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Idea Exchange and Wage Inequality

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This paper investigates how idea exchange affects skilled-unskilled wage inequality. In the basic model with two sectors and full employment, an increase in the intensity of idea exchange in the skilled sector will expand wage inequality; however, an increase in the intensity of idea exchange in the unskilled sector will narrow down wage inequality. In most of the extended models, the main results are almost the same as those of the basic model. However, in the extended model with the skilled product employed in the unskilled sector, the main results are greatly different from those of the basic model. This paper embeds idea exchange into the framework of skilled-unskilled wage inequality.

Key Words: Idea exchange; Skilled-unskilled wage inequality; General equilibrium approach.

JEL Classification Numbers: J31, O12, O31.

1. INTRODUCTION

In the real world, the exchange of ideas is common in enterprises. Ideas and communication are hot research points in economics. Arrow (1962) finds that ideas in the sense of knowledge are the product of experience, and knowledge will grow over time. Based on this, he proposes an endogenous theory of knowledge change and emphasizes that inefficiencies arise if "doing" externally affects the "learning" of others. Shell (1966) points out that knowledge can be regarded as a produced public product, and the level of invention activities (i.e., the knowledge production process) depends on the quantitative activities of economic resources used for the product. After that, Romer (1990) constructively emphasizes the nonrivalry of ideas, which is in a way that other papers have never done before. He points out that ideas do not dry up with use, and once an idea is invented, it is

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technically feasible and can be used by any number of people at the same time. Since then, many economists have developed new growth theories based on Romer (1990), the most important of which is Schumpeterian growth theory. At present, many papers focus on the exchange of ideas to study its impact on economic growth (e.g., Romer, 1993; Jones and Romer, 2010; Jones, 2005; Alvarez et al., 2013; Buera and Lucas, 2018; Akcigit et al., 2016). For example, Jones (2005) believes that the nonrivalry of ideas means that the possibility of production can be characterized by increasing returns to scale, and this view has a profound impact on economic growth. Akcigit et al. (2016) establish an endogenous growth model to try to measure how the efficiency of the patent market affects growth. In their model, in order to increase productivity, the firm will invest in the research and development of new ideas, and these ideas can be bought and sold on the patent market. Alvarez et al. (2013) add an endogenous growth theory to the standard Ricardian model, making the flow of ideas an engine of growth and assuming that people can acquire new ideas by learning from people they do business with or compete with. Ideas can be seen as a source of economic growth, and at the same time idea exchange can be seen as a source of wage inequality in the sense of different labor productivities. In the real world, many firms stress the role of idea exchange, e.g., the firms in Silicon Valley in the United States and Haier and Huawei in China. However, the existing literature ignores the impact of idea exchange on skilled-unskilled wage inequality and does not embed idea exchange into the framework of skilled-unskilled wage inequality.

Rising skilled-unskilled wage inequality is prevalent in many countries, and it is also a hot research topic in labor economics. The studies on this topic focus mainly on two aspects. One is to provide empirical evidence for this phenomenon (see Wood, 1997; Khan, 1998; Feenstra and Hanson, 2003; Horgos, 2009), and the other is to theoretically explain the causes of this phenomenon from different perspectives. Specifically, there are three theoretical strands. The first one emphasizes the role of international factor mobility on wage inequality (e.g., Marjit and Kar, 2005; Anwar, 2006, 2008; Beladi et al., 2013; Pi and Zhou, 2014). The second one highlights the effect of technological change on wage inequality (e.g., Ethier, 2005; Moore and Ranjan, 2005; Fang et al. 2008; Pi and Zhang, 2018b). The third one stresses the impact of government behavior on wage inequality (e.g., Chaudhuri and Yabuuchi, 2007; Anwar and Sun, 2015; Pi and Zhou, 2012, 2014). However, the existing literature neglects the role of idea exchange on skilled-unskilled wage inequality, and idea exchange should have an important place in the framework of skilled-unskilled wage inequality.

This paper explores how idea exchange affects wage inequality. Following Romer (1990), we believe that due to the nonrivalry of ideas, the collision and exchange of new ideas will provide a strong impetus for tech-

nological change, and ultimately promote economic development and living standards. We establish a two-sector model consisting of an urban skilled sector and an urban unskilled sector. We assume that idea exchange takes place within each sector through experience exchange meetings and daily communication. We study the exchange of ideas at the social level, and so the intensity of idea exchange is mainly determined by policies and atmosphere. The dissemination of ideas between societies is accomplished through the interaction between people, and so the enhancement of the intensity of idea exchange can be achieved by increasing the probability of people interacting. For example, infrastructure can be used to facilitate more diverse physical interactions of people, and building community bridges for interactions between communities can increase access to productive activities, economic growth and ideas. In fact, a city's infrastructure is more restrictive to physical mixing and idea exchange than we usually think (see Pentland, 2020). Here, how to improve the ease of idea exchange is not the focus of our study, and this paper aims to examine the effect of idea exchange on the wage gap. Therefore, we only provide a few policy methods that can increase the intensity of idea exchange. At the same time, because the two sectors in the model employ different types of labor, the impact of idea exchange on productivity is different. Our economic intuition is as follows. When the degree of idea exchange in a certain sector changes, the productivity of the corresponding sector will change accordingly. Therefore, there will be an imbalance between the supply and demand of labor, which leads to changes in the absolute and relative wages.

The rest of this paper is organized as follows. Section 2 provides the basic model. Section 3 gives four extended models, including the economy with the individual effect of idea exchange, the economy with a totally open capital market, the economy with the skilled product employed in the unskilled sector, and the economy with a mixed sector. Section 4 makes some concluding remarks.

2. BASIC MODEL

Consider a small open economy consisting of two sectors, a skilled sector and an unskilled sector. The skilled sector employs skilled workers L_S and capital K_X to produce an exportable manufacturing product X, which can be seen as the skilled product. The unskilled sector employs unskilled workers L_U and capital K_Y to produce an import-competing product Y, which can be seen as the unskilled product. In the capital market, capital can flow freely between the two sectors. In the labor market, skilled labor is specific to the skilled sector and unskilled labor is specific to the unskilled sector.

Furthermore, we assume that there exists a mechanism for the exchange of ideas in both sectors, which means the two sectors will organize meetings for workers at regular intervals, during which workers can communicate with each other and share their work experiences. An idea is a non-rival commodity, it doesn't disappear like labor, capital, and land. On the contrary, ideas can be widely disseminated and used simultaneously by any number of people, and the emergence of new ideas within one industry can bring about a sea-change in the industry as a whole. Here, we do not study how ideas are generated, but the degree to which ideas are communicated and exchanged. The generation of ideas is an internal process, while the exchange of ideas works through externalities. The higher the degree of idea exchange, the greater the positive externality and the better it is for the industry as a whole. We use the intensity of idea exchange to express the opportunity and depth of communication. The more meetings held and the deeper the communication, the greater the intensity. We let α and β denote the intensity of idea exchange in the skilled sector and the unskilled sector, respectively. We can visualize the fruits of the exchange of ideas as technological change. From idea exchange, workers will improve their work methods and increase their work efficiency. As a result, the productivity will rise. We assume that the effect of idea exchange on output is $g^i(\cdot) \quad (i = X, Y)$, which is determined by the intensity of idea exchange (i.e., α or β). The effect can be expressed as $g^X(\alpha)$ and $g^Y(\beta)$, respectively. Thus, the production functions can be expressed as follows: $X = g^X(\alpha) \cdot F^1(L_S, K_X)$, and $Y = g^Y(\beta) \cdot F^2(L_U, K_Y)$, where $g^X(\alpha) > 1, g^Y(\beta) > 1$, and the derivatives of these two functions satisfy: $g^X_\alpha = \frac{dg^X(\alpha)}{d\alpha}$, $g^Y_\beta = \frac{dg^Y(\beta)}{d\beta} > 0$. Because the work of unskilled labor is relatively simple, it does not need too much communication, the intensity of idea exchange in the unskilled sector is much smaller than that in the skilled sector. So α is much larger than β. Furthermore, the work of the skilled sector is highly technical, and so there is much room for improvement in skills for skilled labor and the sharing of work experience has a great impact on the productivity of this sector. However, the work of the unskilled sector is lowly technical, and so communication with each other has little effect on the productivity. In other words, technological change brought about by idea exchange has a greater positive impact on the skilled sector than the unskilled sector. Thus, we have $g^X > g^Y$. In addition, all the production functions are quasi-concave and linearly homogeneous.

Finally, we assume that the two sectors in our model are both private and take profit maximization as the objective, which means each sector will choose its appropriate use of production factors to maximize its own profit. The profit functions of the two sectors are expressed as $\pi_X = p_X X - w_S L_S - r K_X$ and $\pi_Y = p_Y Y - w_U L_U - r K_Y$, where p_X and p_Y respectively denote the prices of the final products X and Y. Since the small economy

has no pricing power, p_X and p_Y are exogenously given. w_S and w_U are the wage rates of skilled and unskilled labor, respectively. r is the interest rate of capital. Here, the amounts of production factors employed in production (L_S, L_U, K_X, and K_Y) and the prices of production factors (w_S, w_U, and r) are endogenous variables.

According to the zero-profit conditions, we obtain the following equations:

$$p_X = a_{SX}w_S + a_{KX}r, \tag{1}$$
$$p_Y = a_{UY}w_U + a_{KY}r. \tag{2}$$

In line with Jones (1965), a_{ij} ($i = S, U, K; j = X, Y$) denotes the amount of factor i employed by sector j to produce one unit of the product, i.e., $a_{SX} = \frac{L_S}{X}$, $a_{KX} = \frac{K_X}{X}$, $a_{UY} = \frac{L_U}{Y}$, and $a_{KY} = \frac{K_Y}{Y}$. Here, it should be noted that a_{ij} can be understood as the envelope solution of a sector's cost minimization at a given output level. Thus, a_{ij} is a function of the input factors' relative prices of the corresponding sector and the intensity of idea exchange, i.e., $a_{SX} = a_{SX}(w_S, r, \alpha)$, $a_{KX} = a_{KX}(w_S, r, \alpha)$, $a_{UY} = a_{UY}(w_U, r, \beta)$, and $a_{KY} = a_{KY}(w_U, r, \beta)$.

According to the market-clearing conditions, we have the equations as below:

$$a_{SX}X = \overline{L}_S, \tag{3}$$
$$a_{UY}Y = \overline{L}_U, \tag{4}$$
$$a_{KX}X + a_{KY}Y = \overline{K}, \tag{5}$$

where $\overline{L}_S$, $\overline{L}_U$, and $\overline{K}$ denote the skilled labor, unskilled labor and capital endowments, respectively.

So far, we have built the basic model. Eqs. (1)-(5) determine 5 endogenous variables, namely w_S, w_U, r, X, and Y. Since we plan to explore the impact of the intensity of idea exchange on wage inequality, α and β are the exogenous variables that we focus on in this paper. Other variables are parameters.

Totally differentiating Eqs. (1)-(5), and substituting the results of Eqs. (3) and (4) into Eq. (5), we can rewrite Eqs. (1), (2) and (5) in the matrix form as follows:

$$\begin{pmatrix} \theta_{SX} & 0 & \theta_{KX} \\ 0 & \theta_{UY} & \theta_{KY} \\ \lambda_{KX}\sigma_X & \lambda_{KY}\sigma_Y & -\lambda_{KX}\sigma_X - \lambda_{KY}\sigma_Y \end{pmatrix} \begin{pmatrix} \hat{w}_S \\ \hat{w}_U \\ \hat{r} \end{pmatrix} = \frac{g_\alpha^X}{g^X} \begin{pmatrix} \alpha \\ 0 \\ 0 \end{pmatrix} \hat{\alpha} + \frac{g_\beta^Y}{g^Y} \begin{pmatrix} 0 \\ \beta \\ 0 \end{pmatrix} \hat{\beta}, \tag{6}$$

where θ_{ij} ($i = S, U, K; j = X, Y$) represents the distributive share of factor i employed in sector j (e.g., $\theta_{SX} = \frac{a_{SX}w_S}{p_X}$). λ_{Kj} ($j = X, Y$) is the allocative

share of K used in sector j (e.g., $\lambda_{KX} = \frac{a_{KX}X}{\bar{K}}$). σ_j $(j = X, Y)$ indicates the substitution elasticity between the two factors used in sector j. The symbol "$\hat{}$" denotes the relative rate of the change of a variable (e.g., $\hat{w}_S = \frac{dw_S}{w_S}$).

Firstly, we judge the sign of coefficient determinant (denoted as Δ_1). After calculation, we obtain: $\Delta_1 = -\theta_{UY}\lambda_{KX}\sigma_X - \theta_{SX}\lambda_{KY}\sigma_Y < 0$.

In order to examine the impact of idea exchange in the skilled sector on wage inequality, we set $\hat{\beta} = 0$ and $\hat{\alpha} > 0$.

Now, we use proposition 1 to show the impact.

PROPOSITION 1. *In the economy with two sectors and full employment, an increase in the intensity of idea exchange in the skilled sector will: (i) increase the skilled wage rate; (ii) decrease the unskilled wage rate; and (iii) expand wage inequality.*

Proof. Using Cramer's rule to solve Eq. (6), we obtain:

$$\begin{aligned}\frac{\hat{w}_S}{\hat{\alpha}} &= -\frac{g^X_\alpha}{g^X} \cdot \frac{\alpha(\theta_{UY}\lambda_{KX}\sigma_X + \lambda_{KY}\sigma_Y)}{\Delta_1} > 0,\\ \frac{\hat{w}_U}{\hat{\alpha}} &= \frac{g^X_\alpha}{g^X} \cdot \frac{\alpha\theta_{KY}\lambda_{KX}\sigma_X}{\Delta_1} < 0.\end{aligned}$$

Thus, the relative change of skilled-unskilled wage inequality can be described as:

$$\frac{\hat{w}_S - \hat{w}_U}{\hat{\alpha}} = -\frac{g^X_\alpha}{g^X} \cdot \frac{\alpha(\lambda_{KX}\sigma_X + \lambda_{KY}\sigma_Y)}{\Delta_1} > 0.$$

∎

The economic intuition behind Proposition 1 is straightforward. When there is an increase in the intensity of idea exchange in the skilled sector, skilled workers will have more opportunities to share ideas with each other. Through communication with other workers, the group will be more skilled, which leads to an increase in the productivity of the skilled sector. As a result, the marginal product of skilled workers will increase, and so the skilled wage rate will increase relatively. At the same time, this sector will employ more capital to expand production. As the amount of capital is unchanged, it will increase the interest rate. For the unskilled sector, everything remains the same as before except the interest rate, and so the cost of employing capital will increase. Due to profit maximization, this sector will reduce production. Correspondingly, the demand for unskilled labor will decrease, and so the unskilled wage rate will decrease. As a result, wage inequality will be expanded.

In order to examine the impact of idea exchange in the unskilled sector on wage inequality, we set $\hat{\alpha} = 0$ and $\hat{\beta} > 0$.

Now, we can use Proposition 2 to show the impact.

PROPOSITION 2. *In the economy with two sectors and full employment, an increase in the intensity of idea exchange in the unskilled sector will: (i) decrease the skilled wage rate; (ii) increase the unskilled wage rate; and (iii) narrow down wage inequality.*

Proof. Using Cramer's rule to solve Eq. (6), we obtain:

$$\begin{aligned}\frac{\hat{w}_S}{\hat{\beta}} &= \frac{g_\beta^Y}{g^Y} \cdot \frac{\beta\theta_{KX}\lambda_{KY}\sigma_Y}{\Delta_1} < 0,\\ \frac{\hat{w}_U}{\hat{\beta}} &= -\frac{g_\beta^Y}{g^Y} \cdot \frac{\beta(\lambda_{KX}\sigma_X + \theta_{SX}\lambda_{KY}\sigma_Y)}{\Delta_1} > 0.\end{aligned}$$

Thus, the relative change of skilled-unskilled wage inequality can be depicted as:

$$\frac{\hat{w}_S - \hat{w}_U}{\hat{\beta}} = \frac{g_\beta^Y}{g^Y} \cdot \frac{\beta(\lambda_{KY}\sigma_Y + \lambda_{KX}\sigma_X)}{\Delta_1} < 0.$$

∎

The economic intuition behind Proposition 2 is the opposite process of that behind Proposition 1. An increase in the intensity of idea exchange in the unskilled sector will increase the productivity of this sector and decrease the productivity of the skilled sector. Due to profit maximization, the wage rate of skilled labor will decrease and that of unskilled labor will increase. Therefore, wage inequality will be reduced.

At the same time, the results predicted by the model match the empirical facts. Through the empirical analysis, Luo et al. (2017) point out that the diversity of social interaction links of individuals, which can be used as a proxy variable for the degree of exchange of ideas, is highly correlated with their economic status. Jahani et al. (2016) empirically find that even after controlling for education, occupation, age, and gender, the diversity of social network structure has a significant positive impact on individual income, and that this effect is greater for highly educated people (i.e., skilled workers). Therefore, when the exchange of ideas in the skilled sector increases, the earnings of skilled workers will increase and the wage gap will be widened. However, when the exchange of ideas in the unskilled sector increases, the earnings of unskilled workers will rise and the wage gap will be shrunken.

3. EXTENDED MODELS

3.1. Economy with the Individual Effect of Idea Exchange

In the above model, we regard the effect of idea exchange as a whole. Under this effect, an increase in the intensity of idea exchange will have a positive impact on the overall productivity of the related sector. Now, we assume that through communication and learning from each other, everyone's productivity in this sector will be improved. Under this assumption, the impact of idea exchange will be individualized. Similar to the basic model, we let the effect of idea exchange on each worker be $g^i(\cdot)$ $(i = X, Y)$, which is only determined by the intensity of idea exchange (α or β), and so the effect can be expressed as $g^X(\alpha)$ and $g^Y(\beta)$, respectively. Thus, the production functions can be rewritten as follows: $X = F^1(g^X(\alpha)L_S, K_X)$, and $Y = F^2(g^Y(\beta)L_U, K_Y)$, where $g^X(\alpha) > 0$ and $g^Y(\beta) > 0$. The derivatives of the two functions satisfy: $g^X_\alpha = \frac{dg^X(\alpha)}{d\alpha} > 0$, and $g^Y_\beta = \frac{dg^Y(\beta)}{d\beta} > 0$.

In this model, the zero-profit and the market-clearing conditions are the same as those in the basic model, which means that Eqs. (1)-(5) still hold. Using the same method as in Section 2, we get the matrix form of these equations as follows:

$$\begin{pmatrix} \theta_{SX} & 0 & \theta_{KY} \\ 0 & \theta_{UY} & \theta_{KY} \\ \lambda_{KX}\sigma_X & \lambda_{KY}\sigma_Y & -\lambda_{KX}\sigma_X - \lambda_{KY}\sigma_Y \end{pmatrix} \begin{pmatrix} \hat{w}_S \\ \hat{w}_U \\ \hat{r} \end{pmatrix} = \theta_{SX}\frac{g^X_\alpha}{g^X}\begin{pmatrix} \alpha \\ 0 \\ 0 \end{pmatrix}\hat{\alpha} + \theta_{UY}\frac{g^Y_\beta}{g^Y}\begin{pmatrix} 0 \\ \beta \\ 0 \end{pmatrix}\hat{\beta}, \tag{7}$$

Compare this result with that in the basic model, and we find that the only differences of them are the coefficients of $\hat{\alpha}$ and $\hat{\beta}$.

Firstly, in order to examine the impact of the intensity of idea exchange in the skilled sector on wage inequality, we set $\hat{\beta} = 0$ and $\hat{\alpha} > 0$.

Now, we use Proposition 3 to show the impact.

PROPOSITION 3. *In the economy with two sectors and full employment, considering the individual effect of idea exchange, an increase in the intensity of idea exchange in the skilled sector has almost the same impacts on the skilled and unskilled wage rates and wage inequality as those in Proposition 1.*

Proof. The proof is almost the same as that of Proposition 1. ▌

The economic intuition is similar to that behind Proposition 1. Because the effect of the intensity of idea exchange is individualized, an increase in the intensity in the skilled sector will increase the productivity of each skilled worker. Since the skilled labor's marginal product increases, the skilled wage rate will rise. At the same time, the interest rate will increase, which makes the unskilled sector reduce production. In order to control

costs, the wage rate of unskilled labor will decrease. Consequently, wage inequality will be expanded.

Secondly, in order to examine the impact of the intensity of idea exchange in the unskilled sector on wage inequality, we set $\hat{\alpha} = 0$ and $\hat{\beta} > 0$. Now, we can use Proposition 4 to show the impact.

PROPOSITION 4. *In the economy with two sectors and full employment, considering the individual effect of idea exchange, an increase in the intensity of idea exchange in the skilled sector has almost the same impacts on the unskilled and unskilled wage rates and wage inequality as those in Proposition 2.*

Proof. The proof is almost the same as that in Proposition 2. ▌

The economic intuition is similar to that behind Proposition 2 and is opposite to that behind Proposition 3.

3.2. Economy with a Totally Open Capital Market

In the basic model, we assume that the good market is open but the capital market is closed. In this section, we will consider another economic system with an open capital market. To be specific, capital can freely flow domestically and abroad. Under this assumption, the interest rate refers to the international one, which is exogenous.

We let $\overline{r}$ denote the fixed interest rate, then Eqs. (1)-(2) should be rewritten as follows:

$$p_X = a_{SX} w_S + a_{KX} \overline{r}, \tag{8}$$

$$p_Y = a_{UY} w_U + a_{KY} \overline{r}. \tag{9}$$

In this model, we can find that the two sectors are both independent. We firstly set $\hat{\beta} = 0$ and $\hat{\alpha} > 0$ to examine the impact of the intensity of idea exchange in the skilled sector on wage inequality.

Now, we use Proposition 5 to show the result.

PROPOSITION 5. *In the economy with two sectors, full employment and an open capital market, an increase in the intensity of idea exchange in the skilled sector will: (i) increase the skilled wage rate; (ii) not change the unskilled wage rate; and (iii) expand wage inequality.*

Proof. Totally differentiating Eqs. (8)-(9), we obtain:

$$\begin{aligned}\theta_{SX}\hat{w}_S &= \frac{g_\alpha^X}{g^X}\alpha\cdot\hat{\alpha},\\ \theta_{UY}\hat{w}_U &= \frac{g_\alpha^Y}{g^Y}\beta\cdot\hat{\beta}.\end{aligned}$$

In the case that $\hat{\beta}=0$, we know that $\frac{\hat{w}_S}{\hat{\alpha}}=\frac{g_\alpha^X}{g^X}\cdot\frac{\alpha}{\theta_{SX}}>0$ and $\frac{\hat{w}_U}{\hat{\alpha}}=0$. Thus, $\frac{\hat{w}_S-\hat{w}_U}{\hat{\alpha}}=\frac{g_\alpha^X}{g^X}\cdot\frac{\alpha}{\theta_{SX}}>0$. ▌

The economic intuition behind Proposition 5 is as follows. Similar to Proposition 1, an increase in the intensity of idea exchange will make skilled labor more skilled, and so the marginal product of skilled labor will increase. As a result, there will be an increase in the wage rate of skilled labor. Since the capital market is open and the interest rate is fixed, the productivity expansion in the skilled sector has nothing to do with the productivity of the unskilled sector. So, each parameter in the unskilled sector is unchanged when there is an increase in the intensity of idea exchange in the skilled sector. Finally, wage inequality will be expanded.

Secondly, in order to examine the impact of idea exchange in the unskilled sector on wage inequality, we set $\hat{\alpha}=0$ and $\hat{\beta}>0$.

Now, we establish Proposition 6 to show the impact.

PROPOSITION 6. *In the economy with two sectors, full employment and an open capital market, an increase in the intensity of idea exchange in the unskilled sector will: (i) not change the skilled wage rate; (ii) increase the unskilled wage rate; and (iii) reduce wage inequality.*

Proof. Using the proof of Proposition 3, under the condition that $\hat{\alpha}=0$, we know that $\frac{\hat{w}_S}{\hat{\beta}}=0$ and $\frac{\hat{w}_U}{\hat{\beta}}=\frac{g_\alpha^Y}{g^Y}\cdot\frac{\beta}{\theta_{UY}}>0$. Thus, $\frac{\hat{w}_S-\hat{w}_U}{\hat{\beta}}=-\frac{g_\alpha^Y}{g^Y}\cdot\frac{\beta}{\theta_{UY}}<0$. ▌

The economic intuition behind Proposition 6 is the opposite to that behind Proposition 5.

3.3. Economy with the Skilled Product Employed in the Unskilled Sector

In this subsection, we assume that the skilled sector is the upstream industry of the unskilled sector, and so the skilled product produced by the skilled sector is an intermediate good, which is used by the unskilled sector to produce the product Y. The production function should be specified firstly. Just as before, the skilled sector employs skilled labor L_S and capital K_X to produce the intermediate product X. The unskilled sector

employs unskilled labor L_U, capital K_Y and the intermediate product X to produce the final product Y. In the capital market, capital can flow freely between the two sectors. In the labor market, skilled labor is specific to the skilled sector and unskilled labor is specific to the unskilled sector. In this case, the production functions of the two sectors should be rewritten as follows: $X = g^X(\alpha) \cdot F^1(L_S, K_X)$, and $Y = g^Y(\beta) \cdot F^2(L_U, K_Y, X)$.

In line with Pi and Zhang (2018a), we assume that all the intermediate product X employed by the unskilled sector is provided by the skilled sector, which means that the unskilled sector cannot import X from abroad. Correspondingly, we assume that the product X can only be sold domestically. As a result, the price of the product X is determined by the domestic market.

Now, according to the zero-profit conditions, Eq. (1) still holds and we can rewrite Eq. (2) as follows:

$$p_Y = a_{UY} w_U + a_{KY} r + a_{XY} p_X, \tag{10}$$

where p_X is an endogenous variable and denotes the domestic price of the intermediate product X. a_{XY} denotes the amount of the product X employed by the unskilled sector to produce one unit of the final product Y.

According to the market-clearing conditions, Eqs. (3)-(5) still hold and the condition of the intermediate product X yields:

$$a_{XY} Y = X. \tag{11}$$

So far, a new general equilibrium model with the product X employed in the unskilled sector has been established. Eqs. (1), (3)-(5) and (10)-(11) determine 6 endogenous variables, namely w_S, w_U, r, p_X, X, and Y. α and β are still the exogenous variables that we focus on in this paper. Other variables are parameters.

Totally differentiating Eqs. (1), (3)-(5) and (10)-(11), and substituting the results of Eqs. (3) and (4) into Eqs. (5) and (11), we rewrite Eqs. (1), (5) and (10)-(11) in the matrix form as follows:

$$\begin{pmatrix} \theta_{SX} & 0 & \theta_{KX} & -1 \\ 0 & \theta_{UY} & \theta_{KY} & \theta_{XY} \\ -\sigma_X \theta_{KX} & \sigma_{UX}^Y & \sigma_X \theta_{KX} & -\sigma_{UX}^Y \\ \lambda_{KX}\sigma_X & \lambda_{KY}\sigma_{UK}^Y & -\lambda_{KX}\sigma_X - \lambda_{KY}\sigma_{UK}^Y & 0 \end{pmatrix} \begin{pmatrix} \hat{w}_S \\ \hat{w}_U \\ \hat{r} \\ \hat{p}_X \end{pmatrix}$$
$$= \frac{g_\alpha^X}{g^X} \begin{pmatrix} \alpha \\ 0 \\ \alpha \\ 0 \end{pmatrix} \hat{\alpha} + \frac{g_\beta^Y}{g^Y} \begin{pmatrix} 0 \\ \beta \\ 0 \\ 0 \end{pmatrix} \hat{\beta}, \tag{12}$$

where θ_{XY} denotes the distributive share of the intermediate product X employed in the unskilled sector (i.e., $\theta_{XY} = \frac{a_{XY}Y}{p_X}$). σ_{ij}^Y $(i, j = U, K, X)$ indicates the substitution elasticity between the two factors used in the unskilled sector.

By calculating the coefficient determinant (denoted as Δ_2), we obtain: $\Delta_2 = -\theta_{KX}\lambda_{KY}\sigma_X\sigma_{UK}^Y - (\lambda_{KX}\sigma_X + \theta_{SX}\lambda_{KY}\sigma_{UK}^Y)\sigma_{UX}^Y < 0$.

Firstly, in order to examine the impact of the intensity of idea exchange in the skilled sector on wage inequality, we set $\hat{\alpha} > 0$ and $\hat{\beta} = 0$.

Now, we use Proposition 7 to show the impact.

PROPOSITION 7. *In the economy with two sectors, full employment and the skilled product employed in the unskilled sector, when the intensity of idea exchange in the skilled sector increases, if the substitution elasticity of unskilled labor and the skilled product in the unskilled sector is large (resp., small) enough, then: (i) the skilled wage rate will increase; (ii) the unskilled wage rate will decrease (resp., increase); and (iii) wage inequality will be expanded (resp., narrowed down).*

Proof. Using Cramer's rule to solve Eq. (12), we obtain:

$$\begin{aligned}\frac{\hat{w}_S}{\hat{\alpha}} &= -\frac{\alpha g_\alpha^X}{g^X}\cdot\frac{B_1}{\Delta_2},\\ \frac{\hat{w}_U}{\hat{\alpha}} &= -\frac{\alpha g_\alpha^X}{g^X}\cdot\frac{B_2}{\Delta_2},\end{aligned}$$

where $B_1 = [\theta_{KY}\lambda_{KY}\sigma_{UK}^Y + \theta_{UY}(\lambda_{KX}\sigma_X + \lambda_{KY}\sigma_{UK}^Y)](\sigma_{UX}^Y - 1) + \theta_{XY}(\lambda_{KX}\sigma_X + \lambda_{KY}\sigma_{UK}^Y)\sigma_{UX}^Y + \theta_{KX}\theta_{XY}\lambda_{KY}\sigma_{UK}^Y(\sigma_X - 1)$, and $B_2 = \theta_{XY}\lambda_{KX}\sigma_X + \theta_{XY}\lambda_{KY}(\theta_{SX} + \theta_{KX}\sigma_X)\sigma_{UK}^Y + \theta_{KY}\lambda_{KX}\sigma_X(1 - \sigma_{UX}^Y)$.

Thus, the relative change of skilled-unskilled wage inequality can be described as:

$$\frac{\hat{w}_S - \hat{w}_U}{\hat{\alpha}} = \frac{\alpha g_\alpha^X}{g^X}\cdot\frac{(\lambda_{KX}\sigma_X + \lambda_{KY}\sigma_{UK}^Y)(1 - \sigma_{UX}^Y)}{\Delta_2}.$$

Firstly, we focus on the change of the skilled wage rate. We know that B_1 is an increasing function of σ_{UX}^Y. Suppose that σ_1^* $(\sigma_1^* < 1)$ solves $B_1 = 0$, which makes $\frac{\hat{w}_S}{\hat{\alpha}} = 0$. Then, if $\sigma_{UX}^Y > \sigma_1^*$, then $B_1 > 0$ and $\frac{\hat{w}_S}{\hat{\alpha}} > 0$. However, if $\sigma_{UX}^Y < \sigma_1^*$, then $B_1 < 0$ and $\frac{\hat{w}_S}{\hat{\alpha}} < 0$. Since in actual production $\sigma_{UX}^Y > 0$ is a hard condition but σ_1^* is not necessarily greater than 0, it is uncertain whether $\frac{\hat{w}_S}{\hat{\alpha}} < 0$ holds.

Secondly, for the unskilled wage rate, B_2 is a decreasing function of σ_{UX}^Y. Suppose that σ_2^* $(\sigma_2^* > 1)$ solves $B_2 = 0$, which makes $\frac{\hat{w}_U}{\hat{\alpha}} = 0$. Then, if $\sigma_{UX}^Y > \sigma_2^*$, then $B_2 < 0$ and $\frac{\hat{w}_U}{\hat{\alpha}} < 0$; and if $0 < \sigma_{UX}^Y < \sigma_2^*$, then $B_2 > 0$ and $\frac{\hat{w}_U}{\hat{\alpha}} > 0$.

Finally, we know that wage inequality is also determined by the substitution elasticity of L_U and X in the unskilled sector (i.e., σ_{UX}^Y). If $\sigma_{UX}^Y > 1$, then $\frac{\hat{w}_S - \hat{w}_U}{\hat{\alpha}} > 0$; and if $\sigma_{UX}^Y < 1$, then $\frac{\hat{w}_S - \hat{w}_U}{\hat{\alpha}} < 0$. ▌

The economic intuition behind Proposition 7 is as follows. When the intensity of idea exchange increases in the skilled sector, then the marginal product of skilled labor increases, and so the production of X will be expanded. Since the price of X is determined endogenously, it will decrease due to more supply. With a decrease in the price of X, driven by the principle of profit maximization, the unskilled sector will employ more X. If the substitution elasticity of L_U and X in the unskilled sector is large enough, the unskilled sector will reduce the employment of unskilled labor and employs more X instead. This decision will lead to two changes. First, more demand for X will balance a part of the supply and make the price of X rise not so much. As a result, under the zero-profit condition, the wage rate of skilled labor will increase. Second, although the unskilled sector will expand its production, it does not need to employ more unskilled workers, because they can be replaced by X that is cheaper. Thus, the wage rate of unskilled labor does not rise but fall. Of course, this result requires that the substitution elasticity of L_U and X in the unskilled sector is very large. Combining the two changes together, if the substitution elasticity of L_U and X in the unskilled sector is large enough (i.e., larger than 1), then wage inequality will be widened.

However, when the substitution elasticity of L_U and X in the unskilled sector is small enough, unskilled workers cannot be replaced by X on a large scale, and so the unskilled sector will employ more unskilled labor. Then, the unskilled wage rate will increase. As for skilled labor, the price of X decreases but the marginal product of skilled labor increases, and so the change of the skilled wage rate is not certain. Since the change in the unskilled wage rate is significant, wage inequality will be narrowed down.

Then, in order to examine the impact of the intensity of idea exchange in the unskilled sector on wage inequality, we set $\hat{\beta} > 0$ and $\hat{\alpha} = 0$.

Now, we use Proposition 8 to describe the impact.

PROPOSITION 8. *In the economy with two sectors, full employment and the skilled product employed in the unskilled sector, when the intensity of idea exchange in the unskilled sector increases, then: (i) the skilled wage rate will increase; (ii) the unskilled wage rate will increase; and (iii) wage inequality will be unchanged.*

Proof. Using Cramer's rule to solve Eq. (12), we obtain:

$$\frac{\hat{w}_S}{\hat{\beta}} = \frac{\hat{w}_U}{\hat{\beta}} = -\frac{\beta g^Y_\beta}{g^Y} \cdot \frac{\lambda_{KY}\sigma^Y_{UK}(\theta_{KX}\sigma_X + \theta_{SX}\sigma^Y_{UX}) + \lambda_{KX}\sigma_X\sigma^Y_{UX}}{\Delta_2} > 0.$$

Thus, the relative change of skilled-unskilled wage inequality can be described as:

$$\frac{\hat{w}_S - \hat{w}_U}{\hat{\beta}} = 0.$$

∎

The economic intuition behind Proposition 8 is as follows. When the intensity of idea exchange increases in the unskilled sector, the productivity of this sector will rise. In pursuit of profit maximization, the unskilled sector will employ more factors to expand its production, including unskilled labor L_U, capital K_Y, and the intermediate product X. Since the marginal product of unskilled labor increases, the unskilled wage rate will increase. As for the skilled sector, more demand for the intermediate product X will make this sector employ more skilled labor to expand their production at the same time, and the scale of expansion is exactly the scale of the increased demand for X in the unskilled sector. As a result, the skilled wage rate will increase and the increase is the same as the unskilled wage rate. Thus, wage inequality will not be changed.

3.4. Economy with a Mixed Sector

In the above models, we assume that skilled and unskilled workers are specific to the skilled sector and the unskilled sector, respectively. In this subsection, we relax the assumption and change the skilled sector to a mixed sector, which employs both skilled and unskilled workers. In this sector, skilled workers are engaged in high-skill design or production work and unskilled workers are engaged in low-skill assembly work. In addition, we assume that the staff in the mixed sector are mainly composed of skilled labor. Thus, the mixed sector employs skilled labor L_S, unskilled sector L_{UX}, and capital K_X to produce the product X. As for the unskilled sector, our assumption is unchanged, which means that this sector employs unskilled labor L_{UY} and capital K_Y to produce the product Y.

Just as before, we let α and β denote the intensity of idea exchange in the mixed sector and the unskilled sector, respectively. Thus, the production functions should be rewritten as: $X = g^X(\alpha) \cdot F^1(L_S, L_{UX}, K_X)$, and $Y = g^Y(\beta) \cdot F^2(L_{UY}, K_Y)$, where $g^X(\alpha) > 1$, and $g^Y(\beta) > 1$. The signs of derivatives of these two functions are both positive.

Now, Eq. (2) still holds and the zero-profit condition of the mixed sector is as follows:

$$p_X = a_{SX}w_S + a_{UX}w_U + a_{KX}r, \tag{13}$$

where a_{UX} denotes the amount of unskilled labor employed by the mixed sector to produce one unit of the final product X.

The market-clearing conditions of skilled labor and capital (i.e., Eqs. (3) and (5)) still hold and that of unskilled labor yields:

$$a_{UX}X + a_{UY}Y = \overline{L}_U. \tag{14}$$

So far, we have constructed a new economic system with a mixed sector. Eqs. (2), (3), (5), and (13)-(14) determine 5 endogenous variables, namely w_S, w_U, r, X, and Y. α and β are still the exogenous variables that we focus on in this paper. Other variables are parameters.

Totally differentiating Eqs. (2), (3), (5), and (13)-(14), we can rewrite the results of them in the following matrix form:

$$\begin{pmatrix} \theta_{SX} & \theta_{UX} & \theta_{KX} & 0 & 0 \\ 0 & \theta_{UY} & \theta_{KY} & 0 & 0 \\ -\sigma^X_{SU}\theta_{UX} - \sigma^X_{SK}\theta_{KX} & \sigma^X_{SU}\theta_{UX} & \sigma^X_{SK}\theta_{KX} & 1 & 0 \\ \lambda_{UX}\sigma^X_{SU}\theta_{SX} & A & B & \lambda_{UX} & \lambda_{UY} \\ \lambda_{KX}\sigma^X_{SK}\theta_{SX} & C & D & \lambda_{KX} & \lambda_{KY} \end{pmatrix} \begin{pmatrix} \hat{w}_S \\ \hat{w}_U \\ \hat{r} \\ \hat{X} \\ \hat{Y} \end{pmatrix}$$
$$= \alpha\frac{g^X_\alpha}{g^X}\begin{pmatrix} 1 \\ 0 \\ 1 \\ \lambda_{UX} \\ \lambda_{KX} \end{pmatrix}\hat{\alpha} + \beta\frac{g^Y_\beta}{g^Y}\begin{pmatrix} 0 \\ 1 \\ 0 \\ \lambda_{UY} \\ \lambda_{KY} \end{pmatrix}\hat{\beta}, \tag{15}$$

where $A = -\lambda_{UX}(\sigma^X_{SU}\theta_{SX}+\sigma^X_{UK}\theta_{KX})-\lambda_{UY}\sigma_Y\theta_{KY} < 0$, $B = \lambda_{UX}\sigma^X_{UK}\theta_{KX}+\lambda_{UY}\sigma_Y\theta_{KY} > 0$, $C = \lambda_{KX}\sigma^X_{UK}\theta_{UX}+\lambda_{KY}\sigma_Y\theta_{UY>0}$, and $D = -\lambda_{KX}(\sigma^X_{SK}\theta_{SX}+\sigma^X_{UK}\theta_{UX})-\lambda_{KY}\sigma_Y\theta_{UY} < 0$. θ_{UX} is the distributive share of unskilled labor L_{UX} used in the mixed sector. σ^X_{ij} $(i, j = S, U, K)$ denotes the elasticity of substitution between factor i and factor j in the unskilled sector.

To begin with, we should judge the sign of the coefficient determinant of Eq. (15) (denoted as Δ_3). Using the dynamic adjustment process, $\Delta_3 < 0$ should be required to ensure the local stability (see Appendix A).

Compared with the unskilled sector Y that employs unskilled workers, the mixed sector, which uses capital on a large scale, only employs a small part of unskilled workers. Therefore, it is reasonable to assume that the mixed sector is more capital-intensive than the unskilled sector. Thus, the

following conditions should be satisfied:

$$\lambda_{KX}\lambda_{UY} - \lambda_{KY}\lambda_{UX} > 0, \tag{16}$$
$$\theta_{KX}\theta_{UY} - \theta_{KY}\theta_{UX} > 0. \tag{17}$$

Firstly, in order to examine the impact of the intensity of idea exchange in the mixed sector on wage inequality, we set $\hat{\alpha} > 0$ and $\hat{\beta} = 0$.

Now, we use Proposition 9 to show the impact.

PROPOSITION 9. *In the economy with a mixed sector, an unskilled sector and full employment, an increase in the intensity of idea exchange in the mixed sector has almost the same impacts on the skilled and unskilled wage rates and wage inequality as those in Proposition 1.*

Proof. Using Cramer's rule to solve Eq. (15), we obtain:

$$\frac{\hat{w}_S}{\hat{\alpha}} = -\frac{\alpha g_\alpha^X}{g^X} \cdot \frac{\Omega_1}{\Delta_3},$$
$$\frac{\hat{w}_U}{\hat{\alpha}} = \frac{\alpha g_\alpha^X}{g^X} \cdot \frac{\Omega_2}{\Delta_3},$$

where $\Omega_1 = \theta_{UY}[(\theta_{KX}+\theta_{SX})\lambda_{KX}\lambda_{UY} - \theta_{KX}\lambda_{KY}\lambda_{UX}]\sigma_{SK}^X + (\theta_{KX}\lambda_{KY}\lambda_{UX} + \theta_{UX}\lambda_{KX}\lambda_{UY})\sigma_{UK}^X + \lambda_{KY}\lambda_{UY}\sigma_Y - \theta_{KY}[\theta_{UX}\lambda_{KX}\lambda_{UY} - (\theta_{UX}+\theta_{SX})\lambda_{KY}\lambda_{UX}]\sigma_{SU}^X$, and $\Omega_2 = \theta_{KY}[(\theta_{KX}+\theta_{SX})\lambda_{KX}\lambda_{UY} - \theta_{KX}\lambda_{KY}\lambda_{UX}]\sigma_{SK}^X + \theta_{KY}[\theta_{UX}\lambda_{KX}\lambda_{UY} - (\theta_{UX} + \theta_{SX})\lambda_{KY}\lambda_{UX}]\sigma_{SU}^X$.

Thus, the relative change of skilled-unskilled wage inequality can be described as:

$$\frac{\hat{w}_S - \hat{w}_U}{\hat{\alpha}} = -\frac{\alpha g_\alpha^X}{g^X} \cdot \frac{\Omega_3}{\Delta_3},$$

where $\Omega_3 = \theta_{KX}\sigma_{SK}^X(\lambda_{KX}\lambda_{UY} - \lambda_{KY}\lambda_{UX}) + \theta_{KX}\lambda_{KY}\lambda_{UX}\sigma_{UK}^X + \lambda_{UY}[\theta_{SX}\lambda_{KX}\sigma_{SK}^X + \theta_{UX}\lambda_{KX}\sigma_{UK}^X + \lambda_{KY}\sigma_Y]$.

Firstly, we focus on the change of the skilled wage rate. We know that the work of skilled and unskilled workers is completely different, and so they can hardly be replaced by each other, which means that the substitution elasticity of L_S and L_{UX} in the skilled sector is very small (i.e., close to 0). Then from Eq. (16), we have $(\theta_{KX} + \theta_{SX})\lambda_{KX}\lambda_{UY} - \theta_{KX}\lambda_{KY}\lambda_{UX} > 0$. So we get $\Omega_1 > 0$, then $\frac{\hat{w}_S}{\hat{\alpha}} > 0$ is ensured.

Secondly, for the unskilled wage rate, since the substitution elasticity of L_S and L_{UX} in the skilled sector is very small (i.e., close to 0), we have $\Omega_2 > 0$. Substituting it to the expression, we obtain $\frac{\hat{w}_U}{\hat{\alpha}} < 0$.

Finally, from Eq. (16), we know that $\lambda_{KX}\lambda_{UY} - \lambda_{KY}\lambda_{UX} > 0$, and so $\Omega_3 > 0$. As a result, we get $\frac{\hat{w}_S - \hat{w}_U}{\hat{\alpha}} > 0$. ▌

The economic intuition behind Proposition 9 is as follows. When there is an increase in the intensity of idea exchange in the mixed sector, the productivity of this sector will increase. Then, this sector will employ more skilled workers, unskilled workers, and capital to expand production. More demands for skilled workers and capital will make the skilled wage rate and the interest rate increase. With an increase in the interest rate, the cost of the unskilled sector will increase. To pursue profit maximization, the unskilled sector will employ less unskilled labor. We know that the unskilled sector is the main sector for unskilled labor to work and the mixed sector only employs a small part of unskilled labor, and so the demand for unskilled labor decreases on the whole, which makes the unskilled wage rate lower. Combining the changes of the two types of labor, we know that wage inequality will be expanded.

Then, in order to examine the impact of the intensity of idea exchange in the unskilled sector on wage inequality, we set $\hat{\beta} > 0$ and $\hat{\alpha} = 0$.

Now, we use Proposition 10 to show the impact.

PROPOSITION 10. *In the economy with a mixed sector, an unskilled sector and full employment, an increase in the intensity of idea exchange in the unskilled sector has almost the same impacts on the skilled and unskilled wage rates and wage inequality as those in Proposition 2.*

Proof. Using Cramer's rule to solve Eq. (15), we obtain:

$$\begin{aligned}\frac{\hat{w}_S}{\hat{\beta}} &= \frac{\beta g^Y_\beta}{g^Y} \cdot \frac{\Omega_4}{\Delta_3},\\ \frac{\hat{w}_U}{\hat{\beta}} &= -\frac{\beta g^Y_\beta}{g^Y} \cdot \frac{\Omega_5}{\Delta_3},\end{aligned}$$

where $\Omega_4 = [\theta_{UX}\theta_{SX}\lambda_{KX}\lambda_{UY} + \theta_{KX}\theta_{UX}(\lambda_{KX}\lambda_{UY} - \lambda_{KY}\lambda_{UX})]\sigma^X_{SX} + (\theta_{KX}+\theta_{UX})(\theta_{KX}\lambda_{KY}\lambda_{UX}+\theta_{UX}\lambda_{KX}\lambda_{UY})\sigma^X_{UK}+(\theta_{KX}+\theta_{UX})\lambda_{KY}\lambda_{UY}\sigma_Y+\theta_{KX}[(\theta_{SX}+\theta_{UX})\lambda_{KY}\lambda_{UX}-\theta_{UX}\lambda_{KX}\lambda_{UY}]\sigma^X_{SU}$, and $\Omega_5 = (\theta_{KX}+\theta_{SX})[(\theta_{KX}+\theta_{SX})\lambda_{KX}\lambda_{UY}-\theta_{KX}\lambda_{KY}\lambda_{UX}]\sigma^X_{SK}+\theta_{SX}(\theta_{UX}\lambda_{KX}\lambda_{UY}+\theta_{KX}\lambda_{KY}\lambda_{UX})\sigma^X_{UK}+\theta_{SX}\lambda_{UY}\lambda_{KY}\sigma_Y + \theta_{KX}[\theta_{UX}\lambda_{KX}\lambda_{UY} - (\theta_{UX}+\theta_{SX})\lambda_{KY}\lambda_{UX}]\sigma^X_{SU}$.

Thus, the relative change of skilled-unskilled wage inequality can be described as:

$$\frac{\hat{w}_S - \hat{w}_U}{\hat{\beta}} = \frac{\beta g^Y_\beta}{g^Y} \cdot \frac{\Omega_6}{\Delta_3},$$

where $\Omega_6 = [(\theta_{KX}+\theta_{SX})\lambda_{KX}\lambda_{UY} - \theta_{KX}\lambda_{KY}\lambda_{UX}]\sigma_{SK}^X + (\theta_{UX}\lambda_{KX}\lambda_{UY} + \theta_{KX}\lambda_{KY}\lambda_{UX})\sigma_{UK}^X + \lambda_{KY}\lambda_{UY}\sigma_Y$.

Firstly, for the skilled wage rate, since the substitution elasticity of L_S and L_{UX} in the skilled sector is very small (i.e., close to 0), then from Eq. (16), we have $\Omega_4 > 0$. Thus, $\frac{\hat{w}_U}{\hat{\beta}} < 0$ is ensured.

Secondly, for the unskilled wage rate, in the case that the substitution elasticity of L_S and L_{UX} in the skilled sector is very small (i.e., close to 0), we have $\Omega_5 > 0$. Furthermore, we obtain $\frac{\hat{w}_U}{\hat{\beta}} > 0$.

Finally, from Eq. (16), $\lambda_{KX}\lambda_{UY} - \lambda_{KY}\lambda_{UX} > 0$ is ensured, and so we have $(\theta_{KX}+\theta_{SX})\lambda_{KX}\lambda_{UY} - \theta_{KX}\lambda_{KY}\lambda_{UX} > 0$. As a result, $\Omega_6 > 0$. So we get $\frac{\hat{w}_S - \hat{w}_U}{\hat{\beta}} < 0$. ▌

The economic intuition behind Proposition 10 is as follows. When there is an increase in the intensity of idea exchange in the unskilled sector, the productivity of this sector will increase. To expand production, the unskilled sector will employ more unskilled labor and capital. More demands will make the unskilled wage rate and the interest rate increase, and so the cost of the mixed sector will rise. Thus, the demand for skilled labor will decrease, which makes the skilled wage rate decrease. Combining with the changes of the two types of labor, we know that wage inequality will be narrowed down.

4. CONCLUDING REMARKS

This paper investigates how the intensities of idea exchange in different sectors affect skilled-unskilled wage inequality. In the basic model with two sectors and full employment, an increase in the intensity of idea exchange in the skilled sector will expand wage inequality, and an increase in the intensity of idea exchange in the unskilled sector will reduce wage inequality. In most of the extended models, the results are almost the same as those of the basic model. However, in the extended model with the skilled product employed in the unskilled sector, the effect of idea exchange in the skilled sector on wage inequality depends on the substitution elasticity of unskilled labor and the skilled product in the unskilled sector, and the degree of idea exchange in the unskilled sector cannot affect the wage gap.

The role of idea exchange in the change of skilled-unskilled wage inequality should be considered seriously. In future research, this paper can be extended in the following directions. First, we can lay a micro-foundation for idea exchange by introducing the communication mechanisms (see e.g., Segal, 2006) for the framework of skilled-unskilled wage inequality. Second, we can introduce geographic agglomeration to the framework developed by this paper. Geographic agglomeration and knowledge spillovers may pro-

vide a mechanism for cross-sector idea exchange. Third, we can adopt a dynamic general equilibrium approach to analyze this problem, and such a way may integrate the growth theory into a framework with idea exchange and wage inequality.

APPENDIX: A. DYNAMIC ADJUSTMENT PROCESS OF SUBSECTION 3.4

In line with Beladi et al. (2008), we build the excess demand functions. The differential equations are expressed as follows:

$$\dot{X} = d_1[p_X - (a_{SX}w_S + a_{UX}w_U + a_{KX}r)], \tag{A.1}$$
$$\dot{Y} = d_2[p_Y - (a_{UX}w_U + a_{KY}r)], \tag{A.2}$$
$$\dot{w}_S = d_3(a_{SX}X - \overline{L}_S), \tag{A.3}$$
$$\dot{w}_U = d_4(a_{UX}X + a_{UY}Y - \overline{L}_U), \tag{A.4}$$
$$\dot{r} = d_5(a_{KX}X + a_{KY}Y - \overline{K}), \tag{A.5}$$

where $d_i > 0$ $(i = 1, 2, \ldots, 5)$ represents the speed of adjustment. The notation " $\dot{}$ " denotes the differentiation with respect to time (e.g., $\dot{w}_S = \frac{dw_S}{dt}$). The determinant of the Jacobian matrix of Eqs. (A.1)-(A.5) are given by:

$$|J| = H \begin{vmatrix} 0 & 0 & \theta_{SX} & \theta_{UX} & \theta_{KX} \\ 0 & 0 & 0 & \theta_{UY} & \theta_{KY} \\ 1 & 0 & -\sigma^X_{SU}\theta_{UX} - \sigma^X_{SK}\theta_{KX} & \sigma^X_{SU}\theta_{UX} & \sigma^X_{SK}\theta_{KX} \\ \lambda_{UX} & \lambda_{UY} & \lambda_{UX}\sigma^X_{SU}\theta_{SX} & A & B \\ \lambda_{KX} & \lambda_{KY} & \lambda_{KX}\sigma^X_{SK}\theta_{SX} & C & D \end{vmatrix} = H\Delta_3,$$

where $H = \frac{\prod_{i=1}^{5} d_i \lambda_{SX} p_X p_Y \overline{L}_S \overline{L}_U \overline{K}}{w_S w_U r XY} > 0$.

According to the Routh-Hurwitz theorem, the local stability can be achieved if the sign of $|J|$ equals that of $(-1)^n$, where n denotes the order of the Jacobian matrix. In this case, we have $n = 5$ and $|J| < 0$. Since $H > 0$, we have $\Delta_3 < 0$. As a result, in order to ensure the local stability of the economic system, we need $\Delta_3 < 0$.

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