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Asset Pricing and Microcaps

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I study the pricing power of the microcap stocks with characteristics including accruals, new share issues, momentum and volatility, in addition to asset growth and profitability. After adjusting for the market excess return, I find that the return spreads formed from microcap stocks subsume the pricing power of those formed from other stocks. A microcap-based factor model outperforms many alternative models. The results are consistent with what MacKinlay and Pastor (2000) find that the additional factor that completes the pricing job of a factor model is a portfolio weighted towards mispriced securities.

Key Words: Anomalies; Microcaps; Factor models.
JEL Classification Numbers: G11, G12.

1. INTRODUCTION

Researchers in the last few decades have documented that the average returns on common stocks associated with a large number of characteristics are not explained by the market-based CAPM or the three-factor model of Fama and French (1993). Recently, Hou, Xie and Zhang (HXZ, 2015) propose a four-factor model inspired by the q-theory of investment, which consists of the market excess return, the size factor, the investment factor and the profitability factor. Motivated by a valuation model in which the market value of a firm is the present value of its future dividends, which are related to future investment and profitability, Fama and French (2015) enhance their three-factor model to a five-factor model with alternative investment and profitability factors. Stambaugh and Yuan (2017) propose a four-factor mispricing factor model, by constructing two factors from a set of 11 prominent characteristics (anomalies) unexplained by the CAPM or the Fama-French (1993) three-factor model.

While other researchers mostly examine their models' abilities to explain anomalies by using portfolios formed from univariate sorts on individual

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characteristics, Fama and French (2015, 2016) find that the cross section of average returns on portfolios formed from bivariate sorts on both size (market capitalization) and one of the many characteristics still pose a challenge to their five-factor model, or the model augmented by a momentum factor. The most serious challenge is that microcap stocks, which are stocks in the bottom size quintile, are extremely mispriced by the five or six-factor model. As Fama and French (2008) note, microcap stocks account for a substantial number of stocks, so it is important to find a model that can better explain the average returns on microcaps. MacKinlay and Pastor (2000) show that when a risk factor is missing from an asset pricing model, the missing factor that completes the pricing job of the missing factor model is a portfolio weighted towards mispriced securities. The preceding literature suggests that it is important to compare the pricing power of the microcap stocks with that of non-microcap stocks.

Barillas and Shanken (2017) show that a traded factor is redundant if the intercept (alpha) in the regression of the factor on other factors is zero. Li (2018) explores the link between the alpha- and risk priced-based approaches to testing the statistical significance of the additional factor. I find that when long-short return spreads on non-microcap stocks are regressed on those of microcap stocks and the market excess return, the alphas are statistically insignificant. The regressions are performed with multiple characteristics including accruals, new share issues, momentum and volatility, in addition to asset growth and profitability. The result suggests that factors formed from non-microcap stocks are no longer necessary once factors formed from microcap stocks are included. I find that an asset pricing model with factors formed from microcap stocks outperforms many alternative models, including models with factors formed from non-microcap stocks and existing models in the literature.

Fama and French (2018) analyze variations of the Fama French five and six factor models. The variations they consider include forming factors using small stocks below the NYSE size median. Their small stock factors contain much larger stocks than the bottom size quintile considered in this paper. The superior performance of the microcaps-based model with multiple characteristics contrasts sharply from what they find that their base model that combines small and big stocks in its spread factors performs as well as their small stock model. One contribution of this paper is to illustrates that using microcap stocks that are smaller than stocks below the NYSE size median can make a significant difference, because the evidence in this paper shows that microcap stocks subsume the pricing power of non-microcap stocks, but not vice versa.

The existence of hundreds of anomalies (Harvey, Liu and Zhu, 2016) makes it increasingly dif?cult to have a parsimonious model that can account for patterns in average returns in the cross section. Like most of the preceding researchers, I conduct model comparisons based on a limited number of anomalies examined in the recent literature. Moreover, there are quite a few papers that combine multiple characteristics or use the resulting combination portfolios in factor models, including the work of Light, Maslov and Rytchkov (2017) and Green, Hand and Zhang (2017). While the results of this paper concerning multiple characteristics are largely consistent with those found by other researchers, the motivation and construction method of factors are quite different.

The reminder of the article is organized as follows. In the next section, I discuss the motivation for my factor construction and estimation method. I then describe the data sources and the factor construction. I then examine the pricing power of microcap stocks. Finally, I conduct tests of various models with different characteristics, before conclusions.

2. THE MODEL

Consider each factor model as a linear regression of N excess returns, R_j on a vector of factors, F_j:

$$R_j = \alpha_j + \beta_j F_j + \varepsilon_j, \quad j = 1, 2, \ldots, J \tag{1}$$

where R_j, ε_j, and α_j are $N \times 1$ vectors, F_j is a $K_j \times 1$ vector of factors and β_j is an $N \times K_j$ matrix of factor loadings. The residual vector ε_j has zero means and an invertible covariance matrix, Σ_j. In equation (1), each excess return is the difference between the return on a portfolio and the riskfree asset, or between returns on two portfolios (return spread). Throughout this paper, I only consider spread factors as differences in returns.

Let $Sh(F_j)$ denote the Sharpe ratio of a mean-variance efficient frontier that can be constructed from the factors F_j and the riskfree asset, and $Sh(R_j, F_j)$ denote the Sharpe ratio of a frontier that can be constructed from R_j, F_j and the riskfree asset. Then exact factor pricing with $\alpha_j = 0$ is equivalent to the equality of the frontiers, or, $Sh(F_j) = Sh(R_j, F_j)$ (Huberman and Kandel, 1987, p878). Gibbons, Ross, and Shanken (GRS, 1989) show that, a quadratic form of α_j is the difference between two squared Sharpe ratios:

$$\alpha_j' \Sigma^{-1} \alpha_j = Sh^2(R_j, F_j) - Sh^2(F_j). \tag{2}$$

All alphas in equation (2) collapse to zero, if and only if the difference between the two squared Sharpe ratios vanishes, or equivalently, the two frontiers merge into one. Suppose the goal is to minimize the difference between the squared Sharpe ratios in the right side of equation (2). Barillas and Shanken (2017) argue that, the test assets used to evaluate the models should be augmented by the factors from the other models. If the set of test assets R_j are just the factors in other models under consideration, then (R_j, F_j) and hence $Sh^2(R_j, F_j)$ become common for all models, regardless of the test assets. Since each factor model prices the factors in its own model perfectly with zero alphas, testing each model against all factors in other models is equivalent to testing each model against a complete model that includes the factors in all models under consideration, including the model to be tested. As a result, this type of test can be viewed as a nested test. An alternative way of the model comparison is to use test assets $R_j = R$, which contains all portfolios used to construct the factors in all models under consideration. Since in this case, $Sh^2(R_j, F_j) = Sh^2(R)$, $Sh^2(R_j, F_j)$ are common for all models under consideration, just like the case when the test assets include all factors in other models. Whenever $Sh^2(R_j, F_j)$ are common for all models, the goal of minimizing the difference between the squared Sharpe ratios in the right side of equation (2) is equivalent to the goal of maximizing the squared Sharpe ratio, $Sh^2(F_j)$, of the factor model. In what follows, I include the two types of test assets in comparing factor models.

Consider a benchmark model, such as the CAPM or the three-factor model of Fama and French (1993). This model can be described in the same way as equation (1) with $j = B$. An exact pricing implies that each element of the mispricing vector α_B is zero. MacKinlay and Pastor (2000) show that if exact pricing does not hold due to a missing factor, then there is a unique portfolio of assets that can be combined with the factor portfolios F_B to form a tangency portfolio. The unique portfolio is orthogonal to the factor portfolios and when it is added to the benchmark model, the mispricing vanishes. They further show that if the residual covariance matrix of the extended model with the additional orthogonal factor is assumed to be diagonal and proportional to the identity matrix, then the weights on the assets in the portfolio are proportional to the mispricing vector in the benchmark model. They argue that their result justifies using the spread factors like SMB and HML because these portfolios essentially assume long positions in stocks with positive alphas and short positions in stocks with negative alphas in the CAPM. Following this argument, I form spread factors, like most of the literature. However, I take into consideration of microcap stocks, which are most mispriced, and multiple characteristics in factor construction.

3. DATA AND FACTORS

3.1. Data Description

To compare with the recent literature, especially Fama and French (2015, 2016), I use data on the value-weighted portfolios formed by Fama and

French (2015, 2016) to construct factors and limit the sample period to January 1967?December 2015 (588 months). The data are provided by Kenneth French. The breakpoints use only NYSE stocks, but the sample is all NYSE, Amex, and NASDAQ stocks. The portfolios are formed from 5×5 quintile sorts: first on market equity (size), and then on each of the following characteristics: growth in total assets (AG), accruals (AC), net share issues (NI), operating profitability (OP), prior (2-12 month) return (PR), the variance of daily returns (Var), the variance of daily residuals ($RVar$) in the Fama-French (1993) three-factor model, or the book-to-market ratio (B/M). For the sake of consistency, portfolios formed from sorts with negative or zero net share issues (repurchases) are excluded. The first- and the second-pass sorts are independent for all characteristics, except that the second pass sorts on Var and $RVar$ are conditional on size quintile. Portfolios are formed at the end of June each year, except for PR, Var and $RVar$, which are formed monthly.[1] See Appendix for a detailed description of the definitions of the characteristics.

I also use data on five factors in the model of Fama and French (FF-5, 2015). The data on the five factors are also provided by Kenneth French. The five factors are the value-weighted return on a market portfolio in excess of the riskfree rate (MKT), the return on a small stock portfolio minus the return on a big stock portfolio (SMB), the return on a conservative investment (asset growth) portfolio minus the return on an aggressive investment portfolio (CMA), the return on a portfolio of stocks with robust operating profitability minus the return on a portfolio of stocks with weak operating profitability (RMW), and the return on a portfolio of stocks with high B/M ratios minus the return on a portfolio of stocks with low B/M ratios (HML). Except for MKT, Fama and French (2015) construct the factors from six value-weighted portfolios formed from 2×3 sorts on size and a characteristic such as B/M, AG or OP. All three portfolios, CMA, RMW and HML, are formed at the end of June each year.

Other data include factors in the q-factor model of HXZ (q-4, 2015): The data are provided by Lu Zhang. The size factor (ME) is the difference between the return on a small stock portfolio and the return on a big stock portfolio. The investment (I/A) factor is the difference between the return on a low I/A portfolio and the return on a high I/A portfolio. The profitability (ROE) factor is the difference between the return on a high ROE portfolio and the return on a low ROE portfolio. The three factors are constructed from value-weighted portfolios formed from $2 \times 3 \times 3$ sorts on ME, I/A, and ROE. While the I/A factor is formed at the end of June each year, the ROE factor is formed monthly.

[1] Another characteristic used by Fama and French (2016) is the market beta, β_M, in the market model. Given the lack of difference between average returns on low and high β_M portfolios, I exclude β_M in factor constructions and subsequent analyses.

Finally, I use data, provided by Yu Yuan, on the factors in the mispricing factor model of Stambaugh and Yuan (M-4, 2017), who use 2×3 sorts on size and another sorting variable to construct a size factor and two mispricing factors: $MGMT$ and $PERF$. Unlike Fama and French (2015), who use a single characteristic, Stambaugh and Yuan (2017) use the average stock rankings with respect to a cluster of 11 characteristics as a sorting variable. $MGMT$ is based on a cluster of six characteristics: net stock issues, composite equity issues, accruals, net operating assets, asset growth, and investment to assets. $PERF$ is based on a cluster of five characteristics: distress, O-score, momentum, gross profitability, and return on assets. As Stambaugh and Yuan (2017) construct their size factor differently by using only stocks not used in forming $MGMT$ and $PERF$, I denote their size factor as SMB'. Both $MGMT$ and $PERF$ are formed monthly.

3.2. Factor Construction

After describing the data sources, I move to factor constructions. Some notation is necessary. Let $R_j(Y_j)$ denote the return on a portfolio formed from 5×5 quintile sorts on size and a characteristic, Y. Subscript i refers to a size quintile and j refers to a Y quintile, $i, j = 1, \ldots, 5$. Size and Y are in ascending orders. Following Fama and French (2015, 2016), size quintile 1 refers to microcap stocks, or simply microcaps and size quintile 5 refers to megacap stocks, or simply megacaps. To maintain nonnegativity of average spreads as much as possible, I define long-short return spreads for each of the size quintiles:

$$\text{(Low-high)} \quad S_i(Y) = R_i(Y_1) - R_i(Y_5) \quad \text{for} \quad Y = AG, AC, NI, Var, RVar; \tag{3}$$

$$\text{(High-low)} \quad S_i(Y) = R_i(Y_5) - R_i(Y_1) \quad \text{for} \quad Y = OP, PR, B/M. \tag{4}$$

Fama and French (2018) argue that, if the test assets in equation (2) include all assets, then the model with the highest Sharpe ratio is the best one since its squared Sharpe ratio is closest to that produced by all assets. Equation (2) implies that the model with the highest Sharpe ratio produces the lowest pricing errors for all assets. The Sharpe ratio of each factor model is the average excess return on a portfolio of factors in the model divided by the volatility (standard deviation) of the portfolio return. This suggests that there are two considerations in choosing the factors. The first is to the average factor returns and the second is to the risk of the factors. To increase the average returns, I include microcap spreads to form factors. To lower the risk of the factors, I average return spreads over characteristics to effectively diversify the risks. I now describe the details of factor construction.

I define an investment-related factor and a profitability-related factor as follows:

$$INV_{IJ} = \begin{cases} \frac{1}{I}\sum_{i=1}^{I}(S_i(AG)), & J=1 \\ \frac{1}{2I}\sum_{i=1}^{I}(S_i(AG)+S_i(AC)), & J=2 \\ \frac{1}{3I}\sum_{i=1}^{I}(S_i(AG)+S_i(AC)+S_i(NI)), & J=3 \end{cases} \quad (5)$$

$$PPR_{IJ} = \begin{cases} \frac{1}{I}\sum_{i=1}^{I}(S_i(OP)), & J=1 \\ \frac{1}{2I}\sum_{i=1}^{I}(S_i(OP)+S_i(PR)), & J=2 \\ \frac{1}{3I}\sum_{i=1}^{I}(S_i(OP)+S_i(PR)+S_i(Var)). & J=3 \end{cases} \quad (6)$$

In equations (5)-(6), the factors combine return spreads from up to six characteristics and size quintiles from one to I, $I \leq 5$ including microcaps.

The factor, INV_{I3}, can be regarded as a broad-based investment factor, since the three characteristics in equation (5) capture information about firms' growth in total assets, changes in working capital and changes in capital expenditure through net share issues. As discussed in the section on the valuation model, the expected stock return is negatively related to accruals when operating profitability is a proxy for total earnings. Equation (4) suggests that accruals can be either subtracted from operating profitability (to form cash profitability) or added into the change in book equity. Instead of replacing operating profitability with cash profitability, I treat accruals separately from operating profitability here and use accruals to form a broad-based investment factor since accruals reflect the investment in the short-term assets like the operating working capital, which is current assets (excluding cash and marketable securities) minus current liabilities (excluding short-term debt).[2] Titman, et al. (2004), Cooper, et al. (2008), and Polk and Sapienza (2009) document the negative relation between investment and average return. Sloan (1996) finds that low returns are associated with high accruals, similar to low returns associated with total asset growth.

As discussed earlier (see equation (5)), new share issues (NI) offer supplemental information about the change in equity that is not reflected in AG and AC. This is especially important for leveraged firms paying no or fixed dividends. Ritter (1991) and Loughran and Ritter (1995) show that, the underpricing of initial public offerings is a short-run phenomenon, and in post-issue years, equity issuers underperform matching non-issuers with similar characteristics. The negative relation between new share issues and average future returns is consistent with the relation predicted by

[2]Ball et al (2016) define components of the working capital accruals to include changes in accounts receivable, inventory, prepaid expenses, deferred revene, accounts payable, and accrued expenses.

the valuation equation (2), given everything else including dividends. The difference between short- and long-run stock performance after new share issues is also consistent with the implication of the valuation model (2) and hence suggests that the new share issues may also offer information about the variation of expected returns. As NI mainly reflects equity investment through external financing it is grouped together along with AG and AC rather than profitability.

The second factor, PPR_{I3}, is measures profitability, performance and risk. Haugen and Baker (1996), Fama and French (2006) and Novy-Marx (2013) find that average stock return is positively related to expected profitability. HXZ (2015) show that their profitability factor formed from most recent quarterly earnings explains the average return on the momentum portfolio. Novy-Marx (2015) argues that the stock price momentum is explained by the profitability factor in the q-factor model because the factor reflects momentum in firm fundamentals. As I use the profitability variable, OP, that is based on annually rather than quarterly updated earnings, adding the variable, PR, helps to capture the momentum in firm fundamentals like earnings.

Jegadeesh and Titman (1993) document that stock price exhibits momentum in the intermediate horizon but the stock price performance tends to be reversed in the long run. Liu and Zhang (2008) conclude that risk plays an important role in driving momentum profits. Daniel and Moskowitz (2016) show that momentum portfolios crash in panic states, following market declines and when market volatility is high. The behavior of the momentum portfolios is in accord with the valuation model (2) with time-varying expected returns. As a result, returns on the size- and prior return-sorted portfolios provide insights about the variation of expected returns, in addition to momentum in firm fundamentals.

Ang et al. (2006) ?nd that stocks with highly volatile returns tend to have low average returns whether volatility is measured as the variance of daily returns or as the variance of the residuals from the Fama-French (1993) three-factor model. Jiang et al. (2009) document that idiosyncratic volatility is inversely related to future earning shocks. Moreno and Rodríguez. (2015) document that idiosyncratic volatility anomaly is related to investment and profitability. Therefore, the negative relation between average returns and the volatility is consistent with the positive relation between average returns and future earnings in the valuation model. As the volatility of total stock returns (expected plus unexpected) contain information about the variation of expected returns, the size- and volatility-sorted portfolios also provide insights about the variation of expected returns. Returns on the prior return- and volatility-sorted portfolios are among the most mispriced and difficult to explain, as documented by Fama and French (2016).

INV_{I3} and PPR_{I3} are broader measures of investment and profitability than CMA and RMW used by Fama and French (2015) or I/A factor or ROE factor used by HXZ (2015), who construct each factor based only on one characteristic. Most of the additional characteristics like NI, PR and Var may capture the time variation of expected returns. Since the return spreads in the right side of equations (3)-(4) are less than perfectly correlated, averaging the spreads lowers the volatility of each of the two factors through the benefit of diversification and increases the Sharpe ratios of the factors.

To construct the size factor SMB^*, I first define a size spread for each characteristic Y

$$SMB(Y) = \frac{1}{3}\sum_{j=2}^{4}(R_1(Y_j) - R_5(Y_j)), \tag{7}$$

and then average the spreads over six characteristics:

$$SMB^* = \frac{1}{6}(SMB(AG)+SMB(AC)+SMB(NI)+SMB(OP)+SMB(PR)+SMB(Var)). \tag{8}$$

The construction of the size factor here uses only microcaps and megacaps, unlike other researchers who use small and big stocks which together account for stocks in all size quintiles. The purpose is to increase the average return on the factor. Following Stambaugh and Yuan (2017), I exclude the lowest and highest characteristic quintiles that are used to calculate the spreads and the resulting factors INV_{I3} and PPR_{I3}, so that the size factor is neutral to extreme fluctuations in characteristics.

Following most of the literature, portfolio returns are evaluated before transaction costs, which tend to be negatively related to the firm size, but positively related to portfolio rebalancing frequencies. To mitigate the concern of transaction costs, all portfolios used as test assets or used to construct factors are value-weighted. While the profitability factor in the q-factor model of HXZ (2015) and all of the 11 anomaly variables in the mispricing factor model of Stambaugh and Yuan (2017) are formed monthly, the factors in this paper are constructed from portfolios formed annually, except for those related to momentum and volatility.

3.3. Summary Statistics for Spreads and Factors

Panel A of Table 1 presents the average and standard deviation of the return spread for each size quintile and each characteristic. Except for AC, the average spreads for seven characteristics are highest for microcaps. For example, for AG, the average spread is 62 bps for microcaps but 33 bps for megacaps. For PR, the average spread is 1.38 percent for microcaps but 62 bps for megacaps. For Var and $RVar$, the average spreads for microcaps are 1.30-1.32 percent but only 4-7 bps for megacaps. Of special interest is that, for five characteristics: NI, PR, Var, $RVar$ and $B/$M, the average spreads are inversely related to size.

TABLE 1.

Summary Statistics for Return Spread for Each Size Quintile

Panel A. Average and standard deviation, %										
	Average for size quintile					Standard deviation for size quintile				
Y	1	2	3	4	5	1	2	3	4	5
AG	0.62	0.39	0.36	0.25	0.33	2.34	2.73	2.96	3.37	3.54
AC	0.28	0.18	0.27	0.02	0.38	1.66	2.07	2.44	2.43	3.19
NI	0.66	0.56	0.53	0.33	0.27	3.16	3.33	3.57	3.54	3.10
OP	0.32	0.40	0.41	0.23	0.18	3.22	3.36	3.61	3.23	3.42
PR	1.38	1.07	0.95	0.83	0.62	4.89	5.22	5.63	6.01	6.09
Var	1.30	0.68	0.38	0.28	0.04	6.69	6.37	6.12	6.17	5.42
$RVar$	1.32	0.77	0.46	0.37	0.07	6.42	5.97	5.75	5.75	4.70
B/M	0.86	0.50	0.53	0.20	0.22	4.15	4.13	4.37	4.21	4.21

Panel B. Correlation for microcaps (size quintile 1)							
	AG	AC	NI	OP	PR	Var	$RVar$
AC	0.47						
NI	0.08	0.09					
OP	−0.49	−0.21	0.58				
PR	0.01	0.06	0.05	0.05			
Var	−0.14	−0.01	0.63	0.52	0.37		
$RVar$	−0.13	−0.01	0.65	0.54	0.36	0.99	
B/M	0.21	0.06	0.76	0.43	0.01	0.61	0.62

The breakpoints use only NYSE stocks, but the sample is all NYSE, Amex, and NASDAQ stocks. The portfolios are formed from 5×5 quintile sorts: first on market equity (size), and then on each of the following characteristics (Y): growth in total assets (AG), accruals (AC), net share issues (NI), operating profitability (OP), prior (2-12 month) return, (PR), the variance of daily returns (Var), the variance of daily residuals ($RVar$) in the Fama-French (1993) three-factor model, or the book-to-market ratio (B/M). For the sake of consistency, portfolios formed from sorts with negative or zero net share issues (repurchases) are excluded. Portfolios are formed at the end of June each year, except for PR, Var and $RVar$, which are formed monthly. $R_i(Y_j)$ is the return on a portfolio in size quintile i and Y quintile j. The return spread, $S_i(Y)$, for size quintile i and characteristic Y is constructed as follows:

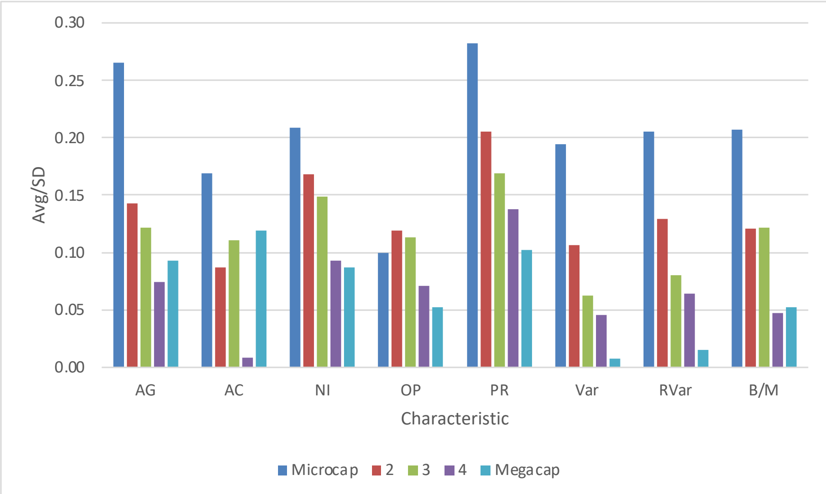
$$\text{(Low-high) } S_i(Y) = R_i(Y_1) - R_i(Y_5) \text{ for } Y = AG, AC, NI, Var, RVar; \quad (3)$$

$$\text{(High-low) } S_i(Y) = R_i(Y_5) - R_i(Y_1) \text{ for } Y = OP, PR, B/M. \quad (4)$$

Size and Y are in ascending orders. The sample period is January 1967 — December 2015 (588 months).

The standard deviations tell an opposite story. The standard deviations of the spreads for microcaps are the lowest for four characteristics: AG, AC, OP, and PR. and. For example, for AG, the standard deviation is 2.34 percent for microcaps but 2.73-3.54 percent for non-microcaps. For AC, the standard deviation is 1.66 percent for microcaps but 2.07-3.19 percent for non-microcaps. More strikingly, for characteristics: AG and AC, standard deviations increase with size. For other four characteristics: NI, Var, $RVar$ and B/M, standard deviations do not show much variability across the size quintiles, despite the enormous differences in the average spreads between microcaps and megacaps. The summary statistics here show that microcaps tend to have high average spreads and low standard deviations.

FIG. 1. Average Return Spread per unit of Standard Deviation.



Microcap refers to size quintile 1 and megacap refers to size quintile 5. Characteristic is growth in total assets (AG), operating profitability (OP), prior (2-12 month) return, (PR), accruals (AC), net share issues (NI), the variance of daily returns (Var), book-to-market (B/M), or the variance of daily residuals ($RVar$) from the Fama-French (1993) three-factor model. The sample period is January 1967 — December 2015 (588 months).

Figure 1 illustrates the average spread per unit of standard deviation for each of the five size quintiles and each of the nine characteristics. Except for OP, microcaps have the highest average spread per unit of standard deviation among all size quintiles. The disparity between microcaps and non-microcaps is quite substantial for most characteristics. The results suggest that using the spreads for microcaps to construct factors can potentially increase the Sharpe ratios associated with the factors. Panel B of Table 1 presents the cross correlations between the spreads for microcaps. The highest correlation (0.99) is between spreads formed from Var and $RVar$-sorted portfolios. Given this nearly perfect correlation, it is unnecessary to include low and high $RVar$ portfolios in constructing factors.

TABLE 2.

Summary Statistics for Factors

Panel A. Investment- and profitability-related factors in the C-4 model

	INV_{11}	INV_{12}	INV_{13}	PPR_{11}	PPR_{12}	PPR_{13}
Average	0.625	0.453	0.521	0.322	0.851	1.002
Std. dev.	2.340	1.721	1.631	3.216	2.991	3.759
Sharpe	0.267	0.263	0.319	0.100	0.285	0.267
t-stat	6.47	6.38	7.75	2.43	6.90	6.46
	INV_{51}	INV_{52}	INV_{53}	PPR_{51}	PPR_{52}	PPR_{53}
Average	0.392	0.310	0.363	0.310	0.639	0.605
Std. dev.	2.240	1.500	1.600	2.633	2.994	3.299
Sharpe	0.175	0.206	0.227	0.118	0.214	0.183
t-stat	4.24	5.01	5.50	2.85	5.18	4.45

Panel B. Factors in other models

	FF-5	q-4	M-4	FF-5	q-4	M-4
	CMA	I/A	$MGMT$	RMW	ROE	$PERF$
Average	0.323	0.411	0.606	0.258	0.561	0.706
Std. dev.	2.031	1.882	2.892	2.292	2.53	3.798
Sharpe	0.159	0.218	0.210	0.113	0.222	0.186
t-stat	3.85	5.30	5.08	2.73	5.38	4.51
			C-4	FF-5	q-4	M-4
			SMB^*	SMB	ME	SMB'
Average			0.404	0.243	0.305	0.429
Std. dev.			4.016	3.090	3.100	2.895
Sharpe			0.101	0.079	0.098	0.148
t-stat			2.44	1.91	2.39	3.59

Panel C. Correlation between pairs of size factors

	SMB^*	SMB	ME
SMB	0.958		
ME	0.934	0.974	
SMB'	0.906	0.942	0.926

The factor model (C-4) consists of four factors: MKT, SMB^*, INV_{IJ} and PPR_{IJ}. MKT is the excess return on the value-weighted market portfolio. SMB^* (see eq. (8)) is difference in returns between smallest and biggest size quintiles, averaged over middle three quintiles of six characteristics. INV_{IJ} (see eq. (5)) is the average of return spreads from size quintile 1 to I ($I = 1, 5$), including up to J ($J \leq 3$) characteristics: asset growth (AG), accruals (AC) and net share issues (NI). PPR_{IJ} (see eq. (6)) is the average of return spreads from size quintile 1 to I, including up to J characteristics: operating profitability (OP), prior return (PR) and variance of daily return (Var). CMA and RMW are, respectively, the investment and profitability factors in the five-factor model of Fama and French (FF-5, 2015). I/A and ROE are alternative investment and profitability factors in the q-factor model of HXZ (q-4, 2015). $MGMT$ and $PERF$ are mispricing factors in the model of Stambaugh and Yuan (M-4, 2017). SMB, ME and SMB' are size factors in other models. The HML factor in the FF-5 model is not reported. The Sharpe ratio of each factor is the average divided by the standard deviation. The sample period is January 1967 — December 2015 (588 months).

In Table 2, I present summary statistics for four factors in the proposed (characteristics) model (C-4) along with factors in competing models. Panel A reports the average, the standard deviation, the Sharpe ratio (the average divided by the standard deviation), and the t-statistic for each factor in the C-4 model and panel B reports the statistics for the factors in the FF-4, q-4 and M-4 models. Panel C reports the correlations between pairs of size factors. Given the same sample size, the *t*-statistic is proportional to the ratio of the average return to the standard deviation. As a result, a high *t*-statistic associated with a factor is indicative of a high Sharpe ratio.

I report summary statistics for the investment- and profitability-related factors for microcaps ($I = 1$) or all size quintiles ($I = 5$) for each given subset of one to three characteristics for each factor. The investment-related factor (INV_{1J}) with microcaps only tends have a much higher average (52-63 bps) than the factor (INV_{5J}) that includes all size quintiles (31-39 bps), while the difference in the standard deviation is relatively small. As a result, the Sharpe ratio of the microcap factor (0.267-0.319) is higher than that of the factor formed on all size (0.175-0.227). It is intriguing to note that the standard deviation of INV_{I3} is lower than that of INV_{I1}, and the Sharpe ratio of INV_{I3} is higher than that of INV_{I1} for $I = 1, 5$.[3]

The results tell a similar story for the profitability-related factor, PPR_{IJ}. The average and the Sharpe ratio are higher for factors formed on microcaps only than those formed on all size, in most cases. The Sharpe ratio is higher with three characteristics than with just one. The Sharpe ratios of the factors in the C-4 model with $I = 1$ and $J \geq 2$ are higher than those of the corresponding investment (management) orr profitability (performance) factors INV_{51}, in other models. The factor, in the C-5 model, is close to the factor, CMA, in the FF-5 model. Similarly, the factor, PPR_{51} is close to the factor, RMW. However, since the factors in the C-5 model here are formed from 5×5 sorts while the factors in the FF-5 model are from 2×3 sorts, the factors here exhibit somewhat higher Sharpe ratios mostly because the factors here are weighted more towards small stocks, even though they are all formed from stocks of all market capitalizations and formed on the same characteristics, AG and OP. The factors in the C_4 model, however, are more difficult to compare with those in other models (q-4 and M-4) because of differences in sorting variables and portfolio formation frequencies.

Finally, a comparison of the size factors reveals that the new size factor, SMB^*, has an average of 40 bps, higher than the average of SMB (24 bps) and the average of ME (31 bps), and similar to the average of SMB'

[3]For factors INV_{IJ} and PPR_{IJ} ($I = 2, 3, 4$), which are formed from size quintiles 1-I, the results lie between those for factors formed from microcaps and those for factors formed from all size quintiles. As a result, I omit the results for $I = 2, 3, 4$, to save space.

(43 bps). However, the high standard deviation of SMB^* (4.02 percent) implies a t-statistic of 2.44, which implies a significant factor return at the 5 percent level. The t-statistic of SMB^* is higher than the t-statistic of SMB (1.91) or ME (2.39) but lower than that of SMB' (3.59). All pairs of size factors are highly correlated (0.91 or higher). Even though SMB^* is constructed like SMB', it is more correlated with SMB (0.96) and ME (0.93) than SMB' (0.91). In the rest of paper, the results are not sensitive to the choice of the size factor, as in the previous studies.

4. PERFORMANCE COMPARISONS

4.1. Pricing Power of Microcaps vs. Non-microcaps

Given the evidence on the difference between microcap spreads and non-microcap spreads, it is of interest to compare the pricing powers of microcap stocks and non-microcap stocks. I first examine how the microcap return spread, $S_1(Y)$, explain the return spreads, $S_i(Y)$ of other size quintile $i = 2, \ldots, 5$. Motivated by Barillas and Shanken (2017), in panel A of Table 3, I report the results of estimating following equation:

$$S_i(Y) = \alpha_i + \beta_{iM} MKT + \gamma_i S_i(Y) + \varepsilon_i, \quad i = 2, \ldots, 5 \tag{9}$$

In equation (9), the microcap return spread is the one of the independent variables while the return spread of one of the other size quintiles is the dependent variables. Y refers to each of the six characteristics that are included in constructing the investment- or profitability-related factors: growth in total assets (AG), accruals (AC), net share issues (NI), operating profitability (OP), prior (2-12 month) return, (PR), the variance of daily returns (Var). Adding other characteristics like B/M or RVar does not change the results in any significant way. I control the market excess return in the right side of equation (9). The size factor is omitted without changing the estimate of the alpha in the equation in any significant way. Each of the error term ε_i has a mean of zero. If microcap stocks subsume the pricing power of non-microcap stocks, then $\alpha_i = 0,\ i = 2, \ldots, 5$; which says that average non-microcap spreads should be explained by the average market excess return and the average microcap spreads:

$$\overline{S}_i(Y) = \beta_{iM} \overline{MKT} + \gamma_i \overline{S}_1(Y), \quad i = 2, \ldots, 5 \tag{10}$$

I estimate equation (9) for each characteristic individually or all characteristics jointly. The results are qualitatively similar for most characteristics. To save space, I report the results of estimating of the system of equations for all six characteristics with cross-equation restrictions on all coefficients by the method of seemingly unrelated regressions (SUR). I do the exercise for the full sample period: January 1967 — December 2015.

To check the robustness of the results to different sample periods, I repeat the exercise for two equally divided subperiods (294 months).

TABLE 3.

The Pricing Power of Microcap vs. Non-Microcap Return Spreads

	Size quintile i							
	2		3		4		5	
Panel A. $S_i(Y) = \alpha_i + \beta_{iM} MKT + \gamma_i S_1(Y) + \varepsilon_i$ (9)								
Full: 1967:1-2015:12								
α_i	0.069	(1.48)	0.075	(1.31)	−0.012	(−0.19)	0.080	(1.25)
β_{iM}	−0.092	(−9.20)	−0.117	(−9.39)	−0.126	(−8.91)	−0.154	(−10.99)
γ_i	0.735	(63.21)	0.663	(49.28)	0.563	(39.17)	0.448	(29.92)
1967:1-1991:6								
α_i	0.055	(0.93)	0.121	(1.89)	−0.066	(−1.03)	0.024	(0.30)
β_{iM}	−0.074	(−6.16)	−0.054	(−4.11)	−0.047	(−3.57)	−0.099	(−6.05)
γ_i	0.707	(38.12)	0.666	(29.16)	0.533	(22.43)	0.387	(15.16)
1991:7-2015:12								
α_i	0.053	(0.77)	0.037	(0.42)	0.083	(0.80)	0.154	(1.66)
β_{iM}	−0.122	(−7.58)	−0.177	(−8.57)	−0.238	(−9.90)	−0.220	(−10.22)
γ_i	0.751	(49.77)	0.649	(38.09)	0.569	(31.65)	0.469	(25.26)
Panel B. $S_1(Y) = \alpha_i + \beta_{iM} MKT + \gamma_i S_i(Y) + \varepsilon_i$ (10)								
Full: 1967:1-2015:12								
α_i	0.306	(7.25)	0.376	(7.83)	0.503	(9.59)	0.510	(9.50)
β_{iM}	0.007	(0.78)	−0.021	(−1.99)	−0.028	(−2.43)	−0.058	(−4.89)
γ_i	0.653	(60.19)	0.524	(46.80)	0.423	(36.04)	0.288	(24.45)
1967:1-1991:6								
α_i	0.317	(6.00)	0.336	(6.39)	0.478	(8.55)	0.500	(8.54)
β_{iM}	0.006	(0.51)	−0.027	(−2.46)	−0.049	(−4.25)	−0.060	(−4.96)
γ_i	0.572	(34.81)	0.447	(28.17)	0.352	(21.02)	0.202	(13.61)
1991:7-2015:12								
α_i	0.320	(5.05)	0.409	(5.20)	0.474	(5.51)	0.492	(5.68)
β_{iM}	0.001	(0.07)	−0.013	(−0.70)	−0.005	(−0.23)	−0.049	(−2.38)
γ_i	0.691	(47.16)	0.565	(35.10)	0.460	(28.12)	0.340	(19.65)

The return spreads, are for size quintile i and the following six characteristics (Y): growth in total assets (AG), accruals (AC), net share issues (NI), operating profitability (OP), prior (2-12 month) return, (PR), the variance of daily returns (Var). MKT is the excess return on the value-weighted market portfolio. The system of equations for all six characteristics are estimated with cross-equation restrictions on all coefficients by the method of seemingly unrelated regressions. In the parentheses are t-statistics.

Interestingly, as shown in panel A, the estimates of the loadings γ_i of $S_1(Y)$ are all positive and significant at very low levels for all three periods. They are inversely related to the size, implying that return spreads for smaller size quintiles behave more like the microcap return spread than bigger size quintiles. They are greater than 0.7 for size quintile 2 and less than 0.5 for size quintile 5. The estimates of the market beta, β_{iM}, are all negative and significant, though much smaller in magnitude as compared to the estimates of the loadings γ_i. Most importantly, the alphas are all small and statistically insignificant at the 10 percent level for all three periods. For instance, for the full period, the alphas range from -1.2 bps to 8 bps. As a result, average return spreads on microcap stocks along with the average market return explain those of non-microcap stocks. In other words, microcap stocks subsume the pricing power of non-microcap stocks.

The results in panel B of Table 3 are from estimating the following system of equations by SUR:

$$S_1(Y) = \alpha_i + \beta_{iM} MKT + \gamma_i S_i(Y) + \varepsilon_i, \quad i = 2, \ldots, 5. \tag{11}$$

Here, the microcap return spread is the dependent variable while the market excess return and the return spread for one of the other size quintiles are the independent variables. Although the loadings on return spreads are all positive and significant and the market betas are often negative and significant for bigger size quintiles, the alphas here are all positive and significant at the 1 percent level. The magnitudes of the alphas are quite large compared with the average spreads reported in Table 1. For instance, for the full period, the alphas range from 31 bps to 51 bps. In the first subperiod, they range from 32-50 bps and the in the second subperiod, they range from 32-49 bps. Overall, the results here imply that non-microcap stocks do not subsume the pricing power of microcap stocks. In other words, average return spreads of microcap stocks are largely unexplained by the average market excess return and the average return spreads of non-microcap stocks.

4.2. Multivariate GRS Tests

I now report the results of multivariate GRS tests of the joint hypothesis that all alphas in a factor model, $R = \alpha + \beta F + \varepsilon$, in equation (1) are zeros. The factor model (C-4) consists of four factors: MKT, SMB^*, INV_{IJ} and PPR_{IJ}. Here I drop the subscripts in equation (1). GRS (1989) show that, for a benchmark factor F, the statistic is related to the ratio of one plus $Sh^2(R, F)$ to one plus $Sh^2(F)$, or equivalently, the percentage difference in one plus the squared Sharpe ratios.[4]

In Table 4, I present the GRS statistics for various test assets. As discussed in an earlier section, one way to compare models is to use the factors

[4]The GRS statistic, $F = (T-N-K)[N(T-K-1)]^{-1}\alpha'\Sigma^{-1}\alpha/\omega_{11}$ has an F distribution with degrees of freedom N and $T-N-K$. ω_{11} is the (1,1)-element of $(X'X)^{-1}$. X is a $T \times (K+1)$ matrix, whose first column contains ones and the remaining K columns contain factors. Here T is the number of monthly observations, N is the number of test assets and K is the number of factors.

TABLE 4.

Model Comparison based on GRS Tests

		GRS for C-4 model with $(I, J) =$			Average absolute alphas, $A|\cdot|$, %			1% Critical
Panel A. Test assets are $INV_{I'J'}$ and $PPR_{I;J'}$ for $I' = 1, 5$ and $J' = 1, 2, 3$, $I' \neq I$ and $J' \neq J$								
	N	(1,1)	(1,2)	(1,3)	(1,1)	(1,2)	(1,3)	
Other C-4	10	11.13	7.13	4.14	0.36	0.18	0.13	2.35
		(5,1)	(5,2)	(5,3)	(5,1)	(5,2)	(5,3)	
Other C-4	10	16.22	11.59	10.76	0.43	0.25	0.21	2.35
Panel B. Tests assets are excess returns								
		(1,1)	(1,2)	(1,3)	(1,1)	(1,2)	(1,3)	
Size 1	40	6.75	5.57	4.68	0.22	0.23	0.12	1.63
Size 1-2	80	4.28	3.66	3.20	0.19	0.20	0.10	1.45
All size	200	2.59	2.33	2.13	0.14	0.16	0.10	1.33
		(5,1)	(5,2)	(5,3)	(5,1)	(5,2)	(5,3)	
Size 1	40	8.34	6.91	6.73	0.22	0.20	0.14	1.63
Size 1-2	80	5.08	4.35	4.25	0.17	0.16	0.10	1.45
All size	200	2.92	2.62	2.56	0.14	0.12	0.11	1.33
Panel C. Test assets are CMA, RMW, I/A, ROE, $MGMT$ and $PERF$								
		(1,1)	(1,2)	(1,3)	(1,1)	(1,2)	(1,3)	
$FF/q/M$	6	17.61	10.47	5.80	0.40	0.27	0.15	2.83
		(5,1)	(5,2)	(5,3)	(5,1)	(5,2)	(5,3)	
$FF/q/M$	6	17.58	12.80	9.52	0.33	0.22	0.16	2.83

The table reports the GRS F-test of the joint hypothesis that all intercepts (alphas) are zero in equation: $R = \alpha + \beta F + \varepsilon$. The factor model (C-4) consists of four factors: MKT, SMB^*, INV_{IJ} and PPR_{IJ}. MKT is the excess return on the value-weighted market portfolio. SMB^* is difference in returns between smallest and biggest size quintiles, averaged over middle three quintiles of six characteristics. INV_{IJ} is the average of return spreads from size quintile 1 to I $(I = 1, 5)$, including up to J $(J \leq 3)$ characteristics: asset growth (AG), accruals (AC) and net share issues (NI). PPR_{IJ} is the average of return spreads from size quintile 1 to I and including up to J characteristics: operating profitability (OP), prior return (PR) and variance of daily return (Var). CMA and RMW are, respectively, the investment and profitability factors in the five-factor model of Fama and French (FF-5, 2015). I/A and ROE are alternative investment and profitability factors in the q-factor model of HXZ (q-4, 2015). $MGMT$ and $PERF$ are mispricing factors in the model of Stambaugh and Yuan (M-4, 2017). The statistic with the lowest value (best) among all six models is highlighted in bold. N is the number of test assets. The critical value is for the GRS F-statistic. The sample period is January 1967 — December 2015 (588 months).

in other models under consideration as the set of test assets. For the factors, INV_{IJ} and PPR_{IJ} $(I = 1, 5,\ J = 1, 2, 3)$, in equations (5)-(6), there are six models and 12 different factors, plus two common factors, MKT and SMB^*. Excluding the four factors in each model, there are 10 alternative factors in other models. When testing against the 10 factors, a low GRS statistic of a model is indicative of a small percentage difference between one plus the squared Sharpe ratio of the factors in the model and one plus the squared Sharpe ratio of the factors in all of the six models. In other words, when test assets are factors in other models, comparing models based on the GRS statistics is equivalent to comparing them based on the Sharpe ratios.

Panel A presents the GRS statistics for this set of test assets and the average absolute alphas,$A|\cdot|$. The statistic with the lowest value among all six models is highlighted in bold. N is the number of test assets. The critical values at the 1 percent level for the GRS statistics are presented in the right column. Given the critical values, all models are rejected based on the test statistics in the table at the 1 percent level. As Fama and French (2015) argue, all models are rejected if the test has enough power, so the focus is on the relative performance of the models. It is striking that for each set of test assets, the model with $(I, J) = (1, 3)$ produces the lowest GRS statistic and the average absolute alpha, as shown in bold.

When INV_{IJ} and PPR_{IJ} are constructed from microcaps only ($I = 1$), the GRS statistics are 11.13 ($J = 1$), 7.13 ($J = 2$) and 4.14 ($J = 3$). However, when the two factors are constructed from stocks of all size ($I = 5$), the GRS statistics are 16.22 ($J = 1$), 11.59 ($J = 2$) and 10.76 ($J = 3$). With a common critical value of 2.35, the results suggest that constructing factors with microcaps only or adding more characteristics lowers the GRS statistics and improve the performance of the models considerably. The average absolute alphas, $A|\cdot|$, confirm the findings. They are 0.36 ($J = 1$), 0.18 ($J = 2$), 0.13 ($J = 3$) (in percent) with $I = 1$; but 0.43 ($J = 1$), 0.25 ($J = 2$) and 0.21 ($J = 3$) with $J = 5$. It is interesting to note that the model performance improves more noticeably when additional characteristics are added in the microcaps-based models than the all size-based models. The best performing model with the lowest GRS statistic or the lowest $A|\cdot|$ is the microcaps-based model including all characteristics, $(I, J) = (1, 3)$.

Alternatively, I use test assets including all portfolios used to construct the factors in the models under consideration. Here a low GRS statistic of a model is indicative of a small percentage difference between one plus the squared Sharpe ratio of the factors in the model and one plus the squared Sharpe ratio of the common test assets. In panel B, I report the results of using 200 portfolios as the test assets, which include all portfolios formed from bivariate 5×5 sorts on size and each of the eight characteristics which contain the six characteristics used for factor construction described earlier. To see the sources of improvements, I also report the results of using 40 microcap portfolios in the bottom size quintile or 80 portfolios in the bottom two size quintiles as the sets of test assets. Since both sets of the test assets include all portfolios used to form the microcap factors, they are useful for comparing microcaps-based models ($I = 1, \quad J = 1, 2, 3$)

With $N = 40$, 80 or 200, the results are fully consistent with those obtained in panel A. Constructing factors with microcaps only or adding more characteristics lowers the GRS statistics and improves the performance of the models. The GRS statistics show that the improvements are more apparent when $N = 40$ than when $N = 80$ or 200. For instance, when $N = 40$, the GRS statistic declines from 6.73 for $(I, J) = (5, 3)$ to 4.68 for $(I, J) = (1, 3)$, while, when $N = 200$, the GRS statistic declines only from 2.56 to 2.13. This implies that the microcaps-based model help to improve the performance of the models for microcaps more than stocks of all market capitalizations. The average absolute alphas are not strictly monotonic with respect to I or J but they show dramatic declines when J increases from 2 to 3. For example, when $N = 40$, declines from 23 bps for $(I, J) = (1, 2)$ to 12 bps for $(I, J) = (1, 3)$; when $N = 200$, $A| \cdot |$ declines from 16 bps to 10 bps. The results underscore the importance of the two characteristics: new share issues and the variance of daily returns.[5]

To examine the sensitivity of the results to test assets, I use the investment and profitability factors in the FF-5 and q-4 models and the management and performance factors in the M-4 model. Since the factors in the M-4 model are formed from 11 anomaly variables, this set of the six test assets reveal the performance of the models for a large cross section of assets. The results are presented in panel C. For the microcaps-based models ($I = 1$), the GRS statistics are 17.61 for model $(I, J) = (1, 1)$, 10.47 for model $(I, J) = (1, 2)$ and 5.80 for model $(I, J) = (1, 3)$. For the all size-based models ($I = 5$), the GRS statistics are 17.58 for model $(I, J) = (5, 1)$, 12.80 for model $(I, J) = (5, 2)$ and 9.52 for model $(I, J) = (5, 3)$. The results further confirm the earlier finding that constructing factors with microcaps only or adding more characteristics lowers the GRS statistics and improve the performance of the models. The lowest GRS statistic (5.80) and the lowest average absolute alpha (15 bps) both occur when $(I, J) = (1, 3)$, although the averages are not always consistent with the GRS statistics, which take into account the residual covariance matrix.

5. CONCLUSIONS

I find that, return spreads on microcap stocks subsume the pricing power of those of other non-microcap stocks. The performance of the model significantly improves when the factors are formed with microcap stocks than with broader-based stock portfolios. The results are consistent with what MacKinlay and Pastor (2000) find that the additional factor that

[5] I also used test assets including 40 portfolios of each size quintile. The model with $(I, J) = (1, 3)$ has the lowest GRS statistic of 4.68 for size quintile 1, 2.02 for size quintile 2, ,2.42 for size quintile 3 and 1.55 for size quintile 4. The model with $(I, J) = (1, 1)$ has the lowest GRS statistic of 1.75 for size quintile 5.

completes the pricing job of a factor model is a portfolio weighted towards mispriced securities. One practical implication of the results here is that the microcaps-based model should be useful in applications such as mutual fund performance evaluation, especially for small-cap and mid-cap mutual funds, since the improvement of the model performance is much greater for small stocks than for big stocks. Another implication is that the factor model that combine multiple characteristics in its factors should be used in the performance evaluation or other studies that include samples of stocks with various characteristics.

APPENDIX

The Definition of Characteristics

For portfolios formed in June of year t:

AG (investment) is the change in total assets from the fiscal year ending in year $t-2$ to the fiscal year ending in $t-1$, divided by $t-2$ total assets;

AC is the change in operating working capital per split-adjusted share from the fiscal year-end in $t-2$ to $t-1$ divided by book equity per share in $t-1$;

NI is the change in the natural log of split-adjusted shares outstanding from the fiscal year-end in $t-2$ to the fiscal year-end in $t-1$;

OP is annual revenues minus cost of goods sold, interest expense, and selling, general, and administrative expenses divided by book equity for the last fiscal year end in $t-1$; and B/M is the book equity for the last fiscal year end in $t-1$ divided by market equity for December of $t-1$.

For portfolios for month t formed at the end of month $t-1$:

PR is prior (2-12) return;

Var is estimated using 60 days (minimum 20) of lagged returns;

$RVar$ is estimated using 60 days (minimum 20) of lagged residuals in the Fama-French (1993) three-factor model.

See the web site of Kenneth French for details.

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