tag{9}$$

subject to the resource constraint

$$\dot{K}(t)+\delta K(t)+c(t)L(t)=Y(t) \tag{10}$$

together with (2)-(3) and initial conditions $L(0)>0, K(0)>0, M_i(0)>0,\ i\in\{a,s,ns\}$.

The solution to the problem above can be broken down into two steps. First, given $K(t), L(t)$ and $M_i(t)$, choose the allocation of factors across sectors $K_i(t)$ and $L_i(t),\ i\in\{a,s,ns\}$ to maximize final output $Y(t)$. Second, given this choice of factor allocations at each date, choose the optimal $K(t)$ and $c(t)$ to maximize the value of the objective function. These two steps correspond to the characterization of the static and dynamic optimal allocations.

We first characterize the static equilibrium problem and then turn to dynamic equilibriums.

3.3. The static equilibrium allocation

Here, we choose the optimal sectoral factor allocations to maximize the final output and subject to market clearing conditions. Given capital stock $K(t)$ at time t, we define the maximized value of the final output as

$$\Phi(K(t),t)=\max_{\{L_i(t),K_i(t)\}} Y(t)=F[Y_a(t),Y_s(t),Y_{ns}(t)]. \tag{11}$$

Since factors can be freely mobile across sectors, it is easy to verify that the first order condition is the equalization of the marginal products of labor and capital in each sector, which equal factor prices $w(t)$ and $R(t)$, respectively.

$$\begin{aligned}
&\gamma\alpha\left[\frac{Y(t)}{Y_a(t)}\right]^{1/\varepsilon}\frac{Y_a(t)}{L_a(t)}\\
=&\ (1-\gamma)\varphi\beta\left[\frac{Y(t)}{Y_{na}(t)}\right]^{1/\varepsilon}\left[\frac{Y_{na}(t)}{Y_s(t)}\right]^{1/\eta}\left[\frac{Y_s(t)}{L_s(t)}\right]\\
=&\ (1-\gamma)(1-\varphi)\beta\left[\frac{Y(t)}{Y_{na}(t)}\right]^{1/\varepsilon}\left[\frac{Y_{na}(t)}{Y_{ns}(t)}\right]^{1/\eta}\left[\frac{Y_{ns}(t)}{L_{ns}(t)}\right] \qquad (12)
\end{aligned}$$

$$\begin{aligned}
&\gamma(1-\alpha)\left[\frac{Y(t)}{Y_a(t)}\right]^{1/\varepsilon}\frac{Y_a(t)}{K_a(t)}\\
=&\ (1-\gamma)\varphi(1-\beta)\left[\frac{Y(t)}{Y_{na}(t)}\right]^{1/\varepsilon}\left[\frac{Y_{na}(t)}{Y_s(t)}\right]^{1/\eta}\left[\frac{Y_s(t)}{K_s(t)}\right]\\
=&\ (1-\gamma)(1-\varphi)(1-\beta)\left[\frac{Y(t)}{Y_{na}(t)}\right]^{1/\varepsilon}\left[\frac{Y_{na}(t)}{Y_{ns}(t)}\right]^{1/\eta}\left[\frac{Y_{ns}(t)}{K_{ns}(t)}\right] \qquad (13)
\end{aligned}$$

Since the key static decision involves the allocation of labor and capital across sectors, we define the following sectoral shares of capital and labor as

$$\begin{aligned}
\frac{K_a(t)}{K(t)} &= \kappa_1(t),\ \ \frac{K_s(t)}{K_s(t)+K_{ns}(t)}=\kappa_2(t)\\
\frac{L_a(t)}{L(t)} &= \lambda_1(t),\ \ \frac{L_s(t)}{L_s(t)+L_{ns}(t)}=\lambda_2(t)
\end{aligned}$$

That is, the agriculture sector's shares of the total capital and labor are κ_1, λ_1, respectively, and the state sector's shares of the non-agriculture sector's capital and labor are κ_2, λ_2 respectively. Obviously, the state sector's shares of total capital and labor are

$$\frac{K_s(t)}{K(t)}=\kappa_2(t)[1-\kappa_1(t)],\quad \frac{L_s(t)}{L(t)}=\lambda_2(t)[1-\lambda_1(t)]$$

Thus, (12) and (13) can be simplified as

$$\frac{\lambda_2(t)}{1-\lambda_2(t)}=\frac{\varphi}{1-\varphi}\left[\frac{Y_s(t)}{Y_{ns}(t)}\right]^{(\eta-1)/\eta} \qquad (14)$$

$$\frac{\lambda_1(t)}{(1-\lambda_1(t))(1-\lambda_2(t))}=\frac{\gamma\alpha}{(1-\gamma)(1-\varphi)\beta}\left[\frac{Y_a(t)}{Y_{na}(t)}\right]^{(\varepsilon-1)/\varepsilon}\left[\frac{Y_{na}(t)}{Y_{ns}(t)}\right]^{(\eta-1)/\eta} \qquad (15)$$

$$\kappa_2(t) = \lambda_2(t) \tag{16}$$

$$\kappa_1(t) = \left\{1 + \frac{\alpha(1-\beta)}{\beta(1-\alpha)}\left[\frac{1-\lambda_1(t)}{\lambda_1(t)}\right]\right\}^{-1} \tag{17}$$

From (16) and (17), at each time t, the state sector's share of capital $\kappa_2(t)$ equals its labor share $\lambda_2(t)$, the agriculture sector's share of capital $\kappa_1(t)$ is strictly increasing in $\lambda_1(t)$, $\lambda_1(t) > \kappa_1(t)$. Since capital share is correlated to labor share, next we then focus on how these two shares $\lambda_1(t)$ and $\lambda_2(t)$ change over time.

Proposition 1. *The dynamic functions of sectoral labor share are*

$$\frac{\dot{\lambda}_2(t)}{\lambda_2(t)(1-\lambda_2(t))} = (m_s - m_{ns})(\eta - 1) \tag{18}$$

$$\frac{\dot{\lambda}_1(t)}{\lambda_1(t)(1-\lambda_1(t))} = \frac{[m_a - \lambda_2(t)m_s - (1-\lambda_2(t))m_{ns}] - (\alpha-\beta)[\dot{k}(t)/k(t)]}{(\varepsilon-1)^{-1} + (\alpha-\beta)(\lambda_1(t) - \kappa_1(t))} \tag{19}$$

where $k(t) = \frac{K(t)}{L(t)}$ *is the capital stock per capita.*

Proposition 1 states that the allocation of labor between the state sector and the nonstate sector is simply determined by the TFP growth and the elasticity of substitution, while between the agriculture sector and the non-agriculture sector, it also involves the factor proportion and capital deepening. However, proposition 1 does not predict the labor transition directions across sectors. Next, we make some additional parameter assumptions.

Assumption 2. $\eta > 1$, $\varepsilon > 1$.[4]

Chinese data shows that the agriculture sector grows more slowly than the non-agriculture sector, and the state sector grows more slowly within the non-agriculture sector. In addition, labor flows from the agriculture sector to the non-agriculture sector and from the state sector to the nonstate sector, which indicates that the elasticity of substitution of inter-sectors and intra-sectors are both larger than 1; that is, factors will reallocate toward the sectors growing much faster. Hence, assumption 2 is consistent with Chinese data, which will be further verified in the calibration section.

Proposition 2. *Under assumptions 1 and 2, the state sector's share of labor* $\lambda_2(t)$ *will strictly decline.*

[4]It will be verified that the elasticity of substitution across Chinese sectors is larger than 1 in the following calibration section.

Under assumptions 1 and 2, equation implies $\dot{\lambda}_2(t) < 0$, which means there is a continuous transition of labor from the state to the nonstate sectors. Intuitively, the state and nonstate sectors are substitutable, so factors will reallocate toward the sector that grows faster. Moreover, the nonstate sector enjoys higher technological progress, inducing faster sectoral growth, which leads to a larger labor fraction.

PROPOSITION 3. *Under assumptions 1 and 2, with regard to the agriculture and non-agriculture sectors, if the difference in TFP growth* $[m_a - \lambda_2(t)m_s - (1-\lambda_2(t))m_{ns}]$ *is smaller than the effect of factor propositions and capital deepening* $(\alpha-\beta)[\dot{k}(t)/k(t)]$*, then the agriculture sector's share of labor* $\dot{\lambda}_1(t)$ *will decrease; otherwise,* $\lambda_1(t)$ *will increase.*

From equation (19), $\dot{\lambda}_1(t)$ is determined by the sign of the numerator to the right of the equation. When the difference in technological progress dominates, labor will flow into the agriculture sector, and labor will reallocate toward the non-agriculture sector when factor income share and capital deepening dominate.

In China, the agriculture sector has the highest TFP growth and the smallest capital income share, which will lead to lower growth in the process of capital deepening. The data show that the agriculture sector grows the least among all three sectors, which implies that it is the factor proportion that dominates the growth of the agriculture sector. When the elasticity of substitution between the agriculture and non-agriculture sectors is larger than 1, a smaller factor income share causes lower growth in the agriculture sector, inducing a reallocation of labor away from this sector.

Therefore, when the elasticity of substitutions across sectors is larger than 1, it is factor income share that determines labor reallocation between the agriculture and non-agriculture sectors, and it is technological progress between the state and nonstate sectors.

Combining (14) and (15), we obtain

$$\frac{Y_s(t)}{Y_{ns}(t)} = \left[\frac{(1-\varphi)}{\varphi}\frac{\lambda_2(t)}{(1-\lambda_2(t))}\right]^{\eta/(\eta-1)} \tag{20}$$

$$\frac{Y_a(t)}{Y_{na}(t)} = \left[\frac{\beta(1-\gamma)}{\alpha\gamma}\frac{\lambda_1(t)}{(1-\lambda_1(t))}\right]^{\varepsilon/(\varepsilon-1)} \tag{21}$$

From the equations above, we obtain the following proposition:

PROPOSITION 4. *Under assumptions 1 and 2, the sectoral output ratio changes in the same direction as sectoral labor ratio, that is, the real output ratio of state sector and nonstate sector and of agriculture sector and non-agriculture sector both decrease gradually.*

Under the condition that $\varepsilon, \eta > 1$, equations (20) and (21) show that sectoral output ratio is strictly increases with sectoral labor ratio. Since labor is continuously reallocated from the agriculture sector to the non-agriculture sector and from the state sector to the nonstate sector, and capital has the same transition trend as labor, inducing real output also presents similar changing directions.

Propositions 2 and 4 describe the "dual" structural change in labor and real output, that is, the reallocation from the agriculture sector to the non-agriculture sector and from the state sector to the nonstate sector.

3.4. The dynamic optimal allocation

The previous section characterized sectoral factor allocations and the maximized value of final output $\Phi(K(t), t)$. Given $\Phi(K(t), t)$, the social planner's problem is

$$\max_{\{K(t),c(t)\}} \int_0^\infty \exp[-(\rho - n)t] \frac{c(t)^{1-\theta} - 1}{1-\theta} dt \tag{22}$$

subject to

$$\dot{K}(t) = \Phi(K(t), t) - \delta K(t) - c(t)L(t) \tag{23}$$

and the initial conditions $L(0) > 0, K(0) > 0, M_i(0) > 0,\ i \in \{a, s, ns\}$.

Note that constraint (23) is not an autonomous system; we need to de-trend variables. To determine the trend in the economy, we first make the following assumption.

Assumption 3. $\frac{m_a}{\alpha} < \frac{m_{ns}}{\beta}$.

Since the nonstate sector has a higher TFP growth than the state sector, which causes faster output growth, the nonstate sector becomes the dominant sector of the non-agriculture sector. For the agriculture and non-agriculture sectors, they differ in TFP growth and factor proportion; thus we need to consider these two heterogeneities together. Assumption 3 indicates that the nonstate sector has a higher augmented rate of technological progress, which is adjusted by factor income share, compared with the agriculture sector. Under the assumption of substitutability across sectors, the sector with the highest rate of augmented TFP growth will be the asymptotically dominant sector. Therefore, the nonstate sector becomes the dominant sector of the economy and determines the long-term growth rate.

Admittedly, assumptions 1 to 3 are rather strong. However, the mechanism here does not rely on these three assumptions. Changing the direction of assumptions, we can obtain a parallel transition process. To fit with the empirical facts in China, we make the three assumptions above, which can be viewed as a good approximation of the Chinese economy.

After determining the long-term trend, we introduce the following transformed variables

$$\tilde{c}(t) = \frac{c(t)}{M_{ns}^{1/\beta}},\ \chi(t) = \frac{K(t)}{L(t)M_{ns}^{1/\beta}}$$

where $\tilde{c}(t)$ and $\chi(t)$ represent the normalized consumption and capital per capita. Thus the solution to the social planner's problem can be expressed in terms of four differential equations in $\{\tilde{c}(t), \chi(t), \lambda_1(t), \lambda_2(t)\}$.

PROPOSITION 5. *The competitive equilibrium satisfies the following four differential equations:*

$$\begin{aligned}
\frac{\dot{\tilde{c}}(t)}{\tilde{c}(t)} &= \frac{1}{\theta}\left\{(1-\gamma)(1-\varphi)(1-\beta)P_1(t)^{1/\varepsilon}P_2(t)^{1/\eta}(1-\lambda_1(t))^{\beta}(1-\kappa_1(t))^{-\beta}\chi(t)^{-\beta} - \delta - \rho\right\} - \frac{m_{ns}}{\beta} \\
\frac{\dot{\chi}(t)}{\chi(t)} &= P_1(t)P_2(t)(1-\lambda_1(t))^{\beta}(1-\lambda_2(t))(1-\kappa_1(t))^{1-\beta}\chi(t)^{-\beta} - \delta - \frac{\tilde{c}(t)}{\chi(t)} - n - \frac{m_{ns}}{\beta} \\
\frac{\dot{\lambda}_1(t)}{\lambda_1(t)} &= \frac{(1-\lambda_1(t))\{[m_a - \lambda_2(t)m_s - (1-\lambda_2(t))m_{ns}] - (\alpha-\beta)(\dot{\chi}(t)/\chi(t) + m_{ns}/\beta)\}}{(\varepsilon-1)^{-1} + (\alpha-\beta)(\lambda_1 - \kappa_1)} \\
\frac{\dot{\lambda}_2(t)}{\lambda_2(t)} &= (1-\lambda_2(t))(m_s - m_{ns})(\eta - 1)
\end{aligned}$$

where

$$P_1(t) = \left\{\frac{(1-\gamma)[\beta\lambda_1(t) + \alpha(1-\lambda_1(t))]}{\alpha(1-\lambda_1(t))}\right\}^{\varepsilon/(\varepsilon-1)} \quad (24)$$

$$P_2(t) = \left[\frac{1-\varphi}{1-\lambda_2(t)}\right]^{\eta/(\eta-1)} \quad (25)$$

and the transversality condition

$$\lim_{t\to\infty} \exp\left\{-\left[\rho - n - \frac{(1-\theta)m_{ns}}{\beta}\right]t\right\}\chi(t) = 0 \quad (26)$$

Further, it can be proved that under assumptions 1 to 3, any solution to proposition 5 must be the solution to the social planner's problem.

3.5. The Balanced Growth Path (BGP)

This section will show that there exists a unique BGP that is the solution to the social planner's problem, where consumption, capital and output all grow at a constant rate. In addition, growth will be nonbalanced due to sectoral heterogeneity.

First, let us define

$$\frac{\dot{Y}(t)}{Y(t)} = g(t), \qquad \frac{\dot{K}(t)}{K(t)} = z \tag{27}$$

$$\frac{\dot{Y}_i(t)}{Y_i(t)} = g_i(t), \qquad \frac{\dot{K}_i(t)}{K_i(t)} = z_i, \qquad \frac{\dot{L}_i(t)}{L_i(t)} = n_i, \ i = a, s, ns \tag{28}$$

here g_i, z_i, n_i denote the growth rates of output, capital and labor, respectively, in sector i, and g, z is the growth rate of aggregate output and capital. Moreover, we denote the corresponding asymptotic growth rates by asterisks, so that $g_i^* = \lim g_i(t), z_i^* = \lim z_i(t), n_i^* = \lim n_i(t)$. Similarly, the asymptotic capital and labor allocations are denoted as

$$\lambda_i^* = \lim_{t\to\infty} \lambda_i(t), \quad \kappa_i^* = \lim_{t\to\infty} \kappa_i(t), \quad i = 1, 2$$

THEOREM 1. *Suppose assumptions 1 to 3 hold. Then, there exists a unique BGP in which*

$$\lambda_1^* = \lambda_2^* = \kappa_1^* = \kappa_2^* = 0 \tag{29}$$

$$\chi^* = \left(\frac{\theta m_{ns}}{\beta} + \delta + \rho\right)^{-1/\beta} [(1-\beta)(1-\gamma)^{\varepsilon/(\varepsilon-1)}(1-\varphi)^{\eta/(\eta-1)}]^{1/\beta} \tag{30}$$

$$\tilde{c}^* = \left[(1-\gamma)^{\varepsilon/(\varepsilon-1)}(1-\varphi)^{\eta/(\eta-1)}(\chi^*)^{-\beta} - \delta - n - \frac{m_{ns}}{\beta}\right]\chi^* \tag{31}$$

Moreover, the growth rates of output, capital and labor in three sectors are

$$g^* = g_{ns}^* = z^* = z_{ns}^* = n + \frac{m_{ns}}{\beta} \tag{32}$$

$$g_a^* = \varepsilon\left(m_a - \frac{\alpha}{\beta} m_{ns}\right) + g^* \tag{33}$$

$$g_s^* = \eta(m_s - m_{ns}) + g^* \tag{34}$$

$$z_a^* = (\varepsilon - 1)\left(m_a - \frac{\alpha}{\beta} m_{ns}\right) + z^* \tag{35}$$

$$z_s^* = (\eta - 1)(m_s - m_{ns}) + z^* \tag{36}$$

$$n_a^* = (\varepsilon - 1)\left(m_a - \frac{\alpha}{\beta} m_{ns}\right) + n \tag{37}$$

$$n_s^* = (\eta - 1)(m_s - m_{ns}) + n, \ n_{ns}^* = n \tag{38}$$

There are a number of important implications of this theorem. First, $\lambda_1^* = \lambda_2^* = \kappa_1^* = \kappa_2^* = 0$ implies that in the BGP, capital and labor will be entirely reallocated to the nonstate sector, and so the economy will be reduced to the nonstate sector. Nevertheless, at all points in time, each sector possesses positive factors and makes productions, so $\lambda_i^* = \kappa_i^* = 0$ is just the limit point of the economy.

Second, there exists nonbalanced growth, in the sense that each sector grows at different asymptotic rate. Under assumptions 1 to 3, we have $g_a^* < g_{ns}^*, g_s^* < g_{ns}^*$, that is, the growth rates of the agriculture sector and the state sector are both smaller than that of the nonstate sector, and similarly for the growth of capital and labor. The nonstate sector becomes the asymptotically dominant sector, and the growth of aggregate output, capital and labor all converge to the corresponding growth of the nonstate sector.

The intuition for this result is quite straightforward, which is mainly due to the heterogeneity across sectors. Since these sectors are highly substitutable, the fastest-growing sector will determine the asymptotic growth of the economy. Within the non-agriculture sector, the nonstate sector has a higher TFP growth, which determines the long-term growth of the non-agriculture sector. In addition, the nonstate sector has a higher augmented technological progress than the agriculture sector from assumption 3. Thus, the nonstate sector grows fastest and becomes the dominant sector of the economy. This process is continuously accompanied by the transition of capital and labor, which leads to a slower growth of the agriculture and state sectors.

Finally, equation states that aggregate growth is related to population growth, TFP growth and factor income share of the dominant sector. The higher the growth of the population, the faster the technological progress, the smaller the capital income share β, then the faster the aggregate growth. This result again links the differences in sectoral TFP growth and factor proportion to the nonbalanced growth. Moreover, it can be verified that the labor income share and the real interest rate are constant in the BGP.

$$\sigma^* = \frac{w(t)L(t)}{Y(t)} = \beta, \quad r^* = R(t) - \delta = \frac{\theta m_{ns}}{\beta} + \rho$$

From the findings above, the asymptotic labor share in national income reflects the labor share of the dominant sector. In addition, the interest rate is constant. These results are consistent with the "Kaldor" facts. Therefore, the BGP matches both the "Kaldor" facts at the aggregate level and generates nonbalanced growth at the sectoral level, that is, the economy features long-term growth as well as sectoral structural change.

Next, we establish the stability of the BGP by investigating the stability of the system in proposition 5, and we have another proposition:

PROPOSITION 6. *Suppose assumptions 1 to 3 hold. The competitive equilibrium, given by proposition 5, is saddle-path stable, in the sense that in the neighborhood of* $(\tilde{c}^*, \chi^*, \lambda_1^*, \lambda_2^*)$*, there is a unique optimal path that converges to* $(\tilde{c}^*, \chi^*, \lambda_1^*, \lambda_2^*)$*.*

The proof can be found in the appendix. This proposition states that, given the initial values of the state variables, the economy will converge to the BGP along a unique locally stable path.

4. SIMULATION

In this section, we undertake a simple simulation to investigate whether the main results generated by our model are broadly consistent with the "dual" structural change facts in China.

4.1. Calibration

We use data on employment, wages and nominal GDP by industries and sectors between 1978 and 2011[5]. The annual population growth rate is $n = 0.0145$ for 1978-2011. As shown in Figure 1 to 3, there have been significant "dual" transitions from the agriculture sector to the non-agriculture sector and from the state sector to the nonstate sector in China.

For our calibrations, we take the initial year, $t = 0$, to correspond to the first year we have data by sector, 1978. The initial values of state variables are $\chi(0) = 0.014, \lambda_1(0) = 0.71, \lambda_2(0) = 0.69$ from the current dataset. Obviously, the initial share of labor for the agriculture and state sectors is quite far from the equilibrium values.

For the labor income share, Song (2006), based on extensively empirical study, has calibrated the Chinese capital income share for the non-agriculture sector to be 0.4, and the labor income share for the agriculture sector to be 0.8. Here we follow his work and show that the labor share for the agriculture sector α is 0.8 and the non-agriculture sector β is 0.6.

For the technological progress, Brandt and Zhu (2010) have conducted a careful growth accounting by sector to quantify the sources of China's growth. They estimate that the agriculture sector has the highest TFP growth, which is 6%, followed by the nonstate sector 4.6%, and the TFP grows most slowly in the state sector, only 1.52%. For our benchmark calibration, we first take the estimates in Brandt and Zhu (2010), that is $m_a = 0.06, m_s = 0.0152, m_{ns} = 0.046$, and then change the parameters for the TFP growth in the nonstate sector to test its sensibility. Note that our parameters satisfy the assumption 3 of $m_a/\alpha < m_{ns}/\beta$.

[5]Data source: the National Bureau of Statistics in China (NBS) and the China Compendium of Statistics.

Moreover, we adopt the standard parameter values for the baseline neoclassical model used in Barro and Sala-i-Martin (2004), the annual discount rate $\rho = 0.02$, the annual depreciation rate $\delta = 0.05$, and the elasticity of intertemporal substitution is 0.25, which implies $\theta = 4$.

The most important parameter for our calibration is the elasticity of substitution ε and η. These two parameters are estimated according to two equations in the model.

$$\ln\left[\frac{\lambda_2}{1-\lambda_2}\right] = \ln\left[\frac{\varphi}{1-\varphi}\right] + \frac{\eta-1}{\eta}\ln\left[\frac{Y_s}{Y_{ns}}\right] \tag{39}$$

$$\ln\left[\frac{\lambda_1}{1-\lambda_1}\right] = \ln\left[\frac{\alpha\gamma}{\beta(1-\gamma)}\right] + \frac{\varepsilon-1}{\varepsilon}\ln\left[\frac{Y_a}{Y_{na}}\right] \tag{40}$$

These equations can be obtained by taking logs of equations (20) and (21). We therefore estimate ε and η by regressing the equations above. Since our focus is not on business cycle fluctuations, we use the HP filter to smooth both the dependent and independent variables (with smoothing weight 1600). The simple OLS regressions of labor share and real sectoral output yield $(\eta-1)/\eta = 0.53$, with a standard error of 0.25, and $(\varepsilon-1)/\varepsilon = 0.36$, with a standard error of 0.16. We then obtain $\eta = 2.13, \varepsilon = 1.56$, which verifies that the elasticity of substitution of inter-sectors and intra-sectors are both larger than 1, so that all sectors are substitutable.

For the relative share of intermediate sectors φ and γ, we choose the parameters to ensure that equations (14) and (15) hold at $t = 0$, which gives $\varphi = 0.49$, $\gamma = 0.62$.

All the parameters and initial values of the model are summarized below:

TABLE 1.

Calibration of the model

Parameters	n	φ	γ	$\lambda_1(0)$	$\lambda_2(0)$	$\chi(0)$	α	β
Calibrated Value	0.0145	0.49	0.62	0.71	0.69	0.014	0.8	0.6
Parameter	m_a	m_s	m_{ns}	ρ	δ	θ	η	ε
Calibrated Value	0.06	0.0152	0.046	0.02	0.05	4	2.13	1.56

4.2. Dynamics

Figure 4 shows the results of benchmark calibration with the parameter values described above. The four panels depict the labor share $\lambda_1(t)$ and $\lambda_2(t)$, the real output ratio of the agriculture sector and the non-agriculture sector $Y_a(t)/Y_{na}(t)$, the real output ratio of the state sector and the non-state sector $Y_s(t)/Y_{ns}(t)$ for the first 100 years.

FIG. 4. Simulation results in the benchmark calibration

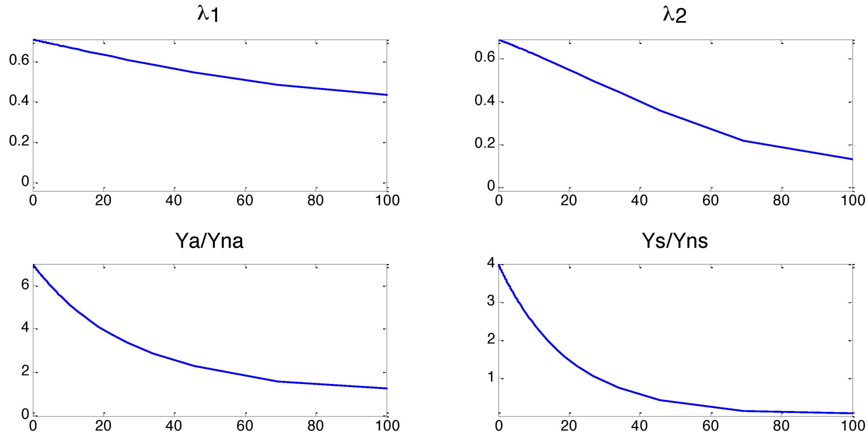


Figure 4 shows that both the labor shares and the real output ratios decline over time, which is consistent with the "dual" structural change in China, a significant reallocation from the agriculture sector to the non-agriculture sector and from the state sector to the nonstate sector. Also note that the transition from the agriculture sector to the non-agriculture sector is relatively modest, while the transition from the state sector to the nonstate sector is quite substantial. This result is simply attributed to a larger elasticity of substitution between the state sector and nonstate sectors, which leads to a faster reallocation.

Moreover, since the initial economy is quite far from the asymptotic equilibrium, the sectoral labor shares do not reach equilibrium in the BGP for the first 100 years. This result means that although there are continuous reallocations along the path, these three sectors still exist and continue to produce. Thus, our model economy generates relatively slow dynamics with a significant "dual" structural change.

4.3. Sensitivity

We next show the results of alternative calibrations of our model. In Tables 2 and 3, we consider different values for the TFP growth rate of the nonstate sector m_{ns} and the elasticity of substitution between the agriculture sector and the non-agriculture sector ε and show the comparison between the model simulation and the actual Chinese data.

The first and second columns of the table show that the labor share of the agriculture sector has declined from 70.5% to 34.8%, and the labor share of the state sector has declined from 68.8% to 14.7%. The third column gives

TABLE 2.

Sensitivity I

	Chinese Data		Model $m_{ns} = 0.46$ $\varepsilon = 1.56$	Model $m_{ns} = 0.51$ $\varepsilon = 1.56$	Model $m_{ns} = 0.56$ $\varepsilon = 1.56$
	1978	2011	2011	2011	2011
	(1)	(2)	(3)	(4)	(5)
λ_1	0.705	0.348	0.473	0.448	0.402
λ_2	0.688	0.147	0.312	0.259	0.212

TABLE 3.

Sensitivity II

	Chinese Data		Model $m_{ns} = 0.46$ $\varepsilon = 1.56$	Model $m_{ns} = 0.46$ $\varepsilon = 2.06$	Model $m_{ns} = 0.46$ $\varepsilon = 2.56$
	1978	2011	2011	2011	2011
	(1)	(2)	(3)	(4)	(5)
λ_1	0.705	0.348	0.473	0.407	0.345
λ_2	0.688	0.147	0.312	0.312	0.312

simulation results at $t = 33$, where the labor shares have a more modest decline than the actual data.

The third to fifth columns of Table 2 show the simulations of different values for the TFP growth of the nonstate sector m_{ns}. When m_{ns} is larger, there are greater changes in labor shares λ_1 and λ_2, which is much closer to the actual data.

Similarly is for the patterns implied by different values of ε in Table 3: as ε gets larger, the labor reallocated to the agriculture sector falls sharply. When $\varepsilon = 2.56$, the simulation of λ_1 is almost the same as the data. Moreover, the variation in ε does not change the results of labor share λ_2. This result is not surprising as ε is not involved in the dynamic equation of λ_2.

Overall, different calibrations do not change the reallocation facts of the labor share. Instead, more appropriate parameters will improve the fitness of the model to empirical data. Therefore, this sensitivity exercise indicates that the mechanism proposed in this model can generate the "dual" structural change that is broadly comparable with the changes we observed in Chinese data.

5. CONCLUSION

This paper proposes a three-sector neoclassical growth model that contains the agriculture sector, the state sector and the nonstate sector based on Acemoglu and Guerrieri (2008). We describe the "dual" structural change from the agriculture sector to the non-agriculture sector and from the state sector to the nonstate sector over the three decades of reform in China and attempt to explain its source.

We attribute the dual structural change to the elasticity of substitution and sectoral heterogeneity. When the sectoral elasticity of substitution is larger than 1, factors will flow toward the sector with a faster growth. The heterogeneity between the agriculture sector and the non-agriculture sector reflects both the factor proportion and technological progress. The effect of factor income share dominates that of TFP growth, inducing a reallocation of labor toward the non-agriculture sector, while within the non-agriculture sector, the nonstate sector has a higher TFP growth than state sector, and thus the labor share of the nonstate sector increases. In addition, there exists a nonbalanced sectoral growth in the BGP, where aggregate growth converges to the growth of the nonstate sector. Finally, we calibrate our model using the Chinese data for 1978 to 2011, and the magnitudes implied by our model are comparable to the sectoral changes in the actual data.

APPENDIX

Proof of Proposition 1.

Equation (18) is easily obtained by differentiating equation (14) with respect to time and combining equation (4).

Rewrite equation (14) as

$$Y_s(t) = \left[\frac{1-\varphi}{\varphi}\frac{\lambda_2(t)}{1-\lambda_2(t)}\right]^{\eta/(\eta-1)} Y_{ns}(t) \tag{A.1}$$

Substitute equation (A.1) into equation (3), we have

$$Y_{na}(t) = \left[\frac{1-\varphi}{1-\lambda_2(t)}\right]^{\eta/(\eta-1)} Y_{ns}(t) = P_2(t)Y_{ns}(t) \tag{A.2}$$

where $P_2(t) = \left[\frac{1-\varphi}{1-\lambda_2(t)}\right]^{\eta/(\eta-1)}$, and then rewrite equation (15) as

$$\frac{\lambda_1(t)}{(1-\lambda_1(t))(1-\lambda_2(t))} = \frac{\gamma\alpha}{(1-\gamma)(1-\varphi)\beta} P_2(t)^{(\eta-\varepsilon)/\eta\varepsilon} \left[\frac{Y_a(t)}{Y_{ns}(t)}\right]^{(\varepsilon-1)/\varepsilon} \tag{A.3}$$

Differentiate the equation above with respect to time and substitute into equation (18), then we can obtain equation (19). ▮

Proof of Proposition 5.

To solve the social planner's problem, we first establish the Hamilton system

$$H = \exp[-(\rho - n)t]\frac{c(t)^{1-\theta} - 1}{1-\theta} + \mu[\Phi(K(t), t) - \delta K(t) - c(t)L(t)] \quad \text{(A.4)}$$

where μ is the Hamiltonian and the shadow price of capital stock.

According to the principle of maximum,

$$\begin{aligned} \frac{\dot{c}}{c} &= \frac{\Phi_K(K(t), t) - \delta - \rho}{\theta} \\ \dot{K} &= \Phi(K(t), t) - \delta K(t) - c(t)L(t) \end{aligned} \quad \text{(A.5)}$$

Rewrite equation (A.3) as

$$Y_a = \left[\frac{(1-\gamma)(1-\varphi)\beta}{\gamma\alpha}\frac{\lambda_1(t)}{(1-\lambda_1(t))(1-\lambda_2(t))}P_2(t)^{(1-\eta)/\eta}\right]^{\varepsilon/(\varepsilon-1)} Y_{na} \quad \text{(A.6)}$$

Thus, the final output can be expressed as

$$Y(t) = \left\{\frac{(1-\gamma)[\beta\lambda_1(t) + \alpha(1-\lambda_1(t))]}{\alpha(1-\lambda_1(t))}\right\}^{\varepsilon/(\varepsilon-1)} Y_{na} = P_1(t)Y_{na} = P_1(t)P_2(t)Y_{ns} \quad \text{(A.7)}$$

where $P_1(t) = \left\{\frac{(1-\gamma)[\beta\lambda_1(t)+\alpha(1-\lambda_1(t))]}{\alpha(1-\lambda_1(t))}\right\}^{\varepsilon/(\varepsilon)}$.

Next, we express output and capital return as the function of the variables of the nonstate sector,

$$Y(t) = \Phi(K(t), t) = P_1(t)P_2(t)M_{ns}(t)L_{ns}(t)^{\beta}K_{ns}(t)^{1-\beta} \quad \text{(A.8)}$$

$$\Phi_K(K(t), t) = R(t) = (1-\gamma)(1-\varphi)(1-\beta)P_1(t)^{1/\varepsilon}P_2(t)^{1/\eta}M_{ns}(t)L_{ns}(t)^{\beta}K_{ns}(t)^{-\beta} \quad \text{(A.9)}$$

Finally, substitute equations (A.8) and (A.9) into (A.5) and express them in transformed variables $(\tilde{c}, \chi)$; then we obtain proposition 5. ▮

Proof of theorem 1.

Equation (18) implies $\lambda_2^* = 0$, and thus $\kappa_2^* = 0$. From equation (19), we can write

$$\lim_{t\to\infty} \dot{\lambda}_1(t) = \lim_{t\to\infty} \frac{\lambda_1(t)(1-\lambda_1(t))\{[m_a - \alpha/\beta m_{ns}] - (\alpha-\beta)\dot{\chi}(t)/\chi(t)}{(\varepsilon-1)^{-1} + (\alpha-\beta)(\lambda_1 - \kappa_1)} \quad \text{(A.10)}$$

A BGP requires that $\dot{\tilde{c}}(t)/\tilde{c}(t), \dot{\chi}(t)/\chi(t)$ are constant, so that the numerator in (A.10) $\{[m_a - \alpha/\beta m_{ns}] - (\alpha-\beta)\dot{\chi}(t)/\chi(t)\}$ is constant. Therefore, the solution to (A.10) is $\lim_{t\to\infty}\dot{\lambda}_1(t) = 0$, which means λ_1^* is constant and κ_1^*, P_1^*, P_2^* are all constant, as well.

Rewrite the Euler equation as

$$\lim_{t\to\infty}\frac{\dot{\tilde{c}}(t)}{\tilde{c}(t)} = \frac{1}{\theta}\{(1-\gamma)(1-\varphi)(1-\beta)(P_1^*)^{1/\varepsilon}(P_2^*)^{1/\eta}(1-\lambda_1^*)^{\beta}(1-\kappa_1^*)^{-\beta}\chi(t)^{-\beta}-\delta-\rho\}-\frac{m_{ns}}{\beta}$$

Since $\lim_{t\to\infty}\frac{\dot{\tilde{c}}(t)}{\tilde{c}(t)}$ is required to be constant, so χ^* must be constant, which induces $\lim_{t\to\infty}\frac{\dot{\chi}(t)}{\chi(t)} = 0$.

Substitute into equation (A.10); since the right hand side of the equation is monotonic decreasing, we can obtain a unique solution $\lambda_1^* = 0$, and also $\kappa_1^* = 0$. Next, according to

$$\lim_{t\to\infty}\frac{\dot{\chi}(t)}{\chi(t)} = P_1^* P_2^* \chi(t)^{-\beta} - \delta - \frac{\tilde{c}(t)}{\chi(t)} - n - \frac{m_{ns}}{\beta}$$

is constant in the BGP, we have $\lim_{t\to\infty}\frac{\tilde{c}(t)}{\chi(t)}$ is constant; thus c^* is constant and $\lim_{t\to\infty}\frac{\dot{\tilde{c}}(t)}{\tilde{c}(t)} = 0$. Combining $\lim_{t\to\infty}\frac{\dot{\tilde{c}}(t)}{\tilde{c}} = 0$ and $\lim_{t\to\infty}\frac{\dot{\chi}(t)}{\chi(t)} = 0$, we can obtain the expression of χ^* and c^*.

From $\lambda_1^* = \lambda_2^* = 0$, we have $z_{ns}^* = z^*, n_{ns}^* = n$. In addition, $\lim_{t\to\infty}\dot{\chi}(t) = 0$ implies $z_{ns}^* = z^* = n + \frac{m_{ns}}{\beta}$.

Equation (A.7) can be written as $Y(t) = P_!^* P_2^* Y_{ns}(t)$, so that the final output is increasing proportionately with the output of the nonstate sector. Combining the expression of the nonstate sector, the growth rate is given by

$$g^* = g_{ns}^* = n + \frac{m_{ns}}{\beta} \tag{A.11}$$

Again, from equations (12) and (13), we obtain

$$\begin{aligned} z_s - z_{ns} &= n_s - n_{ns} = (\eta-1)/\eta(g_s - g_{ns}) \\ z_a - z_{ns} &= n_a - n_{ns} = (\varepsilon-1)/\varepsilon(g_a - g_{ns}) \end{aligned} \tag{A.12}$$

Combining the production function of the agriculture sector and the state sector,

$$\begin{aligned} g_s &= m_s + \beta n_s + (1-\beta) z_s \\ g_a &= m_a + \alpha n_a + (1-\alpha) z_a \end{aligned} \tag{A.13}$$

Finally, we can easily obtain $g_a^*, g_{ns}^*, z_a^*, z_{ns}^*, n_a^*, n_{ns}^*$ from equations (A.12) and (A.13). ∎

Proof of Proposition 6.
Rewrite the system in proposition 4 as

$$\dot{x} = f(x)$$

where $x = (\tilde{c}, \chi, \lambda_1, \lambda_2)'$. To investigate the dynamics in the neighborhood of the steady state, consider the linear system

$$\dot{z} = J(x^*)z$$

where $z = x - x^*$ and x^* is the solution to $f(x^*) = 0$, and $J(x^*)$ is the Jacobian of $f(x)$ evaluated at x^*. First order Taylor expansion gives

$$J(x^*) = \begin{bmatrix} 0 & a_{\tilde{c}\chi} & a_{\tilde{c}\lambda_1} & a_{\tilde{c}\lambda_2} \\ -1 & a_{\chi\chi} & a_{\chi\lambda_1} & a_{\chi\lambda_2} \\ 0 & 0 & a_{\lambda_1\lambda_1} & a_{\lambda_1\lambda_2} \\ 0 & 0 & 0 & a_{\lambda_2\lambda_2} \end{bmatrix}$$

where

$$\begin{aligned} a_{\tilde{c}\chi} &= -\frac{\beta}{\theta}(1-\beta)(1-\gamma)^{\varepsilon/(\varepsilon-1)}(1-\varphi)^{\eta/(eta-1)}\tilde{c}^*(\chi^*)^{-\beta-1} < 0 \\ a_{\lambda_1\lambda_1} &= (\varepsilon-1)\left(m_a - \frac{\alpha}{\beta}m_{ns}\right) < 0 \\ a_{\lambda_2\lambda_2} &= (\eta-1)(m_s - m_{ns}) < 0 \end{aligned}$$

It can be easily obtained that $\det J(x^*) = a_{\tilde{c}\chi}a_{\lambda_1\lambda_1}a_{\lambda_2\lambda_2} < 0$, so either there are three negative and one positive eigenvalues or one negative and three positive eigenvalues.

To determine which one of the possibilities is the case, we further examine the characteristic equation by $\det(J(x^*) - \nu I) = 0$, where ν denotes the eigenvector. Then the characteristic equation can be written as

$$(a_{\lambda_1\lambda_1} - \nu)(a_{\lambda_2\lambda_2} - \nu)[-\nu(a_{\chi\chi} - \nu) + a_{\tilde{c}\chi}] = 0$$

This expression implies that two eigenvalues are equal to $a_{\lambda_1\lambda_1}$ and $a_{\lambda_2\lambda_2}$, which are both negative, so there must be at least two negative eigenvalues. Thus, there are three negative and one positive eigenvalues.

Since the system has three initial values for state variables, $\chi(0), \lambda_1(0), \lambda_2(0)$, the number of negative eigenvalues is equal to the number of initial values. Therefore, the BGP is saddle-path stable, in the sense of the existence of a unique three-dimensional manifold of solutions in the neighborhood of this BGP that converges to it. ▌

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