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Learning About New Eras

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Non-ergodic regime changes or New Eras (wars, inventions and epidemics) create a form of parameter uncertainty that peaks at the time of the regime change and then falls as sample data is collected. This parameter uncertainty can have a large impact on economic behavior. We propose a welfare measure of the importance of this parameter uncertainty and apply it to the post World War II U.S. economy. Our results suggest it took until the middle of the 1970's before this parameter uncertainty was resolved.

Key Words: Non-ergodic Regime Changes; Bayesian Learning; Parameter Uncertainty.
JEL Classification Numbers: C11, C21, C53, E17.

1. INTRODUCTION

Many of the most important economic regime changes are non-ergodic: something new happens that has never been seen before, and the economy never returns to the old regime. Once electricity, the internal combustion engine, the computer, or the internet is discovered, the economic laws of motion are permanently altered. A non-ergodic regime change announces a New Era. New Eras can be both good, like electricity or computers, and bad, like wars or the recent Covid-19 pandemic.

By way of contrast, business cycles as in Hamilton's (1990) Markov switching model, are a good example of ergodic regime changes. An economy in a recession today will eventually switch to an expansion and then eventually switch back to a recession. This repeated switching back and forth means that the historical record and its associated sample data eventually reveal the frequency of these ergodic regime changes and the parameters that characterize them.

When a non-ergodic regime change occurs, however, there is no historical record with which to estimate the parameters that characterize the

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new laws of motion of the economy. When electricity was first discovered there was no historical record that revealed what the post-electrical world would look like. Instead people had to wait to collect enough data from the New Era with electricity. The uncertainty associated with a New Era then has a distinctive time profile: it is at a maximum at the start of the New Era, and then steadily declines over time as sample data from the New Era is collected. The rather slow parametric rate of learning of $n^{-\frac{1}{2}}$, where n is the sample size, means that it may take considerable time before this parameter uncertainty becomes negligible. In the meantime this parameter uncertainty will influence economic behavior as seen in investment, consumption, saving, and asset prices, as we have explored elsewhere (see Sampson, 1998, 2003, 2022). Given the obvious importance of many of these New Eras, the effects on economic behavior can be very large. Romer (1990 a, b) argues that households delaying investment in consumer durables was an important factor in explaining the Great Depression.

In this paper we develop a measure of the welfare cost of the parameter uncertainty caused by a non-ergodic regime change. The measure is constructed in a manner similar to Lucas's (1987) attempt to measure the potential welfare benefits of stabilization policy. By studying how this welfare measure changes over time, we can answer the question of how long it takes for the parameter uncertainty with respect to the New Era to be resolved.

We apply this methodology to the U.S. post-war economy under the assumption that log consumption follows a random walk with unknown growth rate and standard deviation. We take the end of World War II as signaling a non-ergodic regime change or New Era. In 1947 Americans had within living memory experienced three distinct regimes: the prosperity of the 1920's, the Great Depression of the 1930's, and the war years from 1941-1945. From the perspective of an American in 1946 looking ahead to the post-war New Era, the nature of the post-war New Era must have appeared very uncertain. Would drastic reductions in military expenditure, credited by many with bringing the economy out of the Great Depression, now return the economy back to something like the Great Depression? Or would fiscal and monetary policy be able to stabilize the economy and ensure steady growth? The results of this paper suggest, using a variety of scenarios, that it took about 25 years, to about the mid 1970's, for the parameter uncertainty with regard to the post-war New Era to be resolved.

A number of papers appearing in the *Journal of Political Economy* at the time reflected these concerns. Klein (1946) cites press reports in the autumn of 1945 that "Government economists predict 8 million unemployed by 1946" (the US labour force at the time was approximately 60 million). Woytinsky (1947) cites forecasts of unemployment as high as 20 million with a margin of error of 10 million. In 1946 Americans had no way of knowing this: they had to wait until they could collect enough sample data from the post-war economy to convince themselves that the post-war era would be characterized by growth and relative stability.

It is reasonable to suppose that something as fundamental as uncertainty about the nature of the post-war economy would also influence macroeconomic theory. In particular it is a curious fact that while the idea of rational expectations existed in the early 1960's, it took over 10 years for the rational expectations revolution to take place in economics. However the timing of the rational expectations revolution, with its typical assumption that agents know the underlying parameters that characterize the economy, can be explained with our results: that it was only in the 1970's that the parameter uncertainty was resolved with respect to the post-war economy.

2. NON-ERGODIC REGIME CHANGES

Fundamental to this paper is the distinction between ergodic and non-ergodic regime changes. The difference between these two types of regime changes can be illustrated using the theory of Markov chains.

Business cycles, as found in Hamilton (1989), are a good example of ergodic regime changes, where the economy switches back and forth between a recession and an expansion regime. Consider for example an economy with an ergodic business cycle that has two regimes with expected growth rates and standard deviations μ_i, σ_i for $i = 1, 2$. Regime 2 is a recession with lower growth and greater volatility so that $\mu_1 > \mu_2$ and $\sigma_1 < \sigma_2$. The transitions between the two regimes are governed by an ergodic Markov chain with transition matrix P and long-run equilibrium distribution p satisfying $pP = p$ where for example

$$P = \begin{bmatrix} 0.99 & 0.01 \\ 0.09 & 0.91 \end{bmatrix} \text{ and } p = \begin{bmatrix} 0.9 & 0.1 \end{bmatrix}.$$

What makes P here ergodic is that both off-diagonal elements are non-zero, thus ensuring that both regimes occur infinitely often. The vector p indicates the economy will be in an expansion 90% of the time and in a recession 10% of the time. Given a long enough sample there will be enough observations from both regimes to accurately estimate P and μ_i, σ_i for $i = 1, 2.$, thus justifying the typical rational expectations assumption that agents know these parameters.

An example of a non-ergodic transition matrix that generates a New Era in Regime 2 is

$$P = \begin{bmatrix} 0.99 & 0.01 \\ 0 & 1 \end{bmatrix}$$

where μ_1, σ_1 are the growth rate and standard deviation of Regime 1, the old regime, and μ_*, σ_* are the growth rate and standard deviation of Regime 2, the New Era. Assume the economy begins in Regime 1. Each year there is a 0.01 probability of switching to Regime 2, the New Era. This will take about 100 years so that agents likely will be able to accurately infer μ_1, σ_1 from this data. But once the economy switches to Regime 2 it never returns to Regime 1, so that the 100 years of sample data from Regime 1 ceases to be relevant.[1] At the beginning of the New Era agents only have prior information on μ_*, σ_*. They will only know μ_*, σ_* once they have collected enough sample data from Regime 2. In the meantime the economy will be subject to the parameter uncertainty associated with the unknown μ_*, σ_*, which will be at a maximum at the time of the regime change, and will then fall as more sample data is collected. This parameter uncertainty will then influence consumption, savings, investment, and asset prices. The basic question this paper asks is: how long will it take before agents have enough sample data from the New Era that the parameter uncertainty associated with μ_*, σ_* becomes negligible enough not to significantly influence economic behavior?

3. THE WELFARE COST OF PARAMETER UNCERTAINTY

We assume that the New Era begins at $t = 0$ and that time is discrete as $t = 0, 1, 2, \ldots$ so that $t > 0$ is the age of the New Era. In this paper we assume that consumption follows a logarithmic random walk with drift, but the methodology that follows is easily generalized. Let $c_t \equiv \ln(C_t)$ be the log of real consumption C_t and assume constant relative risk aversion with welfare

$$W_t = -E_t \left[\sum_{\tau=0}^{\infty} \exp(-\delta\tau + \theta c_{t+\tau}) \right] \tag{1}$$

where δ is the rate of discount and $1 + \theta$ is the coefficient of relative risk aversion. Consumption growth $\Delta c_t \equiv c_t - c_{t-1}$ is *i.i.d.* and normally distributed as

$$\Delta c_t \sim N\left[\mu, \sigma^2\right]. \tag{2}$$

At $t = 0$ the true μ, σ given by μ_*, σ_* are drawn from a probability distribution with density $p(\mu, \sigma)$. Here μ_*, σ_* characterize the actual but unknown nature of the New Era. Agents know $p(\mu, \sigma)$ which then acts as a Bayesian prior.

[1] A more realistic transition matrix would allow for an exit from Regime 2 to another Regime 3 and so on, but without the possibility of a return to any previous regime.

As time progresses agents combine their prior beliefs $p(\mu, \sigma)$ with the historical record up to time t given by $I_t = \{c_0, c_1, \ldots, c_t\}$ to form a posterior distribution $p(\mu, \sigma | I_t)$. By first conditioning on μ and σ it follows from (1) and (2) that

$$W_t = -\exp(-\theta c_t) E_t \left[f\left(\delta + \theta\mu - \frac{\theta^2}{2}\sigma^2 \right) \right] \tag{3}$$

where the expectation E_t is over the unknown μ and σ using $p(\mu, \sigma | I_t)$ and $f(x) = \frac{1}{1-e^{-x}}$. The actual construction of the posterior and the computation of $E_t \left[f\left(\delta + \theta\mu - \frac{\theta^2}{2}\sigma^2 \right) \right]$ is described in next section.

As t increases the increasing sample information $I_t = \{c_0, c_1, \ldots, c_t\}$ will result in a posterior $p(\mu, \sigma | I_t)$ that is more and more concentrated around the true nature of the post-war economy: μ_*, σ_*. There will thus be some date T by which time for all practical purposes we can assume that μ_*, σ_* is revealed as

$$E_T \left[f\left(\delta + \theta\mu - \frac{\theta^2}{2}\sigma^2 \right) \right] \approx f\left(\delta + \theta\mu_* - \frac{\theta^2}{2}\sigma_*^2 \right).$$

To measure the economic importance of the parameter uncertainty with respect to μ, σ we calculate the permanent proportion of annual consumption y_t that the agent would be willing to give up in return for having the information at T; that is to know the nature of the New Era. This y_t satisfies

$$\begin{aligned} & -\exp(-\theta c_t) E_t \left[f\left(\delta + \theta\mu - \frac{\theta^2}{2}\sigma^2 \right) \right] \\ = & -\exp(-\theta (c_t - y_t)) E_T \left[f\left(\delta + \theta\mu - \frac{\theta^2}{2}\sigma^2 \right) \right] \end{aligned}$$

and so

$$y_t = \frac{1}{\theta} \ln \left(\frac{E_t \left[f\left(\delta + \theta\mu - \frac{\theta^2}{2}\sigma^2 \right) \right]}{E_T \left[f\left(\delta + \theta\mu - \frac{\theta^2}{2}\sigma^2 \right) \right]} \right). \tag{4}$$

A similar approach is used in Lucas (1987) to measure the potential welfare benefits of stabilization policy.

4. COMPUTATION

We first consider the construction of the posterior $p(\mu, \sigma|I_t)$. We assume that the prior density $p(\mu, \sigma)$ is bounded[2] as

$$\mu_{\min} \leq \mu \leq \mu_{\max} \text{ and } \sigma_{\min} \leq \sigma \leq \sigma_{\max}.$$

Now we factor $p(\mu, \sigma)$ as $p(\mu, \sigma) = p(\mu|\sigma) \times p(\sigma)$ where $p(\mu|\sigma)$ and $p(\sigma)$ are

$$\mu|\sigma \sim N\left[\hat{\mu}_o, \frac{\sigma^2}{t_0}, \mu_{\min}, \mu_{\max}\right] \text{ and } \frac{t_0\hat{\sigma}_o^2}{\sigma^2} \sim \chi^2_{t_0}\left[\sigma_{\min}, \sigma_{\max}\right] \tag{5}$$

which are truncated normal and chi-squared distributions with densities defined respectively over $\mu_{\min} \leq \mu \leq \mu_{\max}$ and $\sigma_{\min} \leq \sigma \leq \sigma_{\max}$.

The prior parameters $\hat{\mu}_o$ and $\hat{\sigma}_0$ determine the agent's belief of the most likely values of the growth rate μ and volatility σ. The prior parameter t_0 determines the precision of the prior beliefs. Since $p(\mu, \sigma)$ is a conjugate prior it is equivalent to observing t_0 years of consumption growth. As t_0 increases the priors become more and more concentrated around $\hat{\mu}_0$ and $\hat{\sigma}_0$ with the limit $t_0 = \infty$ corresponding to knowing $\mu = \mu_0$ and $\sigma = \sigma_0$with certainty.

At time t people observe the sample $I_t = \{c_o, c_1, \ldots, c_t\}$. They then combine the sample information I_t with the prior information in (5) to form a posterior for μ and σ as

$$p(\mu, \sigma|I_t) = p(\mu|\sigma, I_t) \times p(\sigma|I_t)$$

where from standard Bayesian results $p(\mu|\sigma, I_t)$ is

$$\mu|\sigma \sim N\left[\hat{\mu}_t, \frac{\sigma^2}{t + t_o}, \mu_{\min}, \mu_{\max}\right] \tag{6}$$

and $p(\sigma|I_t)$ is

$$(t + t_0)\frac{\hat{\sigma}_t^2}{\sigma^2} \sim \chi^2_{t+t_0}\left[\sigma_{\min}, \sigma_{\max}\right]. \tag{7}$$

Here $\hat{\mu}_t$ and $\hat{\sigma}_t$, agents' expectations of the most likely values of μ and σ, are

$$\hat{\mu}_t = \frac{t_0\hat{\mu}_o}{t + t_o} + \frac{t\bar{\mu}_t}{t + t_o}, \; \hat{\sigma}_t^2 = \frac{t_0\hat{\sigma}_o^2 + t\bar{\sigma}_t^2}{t + t_0} + \frac{t_0 t\left(\hat{\mu}_o - \bar{\mu}_t\right)^2}{\left(t + t_0\right)^2} \tag{8}$$

where the sample mean and variance are

$$\bar{\mu}_t = \frac{1}{t}\sum_{\tau=1}^{t} \Delta c_\tau, \; \bar{\sigma}_t^2 = \frac{1}{t}\sum_{\tau=1}^{t} \Delta c_\tau^2 - \bar{\mu}_t^2.$$

[2]Relaxing the assumption of bounded priors results in an infinite y_t.

Now (6) and (7) determines E_t used to calculate the welfare measure y_t in (4) as

$$E_t\left[f\left(\delta+\theta\mu-\frac{\theta^2}{2}\sigma^2\right)\right]=\int_{\mu_{\min}}^{\mu_{\max}}\int_{\sigma_{\min}}^{\sigma_{\max}} f\left(\delta+\theta\mu-\frac{\theta^2}{2}\sigma^2\right)p\left(\mu,\sigma|I_t\right)d\mu d\sigma.$$

We calculate this integral using Monte Carlo integration.

For the i^{th} replication and $t+v>0$, generate $t+v$ squared independent standard normals Z_{ij} for $j=1,2,\ldots,t+v$ and let $\chi^2_{i,t+v}=\sum_{j=1}^{t+v}Z^2_{ij}$, which has a chi-squared distribution with $t+v$ degrees of freedom. If $\sigma_{\min}\le\sigma_i\le\sigma_{\max}$ is satisfied where

$$\sigma_i=\sqrt{\frac{(t+v)\,\hat{\sigma}^2_t}{\chi^2_{i,t+v}}}$$

then this is the i^{th} draw from the posterior distribution of σ given in (7). If the bounds $\sigma_{\min}\le\sigma_i\le\sigma_{\max}$ are not satisfied, this draw is rejected and a new draw from the χ^2 distribution is generated until the bounds are satisfied.

A draw from the conditional posterior of μ in (6) is generated by drawing a standard normal, say $\tilde{Z}_i$, and if $\mu_{\min}\le\mu_i\le\mu_{\max}$ is satisfied, the i^{th} draw of μ will be

$$\mu_i=\hat{\mu}_t+\tilde{Z}_i\frac{\sigma_i}{\sqrt{t+t_o}}.$$

If $\mu_i<\mu_{\min}$ or $\mu_i>\mu_{\max}$ this draw of $\tilde{Z}_i$ is rejected, and new draws are generated until $\mu_{\min}\le\mu_i\le\mu_{\max}$ is satisfied.

Once n draws of μ_i and σ_i for $i=1,2,...,n$ have been generated then $E_t\left[f\left(\delta+\theta\mu-\frac{\theta^2}{2}\sigma^2\right)\right]$ is consistently estimated by

$$\hat{E}_t\left[f\left(\delta+\theta\mu-\frac{\theta^2}{2}\sigma^2\right)\right]\equiv\frac{1}{n}\sum_{i=1}^{n}f\left(\delta+\theta\mu_i-\frac{\theta^2}{2}\sigma_i^2\right).$$

In the calculations in the next section we used $n=50,000$.

5. RESULTS

We assume that $t=0$, the beginning of the post-war New Era begins for the American economy, is 1947. For the consumption growth series Δc_t we use annual U.S. real per-capita consumption growth from 1891-1983.[3]

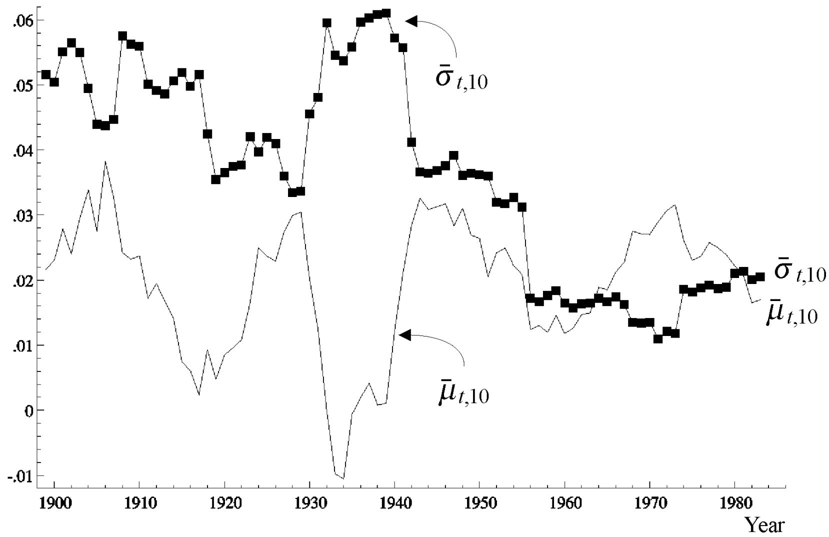
[3] The real consumption series is the Kendrick series for 1890-1945 in 1929 prices and the NIPA series for 1945-1983 as found in the Appendix B constructed by N. Balke

To guide us in the choice of priors we calculate the ten-year rolling sample mean and standard deviation

$$\bar{\mu}_{t,10} = \frac{1}{10}\sum_{i=0}^{9}\Delta c_{t-i},\ \bar{\sigma}_{t,10} = \sqrt{\frac{1}{10}\sum_{i=0}^{9}\Delta c_{t-i}^2 - \bar{\mu}_{t,10}^2} \tag{9}$$

shown in Figure 1.

FIG. 1.



It is apparent from Figure 1 that the economy behaves differently after 1947 with greater stability and growth.

We chose three different priors designed to reflect the range of beliefs in 1946. The Very Pessimistic prior reflects a belief that the post-war economy would contract $\hat{\mu}_o = -0.01$ and be highly unstable $\hat{\sigma}_o = 0.1$, as given by the worst values of $\bar{\mu}_t^{10}$ and $\bar{\sigma}_t^{10}$ in 1932 during the Great Depression. The Pessimistic prior reflects a belief that the post-war economy would be stagnant $\hat{\mu}_o = 0$ and moderately unstable $\hat{\sigma}_o = 0.05$, as suggested by the values of $\bar{\mu}_{t,10}$ and $\bar{\sigma}_{t,10}$ at the end of the Great Depression in 1939. Finally the Optimistic 'prior with $\hat{\mu}_o = \hat{\sigma} = 0.02$ reflects a belief in stable growth

and R. Gordon in Gordon (1986). U.S. population comes from the U.S. Department of Commerce, Bureau of the Census (1977) *Historical Statistics of the United States*, series A7 from 1919-1929, series A6 from 1930-1969, and from the *Economic Report of the President* (1989) from 1970-1983.

as typified by the actual post-war experience. We chose relatively diffuse priors with $t_0 = 10$ to ensure that it is the post-war consumption data stream that reveals that the economy would both grow $\mu_* = 0.02$ and be stable $\sigma_* = 0.02$.[4]

These as well as the other required parameters needed are given below. We take $T = 37$ or the year 1983 as the time by which the nature of the post-war era has been revealed.

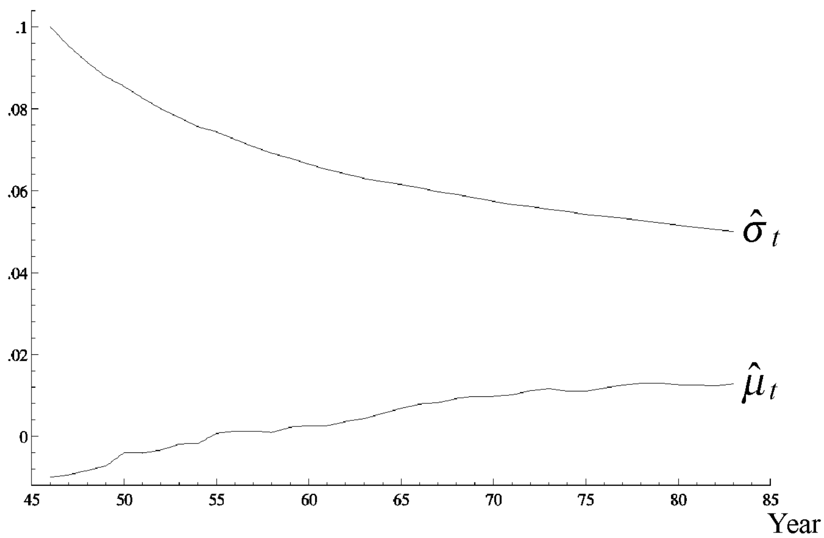
TABLE 1.

Priors

Prior	$\hat{\mu}_o$	$\hat{\sigma}_o$	t_0	θ	δ	$\mu_{\min}$	$\mu_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$
Very Pessimistic	−0.01	0.10	10	1	0.05	−0.03	0.05	0.005	0.15
Pessimistic	0.00	0.05	10	1	0.05	−0.01	0.10	0.005	0.10
Optimistic	0.02	0.02	10	1	0.05	−0.01	0.10	0.005	0.10

In Figure 2 we see how the Very Pessimistic agent learns about the nature of the post-war regime over time, as reflected in $\hat{\mu}_t$ and $\hat{\sigma}_t$. In 1947 the agent believed growth will be very low and volatility very high, but by the early 1980's the agent had learned that growth will be closer to $\mu = 0.02$ and that volatility will be less than initially thought.

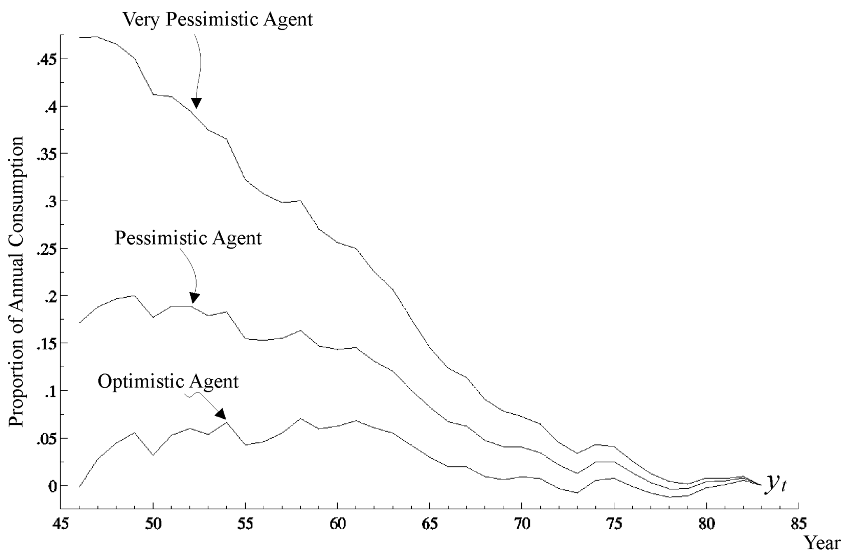
FIG. 2.



[4]In the limit $t_0 = \infty$ agents would believe that $\mu_* = \mu_0$ and $\sigma_* = \sigma_0$ independent of I_t.

In Figure 3 the welfare measure y_t in (4) is plotted for the Very Pessimistic, Pessimistic and Optimistic agents using $T = 37$ or 1983 as the comparison year. All three agents are much less concerned about a return to the Great Depression as time progresses, but the learning process is slow. The agents are concerned about a return to the Great Depression well into the 1960's. In 1961 even the Optimistic agent had $y_t = 0.068$ so that he or she would be willing to sacrifice 6.8% of annual consumption to resolve the uncertainty concerning the post-war economy. The Very Pessimistic and Pessimistic agents were quite concerned with $y_t = 0.25$ and $y_t = 0.15$ respectively.

FIG. 3.



The later part of the 1960's saw most of the concern about the nature of the post-war era disappear as it became clear that the American economy would not slip back into the Great Depression. Thus by 1972 the Optimistic agent has stopped worrying with $y_t \approx 0.00$. The Pessimistic agent was still concerned with $y_t = 0.021$ and even the Very Pessimistic agent had $y_t = 0.045$.

These results can be used to explain some of the development of macroeconomic theory in the post-war era. The year 1961 saw Muth's (1961) publication of the idea of rational expectations. Even though Muth's paper was published in a prominent journal, the idea of rational expectations was not incorporated into macroeconomics until the 1970's with the publication of Lucas (1972). While one could always appeal to intellectual inertia in explaining this decade long delay, it seems more than reasonable to suppose that it was the fact that most macroeconomists, like the three agents in this paper, had not yet settled on what the nature of the post-war New Era would be, and until that happened, would not find it credible to adopt rational expectations where agents are assumed to know the underlying parameters that characterize the economy.

6. CONCLUSION

A non-ergodic regime change or New Era results in a spike in uncertainty at the time of the regime change, with only a slow reduction in uncertainty as agents collect sample data from the new regime. In this paper we have developed a way of measuring the welfare cost of the associated parameter uncertainty. When applied to the US post-war economy we find that it took about 25 years, that is to the middle of the $1970's$, for the effect of this welfare cost of this parameter uncertainty to become economically insignificant.

This methodology in this paper can be easily applied to other historical events where the time of the New Era is easily identified, and the New Era exists long enough for sample data to effectively reveal its underlying parameters. In this paper we assumed a logarithmic random walk for the data generating mechanism for consumption, but it is straightforward using Bayesian numerical methods to apply this methodology to other time series models with deterministic trends, serial correlation, or conditional volatility.

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