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Towards a More Complete Theory of Structural Transformation

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This paper constructs a unified model on how population growth, technological progress and capital deepening induce structural transformation. We solve analytically the closed-form solution for the model to unearth the mechanisms and crucial assumptions underlying how these factors interact and foster structural transformation. When agricultural productivity growth is fast enough to outweigh the relative price effects contributed by non-agricultural productivity growth, population growth and capital deepening, production factors will move out of agriculture. We clarify the condition to sustain growth in asymptotic growth path. We found empirical validity of the model with sectoral data from the United States in 1970-2020.

Key Words: Structural transformation; Sectoral shares; Population growth; Technological progress; Capital deepening; Growth theory.
JEL Classification Numbers: E10, N10.

"Population increases, and the demand for corn raises its price relatively to other things — more capital is profitably employed on agriculture, and continues to flow towards it". (David Ricardo 1821, 361)

1. INTRODUCTION

Structural transformation is an important topic in growth theory.[1] In the recent two decades, economists have been proposing different causes of structural transformation, including technological progress (Ngai, and

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[1]Structural transformation refers to factor reallocation across different sectors in the economy. More broadly, Chenery (1988, 197) defined structural transformation as "changes in economic structure that typically accompany growth during a given period or within a particular set of countries". He considered industrialization, agricultural transformation, migration and urbanization as examples of structural transformation.

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Pissarides 2007), capital deepening (Acemoglu, and Guerrieri 2008), and population growth (Leukhina, and Turnovsky 2016). How these factors interact is critical for any structural transformation model that includes these three causes, and the absence of a theoretical base for this in the current literature means that it is an incomplete theory. In this paper, we will include all these factors in a structural transformation model, and solve analytically how they interact and foster structural transformation and growth in the long run. This can help us to track the mechanisms and crucial assumptions by how structural transformation and sustained growth occur.

We will develop a two-sector ("agricultural" and "non-agricultural"), three-factor (labor, capital and land) model to study structural transformation. In the model, the representative household views agricultural and non-agricultural goods as consumption complements, while agricultural production possesses stronger diminishing returns to labor. Take population growth as an example. Holding sectoral factor shares constant, population growth will increase non-agricultural output relative to agricultural output, raising the relative price of agricultural goods (relative price effect). At the same time, the increase in labor input in the two sectors will reduce the relative marginal product in the agricultural sector (relative marginal product effect). Given that the two sectoral goods are consumption complements, the relative price effect will outride the relative marginal product effect. Since factor return equals sectoral price times marginal product, this will relatively boost agricultural factor returns and draw production factors towards the agricultural sector. We call this the *population growth effect* on structural transformation.[2] Similarly, we will have the *agricultural technology growth effect*, the *non-agricultural technology growth effect*, and the *capital deepening effect* on structural transformation, which all operates through the relative price effect that overweighs the relative marginal product effect.

From our model's analytical solution, to move production factors away from agriculture, we need a fast enough agricultural technology growth rate so that the agricultural technology growth effect overrides the combination of the other three effects. On the other hand, to sustain growth in the long run, technological progress in the non-agricultural sector needs to outpace the population growth drag. We will develop important hypotheses from our model and test the empirical validity of our theory using the United States sectoral data in 1970-2020.

[2]David Ricardo (1821) mentioned that population growth attracts capital towards the agricultural sector through the relative price effect. See his quote ahead of the Introduction.

2. RELEVANT LITERATURE

Our work is related to the literature of structural transformation, which can be traced back to the work by Harris, and Todaro (1970). They hypothesized that when the rural wage is lower than the expected urban wage, labor will migrate from the rural to the urban sector. In their model labor movement is a disequilibrium phenomenon in the sense that unemployment exists. The literature has evolved to consider how structural transformation occurs within frameworks where full employment and allocation efficiency are achieved. Income effect and relative price effect have become standard channels to explain structural transformation within these frameworks. Income effect is a demand-side approach, which assumes a non-homothetic household utility function, usually with a lower income elasticity on agricultural goods than on non-agricultural goods. Hence income growth throughout development process will shift demand away from the agricultural goods, fostering a relative agricultural decline in the economy. For example, Matsuyama (1992), Laitner (2000), Kongsamut, Rebelo, and Xie (2001), Gollin, Parente, and Rogerson (2002, 2007), Foellmi, and Zweimüller (2008), Gollin, and Rogerson (2014) shared this property. Relative price effect is a supply-side approach, which emphasizes that differential productivity growth across sectors will bring along relative price changes among consumption goods. And the resulting direction of sectoral shift will depend on the degree of substitutability among different consumption goods. For example, Doepke (2004), Ngai, and Pissarides (2007, 2008), Acemoglu, and Guerrieri (2008), Bar, and Leukhina (2010), Lagerlöf (2010), Hansen, and Prescott (2002) shared this feature. Acemoglu, and Guerrieri (2008) proposed capital deepening as an additional cause that generates structural transformation through the relative price effect.

In the recent years, the literature has evolved to look into alternative causes for structural transformation. For example, models with education/training costs (Caselli, and Coleman 2001), tax changes (Rogerson 2008), barriers to labor reallocation and adoption of modern agricultural inputs (Restuccia, Yang, and Zhu 2008), transportation improvement (Herrendorf, Schmitz, and Teixeira 2012), scale economies (Buera, and Kaboski 2012b), human capital (Tamura 2002, Buera, and Kaboski 2012a), international trade (Uy, Yi, and Zhang 2013), population growth (Leukhina, and Turnovsky 2016), child mortality (Adams 2022) and migration costs (Baudin, and Stelter 2022) have been proposed. See Herrendorf, Rogerson, and Valentinyi (2014) for a survey.

Due to the complexity of modeling, the structure transformation literature usually either employed micro-founded models to calibrate cross-sectional or time-evolving sectoral share patterns, or evaluated the relative importance of the above causes in accounting for historical structural changes. Some examples are Echevarría (1997), Hansen, and Prescott (2002), Dennis, and İşcan (2009), Duarte, and Restuccia (2010), Alvarez - Cuadrado, and Poschke (2011), Guilló, Papageorgiou, and Perez-Sebastian (2011), Leukhina, and Turnovsky (2016) and Pan (2019)'s works. Yet the drawback of such methodology is that there were no closed-form solutions to provide a solid analytical base on the mechanisms and crucial assumptions of how important factors like population growth, capital deepening and technological progress interact and foster structural transformation. Also it might be difficult to develop hypotheses based on the parameters and assumptions from the models to test their empirical validity. This paper aims to fill these research gaps. In particular, by obtaining the closed-form solution of the model, we can highlight the mechanisms by which different factors interact to foster sectoral shift. We can also develop hypotheses based on the closed-form solution of the model, and investigate the model's empirical validity by testing the hypotheses against the United States data in 1970-2020.

3. THE UNIFIED MODEL

3.1. Model setup

We construct a unified model to examine how population growth, technological progress and capital accumulation induce structural transformation. There are two sectors ("agricultural" and "non-agricultural") and three production factors (labor, capital and land) in the economy. Technological progress occurs in both sectors. Markets are complete and competitive. Factors are freely mobile across sectors.

Consider the economy which starts with L_0 identical households, and the population growth rate is n. Population at time t is:

$$L_t = L_0 e^{nt}. \tag{1}$$

Each household is endowed with one unit of labor, which is supplied inelastically. The representative household holds utility function in the form of:

$$\int_0^\infty e^{-(\rho-n)t} \frac{\tilde{c}_t^{1-\theta} - 1}{1-\theta} dt, \tag{2}$$

where ρ is the discount rate, θ is the inverse of elasticity of intertemporal substitution, $\tilde{c}_t$ is per capita consumption composite at time t.

The representative household makes his or her consumption decisions subject to budget constraints at $t \in [0, \infty)$:

$$\frac{\dot{K}_t}{L_t} = w_t(1) + r_t \frac{K_t}{L_t} + \Omega_t \frac{T}{L_t} - \tilde{c}_t, \tag{3}$$

where $\frac{\dot{K}_t}{L_t}$ is the instantaneous change in per capita capital stock at time t, $\frac{K_t}{L_t}$ and $\frac{T}{L_t}$ are capital and land each household owns at time t, $w_t, r_t (= R_t - \delta)$ and Ω_t are real wage rate, interest rate and land rental rate at time t. At each time point t, the instantaneous change in per capita capital stock equals the sum of individual real wage, capital interest and land rental incomes, minus real individual spending on consumption composite.

Agricultural and non-agricultural goods, Y_{At} and Y_{Mt}, are produced competitively according to Cobb-Douglas technologies, using labor, capital and land as inputs:

$$
\begin{aligned}
Y_{At} &= A_t L_{At}^{\alpha_A} K_{At}^{\beta_A} T_{At}^{\gamma_A}, && (4)\\
&\quad \alpha_A, \beta_A, \gamma_A \in (0,1), \alpha_A + \beta_A + \gamma_A = 1, g_A \equiv \frac{\dot{A}_t}{A_t}, \\
Y_{Mt} &= M_t L_{Mt}^{\alpha_M} K_{Mt}^{\beta_M} T_{Mt}^{\gamma_M}, && (5)\\
&\quad \alpha_M, \beta_M, \gamma_M \in (0,1), \alpha_M + \beta_M + \gamma_M = 1, g_M \equiv \frac{\dot{M}_t}{M_t},
\end{aligned}
$$

where L_{At} and L_{Mt}, K_{At} and K_{Mt}, T_{At} and T_{Mt} are labor, capital and land employed by the two sectors at time t; α_A and α_M, β_A and β_M, γ_A and γ_M are labor intensities, capital intensities and land intensities in the two sectors; A_t and M_t are agricultural and non-agricultural productivities at time t, g_A and g_M are technology growth rates in the two sectors. Note that α_A and α_M measure the degree of diminishing returns to labor in the two sectors: the greater the values of these parameters are, the weaker diminishing returns to labor are. Population growth and technological progresses are the exogenous driving forces across time in the unified model.

Factor market clearing implies that the sum of factor demands from the two sectors equals aggregate factor supplies at each time t:

$$
\begin{aligned}
L_{At} + L_{Mt} &= L_t, && (6)\\
K_{At} + K_{Mt} &= K_t, && (7)\\
T_{At} + T_{Mt} &= T, && (8)
\end{aligned}
$$

where K_t is the aggregate capital stock at time t, T is the aggregate land supply in the economy, which is fixed over time.

We solve the allocation by considering the problem faced by the social planner. We define Y_t as the unique final output at time t, which is produced competitively using agricultural and non-agricultural goods as intermediate inputs:[3]

$$Y_t = \left(\omega_A Y_{At}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M Y_{Mt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{9}$$

where ω_A and ω_M are the relative weight of the two intermediate inputs respectively, and ε is elasticity of substitution between the two sectoral goods.

We normalize the price of final output as the numéraire in the economy for all time t, that is:

$$1 \equiv (\omega_A^{\varepsilon} P_{At}^{1-\varepsilon} + \omega_M^{\varepsilon} P_{Mt}^{1-\varepsilon})^{\frac{1}{1-\varepsilon}}, \tag{10}$$

where the associated prices of agricultural and non-agricultural goods at time t, P_{At} and P_{Mt}, are respectively:[4]

$$P_{At} = \omega_A \left(\frac{Y_t}{Y_{At}}\right)^{\frac{1}{\varepsilon}}, \tag{11}$$

$$P_{Mt} = \omega_M \left(\frac{Y_t}{Y_{Mt}}\right)^{\frac{1}{\varepsilon}}. \tag{12}$$

Also, (3) can be aggregated to give an economy-wide resource constraint:[5]

$$\dot{K}_t + \delta K_t + L_t \tilde{c}_t = Y_t, \quad \delta \in [0, 1], \tag{13}$$

where δ is the capital depreciation rate. Note that (13) differs from the literature of assuming all investments being only produced by the manufacturing sector, and the output of the other sectors can only be used as consumption. The justification is that such assumption has become increasingly at odd with the data over time, because many innovations nowadays

[3]Final output is an aggregator of agricultural and non-agricultural output that represents the representative household's consumption composite preference. Combining (9) and (13) yields per capita consumption composite at time t:

$$\tilde{c}_t = \left(\omega_A \tilde{y}_{At}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M \tilde{y}_{Mt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} - \frac{\dot{K}_t}{L_t} - \frac{\delta K_t}{L_t},\ \omega_A, \omega_M \in (0,1), \omega_A + \omega_M = 1, \varepsilon \in [0, \infty),$$

where $\tilde{y}_{At} \equiv \frac{Y_{At}}{L_t}$ and $\tilde{y}_{Mt} \equiv \frac{Y_{Mt}}{L_t}$ are per capita purchase of agricultural and non-agricultural goods at time t respectively, ω_A and ω_M are the relative strengths of demand for the two sectoral goods respectively, and ε is the elasticity of substitution between the two sectoral goods. Note that the representative household only values a portion of the CES aggregator of purchased sectoral goods, after investment and depreciation have been deducted from it, as the consumption composite. This is also the utility function implicitly embedded in Acemoglu, and Guerrieri (2008)'s model.

[4]See Appendix 3 for the proof.

[5]See Appendix 3 for the proof.

take place outside the manufacturing sector (Herrendorf, Rogerson, and Valentinyi 2014).

The social planner's problem is hence:

$$\max_{\{\tilde{c}_t, K_t, L_{At}, L_{Mt}, K_{At}, K_{Mt}, T_{At}, T_{Mt}\}_{t=0}^{\infty}} \int_0^{\infty} e^{-(\rho-n)t} \frac{\tilde{c}_t^{1-\theta} - 1}{1-\theta} dt \tag{14}$$

subject to (1),(4)-(13), given $K_0, L_0, T, A_0, M_0 > 0$.

The maximization problem (14) can be divided into two layers: the intertemporal and intratemporal allocation. In the intertemporal level, the social planner chooses paths of per capita consumption composite and aggregate capital stock over the entire time horizon $t \in [0, \infty)$. In the intratemporal level, the social planner divides the aggregate capital stock, total population and land between agricultural and non-agricultural production to maximize final output at each time point t. We solve the problem starting from the lower level first, that is, the intratemporal level, and then move on to the higher intertemporal level.

3.2. Intratemporal level: Factor allocation across sectors

In the intratemporal level, at each time point t, the social planner decides factor allocation between agricultural and non-agricultural sectors to maximize the value of final output, which will allow him/her to choose among the largest possible choice set (13) in solving the intertemporal consumption-saving problem in the next subsection:

$$\max_{L_{At}, L_{Mt}, K_{At}, K_{Mt}, T_{At}, T_{Mt}} Y_t \text{ subject to (4)-(12), given } K_t, L_t, T. \tag{15}$$

This results in wages w_t, capital rentals R_t and land rentals Ω_t being equalized across the agricultural and non-agricultural sectors:

$$w_t = \omega_A \alpha_A \left(\frac{Y_t}{Y_{At}}\right)^{\frac{1}{\varepsilon}} \frac{Y_{At}}{L_{At}} = \omega_M \alpha_M \left(\frac{Y_t}{Y_{Mt}}\right)^{\frac{1}{\varepsilon}} \frac{Y_{Mt}}{L_{Mt}}, \tag{16}$$

$$R_t = \omega_A \beta_A \left(\frac{Y_t}{Y_{At}}\right)^{\frac{1}{\varepsilon}} \frac{Y_{At}}{K_{At}} = \omega_M \beta_M \left(\frac{Y_t}{Y_{Mt}}\right)^{\frac{1}{\varepsilon}} \frac{Y_{Mt}}{K_{Mt}}, \tag{17}$$

$$\Omega_t = \omega_A \gamma_A \left(\frac{Y_t}{Y_{At}}\right)^{\frac{1}{\varepsilon}} \frac{Y_{At}}{T_{At}} = \omega_M \gamma_M \left(\frac{Y_t}{Y_{Mt}}\right)^{\frac{1}{\varepsilon}} \frac{Y_{Mt}}{T_{Mt}}. \tag{18}$$

Defining the non-agricultural labor, capital and land shares as $l_{Mt} \equiv \frac{L_{Mt}}{L_t}$, $k_{Mt} \equiv \frac{K_{Mt}}{K_t}$ and $\tau_{Mt} \equiv \frac{T_{Mt}}{T}$ respectively, (16)-(18) can be rewritten as:

$$l_{Mt} = \left[1 + \frac{\omega_A \alpha_A}{\omega_M \alpha_M} \left(\frac{Y_{Mt}}{Y_{At}}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right]^{-1}, \tag{19}$$

$$k_{Mt} = \left[1 + \frac{\alpha_M}{\alpha_A} \frac{\beta_A}{\beta_M} \left(\frac{1 - l_{Mt}}{l_{Mt}}\right)\right]^{-1}, \tag{20}$$

$$\tau_{Mt} = \left[1 + \frac{\alpha_M}{\alpha_A} \frac{\gamma_A}{\gamma_M} \left(\frac{1 - l_{Mt}}{l_{Mt}}\right)\right]^{-1}. \tag{21}$$

Note that the agricultural labor, capital and land shares are $l_{At} = (1-l_{Mt})$, $k_{At} = (1 - k_{Mt})$ and $\tau_{At} = (1 - \tau_{Mt})$ respectively. Equations (19)-(21) characterize the intratemporal equilibrium conditions.

Manipulating (19)-(21) and we obtain the following four propositions, which show how the sectoral shares l_{Mt}, k_{Mt} and τ_{Mt} respond to population growth, technological progresses and capital deepening. We will focus on the $\varepsilon < 1$ case.[6]

PROPOSITION 1 (Population growth effect). *In equilibrium,*

$$\frac{d \ln l_{Mt}}{d \ln L_t} = \tag{22}$$

$$-\frac{(1-\varepsilon)(\alpha_M - \alpha_A)(1 - l_{Mt})}{\varepsilon + (1-\varepsilon)[\alpha_M(1 - l_{Mt}) + \alpha_A l_{Mt} + \beta_M(1 - k_{Mt}) + \beta_A k_{Mt} + \gamma_M(1 - \tau_{Mt}) + \gamma_A \tau_{Mt}]} \begin{matrix} < 0 \text{ if } \varepsilon < 1 \text{ and } \alpha_M > \alpha_A \\ > 0 \text{ if } \varepsilon < 1 \text{ and } \alpha_M < \alpha_A \end{matrix}$$

$$\frac{d \ln k_{Mt}}{d \ln L_t} = \frac{1 - k_{Mt}}{1 - l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln L_t}, \tag{23}$$

$$\frac{d \ln \tau_{Mt}}{d \ln L_t} = \frac{1 - \tau_{Mt}}{1 - l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln L_t}. \tag{24}$$

Proof. See Appendix 1. ▌

Equations (22)-(24) illustrates the population growth effect on structural transformation. From (22), when $\varepsilon < 1$, population growth pushes labor towards the sector characterized by stronger diminishing returns to labor. The mechanism works through relative price effect on sectoral goods that dominates the relative marginal product effect. Combine (11), (12), take

[6]Using the United States data from 1870-2000, Buera, and Kaboski (2009) calibrated the elasticity of substitution across sectoral goods, ε, to be 0.5.

log and differentiate to get the relative price effect:

$$\left.\frac{\partial \ln\left(\frac{P_{Mt}}{P_{At}}\right)}{\partial \ln L_t}\right|_{\text{constant } l_{Mt},k_{Mt},\tau_{Mt}} = \frac{1}{\varepsilon}(\alpha_A - \alpha_M) \begin{matrix} <0 \text{ if } \alpha_M > \alpha_A \\ >0 \text{ if } \alpha_M < \alpha_A \end{matrix}. \quad (25)$$

Holding factor shares allocated to the two sectors constant, population growth will lead to a relative price drop in the sector characterized by weaker diminishing returns to labor. On the other hand, combining (4), (5), taking log and differentiating gives the relative marginal product effect:

$$\left.\frac{\partial \ln\left(\frac{MPL_{Mt}}{MPL_{At}}\right)}{\partial \ln L_t}\right|_{\text{constant } l_{Mt},k_{Mt},\tau_{Mt}} = (\alpha_M - \alpha_A) \begin{matrix} >0 \text{ if } \alpha_M > \alpha_A \\ <0 \text{ if } \alpha_M < \alpha_A \end{matrix}, \quad (26)$$

where MPL_{At} and MPL_{Mt} are marginal products of labor in the two sectors. Marginal product of labor will rise relatively in the weaker diminishing returns sector.

From (25)-(26), if $\varepsilon < 1$, when population increases, the aforementioned relative price drop in the weaker diminishing returns sector will be proportionately more than the rise in relative marginal product of labor in the same sector. Since wage equals sectoral price times marginal product of labor, wage will fall relatively in the weaker diminishing returns sector. This will induce labor to move out of the weaker diminishing returns sector, until the wage parity condition (16) is restored.[7] Intuitively, we can also understand the population growth effect as follows: when the two sectoral goods are consumption complements, households do not want to consume too few of either one of them. When population grows, if sectoral labor shares stay constant, sectoral output grows slower in the sector with stronger diminishing returns to labor. Hence labor will shift to this sector to maximize the value of per capita consumption composite.

The crux importance of $\varepsilon < 1$ is making sure that the relative price effect (equation (25)) dominates over the relative marginal product effect (equation (26)). This assumption has been explicitly stated in Ngai, and Pissarides (2007, 2008), Acemoglu, and Guerrieri (2008), Buera, and Kaboski (2009)'s papers, allowing sectors with slower productivity growth and lower capital intensity to draw in production inputs throughout economic development. On the other hand, if ε is sufficiently large, the relative marginal product effect would outweigh the relative price effect, reversing the directions of sectoral shifts in propositions 1-4. Hansen, and Prescott (2002), Doepke (2004) and Lagerlöf (2010) implicitly assumed perfect consumption substitutability between two sectoral goods ($\varepsilon \to \infty$). Given the

[7]Note (16) can be rewritten as $P_{At}MPL_{At} = P_{Mt}MPL_{Mt}$.

parameter assumptions in their papers, sectors with faster technological progress will attract production factors throughout development process.

Since labor, capital and land are complementary inputs during production of sectoral goods, capital and land use also shift in the same direction as labor.

PROPOSITION 2 (Agricultural technology growth effect). *In equilibrium,*

$$\frac{d\ln l_{Mt}}{d\ln A_t} = \frac{(1-\varepsilon)(1-l_{Mt})}{\varepsilon + (1-\varepsilon)[\alpha_M(1-l_{Mt}) + \alpha_A l_{Mt} + \beta_M(1-k_{Mt}) + \beta_A k_{Mt} + \gamma_M(1-\tau_{Mt}) + \gamma_A \tau_{Mt}]} > 0 \text{ if } \varepsilon < 1, \tag{27}$$

$$\frac{d\ln k_{Mt}}{d\ln A_t} = \frac{1-k_{Mt}}{1-l_{Mt}} \cdot \frac{d\ln l_{Mt}}{d\ln A_t}, \tag{28}$$

$$\frac{d\ln \tau_{Mt}}{d\ln A_t} = \frac{1-\tau_{Mt}}{1-l_{Mt}} \cdot \frac{d\ln l_{Mt}}{d\ln A_t}. \tag{29}$$

Proof. See Appendix 1. ▌

PROPOSITION 3 (Non-agricultural technology growth effect). *In equilibrium,*

$$\frac{d\ln l_{Mt}}{d\ln M_t} = -\frac{(1-\varepsilon)(1-l_{Mt})}{\varepsilon + (1-\varepsilon)[\alpha_M(1-l_{Mt}) + \alpha_A l_{Mt} + \beta_M(1-k_{Mt}) + \beta_A k_{Mt} + \gamma_M(1-\tau_{Mt}) + \gamma_A \tau_{Mt}]} < 0 \text{ if } \varepsilon < 1, \tag{30}$$

$$\frac{d\ln k_{Mt}}{d\ln M_t} = \frac{1-k_{Mt}}{1-l_{Mt}} \cdot \frac{d\ln l_{Mt}}{d\ln M_t}, \tag{31}$$

$$\frac{d\ln \tau_{Mt}}{d\ln M_t} = \frac{1-\tau_{Mt}}{1-l_{Mt}} \cdot \frac{d\ln l_{Mt}}{d\ln M_t}. \tag{32}$$

Proof. See Appendix 1. ▌

The mechanism for propositions 2 and 3 goes as follows. Ceteris paribus, if $\varepsilon < 1$, technological progress in one sector induces a more than proportionate relative price drop (compared to the relative marginal product of labor rise) in the same sector. Hence labor shifts out this sector to preserve the wage parity condition (16). Capital and land use shift in the same direction due to their complementarity during sectoral production. These two propositions correspond to "Baumol's cost disease" being highlighted in Ngai, and Pissarides (2007)'s paper: production inputs move in the direction of the relatively technological stagnating sector.

Proposition 4 (Capital deepening effect). *In equilibrium,*

$$\frac{d\ln l_{Mt}}{d\ln K_t} = -\frac{(1-\varepsilon)(\beta_M-\beta_A)(1-l_{Mt})}{\varepsilon+(1-\varepsilon)[\alpha_M(1-l_{Mt})+\alpha_A l_{Mt}+\beta_M(1-k_{Mt})+\beta_A k_{Mt}+\gamma_M(1-\tau_{Mt})+\gamma_A\tau_{Mt}]} \begin{matrix} <0 \textit{ if } \varepsilon<1 \textit{ and } \beta_M>\beta_A \\ >0 \textit{ if } \varepsilon<1 \textit{ and } \beta_M<\beta_A \end{matrix}, \quad (33)$$

$$\frac{d\ln k_{Mt}}{d\ln K_t} = \frac{1-k_{Mt}}{1-l_{Mt}}\cdot\frac{d\ln l_{Mt}}{d\ln K_t}, \quad (34)$$

$$\frac{d\ln \tau_{Mt}}{d\ln K_t} = \frac{1-\tau_{Mt}}{1-l_{Mt}}\cdot\frac{d\ln l_{Mt}}{d\ln K_t}. \quad (35)$$

Proof. See Appendix 1. ▌

The mechanism for proposition 4 is similar to those in propositions 1-3. Ceteris paribus, if $\varepsilon < 1$, capital deepening induces a more than proportionate relative price drop (compared to the relative marginal product of capital rise) in the sector with higher capital intensity. Hence capital shifts out this sector to retain the capital rental parity condition (17). Labor and land use also move in the same direction. This is the channel highlighted by Acemoglu, and Guerrieri (2008): capital deepening leads to factor reallocation towards the sector with lower capital intensity.

To summarize, given $\varepsilon < 1$, the above four mechanisms all work through the relative price effect that dominates over the relative marginal product effect. Population growth effect pushes production factors towards the sector with stronger diminishing returns to labor.[8] Technology growth effects push factors towards the sector experiencing slower technological progress. Capital deepening effect pushes factors towards the sector with lower capital intensity.[9]

3.3. Intertemporal level: Consumption-saving across time

In the intertemporal level, at each time point t, the social planner solves the consumption-saving problem to maximize the objective function:

$$\max_{\{\tilde{c}_t, K_t\}_{t=0}^{\infty}} \int_0^{\infty} e^{-(\rho-n)t}\left(\frac{\tilde{c}_t^{1-\theta}-1}{1-\theta}\right)dt, \text{subject to} \quad (36)$$

[8] Note that population growth effect depends on the difference between degrees of diminishing returns to labor in the two sectors (($\alpha_M - \alpha_A$) in (22)), but not the difference between land intensities between the two sectors ($\gamma_M - \gamma_A$). So a statement like "population growth effect pushes production factors towards the sector with higher land intensity" is not precise, and sometimes incorrect.

[9] We might also consider how an exogenous increase in land supply could contribute to a "land expansion effect" on structural transformation. See Appendix 2 for details.

$$\dot{K}_t = \Phi(K_t, t) - \delta K_t - e^{nt} L_0 \tilde{c}_t, \tag{37}$$

where $\Phi(K_t, t)$ is the maximized value of current output at time t (equation (15)), which is a function of the capital stock at time t:

$$\Phi(K_t, t) \equiv \max_{L_{At}, L_{Mt}, K_{At}, K_{Mt}, T_{At}, T_{Mt}} Y_t, \text{ given } K_t > 0.$$

Note that $\Phi(K_t, t)$ contains trending variables such as L_t and M_t (or A_t), and sectoral shares l_{Mt}, k_{Mt} and τ_{Mt} which evolve over time.[10]

Maximizing (36) subject to (37) is a standard optimal control problem. It yields the consumption Euler equation:

$$\frac{\dot{\tilde{c}}_t}{\tilde{c}_t} = \frac{1}{\theta}[\Phi_K - \delta - \rho], \tag{38}$$

where Φ_K is the marginal product of capital of the maximized production function, which equals the capital rental R_t in the economy. Equations (38) and (37) characterize how per capita consumption composite and aggregate capital stock evolve over time.

To characterize the equilibrium dynamics of the system, we need to impose certain assumptions, appropriately normalize per capita consumption composite and aggregate capital stock, and include sectoral share evolution equations.[11] For the first purpose, we assume that:

(A1) $\varepsilon < 1$,

(A2) $\beta_M > \beta_A$,

(A3) $g_A > \left(\frac{1-\beta_A}{1-\beta_M}\right) g_M + \left[\alpha_M - \alpha_A + \frac{\alpha_M(\beta_M - \beta_A)}{1-\beta_M}\right] n$.

Assumption (A1) states that agricultural and non-agricultural goods are consumption complements. Assumption (A2) states that the non-agricultural sector is the capital-intensive sector in the economy. We denote $\left(\frac{1-\beta_A}{1-\beta_M}\right) g_M$ as the augmented non-agricultural technology growth rate, and $\left[\alpha_M - \alpha_A + \frac{\alpha_M(\beta_M - \beta_A)}{1-\beta_M}\right] n$ as the augmented population growth rate. Assumption (A3) states that the agricultural technology growth rate is greater than the sum of augmented non-agricultural technology growth rate and augmented population growth rate (we will explain this assumption in more detail in proposition 6). These three assumptions assure that the non-agricultural sector is the asymptotically dominant sector.[12]

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[10] See equation (A.6) in Appendix 1 for the reduced-from expression of $\Phi(K_t, t)$.

[11] Mathematically, we want to remove the trending terms in (37)-(38) and include a sufficient number of equations to capture the evolution of per capita consumption composite, aggregate capital stock and sectoral shares in an autonomous system of differential equations.

[12] We adopt Acemoglu, and Guerrieri (2008, 479)'s notation that "[t]he asymptotically dominant sector is the sector that determines the long-run growth rate of the economy."

For the second purpose, we normalize per capita consumption composite and aggregate capital stock by population and productivity of the asymptotically dominant sector:

$$c_t \equiv \frac{\tilde{c}_t L_t^{\frac{1-\alpha_M-\beta_M}{1-\beta_M}}}{M_t^{\frac{1}{1-\beta_M}}}, \tag{39}$$

$$\chi_t \equiv \frac{K_t^{\frac{1-\beta_M}{\alpha_M}}}{L_t M_t^{\frac{1}{\alpha_M}}}. \tag{40}$$

With these two normalized variables, given the initial conditions χ_0 and k_{M0}, we can characterize the equilibrium dynamics of the economy by an autonomous system of three differential equations in c_t, χ_t and k_{Mt}, as stated in proposition 5.

Proposition 5 (Equilibrium dynamics). *Suppose (A1)-(A3) hold. The equilibrium dynamics of the economy is characterized by the following three differential equations:*

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\theta}\left(\omega_M \beta_M \eta_t^{\frac{1}{\varepsilon}} \chi_t^{-\alpha_M} T^{\gamma_M} l_{Mt}^{\alpha_M} k_{Mt}^{\beta_M - 1} \tau_{Mt}^{\gamma_M} - \delta - \rho\right) - \frac{g_M}{1-\beta_M} + \left(\frac{1-\alpha_M-\beta_M}{1-\beta_M}\right) n, \tag{41}$$

$$\frac{\dot{\chi}_t}{\chi_t} = \frac{1-\beta_M}{\alpha_M}\left(\eta_t \chi_t^{-\alpha_M} T^{\gamma_M} l_{Mt}^{\alpha_M} k_{Mt}^{\beta_M} \tau_{Mt}^{\gamma_M} - \chi_t^{\frac{\alpha_M}{1-\beta_M}} \cdot c_t - \delta\right) - n - \frac{g_M}{\alpha_M}, \tag{42}$$

$$\frac{\dot{k}_{Mt}}{k_{Mt}} = \frac{(1-k_{Mt})\left\{(g_M - g_A) + (\alpha_M - \alpha_A)n + (\beta_M - \beta_A)\left[\frac{\alpha_M}{1-\beta_M} \cdot \frac{\dot{\chi}_t}{\chi_t} + \frac{\alpha_M}{1-\beta_M} n + \frac{g_M}{1-\beta_M}\right]\right\}}{\left(\frac{\varepsilon}{\varepsilon-1}\right) - [(\beta_M - \beta_A)(1-k_{Mt}) + (\alpha_M - \alpha_A)(1-l_{Mt}) + (\gamma_M - \gamma_A)(1-\tau_{Mt}) + \alpha_A + \beta_A + \gamma_A]}, \tag{43}$$

where

$$\eta_t \equiv \omega_M^{\frac{\varepsilon}{\varepsilon-1}} \left[1 + \left(\frac{\beta_M}{\beta_A}\right)\left(\frac{1-k_{Mt}}{k_{Mt}}\right)\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{44}$$

given $\chi_0, k_{M0} > 0$, and the transversality condition is satisfied:

$$\lim_{t\to\infty} \exp\left(\left\{-\rho + \left[\frac{\alpha_M + (1-\alpha_M-\beta_M)\theta}{1-\beta_M}\right] n + \left(\frac{1-\theta}{1-\beta_M}\right) g_M\right\} t\right) c_t^{-\theta} \chi_t^{\frac{\alpha_M}{1-\beta_M}} = 0. \tag{45}$$

Proof. See Appendix 1. ▌

The dynamic system (41)-(43) in proposition 5 is a three-dimensional generalization of the per capita consumption-effective capital-labor ratio dynamic system in Ramsey (1928)-Cass (1965)-Koopmans (1965) model, where we add in features of sectoral production and land as a fixed production factor.[13] Note that l_{Mt} and τ_{Mt} in (41)-(43) are functions of k_{Mt} at each time t (see intratemporal equilibrium conditions (20)-(21)). They evolve according to:

$$\frac{\dot{l}_{Mt}}{l_{Mt}} = \left(\frac{1-l_{Mt}}{1-k_{Mt}}\right)\frac{\dot{k}_{Mt}}{k_{Mt}}, \tag{46}$$

$$\frac{\dot{\tau}_{Mt}}{\tau_{Mt}} = \left(\frac{1-\tau_{Mt}}{1-k_{Mt}}\right)\frac{\dot{k}_{Mt}}{k_{Mt}}. \tag{47}$$

We give the interpretations of the above equations: (41) and (42) are the consumption Euler equation and capital accumulation equation transformed to sort out the trending population and productivity terms; (43), (46) and (47) come from taking log and differentiating the intratemporal equilibrium conditions (19)-(21), and they show how the sectoral shares evolve over time.[14] We impose the following parameter restriction to guarantee the transversality condition (45):

(A4) $\rho - \left[\frac{\alpha_M+(1-\alpha_M-\beta_M)\theta}{1-\beta_M}\right] n > \left(\frac{1-\theta}{1-\beta_M}\right) g_M$.

3.4. Constant growth path (CGP)

We focus on one particular equilibrium path characterized by proposition 5: the constant growth path (CGP), which is defined as a path featured with constant normalized per capita consumption composite growth rate. Proposition 6 shows the closed-form solution of sectoral share evolution equations in CGP.

Proposition 6 (Structural transformation in CGP). *Suppose (A1)-(A4) hold. In a constant growth path, sectoral shares evolve according to:*

$$\frac{\dot{k}_{Mt}}{k_{Mt}} = G(k_{Mt})\left\{g_A - \left(\frac{1-\beta_A}{1-\beta_M}\right)g_M - \left[\alpha_M - \alpha_A + \frac{\alpha_M(\beta_M-\beta_A)}{1-\beta_M}\right]n\right\}, \tag{48}$$

where $G(k_{Mt}) > 0$ is a function of k_{Mt} and is unrelated to g_A, g_M and n; and (46)-(47), given $k_{M0} > 0$.

As $t \to \infty, k_{Mt} \to k_M^* = 1$, $l_{Mt} \to l_M^* = 1$ and $\tau_{Mt} \to \tau_M^* = 1$.

---

[13] Setting $\gamma_M = 0, k_{Mt} = l_{Mt} = \tau_{Mt} = 1$ reduces (39)-(45) to Ramsey (1928)-Cass (1965)-Koopmans (1965) model's two-dimensional dynamic equation system. Zou et al. (2010) incorporated health and tax in the Ramsey model to examine the effects of consumption tax and income tax.

[14] For relative sectoral output evolution, rewrite (19) as $(l_{Mt}^{-1} - 1) = \frac{\omega_A\alpha_A}{\omega_M\alpha_M}\left(\frac{Y_{Mt}}{Y_{At}}\right)^{\frac{1-\varepsilon}{\varepsilon}}$. Take log and differentiate with respect to time to get $\frac{\dot{l}_{Mt}}{l_{Mt}} = (1-l_{Mt})\left(\frac{\varepsilon-1}{\varepsilon}\right)\left(\frac{\dot{Y}_{Mt}}{Y_{Mt}} - \frac{\dot{Y}_{At}}{Y_{At}}\right)$. Given $\varepsilon < 1$, $\frac{\dot{Y}_{Mt}}{Y_{Mt}} - \frac{\dot{Y}_{At}}{Y_{At}}$ and $\frac{\dot{l}_{Mt}}{l_{Mt}}$ follow different signs.

Proof. See Appendix 1. ▌

Proposition 6 highlights the result of interplay among population growth effect, technology growth effects and capital deepening effect in fostering structural transformation in CGP. Equation (48) explains why assumption (A3) guarantees that the non-agricultural sector is the asymptotically dominant sector: assumption (A3) guarantees a strong enough agricultural technology growth effect which overrides non-agricultural technology growth effect, population growth effect and capital deepening effect to ensure factor reallocations towards the non-agricultural sector. The technology growth effects from the two sectors are represented by the g_A and g_M terms. The population growth effect is represented by the $[\alpha_M - \alpha_A]n$ term. Capital accumulation is endogenous in the model and the capital deepening effect is captured by the "wedge" coefficients $\frac{1-\beta_A}{1-\beta_M}$ and $\frac{\alpha_M(\beta_M-\beta_A)}{1-\beta_M}$, which respectively augment the non-agricultural technology growth effect and population growth effect terms relative to the agricultural technology growth effect term. We reinforce our result in the following corollary.

Corollary 1 (Escape from land). *Suppose there are two sectors producing consumption complements in the economy: one is land-intensive and the other is capital-intensive. Production factors shift from the land-intensive sector to the capital-intensive sector if the technology growth rate in the land-intensive sector is greater than the sum of augmented technology growth rate in the capital-intensive sector and augmented population growth rate.*[15]

Corollary 1 highlights structural transformation in an economy that features population growth, technological progress and capital accumulation. Given agriculture is the land-intensive sector, the key to move production factors out of agriculture is fast enough agricultural productivity growth that outweighs the combination of non-agricultural technology growth effect, population growth effect and capital deepening effect.

Next, we investigate a "razor's edge" condition, which illustrates why the literature usually neglects the effect of population growth on structural transformation. From (48), unless the following "razor's edge" condition

[15] Due to model symmetry, suppose instead the agricultural sector is capital-intensive ($\beta_M < \beta_A$) and the non-agricultural sector is land-intensive ($\gamma_M > \gamma_A$). Given that the two sectors produce consumption complements, the condition to ensure "escape from land" is $g_M > \left(\frac{1-\beta_M}{1-\beta_A}\right) g_A + \left[\alpha_A - \alpha_M + \frac{\alpha_A(\beta_A-\beta_M)}{1-\beta_A}\right] n$.

holds:

$$\alpha_M - \alpha_A + \frac{\alpha_M(\beta_M - \beta_A)}{1 - \beta_M} = 0, \qquad (49)$$

otherwise a change in population growth rate will affect the direction and pace of structural transformation in CGP. The "razor's edge" condition (49) can be reduced to either $\gamma_A = \gamma_M = 0$ or $\frac{\alpha_A}{\alpha_M} = \frac{\gamma_A}{\gamma_M}$. The former means land intensities equal zero in the two sectors. The latter means the ratio of labor intensity equals the ratio of land intensity in the two sectors.

Acemoglu, and Guerrieri (2008)'s model is a special case of ours, where the "razor's edge" condition (49) applies. In their paper, they did not include land as an input in sectoral production. This is equivalent to setting $\gamma_A = \gamma_M = 0$, $\alpha_A = 1 - \beta_A$ and $\alpha_M = 1 - \beta_M$ in our model. Equation (48) is reduced to $\frac{\dot{k}_{Mt}}{k_{Mt}} = G(k_{Mt}) \left\{ g_A - \left(\frac{\alpha_A}{\alpha_M} \right) g_M \right\}$. It happens that the population growth effect is cancelled out by some part of the capital deepening effect, and population growth rate does not show up in the sectoral share evolution equation. Also, as a special case of our (A3), they assume $g_A - \left(\frac{\alpha_A}{\alpha_M} \right) g_M > 0$ to make sure that the non-agricultural sector is the asymptotically dominant sector.[16]

Our model can also be collapsed to the two-sector version of Ngai, and Pissarides (2007)'s one, which again fulfils the "razor's edge" condition. In their paper, land is not an input to sectoral production. They also assumed same capital intensity for all sectoral production functions. This makes $\gamma_A = \gamma_M = 0$, $\alpha_A = 1 - \beta_A$, $\alpha_M = 1 - \beta_M$, $\alpha_A = \alpha_M$ and $\beta_A = \beta_M$ in our model. Equation (48) is reduced to $\frac{\dot{k}_{Mt}}{k_{Mt}} = G(k_{Mt})\{g_A - g_M\}$. There was neither population growth effect nor capital deepening effect in the reduced model. By assuming $g_A - g_M > 0$, we get their result that the sector with the slowest technology growth will continuously draw in employment in the aggregate balanced growth path.[17]

From (48), we can also study the effects of changes in technology growth rates and population growth rate on the pace of agricultural-to-non-agricultural transformation, given that (A1)-(A4) hold. Straightforward differentiation

[16]See Proposition 3 (sectoral share evolution equation) and Assumption 2(i) in Acemoglu, and Guerrieri (2008)'s paper.

[17]See Proposition 2 in Ngai, and Pissarides (2007)'s paper. Note that Ngai, and Pissarides (2007) have examined the inclusion of a fixed production factor in at least one production sector in their appendix.

yields:

$$\frac{d\left(\frac{\dot{k}_{Mt}}{k_{Mt}}\right)}{dg_A} = G(k_{Mt}) > 0, \tag{50}$$

$$\frac{d\left(\frac{\dot{k}_{Mt}}{k_{Mt}}\right)}{dg_M} = -G(k_{Mt})\left(\frac{1-\beta_A}{1-\beta_M}\right) < 0, \tag{51}$$

$$\frac{d\left(\frac{\dot{k}_{Mt}}{k_{Mt}}\right)}{dn} = -G(k_{Mt})\left[\alpha_M - \alpha_A + \frac{\alpha_M(\beta_M-\beta_A)}{1-\beta_M}\right] \gtrless 0. \tag{52}$$

Speeding up agricultural technological progress accelerates sectoral shift, while boosting non-agricultural technology growth rate decelerates it. Increasing population growth rate has a theoretically ambiguous effect on the pace of sectoral shift, and we resolve the sign by relying on the estimates of sectoral production function parameters. We consider the modern agricultural and Solow production technologies calibrated by Yang, and Zhu (2013) and Hansen, and Prescott (2002):[18]

$$Y_{At} = A_t L_{At}^{0.36} K_{At}^{0.4} T_{At}^{0.24}, \text{ that is, } \alpha_A = 0.36, \beta_A = 0.4, \gamma_A = 0.24. \tag{53}$$

$$Y_{Mt} = M_t L_{Mt}^{0.58} K_{Mt}^{0.41} T_{Mt}^{0.01}, \text{ that is, } \alpha_M = 0.58, \beta_M = 0.41, \gamma_M = 0.01. \tag{54}$$

Plug the coefficients from (53) and (54) into (52) to get $\frac{\partial\left(\frac{\dot{k}_{Mt}}{k_{Mt}}\right)}{\partial n} = -0.23 \cdot G(k_{Mt}) < 0$. An increase in population growth rate will slow down structural transformation.[19]

3.5. Asymptotic growth path

Lastly, we study the properties of the economy in its asymptotic growth path. The economy converges to a unique, saddle-path stable CGP with non-balanced sectoral growth, which is summarized in proposition 7.

PROPOSITION 7 (Asymptotic growth path). *Suppose (A1)-(A4) hold, denote* $a^* \equiv \lim_{t\to\infty} a_t$, $g_a^* \equiv \lim_{t\to\infty}\left(\frac{\dot{a}_t}{a_t}\right)$, $y_t \equiv \frac{Y_t}{L_t}$ *as per capita final*

[18]Hansen, and Prescott (2002) stated the non-agricultural production function in the form of $Y_{Mt} = M_t L_{Mt}^{0.6} K_{Mt}^{0.4}$. We slightly modify the production function to $Y_{Mt} = M_t L_{Mt}^{0.58} K_{Mt}^{0.41} T_{Mt}^{0.01}$ to include land as a factor of production and ensure Assumption (A2) holds.

[19]In other words, our model implies that an increase in population growth rate would slow down economic development in terms of counteracting agricultural technology growth effect, retaining production factors in agriculture. This is in analogy to unified growth theories' mechanism in which an increase in population growth rate would neutralize the effect of technological progress, hence retain per capita income in a Malthusian Trap (Galor, and Weil 2000, Galor, and Moav 2002).

output or per capita income in the economy, then there exists a unique, saddle-path stable asymptotic growth path such that:

$$k_M^* = l_M^* = \tau_M^* = 1, \quad \eta^* = \omega_M^{\frac{\varepsilon}{\varepsilon-1}}.$$

$$\chi^* = \left[\frac{\frac{\theta g_M - (1-\alpha_M-\beta_M)\theta n}{1-\beta_M} + \delta + \rho}{\beta_M \omega_M^{\frac{\varepsilon}{\varepsilon-1}} T^{\gamma M}}\right]^{\frac{1}{\alpha_M}},$$

$$c^* = \left[\eta^*(\chi^*)^{-\alpha_M} T^{\gamma_M} - \delta - \frac{\alpha_M}{1-\beta_M}\left(n + \frac{g_M}{\alpha_M}\right)\right](\chi^*)^{\frac{\alpha_M}{1-\beta_M}}.$$

For the aggregate variables,

$$\begin{aligned}
g_Y^* =& \frac{\alpha_M}{1-\beta_M} n + \frac{g_M}{1-\beta_M}, \\
g_y^* =& \frac{g_M}{1-\beta_M} - \left(\frac{1-\alpha_M-\beta_M}{1-\beta_M}\right) n, \\
g_{\tilde{c}}^* =& \frac{g_M}{1-\beta_M} - \left(\frac{1-\alpha_M-\beta_M}{1-\beta_M}\right) n, \\
g_L^* =& n, \\
g_K^* =& \frac{\alpha_M}{1-\beta_M} n + \frac{g_M}{1-\beta_M}, \\
g_T^* =& 0.
\end{aligned}$$

For the agricultural sector,

$$\begin{aligned}
g_{Y_A}^* =& g_A + \alpha_A n + \beta_A \left(\frac{\alpha_M}{1-\beta_M} n + \frac{g_M}{1-\beta_M}\right), \\
g_{L_A}^* =& \left(1 - \frac{1}{\varepsilon}\right)(g_{Y_A}^* - g_{Y_M}^*) + n, \\
g_{K_A}^* =& \left(1 - \frac{1}{\varepsilon}\right)(g_{Y_A}^* - g_{Y_M}^*) + \frac{\alpha_M}{1-\beta_M} n + \frac{g_M}{1-\beta_M}, \\
g_{T_A}^* =& \left(1 - \frac{1}{\varepsilon}\right)(g_{Y_A}^* - g_{Y_M}^*).
\end{aligned}$$

For the non-agricultural sector,

$$\begin{aligned} g^*_{Y_M} =& \frac{\alpha_M}{1-\beta_M} n + \frac{g_M}{1-\beta_M} < g^*_{Y_A}, \\ g^*_{L_M} =& n > g^*_{L_A}, \\ g^*_{K_M} =& \frac{\alpha_M}{1-\beta_M} n + \frac{g_M}{1-\beta_M} > g^*_{K_A}, \\ g^*_{T_M} =& 0 > g^*_{T_A}. \end{aligned}$$

Proof. See Appendix 1. ▮

There is non-balanced growth in the sense that the non-agricultural sector will tend to draw away all production resources (capital, labor and land) in the economy and become the asymptotically dominant sector. On the other hand, the agricultural output will grow at a faster rate than the non-agricultural output.

Given $\alpha_M + \beta_M \neq 1$, population growth puts a drag $-\left(\frac{1-\alpha_M-\beta_M}{1-\beta_M}\right) n$ on per capita income growth rate in the asymptotic growth path. The higher the population growth rate is, the faster per capita income diminishes. This differs from the literature's prediction of a non-negative effect of population growth rate on per capita income growth rate in the steady states.[20] The drag on per capita income growth rate originates from the presence of land as a fixed factor of sectoral production. Due to diminishing returns to labor, the limitation land puts on per capita income growth becomes more and more severe as population grows over time. The faster population grows, the quicker per capita income deteriorates due to this problem, and the larger is the resulting drag. Without fast enough technological progress in the asymptotically dominant sector, per capita income keeps on shrinking over time, and the economy ultimately ends up with stagnation.[21] To ensure a sustainable per capita income growth, technological progress in

[20] In Solow (1956), Cass (1965) and Koopmans (1965)'s exogenous growth models, diminishing marginal product of capital assures saving in the economy just to replenish capital depreciation and population growth in the steady state. A change in population growth rate has just a level effect but no growth effect on per capita income evolution in the long run. In the 1990s, Jones (1995), Kortum (1997) and Segerstrom (1998) proposed semi-endogenous growth models, which incorporate R&D and assume diminishing returns to R&D. In steady states, these models predict that per capita income (or real wage) growth rate increases linearly with population growth rate. To summarize, the above literature predicts a non-negative effect of population growth rate on per capita income growth rate in steady states. In contrast, our model predicts that population growth rate will have a negative effect on per capita income growth rate even in the asymptotic growth path.

[21] According to Ricardo (1821), in the absence of technological progress, with diminishing returns to land use, population growth will eventually drain up the entire agri-

the asymptotically dominant sector needs to outpace the population growth drag.[22]

4. MODEL CALIBRATION

We apply the unified model to explore whether the equilibrium dynamics implied by our model are broadly consistent with the patterns of the United States data between 1970-2020. We assemble the data of agricultural capital share as U.S. Bureau of Economic Analysis (2023)'s proportion of net stock of private fixed asset allocated to agriculture, forestry, fishing and hunting sectors; agricultural labor share as U.S. Bureau of Economic Analysis (2023)'s proportion of full-time and part-time employees allocated to the same sectors; agricultural land share as The World Bank (2023)'s proportion of land area devoted to agriculture; agricultural output share as U.S. Bureau of Economic Analysis (2023)'s proportion of real gross value added contributed from the farm sector; relative agricultural price as U.S. Bureau of Economic Analysis (2023)'s farm sector price index divided by nonfarm sector price index; per capita income growth as The Federal Reserve Bank of St. Louis (2023)'s annualized real gross domestic product per capita growth rate. The patterns of the data are shown as dots in Figure 1.

In our model, K_{At}/K_t corresponds to agricultural capital share, L_{At}/L_T corresponds to agricultural labor share, T_{At}/T corresponds to agricultural land share, Y_{At}/Y_t corresponds to agricultural output share, P_{At}/P_{Mt} corresponds to relative agricultural price, $(y_{t+1} - y_t)/y_t$ corresponds to per capita income growth.

We first examine whether the United States was characterized by CGP, so that we can apply the model equations (46)-(48). Figure 2 depicts the annual growth rate of normalized real personal consumption expenditures per capita series (by (39); solid line) and its ten-year average series (dashed line) in the United States during 1948-2021. Since the 1970s, the ten-year average series has fluctuated within a narrow range of about 0-2%. Therefore we assume that the United States has been growing along a CGP since the 1970s, and choose 1970 as the starting year for the calibration exercise.

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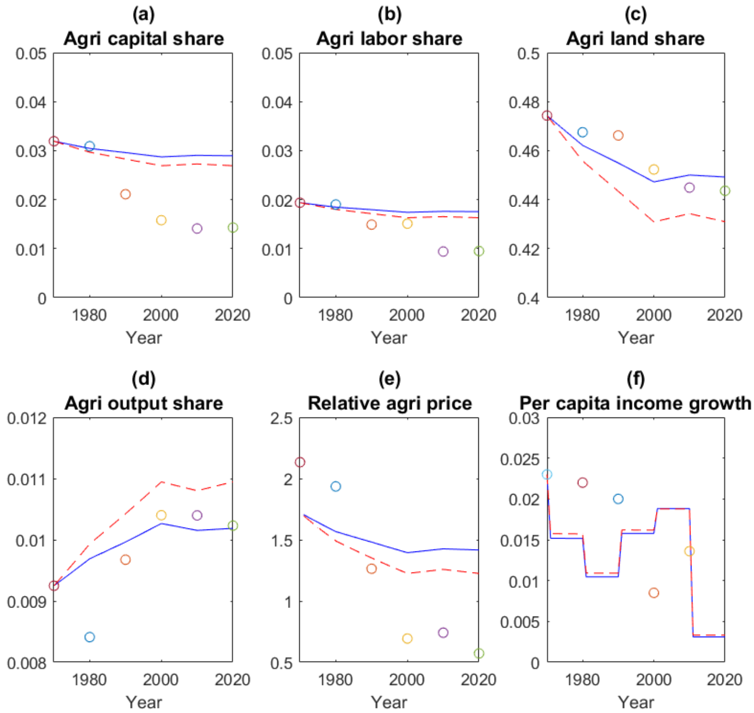
cultural surplus, cutting off the incentive for agricultural capitalists to accumulate fixed capital. The economy ends up with agricultural stagnation. Malthus (1826) pointed out that, as population multiplies geometrically and food arithmetically, population growth will eventually lead to falling wage (and rising food price), pressing the people to the subsistence level.

[22]Mathematically, we require $g_M > (1-\alpha_M-\beta_M)n$ to ensure a sustainable per capita income growth in the asymptotic growth path.

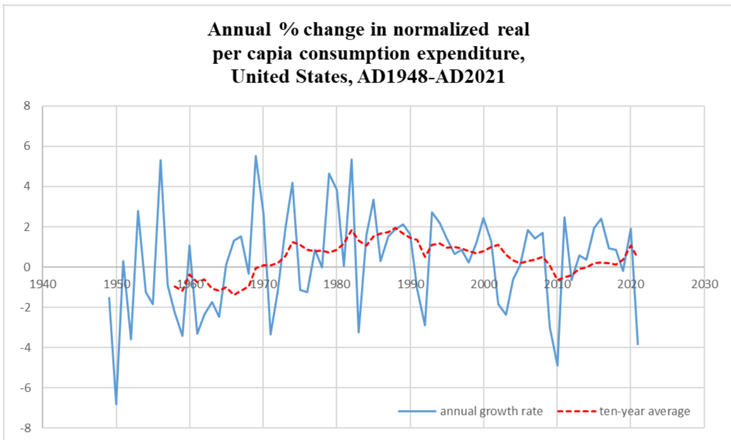
FIG. 1. Behavior of agricultural labor, capital, land and output shares, relative agricultural price and per capita income growth, 1970-2020



Note: Solid (blue) line: benchmark calibration. Dashed (red) lines: the counterfactual economy, $n = 0$ otherwise benchmark parameters from Table 1. Dots: data, see text.

The model economy is characterized by 12 parameters: $\alpha_A, \beta_A, \gamma_A$, α_M, $\beta_M, \gamma_M, \varepsilon, \omega_A, T, g_A, g_M, n$, and initial values of $A_0, M_0, k_{A0}, l_{A0}, \tau_{A0}$. Table 1 shows the benchmark parameters and initial values. We follow Yang, and Zhu (2013) and Hansen, and Prescott (2002) to let the sectoral production functions to take the forms of (53)-(54). We fix $\varepsilon = 0.5$ (Buera, and Kaboski 2009) and $\omega_A = 0.01$ to match agricultural output share in 2020. Aggregate land supply T is normalized to 1. The initial agricultural and non-agricultural productivities are calibrated using the agricultural and non-agricultural production functions (4)-(5). We set initial agricultural capital share as U.S. Bureau of Economic Analysis (2023)'s 1970 estimate of proportion of net stock of private fixed assets held by agricultural, forestry, fishing and hunting sectors, initial agricultural labor share as U.S. Bureau of Economic Analysis (2023)'s 1970 estimate of proportion of full-time and part-time employees in the same sectors, initial agricultural

FIG. 2. Annual percentage change in normalized real per capita consumption expenditure and ten-year average series, United States, 1948-2021



Note: Solid (blue) line: Annual percentage change in normalized real per capita consumption expenditure, the United States, 1948-2021. Dashed (red) line: ten-year average series. Source: The Federal Reserve Bank of St. Louis (2023), real personal consumption expenditures per capita, chained 2012 dollars, quarterly, seasonally adjusted annual rate.

land share as The World Bank (2023)'s 1970 estimate of proportion of land area devoted to agriculture. We set agricultural technology growth rate as the annualized growth rate of farm total factor productivity in each decade over 1970-2019, provided by U.S. Department of Agriculture (2023), and the non-agricultural technology growth rate as the annualized multifactor productivity growth rate for private nonfarm business sector in each decade over 1970-2020, provided by The Federal Reserve Bank of St. Louis (2023). We set the population growth rate as the annualized growth rate of full-time and part-time employees in each decade over 1970-2020, provided by U.S. Bureau of Economic Analysis (2023).

Each model period represents a year. We employ the Euler method to discretize (46)-(48) into difference equations in k_{Mt}, l_{Mt} and τ_{Mt}. Together with $l_{At} = (1-l_{Mt})$, $k_{At} = (1-k_{Mt})$ and $\tau_{At} = (1-\tau_{Mt})$, we have a system of difference equations in six sectoral shares k_{At}, k_{Mt}, l_{At}, l_{Mt}, τ_{At} and τ_{Mt}. Through (40), (A.13)-(A.14) we can obtain the evolution of K_t over time, and with (4)-(5) we can obtain the evolution of sectoral outputs. With (11)-(12) we can obtain the evolution of relative agricultural price.

Figure 1 (solid lines) depicts the benchmark calibration results. Overall our model is capable of generating the main features of structural transformation in the United States during 1970-2020, namely the general fall

TABLE 1.

Benchmark parameter values for the calibrated model

	Interpretation	Value	Comments
Parameters			
α_A	Labor intensity in agricultural sector	0.36	Yang, and Zhu (2013)
β_A	Capital intensity in agricultural sector	0.4	Yang, and Zhu (2013)
γ_A	Land intensity in agricultural sector	0.24	Yang, and Zhu (2013)
α_M	Labor intensity in non-agricultural sector	0.58	Hansen, and Prescott (2002)
β_M	Capital intensity in non-agricultural sector	0.41	Hansen, and Prescott (2002)
γ_M	Land intensity in non-agricultural sector	0.01	Hansen, and Prescott (2002)
ε	Elasticity of substitution	0.5	Buera, and Kaboski (2009)
ω_A	Relative weight of agriculture as intermediate input in final output	0.01	
T	Aggregate land supply	1	
g_A	Agricultural technology growth rate	0.0232 for 1970-1980 0.0158 for 1980-1990 0.0191 for 1990-2000 0.0087 for 2000-2010 0.0045 for 2010-2019	U.S. Department of Agriculture (2023), annualized farm TFP growth rate
g_M	Non-agricultural technology growth rate	0.0089 for 1970-1980 0.0062 for 1980-1990 0.0093 for 1990-2000 0.0111 for 2000-2010 0.0019 for 2010-2020	The Federal Reserve Bank of St. Louis (2023), annualized private nonfarm business sectors multifactor productivity growth rate
n	Population growth rate	0.0212 for 1970-1980 0.0175 for 1980-1990 0.0161 for 1990-2000 -0.00185 for 2000-2010 0.00845 for 2010-2020	U.S. Bureau of Economic Analysis (2023), annualized full-time and part-time employees growth rate
Initial values			
k_{A0}	Initial agricultural capital share	0.031919	U.S. Bureau of Economic Analysis (2023), proportion of net stock of private fixed assets held by agricultural, forestry, fishing and hunting sectors in 1970
l_{A0}	Initial agricultural labor share	0.019373	U.S. Bureau of Economic Analysis (2023), proportion of full-time and part-time employees in agricultural, forestry, fishing and hunting sectors in 1970
t_{A0}	Initial agricultural land share	0.4743	The World Bank (2023), % of land area in agriculture in 1970

in agricultural capital, labor and land shares, the mild rise in agricultural output share and the general fall in relative agricultural price. We deem this as a considerable success of our model. The general fall in agricultural labor share is consistent with Duarte, and Restuccia (2010)'s calibrated path of declining hours devoted to agriculture throughout the development process. Also, our calibration quantitatively matches the fall in agricultural land share in the United States during 1970-2020. To our knowledge, such a calibration exercise to match the evolution of agricultural land share has not been performed in the structural transformation literature.

To gain more insights from our calibration exercise, we can compare the calibration results against four hypotheses implied by our model.

Hypothesis 1: If the condition $g_A > \left(\frac{1-\beta_A}{1-\beta_M}\right) g_M + \left[\alpha_M - \alpha_A + \frac{\alpha_M(\beta_M-\beta_A)}{1-\beta_M}\right] n$ holds, then production factors will shift from the agricultural sector to the non-agricultural sector.

TABLE 2.

Model prediction of sectoral share evolution against data pattern

Period	$g_A - \left(\frac{1-\beta_A}{1-\beta_M}\right) g_M - \left[\alpha_M - \alpha_A + \frac{\alpha_M(\beta_M-\beta_A)}{1-\beta_M}\right] n$	Model prediction	Data
1970-1980	0.0141	$k_{At}, l_{At}, \tau_{At}$ fall	k_{At} falls l_{At} falls τ_{At} falls
1980-1990	0.0095	$k_{At}, l_{At}, \tau_{At}$ fall	k_{At} falls l_{At} falls τ_{At} falls
1990-2000	0.0096	$k_{At}, l_{At}, \tau_{At}$ fall	k_{At} falls l_{At} falls τ_{At} falls
2000-2010	-0.0026	$k_{At}, l_{At}, \tau_{At}$ rise	k_{At} falls l_{At} rises τ_{At} falls
2010-2020	0.0026	$k_{At}, l_{At}, \tau_{At}$ fall	k_{At} falls l_{At} falls τ_{At} falls

Hypothesis 1 is the consequence of Corollary 1: as long as agricultural productivity growth outweighs the combination of non-agricultural technology growth effect, population growth effect and capital deepening effect, production factors will move out of the agricultural sector, implying a fall in agricultural capital, labor and land shares. Table 2 shows that, except for the decade of 2000-2010, the condition $g_A > \left(\frac{1-\beta_A}{1-\beta_M}\right) g_M +$

$\left[\alpha_M - \alpha_A + \frac{\alpha_M(\beta_M-\beta_A)}{1-\beta_M}\right] n$ held for all other decades from 1970-2020. Therefore our model would predict a fall in agricultural capital, labor and land shares in all decades except 2000-2010, which is consistent with the pattern shown in real data. This result is also in accordance with Leukhina & Turnovsky (2016, 213)'s England 1650-1920 study that "total factor productivity growth in the manufacturing sector is expected to work against the observed process of labor reallocation [away from agriculture]. However ... technological progress in agriculture are likely to contribute to this process."

Hypothesis 2: If relative agricultural price is falling, then production factors will shift from the agricultural sector to the non-agricultural sector.

Hypothesis 2 is the consequence of Proportions 1-4: From (25)-(26), we require the relative agricultural price to drop (that is proportionately more than the rise in marginal product in agriculture) and induce production factors to shift out from agriculture. Table 3 shows that, for all decades from 1970-2020 (except the 2000-2010 decade), when we input the estimates of agricultural and non-agricultural productivity growth and population growth, the model would predict the fall in relative agricultural price. Through the aforementioned relative price effect (that dominated the relative marginal product effect) this would induce production factors to shift away from the agriculture. This is consistent with the pattern shown in real data for each decade during 1970-2020. This result stands in contrast to Leukhina & Turnovsky (2016, 205)'s England 1650-1920 study that ""[t]o support the shift in consumption away from farming ... the relative [agricultural] price ... must rise."

TABLE 3.

Model prediction of relative agricultural price evolution against data pattern

Period	Model prediction	Data
1970-1980	P_{At}/P_{Mt} falls	P_{At}/P_{Mt} falls
1980-1990	P_{At}/P_{Mt} falls	P_{At}/P_{Mt} falls
1990-2000	P_{At}/P_{Mt} falls	P_{At}/P_{Mt} falls
2000-2010	P_{At}/P_{Mt} rises	P_{At}/P_{Mt} rises
2010-2020	P_{At}/P_{Mt} falls	P_{At}/P_{Mt} falls

Hypothesis 3: If the condition $g_A > \left(\frac{1-\beta_A}{1-\beta_M}\right) g_M + \left[\alpha_M - \alpha_A + \frac{\alpha_M(\beta_M-\beta_A)}{1-\beta_M}\right] n$ holds, the agricultural output will grow at a faster rate than the non-agricultural output.

Hypothesis 3 is the consequence of Proportion 7: As the condition $g_A > \left(\frac{1-\beta_A}{1-\beta_M}\right) g_M + \left[\alpha_M - \alpha_A + \frac{\alpha_M(\beta_M-\beta_A)}{1-\beta_M}\right] n$ holds, over time the agricultural output will grow at a faster rate than non-agricultural output. The hypothesis is consistent with the general (slow) rise in agricultural output share throughout 1970-2020, as depicted in panel (d) of Figure 1. Such non-balanced growth, where the non-agricultural sector tends to draw away all production resources while agricultural output grows at a relatively faster rate, is similar to Acemoglu, and Guerrieri (2008)'s calibration exercise that shows the less capital-intensive sector tends to draw away production factors while the more capital-intensive sector grows at a relatively faster rate when capital deepening is occurring in the economy.

Hypothesis 4: Population growth slows down the pace of structural transformation.

Hypothesis 4 is the consequence of (52): given parameter estimates in our model, an increase in population growth rate will slow down structural transformation. We follow Leukhina & Turnovsky (2016) to perform a counterfactual experiment to investigate such an effect. The dashed lines in Figure 1 depict the calibration paths by adopting all benchmark parameters and initial values in Table 1, except adjusting population growth rate to $n = 0$ for all time periods. Consistent with the hypothesis, removing population growth speeds up sectoral shift out of the agriculture. Yet this would generate a decline that is too fast in the agricultural land share when compared to the real data (Figure 1 panel (c)). This result stands in contrast to Leukhina & Turnovsky (2016, 216)'s England 1650-1920 study that "population growth was unambiguously a major factor behind labor movement away from the farming sector, especially during the period of 1750-1850". Again, such result is perhaps not surprising for as shown in (52), the effect of population growth on the growth rate of agricultural capital share can be either positive or negative depending on the parameters of the model.

However, the main drawback of our model is that it generates overly sluggish structural transformation for the efflux of capital and labor out of agriculture. The calibrated agricultural capital and labor shares have stayed well above their empirical counterparts since 1990 and 2010 respectively (depicted in panels (a) and (b) in Figure 1). That means, it is not sufficient to focus only on the relative price effects brought about by population growth, technological progress and capital deepening to quantitatively reconcile capital and labor movements out of the agricultural sector in the United States in the later calibration periods. We might need to take the income effect and other supply-side channels or institutions (section 2) into account to quantitatively explain the evolution of these sectoral shares.

To summarize: when we input the estimates of agricultural and non-agricultural productivity growth rates and population growth rate into our unified model, it generates plausible empirical trends, especially reconciling the fall in agricultural land share, the rise in agricultural output share and the fall in relative agricultural price. We deem our model to be a considerable success. Yet, there is still room for improvement by incorporating other causes of structural transformation to quantitively reconcile the movement of agricultural capital and labor shares. Lastly, the conclusion we arrived at regarding the effect of population growth on the pace of sectoral shift remains an interesting topic for future research on structural transformation across other regions and/or time dimensions.

5. CONCLUSION

This paper develops a more complete theory of structural transformation, by solving analytically the closed-form solutions for a structural transformation model with population growth (Leukhina, and Turnovsky 2016), technological progress (Ngai, and Pissarides 2007) and capital deepening (Acemoglu, and Guerrieri 2008). Our work unearths the underlying logics and crucial assumptions on how the relative price effects originating from these factors interact and explain sectoral shifts (propositions 1-6). We also clarify the conditions under which production factors will escape from the land-intensive sector throughout the development process (corollary 1), as well as to sustain long-run growth along an asymptotic growth path (proposition 7).

We then calibrated the model using sectoral data from the United States during 1970-2020. In the calibration exercise, we found empirical validity for the hypotheses implied by the above propositions and corollaries, and reached a conclusion regarding the effect of population growth on the pace of structural transformation that is different from what is available in the current literature.

A unified explanation for structural transformation is a challenging and fascinating topic. Future work on combining the relative price effects with the other mechanisms fostering sectoral shifts to reconcile non-balanced economic growth will be a fruitful area of research. Hopefully our analysis has shed light on building a more complete theory of structural transformation.

APPENDIX: PROOFS FOR THE PROPOSITIONS

Proposition 1

Proof. Use (4) and (5) to obtain $\frac{Y_{Mt}}{Y_{At}} = l_{Mt}^{\alpha_M} l_{At}^{-\alpha_A} k_{Mt}^{\beta_M} k_{At}^{-\beta_A} \tau_{Mt}^{\gamma_M} \tau_{At}^{-\gamma_A} \frac{Mt}{At} L_t^{\alpha_M-\alpha_A} K_t^{\beta_M-\beta_A} T^{\gamma_M-\gamma_A}$.

Plug it to (19) and we get

$$l_{Mt}^{-1}-1 = \frac{\omega_A \alpha_A}{\omega_M \alpha_M} \left(l_{Mt}^{\alpha_M} l_{At}^{-\alpha_A} k_{Mt}^{\beta_M} k_{At}^{-\beta_A} \tau_{Mt}^{\gamma_M} \tau_{At}^{-\gamma_A} \frac{M_t}{A_t} L_t^{\alpha_M-\alpha_A} K_t^{\beta_M-\beta_A} T^{\gamma_M-\gamma_A} \right)^{\frac{1-\varepsilon}{\varepsilon}}. \tag{A.1}$$

Take log and differentiate (A.1) with respect to $\ln L_t$, we get

$$\frac{-1}{1-l_{Mt}}\cdot\frac{d\ln l_{Mt}}{d\ln L_t}=\left(\frac{1-\varepsilon}{\varepsilon}\right)\left[\alpha_M\frac{d\ln l_{Mt}}{d\ln L_t}+\alpha_A\frac{l_{Mt}}{1-l_{Mt}}\cdot\frac{d\ln l_{Mt}}{d\ln L_t}+\beta_M\frac{d\ln k_{Mt}}{d\ln L_t}\right.$$
$$\left.+\beta_A\frac{k_{Mt}}{1-k_{Mt}}\cdot\frac{d\ln k_{Mt}}{d\ln L_t}+\gamma_M\frac{d\ln \tau_{Mt}}{d\ln L_t}+\gamma_A\frac{\tau_{Mt}}{1-\tau_{Mt}}\cdot\frac{d\ln \tau_{Mt}}{d\ln L_t}+(\alpha_M-\alpha_A)\right]. \tag{A.2}$$

Also, take log and differentiate (20) and (21) with respect to $\ln L_t$, we get (23) and (24) respectively.

Plug (23) and (24) into (A.2), we get (22). ∎

Proposition 2

Proof. Take log and differentiate (A.1) with respect to $\ln A_t$, we get

$$\frac{-1}{1-l_{Mt}}\cdot\frac{d\ln l_{Mt}}{d\ln A_t}=\left(\frac{1-\varepsilon}{\varepsilon}\right)\left[\alpha_M\frac{d\ln l_{Mt}}{d\ln A_t}+\alpha_A\frac{l_{Mt}}{1-l_{Mt}}\cdot\frac{d\ln l_{Mt}}{d\ln A_t}+\beta_M\frac{d\ln k_{Mt}}{d\ln A_t}\right.$$
$$\left.+\beta_A\frac{k_{Mt}}{1-k_{Mt}}\cdot\frac{d\ln k_{Mt}}{d\ln A_t}+\gamma_M\frac{d\ln \tau_{Mt}}{d\ln A_t}+\gamma_A\frac{\tau_{Mt}}{1-\tau_{Mt}}\cdot\frac{d\ln \tau_{Mt}}{d\ln A_t}-1\right]. \tag{A.3}$$

Also, take log and differentiate (20) and (21) with respect to $\ln A_t$, we get (28) and (29) respectively.

Plug (28) and (29) into (A.3), we get (27). ∎

Proposition 3

Proof. Take log and differentiate (A.1) with respect to $\ln M_t$, we get

$$\frac{-1}{1-l_{Mt}}\cdot\frac{d\ln l_{Mt}}{d\ln M_t}=\left(\frac{1-\varepsilon}{\varepsilon}\right)\left[\alpha_M\frac{d\ln l_{Mt}}{d\ln M_t}+\alpha_A\frac{l_{Mt}}{1-l_{Mt}}\cdot\frac{d\ln l_{Mt}}{d\ln M_t}+\beta_M\frac{d\ln k_{Mt}}{d\ln M_t}\right.$$
$$\left.+\beta_A\frac{k_{Mt}}{1-k_{Mt}}\cdot\frac{d\ln k_{Mt}}{d\ln M_t}+\gamma_M\frac{d\ln \tau_{Mt}}{d\ln M_t}+\gamma_A\frac{\tau_{Mt}}{1-\tau_{Mt}}\cdot\frac{d\ln \tau_{Mt}}{d\ln M_t}+1\right]. \tag{A.4}$$

Also, take log and differentiate (20) and (21) with respect to $\ln M_t$, we get (31) and (32) respectively.

Plug (31) and (32) into (A.4), we get (30). ∎

Proposition 4

Proof. Take log and differentiate (A.1) with respect to $\ln K_t$, we get

$$\frac{-1}{1-l_{Mt}}\cdot\frac{d\ln l_{Mt}}{d\ln K_t}=\left(\frac{1-\varepsilon}{\varepsilon}\right)\left[\alpha_M\frac{d\ln l_{Mt}}{d\ln K_t}+\alpha_A\frac{l_{Mt}}{1-l_{Mt}}\cdot\frac{d\ln l_{Mt}}{d\ln K_t}+\beta_M\frac{d\ln k_{Mt}}{d\ln L_t}\right.$$
$$\left.+\beta_A\frac{k_{Mt}}{1-k_{Mt}}\cdot\frac{d\ln k_{Mt}}{d\ln K_t}+\gamma_M\frac{d\ln \tau_{Mt}}{d\ln K_t}+\gamma_A\frac{\tau_{Mt}}{1-\tau_{Mt}}\cdot\frac{d\ln \tau_{Mt}}{d\ln K_t}+(\beta_M-\beta_A)\right]. \tag{A.5}$$

Also, take log and differentiate (20) and (21) with respect to $\ln K_t$, we get (34) and (35) respectively.

Plug (34) and (35) into (A.5), we get (33). ∎

Proposition 5

Proof. Since the non-agricultural sector is the asymptotically dominant sector, use the modified form of (20) $k_{Mt}=\left[1+\frac{\omega_A\beta_A}{\omega_M\beta_M}\left(\frac{Y_{Mt}}{Y_{At}}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right]^{-1}$, with

(9), we can verify that $\eta_t \equiv \omega_M^{\frac{\varepsilon}{\varepsilon-1}} \left[1 + \left(\frac{\beta_M}{\beta_A}\right)\left(\frac{1-k_{Mt}}{k_{Mt}}\right)\right]^{\frac{\varepsilon}{\varepsilon-1}} = \frac{Y_t}{Y_{Mt}}$. This implies $Y_t = \eta_t Y_{Mt}$.

Plug (5) into $Y_t = \eta_t Y_{Mt}$ to get

$$Y_t = \eta_t M_t L_t^{\alpha_M} K_t^{\beta_M} T^{\gamma_M} l_{Mt}^{\alpha_M} k_{Mt}^{\beta_M} \tau_{Mt}^{\gamma_M}. \tag{A.6}$$

By $\Phi_K = R_t$ and (17), $\Phi_K = \omega_M \beta_M \left(\frac{Y_t}{Y_{Mt}}\right)^{\frac{1}{\varepsilon}} \frac{Y_{Mt}}{K_{Mt}}$. Using (5) and $\frac{Y_t}{Y_{Mt}} = \eta_t$, we get

$$\Phi_K = \omega_M \beta_M \eta_t^{\frac{1}{\varepsilon}} M_t L_{Mt}^{\alpha_M} K_{Mt}^{\beta_M - 1} T_{Mt}^{\gamma_M}. \tag{A.7}$$

Now take log and differentiate (39) and (40) with respect to time, we obtain

$$\frac{\dot{c}_t}{c_t} = \frac{\dot{\tilde{c}}_t}{\tilde{c}_t} - \frac{1}{1-\beta_M} g_M + \left(\frac{1-\alpha_M-\beta_M}{1-\beta_M}\right) n, \tag{A.8}$$

$$\frac{\dot{\chi}_t}{\chi_t} = \frac{1-\beta_M}{\alpha_M}\frac{\dot{K}_t}{K_t} - n - \frac{g_M}{\alpha_M}. \tag{A.9}$$

Plug (A.7) and (38) into (A.8) to obtain (41). Plug (A.6) and (37) into (A.9) to get (42).

Manipulate (17) to get $k_{Mt} = \left[1 + \frac{\omega_A \beta_A}{\omega_M \beta_M}\left(\frac{Y_{Mt}}{Y_{At}}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right]^{-1}$. Take log and differentiate the expression with respect to time to get

$$\frac{\dot{k}_{Mt}}{k_{Mt}} = (1-k_{Mt})\left(\frac{\varepsilon-1}{\varepsilon}\right)\left(\frac{\dot{Y}_{Mt}}{Y_{Mt}} - \frac{\dot{Y}_{At}}{Y_{At}}\right). \tag{A.10}$$

Take log and differentiate (4), (5) with respect to time and plug into (A.10), we get $\frac{\dot{k}_{Mt}}{k_{Mt}(1-k_{Mt})}\left(\frac{\varepsilon}{\varepsilon-1}\right) = \left(\frac{\dot{M}_t}{M_t} - \frac{\dot{A}_t}{A_t}\right) + \beta_M\left(\frac{\frac{d(k_{Mt}K_t)}{dt}}{k_{Mt}K_t}\right) - \beta_A\left(\frac{\frac{d(k_{At}K_t)}{dt}}{k_{At}K_t}\right) + \alpha_M\left(\frac{\frac{d(l_{Mt}L_t)}{dt}}{l_{Mt}L_t}\right) - \alpha_A\left(\frac{\frac{d(l_{At}L_t)}{dt}}{l_{At}L_t}\right) + \gamma_M\left(\frac{\frac{d(\tau_{Mt}T)}{dt}}{\tau_{Mt}T}\right) - \gamma_A\left(\frac{\frac{d(\tau_{At}T)}{dt}}{\tau_{At}T}\right)$.

Apply product rule and manipulate the above equation to obtain

$$\begin{aligned}\frac{\dot{k}_{Mt}}{k_{Mt}(1-k_{Mt})}\left(\frac{\varepsilon}{\varepsilon-1}\right) &= (g_M - g_A) + (\beta_M - \beta_A)\left(\frac{\dot{k}_{Mt}}{k_{Mt}} + \frac{\dot{K}_t}{K_t}\right)\\ &+\beta_A\left(\frac{\dot{k}_{Mt}}{k_{Mt}(1-k_{Mt})}\right) + (\alpha_M - \alpha_A)\left(\frac{\dot{l}_{Mt}}{l_{Mt}} + \frac{\dot{L}_t}{L_t}\right)\\ &+\alpha_A\left(\frac{\dot{l}_{Mt}}{l_{Mt}(1-l_{Mt})}\right) + (\gamma_M - \gamma_A)\left(\frac{\dot{\tau}_{Mt}}{\tau_{Mt}}\right) + \gamma_A\left(\frac{\dot{\tau}_{Mt}}{\tau_{Mt}(1-\tau_{Mt})}\right).\end{aligned} \tag{A.11}$$

Take log and differentiate (20), (21) with respect to time to get (46) and (47) respectively. Plug (46), (47) and (A.9) into (A.11), we obtain (43).

To ensure the transversality condition is satisfied, we require

$$\lim_{t\to\infty} \mu_t K_t = 0, \tag{A.12}$$

where μ_t is the costate variable in the Hamiltonian $H(\tilde{c}_t, K_t, \mu_t) \equiv e^{-(\rho-n)t}\left(\frac{\tilde{c}_t^{1-\theta}-1}{1-\theta}\right) + \mu_t[\Phi(K_t,t) - \delta K_t - e^{nt}L_0\tilde{c}_t]$.

Maximum principle requires $H_{\tilde{c}_t} = 0$, which implies $e^{-(\rho-n)t}\tilde{c}_t^{-\theta} - \mu_t e^{nt}L_0 = 0$. Using (39), $\mu_t = \frac{e^{-\rho t}}{L_0} c_t^{-\theta} M_t^{\frac{-\theta}{1-\beta_M}} L_t^{\left(\frac{1-\alpha_M-\beta_M}{1-\beta_M}\right)\theta}$. Plugging it into (A.12), and noting from (40) that $K_t = L_t^{\frac{\alpha_M}{1-\beta_M}} M_t^{\frac{1}{1-\beta_M}} \chi_t^{\frac{\alpha_M}{1-\beta_M}}$, with some algebra we get (45). ∎

Proposition 7 will state that $c_t \to c^*$ and $\chi_t \to \chi^*$ asymptotically, and the transversality is equivalent to (A4).

Proposition 6

Proof. From (A.8) CGP requires $\frac{\dot{c}_t}{c_t}$ to be a constant. By (41) it implies $\eta_t^{\frac{1}{\varepsilon}} \chi_t^{-\alpha_M} l_{Mt}^{\alpha_M} k_{Mt}^{\beta_M-1} \tau_{Mt}^{\gamma_M}$ is constant or

$$\frac{1}{\varepsilon}\frac{\dot{\eta}_t}{\eta_t} - \alpha_M \frac{\dot{\chi}_t}{\chi_t} + \alpha_M \frac{\dot{l}_{Mt}}{l_{Mt}} + (\beta_M - 1)\frac{\dot{k}_{Mt}}{k_{Mt}} + \gamma_M \frac{\dot{\tau}_{Mt}}{\tau_{Mt}} = 0. \tag{A.13}$$

On the other hand, take log and differentiate (44) with respect to time to get

$$\frac{\dot{\eta}_t}{\eta_t} = \frac{\left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{\beta_M}{\beta_A}\right)\left(\frac{-1}{k_{Mt}} \cdot \frac{\dot{k}_{Mt}}{k_{Mt}}\right)}{1 + \frac{\beta_M}{\beta_A}\left(\frac{1-k_{Mt}}{k_{Mt}}\right)} = \frac{-\left(\frac{\varepsilon}{\varepsilon-1}\right)\beta_M \frac{\dot{k}_{Mt}}{k_{Mt}}}{\beta_A k_{Mt} + \beta_M(1-k_{Mt})}. \tag{A.14}$$

Plug (A.13), (A.14), (46) and (47) into (43) to get (48), where $G(k_{Mt}) \equiv \frac{\left(\frac{1-\beta_M}{\beta_M-\beta_A}\right)(1-k_{Mt})}{\frac{-\left(\frac{1}{\varepsilon-1}\right)\beta_M(1-k_{Mt})}{\beta_A k_{Mt}+\beta_M(1-k_{Mt})} + \alpha_M(1-l_{Mt}) + \gamma_M(1-\tau_{Mt}) - \left(\frac{1-\beta_M}{\beta_M-\beta_A}\right)\left(\frac{\varepsilon}{\varepsilon-1}\right) + \left(\frac{1-\beta_M}{\beta_M-\beta_A}\right)[(\alpha_M-\alpha_A)(1-l_{Mt}) + (\gamma_M-\gamma_A)(1-\tau_{Mt}) + \alpha_A + \beta_A + \gamma_A]}$.

Observe that $G(1) = 0$, and $G(k_{Mt}) > 0 \ \ \forall k_{Mt} \in [0,1)$, given (A1), (A2) hold. Hence when (A3) is also satisfied, by (48) $\dot{k}_{Mt} > 0$ and $k_{Mt} \to 1$ as $t \to \infty$. By (20) and (21), $l_{Mt} \to 1$ and $\tau_{Mt} \to 1$ as $t \to \infty$ too. ∎

Proposition 7

Proof. We first solve for the steady state allocation in CGP. From proposition 6, given (A1)-(A4), $k_{Mt} \to k_M^* = 1$, $l_{Mt} \to l_M^* = 1$ and $\tau_{Mt} \to \tau_M^* = 1$ as $t \to \infty$. By (44) $\eta_t \to \omega_M^{\frac{\varepsilon}{\varepsilon-1}}$.

From (A.13), $\chi_t \to \chi^*$ also exists. We can solve for χ^* by (41), c^* by (42), $g^*_{\tilde{c}}$ by (A.8), g^*_K by (A.9). Note that by construction, there are no other steady state allocations with constant $\frac{\dot{c}_t}{c_t}$.

Since non-agricultural sector is the asymptotically dominant sector, by (6)-(8) and $k^*_M = l^*_M = \tau^*_M = 1$, we have $g^*_{K_M} = g^*_K$, $g^*_{L_M} = g^*_L = n$ and $g^*_{T_M} = g^*_T = 0$.

By (5), $g^*_{Y_M} \equiv \frac{\dot{Y}_M}{Y_M} = \frac{\dot{M}}{M} + \alpha_M \frac{\dot{L}}{L} + \beta_M \frac{\dot{K}}{K} = g_M + \alpha_M n + \beta_M \left(\frac{\alpha_M}{1-\beta_M} n + \frac{g_M}{1-\beta_M} \right)$.

For the agricultural sector, by (4), $g^*_{Y_A} \equiv \frac{\dot{Y}_A}{Y_A} = \frac{\dot{A}}{A} + \alpha_A \frac{\dot{L}}{L} + \beta_A \frac{\dot{K}}{K} = g_A + \alpha_A n + \beta_A \left(\frac{\alpha_M}{1-\beta_M} n + \frac{g_M}{1-\beta_M} \right)$.

Note $g^*_{Y_A} - g^*_{Y_M} = \left[g_A - \left(\frac{1-\beta_A}{1-\beta_M} \right) g_M \right] + \left[\alpha_A - \left(\frac{1-\beta_A}{1-\beta_M} \right) \alpha_M \right] n > 0$ by (A3).

From the second equality in (16) we have $\frac{1}{\varepsilon} \frac{\dot{Y}}{Y} + \left(1 - \frac{1}{\varepsilon}\right) \frac{\dot{Y}_A}{Y_A} - \frac{\dot{L}_A}{L_A} = \frac{1}{\varepsilon} \frac{\dot{Y}}{Y} + \left(1 - \frac{1}{\varepsilon}\right) \frac{\dot{Y}_M}{Y_M} - \frac{\dot{L}_M}{L_M}$, which implies $g^*_{L_A} \equiv \frac{\dot{L}_A}{L_A} = \left(1 - \frac{1}{\varepsilon}\right) (g^*_{Y_A} - g^*_{Y_M}) + g^*_{L_M} < g^*_{L_M}$, given (A1).

Similarly, from the second equality in (17) we have $\left(1 - \frac{1}{\varepsilon}\right) \frac{\dot{Y}_A}{Y_A} - \frac{\dot{K}_A}{K_A} = \left(1 - \frac{1}{\varepsilon}\right) \frac{\dot{Y}_M}{Y_M} - \frac{\dot{K}_M}{K_M}$, which implies $g^*_{K_A} \equiv \frac{\dot{K}_A}{K_A} = \left(1 - \frac{1}{\varepsilon}\right) (g^*_{Y_A} - g^*_{Y_M}) + g^*_{K_M} < g^*_{K_M}$, given (A1). And from the second equality in (18) we have $\left(1 - \frac{1}{\varepsilon}\right) \frac{\dot{Y}_A}{Y_A} - \frac{\dot{T}_A}{T_A} = \left(1 - \frac{1}{\varepsilon}\right) \frac{\dot{Y}_M}{Y_M} - \frac{\dot{T}_M}{T_M}$, which implies $g^*_{T_A} \equiv \frac{\dot{T}_A}{T_A} = \left(1 - \frac{1}{\varepsilon}\right) (g^*_{Y_A} - g^*_{Y_M}) + g^*_{T_M} < g^*_{T_M}$, given (A1).

From (9), we have

$$g^*_Y \equiv \frac{\dot{Y}}{Y} = \frac{\omega_A Y_A^{\frac{\varepsilon-1}{\varepsilon}}}{\omega_A Y_A^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M Y_M^{\frac{\varepsilon-1}{\varepsilon}}} \frac{\dot{Y}_A}{Y_A} + \frac{\omega_M Y_M^{\frac{\varepsilon-1}{\varepsilon}}}{\omega_A Y_A^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M Y_M^{\frac{\varepsilon-1}{\varepsilon}}} \frac{\dot{Y}_M}{Y_M}$$
$$= \begin{cases} \min\left\{ \frac{\dot{Y}_A}{Y_A}, \frac{\dot{Y}_M}{Y_M} \right\} & \text{if } \varepsilon < 1 \\ \max\left\{ \frac{\dot{Y}_A}{Y_A}, \frac{\dot{Y}_M}{Y_M} \right\} & \text{if } \varepsilon > 1 \end{cases}$$

Hence $g^*_y = \frac{\dot{y}}{y} = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = \frac{\dot{Y}}{Y} - n = g^*_Y - n = \frac{g_M}{1-\beta_M} - \left(\frac{1-\alpha_M-\beta_M}{1-\beta_M} \right) n$.

(A4) plus $c_t \to c^*$ and $\chi_t \to \chi^*$ ensures the transversality condition (45) is satisfied. Together with household's period utility function $\frac{\tilde{c}_t^{1-\theta}-1}{1-\theta}$ being strictly concave in $\tilde{c}_t$, (41)-(43) is the unique dynamic equilibrium characterizing the social planner's solution to (14). (Acemoglu 2009, Thm. 7.8).

To prove that the dynamic equilibrium converges to the unique CGP steady state, we study the saddle-path property of the linearized dynamic system around the asymptotic state (CGP steady state). We rewrite the system (41)-(43) as

$$\dot{X}_t = f(X_t), \quad \text{where } X_t \equiv \begin{pmatrix} c_t \\ \chi_t \\ k_{Mt} \end{pmatrix}. \tag{A.15}$$

Define $z_t \equiv X_t - X^*$. We linearize (A.15) around the asymptotic state X^* to get

$$\dot{z}_t = J(X^*)z_t, \quad \text{where } J(X^*) = \begin{pmatrix} 0 & a_{c\chi} & a_{ck_M} \\ -\frac{1-\beta_M}{\alpha_M}(\chi^*)^{1-\frac{\alpha_M}{1-\beta_M}} & a_{\chi\chi} & a_{\chi k_M} \\ 0 & 0 & a_{k_M k_M} \end{pmatrix} \tag{A.16}$$

is value of the Jacobian matrix of the system (A.15) at the asymptotic state X^*.

- From (41), $a_{c\chi} = \frac{-\alpha_M}{\theta}\left[\omega_M \beta_M (\eta^*)^{\frac{1}{\varepsilon}}(\chi^*)^{-\alpha_M - 1}T^{\gamma_M}\right]c^* < 0$.
- From (43), $a_{k_M k_M} = \frac{g_A - \left(\frac{1-\beta_A}{1-\beta_M}\right)g_M - \left[\alpha_M - \alpha_A + \frac{\alpha_M(\beta_M - \beta_A)}{1-\beta_M}\right]n}{\frac{\varepsilon}{\varepsilon - 1} - (\alpha_A + \beta_A + \gamma_A)} < 0$, given (A1)-(A3).

This implies that $|J(X^*)| = \frac{1-\beta_M}{\alpha_M}(\chi^*)^{1-\frac{\alpha_M}{1-\beta_M}} a_{c\chi} a_{k_M k_M} > 0$ and all eigenvalues of $J(X^*)$ have non-zero real parts.[1] Hence the asymptotic state X^* is hyperbolic. By the Grobman-Hartman Theorem, the dynamics of the nonlinear system (A.15) in the neighborhood of X^* is qualitatively the same as the dynamics of the linearized system (A.16). (Acemoglu 2009, Thm. B.7).

Next we set up the characteristics equation for the Jacobian matrix evaluated at the asymptotic state:

$$|J(X^*) - \nu I| = 0 \Longrightarrow \begin{vmatrix} -\nu & a_{c\chi} & a_{ck_M} \\ -\frac{1-\beta_M}{\alpha_M}(\chi^*)^{1-\frac{\alpha_M}{1-\beta_M}} & a_{\chi\chi} - \nu & a_{\chi k_M} \\ 0 & 0 & a_{k_M k_M} - \nu \end{vmatrix} = 0.$$

$$\Longrightarrow (a_{k_M k_M} - \nu)\left[\nu^2 - a_{\chi\chi}\nu + \frac{1-\beta_M}{\alpha_M}(\chi^*)^{1-\frac{\alpha_M}{1-\beta_M}} \cdot a_{c\chi}\right] = 0.$$

Since $a_{k_M k_M}, a_{c\chi} < 0$, the above characteristic equation has two negative roots and one positive root, which implies that the asymptotic state is saddle-path stable. That means, there exists a unique two-dimensional manifold of solutions to the dynamic system (41)-(43) converging to the CGP steady state. ∎

[1] We directly assume $c^*, \chi^* > 0$. Note that assuming $\chi^* > 0$ also assures sustainable per capita income growth.

APPENDIX: LAND EXPANSION EFFECT

Proposition 8 states how a one-time increase in land supply in an economy affects sectoral shares in the unified model.

PROPOSITION 8 (Land expansion effect). *In equilibrium,*

$$\frac{d\ln l_{Mt}}{d\ln T} = \tag{A.17}$$

$$-\frac{(1-\varepsilon)(\gamma_M-\gamma_A)(1-l_{Mt})}{\varepsilon+(1-\varepsilon)[\alpha_M(1-l_{Mt})+\alpha_A l_{Mt}+\beta_M(1-k_{Mt})+\beta_A k_{Mt}+\gamma_M(1-\tau_{Mt})+\gamma_A\tau_{Mt}]}\ \begin{matrix} <0 & \text{if } \varepsilon<1 \text{ and } \gamma_M>\gamma_A \\ >0 & \text{if } \varepsilon<1 \text{ and } \gamma_M<\gamma_A \end{matrix},$$

$$\frac{d\ln k_{Mt}}{d\ln T} = \frac{1-k_{Mt}}{1-l_{Mt}}\cdot\frac{d\ln l_{Mt}}{d\ln T}, \tag{A.18}$$

$$\frac{d\ln \tau_{Mt}}{d\ln T} = \frac{1-\tau_{Mt}}{1-l_{Mt}}\cdot\frac{d\ln l_{Mt}}{d\ln T}. \tag{A.19}$$

Proof. Take log and differentiate (A.1) with respect to $\ln T$, we get

$$\frac{-1}{1-l_{Mt}}\cdot\frac{d\ln l_{Mt}}{d\ln T} = \left(\frac{1-\varepsilon}{\varepsilon}\right)\Bigg[\alpha_M\frac{d\ln l_{Mt}}{d\ln T}+\alpha_A\frac{l_{Mt}}{1-l_{Mt}}\cdot\frac{d\ln l_{Mt}}{d\ln T}$$
$$+\beta_M\frac{d\ln k_{Mt}}{d\ln T}+\beta_A\frac{k_{Mt}}{1-k_{Mt}}\cdot\frac{d\ln k_{Mt}}{d\ln T}$$
$$+\gamma_M\frac{d\ln \tau_{Mt}}{d\ln T}+\gamma_A\frac{\tau_{Mt}}{1-\tau_{Mt}}\cdot\frac{d\ln \tau_{Mt}}{d\ln T}+(\gamma_M-\gamma_A)\Bigg] \tag{A.20}$$

Also, take log and differentiate (20) and (21) with respect to $\ln T$, we get (A.18) and (A.19) respectively.

Plug (A.18) and (A.19) into (A.20), we get (A.17). ∎

Similar to population growth effect, technology growth effects and capital deepening effect (propositions 1-4), land expansion effect operates through the relative price effect. Ceteris paribus, if ε <1, land expansion generates a more than proportionate relative price drop (compared to the relative marginal product of land rise) in the sector with higher land intensity. Land use shifts out of this sector until the land rental parity condition (18) is restored. Due to input complementarity, capital and labor also move in the same direction.

APPENDIX: OTHER PROOFS

A.1. DERIVING THE PRICE INDICES (10)-(12)

Consider the choice problem of the final output producer. For whatever final output level Y_t the producer decides on, it is always optimal to purchase the combination of agricultural and non-agricultural goods that minimize the cost of achieving the level Y_t, that is:

$$\min_{Y_{At},Y_{Mt}} P_{At}Y_{At} + P_{Mt}Y_{Mt} \text{ subject to } \left(\omega_A Y_{At}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M Y_{Mt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \geq Y_t. \tag{A.21}$$

We set up the Lagrangian for the problem (A.21):

$$\Pi_t = P_{At}Y_{At} + P_{Mt}Y_{Mt} + F_t\left[Y_t - \left(\omega_A Y_{At}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M Y_{Mt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right], \tag{A.22}$$

where the Lagrangian multiplier F_t shows the shadow price of Y_t, that is, the price of final output at time t.

First order conditions with respect to Y_{At} and Y_{Mt} yields:

$$Y_{At} = \left(\frac{P_{At}}{F_t}\right)^{-\varepsilon} \omega_A^{\varepsilon} Y_t, \tag{A.23}$$

$$Y_{Mt} = \left(\frac{P_{Mt}}{F_t}\right)^{-\varepsilon} \omega_M^{\varepsilon} Y_t. \tag{A.24}$$

Plug (A.23) and (A.24) into the definition of Y_t (equation (9)), solving for F_t,

$$F_t = (\omega_A^{\varepsilon} P_{At}^{1-\varepsilon} + \omega_M^{\varepsilon} P_{Mt}^{1-\varepsilon})^{\frac{1}{1-\varepsilon}}. \tag{A.25}$$

By normalizing $F_t \equiv 1$ in (A.25), (A.23), (A.24), we obtain (10)-(12).

We note that the consumption composite price P_t always equals final output price F_t. To see this, consider the choice problem of the representative household. For whatever consumption composite $\tilde{c}_t$ the household chooses, it is always optimal to purchase the combination of agricultural and non-agricultural goods that minimizes the cost of achieving the level $\tilde{c}_t$, that is:

$$\min_{\tilde{y}_{At},\tilde{y}_{Mt}} P_{At}\tilde{y}_{At} + P_{Mt}\tilde{y}_{Mt} \tag{A.26}$$

subject to $(\omega_A \tilde{y}_{At}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M \tilde{y}_{Mt}^{\frac{\varepsilon-1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}} - \frac{\dot{K}_t}{L_t} - \frac{\delta K_t}{L_t} \geq \tilde{c}_t$, given $L_t, K_t > 0$, $\dot{K}_t$.

We set up the Lagrangian for the problem (A.26):

$$\Pi_t' = P_{At}\tilde{y}_{At} + P_{Mt}\tilde{y}_{Mt} + P_t\left[\tilde{c}_t - \left(\omega_A \tilde{y}_{At}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M \tilde{y}_{Mt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\dot{K}_t}{L_t} + \frac{\delta K_t}{L_t}\right], \tag{A.27}$$

where P_t is the shadow price of $\tilde{c}_t$, which is in the same form as in (A.21). By similar procedures as in (A.22)-(A.25), we get $P_t = (\omega_A^{\varepsilon} P_{At}^{1-\varepsilon} + \omega_M^{\varepsilon} P_{Mt}^{1-\varepsilon})^{\frac{1}{1-\varepsilon}} = F_t = 1$ for all t.

A.2. DERIVING THE ECONOMY-WIDE RESOURCE CONSTRAINT (13)

First, consider the choice problem faced by the agricultural producer:

$$\max_{L_{At}, K_{At}, T_{At}} P_{At}Y_{At} - w_t L_{At} - R_t K_{At} - \Omega_t T_{At} \text{ subject to (4).} \tag{A.28}$$

First order conditions imply:

$$w_t L_{At} = \alpha_A P_{At} Y_{At}, \tag{A.29}$$
$$R_t K_{At} = \beta_A P_{At} Y_{At}, \tag{A.30}$$
$$\Omega_t T_{At} = \gamma_A P_{At} Y_{At}. \tag{A.31}$$

Similarly, consider the choice problem faced by the non-agricultural producer:

$$\max_{L_{Mt}, K_{Mt}, T_{Mt}} P_{Mt}Y_{Mt} - w_t L_{Mt} - R_t K_{Mt} - \Omega_t T_{Mt} \text{ subject to (5).} \tag{A.32}$$

First order conditions imply:

$$w_t L_{Mt} = \alpha_M P_{Mt} Y_{Mt}, \tag{A.33}$$
$$R_t K_{Mt} = \beta_M P_{Mt} Y_{Mt}, \tag{A.34}$$
$$\Omega_t T_{Mt} = \gamma_M P_{Mt} Y_{Mt}. \tag{A.35}$$

Now we multiple both sides of (3) by L_t and apply $r_t = R_t - \delta$, (6)-(8) to get:

$$\dot{K}_t = w_t(L_{At} + L_{Mt}) + R_t(K_{At} + K_{Mt}) - \delta K_t + \Omega_t(T_{At} + T_{Mt}) - L_t\tilde{c}_t. \tag{A.36}$$

Apply (A.29)-(A.31) and (A.33)-(A.35) to (A.36). Using $\alpha_A + \beta_A + \gamma_A = 1$ and $\alpha_M + \beta_M + \gamma_M = 1$ to get:

$$\dot{K}_t = P_{At}Y_{At} + P_{Mt}Y_{Mt} - \delta K_t - L_t\tilde{c}_t. \tag{A.37}$$

Note that

$$P_{At}Y_{At} + P_{Mt}Y_{Mt} = \left(\omega_A Y_{At}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M Y_{Mt}^{\frac{\varepsilon-1}{\varepsilon}}\right) Y_t^{\frac{1}{\varepsilon}} = Y_t^{\frac{\varepsilon-1}{\varepsilon}} Y_t^{\frac{1}{\varepsilon}} = Y_t, \quad \text{(A.38)}$$

where the first equality comes from (11)-(12), and the second equality follows from the definition of Y_t (equation (9)). Plug (A.38) into (A.37) and we obtain (13).

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