ANNALS OF ECONOMICS AND FINANCE **24-1**, 91–117 (2023)

Forming Stable R&D Networks in Different Market Structures

Mohamad Alghamdi*

The paper investigates the influence of dense collaborative R&D structures between firms as stable standard networks on equilibrium outcomes. Considering the different levels of competition, the discussion focuses on two topics: the effect of dense structure growth on outcomes and the effect of dense component formation on both the cooperative structure and economic outcomes. In the first topic, the competition limits cooperation between firms, which leads to the creation of less dense cooperative structures. In the second topic, although dense components are a better structure in both individual and social perspectives, they do not affect the overall cooperation structure.

Key Words: Stable networks; Network size; Market structures; Equilibria; Maximum outcomes.

JEL Classification Numbers: D21, L14, L22.

1. INTRODUCTION

Recently, R&D networks have attracted great attention due to the large increase in the number of collaborators worldwide (Autant et al., 2007; Narula and Santangelo, 2009; Sanou et al., 2016, Tomasello et al., 2013; van and Rameshkoumar, 2018.) In empirical point of view and based on different databases, many studies have indicated the significant growth in the number of firms that entered the field of R&D cooperation. On the other hand, the results indicated that the number of cooperative relations between firms was small compared to the number of cooperating firms in various industrial sectors. Though, some authors have been able to describe the overall structure of the R&D network as a small network characterized by short distances between agents (e.g., Autant et al., 2007; Fleming, 2007; Tomasello et al., 2013; Verspagen and Duysters, 2004).

In theoretical point of view, the network game consists of simultaneous competition and cooperation between firms. Based on the different network models, the authors' discussions took many aspects. Among these

* Department of Mathematics, King Saud University, Riyadh, Saudi Arabia. E-mail: almohamad@ksu.edu.sa.

1529-7373/2023

All rights of reproduction in any form reserved.

aspects, for example, is the study of changes in economic variables (R&D investment, production, profit and total welfare) with the growth of cooperative agreements (e.g., Alghamdi, 2020; Alghamdi et al., 2020; Goyal and Moraga-Gonzalez, 2001; Song and Vannetelbosch, 2007). The other aspect that has caught the attention of the authors is tracking changes in the overall structure of collaboration and its impact on individual and social outcomes (e.g., Alghamdi, 2016; Alghamdi, 2017; Konig et al., 2012). In addition, other authors have paid more attention to bringing individual and social incentives closer to making the R&D network more profitable (e.g., Alghamdi, 2017; Alghamdi, 2019; Goyal and Moraga-Gonzalez, 2001; Konig et al., 2012).

Studying the cooperation of firms in the field of R&D within the framework of the concept of the network is an approach to examine the general structure of cooperation that can be expressed in one of the distinct types of networks. For example, theoretical studies focused on complete networks, which are characterized by the interconnection of all firms with each other, as well as star networks that feature a firm in the center linked to other firms distributed in the periphery (e.g., Alghamdi, 2016; Goyal and Joshi, 2003; Goyal and Moraga-Gonzalez, 2001; Roketskiy, 2018). In this paper, we will start from the principle that the complete network is stable in all market structures Alghamdi, 2016. The compete network is an explicit example of the small world network that has become more prominent than other structures in empirical contexts. While Goyal and Moraga-Gonzalez (2001) found that this type of the networks is not the only stable network when goods are homogeneous, this result can be extended to the case when the goods are substitutes. For this reason, we call the complete network a standard stable network, which means there may be another stable network.

In this paper, we will address the following two questions. First, how does the growth of a standard stable network affect the economic outcomes? Second, how does the stability of the components of the cooperative network affect the economic outcomes? We answer these two questions taking into account the different levels of competition. When the goods are complementary or independent, we assume that firms are in a weak competitive market; while they are in a competitive market when goods are substitutes.

The results of this paper can be summarized as follows. First, the growth of the complete network leads to economic changes that affect individual and social outcomes. The changes are not only the result of new firms entering the network, but also of establishing new links with all firms in the network. From an individual perspective, the growth of the entire network improves investments in R&D, production quantities and firms' profits if they are in a weak competitive market. However, if firms are in a competitive market, the opposite occurs in the sense that economic variables will decrease in value as the number of firms increases. Additionally, when focusing on corporate profits, the change will not be a steady procedure despite increasing links that are consistent for all firms. This means that with every new firm in the complete network, the change will not be a constant value. In a social perspective, the effect of network growth depends on the economic variable to be examined. In terms of the consumer surplus, the network growth improves the results if firms are in a competitive market and in a weak competitive market, this result is obtained if the number of firms is not large. In terms of the industry profit, the complete network growth improves the results when firms are in a weak competitive market; whereas the total welfare always increases with the growth of the complete network, regardless of the market structure.

Second, the stability of the network components does not affect the stability of the overall network. This means that if the cooperative network consists of stable components (complete components), then the overall network is not necessarily stable. Also, the production of firms and their profits in a large complete component are higher compared to other components, which in turn will encourage firms to form larger components within the cooperation network. In addition, when comparing the results of the complete components with a complete network consisting of the same number of firms in each component, we found that the outcomes are higher in the complete network if firms are in a weak competitive market; whereas the opposite occurs in a competitive market. This leads us to say that if the competition between firms is weak, they prefer a complete cooperation structure so that there are no other cooperative structures in the market, while in the competitive market, firms prefer separate cooperative structures.

The paper is structured as follows. In the second section, we provide a review of the social network and microeconomics. In the third section, we present our results. In the fourth section, we conclude our study.

2. BACKGROUND

2.1. Network

Network theory is the study of graphs as a representation of relationships between objects. A **network** consists of two parts vertices or nodes and links or edges Newman, (2010). If $N = \{i, j, k, \ldots\}$ and $L = \{ij, jk, \ldots\}$ are sets of the nodes and the links, then $G(N, L)$ (or G for simplicity) defines a network. For the purpose of this article, we focus on simple networks that have neither parallel links (links that have the same end nodes) nor loops (links where their start and end nodes are the same).

Let N_i be a set of nodes that linked to node i. Then, the set N_i denotes a neighbor set of the node i and its length is the degree of that node:

$$N_i = \{j \in N : ij \in L\}, \tag{1a}$$
$$deg(i) = |N_i|, \text{ and } 0 \le deg(i) \le n-1, \tag{1b}$$

where n is the number of nodes in the network G. If each two nodes in the network are linked, the graph is called **complete network** (denoted by K_n) and the degree of each node is $deg(i) = n-1$. If the network consists of nodes without links between them, the graph is called **empty network** (denoted by E_n) and the degree of each node is $deg(i) = 0$. A **star network** is prominent structure, which has a node at the center linked to all other nodes (periphery) such that the latter nodes are not linked to each other.

A subgraph $G'(N', L')$ of the network $G(N, L)$ is a graph such that $N' \subseteq N$ and $L' \subseteq L$. A **component** of the graph G is defined as a connected subgraphs where the largest connected component is called the **giant component**.

2.2. The Economic Model

The emphasis in this paper is on the utility function of consumers that is a generalisation of the quadratic utility function given by Singh and Vives (1984).[1] Hackner (2000) considers n firms with the following utility function:

$$U = \alpha \sum_{i=1}^{n} q_i - \frac{1}{2}\left(\lambda \sum_{i=1}^{n} q_i^2 + 2\delta \sum_{j \neq i} q_i q_j\right) + I\ , \tag{2}$$

where q_i is the quantity of good produced by firm i and I is the consumer's consumption of all other goods. $\alpha > 0$ represents the maximum price of a unit's good, and $\lambda > 0$ represents the amount of its price decreases when the price of the product increases by one unit. The parameter $-1 \le \delta \le 1$ represents the marginal rate of differentiation between goods where if $\delta < 0$ ($\delta > 0$), goods are complementary (substitutes) and if $\delta = 0$ (1), goods are independent (homogeneous).

The differentiation degree also refers to the extent to the strength of competition between firms in a market. If goods are complementary or independent, firms are in a weak competitive market. As the degree of substitution between goods increases, the competition increases, so if the goods are homogeneous, firms are in a strong competitive market.

[1] The consumer maximizes the function $U(q, I) - \sum_{i=1}^{n} p_i q_i$, where $U(q, I)$ is the utility function, q_i is the amount of good i and p_i its price. The utility function is assumed to be concave and leads to a linear demand structure from which inverse demand can be inferred Singh and Vives (1984).

Payoffs. Let m be a consumer's income and p_i be the price of good i produced by firm i. When consuming q_i of good i, the money spent is $p_i q_i$ and the balance is $I = m - p_i q_i$. By substituting into the utility function (2) and calculating $\frac{\partial U}{\partial q_i} = 0$, we find the optimal consumption of good i:

$$\alpha - q_i - \delta \sum_{j \neq i} q_j - p_i = 0 \ . \tag{3}$$

Thus, the inverse demand function for each good i (D_i^{-1}) is

$$p_i = \alpha - q_i - \delta \sum_{j \neq i} q_j, \quad i = 1, \ldots, n \ . \tag{4}$$

If c_i is the cost of producing good i, then the profit of firm i is

$$\pi_i = (p_i - c_i) q_i = \left(\alpha - q_i - \delta \sum_{j \neq i}^{n} q_j - c_i \right) q_i, \quad i = 1, \ldots, n \ . \tag{5}$$

The total profits of all firms in the market represent the industry surplus: $\Pi = \sum_{i=1}^{n} \pi_i$. Consumer surplus is defined as the difference between the price consumers are willing to pay for a good and the actual market price:

$$CS = \frac{1 - \delta}{2} \sum_{i=1}^{n} q_i^2 + \frac{\delta}{2} (\sum_{i=1}^{n} q_i)^2 \ . \tag{6}$$

Total welfare is the sum of the industry surplus and the consumer surplus:

$$TW = CS + \Pi \ . \tag{7}$$

2.3. R&D Network Model

The cooperation between firms in R&D can be perceived as a network where nodes represent firms and links represent the R&D partnerships (Goyal and Moraga-Gonzalez, 2001). The cooperation network is based on mutual benefits between firms; meaning that if any two firms decide to cooperate in R&D, they agree to share the results of the investment in R&D. This is interpreted in the network theory as an undirected network (that is, each link between any two firms should serve both sides).

Network Structure. When focusing on network architecture, there are two main types. The first type is regular (symmetric) networks in which all firms have the same number of links k i.e. $deg(i) = k \ \forall i \in N$. For example, in the complete R&D network, each firm has the same number of links and in the empty network, firms do not have links. The second type is irregular (asymmetric) networks where firms have different numbers of links. In this paper, we focus on the regular R&D networks.

Game Stages. In the R&D network game, there are three stages as follows:

The First Stage Firms strategically choose their partners in R&D by forming bilateral collaborative links. At the end of this stage, the cooperation network G will be constructed and firms will identify their locations in the network.

The Second Stage Firms choose their amounts of investment (effort) in R&D simultaneously and independently in order to reduce the cost of production. If s_i denotes R&D investment of firm i, then the effective investment of that firm in R&D is

$$S_i = s_i + \sum_{j \in N_i} s_j + \phi \sum_{k \notin N_i} s_k, \quad i = 1, \ldots, n \ , \tag{8}$$

where N_i is the set of firms participating in R&D with the firm i (Goyal and Moraga-Gonzalez, 2001). The parameter $\phi \in [0, 1]$ is an exogenous parameter that captures knowledge spillovers acquired from firms not engaged in R&D with firm i.

According to equation (8), the effective amount of investment of each firm consists of two parts: an individual expenditure on R&D and expenditures of other firms in the market. The benefit of others' expenditures depends on the R&D spillover ϕ, which is equal to one between any two cooperating companies; otherwise, it takes a positive value less than one (D'Aspremont and Jacquemin, 1988).

The effective investment reduces the marginal production cost of firm $i \in N_i$. If c_0 is the marginal cost, then the cost function becomes

$$c_i = c_0 - S_i = c_0 - s_i - \sum_{j \in N_i} s_j - \phi \sum_{k \notin N_i} s_k, \quad i = 1, \ldots, n \ , \tag{9}$$

where the marginal cost c_0 is assumed to be constant and equal for all firms.

Since we consider regular networks with $\phi = 0$, the cost function (9) becomes

$$c_i = c_0 - s_i - \sum_{j \in N_i} s_j, \quad i = 1, \ldots, n \ . \tag{10}$$

The Third Stage Firms compete in the product market by setting production quantities (Cournot competition). At this stage, firms choose their levels of production in order to maximize their profits.

The investment in R&D is assumed to be costly and the function of the cost is quadratic. Thus, if firm i invests $s_i \in [0, c_0]$, the cost of R&D is $C(s_i) = \mu s_i^2$, where $\mu > 0$ refers to the effectiveness of R&D expenditure (D'Aspremont and Jacquemin, 1988). Thus, the profit function (5) becomes

$$\pi_i = (p_i - c_i)q_i - C(s_i) = \left(\alpha - c_0 - q_i - \delta \sum_{j \neq i}^{n} q_j + S_i \right) q_i - C(s_i), \quad i = 1, \ldots, n \,, \tag{11}$$

where the marginal cost satisfies $\alpha > c_0$.

2.4. Nash Equilibria

We identify the sub-game perfect Nash equilibrium by using backwards induction. From the profit function (11), we calculate $\frac{\partial \pi_i}{\partial q_i} = 0$ to have the best response function of quantity for good i:

$$q_i = \frac{(\alpha - c_0) + S_i - \delta \sum_{j \neq i} q_j}{2}, \quad i = 1, \ldots, n \,. \tag{12}$$

By substituting the best response functions into each other, we have the symmetric equilibrium output for each good i

$$q_i^* = \frac{(2 - \delta)(\alpha - c_0) + (2 + (n-2)\delta)S_i + \delta \sum_{j \neq i} S_j}{(2 - \delta)((n-1)\delta + 2)}, \quad i = 1, \ldots, n \,. \tag{13}$$

To find the symmetric equilibrium profit, we substitute the equilibrium output (13) into the profit function (11) which gives

$$\pi_i^* = \left[\frac{(2 - \delta)(\alpha - c_0) + (2 + (n-2)\delta)S_i + \delta \sum_{j \neq i} S_j}{(2 - \delta)((n-1)\delta + 2)} \right]^2 - C(s_i) \,. \tag{14}$$

For convenience, the profit function (14) can be expressed in the following form:

$$\pi_i^* = {q_i^*}^2 - C(s_i), \quad i = 1, \ldots, n \,. \tag{15}$$

The final list of the equilibria depends on the network structure. By knowing the structure, we have the effective investment of each firm i. By substituting into the profit function (15) and calculating $\frac{\partial \pi_i^*}{\partial s_i} = 0$, we have the best response function of the R&D investment for each firm i. By plugging the best response functions into each other, we have the symmetric equilibrium investment s_i^*. Then, we use the backwards induction to have the final equilibria. In Appendix A, we provide the final equilibrium equations for R&D investment and production quantity. To find the final equilibrium equations for the profit, consumer surplus and total welfare, we substitute the results in equations (15), (6) and (7); respectively.

R&D Effectiveness. Assume n firms compete in the market by setting their own production quantities. If the cooperation in R&D forms k-regular network such that the spillover is zero, the R&D effectiveness must satisfy the following condition (Alghamdi, 2019):

$$\mu > \mu^* = max\left\{\frac{(k+1)\big((n-(k+2))\delta+2\big)\alpha}{c_0\big((n-1)\delta+2\big)^2(2-\delta)}, \left[\frac{\big(n-(k+2)\big)\delta+2}{\big((n-1)\delta+2\big)(2-\delta)}\right]^2\right\}. \tag{16}$$

Differentiation Degree. Alghamdi et al. (2020) found that under Cournot competition, the equilibrium quantity provides positive outcomes if the complementary degree is greater than a certain value (δ^*) given in the following inequality:

$$\delta > \delta^* = \frac{2}{1-n}\ . \tag{17}$$

Pairwise Stability. The pairwise stability is dependent on corporate profit functions and is a necessary condition for strategic stability (Jackson and Wolinsky, 1996). We say that the network G is stable if no firm can obtain higher profit from deleting one of its links; and any other link between two firms would benefit only one of them.

Definition 2.1. For any network G to be stable, the following two conditions need to be satisfied for any two firms i, $j \in G$:

1. If $ij \in G$, $\pi_i(G) \geq \pi_i(G-ij)$ and $\pi_j(G) \geq \pi_j(G-ij)$,
2. If $ij \notin G$ and if $\pi_i(G) < \pi_i(G+ij)$, then $\pi_j(G) > \pi_j(G+ij)$,

$G-ij$ is the network resulting from deleting a link ij from the network G and $G+ij$ is the network resulting from adding a link ij to the network G.

3. THE RESULTS

In this section, we discuss two issues related to the structure and development of the R&D network. In the first issue, we study the effect of standard stable network growth (the complete network) on equilibrium outcomes. In the second issue, we study the effect of the stability of network components on the stability of the entire network.

3.1. Growing Standard Stable Network Size

As stated in Alghamdi et al. (2020), firms prefer to form multiple collaborative links for high profits. This indicates that the complete network is a stable network; regardless of the market structure. In this section, we examine the impact of complete network expansion on the equilibrium outcomes.

The following proposition concerns firms' performance in the standard stable network when increasing the network size.[2] It states that individual economic variables (investment, quantity, and profit) in the complete network are affected by each new firm that enters the network and establishes links with all existing firms. The effect depends on the market structure, if firms are in a weak competitive market, the network growth always improves the results, while the opposite occurs in the market which consists of more competitive actions.

PROPOSITION 1. *Given a complete R&D network K_n, then for any value of the R&D effectiveness $\mu > \mu^*$ satisfied with all values of $\delta > \delta^*$, we have the following:*

1.$s^*(K_{n+1}) > (<)\ s^*(K_n)$ *if* $\delta^* < \delta \leq 0\ \ (\delta > 0)$.
2.$q^*(K_{n+1}) > (<)\ q^*(K_n)$ *if* $\delta^* < \delta \leq 0\ \ (\delta > 0)$.
3.$\pi^*(K_{n+1}) > (<)\ \pi^*(K_n)$ *if* $\delta^* < \delta \leq 0\ \ (\delta > 0)$.

The proof is given in Appendix B.

The following results concern the change rate in the profit of firms with growing the size of the complete network K_n. The first result shows that the change in the profit is not a constant amount; regardless of the market structure. Meaning that for any network size n, $|\pi_i(K_n) - \pi_i(K_{n+1})| \neq |\pi_i(K_{n+1}) - \pi_i(K_{n+2})|$. Figure 1 shows the equilibrium profit corresponding to different sizes of the complete networks.

COROLLARY 1. *Given complete R&D networks K_n and K_{n+1} with cooperative activity links k_n and k_{n+1}, respectively. For any $\mu > \mu^*$ satisfied with all values of $\delta > \delta^*$, $\pi_i(K_{n+1}) = \pi_i(K_n) \pm \epsilon_{k_{n+1},k_n}$ such that ϵ_{k_{n+1},k_n} is not a fixed amount among different standard stable networks.*

The following result suggests that there are potential advantages and challenges in forming and growing the complete networks. The greatest advantage in establishing such networks is to maximize the individual profits. When the competition between firms is weak, the profits increase as the

[2]In this paper, the size of the organization (network or market) means the number of firms in that organization.

complete network grows, and this indicates that profits reach the highest amount in the complete network that consists of a large number of firms. Therefore, forming and maintaining R&D cooperative relationships in a weak competitive market is a necessary requirement for firms to conduct strong business.

The main challenge occurs in the individual profits when the strength of competition increases. Although corporate profits increase through their cooperative links, increasing network size reduces profits. Thus, the profits in the competitive market are expected to reach the lowest amount as the complete network expands. This, in turn, indicates that when forming a complete R&D network, firms must be aware that increasing the network will have a negative impact on their earnings.

Corollary 2. *Given a complete R&D network K_n, then for any $\mu > \mu^*$ satisfied with all values of $\delta > \delta^*$, we have the following:*

1.If $\delta^* < \delta \leq 0$, $\lim_{n\to\infty} \pi_i(K_n) = \kappa$ where $\kappa > 0$ is very large.
2.If $0 < \delta$, $\lim_{n\to\infty} \pi_i(K_n) = \kappa$ where $\kappa > 0$ is very small.

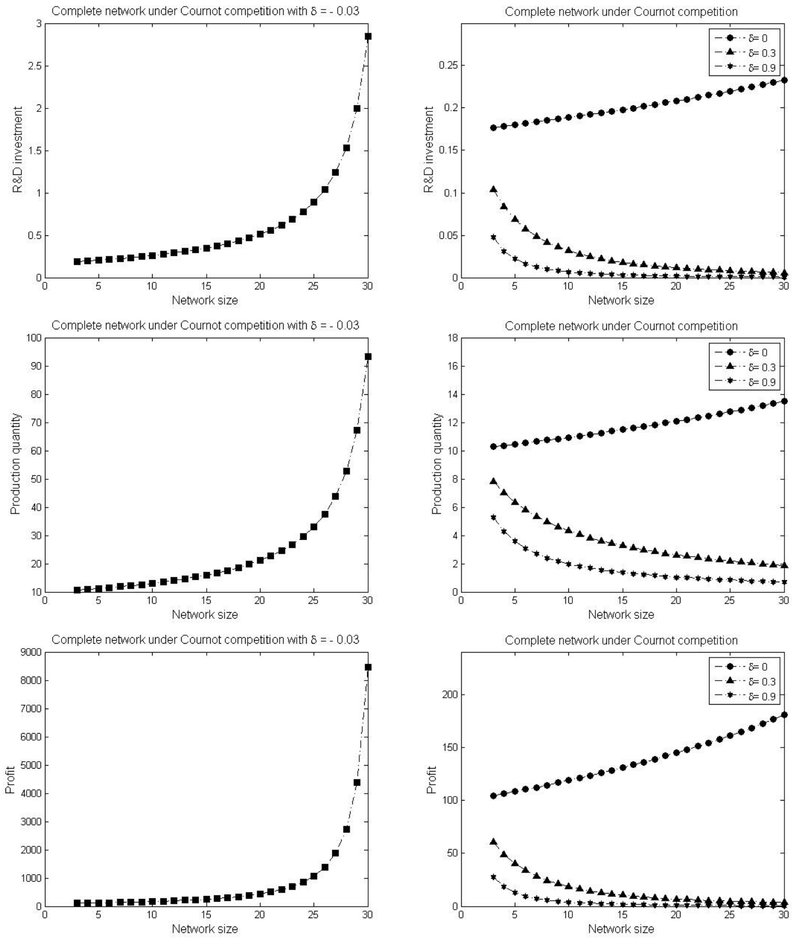
The previous results were inferred directly from Proposition 1, so instead of proving them, we provide an example.

Example 3.1. Assume n firms cooperate in R&D such that the final structure forms a complete network K_n. Suppose that the size of the network increases from $n = 3$ to $n = 30$ such that the resulting network with each new firm is a complete network $(K_3, K_4, \ldots, K_{30})$. Figure 1 shows the change in the economic variables with increasing the network size for different values of the differentiation degree.

Now, while the profits in a weak competitive market increase with the complete network size (Corollary 2), do firms prefer network hubs? In other words, what is the difference between the firms' gains if they form centers like that in star networks or form a complete network? According to several studies (e.g., Meagher and Rogers, 2004; Sanou et al., 2016; Silipo and Weiss, 2005), the cooperation provides firms with opportunities to access others knowledge plus the benefits of gains when they capture key positions in the collaboration network. However, the positive relationship between profits and the cooperative links pushes firms to create new R&D agreements that in turn may allow the formation of the complete network. Example 2 compares the corporate profits in the complete and star networks for different market sizes.

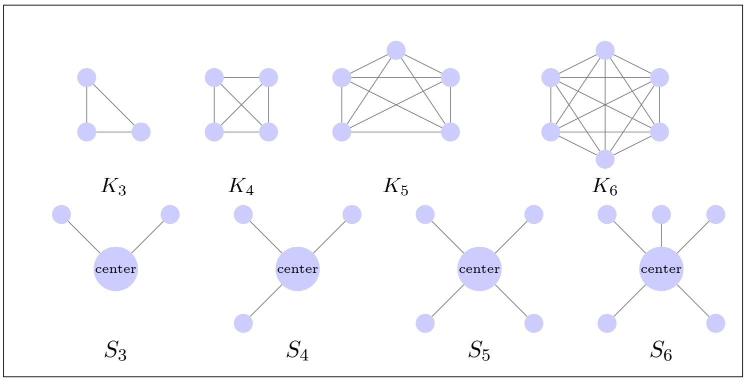
Example 3.2. Assume n firms cooperate in R&D such that the final structure forms either a star network S_n or a complete network K_n. Assume

FIG. 1. The economic variables with increasing the complete network size for different values of the differentiation degree. The parameters used to plot the figures are $\alpha = 120$, $c_0 = 100$ and $\mu = 29$ where $\mu \geq max\{\mu_n : 3 \leq n \leq 30\}$ and μ_n is the R&D effectiveness corresponding to the network size n.



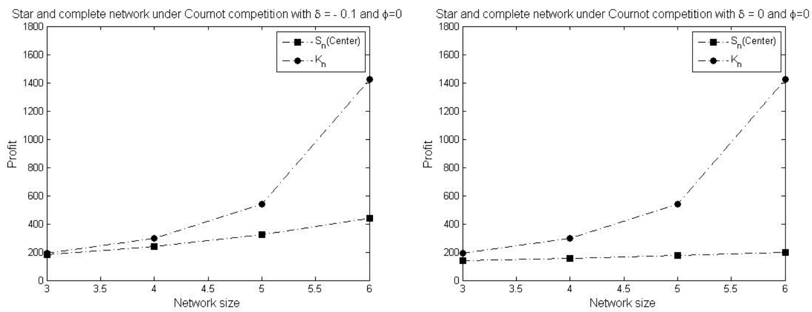
that the network size increases from $n = 3$ to $n = 6$ in both networks as shown in Figure 6. For two values of the differentiation degree $\delta = -0.1$ and $\delta = 0$, Figure 3 displays the profits of the central firms in the star networks and the profits in the complete network such that the spillover is assumed to be zero. In Appendix A, we provide the equilibria for the central firm in the star networks.

FIG. 2. The complete network K_n and the star S_n with different numbers of firms. The cooperation network starts with three firms, then the network size increases by entering one firm each time.



As mentioned in Corollary 2, if firms are in a competitive market, the profits decrease with the size of the complete network. Thus, what is the difference between the firms' gains if they form an empty network or a complete network? First, according to Alghamdi et al. (2020), the profits of firms in the complete network are always higher than those in the empty network. Second, from Corollary 1, the profit gap between the two networks as the number of firms grows is not fixed. From this, firms will not prefer to form an empty network, regardless of the number of firms on the market.

FIG. 3. The profits of firms under the complete network K_n and the star network S_n for different values of the substitution degree. The parameters used to plot the figures are $\alpha = 120$, $c_0 = 100$ and $\mu = 4$ where $\mu \geq max\{\mu_n : 3 \leq n \leq 6\}$ and μ_n is the R&D effectiveness corresponding to the network size n.



Proposition 2. *Given the R&D networks K_n and E_n, let $\Gamma = \pi^*(K_n) - \pi^*(E_n))$ be the profit gap between the two networks. Then, for any $\mu > \mu^*$ satisfied with all values of $\delta > 0$, we have the following:*

1.If δ is not small, the profit gap Γ is decreasing function with respect to the number of firms n.

2.The profit gap Γ is decreasing function with respect to the substitution degree δ.

The proof is given in Appendix B.

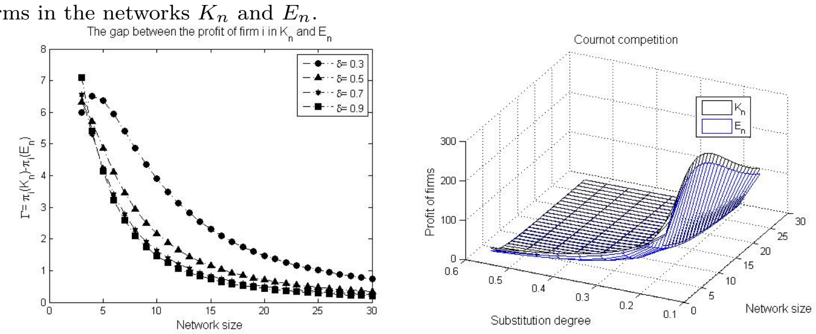
The following example compares corporate profits in the complete and empty networks when the firms are in a competitive market in terms of the network size and substitution degree.

Example 3.3. Assume n firms participate in R&D such that if they cooperate, the final structure forms a complete network K_n; otherwise the final structure is an empty network. Suppose that the size of the network increases from $n = 3$ to $n = 30$ in both networks.

1. Figure 4 (left) shows the corporate profits in the two networks with increasing n for different values of the substitution degree.

2. Figure 4 (right) shows the corporate profits in the two networks with increasing n and δ.

FIG. 4. The profits of firms in the complete network K_n and the empty network E_n in terms of the network size and the substitution degree. The parameters used to plot the figures are $\alpha = 120$, $c_0 = 100$ and $\mu = 2$ where $\mu \geq max\{\mu_n : 3 \leq n \leq 30\}$ and μ_n is the R&D effectiveness corresponding to the number of firms in the networks K_n and E_n.



In a social perspective, the growth of the standard stable network has various impacts depending on the final size of the network and the differentiation degree. First, the consumer surplus in the complete network generally improves with the size of the network unless the network size is large in the case of complementary goods. Second, the industry profit increases with the complete network size if firms are in a weak competitive market or in a competitive market provided that the degree of substitution is small. Finally, the total welfare improves with the complete network size; regardless of the rivalry strength.

PROPOSITION 3. *Given a complete network K_n, for any $\mu > \mu^*$ satisfied with all values of $\delta > \delta^*$, we have the following:*

1.$CS^*(K_{n+1}) > CS^*(K_n)$ *for all* $\delta^* < \delta \leq 1$.
2.$\Pi^*(K_{n+1}) > \Pi^*(K_n)$ *if*

(i)the differentiation degree $\delta^* < \delta \leq 0$ *or*

(ii)$\delta > 0$ *is small.*

3.$TW^*(K_{n+1}) > TW^*(K_n)$ *for all* $\delta^* < \delta \leq 1$.

The proof is given in Appendix B.
The following example illustrates the changes in the social outcomes as the complete network grows.

EXAMPLE 3.4. Assume n firms cooperate in R&D such that the final structure forms a complete network K_n. Suppose that the size of the network increases from $n = 3$ to $n = 30$ such that the resulting network with each new firm is a complete network. Figure 5 illustrates the change in the consumer surplus, the industry profit and the total welfare as the network size increases.

3.2. Network Components Stability vs. Overall Network Outcomes

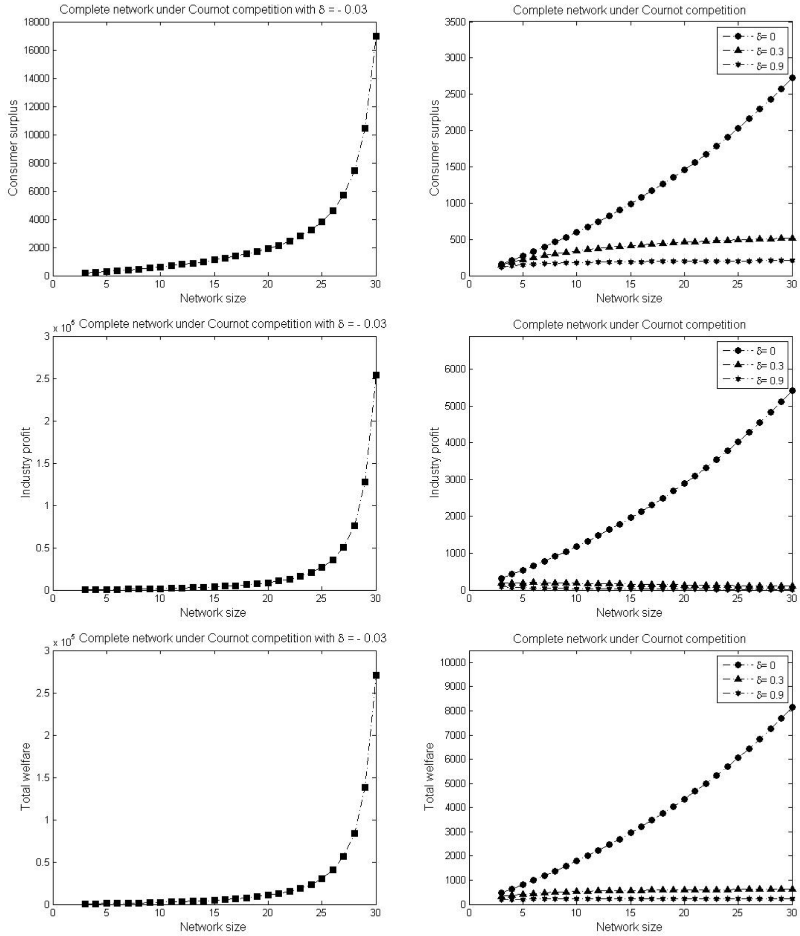
Considering networks with multiple components, we study the effect of stability of these components on the equilibrium outcomes of the overall network. Alghamdi (2016) found that for homogeneous goods, stable components do not generate a stable overall network. Also, the author found that if the cooperators in the giant component form a complete structure, the overall network is not necessarily stable. In this paper, we discuss the two aspects for all degrees of goods differentiation.

PROPOSITION 4. *Given a network G consists of n firms, then for any $\delta > \delta^*$ and $\mu > \mu^*$, we have the following:*

1.Suppose the network G contains a set of components $C_1, C_2, \ldots, C_t$. If these components are stable, the overall network G is not necessarily stable.

2.Suppose the network G contains a giant component GC. If GC is stable, the overall network G is not necessarily stable.

FIG. 5. The changes in the social equilibrium outcomes as the complete network grows. The parameters used to plot the figures are $\alpha = 120$, $c_0 = 100$ and $\mu = 29$ where $\mu \geq max\{\mu_n : 3 \leq n \leq 30\}$ and μ_n is the R&D effectiveness corresponding to the network size n.



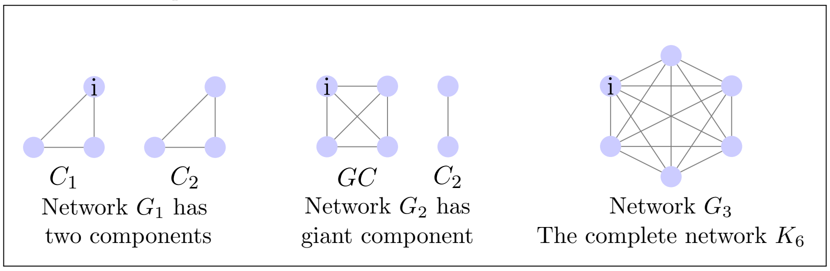
The above result is directly derived from the positive relationship between individual gains and cooperative relationships (Alghamdi et al., 2020). For this reason, we will suffice with an example that compares corporate gains in stable components and in the complete network.

EXAMPLE 3.5. Figure 6 shows three types of R&D networks, each consists of six firms. The first network G_1 contains two complete components

(C_1 and C_2) while the second network G_2 contains a giant component. The third network G_3 is the complete network K_6.

1. Each component of the network G_1 is stable since it forms a complete structure. However, the overall network is not stable because each firm will strive to form many cooperative relationships to have higher profits (Alghamdi et al., 2020). Figure 7 (left) compares the profit of firm i in the networks G_1 and G_3. In the first case, assume that firms sell complementary goods with a differentiation degree $\delta = -0.1$. The profit of firm i in the complete network is always higher compared to its profit in other networks. In the second case, assume that the goods are substitutes with a substitution degree $\delta = 0.5$. The profit of firm i is higher in the network G_1, but will decrease when other firms establish new cooperative links to maximize their profits.
2. The network G_2 has a giant component GC of size four firms and another component C_2. The giant component is stable since it forms a complete structure, but the overall network is not stable. Figure 7 (right) compares the profit of firm i in the networks G_2 and G_3. When the differentiation degree $\delta = -0.1$, the profit of firm i in the network G_3 is always higher than in other networks. However, when $\delta = 0.5$, the profit of firm i is higher in the network G_2, but the cooperation of other firms in that network will reduce the profit of firm i.

FIG. 6. The figure shows three types of R&D networks, the size of each of which is six firms. The network G_1 consists of two complete components and the network G_2 consists of a giant component GC and another component C_2, and the third network is the complete network K_6.



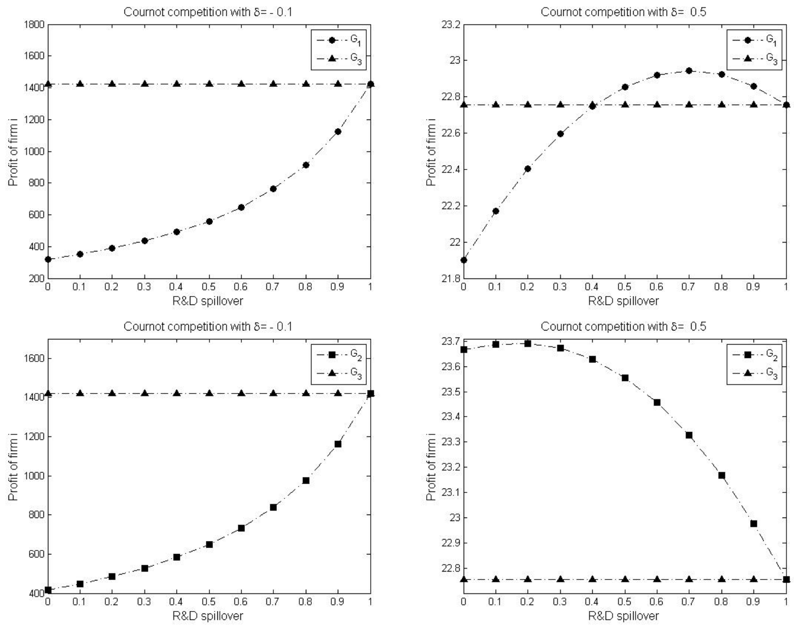
Alghamdi et al. (2020) stated that for any market size, the complete R&D network is stable; regardless of the market structure. This indicates that the cooperative links have a positive impact on the production quantity and the profit of firms. This leads us to state that these two economic variables are higher in the complete giant component than in any other components.

Proposition 5. *Given a network G contains a giant component GC and other components $C_1, C_2, \ldots, C_t$. If GC forms a complete structure, then for any $\delta > \delta^*$ and $\mu > \mu^*$, we have the following:*

1.$q^*(GC) > q^*(C_i)$, $\forall i \in [1, t]$.

2.$\pi^*(GC) > \pi^*(C_i)$, $\forall i \in [1, t]$.

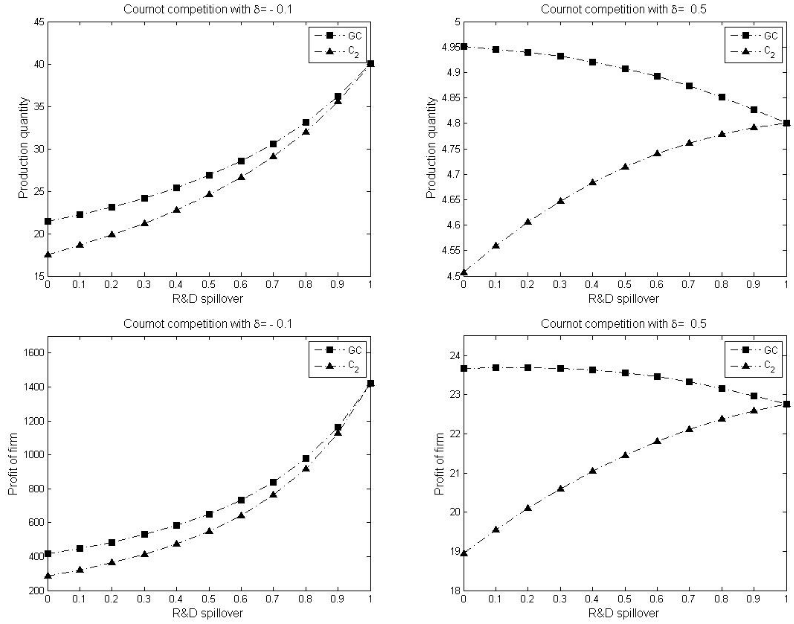
FIG. 7. The profit of firm i in the networks given in Figure 6. The figures on the top compare the profit of firm i in the networks G_1 and G_2. The figures on the bottom compare the profit of firm i in the networks G_2 and G_3. The parameters used to plot the figures are $\alpha = 120$, $c_0 = 100$ and $\mu = 4$ where $\mu \geq max\{\mu(G_1), \mu(G_2), \mu(G_3)\}$.



Example 3.6. Consider the network G_2 given in Figure 6. Figure 8 compares the production quantity and the profit in the giant component GC and the component C_2.

Now, suppose the cooperative network consists of complete components, each of which contains n firms. Proposition 6 compares the quantity production and the profit of firms in these components with the results in the complete network K_n.

FIG. 8. The production quantity and the profit of firms in the components ***GC*** **and** C_2 **in the network** G_2 **given in Figure 6.** The parameters used to plot the figures are $\alpha = 120$, $c_0 = 100$ and $\mu = 4$.



Proposition 6. *Given a network G consists of a set of complete components $C_1, C_2, \ldots, C_t$ where each component is of size n. For any $\mu > \mu^*$, we have the following:*

1.$q^*(C_i) < (>)\ q^*(K_n)$ if $\delta^* < \delta \leq 0$ $(\delta > 0)$, $i = 1, \ldots, t$.
2.$\pi^*(C_i) < (>)\ \pi^*(K_n)$ if $\delta^* < \delta \leq 0$ $(\delta > 0)$, $i = 1, \ldots, t$.

The previous results (from Proposition 4 to Proposition 6) are naturally resulting from the positive relationships between the cooperative links and both the production quantity and the profit.

Example 3.7. Figure 9 shows two types of R&D networks. The first network G_1 contains two complete components C_1 and C_2. The second network is the complete network with three firms K_3. Figure 10 compares the production quantity and the profit of firms in both networks.

1. In a weak competitive market, the production quantity and the profit of firms in each complete component is higher than the outcomes in the complete network K_3.

2. In a competitive market, the production quantity and the profit of firms in each complete component is lower than the outcomes in the complete network.

FIG. 9. The figure shows the network G_2 that consists of two disconnected components and the complete network K_3. The size of each component in the network G_2 and the complete network K_3 is three firms.

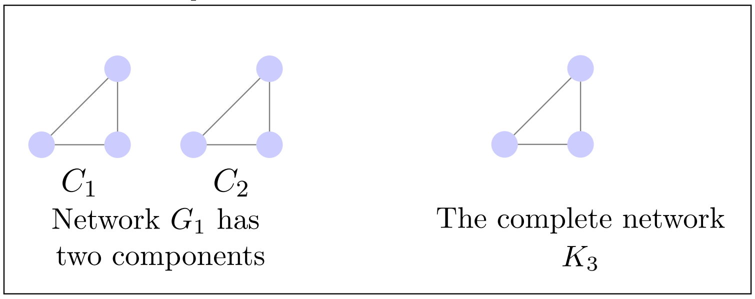
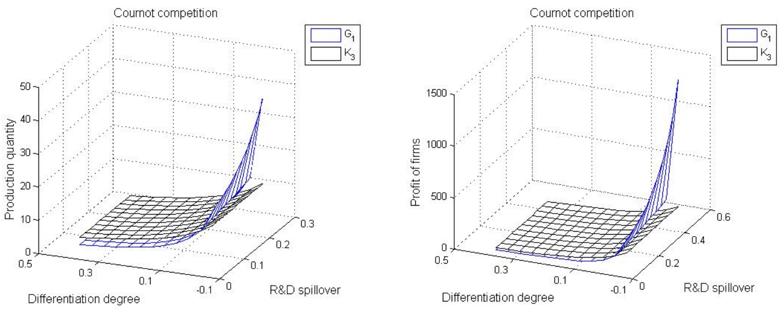


FIG. 10. The production quantity and the profit of firms in the networks G_1 and K_3 for different values of the differentiation degree. The parameters used to plot the figures are $\alpha = 120$, $c_0 = 100$ and $\mu = 4$ where $\mu \geq max\{\mu(G_2), \mu(K_3)\}$.



4. CONCLUSION

The aim of this paper was to extend the analysis of stable R&D networks taking into account different levels of competition between firms. By considering the complete network as a standard stable network, the paper presented interesting results within the framework of the Cournot contest.

The results indicated that the feature of the complete network growth is mostly related to the market structure. In a weak competitive market, the

R&D investments, production quantities and profits closely related to the growth of the complete network, which in turn stimulates firms to build and develop such cooperative networks. But in a competitive market, firms will not prefer large complete networks because the economic benefits will be less in those networks. In terms of the total welfare, the results suggested that the outcomes always increase as the complete network grows; regardless of the market structure.

In addition, the paper examines the effect of stability of network components on the economic outcomes. The results indicated that the stability of the components has no effect on the overall network; regardless of the market structure. Also, the complete network in a weak competitive market is a better structure in both individual and social perspectives compared to other structures of the cooperative network.

COMPLIANCE WITH ETHICAL STANDARDS

Acknowledgment. This research was supported by King Saud University, Deanship of Scientific Research, College of Science Research Center.

Funding. This work was supported by King Saud University, Deanship of Scientific Research, College of Science Research Center.

Conflict of interest. The authors declare that they have no conflict of interest.

Informed consent. Not applicable.

APPENDIX

A.1. THE EQUILIBRIUM RESULTS

The parameters used in this study:

α: consumers willingness to pay	c_0: Marginal cost	n: Number of firms
δ: Differentiation degree of goods	k: Activity level	μ: R&D effectiveness

(1) Regular R&D Networks

The following list is the equilibria for any regular network size in different market structures as given in Alghamdi et al. (2020):

R&D investment: $$s^* = \frac{\big((n-(k+2))\,\delta+2\big)(\alpha-c_0)}{\mu\big((n-1)\delta+2\big)^2(2-\delta)-(k+1)\big((n-(k+2))\,\delta+2\big)},$$

Production quantity: $$q^* = \frac{\mu(2-\delta)\big((n-1)\delta+2\big)(\alpha-c_0)}{\mu\big((n-1)\delta+2\big)^2(2-\delta)-(k+1)\big((n-(k+2))\,\delta+2\big)},$$

Profit of firms: $$\pi^* = \frac{\mu\left[\mu(2-\delta)^2\big((n-1)\delta+2\big)^2-\big((n-(k+2))\,\delta+2\big)^2\right](\alpha-c_0)^2}{\left[\mu\big((n-1)\delta+2\big)^2(2-\delta)-(k+1)\big((n-(k+2))\,\delta+2\big)\right]^2},$$

Total welfare: $$TW^* = \frac{n\mu\left[\mu(2-\delta)^2\big((n-1)\delta+2\big)^2\big(3+(n-1)\delta\big)-2\big((n-(k+2))\delta+2\big)^2\right](\alpha-c_0)^2}{2\left[\mu\big((n-1)\delta+2\big)^2(2-\delta)-(k+1)\big((n-(k+2))\,\delta+2\big)\right]^2}.$$

(2) Complete R&D Networks.

The following list is the equilibria for a complete network K_n in different market structures:

R&D investment: $$s^* = \frac{(\alpha-c_0)}{\mu\big((n-1)\delta+2\big)^2-n},$$

Production quantity: $$q^* = \frac{\mu\big((n-1)\delta+2\big)(\alpha-c_0)}{\mu\big((n-1)\delta+2\big)^2-n},$$

Profit of firms: $$\pi^* = \frac{\mu\Big(\mu\big((n-1)\delta+2\big)^2-1\Big)(\alpha-c_0)^2}{\Big(\mu\big((n-1)\delta+2\big)^2-n\Big)^2},$$

Consumer surplus: $$CS^* = \frac{\mu^2 n\big(1+(n-1)\delta\big)\big((n-1)\delta+2\big)^2(\alpha-c_0)^2}{\Big(\mu\big((n-1)\delta+2\big)^2-n\Big)^2}.$$

Total welfare: $$TW^* = \frac{n\mu\Big(\mu\big((n-1)\delta+2\big)^2\big(3+(n-1)\delta\big)-2\Big)(\alpha-c_0)^2}{2\Big(\mu\big((n-1)\delta+2\big)^2-n\Big)^2}.$$

(3) Star R&D Networks

The following list is the equilibria for a star network S_n in different market structures:

(A) Star R&D Network S_3.

$$\text{R\&D investment:} \quad s^*_{S_3} = \frac{-((\alpha - c_0)(\mu\delta^3 - 3\mu\delta^2 + 4\mu + 1))}{(-4\mu^2\delta^5 + 4\mu^2\delta^4 + 20\mu^2\delta^3 - 4\mu^2\delta^2 - 32\mu^2\delta - 16\mu^2 + \mu\delta^3 - 7\mu\delta^2 + 8\mu + 1)}$$

$$\text{Production quantity:} \quad q^*_{S_3} = \frac{-(2\mu(\alpha - c_0)(\delta + 1)(\mu\delta^3 - 3\mu\delta^2 + 4\mu + 1))}{(-4\mu^2\delta^5 + 4\mu^2\delta^4 + 20\mu^2\delta^3 - 4\mu^2\delta^2 - 32\mu^2\delta - 16\mu^2 + \mu\delta^3 - 7\mu\delta^2 + 8\mu + 1)}$$

(B) Star R&D Network S_4.

$$\text{R\&D investment:} \quad s^*_{S_4} = \frac{-((\alpha - c_0)(8\mu + 2\delta + 4\mu\delta - 10\mu\delta^2 + 3\mu\delta^3 + 4))}{(-27\mu^2\delta^5 + 54\mu^2\delta^4 + 72\mu^2\delta^3 - 80\mu^2\delta^2 - 112\mu^2\delta - 32\mu^2 - 6\mu\delta^3 - 28\mu\delta^2 + 8\mu\delta + 16\mu + 2\delta + 4)}$$

$$\text{Production quantity:} \quad q^*_{S_4} = \frac{-(\mu(3\delta + 2)(\alpha - c_0)(8\mu + 2\delta + 4\mu\delta - 10\mu\delta^2 + 3\mu\delta^3 + 4))}{(-27\mu^2\delta^5 + 54\mu^2\delta^4 + 72\mu^2\delta^3 - 80\mu^2\delta^2 - 112\mu^2\delta - 32\mu^2 - 6\mu\delta^3 - 28\mu\delta^2 + 8\mu\delta + 16\mu + 2\delta + 4)}$$

(C) Star R&D Network S_5.

$$\text{R\&D investment:} \quad s^*_{S_5} = \frac{-((\alpha - c_0)(4\mu + 3\delta + 4\mu\delta - 7\mu\delta^2 + 2\mu\delta^3 + 3))}{(-32\mu^2\delta^5 + 80\mu^2\delta^4 + 40\mu^2\delta^3 - 100\mu^2\delta^2 - 80\mu^2\delta - 16\mu^2 - 14\mu\delta^3 - 23\mu\delta^2 + 8\mu\delta + 8\mu + 3\delta + 3)}$$

$$\text{Production quantity:} \quad q^*_{S_5} = \frac{-(2\mu(2\delta + 1)(\alpha - c_0)(4\mu + 3\delta + 4\mu\delta - 7\mu\delta^2 + 2\mu\delta^3 + 3))}{(-32\mu^2\delta^5 + 80\mu^2\delta^4 + 40\mu^2\delta^3 - 100\mu^2\delta^2 - 80\mu^2\delta - 16\mu^2 - 14\mu\delta^3 - 23\mu\delta^2 + 8\mu\delta + 8\mu + 3\delta + 3)}$$

(D) Star R&D Network S_6.

$$\text{R\&D investment:} \quad s^*_{S_6} = \frac{-((\alpha - c_0)(8\mu + 12\delta + 12\mu\delta - 18\mu\delta^2 + 5\mu\delta^3 + 8))}{(-125\mu^2\delta^5 + 350\mu^2\delta^4 + 40\mu^2\delta^3 - 368\mu^2\delta^2 - 208\mu^2\delta - 32\mu^2 - 70\mu\delta^3 - 68\mu\delta^2 + 24\mu\delta + 16\mu + 12\delta + 8)}$$

$$\text{Production quantity:} \quad q^*_{S_6} = \frac{-(\mu(5\delta + 2)(\alpha - c_0)(8\mu + 12\delta + 12\mu\delta - 18\mu\delta^2 + 5\mu\delta^3 + 8))}{(-125\mu^2\delta^5 + 350\mu^2\delta^4 + 40\mu^2\delta^3 - 368\mu^2\delta^2 - 208\mu^2\delta - 32\mu^2 - 70\mu\delta^3 - 68\mu\delta^2 + 24\mu\delta + 16\mu + 12\delta + 8)}$$

A.2. PROOFS OF PROPOSITIONS

Proof of Proposition 1.

For simplicity, we assume that $(\alpha - c_0) = 1$.

1. The equilibrium investment in the complete network with n firms is

$$s^* = \frac{1}{\mu\big((n-1)\delta + 2\big)^2 - n} \ .$$

When calculating the derivative of the equilibrium investment function with respect to the network size n, we obtain

$$\frac{ds}{dn} = \frac{1 - 2\mu\delta\big((n-1)\delta + 2\big)}{\big(\mu\big((n-1)\delta + 2\big)^2 - n\big)^2} \ .$$

For any $\mu > \mu^*$, the numerator of the previous fraction is positive when $\delta^* < \delta \leq 0$ and this implies $ds/dn > 0$, which in turn means that the investment decreases as the network size n increases. When goods are substitutes, we have $\mu\delta\big((n-1)\delta + 2\big) > 1/2$ for any $\delta > 0$ and $\mu > \mu^*$. This implies that $ds/dn < 0$ which means that the investment decreases with increasing n if goods are substitutes.

2. The equilibrium quantity in the complete network with n firms is

$$q^* = \frac{\mu\big((n-1)\delta + 2\big)}{\mu\big((n-1)\delta + 2\big)^2 - n} \ .$$

When calculating dq/dn, we have

$$\frac{dq}{dn} = \frac{\delta\mu\big(\mu((n-1)\delta+2)^2 - n\big) - \mu((n-1)\delta+2)\big(2\delta\mu((n-1)\delta+2) - 1\big)}{\big(\mu\big((n-1)\delta+2\big)^2 - n\big)^2} \ .$$

The term $\big(\mu((n-1)\delta+2)^2 - n\big) > 0$ for any suitable value of $\mu > \mu^*$ also, the term $(n-1)\delta + 2 > 0$ from the inequality (17). Now, if $\delta^* < \delta \leq 0$, we have $\mu((n-1)\delta+2)\big(2\delta\mu((n-1)\delta+2) - 1\big) > 0$, but for each network size n,

$$\delta\mu\big(\mu((n-1)\delta+2)^2 - n\big) > \mu((n-1)\delta+2)\big(2\delta\mu((n-1)\delta+2) - 1\big) \ .$$

This implies $dq/dn > 0$ which indicates that the equilibrium quantity increases with the complete network size. The opposite occurs when $\delta > 0$ where the expression $\mu((n-1)\delta+2)\big(2\delta\mu((n-1)\delta+2) - 1\big)$ will be larger.

3. The equilibrium profit in the complete network with n firms is

$$\pi^* = \frac{\mu\Big(\mu\big((n-1)\delta+2\big)^2 - 1\Big)}{\Big(\mu\big((n-1)\delta+2\big)^2 - n\Big)^2} \ .$$

When calculating the derivative of the equilibrium profit function with respect to the network size n, we have

$$d\pi/dn = \frac{2\mu\Big(\mu((n-1)\delta+2)\big(((n-1)\delta+2)(1-\delta\mu((n-1)\delta+2)) - (n-2)\delta\big) - 1\Big)}{\Big(\mu\big((n-1)\delta+2\big)^2 - n\Big)^3} \ .$$

If $\delta^* < \delta \leq 0$, the expression $((n-1)\delta+2)(1-\delta\mu((n-1)\delta+2)) - (n-2)\delta$ is positive, thus $d\pi/dn > 0$, which means that $\pi^*(K_{n+1}) > \pi^*(K_n)$. If $\delta > 0$, the previous expression is negative, thus $d\pi/dn < 0$ and this indicates that the profit of firms in the complete network decreases with number of firms if goods are substitutes. ∎

Proof of Proposition 2.

The equilibrium profits in the complete network K_n and the empty network E_n are given in the following equations:

$$\pi^*(K_n) = \frac{\mu\Big(\mu\big((n-1)\delta+2\big)^2-1\Big)\big(\alpha-c_0\big)}{\Big(\mu\big((n-1)\delta+2\big)^2-n\Big)^2} \ ,$$

$$\pi^*(E_n) = \frac{\mu\Big(\mu(2-\delta)^2((n-1)\delta+2)^2-((n-2)\delta+2)^2\Big)\big(\alpha-c_0\big)}{\Big(\mu\big((n-1)\delta+2\big)^2(2-\delta)-((n-2)\delta+2)\Big)^2} \ .$$

The difference between the profits in the two networks is

$$\begin{aligned}\Gamma =&\pi^*(K_n)-\pi^*(E_n)\\ =&\frac{\Big(((n-2)\delta+2)-\mu(2-\delta)\big((n-1)\delta+2)^2\big)\Big)^2\Big(\mu\big((n-1)\delta+2\big)^2-1\Big)}{\Big(\mu\big((n-1)\delta+2\big)^2-n\Big)^2\Big(\mu\big((n-1)\delta+2\big)^2(2-\delta)-((n-2)\delta+2)\Big)^2}\\ &+\frac{\Big(\big((n-2)\delta+2\big)^2-\mu(2-\delta)^2\big((n-1)\delta+2\big)^2\Big)\Big(n-\mu\big((n-1)\delta+2\big)^2\Big)^2}{\Big(\mu\big((n-1)\delta+2\big)^2-n\Big)^2\Big(\mu\big((n-1)\delta+2\big)^2(2-\delta)-((n-2)\delta+2)\Big)^2} \ , \end{aligned} \tag{A.1}$$

where we assumed that $(\alpha - c_0) = 1$.

1. We prove that the profit gap Γ is decreasing function with respect to the number of firms n. We show this result for the case when $\delta = 1$ by calculating $d\Gamma/dn$:

$$d\Gamma/dn = \frac{-2\mu\big(\mu(n+1)^2(n-2)+1\big)}{\big(\mu(n+1)^2-n\big)^3} \ .$$

Note that for any $n > 2$, the term $\mu(n+1)^2(n-2)+1 > 0$ and this implies that $d\Gamma/dn < 0$. This indicates that the difference between equilibrium profits under the complete and empty networks decreases as the number of firms increases.

2. We prove that the profit gap Γ is decreasing function with respect to the substitution degree δ. We show this result by finding the derivative of the function Γ with respect to δ. The calculations are very long and for simplicity, we prove the result for the case when there are three firms with $\mu = 1$. Thus,

$$\begin{aligned} d\Gamma/d\delta =& -\frac{128\left(64\delta^{10}+224\delta^9+144\delta^8+24\delta^7+940\delta^6+1770\delta^5+367\delta^4\right)}{\left(4\delta^2+8\delta+1\right)^3\left(4\delta^3-11\delta-6\right)^3}\\ &+\frac{64\left(3019\delta^3+2898\delta^2+1017\delta+132\right)}{\left(4\delta^2+8\delta+1\right)^3\left(4\delta^3-11\delta-6\right)^3} \ . \end{aligned} \tag{A.2}$$

For any value of the substitution degree $\delta > 0$, we have $d\Gamma/d\delta < 0$ and this implies that the profit gap Γ is decreasing function with respect to the substitution degree. ∎

Proof of Proposition 3.

For simplicity, we assume that $(\alpha - c_0) = 1$.

1. We prove the proposition for the case when the goods are substitutes. The consumer surplus in the complete network consists of n firms is

$$CS^* = \frac{\mu^2 n\big(1+(n-1)\delta\big)\big((n-1)\delta+2\big)^2(\alpha-c_0)^2}{\Big(\mu\big((n-1)\delta+2\big)^2-n\Big)^2} .$$

We want to show that CS^* is an increasing function with n for the case when $\delta \geq 0$. When calculating the first derivative with respect to n, we have the following:

$$\frac{dCS^*}{dn} = \frac{\mu^2((n-1)\delta+2)^2((n-1)\delta+1)\big(\mu((n-1)\delta+2)(2-(3n+1)\delta)+n\big)}{2\big(\mu((n-1)\delta+2)^2-n\big)^3} + \frac{n\delta(3(n-1)\delta+4)\big(\mu((n-1)\delta+2)^2-n\big)}{2\big(\mu((n-1)\delta+2)^2-n\big)^3} . \tag{A.3}$$

For any network size $n > 2$, the term $2-(3n+1)\delta > 0$ if $\delta < 0.2$ and this means that the previous term provides negative results most values of the substitution degree. However, the expression $n\delta(3(n-1)\delta+4)\big(\mu((n-1)\delta+2)^2-n\big)$ provides large results for any $\delta \geq 0$ and $\mu > \mu^*$ and this implies that $dCS^*/dn > 0$. This indicates that the consumer surplus increases as the complete network size increases.

2. For the industry profit, the proof is straightforward from the equilibrium profit in Proposition 1.

3. The total welfare in the complete network K_n is

$$TW^* = \frac{n\mu\Big(\mu\big((n-1)\delta+2\big)^2\big(3+(n-1)\delta\big)-2\Big)(\alpha-c_0)^2}{2\Big(\mu\big((n-1)\delta+2\big)^2-n\Big)^2} .$$

When calculating the derivative of the function TW^* with respect to n, we obtain

$$\frac{dTW^*}{dn} = \frac{\mu\big(\mu((n-1)\delta+2)^2((n-1)\delta+3)-2\big)\Big(4n\mu\delta\big(1-((n-1)\delta+2)\big)+\big(\mu((n-1)\delta+2)^2-n\big)\Big)}{2\big(\mu((n-1)\delta+2)^2-n\big)^3} + \frac{n\mu^2\delta((n-1)\delta+2)\big(\mu((n-1)\delta+2)^2-n\big)\big(3(n-1)\delta+8\big)}{2\big(\mu((n-1)\delta+2)^2-n\big)^3} . \tag{A.4}$$

For each complete network size n and $\delta^* < \delta \leq 1$ where the effectiveness $\mu > \mu^*$, we have $\mu((n-1)\delta+2)^2((n-1)\delta+3) > 2$ and $\mu((n-1)\delta+2)^2 > n$. This implies that $dTW^*/dn > 0$ and this indicates that the total welfare increases with growing the complete network size. ▮

REFERENCES

Alghamdi, M., 2016. Economic returns in forming stable R&D networks. *Springerplus* **5(1)** 1570.

Alghamdi, M., 2017. Maximum total welfare versus Growth of R&D networks. *Journal of mathematics in industry* **7:11**.

Alghamdi, M., 2019. Individual and Social Incentives versus R&D Network Restriction. *J. Korean Soc. Ind. Appl. Math.* **23(4)**, 329-350.

Alghamdi, M., 2020. Company Level Variables in R&D Cooperation Perspective. *Communications in Mathematics and applications* **11(1)**, 41-55.

Alghamdi, M., S. McDonald, and B. Pailthorpe, 2020. The Impact of Product Differentiation on Symmetric R&D Networks. In: Yeung D., Luckraz S., Leong C. (eds) Frontiers in Games and Dynamic Games. *Annals of the International Society of Dynamic Games* **16**, Birkhuser, Cham.

Autant-Bernard, C., P. Billand, D. Frachisse, and N. Massard, 2007. Social distance versus spatial distance in R&D cooperation: Empirical evidence from European collaboration choices in micro and nanotechnologies. *Papers in Regional Science* **86**, 495-519.

Belderbos, R., M. Carree, B. Diedern, B. Lokshin, and R. Veugelers, 2004. Heterogeneity in R&D cooperation strategies. *International Journal of Industrial Organization* **22**, 1237-1263.

D'Aspremont, C. and A. Jacquemin, 1988. Cooperative and Noncooperative R&D in Duopoly with Spillovers. *American Economic Review* **78**, 1133-1137.

Deroian, F., 2008. Dissemination of spillovers in cost-reducing alliances. *Research in Economics* **62**, 34-44.

Fleming, L., C. King, and A. Juda, 2007. Small worlds and regional innovative advantage. *Organization Science* **18**, 938-954.

Fritsch, M. and R. Lukas, 2001. Who cooperates on R&D?. *Research Policy* **30(2)**, 297-312.

Goyal, S. and S. Joshi, 2003. Networks of collaboration in oligopoly. *Games and Economic Behavior* **43(1)**, 57-85.

Goyal, S. and J. L. Moraga-Gonzalez, 2001, R&D Networks. *Rand Journal of Economics* **32**, 686-707.

Hackner, J., 2000. A note on price and quantity competition in differentiated oligopolies. *Journal of Economic Theory* **93**, 233-239.

Hagedoorn, J., A. L. Link, and N. Vonortas, 2000. Research partnerships. *Research Policy* **29**, 567-586.

Hagedoorn, J. and M. van Ekert, 2002. The Cooperative Agreements and Technology Indicators Database. *MERIT* Maastricht University.

Jackson, M. O. and A. Wolinsky, 1996. A Strategic Model of Social and Economic Networks. *Journal of Economic Theory* **71**, 44-74.

Konig, M. D., S. Battiston, M. Napoletano, and F. Schweitzer, 2012. The efficiency and stability of R&D networks. *Games and Economic Behavior* **75**, 694-713.

Meagher, K. and M. Rogers, 2004. Network density and R&D spillovers. *Journal of Economic Behavior & Organization* **53(2)**, 237-260.

Miotti, L. and F. Sachwald, 2003. Co-operative R&D: Why and with whom? An integrated framework of analysis. *Research Policy* **32**, 1481-1499.

Narula, R. and G. D. Santangelo, 2009. Location, collocation and R&D alliances in the European ICT industry. *Research Policy* **38(2)**, 393-403.

Newman, M. E. J., 2010. Networks: an introduction. Oxford University Press, Oxford, UK.

Okamuro, H., 2007. Determinants of successful R&D cooperation in Japanese small businesses: The impact of organizational and contractual characteristics. *Research Policy* **36**, 1529-1544.

Roketskiy, N., 2018. Competition and networks of collaboration. *Theoretical Economics* **13**, 1077-1110.

Silipo, D. B. and A. Weiss, 2005. Cooperation and competition in a duopoly R&D market. *Research in Economics* **59**, 41-57.

Singh, N., and X. Vives, 1984. Price and Quantity Competition in a Differentiated Duopoly. *Rand Journal of Economics* **15**, 546-554.

Song, H. and V. Vannetelbosch, 2007. International R&D Collaboration Networks. *The Manchester School* **75(6)**, 742-766.

Sanou, F. H., F. Le Roy, and D. R. Gnyawali, 2016. How does centrality in coopetition networks matter? An empirical investigation in the mobile telephone industry. *British Journal of Management* **27(1)**, 143-160.

Stuart, T. E., 2000. Interorganizational Alliances and the Performance of Firms: A Study of Growth and Innovation Rates in a High-Technology Industry. *Strategic Management Journal* **21**, 791-811.

Tomasello, M. V., M. Napoletano, A. Garasz, and F. Schweitzer, 2013. The Rise and Fall of R&D Networks. arXiv:1304.3623v1.

van der Pol, J. and J. P. Rameshkoumar, 2018. The co-evolution of knowledge and collaboration networks: the role of the technology life-cycle. *Scientometrics* **114(1)**, 307-323.

Verspagen, B. and G. Duysters, 2004. The small worlds of strategic technology alliances. *Technovation* **24**, 563-571.

Westbrock, B., 2010. Natural concentration in industrial research collaboration. *RAND Journal of Economics* **41(2)**, 351-371.

Zu, L., B. Dong, X. Zhao, and J. Zhang, 2011. International R&D Networks. *Review of International Economics* **19**, 325-340.

Zirulia, L., 2012. Economics of Innovation and New Technology: The role of spillovers in R&D network formation. *Economics of Innovation and New Technology* **21**, 83-105.