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Economic Growth Theory in the Twenty-First Century

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Economic growth theory studies the dynamic forces and complex interactions that enable societies to progressively increase their material well-being. Neoclassical and endogenous growth models, of the 1950s-1960s and of the 1980s-1990s, respectively, are unanimously recognized as the main building blocks of this theory. Notwithstanding, recent academic work has been able to contribute, as well, with important new insights, which have decisively strengthened and refreshed the theory. This selective survey of up-to-date literature on growth theory highlights some of the most recent essential contributions for the discipline, offering a critical assessment about its current state and prospects for its future development.

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1. INTRODUCTION: A VERY CONCISE CONSENSUAL HISTORY OF GROWTH THEORY

If asked to tell a brief history of the theory of economic growth, researchers working on this field of knowledge surely would not be able to avoid pointing out two main building blocks: the neoclassical growth theory of the 1950s-1960s and the endogenous growth theory of the 1980s-1990s. Faced precisely with that challenge, Akcigit (2017) follows the predictable plan of action, highlighting how the first theory was decisive in explaining capital accumulation and the exhaustibility of growth, and how the second theory was capable of introducing sustained long-term growth via investment in R&D and / or via accumulation of human capital.

The key features supporting each of the theories are well known. Either assuming an exogenous savings rate (the Solow-Swan model) or endogenous intertemporal consumption choices (the Ramsey-Cass-Koopmans model),

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the neoclassical growth theory characterizes the process under which an economy that accumulates physical capital converges to a steady state of zero growth in the absence of technological progress, given the diminishing marginal returns property associated with the accumulation of the capital input. In this view, sustained growth can only be attributed to technological progress, although the theory is silent on how innovation eventually takes place.

The endogenous growth theory takes one step forward, by investigating the conditions under which diminishing returns may be overturned. Investment in education and purposive R&D effort are the strongest candidates to achieve such desideratum. The sharing of ideas in schools, laboratories, and firms, enables the economy to innovate and to engage in processes of creative destruction, such that relatively old and low-productivity ideas and techniques are systematically replaced by new and more productive ones.

This survey evaluates how growth theory evolved in the last few years, by standing on the shoulders of neoclassical growth and endogenous growth paradigms, and also by exploring new avenues of research. In order to achieve the mentioned goal, a selective review of recent literature published in top economics journals is undertaken.[1] The surveyed literature will allow to address the following topics: innovation-based growth, with an emphasis on industrial organization, firm dynamics and creative destruction (section 2); heterogeneity in innovation, science, and labor skills (sections 3 and 4); the interplay between environmental protection, innovation and growth (section 5); income convergence and divergence dynamics between technological leaders and the rest of the world (section 6); and contagion mechanisms through which ideas and productivity may spread (section 7).

Besides the above topics, the survey also approaches alternative frameworks for growth analysis, namely those associated with local interaction networks and agent-based models (section 8); it will address, as well, other less conventional topics, involving uncertainty and the possibility of undesirable growth outcomes (section 9); sections 10 and 11 are dedicated to the discussion of the role of relevant new inputs, namely robotic capital and digital data. Section 12 concludes with a synthesis and a few remarks on the way ahead.

A familiar framework. In the remainder of this first section, and in order to organize ideas and to set the stage for the discussion along the es-

[1]Thirty articles are surveyed, from thirteen economics journals, published between the years 2009 and 2022. The journals with more mentions on the reference list are the Journal of Political Economy (6 articles), the Journal of Economic Growth (5 articles), the American Economic Review (3 articles), and the Journal of Economic Dynamics and Control (3 articles).

say, a typical optimal growth problem is sketched. The problem involves, in this initial approach, five variables and three parameters. Variables are consumption, $C(t)$; physical capital (a composite measure of the collection of intermediate inputs used in the production of a final good), $K(t)$; the population level, the amount of labor, or the dimension of the representative household (which are identical entities in this setting), $L(t)$; the efficiency of the labor input when applied to the production of final goods, $h_Y(t)$; and the share of labor allocated to the production of final goods, $u_Y(t)$. The relevant parameters are the rate of time preference, $\rho \geq 0$; the depreciation rate of physical capital, $\delta \in (0,1)$; and the exogenous rate of population growth, $n \geq 0$.

The objective function of the representative agent corresponds to the intertemporal flow of instantaneous utilities, duly discounted to the initial date of the planning problem, i.e.,

$$U(0) = \int_0^{+\infty} e^{-\rho t} u \left[\frac{C(t)}{L(t)} \right] L(t) dt \tag{1}$$

Observe, in equation (1), that utility at date t corresponds to individual utility, of each household's member, multiplied by the dimension of the household. The representative agent will maximize (1), by controlling the time trajectory of consumption, and taking into consideration a dynamic resource constraint, which is an equation of motion representing the process of physical capital accumulation. The relevant differential equation is expressed under the form,

$$\dot{K}(t) = F[K(t), h_Y(t)u_Y(t)L(t)] - C(t) - \delta K(t), \quad K(0) = K_0 \text{ given} \tag{2}$$

In equation (2), $Y(t) = F[K(t), h_Y(t)u_Y(t)L(t)]$ defines the aggregate productionfunction. Functions u and F obey to a few standard properties. They are both continuous, differentiable, and concave. Marginal utility is positive and diminishing ($u' > 0$, $u'' < 0$); marginal input returns are also positive and diminishing ($F_K > 0$, $F_{KK} < 0$, $F_L > 0$, $F_{LL} < 0$); and returns to scale in production are constant. Function F is commonly designated neoclassical production function.

Defining per capita variables, the optimal control problem that characterizes aggregate growth is presented in intensive form. Let $k(t) \equiv \frac{K(t)}{L(t)}$ and $c(t) \equiv \frac{C(t)}{L(t)}$. The optimal growth problem, which poses an intertemporal trade-off between consumption and capital accumulation, takes the form

$$\begin{aligned} &\max \int_0^{+\infty} e^{-(\rho-n)t} u[c(t)]dt \\ \text{subject to:}\quad &\dot{k}(t) = f[k(t), h_Y(t)u_Y(t)] - c(t) - (n+\delta)k(t), \\ &k(0) = k_0 \ \text{ given} \end{aligned} \tag{3}$$

The nature of growth, namely if the economy converges to a zero-growth steady state or if it exhibits sustained long-term growth, fundamentally depends on the essence of the production function. In the absence of any process of innovation or skill acquisition capable of reverting the diminishing marginal returns associated with the accumulation of inputs, the economy converges to the zero-growth neoclassical equilibrium. Endogenous growth emerges whenever new and better tools for production and / or new and better skills are generated by the purposive effort of people and institutions in the economy.

Telling the story of contemporaneous growth theory is, to a considerable extent, an exercise in documenting how human ingenuity always finds ways to keep the bicycle of progress running. In other words, contemporaneous research focuses on new and increasingly sophisticated forms of explaining the endogenous and intrinsically sustained nature of the process of growth, as the sections that follow will allow to verify.

2. MICRO-FOUNDATIONS OF FIRM DYNAMICS AND CREATIVE DESTRUCTION

The Schumpeterian growth paradigm, grounded on the notion of creative destruction, is one of the pioneering interpretations of endogenous growth, and probably the one that best survived the test of time (Aghion et al., 2015). This class of growth models is built upon the analysis of purposive R&D effort and, thus, upon the analysis of the determinants of the investment of firms in innovation. The growth result is, in this context, shaped by market competition and firm dynamics; firm dynamics involve entry and exit, expansion to new product lines, and reallocation of resources across production units.

In a benchmark creative destruction model, the economy is structured around three sectors: the final goods sector; the sector that produces intermediate goods (i.e., capital inputs, which come in many different varieties); and the R&D sector. The creative sector (R&D) generates the ideas or blueprints that materialize into intermediate goods. Intermediate goods, in turn, once produced, will form a composite measure of capital that will be employed in the production of the economy's final good,

$$k(t) = \left[\int_0^{\Omega(t)} k(t;\omega)^{(\sigma-1)/\sigma} d\omega\right]^{\sigma/(\sigma-1)}, \quad \sigma > 1 \tag{4}$$

In expression (4), $k(t;\omega)$ is the stock of the input of variety ω, and $\Omega(t)$ represents the technology frontier, which indicates how many differentiated inputs or varieties of intermediate goods are available, at date t, given the state of techniques. Parameter σ is the elasticity of substitution between intermediate inputs.

In this class of models, innovation can signify two different processes: an improvement in the quality of existing product lines or varieties, what allows to increase the productivity in the production of the respective intermediate goods; and an expansion of the number of product lines (i.e., of the number of varieties), what signifies an increase in the value of $\Omega(t)$. Increasing quality and expanding variety both contribute to the process of creative destruction, i.e., to the process of replacement of technologies where the new ones turn the old obsolete.

The notion of creative destruction builds upon the implicit idea that firms are heterogeneous in their capacity to innovate (Acemoglu et al., 2018). The heterogeneity in the ability to develop new technologies will create an asymmetric market, where large and small firms coexist, and where those with the highest technological capabilities potentially thrive and grow, while the remaining firms eventually head to extinction. Such interpretation of the organization of industries is possible under the definition of firm adopted in this context: a firm is a collection of product lines or input varieties. Firms may operate a large or a small number of product lines, depending on the capacity to innovate and, thus, on the capacity to add new product lines to their portfolio.

For each variety, the corresponding production function must consider two arguments. One of them is labor, and the other is a measure of the quality of the product line, which translates, as well, the productivity associated with the production of the input. Analytically,

$$k(t;\omega) = f[q(t;\omega), h_\omega(t)u_\omega(t)], \quad f_q > 0; \quad f_{hu} > 0 \tag{5}$$

In equation (5), $u_\omega(t)$ is the share of labor allocated to the production of variety ω and $h_\omega(t)$ is the corresponding average labor efficiency. Variable $q(t;\omega)$ is the quality / productivity associated with this specific production process. Improvements in $q(t;\omega)$ are the outcome of purposive innovation effort (the next section explains how) and, hence, they are one of the fundamental sources of endogenous growth in Schumpeterian growth models.

Increases in quality promote growth on the aggregate and, at a micro level, they rearrange industries. If the increase in quality takes place in a product line that the firm already develops, this just triggers an internal rearrangement of the firm's activity. However, if the increase in quality that one firm achieves corresponds to a variety that was previously explored by other firm, this conducts to a reallocation of resources and, most importantly, to a destructive shock for the firm that loses the ability to competitively continue to supply the product line in question.

If a firm loses all its product lines because of the more efficient innovation capabilities of the other firms, it will become obsolete and exit the market. Since innovation can be pursued not only by incumbents but also by new firms, a movement of entry and exit in the market will follow the creative destruction process. Because creative destruction implies replacing existing inputs by inputs of higher quality (more productive capital varieties), this is also a process leading to sustained long-term growth.

The evidence that firms are heterogeneous in their innovation capabilities opens the door for the analysis of markets for ideas (Akcigit et al., 2016). Innovation is a process involving uncertainty and firms have no way of knowing, with accuracy and in anticipation, what the outcome of the innovation process will be. Therefore, when discovering a new idea, its owner, i.e., the firm that holds the patent, has a decision to make: to keep the patent and to use it to produce a new input variety, or to sell it to another firm, a firm that will find it advantageous to purchase the patent and to implement the respective idea.

To assess whether the firm will keep or sell the idea, it is important to realize that each firm operates within a specific technology class, and that the discovered idea may fall inside or outside such class. If it falls outside, the firm will find it advantageous to sell it in the market for patents (and eventually to buy patents for ideas that are inside its technological domain). The value of ideas for a specific firm depends on the distance of the idea to the firm's main line of business, and therefore it varies from one firm to another. Let ω be the main line of business and ω' the new idea. The relevant distance is the measure $d(\omega, \omega')$. Therefore, the firm will keep the patent and profit with it if $d(\omega, \omega')$ is low, and the firm will sell it if $d(\omega, \omega')$ is high.

Let $q(t; \omega')$ be the quality of product line ω' at date t. For the firm that develops product line ω, the productivity in the production of the intermediate good will now not only depend on the quality of the idea, as in (5), but also on the distance to its own technology class. This can be formalized as follows, by inserting a new term in equation (5),

$$k(t; \omega|\omega') = f[q(t; \omega')e^{-d(\omega,\omega')}, h_{\omega'}(t)u_{\omega'}(t)] \tag{6}$$

In (6), $k(t;\omega|\omega')$ is the stock of intermediate input ω' that a firm that has ω as its main line of business can produce. Observe that if $d(\omega,\omega')=0$, the firm generates the same value producing any of the two varieties, that is, using any of the two ideas. As the distance increases, the return of the firm when engaging in the production of variety ω' becomes progressively smaller. If the return of selling the patent at date t is some value $\rho(t;\omega')$, the threshold separating the keeping and selling possibilities may be represented as the point in which $\int_{\tau=t}^{\infty} e^{-r\tau}k(\tau;\omega|\omega')d\tau=\rho(t;\omega')$, with $r\geq 0$ a given rate of time preference. The condition signifies that the value of selling the patent must compensate for all the return the firm would get, now and in the future, from producing the intermediate good associated with idea ω'.

The Schumpeterian growth view is flexible enough to account for features that go much beyond the organization of production and firm dynamics in growing economies. Aghion et al. (2016) explore one of such avenues, associated with the study of the impact of innovation on employment. Combining a creative destruction innovation environment with a search and matching labor market framework, the authors argue that innovation simultaneously creates and destroys jobs, thus increasing job turnover, but having no significant net effect on the aggregate level of employment.

Overall, the impact of creative destruction on households' welfare is expected to be positive, because besides the above-mentioned employment dynamics, households unequivocally benefit, in terms of life satisfaction, from the acceleration of growth due to innovation. Although creative destruction is a process of displacement and deployment of workers, with all the inconveniences it may bring, it should be fundamentally perceived as a welfare enhancing process.

3. INNOVATION DYNAMICS: RADICAL VS INCREMENTAL INNOVATION; BASIC VS APPLIED RESEARCH

With the above characterization of the Schumpeterian growth setup, it was highlighted that innovation can have two distinct meanings: it might relate to the improvement of quality of blueprints already in use, or it can be associated with the expansion of the number of blueprints available to produce different varieties of intermediate goods. Such characterization has been, thus far, silent about which factors determine the dynamics underlying the evolution of each of the involved variables, i.e., $q(t;\omega)$ and $\Omega(t)$. In what follows, the mentioned dynamics are approached, what allows as well to briefly discuss the typology of innovations and research.

A popular distinction is the one between incremental and radical innovations. Incremental innovations add new knowledge to existing product lines and are typically subject to diminishing returns; radical innovations have pervasive effects, that typically go much beyond the specific product lines in which they eventually originate. Radical innovations are the drivers of economic growth, i.e., the force behind the expansion of $\Omega(t)$. Acemoglu et al. (2022) investigate the origins of radical innovations, placing their focus on the contribution of younger managers and inventors. Young managers and young inventors allegedly have a comparative advantage in radical innovation, because they have acquired general skills more recently.

Analytically, given the characterized features, the dynamics of each type of innovation can be modeled as follows. Starting with incremental innovations, let:

$$\begin{aligned}\dot{q}(t;\omega) &= f[q(t;\omega),\bar{q}(t),h_{q\omega}(t)u_{q\omega}(t)]\alpha^{\tilde{n}}, \\ &\quad f_q>0; f_{\bar{q}}>0; f_{hu}>0; \alpha\in(0,1); \tilde{n}\in\mathbb{N}\end{aligned} \tag{7}$$

The quality of intermediate good $q(t;\omega)$ will improve under an innovation process that combines, as inputs, the productivity of variety ω, the average quality of all varieties, $\bar{q}(t)=\frac{\int_0^{\Omega(t)} q(t;\omega)d\omega}{\Omega(t)}$, and the share of human capital allocated to the corresponding research activity, $h_{q\omega}(t)u_{q\omega}(t)$. Besides these inputs, the change in quality also depends on a term $\alpha^{\tilde{n}}$ that translates the diminishing returns associated incremental innovation; $\tilde{n}$ is the number of incremental innovations and, as modeled, the term signifies that the gains of quality decrease geometrically with the number of incremental innovations that the quality of the intermediate good has already gone through.

Radical innovations are not subject to diminishing returns. On the contrary, they spill over to new product lines. As these new product lines emerge, others become obsolete and disappear and, thus, the scenario is again one of creative destruction in innovation. Radical innovation triggers novel ideas and can be associated with the expansion of the knowledge frontier, i.e., with the increase in $\Omega(t)$. This can be expressed under the form:

$$\dot{\Omega}(t)=f[\Omega(t),\bar{q}(t),h_{\Omega}(t)u_{\Omega}(t)],\quad f_{\Omega}>0;\quad f_{\bar{q}}>0;\quad f_{hu}>0 \tag{8}$$

In equation (8), $u_{\Omega}(t)$ is the share of labor allocated to the innovation activity that searches for radical innovations, and $h_{\Omega}(t)$ is the average efficiency of labor in this activity. Observe that the quality of existing blueprints, $\bar{q}(t)$, has been considered as an input in the production of radical innovations, meaning that there is a complementarity between the two associated forms of research. Observe, as well, that an expanding frontier $\Omega(t)$ necessarily implies an increase on average quality, $\bar{q}(t)$, and therefore radical innovation also offers an indirect contribution to incremental technical progress. The two types of innovation are intertwined.

In a similar perspective, Akcigit and Kerr (2018) distinguish between internal and external innovations. Internal innovations improve the product lines that existing firms already explore; thus, these correspond to incremental innovations. External innovations allow for the creation of new lines and to seize markets that others would eventually explore; external innovations correspond to major breakthroughs or radical innovations. Again, sustained growth is explained in a Schumpeterian growth scenario, where innovation and firm dynamics are the key factors for sustaining growth in the long run.

A parallel analysis can be made by distinguishing between basic and applied research (Gerbasch et al., 2018; Akcigit et al., 2021). Basic research is the activity that expands the knowledge frontier, generating new ideas, theories, and prototypes, not necessarily with an immediate commercial use; typically, the kind of innovation originating in basic research spills over various economic sectors and has a radical nature. Applied research is developed by firms, with a direct commercial intent, and the innovation it creates clearly overlaps with the characterized notion of incremental innovation. Roughly speaking, basic science generates new ideas, and applied research makes them operational to create value. In these settings, endogenous growth is assured by the reciprocal spillovers across research sectors.

4. WIDESPREAD HETEROGENEITY

As characterized in the above two sections, a significant portion of contemporaneous growth theory relies on the distinct innovative capabilities of firms to explain patterns of accumulation and growth, i.e., it relies on firm heterogeneity. One can attribute additional realism to growth settings if, besides firm heterogeneity, worker heterogeneity is contemplated as well. Workers have different skills and, therefore, they must be sorted across firms and sectors, in search for the most efficient compatibility between human abilities and available operational technologies. Grossman and Helpman (2018) explore a model along the above-mentioned lines, with firm heterogeneity and labor heterogeneity. The first is modeled following the mechanism explained in the previous sections, i.e., different research capabilities make firms capable of expanding their product lines in different extents. The latter implies assuming workers with dissimilar abilities.

Let $h(t;a)$ represent the labor efficiency, or human capital index, of a worker endowed with a level of skills equal to a. The insertion of human capital heterogeneity in the growth framework of the previous sections is straightforward. One just needs to recall that labor or human capital is required in every productive task, i.e., on the production of final goods, on the production of intermediate goods, on incremental innovation, and on the scientific research sector, such that

$$u_Y(t) + \int_0^{\Omega(t)} u_\omega(t)d\omega + \int_0^{\Omega(t)} u_{q\omega}(t)d\omega + u_\Omega(t) = 1 \tag{9}$$

Therefore, the aggregate measures of labor included in expressions (2), (5), (7) and (8), might be decomposed in the following way,[2]

$$h_Y(t)u_Y(t) = \left[\int_0^{u_Y(t)} h_Y(t;a)^{(\sigma-1)/\sigma} da\right]^{\sigma/(\sigma-1)} \tag{10}$$

$$h_\omega(t)u_\omega(t) = \left[\int_0^{u_\omega(t)} h_\omega(t;a)^{(\sigma-1)/\sigma} da\right]^{\sigma/(\sigma-1)} \tag{11}$$

$$h_{q\omega}(t)u_{q\omega}(t) = \left[\int_0^{u_{q\omega}(t)} h_{q\omega}(t;a)^{(\sigma-1)/\sigma} da\right]^{\sigma/(\sigma-1)} \tag{12}$$

$$h_\Omega(t)u_\Omega(t) = \left[\int_0^{u_\Omega(t)} h_\Omega(t;a)^{(\sigma-1)/\sigma} da\right]^{\sigma/(\sigma-1)} \tag{13}$$

In the Grossman-Helpman setup of pervasive heterogeneity, the derived equilibrium presupposes the matching between labor supply and labor demand, such that high-skilled workers are assigned to research tasks, while low-skilled workers will perform activities attached to the plain implementation of productive processes. This matching model allows for endogenous growth with an ever-expanding number of new varieties of intermediate goods.

A similar model is developed by Stokey (2021). Again, innovation and skill accumulation are the engines of growth. Innovation predominantly stimulates quality or productivity growth, while skills are, to a great extent, responsible by variety growth. Also in this scenario, pervasive heterogeneity (of firms and workers) prevails. Furthermore, there are strategic complementarities between the two factors of production: the skills of workers attain higher returns within more productive firms; firms invest in technology to increase productivity, but this is only possible if workers are well endowed with skills. Therefore, sustained growth requires the need for both inputs to systematically receive investment.

[2]To simplify, assume, for this representation, an elasticity of substitution across skills identical to the elasticity of substitution across intermediate inputs in equation (4).

5. TECHNOLOGICAL PARADIGMS AND ENVIRONMENTAL PRESERVATION

When addressing the long-term performance of economies, the preservation of the environment emerges as a central concern, alongside with the improvement of material living conditions. Environmental degradation impacts the welfare of current and future generations and, as such, it is an obvious candidate to integrate the prototypical growth model as an argument of the utility function, together with consumption.

If concerns with the quality of the environment are taken into consideration, the objective function of the agent in the optimal growth model is modified to account for the stock of pollution or the concentration of carbon in the atmosphere, a variable that represents the extent of environmental degradation and which is, in what follows, denoted by $z(t)$,

$$U(0) = \int_0^{+\infty} e^{-(\rho-n)t} u[c(t), z(t)]dt, \quad u_z < 0, \quad u_{zz} < 0 \tag{14}$$

Besides the impact over utility, pollution also has a possible detrimental effect on the ability to create value through production. Recover the production function of final goods (with $y(t)$ representing per capita income) and append to it a term reflecting the negative impact of excessive emissions,

$$y(t) = e^{-\xi[z(t)-\bar{z}]} f[k(t), h_Y(t)u_Y(t)], \quad \xi > 0 \tag{15}$$

In equation (15), $\bar{z}$ might be interpreted as a residual environmental degradation level or the pre-industrial level of atmospheric carbon concentration.

Variable $z(t)$ can be converted in an endogenous variable of the growth problem in different ways. One possibility, in the line of the model developed by Brock and Taylor (2010), is to consider a control abatement variable. In this case, the representative agent has the direct possibility of (optimally) choosing the amount of resources she desires to divert from output to fight emissions. Let $\tilde{a}(t) \in (0, 1)$ be the abatement variable or environmental preservation variable. This is a share of the level of output, and it should be interpreted in the following way: by allocating $\tilde{a}(t)$ units of output to the fight against pollution, pollution will decrease, given the functional relation between the two variables, $z(t) = z[\tilde{a}(t)]$, such that $z' < 0$, $z'' > 0$. The function predicts the presence of diminishing returns of abatement: as the abatement effort intensifies, the reduction in pollution falls. Under the above reasoning, the expression for income sophisticates further:

$$y(t) = e^{-\xi\{z[\tilde{a}(t)]-\bar{z}\}}[1 - \tilde{a}(t)] f[k(t), h_Y(t)u_Y(t)] \tag{16}$$

Expression (16) encloses a trade-off: the increase in the abatement rate directly lowers output, given the diverted resources, but it also increases output through the mitigation of the negative effect of the first term. In this circumstance, the optimal abatement rate can be derived, and it will correspond to the value of $\tilde{a}(t)$ that maximizes $y(t)$. The problem becomes more complex (and dynamic) once one takes agents' preferences as described in equation (14).

A more sophisticated approach, and closer to the creative destruction growth paradigm, is the one advanced by Acemoglu et al. (2016). This study assumes a production function for final goods similar to (15); however, it associates environmental protection concerns to the production of intermediate inputs. In the model, intermediate producers may resort to 'dirty' or to 'clean' technologies. Although the social choice directed to 'clean' technologies is obvious, it is not so evident for producers, who react to private economic incentives. Because the transition from pollutant technologies to non-pollutant production processes is socially desirable, policy considerations are relevant, and public policies might be pondered (e.g., carbon taxes and research subsidies that stimulate the discovery and implementation of clean technologies).

In the mentioned framework, production of intermediate goods takes place through a production function similar to (5). In this equation, the development of product line ω may originate on a clean technology, $q^c(t;\omega)$, or on a dirty technology, $q^d(t;\omega)$. Which one will prevail depends on the pace of innovation attached to each of the two types of technology. As remarked in previous sections, quality of product lines is subject to incremental innovations and it also depend on a series of factors, as characterized in equation (7).

The point in favor of clean technologies, from a strict decentralized market point of view, is that they allow to reduce the value of $z(t)$ in equation (15) and, therefore, the producer of final goods will find it advantageous to resort to intermediate goods produced with clean technologies. This advantage only materializes in a replacement of 'dirty' for 'clean' technologies if the original difference in costs pays off. Beyond market mechanisms, an evident externality exists, implying that the technological transition requires an active role of governments, not only through direct taxation of producers but fostering radical innovations and basic science.

Peretto and Valente (2015) also develop a Schumpeterian growth model of environment and resource use, but with a slightly different focus. They endogenize population growth and combine population dynamics, resource depletion and innovation in a single framework. The main conclusion is that it is possible to devise a growth setting in which a constant population is compatible with a growing economy. In the model, the obtained steady state is one of zero population growth, driven by resource scarcity, and positive economic growth, driven by innovation.

6. THE WORLD TECHNOLOGY FRONTIER, LEADERS AND FOLLOWERS

As suggested thus far, technological progress emerges in growth theory as the outcome of a purposive effort of firms and public institutions directed to innovation. The bulk of this innovation is incremental, but every time a radical innovation takes place, the frontier of knowledge, $\Omega(t)$, expands. In this section, it is particularly relevant to distinguish between the knowledge or technology frontier of an individual country j, $\Omega_j(t)$, and the world technology frontier, $\widehat{\Omega}(t)$.

Following Acemoglu et al. (2017), the world technology frontier ca be defined in two distinct ways. It can simply be the state of knowledge of the technological leader,

$$\widehat{\Omega}(t) = \max\{\Omega_1(t), \ldots, \Omega_j(t), \ldots\} \tag{17}$$

Alternatively, the frontier might be conceived as a convex aggregator across the technological capacities of every country:

$$\widehat{\Omega}(t) = \left[\frac{\sum_{j=1}^{J} \Omega_j(t)^{\frac{\widehat{\sigma}-1}{\widehat{\sigma}}}}{J}\right]^{\frac{\widehat{\sigma}}{\widehat{\sigma}-1}}, \quad \widehat{\sigma} < 0 \tag{18}$$

Recovering equation (8), which lists the determinants of the expansion of an economy's technology frontier, a new element can now be added, namely the distance of the national frontier to the world frontier, i.e.,[3]

$$\dot{\Omega}(t) = f\left[\Omega(t), \bar{q}(t), h_{\Omega}(t)u_{\Omega}(t), \frac{\Omega(t)}{\widehat{\Omega}(t)}\right], \quad f_{\Omega/\widehat{\Omega}} < 0 \tag{19}$$

The negative sign of the derivative in equation (19) reflects a predictable process of convergence: the further away $\Omega(t)$ is from $\widehat{\Omega}(t)$ (smaller ratio), the faster $\Omega(t)$ will potentially increase.

Acemoglu et al. (2017) resort to the notion of world technology frontier, and to the creative destruction benchmark growth model, to address the institutional and technological choices of countries in an interdependent international economy. The authors highlight the existence of two types of institutional arrangements, which might take the designations of 'cutthroat'

[3]For simplicity of notation, the j index is dropped.

capitalism and 'cuddly' capitalism. The former allows for fast growth, because it concentrates on the incentives to innovate, while the latter is willing to accept lower growth in return for some degree of social protection, insurance, and cohesion.

The appeal of the aforementioned model is associated with its capacity to generate an asymmetric world equilibrium that, to some extent, replicates observable international economic organization: there is a technologically leading country (or a small group of leading countries), which sustains its supremacy on a highly competitive economy that tolerates high levels of income inequality. The other capitalist countries choose a softer and more protective form of capitalism, which succeeds if these economies free ride on the leadership of those who stay at the world technology frontier. The obtained equilibrium is stable, in the sense that there is no incentive to change it: the equilibrium is convenient for 'cuddly' capitalists, who achieve high levels of discounted utility, regardless of their secondary role in innovation and creation of wealth; the equilibrium is also stable for the richest and most innovative countries, because if they choose to lower the 'cutthroat' incentives in the economy, the world growth rate will slow down, with damaging implications for all countries including the leader.

Additional and alternative interpretations of the leader-followers tension can be found in the literature. It is the case of Benhabib et al. (2014), who approach the innovation — imitation trade-off by devising a simple optimal control problem. In this problem, the economy maximizes an intertemporal stream of utility functions that have, as argument, the net benefits of holding a high level of technology. Let B represent the constant unitary gross benefit, to which one must subtract the costs of innovation, $\gamma(t)$, and the costs of imitation, $s(t)$; these two variables are both control variables of the economy's problem. The relevant objective function is, in this case,

$$V(0) = \int_0^{+\infty} e^{-rt} u\{[B - \gamma(t) - s(t)]\Omega(t)\} dt \tag{20}$$

The maximization of $V(0)$ is subject to a constraint that translates the expansion of the national knowledge frontier as the economy engages in both innovation and imitation. The equation is,

$$\dot{\Omega}(t) = [\zeta\gamma(t) + D(t, \Omega)s(t)]\Omega(t) \tag{21}$$

In differential equation (21), the term $\zeta\gamma(t)$ corresponds to the gains from innovation and $D(t, \Omega)s(t)$ respects to the gains of imitation. Central in this analysis is $D(t, \Omega)$, which is a diffusion function or a catch-up function; it characterizes how investment in technology adoption influences productivity and the distance to the frontier. Taking parameters $c > 0$

and $m \in \mathbb{R}^* \quad (m \neq 0)$, the diffusion function takes the following generic functional form,

$$D(t, \Omega) = \frac{c}{m}\left\{1 - \left[\frac{\Omega(t)}{\widehat{\Omega}(t)}\right]^m\right\} \tag{22}$$

The optimal control problem can be solved in two steps. The first step consists in determining the growth rate of the world technology frontier; this can be done by abstracting from diffusion, i.e., taking $s(t) = 0$ (the leader does not imitate; it just innovates). The outcome of such simple model is a constant long-term growth rate equal to:

$$\frac{\dot{\widehat{\Omega}}}{\widehat{\Omega}} = \zeta B - r \tag{23}$$

The followers will solve the same problem, but for $s(t) > 0$ and under a world technology frontier that grows at rate (23). The solution of the model yields a constant steady state value for ratio $\frac{\Omega t}{\widehat{\Omega}(t)}$, meaning that in the long run the technology level will grow at the same rate as the frontier. Furthermore, as designed, the model allows for the computation of a threshold ratio above which firms innovate and below which they imitate. Given the parameters and functional forms of the model, this ratio is:

$$\left(\frac{\Omega}{\widehat{\Omega}}\right)^* = \left(1 - \frac{\zeta m}{c}\right)^{1/m} \tag{24}$$

For $\frac{\Omega(t)}{\widehat{\Omega}(t)} < \left(\frac{\Omega}{\widehat{\Omega}}\right)^*$, the economy will be an imitator, solving the optimization problem for $s > 0$, $\gamma = 0$. Under $\frac{\Omega(t)}{\widehat{\Omega}(t)} > \left(\frac{\Omega}{\widehat{\Omega}}\right)^*$, the economy will be an innovator, will place itself in the frontier, and it will solve the optimization problem for $s = 0$, $\gamma > 0$.

In a framework that shares similarities with the aforementioned models, Stokey (2015) also approaches patterns of catching up and falling behind in the global economy. Contingent on initial conditions and the quality of public policies, developing economies may keep pace with the technology frontier and, hence, maintain sustained growth; alternatively, the economy may fall into a state of stagnation, where it subsists a minimal technology level that is independent of the world frontier.

In Stokey's setting, the world technology frontier grows at an exogenous constant rate, $\dot{\widehat{\Omega}}(t) = \widehat{g}\widehat{\Omega}(t)$. The state of knowledge of the economy evolves under a rule of technological accumulation that has some points in common with equation (19), but that displays, as well, some distinguishing features.

Given its main arguments, the equation of motion may be presented as

$$\dot{\Omega}(t) = f\left[\Omega(t), b(t), \frac{h_{\Omega}(t)u_{\Omega}(t)}{\widehat{\Omega}(t)}, \frac{\Omega(t)}{\widehat{\Omega}(t)}\right],\ f_{\Omega} > 0, f_b < 0, f_{hu/\widehat{\Omega}} > 0, f_{\Omega/\widehat{\Omega}} < 0 \tag{25}$$

Expression (25) indicates that the expansion of the country's technological capabilities is positively associated with the state of the technology in the country and with the ratio between human capital and the world technology frontier. There is also a negative relation between the domestic - worldwide technology ratio and the expansion of domestic technology. Variable $b(t)$ represents barriers to adoption (e.g., protectionist trade policies, or political and institutional instability and conflict) whose increase prevents the technology frontier to expand.

Observe that two ratios having as denominator the world technology frontier determine the evolution of $\Omega(t)$: the human capital — world frontier ratio reflects the capacity of the labor force in the economy to absorb technologies close to the frontier; the local technology — world technology ratio represents, as earlier stated, the gap between the two, and the larger the gap the stronger will be, in principle, the pace of convergence. Therefore, the world frontier enters equation (25) in two distinct ways, with countervailing effects: a higher value of $\widehat{\Omega}(t)$ lowers the absorption capacity of domestic human capital, retarding growth; a higher value of $\widehat{\Omega}(t)$ widens the technology gap, thus leading to faster convergence and, concomitantly, to faster growth.

Whether the economy can keep up with the evolution of international knowledge or will simply stagnate and fall behind depends on the trajectories followed by the inputs of the function in equation (25). Because human capital and technology are intimately related (a production function for human capital will contain, as inputs, human capital and also technology), the isolated factor that appears to be more relevant is the measure of barriers to adoption, which can be decisive for whether the economy catches up or falls behind: the quality of institutions and how they promote the absorption of techniques and ideas from abroad is, in this perspective, a fundamental driver of growth.

7. THE PROPAGATION OF IDEAS AND PRODUCTIVITY

Most contemporaneous growth theory concentrates on the mechanisms through which formal and purposive innovation effort, undertaken by profit-maximizing firms, can generate sustained endogenous growth. Although institutionally framed research is relevant in uncovering new ideas, one may adopt a broader view about the creation of knowledge. In a certain sense, knowledge is generated every time people interact, and most of the value-creating activities in the world today are somehow associated with how people connect, interact, share ideas, and solve problems. This is the view that Lucas (2009) takes to interpret reality and to analytically approach economic growth. In Lucas' (2009, p.1) own words,

> "What is it about modern capitalist economies that allow them, in contrast to all earlier societies, to generate sustained growth in productivity and living standards? (…) What is central, I believe, is the fact that the industrial revolution involved the emergence (or rapid expansion) of a class of educated people, thousands — now many millions — of people who spend entire careers exchanging ideas, solving work-related problems, generating new knowledge."

Models in Lucas (2009), Lucas and Moll (2014), and Perla and Tonetti (2014) are built upon the intuition that ideas and productivity propagate via direct contact between heterogeneous agents, i.e., agents that hold diverse levels of knowledge. In these models, ideas spread by contagion among people who meet randomly. Returning to the setting in section 4, each member of the labor force is endowed with a level of skills a and, therefore, the corresponding human capital index at date t will be $h(t;a)$. The entire distribution of skills in the economy is known and the productivity of each agent evolves taking independent draws from the distribution of productivities. The agent will then compare its own level of skills a with the level of skills of the matching individual, a', and the new set of ideas is adopted whenever $a' > a$. Individual productivity may, then, remain constant or improve depending on the productivity of each random contact at each date. Therefore, even if in an intermittent way, individual productivity will improve and approach the knowledge frontier (this must grow as well, in order to guarantee endogenous growth).

A more structured version of a model involving the above conceptualization requires thinking about the allocation of time. Consider that every agent is endowed with a unit of time, at every period t, and that the time can be allocated to two activities. A share $u(t;a) \in (0,1)$ of time is allocated to the production of a final good, and the remaining share $1-u(t;a)$ will be assigned to social interaction. Through social interaction, the productivity of the agent may improve, if some of the agents that the individual meet have a productivity $a' > a$. An evident trade-off involving time allocation emerges in this context: people want to produce to earn income that they can spend in consumption, but they need to allocate time to interaction to increase productivity. The more time is allocated to interaction, the higher is the probability of meeting someone that will convey to the agent a set of better ideas, ideas leading to a higher level of productivity.

The individual production function is, in this case, abstracting from capital,

$$y(t;a) = f[h(t;a)u(t;a)] \tag{26}$$

Taking a trivial utility maximization problem, the control variables will be consumption and the share $u(t;a)$. By choosing the trajectory of $u(t;a)$, the agent is also determining the evolution of $h(t;a)$; this happens because more time dedicated to contacts raises the probability of improving skills and, thus, the capabilities of human capital. The trade-off is clear: a higher h requires a lower u, and vice-versa.

8. AGENT-BASED MODELS AND THE NETWORK APPROACH

The models mentioned in the precedent section take an important first step in reconfiguring growth theory, making it evolve beyond its most traditional and orthodox general-equilibrium paradigm. The reason is that such models allow for the possibility of local interaction among agents in a decentralized economy. Once local interaction is recognized as an important feature to explain the growth process, the way one thinks about the analytical frameworks required to study growth radically changes. Network analysis, a prominent line of scientific inquiry, becomes relevant, and the entire approach to economic relations changes, with the traditional top-down characterization of events being replaced by a bottom-up perspective, a perspective that requires a new model apparatus, namely the adoption of agent-based models and other complexity-related frameworks of analysis.

Agent-based models consist in an interpretation of the economy as being a complex evolving system, where macro regularities originate on the decentralized interaction among heterogeneous adaptive agents. This bottom-up perspective is useful to approach a wide variety of economic issues, and it is particularly well-suited to justify aggregate results that emerge naturally from congregating the outcomes of individual agents, within systems with a strong feedback effect: it is the behavior of individual units that conducts to the aggregate result, and the aggregate result is crucial in influencing individual behavior.

Dosi et al. (2019) address economic growth with interdependent economies, taking an agent-based modelling approach. The proposed growth model is a model of Schumpeterian growth with firm dynamics, with agent-based foundations: firms follow a series of steps associated with investment and innovation / imitation decisions; the outcome of innovation determines the following steps about what technology to use and how to organize production. Firms are heterogeneous regarding innovation capabilities and, therefore, running the model, different individual results concerning output, consumption and foreign trade are obtained. These results give origin to an aggregate outcome, whose main feature is the persistence of positive income growth. In the assumed multi-country setting, divergent patterns of growth are admissible; while some countries catch-up, the others diverge from the world's technological frontier.

The structural idea underlying agent-based models and traditional models of growth is the same — innovation through creative destruction generates endogenous growth — but the approach is different. Agent-based models construct an algorithm of economic relations and allow the algorithm to run in order to simulate the behavior of the economy.

Agent-based models can be contrasted with general-equilibrium models. The latter attribute to economic agents a capacity to fully interact with all other agents in the economy, without the need for segmenting markets or consumers. The agent-based paradigm is well-equipped to consider the existence of networks, i.e., of contact groups with which individual agents have privileged relations. Gualdi and Mandel (2018) explore an agent-based model with formation of production networks. In this scenario, networks are not a given element of the model; they emerge through competition and innovation. Therefore, there is a feedback effect: market dynamics shape networks, and the configuration of networks will determine the generation of income and the pattern of growth; growth, in turn, is decisive in shaping market dynamics.

Looking in more detail at the mechanism proposed by the mentioned authors, one identifies the interplay between two opposing forces: process innovation (incremental innovation) increases the number of available varieties, thus stimulating a stronger connectivity within the network; product innovation (radical innovation) reinforces the productivity of firms, what is a potential source of destruction of links in the network. Complex technological dynamics emerge from the combination of the two forces, involving a systematic reconfiguration of the network and of the potential of the economy to generate wealth.

A way to formalize the described mechanism is to consider that it exists more than one producer of final goods, and that each producer of final goods receives a supply of intermediate goods originating on a subset of intermediate good suppliers, namely the subset formed by those that participate in the producer's network. Let the subset of suppliers of final goods' producer $\tilde{m}$ be

$$\Gamma(t) = \{\omega \in \Omega(t) | a_{\tilde{m},\omega} = 1\} \tag{27}$$

In equation (27), intermediate producer ω is a connection in the network of the final producer $\tilde{m}$ if a link between them exists, i.e., if $a_{\tilde{m},\omega} = 1$ rather than $a_{\tilde{m},\omega} = 0$. The composite capital aggregate that firm $\tilde{m}$ will utilize in the production of final goods is a subset of the available intermediate goods, i.e., equation (4) gives place to

$$k(t) = \left[\int_{\omega \in \Gamma(t)} k(t;\omega)^{(\sigma-1)/\sigma} d\omega \right]^{\sigma/(\sigma-1)}, \quad \sigma > 1 \tag{28}$$

Note that $\Gamma(t) \subset \Omega(t)$. Final producers will systematically search for more competitive suppliers, and this will systematically change the configuration of the network. Suppliers become less or more competitive given their capacity to innovate and the type of innovations (incremental vs radical) that they develop.

Another contribution on networks, social interaction and economic growth is the one by Fogli and Veldkamp (2021). These authors develop a theory of networks in which human contact disseminates both ideas and contagious diseases. Networks with few links reduce the risk of disease infection, allowing for longer lives; at the same time, these networks lead to a low diffusion of ideas and, consequently a low rate of innovation and growth.

Countries, namely developing economies, should search for the adequate balance between good and bad propagation flows, and therefore they must choose the adequate configuration for the network of social interactions. As in the Lucas' models mentioned in the previous section, technology spreads easily through the simple contact between agents holding different productivity levels. The downside is that with the same easiness with which ideas spread also do diseases.

In the mentioned model, interesting dynamics come from the systematic potential reshaping of the network that is triggered by a feedback effect from diseases to innovation and the other way around. On one hand, infection lowers innovation, because there will be less healthy people to engage in research; on the other hand, innovation reduces infection because the improvement in technology directly helps (e.g., through better public health and healthcare) and indirectly helps (better socio-economic conditions) to reduce infections. These two effects can be modeled as follows:

$$\lambda(t) = \lambda_0[1 - \Phi(\hat{d}(t))], \quad \lambda_0 > 0 \tag{29}$$

$$\pi(t) = \pi_0[1 - \Phi(\hat{A}(t))], \quad \pi_0 > 0 \tag{30}$$

In equations (29) and (30), $\lambda(t)$ and $\pi(t)$ are, respectively, the probability of innovation and the probability of infection; $\Phi(\hat{d}(t))$ and $\Phi(\hat{A}(t))$ are, respectively, the distribution of the rate of infection and the distribution of technological capabilities.

Growth outcomes depend on the structure and properties of the network (the extent and the intensity with which agents are interconnected) and how such structure changes given the feedback between innovation and spreading of diseases. Networks will inhibit or foster growth depending on how the society and institutions have evolved with the objective of preventing the dissemination of diseases at the same time they feel the necessity to increase contact to stimulate innovation and economic growth.

9. GROWTH UNDER POTENTIAL DISASTERS

Typically, interpretations of growth are optimistic: there is a purposive effort of economic agents to create wealth and increase living standards, and this effort is generally well-succeeded, leading to sustained growth over time. However, things can go wrong, and looking at what can go wrong with growth is not an incipient exercise; it can help in better understanding aggregate outcomes that conventional growth models have difficulties in justifying.

With this scenery as a backdrop, Jones (2016) asks a relevant question: how is growth theory changed by assuming technologies that lead to life and death rather than just higher consumption? The answer to this interrogation encloses a relevant trade-off, which is the trade-off between consumption growth and safety. If safety concerns are seriously taken, the representative agent may be willing to trade consumption growth by reinforced safety conditions. In the words of Charles Jones, many technologies of production are like Russian roulettes; they are developed to promote well-being, but there is some probability of leading to disastrous outcomes (e.g., research on nuclear energy).

Therefore, the agent has a decision to make about engaging or not in research activities. Research will generate, with a high probability, $1-\pi$, an increase in consumption possibilities at a rate g. The probability of research resulting in a disaster occurs with probability π; assume that the utility of the disaster is zero. If the agent does not engage in research, there is no risk of disaster; however, consumption will not grow.

Under an intertemporal perspective, two objective functions should then be alternatively considered,

$$U_r(0) = \int_0^{+\infty} e^{-\rho t}(1-\pi)u[c(0)e^{gt}]dt \tag{31}$$

$$U_{nr}(0) = \int_0^{+\infty} e^{-\rho t}u[c(0)]dt \tag{32}$$

The condition to engage in research is, obviously, $U_r(0) > U_{nr}(0)$. Assuming a logarithmic utility function, the integrals can be computed,

$$U_r(0) = \frac{(1-\pi)\{u[c(0)]\rho + g\}}{\rho^2} \tag{33}$$

$$U_{nr}(0) = \frac{u[c(0)]}{\rho} \tag{34}$$

Expressions (32) and (33) allow to illustrate the type of condition that constitutes the threshold on the economy's choice between innovation and no innovation.[4] Applying condition $U_r(0) > U_{nr}(0)$, one gets

$$g > \frac{\pi}{1-\pi}\rho u[c(0)] \tag{35}$$

Inequality (35) indicates that research pays off whenever the growth rate exceeds the value of the term in the right-hand-side. Taking constant ρ and $u[c(0)]$, the decision is reduced to the comparison between consumption growth triggered by research and the probability of research provoking a disaster. The larger the value of π, the faster consumption has to grow under research, in order for research to take place.

Rearranging inequality (35), another interpretation can be given for the problem at hand,

$$u[c(0)] < \frac{1-\pi}{\pi}\frac{g}{\rho} \tag{36}$$

Under (36), the initial level of consumption can be perceived as a fundamental determinant of the decision on whether to innovate. Relatively low consumption at the planning date implies a desire to engage in research. High levels of consumption at the planning date may imply that inequality (36) will not hold, and in this case no growth and safety are preferred to the opposite scenario of growth and risk of disaster. This observation is compatible with observable empirical evidence and intuition: relatively richer societies and / or individuals (with better initial conditions) are more likely to trade growth for safety; poorer societies and / or individuals will privilege consumption over safety.

Therefore, when solving the optimal growth problem, two outcomes are feasible: sustained growth with risk of disruption, and deliberately chosen stagnation. The rate of innovation, the underlying risk of disruption, the rate of time preference, and the level of consumption at the planning date, will all be decisive factors for the path the economy chooses to undergo.

[4] The configuration of this condition changes if other functional forms for the utility function, rather than logarithmic utility, are assumed.

Jovanovic and Ma (2022) propose a model that also combines growth and uncertainty, although they concentrate their attention on technology adoption and productivity. Taking the reasonable assumption that the adoption of new technologies is an irreversible process, the technological commitment has an uncertain outcome that cannot be reversed. Eventual mismatches between the acquired technology and the needs of the firm's operations can only be observed ex-post. The level of uncertainty will determine the rate of adoption and, consequently, also the rate of growth.

In this setting, the production function of a given good can be formulated as

$$y(t) = \exp\left\{A(t) - \frac{\phi}{2}[\bar{h}(t) - h(t)]^2\right\}, \quad \phi > 0 \tag{37}$$

In equation (37), $A(t)$ represents the level of technology that allows to potentially generate $y(t) = \exp[A(t)]$ units of output. However, the productivity of this level of technology is eventually reduced due to the lack of compatibility between the ideal skills to operate with the technology, $\bar{h}(t)$, and the current skills of the workers of the firm, $h(t)$. The term $\Delta h(t) = \bar{h}(t) - h(t)$ can be designated skill gap. Uncertainty comes from the fact that the firm has no guarantee that switching technologies will allow for a better human capital adjustment. An alternative is to adjust skills, but this has an associated cost that reduces the value of output.

The above-mentioned model departs from the idealist world underlying most optimal growth models, in which it is always possible to choose the best available technology to produce. If the adequacy of the adopted technology cannot be perceived ex-ante and technology adoption is irreversible, endogenous growth cannot be guaranteed, and fluctuations might emerge as the outcome of the systematic adjustment the skills of the firm have to undergo.

10. THE RISE OF ARTIFICIAL INTELLIGENCE AND THE FALL OF THE LABOR SHARE

In recent years, an unprecedented acceleration of the automation of economic activities has taken place. The advancements in artificial intelligence and machine learning allowed for substantial productivity gains and for a transformation of productive processes, with human labor being replaced, at a fast pace, by machines and algorithms. Concerning growth analysis, it is of particular interest to associate the artificial intelligence revolution with the rise of a new input: a form of capital — robotic capital — that, in opposition to typical physical capital, is not a complement of labor in production, but rather a substitute.

The implications of artificial intelligence and automation for economic growth are significant. Nordhaus (2021) conjectures about the possible emergence of a singularity, that is, the formation of a threshold beyond which economic growth will sharply accelerate, creating a world of unimaginable abundance. Although this singularity might be technically feasible, the way the economy and the society are organized does not allow to locate it in the near future. Full automation is not possible in many nonroutine tasks and political and institutional choices may eventually constitute a force of resistance against pushing technical capabilities to their limits. Furthermore, as highlighted by Irmen (2021), automation has paradoxical effects and encloses a relevant trade-off: there is both a rationalization effect (fewer resources are necessary to develop tasks) and a productivity effect (which lowers the costs per task). Which of the effects dominates depends on the specific features of the tasks under consideration and on how their execution is managed.

Independently of the intensity of the increasing returns associated with artificial intelligence, and of the respective consequences for the acceleration of growth, it is meaningful to address how the basic features of automation impact conventional endogenous growth frameworks. Machines and algorithms may enter growth models in various ways. A couple of ideas is highlighted in what follows.

A possible interpretation is the one proposed by Prettner and Strulik (2020). For these authors, the capital input is evolving and acquiring a progressively larger component of automation. Therefore, physical capital, $k(t)$ should be replaced by automated capital, $k_R(t)$, in the interpretation of growth. The difference between the two inputs is that while $k(t)$ is a complement of labor in production, $k_R(t)$ is a complement only of high-skilled labor, and a substitute for low-skilled labor. Formally, this is reflected in the following aggregate production function,

$$y(t) = f[k_R(t) + h_{LY}(t)u_{LY}(t), h_{HY}(t)u_{HY}(t)] \tag{38}$$

In production function (38), $u_{LY}(t)$ and $u_{HY}(t)$ are the shares of low-skilled labor and high-skilled labor, respectively, allocated to the production of final goods; $h_{LY}(t)$ and $h_{HY}(t)$ are the corresponding efficiency levels; by definition, $h_{HY}(t) > h_{LY}(t)$.

Because automation complements only high-skilled labor and substitutes for low-skilled labor, people desire to hold high-quality skills, what leads them to invest in education. However, not all individuals are successful in acquiring additional skills. Therefore, the model predicts income growth, technological progress, and increased average human capital; it predicts, as well, a decline of the labor share and an acceleration of income inequality.

A different approach is pursued by Lu (2021), who considers artificial intelligence as an input distinct from physical capital. This implies that physical capital is now a complement in production of one of two inputs: labor or robotic capital and, consequently, labor and robotic capital emerge as substitutes of one another. In simple terms, an alternative interpretation of the production technology with automation, relative to (38), is the following:

$$y(t) = f[k(t), h_Y(t)u_Y(t) + k_R(t)] \tag{39}$$

In expression (39), the distinction is made between forms of capital and not between types of labor, as in (38). Nevertheless, the implications are similar: a more productive economy is likely to emerge, which is also an economy where people are progressively replaced by automated processes. Robots are not only more productive than people; they are also potentially endowed with another economic appealing feature: in opposition to labor, robotic capital is in part nonrival (e.g., algorithms and software), a characteristic that can help, as well, in further stimulating growth. The nonrival nature of robotic capital allows to imagine one other economic sector, which increments the production of this input by employing rival human capital and the nonrival form of capital,

$$\dot{k}_R(t) = f[h_{kR}(t)u_{kR}(t), k_R(t)] \tag{40}$$

11. DATA: THE ECONOMICS OF THE NEW GROWTH VARIABLE

Machine learning introduced fundamental changes in the organization of the economy. Through machine learning, huge amounts of data are collected and processed to create recognition patterns that can be utilized to develop many activities with much greater efficiency and precision. If this is so, digital data becomes a new input in production. A salient characteristic of data as an input in production is its nonrivalry, meaning that it can be used by many firms simultaneously without losing its intrinsic properties, thus having the potential to generate relevant social gains. Digital data has also the peculiar feature of being a byproduct of consumption; each time consumption takes place, data is created, and it can be transferred (i.e., sold) to firms. Finally, one should note that the transmission of data from consumers to firms involves privacy concerns, which should be accounted for, as well.

The above considerations unveil how pertinent and pressing it is to append a data variable to typical endogenous growth frameworks, with the purpose of better highlighting how this impacts economic decisions and, ultimately, the process of growth. Two studies that formalize the role of data as an input in growth models are those by Jones and Tonetti (2020) and Cong et al. (2022); these studies constitute the support for the discussion that follows.

To better understand the role of data, a first step consists in distinguishing data from ideas. Typically, data is employed in the training of machine learning algorithms to generate ideas and, therefore, in simple terms, one might interpret data as an input, in modern production systems, in the generation of new ideas. The novel ideas may correspond either to incremental innovations or to radical innovations and, therefore, data, d(t), emerges, under the class of models considered in the initial sections, as an additional, nonrival, input in equations (7) and (8), i.e.,

$$\dot{q}(t;\omega) = f[q(t;\omega), \bar{q}(t), h_{q\omega}(t)u_{q\omega}(t), d(t)]\alpha^{\tilde{n}}, \quad f_d > 0 \tag{41}$$

$$\dot{\Omega}(t) = f[\Omega(t), \bar{q}(t), h_{\Omega}(t)u_{\Omega}(t), d(t)], \quad f_d > 0 \tag{42}$$

The nonrival nature of data makes it, simultaneously, a potential input in the expansion of the technological frontier, as well as a factor of production in any of the processes of incremental innovation of each variety ω.

As mentioned, data is a by-product of consumption; hence, for each unit of consumption, a given amount of additional data is created,

$$\dot{d}(t) = f[c(t)], \quad f_c > 0 \tag{43}$$

Consumers may withhold data and not convey it to firms due to privacy concerns (e.g., privacy violations and abuses in data use). These concerns translate into a negative effect over utility, making the objective function of the representative agent to take the form

$$U(0) = \int_0^{+\infty} e^{-(\rho-n)t} u[c(t), d(t)]dt, \quad u_d < 0 \tag{44}$$

The representative agent is confronted with a trade-off, which consists in weighing the additional income generated by selling data to firms against the privacy worries reflected in (44). Because data is an input in the innovation sector, it emerges as a driver of sustained growth. It also raises critical issues concerning externalities and the social value of data: nonrivalry makes data sharing lower than optimal, because consumers will not be able to internalize the benefits that arise from the corresponding knowledge spillovers.

12. CONCLUSION

In the last two decades, economic growth theorists have intensified their research on what they believe to be the main determinant of endogenous growth, namely the many ways in which innovation takes place. Firms have been conceptualized as collections of product lines; these product lines can receive incremental improvements (incremental innovation), although new lines of production can emerge as well (radical innovation). The detailed explanation of how innovation thrives and allows to revert diminishing returns in production constitutes, today, the basis for one of the most credible frameworks to address growth, a framework that is firmly grounded on the Schumpeterian notion of creative destruction. A sizable portion of contemporaneous theoretical work on many issues related to growth takes, as starting point, this structure of analysis. It serves to explore the contribution of diverse types of research, to address the matching between heterogeneous firms and workers, to conceptualize institutional choices of national economies concerning innovation and adoption, and it serves as well, among other purposes, to approach the relationship between the preservation of the environment and economic growth.

Besides innovation, growth theory also continues to explore the role of the other main driver of growth, namely the accumulation of human capital via education and learning. An appealing interpretation is the one that conjectures that most of the growth in productivity observed in the contemporaneous world is associated with the interaction between common people, in common workplaces, that establish contact in an everyday basis to find solutions to common problems. In this view, human contact is interpreted as an instantaneous form of spreading innovative ideas and enhancing productivity. Such an interpretation has led to meaningful growth models, which necessarily require non-orthodox structures of analysis. Because interaction is essential for the propagation of ideas, socio-economic networks emerge as a natural setting for the analysis of the mechanics of growth; studying the topology and dynamics of networks becomes, in this way, an indissociable part of the study of growth. The algorithmic frameworks provided by agent-based computational economics and complexity science are another valuable tool to address economic growth form a different angle, capable of providing new important insights.

The survey has addressed other issues that are of relevance in recent literature. In particular, two final sections have remarked how growth theory is adapting to the inclusion of new growth drivers. These growth drivers are, essentially, inputs whose participation in production was negligeable in the recent past but have nowadays a decisive role in accelerating the pace of productivity growth, namely robotic capital and digital data.

The future of growth theory will certainly be shaped by an increased interconnection between the various elements that were mentioned along the text: the creative destruction framework became a dominant benchmark for the analysis of innovation and growth, but it must be adapted to account for the propagation of ideas via social interaction. As data and automation become increasingly relevant, researchers will have also to further explore their role in the context of conventional and less orthodox economic models. Moreover, recent worldwide events, with a huge impact on growth, as the covid pandemic or the escalation of international conflicts and political tensions, make us wonder if one should continue to concentrate almost exclusively on the factors that promote ever-increasing productivity levels. Perhaps the focus should be, at least partially, redirected to the events and processes that pose a significant risk to sustained growth, which economists became used to assume as an inevitable outcome.

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