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Home Production and Indeterminacy with Variable Income Effects*

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In this paper, we utilize a real business cycle model with home production, productive increasing returns and the Jaimovich-Rebelo's preferences that allow for varying degrees of income effect, to show that when the values of income effect are intermediary and the level of increasing returns is reasonably high, the economy can be subject to local indeterminacy, provided that the other model parameters take reasonable values. In particular, we show that in the indeterminacy region, the minimum level of productive externality that induces instability increases as the income effect increases.

Key Words: Aggregate externalities; Income effects; Home production; Local indeterminacy.

JEL Classification Numbers: E32, E30.

1. INTRODUCTION

It is well known that a one-sector real business cycle (RBC) model with productive externalities can exhibit local indeterminacy, in the sense that a continuum of equilibrium paths converges to a steady state.[1] Benhabib and Farmer (1994) showed that in order to generate indeterminacy, relatively large aggregate externalities are required in their one-sector model. The utility function used by them is characterized by a positive income effect on labor supply. However, Jaimovich (2008) showed that varying degrees of income effect can affect indeterminacy and the indeterminacy result obtained by Benhabib and Farmer (1994) will not hold if the utility function is characterized by no-income effect on labor supply. Particularly, only a small amount of income effect is needed to generate indeterminacy.

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[1] See Benhabib and Farmer (1999) for an excellent survey.

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The preferences that were used by Jaimovich (2008) and Jaimovich and Rebelo (2009; hereafter JR) are recently modified by Nourry, Seegmuller, and Venditti (2013; hereafter NSV) and Abad, Seegmuller, and Venditti (2017; hereafter ASV).

Following Benhabib, Rogerson, and Wright (1991; hereafter BRW) and Greenwood and Hercowitz (1991), Perli (1998) showed that introducing a home sector into a traditional RBC model can make the economy subject to indeterminacy with much smaller externalities than in Benhabib and Farmer (1994). The utility function used by Perli (1998) is derived from King, Plosser, and Rebelo (1988; hereafter KPR). And his formulation is nested as a particular case of the JR class of preferences. In order to investigate the role of income effects in generating indeterminacy, it is natural for us to consider varying degrees of income effect in the model of Perli (1998). To this end, we prove that his results need to be revised when the modified JR preferences proposed by ASV are considered.

Our major contributions are summarized as follows. First of all, we show that when the income effect lies in the intermediary region and the degree of increasing returns is reasonably high, the economy with a home sector can be subject to indeterminacy. Based on the degree of productive externalities (in the market sector) used by Perli (1998) for the U.S. economy, we use numerical examples to demonstrate that our model with home production and reasonable magnitudes of externalities can exhibit indeterminacy. This conclusion is applicable when the other structural parameters also take realistic values. Since our main purpose is to examine the role of income effects in generating indeterminacy, we do not simulate our model and compare our simulated results with real data.

Second, we explain why the income effects coupled with externalities matter for indeterminacy and how they induce indeterminacy through the intratemporal and intertemporal mechanisms between consumption and labor. In a one-sector RBC model with consumption taxes and the JR type of preferences, NSV (2013) used a similar interpretation to analyze their indeterminacy result. Suppose that optimistic expectations induce labor supply to increase tomorrow. This happens only if a higher return to labor is to be expected. In the model with a home sector, the agent can draw labor out of leisure and non-market work at the same time if she wants to work more in the market sector. This will induce the allocation of capital between the two sectors to vary because the marginal product of capital in the home sector decreases. Thus, from the Euler equation, we deduce that the shadow price of the capital stock increases. When the agent wants to invest more in the market sector, the negatively-sloped labor demand curve can move outwards so that in the new equilibrium, both labor and wage rate can exceed their initial values. Therefore, optimistic expectations are self-fulfilled. In the above mechanism, due to the intratemporal condition between labor and consumption, the (intratemporal) income effect will induce consumption and hours worked to move in the same direction because the agent expects her (labor) income to increase, and increases her consumption when her hours worked increase[2]. We also notice that as in NSV (2013) and ASV (2017), in the above mechanism, indeterminacy results if the intertemporal condition between consumption and labor is consistent with the intratemporal mechanism. And the intertemporal mechanism is related to the income effect as well. To be more precise, the intertemporal mechanism directly links the variations of the labor supply in the market sector and consumption with those of the shadow price, the capital stock in the market sector and the labor supply in the home sector. In the above process, the magnitude of the increase in labor supply in the market sector is controlled by the term which we call the first effect. And the magnitude of the decrease in consumption is controlled by the term which we call the second effect. We observe that in the numerical case, the first effect increases in the income effect, and the second effect increases in the income effect as well. The intertemporal mechanism holds only if the first effect is large and positive and the second effect is negative.

Third, we explain why in the indeterminacy region, the minimum level of productive externality that induces instability increases as the income effect increases. In this paper, we show that when optimistic expectations arrive, increasing the income effect and degree of productive externalities have opposite effects on the intertemporal mechanism between consumption and labor (the first and second effects). Therefore, in order to make the intertemporal mechanism valid and consistent with the intratemporal mechanism, the minimum level of productive externality that induces instability should increase as the income effect increases.

The reminder of this paper is organized as follows. Section 2 presents the model. Section 3 presents the analysis of the steady state, elasticities, and local dynamics. Section 4 discusses the calibrated example. Section 5 describes the economic intuition. Section 6 concludes the paper.

2. THE BASIC MODEL

Following BRW (1991) and Perli (1998), we consider the economy that contains two sectors, called market and home. The market good (Y_t) can be either consumed (C_{m_t}) or invested. The home good (C_{n_t}) can only be consumed. The producers in these sectors use capital, K_{m_t} and K_{n_t}, and labor, H_{m_t} and H_{n_t} as inputs, respectively. The composite consumption

[2] In fact, the intratemporal income effect between consumption and labor dominates the intratemporal substitution effect between consumption and labor, thereby making consumption and labor move in the same direction.

good (C_t) is defined as follows:

$$C_t = \left[aC_{m_t}^{\varepsilon} + (1-a)\, C_{n_t}^{\varepsilon}\right]^{1/\varepsilon}, \tag{1}$$

where $a \in (0,1)$ and $1/(1-\varepsilon)$ measures the elasticity of substitution between C_{m_t} and C_{n_t}, which usually varies from 0 to $+\infty$.

Assuming that H_{m_t} and H_{n_t} are perfect substitutes, the agent has the following preference with varying income effects, that was studied by ASV (2017):

$$u(C_t, \mathcal{L}_t) = \frac{[C_t + (\frac{1-H_{m_t}-H_{n_t}}{B})^{1+\chi} C_t^{\gamma}]^{1-\theta} - 1}{1-\theta}, \text{ with } \theta \geq 0,\ \chi \geq 0 \text{ and } \gamma \in [0,1]. \tag{2}$$

Here, C_t and $\mathcal{L}_t \equiv 1 - H_{m_t} - H_{n_t}$ denote consumption and leisure, respectively. The time endowment available to the agent is unity.

Remark 1: The above preference specification was first introduced by Jaimovich and Rebelo (2009) and recently modified by ASV (2017). It allows for varying degrees of income effect. As $\gamma = 0$, it is referred to as the Greenwood-Hercowitz-Huffman (GHH) type that exhibits no income effect on labor supply. As $\gamma = 1$, it is referred to as the King-Plosser-Rebelo (KPR) type that exhibits a large income effect on labor supply. And the parameter γ measures the magnitude of the income effect. The larger γ is, the stronger the income effect is. Moreover, consumption and labor are Edgeworth substitutes when $\gamma > \theta$ whereas they are Edgeworth complements when $\gamma < \theta$.

The Cobb-Douglas production technologies are adopted in two sectors. There are no externalities in the home sector, as in Perli (1998). C_{n_t} is produced according to

$$C_{n_t} = K_{n_t}^{s} H_{n_t}^{1-s}. \tag{3}$$

The production function in the market sector is

$$Y_t = X_t K_{m_t}^{b} H_{m_t}^{1-b}, \tag{4}$$

where X_t is the external effect coming from the aggregate levels of K_{m_t} and H_{m_t}, and takes the following form

$$X_t = \overline{K}_{m_t}^{b_K} \overline{H}_{m_t}^{b_H}, \text{ with } b_K \geq 0 \text{ and } b_H \geq 0. \tag{5}$$

We assume that capital and labor can be freely mobile between the two sectors:

$$\begin{aligned} H_t &= H_{m_t} + H_{n_t} = 1 - \mathcal{L}_t, \\ K_t &= K_{m_t} + K_{n_t}, \end{aligned}$$

where K_t and H_t denote the aggregate levels of capital and labor in the economy, respectively.

Because the home good is nontradeable and only consumed at home, the law of motion of capital is given by

$$\dot{K}_t = Y_t - C_{m_t} - \delta K_t,$$

where δ denotes the depreciation rate of capital.

For a given initial capital stock $K_0 > 0$, the maximization problem of the agent can be written as follows:

$$\max_{C_{i_t}, K_{i_t}, H_{i_t}} \int_{t=0}^{+\infty} e^{-\rho t} \frac{[C_t + (\frac{1-H_{m_t}-H_{n_t}}{B})^{1+\chi} C_t^{\gamma}]^{1-\theta} - 1}{1-\theta} dt,$$

subject to

$$\dot{K}_t = Y_t - C_{m_t} - \delta K_t,$$

$$C_{n_t} = K_{n_t}^{s} H_{n_t}^{1-s},$$

$$C_t = \left[a C_{m_t}^{\varepsilon} + (1-a) C_{n_t}^{\varepsilon} \right]^{1/\varepsilon},$$

$$K_t = K_{m_t} + K_{n_t},$$

$$H_t = H_{m_t} + H_{n_t} = 1 - \mathcal{L}_t,$$

where ρ denotes the time preference.

Using the current-value Hamiltonian, one sees that this system has one state variable K_t and five free variables: C_{m_t}, H_{m_t}, H_{n_t}, K_{m_t}, and λ_t, where λ_t denotes the costate variable associated with the capital accumulation equation.

The first order conditions and Euler equation can be expressed as follows:

$$[C_{m_t}] \;:\; u_C(C_t, \mathcal{L}_t)\, a(\frac{C_t}{C_{m_t}})^{1-\varepsilon} = \lambda_t, \tag{6}$$

$$[C_{n_t}] \;:\; u_C(C_t, \mathcal{L}_t)(1-a)(\frac{C_t}{C_{n_t}})^{1-\varepsilon} = \eta_t, \tag{7}$$

$$[H_{m_t}] \;:\; u_{\mathcal{L}}(C_t, \mathcal{L}_t) = (1-b)\lambda_t X_t K_{m_t}^{b} H_{m_t}^{-b}, \tag{8}$$

$$[H_{n_t}] \;:\; u_{\mathcal{L}}(C_t, \mathcal{L}_t) = (1-s)\eta_t (K_t - K_{m_t})^{s} H_{n_t}^{-s}, \tag{9}$$

$$[K_{m_t}] \;:\; b\lambda_t X_t K_{m_t}^{b-1} H_{m_t}^{1-b} = s\eta_t (K_t - K_{m_t})^{s-1} H_{n_t}^{1-s}, \tag{10}$$

$$[K_t] \;:\; \dot{\lambda}_t = \lambda_t(\rho + \delta) - s\eta_t (K_t - K_{m_t})^{s-1} H_{n_t}^{1-s}, \tag{11}$$

where η_t denotes the costate variable associated with the market clearing condition of the home good, and $u_C(C_t, \mathcal{L}_t)$ and $u_{\mathcal{L}}(C_t, \mathcal{L}_t)$ are given in Appendix A.

Recall that in equilibrium, we have $\overline{K}_{m_t} = K_{m_t}$ and $\overline{H}_{m_t} = H_{m_t}$. Tedious derivations lead to the following equilibrium conditions:

$$\lambda_t = a(\frac{C_t}{C_{m_t}})^{1-\varepsilon}\left[C_t + \left(\frac{1-H_{m_t}-H_{n_t}}{B}\right)^{1+\chi} C_t{}^{\gamma}\right]^{-\theta}[1+\gamma C_t{}^{\gamma-1}\left(\frac{1-H_{m_t}-H_{n_t}}{B}\right)^{1+\chi}], \tag{12}$$

$$\begin{aligned}&\frac{\left[aC_{m_t}^{\varepsilon}+(1-a)\,C_{n_t}^{\varepsilon}\right]^{\gamma/\varepsilon}\left(\frac{1+\chi}{B}\right)\left(\frac{1-H_{m_t}-H_{n_t}}{B}\right)^{\chi}}{1+\gamma\left[aC_{m_t}^{\varepsilon}+(1-a)\,C_{n_t}^{\varepsilon}\right]^{\frac{\gamma-1}{\varepsilon}}\left(\frac{1-H_{m_t}-H_{n_t}}{B}\right)^{1+\chi}}\\ &= \; a\,(1-b)\,\{\frac{[aC_{m_t}^{\varepsilon}+(1-a)\,C_{n_t}^{\varepsilon}]^{1/\varepsilon}}{C_{m_t}}\}^{1-\varepsilon}K_{m_t}{}^{\alpha}H_{m_t}{}^{\beta-1},\end{aligned} \tag{13}$$

$$\frac{1-b}{1-s}\frac{a}{1-a}C_{m_t}^{\varepsilon-1}K_{m_t}^{\alpha}H_{m_t}^{\beta-1} = (K_t - K_{m_t})^{s\varepsilon}H_{n_t}^{\varepsilon(1-s)-1}, \tag{14}$$

$$\frac{b}{s}\frac{a}{1-a}C_{m_t}^{\varepsilon-1}K_{m_t}^{\alpha-1}H_{m_t}^{\beta} = (K_t - K_{m_t})^{s\varepsilon-1}H_{n_t}^{\varepsilon(1-s)}, \tag{15}$$

$$\dot{\lambda}_t = -\lambda_t\left[s\frac{1-a}{a}C_{m_t}^{1-\varepsilon}(K_t - K_{m_t})^{s\varepsilon-1}H_{n_t}^{\varepsilon(1-s)} - (\rho+\delta)\right], \tag{16}$$

$$\dot{K}_t = K_{m_t}^{\alpha}H_{m_t}^{\beta} - C_{m_t} - \delta K_t, \tag{17}$$

where $\alpha \equiv b + b_K$ and $\beta \equiv 1 - b + b_H$.

Notice that four functions $H_{m_t}(C_{m_t}, K_t)$, $H_{n_t}(C_{m_t}, K_t)$, $K_{m_t}(C_{m_t}, K_t)$ and $\lambda_t(C_{m_t}, K_t)$ can be solved implicitly from Eq. (12) to Eq. (15). Therefore, the economy can be reduced to a two dimensional dynamical system.

Dividing (14) by (15) leads to the following equation:

$$\frac{K_{m_t}}{K_t - K_{m_t}}\Delta_1 = \frac{H_{m_t}}{H_{n_t}},\text{ with }\Delta_1 \equiv \frac{1-b}{1-s}\frac{s}{b}. \tag{18}$$

Letting $\mu_t \equiv \frac{K_{m_t}}{K_t}$, the following endogenous variables can be expressed as functions of $\{\mu_t, K_t, H_{m_t}\}$: $C_{n_t} = \Delta_1^{s-1}(1-\mu_t)\mu_t{}^{s-1}K_t{}^{s}H_{m_t}{}^{1-s}$, $C_{m_t} =$

$\Delta_2(1-\mu_t)K_t^{\zeta_1}H_{m_t}^{\zeta_2}{\mu_t}^{\zeta_3}$, $K_{m_t}=\mu_t K_t$, $K_{n_t}=(1-\mu_t)K_t$, $H_{n_t}=\frac{H_{m_t}}{\Delta_1}\frac{1-\mu_t}{\mu_t}$, and $H_t = H_{m_t}\frac{b(1-s)+\mu_t(s-b)}{s(1-b)\mu_t}$, where $\Delta_2 = \left[\frac{\Delta_1^{\varepsilon(1-s)}ba}{s(1-a)}\right]^{\frac{1}{1-\varepsilon}}$, $\zeta_1 = \frac{s\varepsilon-\alpha}{\varepsilon-1}$, $\zeta_2 = \frac{\varepsilon(1-s)-\beta}{\varepsilon-1}$, and $\zeta_3 = \frac{1+s\varepsilon-\varepsilon-\alpha}{\varepsilon-1}$. Thus, the equilibrium conditions can be simplified as equations of $\{\lambda_t,\mu_t,K_t,H_{m_t}\}$:

$$\lambda_t = a[\frac{C_t\left(\mu_t,K_t,H_{m_t}\right)}{C_{m_t}\left(\mu_t,K_t,H_{m_t}\right)}]^{1-\varepsilon} * \tag{19}$$

$$\left[C_t\left(\mu_t,K_t,H_{m_t}\right)+\left(\frac{1-H_{m_t}-\frac{H_{m_t}}{\Delta_1}\frac{1-\mu_t}{\mu_t}}{B}\right)^{1+\chi}C_t\left(\mu_t,K_t,H_{m_t}\right)^{\gamma}\right]^{-\theta} *$$

$$[1+\gamma C_t\left(\mu_t,K_t,H_{m_t}\right)^{\gamma-1}\left(\frac{1-H_{m_t}-\frac{H_{m_t}}{\Delta_1}\frac{1-\mu_t}{\mu_t}}{B}\right)^{1+\chi}],$$

$$\begin{aligned}&\frac{C_t\left(\mu_t,K_t,H_{m_t}\right)^{\gamma}\left(\frac{1+\chi}{B}\right)\left(\frac{1-H_{m_t}-\frac{H_{m_t}}{\Delta_1}\frac{1-\mu_t}{\mu_t}}{B}\right)^{\chi}}{1+\gamma C_t\left(\mu_t,K_t,H_{m_t}\right)^{\gamma-1}\left(\frac{1-H_{m_t}-\frac{H_{m_t}}{\Delta_1}\frac{1-\mu_t}{\mu_t}}{B}\right)^{1+\chi}}\\ &= a\left(1-b\right)\left[\frac{C_t\left(\mu_t,K_t,H_{m_t}\right)}{C_{m_t}\left(\mu_t,K_t,H_{m_t}\right)}\right]^{1-\varepsilon}\left(\mu_t K_t\right)^{\alpha}{H_{m_t}}^{\beta-1},\end{aligned}\tag{20}$$

$$\begin{aligned}\dot{\lambda}_t &= -\lambda_t\{s\frac{1-a}{a}C_{m_t}\left(\mu_t,K_t,H_{m_t}\right)^{1-\varepsilon}\left[(1-\mu_t)K_t\right]^{s\varepsilon-1} *\\ &(\frac{H_{m_t}}{\Delta_1}\frac{1-\mu_t}{\mu_t})^{\varepsilon(1-s)}-(\rho+\delta)\},\end{aligned}\tag{21}$$

$$\dot{K}_t = (\mu_t K_t)^{\alpha}{H_{m_t}}^{\beta} - C_{m_t}\left(\mu_t,K_t,H_{m_t}\right) - \delta K_t, \tag{22}$$

where $C_t\left(\mu_t,K_t,H_{m_t}\right) = (1-\mu_t)\left[\begin{array}{c} a\Delta_2^{\varepsilon}{K_t}^{\varepsilon\zeta_1}{H_{m_t}}^{\varepsilon\zeta_2}{\mu_t}^{\varepsilon\zeta_3} \\ +\left(1-a\right)\Delta_1^{(s-1)\varepsilon}\mu_t^{\varepsilon(s-1)}K_t^{\varepsilon s}H_{m_t}^{\varepsilon(1-s)}\end{array}\right]^{\frac{1}{\varepsilon}}$.

3. STEADY STATE, ELASTICITIES, AND DYNAMICS

At the steady state, we have $\dot{\lambda}_t = \dot{K}_t = 0$. Thus, the steady-state values satisfy the following equations:

$$\lambda = a(\frac{C}{C_m})^{1-\varepsilon}\left[C + \left(\frac{1-H}{B}\right)^{1+\chi} C^{\gamma}\right]^{-\theta}\left[1 + \gamma C^{\gamma-1}\left(\frac{1-H}{B}\right)^{1+\chi}\right], \tag{23}$$

$$\frac{C^{\gamma+\varepsilon-1}\left(\frac{1+\chi}{B}\right)\left(\frac{1-H}{B}\right)^{\chi}}{1+\gamma C^{\gamma-1}\left(\frac{1-H}{B}\right)^{1+\chi}} = a\,(1-b)\,C_m^{\varepsilon-1}(\mu K)^{\alpha} H_m^{\beta-1}, \tag{24}$$

$$s\frac{(1-a)}{a}C_m^{1-\varepsilon}\left[(1-\mu)\,K\right]^{s\varepsilon-1}\left(\frac{H_m}{\Delta_1}\frac{1-\mu}{\mu}\right)^{\varepsilon(1-s)} = \rho+\delta, \tag{25}$$

$$(\mu K)^{\alpha} H_m^{\beta} = \Delta_2(1-\mu)K^{\zeta_1}H_m^{\zeta_2}\mu^{\zeta_3} + \delta K, \tag{26}$$

where $H = H_m \frac{b(1-s)+\mu(s-b)}{s(1-b)\mu}$, $C_m = \Delta_2(1-\mu)K^{\zeta_1}H_m^{\zeta_2}\mu^{\zeta_3}$, and

$$C = (1-\mu)\left[a\Delta_2^{\varepsilon}K^{\varepsilon\zeta_1}H_m^{\varepsilon\zeta_2}\mu^{\varepsilon\zeta_3} + (1-a)\,\Delta_1^{(s-1)\varepsilon}\mu^{\varepsilon(s-1)}K^{\varepsilon s}H_m^{\varepsilon(1-s)}\right]^{\frac{1}{\varepsilon}}.$$

From Eqs. (25), (26), C_m, and (24), we have the following equations:

$$H_m = \left[\Delta_3\frac{\left(\frac{\rho+\delta}{b}\right)\mu-\delta}{1-\mu}\right]^{\xi}, \tag{27}$$

$$K = \left(\frac{b}{\rho+\delta}\right)^{\frac{1}{1-\alpha}}\mu^{-1}\left[\Delta_3\frac{\left(\frac{\rho+\delta}{b}\right)\mu-\delta}{1-\mu}\right]^{\frac{\beta\xi}{1-\alpha}}, \tag{28}$$

and

$$\frac{C^{\gamma+\varepsilon-1}(1+\chi)\,(1-H)^{\chi}}{B^{1+\chi}+\gamma C^{\gamma-1}\,(1-H)^{1+\chi}} = \frac{s\,(1-a)\,(1-b)}{\Delta_1^{\varepsilon(1-s)}b}K^{s\varepsilon}H_m^{\varepsilon(1-s)-1}(1-\mu)^{\varepsilon-1}\mu^{1+s\varepsilon-\varepsilon}, \tag{29}$$

where $\Delta_3 = \frac{1}{\Delta_2}\left(\frac{\rho+\delta}{b}\right)^{\frac{\zeta_3}{1-\alpha}}$ and $\xi = \frac{1-\alpha}{\beta\zeta_3+(1-\alpha)\zeta_2}$.

The following elasticities (at the steady states) are introduced into our model:

$$\begin{aligned}\epsilon_{CC} &= -\frac{U_C(C,\mathcal{L})}{U_{CC}(C,\mathcal{L})C},\ \epsilon_{HC} = -\frac{U_{\mathcal{L}}(C,\mathcal{L})}{U_{\mathcal{L}C}(C,\mathcal{L})C},\\ \epsilon_{CH} &= -\frac{U_C(C,\mathcal{L})}{U_{C\mathcal{L}}(C,\mathcal{L})H},\ \epsilon_{HH} = -\frac{U_{\mathcal{L}}(C,\mathcal{L})}{U_{\mathcal{L}\mathcal{L}}(C,\mathcal{L})H},\end{aligned} \tag{30}$$

where $U_{CC}(C,\mathcal{L})$, $U_{\mathcal{L}C}(C,\mathcal{L})$, $U_{C\mathcal{L}}(C,\mathcal{L})$, $U_{\mathcal{L}\mathcal{L}}(C,\mathcal{L})$ are shown in Appendix A. Notice that the elasticity of substitution in consumption is denoted by ϵ_{CC} and the Frisch labor elasticity (ϵ_{HW}) is derived from Eq. (6) and Eq. (8)

$$\epsilon_{HW} = [\epsilon_{HH}^{-1} - \epsilon_{CH}^{-1}]^{-1}. \tag{31}$$

We should emphasize that the modified JR preferences may not be concave at the steady state. Therefore, the following lemma is required to enable the locally optimal solution to exist.

Lemma 1. *The modified JR preferences are concave at the steady state if and only if*

$$\theta \geq \max\{\theta_1(\gamma), \theta_2(\gamma)\}, \tag{32}$$

where $\theta_1(\gamma) = \frac{C+C^\gamma\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}{C^\gamma(1+\chi)\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}\chi$ *and* $\theta_2(\gamma) = \frac{\frac{(\chi+\gamma)\gamma C^\gamma\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}{C+\gamma C^\gamma\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}}{\frac{\gamma C^\gamma\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}-\chi C}{C+C^\gamma\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}+\frac{\gamma C^\gamma(1+\chi)\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}{C+\gamma C^\gamma\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}}$.

Proof. See Appendix A. ▮

Under the assumption of this Lemma, we can discuss the local dynamics around the steady state and compare our indeterminacy results with those obtained by Perli (1998).

We analyze the local dynamics of Eq. (19) to Eq. (22) around the steady state. Using the linearization method, we obtain the following dynamic system:

$$\begin{bmatrix} \dot{\lambda}_t \\ \dot{K}_t \end{bmatrix} = J \begin{bmatrix} \lambda_t \\ K_t \end{bmatrix}, \tag{33}$$

where four elements of the Jacobian matrix J can be computed via symbolic toolbox. Local indeterminacy requires that the determinant of the Jacobian matrix be positive and the trace negative. Given the difficulty of obtaining an analytical solution from the model, we have to use numerical methods to determine the indeterminacy region.

4. CALIBRATED EXAMPLE

In the calibrated example, we set $(b, \delta, \rho) = (0.3, 0.025, 0.01)$ based on quarterly data. As in Perli (1998), we set $\varepsilon = \frac{1}{3}$, which is close to the point estimate observed by McGrattan, Rogerson, and Wright (1993). The steady-state values of H_m and H_n for the US economy are set at 0.17 and 0.15 respectively, according to Hill (1985). Using Eq. (18), $H_m = 0.17$ and $H_n = 0.15$, we see that the capital share in the home sector (s) is set at 0.354, so that the ratio $\frac{K_m}{K_n}$ is equal to 1/1.13 at the steady state, which is estimated by Greenwood and Hercowitz (1991). The parameter $a = 0.42$ is set to make both $H_m = 0.17$ and $H_n = 0.15$ hold. Similarly to Abad et al. (2017), B is set to make both Eq. (29) and $H_m = 0.17$ hold at the steady state for each combination of $\gamma \in [0, 1]$ and $b_H \in [0, 0.33]$, provided that $b_K = 0.1$[3]. Here, we follow Perli (1998) by setting b_K at 0.1. We select the other parameters (χ and θ) based on the following empirical results:

i) In the empirical literature, most estimates of the elasticity of intertemporal substitution in consumption lie in the interval of 0 to 2. The newest interval of the elasticity in consumption is (2, 3)[4].

ii) Several authors assume that the Frisch wage elasticity of the labor supply is infinity. Rogerson and Wallenius (2009) demonstrate that its range is from 2.25 to 3 at the macroeconomic level. However, Chetty *et al.* (2012) suggest a value of 0.5 based on the intensive margin of hours worked. As in Abad et al. (2017), we let the labor elasticity vary from 2.25 to 3.

Notice that χ and θ are two crucial parameters that affect ϵ_{CC} and ϵ_{HW}. We assume $\chi = 0.43$ and $\theta = 1.1$. In this case, local indeterminacy occurs when $\gamma \in [0.15, 0.77]$ and $b_H \in [0.213, 0.33]$. The consumption elasticity ϵ_{CC} (in the indeterminacy region) lies in the interval of $[0.9285, 1.4344]$. If $b_H = 0.25$, the Frisch elasticity ϵ_{HW} lies in the reasonable interval of $[2, 3.48]$, provided that the income effect lies in the region of $[0.56, 0.76]$. Recall that Kahn and Tsoukalas (2011) get a distribution of γ that has a mean of 0.81 and a 10th-90th percentile interval of $[0.69, 0.95]$. Our indeterminacy result ($\gamma \in [0.56, 0.76]$) with reasonable values of ϵ_{HW}, doesn't violate their estimates.

One may argue that in this numerical case, the degree of returns to scale in the market sector (1.35) seems a bit larger than the estimate (1.33) obtained by Basu and Fernald (1997). However, this value is surely lower than the highest degree of returns to scale in the market sector (1.45) used by Perli (1998). Since we are interested in the role of income effects in

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[3]Notice that in the process of calibrating B, K_m, K_n, and H_n will be affected by the parameters γ and b_H.

[4]See, for example, Vissing-Jorgensen and Attanasio (2003). NSV (2013) also use this estimated interval.

FIG. 1. Indeterminacy region ($\chi = 0.43$ and $\theta = 1.1$).

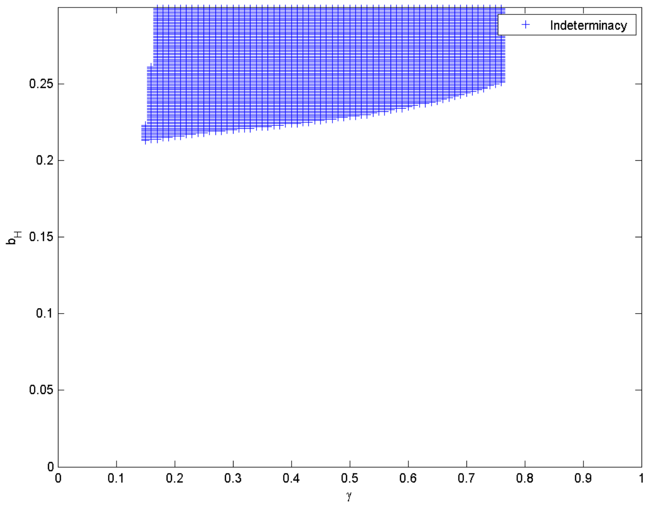
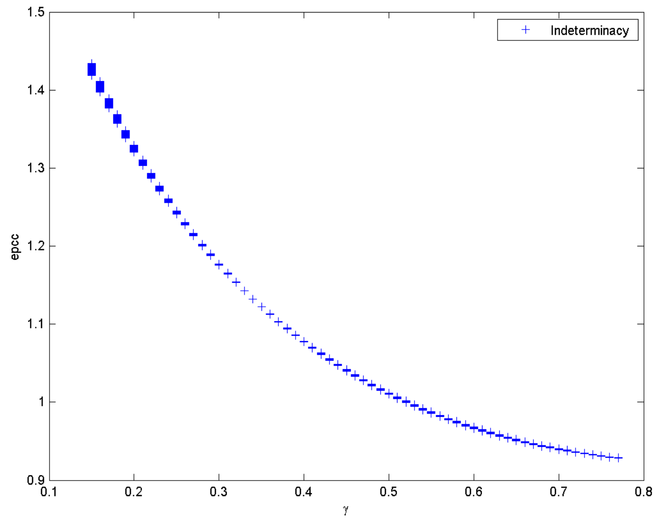
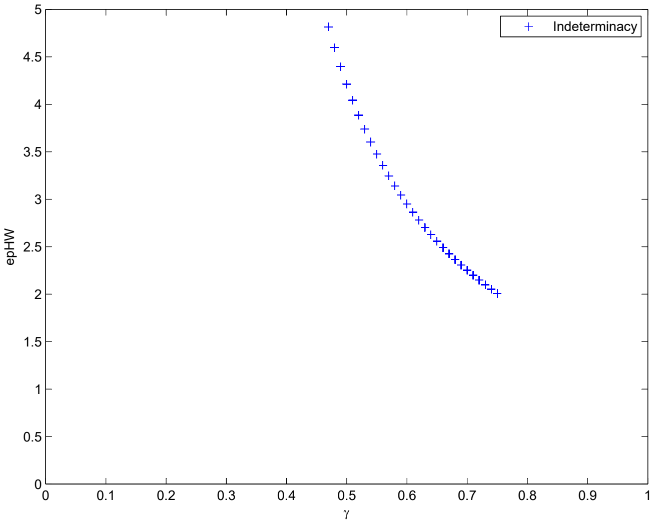


FIG. 2. The consumption elasticity ϵ_{CC} (in the indeterminacy region).



generating indeterminacy, we take $b_H = 0.25$ and $b_K = 0.1$ as our reference point in the next section.

FIG. 3. The labor elasticity ϵ_{HW} (in the indeterminacy region): the case of $b_H = 0.25$ and $b_K = 0.1$.



5. ECONOMIC INTERPRETATION

Following Perli (1998) and ASV (2017), it is not hard to figure out the mechanism behind the indeterminacy result. First of all, the marginal product of labor in the market sector (MPL_{m_t}) equals $(1-b)\, X_t K_{m_t}^{b} H_{m_t}^{-b}$. Using $X_t = \overline{K}_{m_t}^{b_K} \overline{H}_{m_t}^{b_H}$, we deduce that $MPL_{m_t} = (1-b) K_{m_t}^{b+b_K} H_{m_t}^{-b+b_H}$ holds in equilibrium. Then, we have

$$\frac{dMPL_{m_t}}{MPL_m} = (b_H - b)\frac{dH_{m_t}}{H_m} + (b + b_K)\frac{dK_{m_t}}{K_m}, \tag{34}$$

where MPL_m is the steady-state value of MPL_{m_t}, and $dZ_t \equiv Z_t - Z$ represents the deviation of the variable Z_t around its steady-state value.

Using Eq. (34), $u_{\mathcal{L}}(C_t, \mathcal{L}_t) = \lambda_t MPL_{m_t}$, and $H_t = H_{m_t} + H_{n_t}$, we have the intertemporal equilibrium condition which can be expressed by the following equation:

$$[\frac{(-b_H + b)}{H_m} + \frac{1}{H\epsilon_{HH}}]dH_{m_t} - \frac{1}{C\epsilon_{HC}}dC_t = (b+b_K)\frac{dK_{m_t}}{K_m} + \frac{d\lambda_t}{\lambda} - \frac{1}{H\epsilon_{HH}}dH_{n_t}. \tag{35}$$

Second, we follow Perli (1998, p. 106) to provide the economic intuition. Assume that labor supply will increase tomorrow because of optimistic expectations. This might occur if we have a higher return to labor. In the model with a home sector, the agent can draw labor out of leisure and non-

FIG. 4. The first effect (in the determinacy region): the case of $b_H = 0.25$ and $b_K = 0.1$.

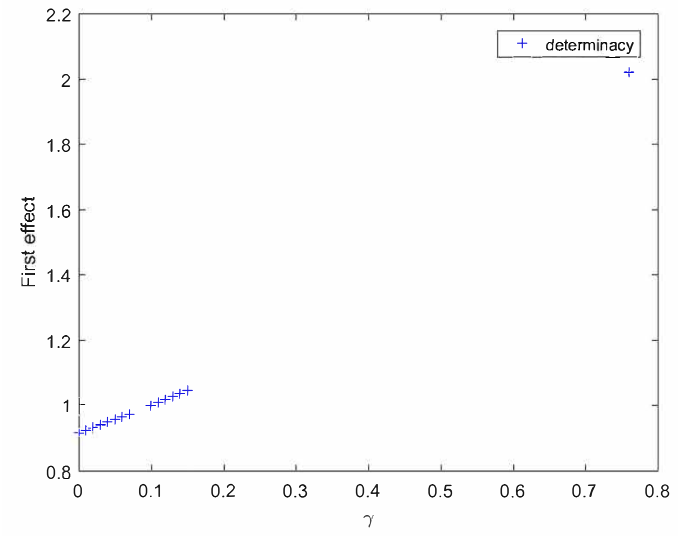
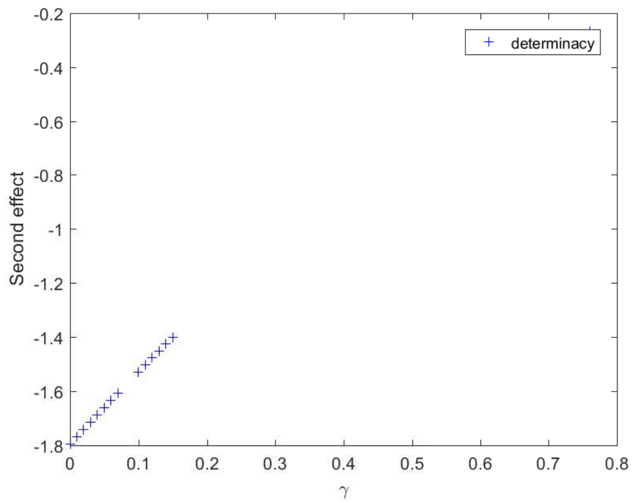


FIG. 5. The second effect (in the determinacy region): the case of $b_H = 0.25$ and $b_K = 0.1$.



market work ($dH_{n_t} < 0$) if she wants to work more in the market sector ($dH_{m_t} > 0$). Her action will induce the allocation of capital between the two sectors to change because the marginal product of capital in the home sector $[MPK_{n_t} = s\,(K_t - K_{m_t})^{s-1}\, H_{n_t}^{1-s}]$ decreases. Thus, from Eq. (11),

FIG. 6. The first effect (in the indeterminacy region): the case of $b_H = 0.25$ and $b_K = 0.1$.

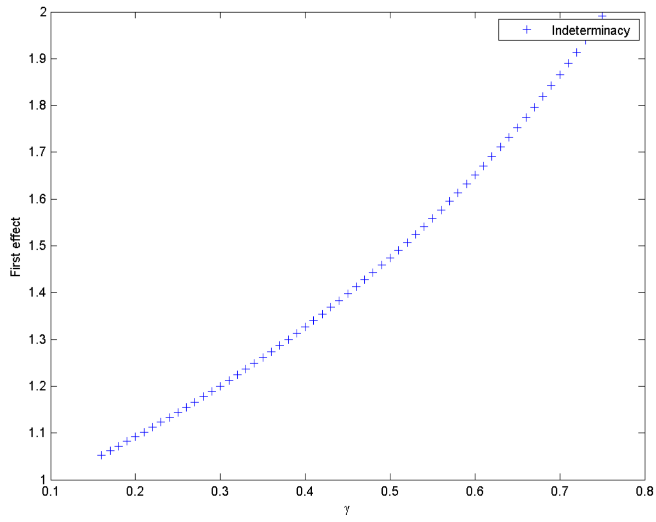
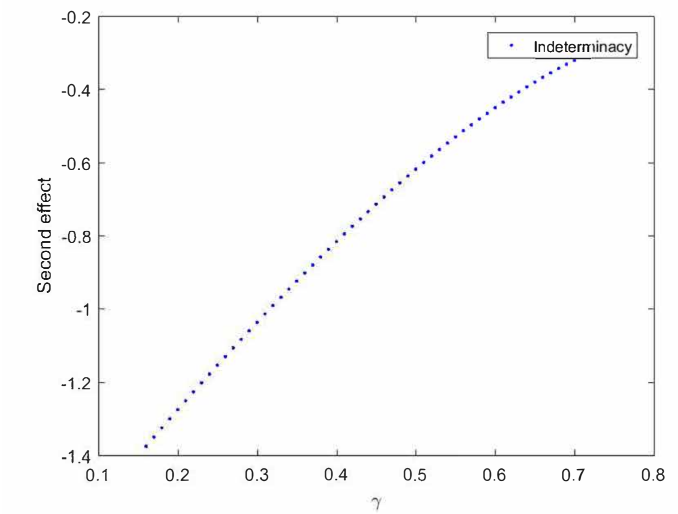


FIG. 7. The second effect (in the indeterminacy region): the case of $b_H = 0.25$ and $b_K = 0.1$.



we deduce that λ_t increases ($d\lambda_t > 0$). When the agent wants to invest more in the market sector ($dK_{m_t} > 0$), the downward-sloping labor demand curve can move outwards so that in the new equilibrium point, both labor

FIG. 8. The first effects (in the indeterminacy region): the cases of $b_H = 0.25$ and $b_H = 0.33$.

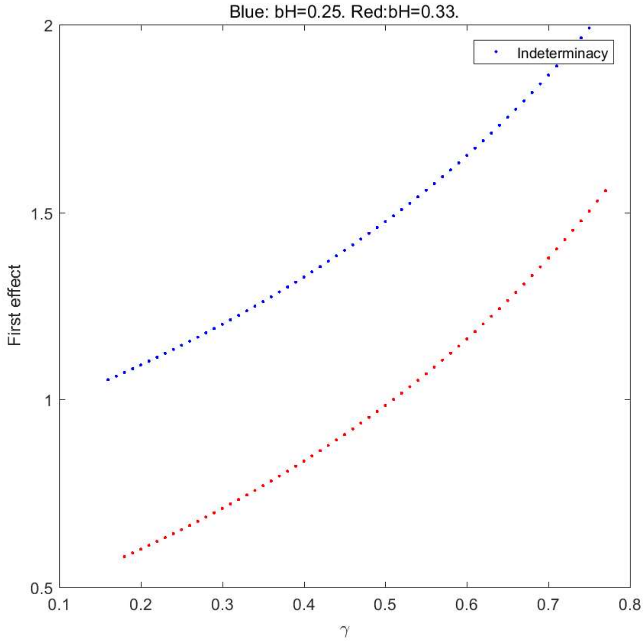
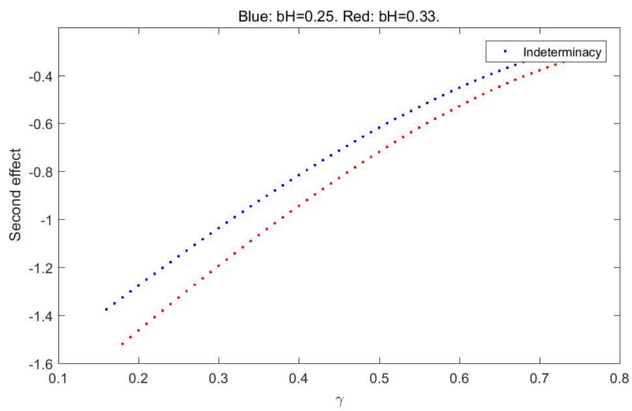


FIG. 9. The second effects (in the indeterminacy region): the cases of $b_H = 0.25$ and $b_H = 0.33$.



and wage rate can be higher than their initial values[5]. Therefore, optimistic expectations are self-fulfilled.

[5]Note that $1 - b + b_H < 1$ holds in this example. This indicates that the aggregate labor demand curve is downward sloping.

Third, Perli (1998) mentioned that consumption and hours worked move in the same direction in the model with a home sector. In the above mechanism, due to the intratemporal condition between labor and consumption $[u_{\mathcal{L}}\left(C_t, \mathcal{L}_t\right) = u_C\left(C_t, \mathcal{L}_t\right) a(\frac{C_t}{C_{m_t}})^{1-\varepsilon} MPL_{m_t}]$[6], the agent will increase her consumption ($dC_t > 0$) because her hours worked increase and she expects her labor income to increase (the intratemporal income effect). When the intratemporal income effect dominates the intratemporal substitution effect, consumption and labor can move in the same direction, which explains why indeterminacy cannot occur if $\gamma = 0$.

Fourth, we notice that as in ASV (2017), in the mechanism described above, indeterminacy results only if the intertemporal condition Eq. (35) holds. And this intertemporal mechanism depends on the income effect. Let us define the first effect as $[\frac{(-b_H+b)}{H_m} + \frac{1}{H\epsilon_{HH}}]$ and the second effect as $[-\frac{1}{C\epsilon_{HC}}]$. Given that $\epsilon_{HH} > 0$ holds under the concavity assumption of the per-period utility function, the optimistic expectations induce the right hand side of Eq. (35) to increase. And indeterminacy arises only if the left hand side of Eq. (35) increases. Letting $b_H = 0.25$, we see that in the determinacy region, the first effect is small and positive and the second effect is large in absolute value and negative. Therefore, Eq. (35) cannot be satisfied because the second effect dominates, thereby making indeterminacy impossible. However, in the indeterminacy region, the second effect is small in absolute value and negative ($\epsilon_{HC} > 0$)[7], and the first effect is large and positive. Thus, Eq. (35) is satisfied when the first effect dominates, and indeterminacy becomes possible.

Lastly, what interests us most is that in the indeterminacy region, the minimum level of the productive externality b_H that induces instability increases as the income effect parameter γ increases. In order to explain this finding in detail, we allow b_H to take two values, 0.25 and 0.33. It is clear that as b_H increases, both of these effects decrease. And the first effect keeps positive and the second effect negative. Thus, these changes make it harder for Eq. (35) to hold. We also notice that increasing γ can increase both of these two effects, thereby making indeterminacy easier to arise. In other words, the larger b_H is, the larger the upper bound of the income effect (γ) that is compatible with local indeterminacy.

6. CONCLUSIONS

We examine a real business cycle with home production, productive externalities, and the Jaimovich and Rebelo's (2009) preferences that allow

[6]This implies that the marginal utility of consumption divided by the marginal utility of leisure equals the marginal product of labor.

[7]This means that consumption and labor are Edgeworth complements.

for a broad range of income effect values. Our findings are summarized as follows. First, we show that when the income effects are intermediary and the level of increasing returns is reasonably high, our model with a home sector can exhibit indeterminacy, provided that the other structural parameters take reasonable values. Second, we explain why the income effect coupled with externalities is crucial to the indeterminacy result and how it induces indeterminacy through the intratemporal and intertemporal mechanisms between consumption and labor. Third, we explain why in the indeterminacy region, the minimum level of the productive externality that induces instability increases as the income effect increases.

APPENDIX A

Because $u(C_t, \mathcal{L}_t) = \frac{[C_t + \left(\frac{\mathcal{L}_t}{B}\right)^{1+\chi} C_t^{\gamma}]^{1-\theta} - 1}{1-\theta}$ is not concave, the local concavity conditions are required. Let $\Delta = C_t + \left(\frac{\mathcal{L}_t}{B}\right)^{1+\chi} C_t^{\gamma}$. We impose the following restriction on $(C_t, \mathcal{L}_t)$. To save space, we omit the time subscript. Condition 1. The domain of $(C, \mathcal{L})$ is that both $C > 0$ and $0 \leq \mathcal{L} \leq 1$ hold.

The local concavity conditions of the utility function are given as follows. $u(C, \mathcal{L})$ is concave if and only if $u_{CC} \leq 0$, $u_{\mathcal{L}\mathcal{L}} \leq 0$, and $u_{CC}u_{\mathcal{L}\mathcal{L}} - u_{C\mathcal{L}}u_{\mathcal{L}C} \geq 0$. First, we calculate the first and second derivatives of $u(C, \mathcal{L})$:

$$u_C(C, \mathcal{L}) = \Delta^{-\theta}[1 + \gamma C^{\gamma-1}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}] > 0, \tag{A1}$$

$$u_{\mathcal{L}}(C, \mathcal{L}) = \Delta^{-\theta} C^{\gamma}\left(\frac{1+\chi}{B}\right)\left(\frac{\mathcal{L}}{B}\right)^{\chi} > 0, \tag{A2}$$

$$u_{CC}(C, \mathcal{L}) = \frac{u_C}{C}\left[-\frac{\gamma(1-\gamma)C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}{C + \gamma C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}} - \theta\frac{C + \gamma C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}{C + C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}\right] \leq 0, \tag{A3}$$

$$u_{\mathcal{L}\mathcal{L}}(C, \mathcal{L}) = \frac{u_{\mathcal{L}}}{\mathcal{L}}\left[\chi - \theta\frac{C^{\gamma}(1+\chi)\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}{C + C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}\right] \leq 0, \tag{A4}$$

$$u_{\mathcal{L}C}(C, \mathcal{L}) = \frac{u_{\mathcal{L}}}{C}\left[\gamma - \theta\frac{C + \gamma C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}{C + C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}\right], \tag{A5}$$

and

$$u_{C\mathcal{L}}(C, \mathcal{L}) = \frac{u_C}{\mathcal{L}}\left[\frac{\gamma C^{\gamma}(1+\chi)\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}{C + \gamma C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}} - \theta\frac{C^{\gamma}(1+\chi)\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}{C + C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}\right]. \tag{A6}$$

To find the local concavity conditions, we have to examine the following conditions:

1. $u_{\mathcal{L}\mathcal{L}} \leq 0 \Leftrightarrow$
$$\frac{C + C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}{C^{\gamma}(1+\chi)\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}\chi \leq \theta. \tag{A7}$$

2. $u_{CC}u_{\mathcal{L}\mathcal{L}} - u_{C\mathcal{L}}u_{\mathcal{L}C} \geq 0$

$\Leftrightarrow$
$$\theta\left[\frac{\gamma C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi} - \chi C}{C + C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}} + \frac{\gamma C^{\gamma}(1+\chi)\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}{C + \gamma C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}\right] \geq \frac{(\chi+\gamma)\gamma C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}{C + \gamma C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}. \tag{A8}$$

Proof of Claim 2. Let $\mathcal{A}_1 = \frac{C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}{C+\gamma C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}$, $\mathcal{A}_2 = \frac{C+\gamma C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}{C+C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}} = \frac{1}{(1-\gamma)\mathcal{A}_1+1}$, $\mathcal{A}_3 = \frac{C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}{C+C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}} = \mathcal{A}_1\mathcal{A}_2$. We get the following equations:

$$\begin{aligned}
u_{CC}(C,\mathcal{L}) &= \frac{u_C}{C}\left[-\gamma(1-\gamma)\mathcal{A}_1 - \theta\mathcal{A}_2\right], \\
u_{\mathcal{L}\mathcal{L}}(C,\mathcal{L}) &= \frac{u_{\mathcal{L}}}{\mathcal{L}}\left[\chi - \theta(1+\chi)\mathcal{A}_3\right], \\
u_{\mathcal{L}C}(C,\mathcal{L}) &= \frac{u_{\mathcal{L}}}{C}(\gamma - \theta\mathcal{A}_2), \\
u_{C\mathcal{L}}(C,\mathcal{L}) &= \frac{u_C}{\mathcal{L}}\left[\gamma(1+\chi)\mathcal{A}_1 - \theta(1+\chi)\mathcal{A}_3\right] \\
&= \frac{u_C}{\mathcal{L}}(1+\chi)\mathcal{A}_1(\gamma - \theta\mathcal{A}_2).
\end{aligned}$$

Then, we have

$$\begin{aligned}
&\frac{u_{CC}u_{\mathcal{L}\mathcal{L}} - u_{C\mathcal{L}}u_{\mathcal{L}C}}{\frac{u_C u_{\mathcal{L}}}{C\mathcal{L}}} = -\gamma(1-\gamma)\chi\mathcal{A}_1 + \gamma(1-\gamma)\theta(1+\chi)\mathcal{A}_1\mathcal{A}_3 \\
&-\theta\chi\mathcal{A}_2 - \gamma^2(1+\chi)\mathcal{A}_1 + 2\gamma\theta(1+\chi)\mathcal{A}_1\mathcal{A}_2 \\
= \;&-\gamma(\chi+\gamma)\mathcal{A}_1 - \theta\chi\mathcal{A}_2 + \gamma\theta(1+\chi)(\mathcal{A}_1+\mathcal{A}_3),
\end{aligned}$$

because $\mathcal{A}_3\left[(1-\gamma)\mathcal{A}_1 + 2\right] = \mathcal{A}_1 + \mathcal{A}_3$. Thus, $u_{CC}u_{\mathcal{L}\mathcal{L}} - u_{C\mathcal{L}}u_{\mathcal{L}C} \geq 0$ $\Leftrightarrow$

$$\theta\left[\frac{\gamma C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi} - \chi C}{C + C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}} + \frac{\gamma C^{\gamma}(1+\chi)\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}{C + \gamma C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}\right] \geq \frac{(\chi+\gamma)\gamma C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}{C + \gamma C^{\gamma}\left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}.$$

We are done.■

Note that B does not depend on θ. The local concavity conditions are simplified as follows:

$$\theta \geq \theta_1(\gamma) = \frac{C + C^\gamma \left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}{C^\gamma (1+\chi) \left(\frac{\mathcal{L}}{B}\right)^{1+\chi}} \chi,$$

and

$$\theta \geq \theta_2(\gamma) = \frac{\frac{(\chi+\gamma)\gamma C^\gamma \left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}{C+\gamma C^\gamma \left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}}{\frac{\gamma C^\gamma \left(\frac{\mathcal{L}}{B}\right)^{1+\chi} - \chi C}{C + C^\gamma \left(\frac{\mathcal{L}}{B}\right)^{1+\chi}} + \frac{\gamma C^\gamma (1+\chi) \left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}{C + \gamma C^\gamma \left(\frac{\mathcal{L}}{B}\right)^{1+\chi}}}.$$

In other words, $\theta \geq \max\{\theta_1(\gamma), \theta_2(\gamma)\}$.

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