

Labor Market Implications of Pension Reform: Analyzing Delayed Retirement Policies and Fiscal Adjustments in Aging Economies*

Tiantian Dai, Hua Fan, and Shenyi Jiang[†]

This paper employs a labor search model with two different skill-intensive markets to examine how pension systems and fiscal policies influence the young and the old unemployment rates in an aging population. In general, adjustments to the meeting rates, statutory contribution rates, and fiscal policies exert ambiguous effects on the labor market, and spillover effects arise as firms cannot discriminate based on age when hiring. The paper calibrates the model using Chinese data to quantify the impact of retirement policy reforms during the first decade of the 21st century, and it highlights each influencing channel using the counterfactual experiments.

Key Words: Aging; Pension reform; Unemployment rates; Labor search.

JEL Classification Numbers: E24, H55, J26, J64.

1. INTRODUCTION

In recent decades, population aging has become a prominent global phenomenon. According to the World Population Prospects 2017, Europe exhibits the most pronounced demographic aging, with global growth rate among individuals aged 60 and above. The 60+ population in Europe has grown by 24%, followed by 21% in North America, 16.5% in Oceania, and 12% in Asia. In response to the economic and social challenges posed by

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[†]Dai: China Economics and Management Academy, Central University of Finance and Economics, Beijing, China. Email address: dait@cufe.edu.cn; Fan: Center for Industrial and Business Organization and Institute for Advanced Economic Research, Dongbei University of Finance and Economics, Dalian, China. Email address: huafan@dufe.edu.cn; Jiang: Corresponding author. China Economics and Management Academy, Central University of Finance and Economics, No. 39 South College Road, Haidian District, Beijing, China. Email address: syijiang@gmail.com.

aging populations, many countries have undertaken reforms to adjust the components and structure of their pension systems.

One widely studied approach to pension reform is the implementation of a delayed retirement policy (Krueger and Pischke, 1992; Diamond and Gruber, 1999; Baker et al., 2009; Banks et al., 2010). The literature suggests that delayed retirement may reduce the labor force participation of old workers, increase their risk of unemployment, and exacerbate pressures on unemployed young workers, leading to higher overall unemployment rates (Gruber and Wise, 2002; Staubli and Zweimüller, 2013). However, contrasting evidence indicates that delayed retirement can also increase labor force participation among old workers, potentially mitigating some negative labor market effects (Rust and Phelan, 1997; Panis et al., 2002). Dai et al. (2022) show that such effects on the young would be ambiguous. Remaining adjustments to pension systems include reinforcing the tax-benefit link, implementing parametric reform, and introducing greater actuarial fairness (Diamond, 2004; Diamond and Orszag, 2005; Fisher and Keuschnigg, 2010).

Existing literature on the labor market impacts of pension reforms primarily focuses on the intensive margin of labor supply, i.e., the number of hours worked. For example, Fisher and Keuschnigg (2010) argue that pension reforms may reduce the labor supply among prime-age workers through the tax-benefit link.¹ Despite this, few studies have examined the effects on the extensive margin, i.e., the number of employed and unemployed workers, even though one of the primary objectives of delayed retirement policies is to increase the number of employed workers, particularly pension contributors to maintain the sustainability of the pension system. Given the significant implications of such policies for young workers in their most productive years, it is essential to analyze the impact on unemployment rates, especially for young workers. While Börsch-Supan and Ludwig (2010) distinguish between the two margins, the extensive margin in their model remains exogenous.

In this paper, we endogenize the extensive margin and aim to explore how different components of pension systems affect the unemployment rates of both young and old workers. We show that changes in the meeting rate serve as an important channel through which policies impact labor market outcomes.²

¹Börsch-Supan and Ludwig (2010) distinguish between labor supply adjustments along both the extensive and the intensive margins, discussing the connections/interactions between pension reforms and broader labor market policies. However, changes along the extensive margin are often treated as exogenous in these models.

²Choi et al. (2015) demonstrate the importance of job-finding rates in explaining high unemployment and low participation rates among young workers and the old's low participation and unemployment rates.

Two key features are necessary to construct a model that effectively captures the impact of pension system components on unemployment rates of both young and old workers: (1) unemployment must arise as an equilibrium outcome, and (2) workers must operate in different skill-intensive markets. This assumption reflects the labor market dynamics in industries where old workers may be displaced into junior positions, and where exceptional younger workers can be matched with senior roles. Given that firms cannot engage in age-based hiring discrimination, there is an inherent interaction between young and old workers within the labor market.

We employ the labor search model of Mortensen's (1982) and Pissarides's (2000), incorporating the two-market structure proposed by Chassamboulli and Palivos (2014). This framework allows us to examine the interactions between demographic aging and pension contribution rates levied on employed workers, unemployed workers, and unskilled-intensive firms alongside two types of taxation: production tax and wage tax. Our analysis reveals that the effects of pension contribution rates and taxes on unemployment rates for both young and old workers are generally ambiguous. Similarly, the impact of delayed retirement policies on the labor market is uncertain, except for the wages of old workers employed in junior markets.

Two opposing channels drive the impact of contribution rates. First, higher contribution rates and taxes negatively affect the extensive margin in both labor markets. Specifically, they reduce workers' flow value of employment and discourage firms from posting vacancies, which exerts downward pressure on market tightness and wages while increasing the unemployment rates of both markets. Second, higher contribution rates and taxes increase the flow value of retirees, which positively influences market tightness via a tax-benefit channel, thereby reducing unemployment rates. Additionally, as firms cannot discriminate based on age when hiring, a positive spillover effect occurs when a firm favors one type of worker, benefiting the other group. Consequently, the overall impact on unemployment rates for both young and old workers remains ambiguous.

We also examine the effects of delayed retirement policies and demonstrate that, by increasing the presence of old workers in both markets, these policies directly influence the size of the labor force and the number of unemployed old workers. This, in turn, affects firms' meeting rates with the old. A higher meeting rate with the old increases firms' flow value of employment and encourages greater vacancy postings. Conversely, a rise in the number of unemployed old workers negatively impacts market tightness, which exerts upward pressure on wages and contributes to higher unemployment rates. A positive spillover effect is observed, whereby an increase in old workers also raises market tightness in both markets. Despite these dynamics, the overall effect of delayed retirement policies on unemployment rates remains ambiguous.

Another line of literature has introduced labor market frictions into the analysis of the pension system. For instance, Bhattacharya et al. (2004) examine the relationship between pension systems and retirement incentives using a labor search model, showing that labor participation among young workers is inefficient in the presence of retirement-inducing policies due to search frictions.³ De la Croix et al. (2013) show that aging shocks or pension reforms reduce interest rates and equilibrium unemployment. However, their model includes only one market and cannot distinguish between policy-induced changes in market tightness across different markets and their effects on unemployment rates. Our counterfactual experiments reveal that different markets respond asymmetrically to policy changes. Jaag et al. (2010) consider both the intensive and extensive margins in a labor search model, though their focus is on structural pension reforms, whereas we concentrate on component-specific reforms. Furthermore, we find that the inability of firms to discriminate in hiring creates a spillover effect, whereby policies targeting old workers indirectly affects the unemployment rates of young workers through changes in market tightness.

We calibrate the model using Chinese labor market data and conduct a series of counterfactual experiments to simulate the potential effects of China's proposed pension and retirement policy reforms, particularly on unemployment rates for both young and old workers in the context of an aging society. Our results indicate that a 5% decrease in the probability of retirement would reduce the unemployment rate of the old by 0.42% while increasing the unemployment rate of the young by 0.23%. Furthermore, a 5% increase in a firm's statutory contribution rate would lead to a significant reduction in unemployment, with the rate for the old decreasing by 11.52% and that for the young decreasing by 7.99%. In contrast, a 5% increase in the statutory contribution rates for unemployed and employed workers, as well as increases in the production tax or wage tax would raise the unemployment rates for both groups, with increases ranging from 0.51% to 9.25%.

The remainder of the paper is organized as follows: Section 2 describes the baseline model. Section 3 defines the equilibrium. Section 4 illustrates the evidence of China and demonstrates some key propositions. Section 5 details calibration, counterfactual experiments, and sensitivity tests. Section 6 concludes.

³Similarly, Hairault (2015) incorporates a search model into a life-cycle framework to study retirement decisions and their impact on unemployment rates, while García-Pérez and Sánchez-Martín (2015) conduct a comparable study for Spain.

2. THE BASELINE MODEL

Time is modeled as continuous, with the economy populated by a continuum of workers and jobs. Workers are categorized as either young (y) or old (o). Young workers transition to the old with probability λ_y , while old workers retire with probability λ_o and face a mortality risk represented by the probability d . The mass of jobs is determined endogenously as part of the equilibrium conditions. All agents in the model are risk-neutral and discount future utility at a common interest rate, $r > 0$.

2.1. Goods Market

Two types of intermediate goods are produced in two distinct sectors: sector j (unskilled-intensive) and sector s (skilled-intensive). Firms operate exclusively within one of these sectors, and production in both sectors employs a linear technology. Firms use old or young workers to produce intermediate inputs Y_j and Y_s , with a one-to-one production relationship in both cases. These intermediate goods are non-storable and sold in competitive markets, where they are immediately utilized to produce final goods (Y).

To simplify the analysis, we eliminate the effect of the production function and assume the prices of one unit intermediate goods $(\xi_j)p_j$, $(\xi_s)p_s$, to be exogenously determined, where ξ_s and ξ_j denote the relative production efficiencies of young and old workers in each sector. In the baseline model, no specific restrictions are imposed on these efficiency parameters.

2.2. Labor Market

2.2.1. Matching Technology

The model features two labor markets with search frictions, where firms post vacancies to hire workers either for the unskilled-intensive sector or the skilled-intensive sector. Both young and old workers compete for jobs within each sector; however, cross-matching between sectors is not permitted. Importantly, firms cannot anticipate the type of workers they will match when they open vacancies. In each labor market, unemployed workers and unfilled vacancies are paired through a stochastic matching technology $M(u_i; v_i) = M_0 u_i^\epsilon v_i^{1-\epsilon}$, where $i = j, s$. M_0 is an efficiency parameter, while u_i and v_i denote the number of unemployed workers and vacancies in the labor market i , respectively. The matching function $M(\cdot)$ exhibits standard properties: it is at least twice continuously differentiable, increasing in both arguments, exhibits constant returns to scale, and satisfies the Inada conditions.

Using the property of constant returns to scale, we can express the flow rate of matching for a worker as $M(u_i; v_i)/u_i = m(\theta_i)$, and the flow rate

of matching for a vacancy as $M(u_i; v_i)/v_i = q(\theta_i)$, where $\theta_i = v_i/u_i = m(\theta_i)/q(\theta_i)$ indicates the tightness in the labor market i . The function $m(\theta_i)$ and $q(\theta_i)$ exhibits the usual properties:

$$\begin{aligned} m'(\theta_i) &> 0, & \lim_{\theta_i \rightarrow 0} m(\theta_i) &= 0, & \lim_{\theta_i \rightarrow \infty} m(\theta_i) &= \infty, \\ q'(\theta_i) &< 0, & \lim_{\theta_i \rightarrow 0} q(\theta_i) &= \infty, & \lim_{\theta_i \rightarrow \infty} q(\theta_i) &= 0. \end{aligned}$$

At any given time, a worker must be in one of the three states: employed (E_i^κ), unemployed (U_i^κ) or retired (R_i^o). Similarly, a vacancy is either filled (J_i^κ) or unfilled (V_i). In this notation, E_i^κ , U_i^κ , R_i^o , J_i^κ and V_i represent present discounted values associated with each respective state, where $i = j; s$ denotes the sector and $\kappa = y; o$ refers to the age group.

2.2.2. Firm's Value Functions

The following equations represent the flow value of firms in both unskilled-intensive and skilled-intensive sectors with respect to vacancies filled by either young or old workers:

$$rJ_j^y = (1 - \tau_p)(p_j - \Phi_j^y) - w_j^y - s_j(J_j^y - V_j) - \lambda_y(J_j^y - J_j^o), \quad (1)$$

$$rJ_j^o = (1 - \tau_p)(\xi_j p_j - \Phi_j^o) - w_j^o - s_j(J_j^o - V_j) - (\lambda_o + d)J_j^o, \quad (2)$$

$$rJ_s^y = (1 - \tau_p)(p_s - \Phi_s^y) - w_s^y - s_s(J_s^y - V_s) - \lambda_y(J_s^y - J_s^o), \quad (3)$$

$$rJ_s^o = (1 - \tau_p)(\xi_s p_s - \Phi_s^o) - w_s^o - s_s(J_s^o - V_s) - (\lambda_o + d)J_s^o, \quad (4)$$

$$rV_j = -c_j + q(\theta_j)[\phi_j J_j^y + (1 - \phi_j)J_j^o - V_j], \quad (5)$$

$$rV_s = -c_s + q(\theta_s)[\phi_s J_s^y + (1 - \phi_s)J_s^o - V_s]. \quad (6)$$

Equations (1) and (2) represent the flow value of the firms in the unskilled-intensive sector with vacancies filled by a young or an old worker, respectively. The flow value depends on the worker's productivity (ξ_j) and wages (w_j^κ). The separation probability (s_j) captures the likelihood of workers leaving their jobs through retirement or job transition. In addition, the term $\lambda_o + d$ reflects the probability of old workers retiring or dying, while λ_y accounts for young workers transitioning into old workers, providing J_j^o to the firm. Equations (3) and (4) are the analogous value functions for firms in the skilled-intensive sector, reflecting similar dynamics in the

unskilled-intensive sector but with potential differences in productivity (ξ_j , ξ_s) and market characteristics. Firms must also pay pension insurance (Φ) and a revenue tax (τ_p). In general, the pension contribution can be simplified as

$$\Phi = \varepsilon_f w,$$

where ε_f denotes the share of the wage w that a firm must contribute towards pension insurance. For instance, in the US Social Security Pension System, $\varepsilon_f = 50\% \times 12.4\%$, while in Japan it is $\varepsilon_f = 50\% \times 18.3\%$, and in Singapore, $\varepsilon_f = 17\%$. Equations (5) and (6) represent the flow value of a firm holding a vacancy in either market where $\phi_i = \frac{u_i^y}{u_i^y + u_i^o}$ denotes the meeting rates, i.e., the probability of a firm matching with a young worker relative to the total number of unemployed workers (young and old) in market i .

2.2.3. Worker's Value Functions

The following equations represent the worker's value functions:

$$rE_j^y = w_j^y - \varphi_j^y - s_j(E_j^y - U_j^y) - \lambda_y(E_j^y - E_j^o), \quad (7)$$

$$rE_j^o = w_j^o - \varphi_j^o - s_j(E_j^o - U_j^o) - \lambda_o(E_j^o - R_j^o) - dE_j^o, \quad (8)$$

$$rE_s^y = (1 - \tau_w)(w_s^y - \varphi_s^y) - s_s(E_s^y - U_s^y) - \lambda_y(E_s^y - E_s^o), \quad (9)$$

$$rE_s^o = (1 - \tau_w)(w_s^o - \varphi_s^o) - s_s(E_s^o - U_s^o) - \lambda_o(E_s^o - R_s^o) - dE_s^o, \quad (10)$$

$$rU_j^y = b_j^y + m(\theta_j)(E_j^y - U_j^y) - \lambda_y(U_j^y - U_j^o) - \eta_j^y, \quad (11)$$

$$rU_j^o = b_j^o + m(\theta_j)(E_j^o - U_j^o) - \lambda_o(U_j^o - R_j^o) - dU_j^o - \eta_j^o, \quad (12)$$

$$rU_s^y = b_s^y + m(\theta_s)(E_s^y - U_s^y) - \lambda_y(U_s^y - U_s^o) - \eta_s^y, \quad (13)$$

$$rU_s^o = b_s^o + m(\theta_s)(E_s^o - U_s^o) - \lambda_o(U_s^o - R_s^o) - dU_s^o - \eta_s^o. \quad (14)$$

Equations (7)-(10) describe the flow values of an employed worker in the labor market. In Equations (7) and (9), a young worker earns a wage (w_i^y ; $i = j, s$), pays pension contributions (φ_i^y ; $i = j, s$), and income tax at a rate τ_w , while facing a separation probability (s_i ; $i = j, s$) and the chance of aging (λ_y). Equations (8) and (10) follow a similar logic, except that an old worker may retire (λ_o) or die (d). Equations (11)-(14) represent the

flow values of an unemployed worker, who receives unemployment benefits (b) and pays pension contribution (η). Equations (11) and (13) reflect that a young unemployed worker may transition to employment with probability $m(\theta_i)$ or age into the old group (λ_y). In Equations (12) and (14) an old unemployed worker either finds a job with a probability $m(\theta_i)$, retires with probability λ_o , or dies with probability d . Pension payments can be simplified as follows:

$$\begin{aligned}\varphi &= \varepsilon_w w, \\ \eta &= \varepsilon_I w,\end{aligned}$$

where ε_w represents the portion of the wage that an employee must contribute towards pension insurance. For example, in the U.S. Social Security Pension System, $\varepsilon_w = 50\% \times 12.4\%$. In Japan, $\varepsilon_w = 50\% \times 18.3\%$, while in Singapore, $\varepsilon_w = 20\%$. The term ε_I denotes the share of the wage that unemployed or self-employed workers must contribute towards pension insurance. In the U.S., this contribution is set at $\varepsilon_I = 12.4\%$ under the Social Security Pension System. In Japan, unemployed workers are required to join the National Pension Fund, where their contribution $\eta = \bar{x}$ is fixed at a predetermined rate, although housewives are exempt from this requirement. In Singapore, the contribution η depends on personal pension savings insurance without a specifically defined contribution rate.

2.2.4. Retiree's Value Functions

The following equations represent the retiree's value functions:

$$rR_j^o = \chi_j^o - d \times R_j^o, \quad (15)$$

$$rR_s^o = \chi_s^o - d \times R_s^o. \quad (16)$$

Equations (15) and (16) describe the flow value of retirees in each sector, where χ represents pension benefits. The calculation of pension benefits varies significantly across countries. For example, in the U.S. Social Security Pension System, the pension benefit formula is as follows: $\chi = 767 \times 90\% + (4624 - 767) \times 32\% + (\bar{w} - 4624) \times 15\%$, where \bar{w} is the average income over the highest-earning 35 years. In Japan, as outlined by Kitao (2017), the pension benefit is $\chi = \bar{s}s + \rho \times e$, where $\bar{s}s$ is a fixed basic pension, and $\rho \times e$ is a portion proportional to an individual's career earnings (e). If a worker remains unemployed their life, they receive only the basic pension $\bar{s}s$. In Singapore, pension benefits depend on the balance

remaining in the Retirement Account, which can be withdrawn in full or in installments.

Finally, we assume free entry into each labor market. Consequently, the expected net payoff from posting a vacancy is zero, implying that

$$V_s = V_j = 0. \tag{17}$$

3. NASH BARGAINING

When a firm and an unemployed worker are matched, they negotiate the total surplus through Nash bargaining, where β represents the worker’s bargaining power. The bargaining problem is solved by maximizing the following objective:

$$Max_{w_i^k} (E_i^\kappa - U_i^\kappa)^\beta (J_i^\kappa - V_i)^{1-\beta}. \tag{18}$$

The solution to this bargaining problem must satisfy the following conditions:

$$(1 - \beta)(E_i^y - U_i^k) = \beta(J_i^\kappa - V_i). \tag{19}$$

This condition reflects the balance of power between the worker and the firm in determining the wage w_i^k , ensuring that the surplus is split according to their respective bargaining strengths.

3.1. Steady-State Composition of the Labor Market

To analyze the steady-state composition of the labor market, we must identify the following 14 variables: $u_j^y, e_j^y, P_j^y, u_j^o, e_j^o, P_j^o, u_s^y, e_s^y, P_s^y, u_s^o, e_s^o, P_s^o, RE_s^o$, and RE_j^o . At the steady state, the inflows and outflows for each variable must be balanced, i.e., net change must be zero. The following relationships hold:

$$u_j^y + e_j^y = P_j^y \equiv 1, \tag{20}$$

$$u_j^o + e_j^o = P_j^o, \tag{21}$$

$$u_s^y + e_s^y = P_s^y \equiv L, \tag{22}$$

$$u_s^o + e_s^o = P_s^o, \tag{23}$$

$$z_j P_j^y + s_j e_j^y = [\lambda_y + m(\theta_j)] u_j^y, \tag{24}$$

$$m(\theta_j)u_j^y = (\lambda_y + s_j)e_j^y, \quad (25)$$

$$s_j e_j^o + \lambda_y u_j^y = (m(\theta_j) + \lambda_o + d)u_j^o, \quad (26)$$

$$\lambda_y e_j^y + m(\theta_j)u_j^o = (\lambda_o + s_j + d)e_j^o, \quad (27)$$

$$z_s P_s^y + s_s e_s^y = [\lambda_y + m(\theta_s)]u_s^y, \quad (28)$$

$$m(\theta_s)u_s^y = (\lambda_y + s_s)e_s^y, \quad (29)$$

$$s_s e_s^o + \lambda_y u_s^y = m(\theta_s)u_s^o + (\lambda_o + d)u_s^o, \quad (30)$$

$$\lambda_y e_s^y + m(\theta_s)u_s^o = (\lambda_o + s_s + d)e_s^o, \quad (31)$$

$$\lambda_o P_j^o = dRE_j^o, \quad (32)$$

$$\lambda_o P_s^o = dRE_s^o. \quad (33)$$

Equations (20)-(23) represent the total labor force, where u_i^r and e_i^r denote the number of unemployed and employed workers in each group. The total labor force in the unskilled-intensive sector is normalized to one, with P_i^r and L being exogenously determined from data. Equations (24)-(27) describe the labor flow in the unskilled-intensive sector. For example, in Equation (24), young unskilled workers enter the labor force at rate z_j and are initially counted as unemployed, together with those who have separated from their jobs. This constitutes the inflow to u_j^y . The right-hand side of the equation represents the outflow, which includes unemployed individuals who either find employment or transition to the older age group. Equations (28)-(33) follow the same logic ensuring that inflows and outflows are balanced for each group at the steady state. From Equations (24), (25), (28), and (29), we derive $z_i = \lambda_y$, where $i = j, s$. Using Equations (26), (27), (20), we obtain $\frac{\lambda_y}{\lambda_o + d} = P_j^o$. From Equations (30), (31), and (22), we similarly derive $\frac{\lambda_y}{\lambda_o + d}L = P_s^o$. With these, we can now express the steady-state unemployment and employment levels as follows:

$$u_j^y = \frac{\lambda_y + s_j}{\lambda_y + s_j + m(\theta_j)}, \quad (34)$$

$$e_j^y = 1 - u_j^y = \frac{m(\theta_j)}{\lambda_y + s_j + m(\theta_j)}, \quad (35)$$

$$u_j^o = \frac{\lambda_y[(d + \lambda_o)(\lambda_y + s_j) + s_j(\lambda_y + m(\theta_j) + s_j)]}{[\lambda_o + d + s_j + m(\theta_j)](\lambda_o + d)[\lambda_y + s_j + m(\theta_j)]}, \quad (36)$$

$$e_j^o = \frac{\lambda_y}{\lambda_o + d} - u_j^o = \frac{\lambda_y m(\theta_j)[d + \lambda_o + \lambda_y + m(\theta_j) + s_j]}{[\lambda_o + d + s_j + m(\theta_j)](\lambda_o + d)[\lambda_y + s_j + m(\theta_j)]}. \quad (37)$$

For the skilled-intensive sector, we have:

$$u_s^y = \frac{(\lambda_y + s_s)L}{\lambda_y + s_s + m(\theta_s)}, \quad (38)$$

$$e_s^y = L - u_s^y = \frac{m(\theta_s)L}{\lambda_y + s_s + m(\theta_s)}, \quad (39)$$

$$u_s^o = \frac{\lambda_y[(d + \lambda_o)(\lambda_y + s_s) + s_s(\lambda_y + m(\theta_s) + s_s)]L}{[\lambda_o + d + s_s + m(\theta_s)](\lambda_o + d)[\lambda_y + s_s + m(\theta_s)]}, \quad (40)$$

$$e_s^o = \frac{\lambda_y L}{\lambda_o + d} - u_s^o = \frac{\lambda_y m(\theta_s)[d + \lambda_o + \lambda_y + m(\theta_s) + s_s]L}{[\lambda_o + d + s_s + m(\theta_s)](\lambda_o + d)[\lambda_y + s_s + m(\theta_s)]}. \quad (41)$$

Finally, for retirees, we have:

$$RE_j^o = \frac{\lambda_o \lambda_y}{d(\lambda_o + d)}, \quad (42)$$

$$RE_s^o = \frac{\lambda_o \lambda_y}{d(\lambda_o + d)} L. \quad (43)$$

These equations summarize the steady-state levels of employment, unemployment, and retirement across both the unskilled-intensive and skilled-intensive sectors.

3.2. Government Budget Constraint: Pay-As-You-Go

We now consider the government's budget constraint under a pay-as-you-go (PAYG) pension system. In contrast to a fund-based pension system, where tax revenues do not subsidize individual pension accounts, the PAYG system uses current tax income to finance pension payments. The following equation gives the budget constraint:

$$\begin{aligned} & (\Phi_j^y + \varphi_j^y) e_j^y + (\Phi_j^o + \varphi_j^o) e_j^o + (\Phi_s^y + \varphi_s^y) e_s^y + (\Phi_s^o + \varphi_s^o) e_s^o \\ & + \eta_j^o u_j^o + \eta_s^y u_s^y + \eta_s^o u_s^o \\ & + \tau_p[(p_j - \Phi_j^y) e_j^y + (\xi_j p_j - \Phi_j^o) e_j^o] + \tau_p[(p_s - \Phi_s^y) e_s^y + (\xi_s p_s - \Phi_s^o) e_s^o] \\ & + \tau_w[(w_s^y - \varphi_s^y) e_s^y + (w_s^o - \varphi_s^o) e_s^o] \\ & = \chi_s^o RE_s^o + \chi_j^o RE_j^o + b_s^y u_s^y + b_s^o u_s^o + G, \end{aligned} \quad (44)$$

where G represents the government's public spending. In this equation, the left-hand side represents the government's revenue, consisting of contributions from employed and unemployed workers (both young and old) in both sectors, as well as taxes on production and wages. The right-hand side reflects government expenditures, including pension payments χ to retirees, unemployment benefits b , and general public spending G . This budget constraint highlights how current labor force participation and taxes fund the pension system under the PAYG framework, which many countries use to support retirees and stimulate economic development through public investments.

3.3. Equilibrium

From Equation (17), we can derive a system of two equations with two unknowns:

$$\begin{aligned} & \frac{c_j (r + s_j + \lambda_o + d)}{q(\theta_j)} \\ &= \phi_j \left\{ \frac{r + s_j + \lambda_o + d}{r + s_j + \lambda_y} [(1 - \tau_p)(p_j - \Phi_j^y) - w_j^y] + \frac{\lambda_y}{r + s_j + \lambda_y} [(1 - \tau_p)(\xi_j p_j - \Phi_j^o) - w_j^o] \right\} \\ &+ (1 - \phi_j) [(1 - \tau_p)(\xi_j p_j - \Phi_j^o) - w_j^o], \end{aligned} \quad (45)$$

$$\begin{aligned} & \frac{c_s (r + s_s + \lambda_o + d)}{q(\theta_s)} \\ &= \phi_s \left\{ \frac{r + s_s + \lambda_o + d}{r + s_s + \lambda_y} [(1 - \tau_p)(p_s - \Phi_s^y) - w_s^y] + \frac{\lambda_y}{r + s_s + \lambda_y} [(1 - \tau_p)(\xi_s p_s - \Phi_s^o) - w_s^o] \right\} \\ &+ (1 - \phi_s) [(1 - \tau_p)(\xi_s p_s - \Phi_s^o) - w_s^o]. \end{aligned} \quad (46)$$

We now define the steady-state equilibrium for this economy.

DEFINITION 3.1. A steady-state equilibrium is a set $\{\theta_i^*, w_i^{\kappa^*}, u^{\kappa^*}\}$, where $i = j, s$ and $\kappa = y, o$, such that: (i) The free-entry (Equation 17) for each sector i is satisfied. (ii) The Nash bargaining optimality (Equation 19) for each sector i and generation κ holds. (iii) The flow of employed and unemployed workers, as well as of filled and unfilled vacancies for each type and generation, remains constant; i.e., Equations (24)-(33) are satisfied.

4. EVIDENCE FROM CHINA

4.1. Chinese Pension System

In the Chinese context, significant differences exist between workers in the unskilled-intensive and skilled-intensive sectors, despite both groups participated in the Basic Old Age Insurance (BOAI) system. In this section,

we explore these differences and adjust the pension-related variables in the general model accordingly.

As mandated by the Chinese government, firms and workers share the cost of pension contributions, with the minimum payment base set at 60% of the average wage. However, according to the 2018 Chinese Enterprise Social Security White Paper by 51Shebao, China’s largest third-party professional social security website, only 27% enterprises fully comply with the required payment standards. Consequently, we assume only firms in the skilled-intensive sector strictly adhere to the government’s requirements. Therefore, the pension-related variables are defined as follows:

$$\begin{aligned}
 \Phi_j^y &= \Phi_j^o = \varepsilon_r \times \bar{w} \times \varepsilon_f, \\
 \Phi_s^y &= w_s^y \times (\varepsilon_f + \varepsilon_u^f), \\
 \Phi_s^o &= w_s^o \times (\varepsilon_f + \varepsilon_u^f), \\
 \varphi_j^y &= \varphi_j^o = \varepsilon_r \times \bar{w} \times \varepsilon_w, \\
 \varphi_s^y &= w_s^y \times (\varepsilon_w + \varepsilon_u^w), \\
 \varphi_s^o &= w_s^o \times (\varepsilon_w + \varepsilon_u^w),
 \end{aligned}
 \tag{47}$$

where \bar{w} is the average annual wage from the previous year and $\varepsilon_r \bar{w}$ represents the minimum payment base for unskilled-intensive firms, where $\varepsilon_r = 60\%$. Firms and workers in the unskilled-intensive sector are required to contribute $\varepsilon_f = 16\%$ and $\varepsilon_w = 8\%$ of the minimum pension insurance payment base, respectively. In the skilled-intensive sector, firms and workers also contribute to unemployment insurance, with firms paying an additional $\varepsilon_u^f = 2\%$ and workers paying $\varepsilon_u^w = 1\%$ of their respective wages.

Unemployed or self-employed workers can also participate in the BOAI as flexible workers (LingHuoJiuYe), as the continuity of pension contributions significantly impacts future pension benefits. To avoid interruptions in their pension contributions, unemployed workers must pay at a higher contribution rate. According to the report by Public Pension Schemes in China, unskilled young workers tend to pay less attention to pension insurance.⁴ As a result, when they become unemployed, they are more likely to suspend payments, a phenomenon referred to as “DuanBao.” These workers typically resume their pension contributions once they reach old age.

Unemployed workers receive unemployment benefits that are approximately 90% of the minimum wage ($\varepsilon_b = 90\%$) in most provinces. Accordingly, we assume the following relationships for pension and unemployment

⁴Please refer to “Public Pension Schemes in China: A Study on Social for Flexible Employment Personnel and Platform Employees in China”, ILO and MOHRSS, 2022.

benefits:

$$\begin{aligned}
 \eta_j^o &= \varepsilon_r \times \bar{w} \times \varepsilon_I, \\
 \eta_s^y &= w_s^y \times \varepsilon_I, \\
 \eta_s^o &= w_s^o \times \varepsilon_I, \\
 b_s^y &= b_s^o = \varepsilon_b \times \underline{w},
 \end{aligned} \tag{48}$$

where $\varepsilon_I = 20\%$ represents the contribution rate required of flexible workers for pension insurance.

In China, pension benefits consist of two components: the basic pension and the personal account pension.⁵ The basic pension is calculated based on an individual's wage and the average social wage, increasing by 1% for every additional year of contributions. The personal account pension is determined by the savings accumulated in the account, divided by the number of counting months. We assume that individual and average social wages remain constant to simplify the model. Therefore, the pension benefits χ are expressed as:

$$\begin{aligned}
 \chi_j^o &= \frac{1}{2} \bar{w} (1 + i_j) \times n_j \times 1\% + \frac{IC_j \times \varepsilon_w}{Denom} \times 12, \\
 \chi_s^o &= \frac{1}{2} \bar{w} (1 + i_s) \times n_s \times 1\% + \frac{IC_s \times \varepsilon_w}{Denom} \times 12,
 \end{aligned} \tag{49}$$

where $n_j = (1 - \frac{u_j^y}{e_j^y + u_j^y}) \times (35 - 16) + (65 - 35)$ and $n_s = 65 - 16$ represent the accumulated years of pension contributions for unskilled workers and skilled workers, respectively. The term $\frac{u_j^y}{e_j^y + u_j^y}$ denotes the probability of unemployment among unskilled young workers, which is also the probability of "DuanBao" (suspension of pension contributions). The wage indices are defined as $i_j = \varepsilon_r$ and $i_s = \frac{w_s^o}{\bar{w}}$.

The individual pension account values are:

$$\begin{aligned}
 IC_j &= \varepsilon_r \times \bar{w} \times n_j, \\
 IC_s &= (35 - 16) \times w_s^y + (65 - 35) \times w_s^o.
 \end{aligned} \tag{50}$$

The government encourages delayed retirement, which reduces the denominator "Denom" in the personal account pension formula. We assume "Denom" to be 101 if workers retire at age 65. This framework captures the basic pension and personal account components, factoring in differences between skilled and unskilled workers, as well as the impact of employment disruptions like "DuanBao" on pension outcomes.

⁵For simplicity, we do not consider the transitional pension in this analysis.

4.2. Equilibrium Analysis

4.2.1. Wage

Substituting the value functions J , E , U , and Equation (19), the equilibrium wage levels can be expressed as follows:

$$w_j^o = \frac{r + s_j + m(\theta_j) + \lambda_o + d}{r + s_j + \beta m(\theta_j) + \lambda_o + d} \beta (1 - \tau_p) (\xi_j p_j - \Phi_j^o) + \frac{r + s_j + \lambda_o + d}{r + s_j + \beta m(\theta_j) + \lambda_o + d} (1 - \beta) (\varphi_j^o - \eta_j^o), \quad (51)$$

$$w_j^y = \frac{r + s_j + \lambda_y + m(\theta_j)}{r + s_j + \beta m(\theta_j) + \lambda_y} \beta \left\{ (1 - \tau_p) (p_j - \Phi_j^y) + \frac{\lambda_y}{r + s_j + \lambda_o + d} [(1 - \tau_p) (\xi_j p_j - \Phi_j^o) - w_j^o] \right\} + \frac{r + s_j + \lambda_y}{r + s_j + \beta m(\theta_j) + \lambda_y} (1 - \beta) \left[\varphi_j^y + \frac{\lambda_y}{r + s_j + m(\theta_j) + \lambda_o + d} (\varphi_j^o - \eta_j^o - w_j^o) \right], \quad (52)$$

$$w_s^o = \frac{\frac{(1-\beta)b_s^o}{r+s_s+m(\theta_s)+\lambda_o+d} + \frac{\beta(1-\tau_p)\xi_s p_s}{r+s_s+\lambda_o+d}}{(1-\beta)\frac{(1-\tau_w)(1-\varepsilon_w-\varepsilon_w^w)+\varepsilon_I}{r+s_s+m(\theta_s)+\lambda_o+d} + \beta\frac{(1-\tau_p)(\varepsilon_f+\varepsilon_u^f)+1}{r+s_s+\lambda_o+d}}, \quad (53)$$

$$w_s^y = \frac{(1-\beta)\frac{b_s^y-\lambda_y\frac{(1-\tau_w)(w_s^o-\varphi_s^o)-b_s^o+\eta_s^o}{r+s_s+m(\theta_s)+\lambda_o+d}}{r+s_s+m(\theta_s)+\lambda_y} + \beta\frac{(1-\tau_p)p_s+\lambda_y\frac{(1-\tau_p)(\xi_s p_s-\Phi_s^o)-w_s^o}{r+s_s+\lambda_o+d}}{r+s_s+\lambda_y}}{(1-\beta)\frac{(1-\tau_w)(1-\varepsilon_w-\varepsilon_w^w)+\varepsilon_I}{r+s_s+m(\theta_s)+\lambda_y} + \beta\frac{(1-\tau_p)(\varepsilon_f+\varepsilon_u^f)+1}{r+s_s+\lambda_y}}. \quad (54)$$

4.2.2. Tax Rates

Rewriting Equation (44), the equilibrium tax rates τ_p^* and τ_w^* , as well as other policy-related rates ε_I^* , ε_f^* , and ε_w^* , are given by:

$$\tau_p^* = \frac{\left\{ \begin{aligned} &\chi_j^o RE_j^o + \chi_s^o RE_s^o + u_s^o (b_s^o - \eta_s^o) + u_s^y (b_s^y - \eta_s^y) + G - e_s^o [\tau_w (w_s^o - \varphi_s^o) + \Phi_s^o + \varphi_s^o] \\ &- e_s^y [\tau_w (w_s^y - \varphi_s^y) + \Phi_s^y + \varphi_s^y] - u_j^o \eta_j^o - e_j^o (\Phi_j^o + \varphi_j^o) - e_j^y (\Phi_j^y + \varphi_j^y) \end{aligned} \right\}}{e_j^y (p_j - \Phi_j^y) + e_s^y (p_s - \Phi_s^y) + e_j^o (\xi_j p_j - \Phi_j^o) + e_s^o (\xi_s p_s - \Phi_s^o)}, \quad (55)$$

$$\tau_w^* = \frac{\left\{ \begin{aligned} &\chi_j^o RE_j^o + \chi_s^o RE_s^o + u_s^o (b_s^o - \eta_s^o) + u_s^y (b_s^y - \eta_s^y) - u_j^o \eta_j^o + G - e_s^o [\Phi_s^o + \varphi_s^o + \tau_p (\xi_s p_s - \Phi_s^o)] \\ &- e_s^y [\Phi_s^y + \varphi_s^y + \tau_p (p_s - \Phi_s^y)] - e_j^y (\Phi_j^y + \varphi_j^y + \tau_p (p_j - \Phi_j^y)) - e_j^o (\Phi_j^o + \varphi_j^o + \tau_p (\xi_j p_j - \Phi_j^o)) \end{aligned} \right\}}{e_s^o (w_s^o - \varphi_s^o) + e_s^y (w_s^y - \varphi_s^y)}, \quad (56)$$

$$\varepsilon_I^* = \frac{\left\{ \begin{array}{l} \chi_j^o RE_j^o + \chi_s^o RE_s^o + u_s^o b_s^o + u_s^y b_s^y + G - e_s^o [\tau_p(\xi_s p_s - \Phi_s^o) + \tau_w(w_s^o - \varphi_s^o) + \Phi_s^o + \varphi_s^o] \\ -e_s^y [\tau_p(p_s - \Phi_s^y) + \tau_w(w_s^y - \varphi_s^y) + \Phi_s^y + \varphi_s^y] - e_j^y [\tau_p(p_j - \Phi_j^y) + \Phi_j^y + \varphi_j^y] \\ -e_j^o [\tau_p(\xi_j p_j - \Phi_j^o) + \Phi_j^o + \varphi_j^o] \end{array} \right\}}{u_j^o \varepsilon_r \bar{w} + u_s^o w_s^o + u_s^y w_s^y}, \quad (57)$$

$$\varepsilon_f^* = \frac{\left\{ \begin{array}{l} \chi_j^o RE_j^o + \chi_s^o RE_s^o + u_s^o (b_s^o - \eta_s^o) + u_s^y (b_s^y - \eta_s^y) + G - u_j^o \eta_j^o \\ -e_s^o [\tau_p(\xi_s p_s - \varepsilon_u^f w_s^o) + \tau_w(w_s^o - \varphi_s^o) + \varphi_s^o + \varepsilon_u^f w_s^o] \\ -e_s^y [\tau_p(p_s - \varepsilon_u^f w_s^y) + \tau_w(w_s^y - \varphi_s^y) + \varepsilon_u^f w_s^y + \varphi_s^y] - e_j^y (\tau_p p_j + \varphi_j^y) - e_j^o (\tau_p \xi_j p_j + \varphi_j^o) \end{array} \right\}}{(1 - \tau_p)[(e_j^y + e_j^o)\varepsilon_r \bar{w} + e_s^y w_s^y + e_s^o w_s^o]}, \quad (58)$$

$$\varepsilon_w^* = \frac{\left\{ \begin{array}{l} \frac{1}{2}(1 + i_j) \bar{w} n_j RE_j 1\% + \frac{1}{2}(1 + i_s) \bar{w} n_s RE_s 1\% + G \\ -e_s^o [\tau_p(\xi_s p_s - \Phi_s^o) + \Phi_s^o + \tau_w(w_s^o - \varepsilon_u^w w_s^o) + \varepsilon_u^w w_s^o] \\ -e_s^y [\Phi_s^y + \tau_p(p_s - \Phi_s^y) + \tau_w(w_s^y - \varepsilon_u^w w_s^y) + \varepsilon_u^w w_s^y] - e_j^y [\tau_p(p_j - \Phi_j^y) + \Phi_j^y] \\ -e_j^o [\tau_p(\xi_j p_j - \Phi_j^o) + \Phi_j^o] - u_j^o \eta_j^o + u_s^o (b_s^o - \eta_s^o) - u_s^y (b_s^y - \eta_s^y) \end{array} \right\}}{\varepsilon_r \bar{w} (e_j^o + e_j^y)\varepsilon_r \bar{w} + (1 - \tau_w)(e_s^o w_s^o + e_s^y w_s^y) - \frac{12(IC_j RE_j^o + IC_s RE_s^o)}{\text{Denom}}}. \quad (59)$$

4.2.3. The Effect of Retirement Delay on the Labor Market

LEMMA 1. *The effects of market tightness θ_j on the labor market are as follows:*

$$\begin{aligned} \frac{\partial w_j^o}{\partial \theta_j} &> 0, & \frac{\partial u_j^y}{\partial \theta_j} &< 0, \\ \frac{\partial e_j^y}{\partial \theta_j} &> 0, & \frac{\partial (u_j^o/P_j^o)}{\partial \theta_j} &< 0, \\ \frac{\partial (e_j^o/P_j^o)}{\partial \theta_j} &> 0, & \frac{\partial RE_j^o}{\partial \theta_j} &= 0, \\ \frac{\partial \phi_j}{\partial \theta_j} &> 0, & \frac{\partial w_j^y}{\partial \theta_j} &\leq 0. \end{aligned}$$

Proof. See appendix. ■

An increase in market tightness θ_j makes it easier for workers to find jobs, resulting in a lower unemployment rate and an increase in w_j^o . The situation is more complex for young workers in this sector, as firms also consider their potential productivity when they age. On one hand, higher θ_j may increase w_j^y for the same reasons as for w_j^o . On the other hand, if w_j^o rises, firms may seek to reduce w_j^y to manage costs, leading to an

ambiguous effect on w_j^y . Additionally, labor market tightness does not affect the number of retirees, which remains constant at the steady state, as demonstrated in Equation (42).

LEMMA 2. *The effects of market tightness θ_s on the labor market are as follows:*

$$\begin{aligned} \frac{\partial u_s^y/P_s^y}{\partial \theta_s} < 0, & \quad \frac{\partial e_s^y/P_s^y}{\partial \theta_j} > 0, \\ \frac{\partial (u_s^o/P_s^o)}{\partial \theta_s} < 0, & \quad \frac{\partial (e_s^o/P_s^o)}{\partial \theta_s} > 0, \\ \frac{\partial (RE_s^o)}{\partial \theta_s} = 0, & \quad \frac{\partial \phi_s}{\partial \theta_s} > 0, \\ \frac{\partial w_s^o}{\partial \theta_s} \leq 0, & \quad \frac{\partial w_s^y}{\partial \theta_s} \leq 0. \end{aligned}$$

Proof. See appendix. ■

The unemployment and employment rates in the skilled-intensive sector exhibit symmetry with those in the unskilled-intensive sector. However, wage dynamics in the skilled sector become more complex due to the wage-dependent pension payments, making the signs of the last two derivatives ambiguous. For instance, an increase in θ_s raises w_s^o by increasing the matching rate. However, higher wages for the old also increase pension payments, reducing overall expected utility for firms. This reduction in expected utility, driven by the increased pension contribution Φ_s^o , may lead firms to lower wages for old workers in order to contain costs.

Similarly, a higher pension payment η_s^o for unemployed (or self-employed) workers also reduces the wage level as the value of the employed worker’s outside option decreases. At the same time, higher pension payments with a larger φ_s^o can reduce the expected utility of employees, potentially increasing wages. Therefore, the effect on the wage level for the young is ambiguous, similar to the dynamics described in Lemma 1. Given the uncertain effect on w_s^o , the effect on the wage level for the young, w_s^y , is also indeterminate.

PROPOSITION 1. *There exists a unique labor market tightness θ_j (in the unskilled-intensive sector) and θ_s (in the skilled-intensive sector).*

Proof. See appendix. ■

We assume $\frac{(1-\tau_p)(\xi_j p_j - \Phi_j^o) - w_j^o}{r+s_j+\lambda_o+d} > \frac{(1-\tau_p)(p_j - \Phi_j^y) - w_j^y}{r+s_j}$, i.e., $J_j^o > J_j^y \Big|_{\lambda_y=0}$,

implying that hiring an old worker is more beneficial than hiring a young worker. As θ_j increases, the probability of matching with a young worker ϕ_j rises, potentially reducing the firm’s expected value. However, an increased θ_j can also reduce wage levels, as analyzed in Lemma 1 and 2, thereby improving the firm’s expected value. The second effect must dominate the first to ensure a unique θ_j .

Similarly, for the skilled-intensive sector, we need $\frac{(1-\tau_p)(\xi_s p_s - \Phi_s^o) - w_s^o}{r+s_s+\lambda_o+d} > \frac{(1-\tau_p)(p_s - \Phi_s^y) - w_s^y}{r+s_s}$ to ensure a unique θ_s , i.e., $J_s^o > J_s^y \Big|_{\lambda_y=0}$, with the negative effect on the firm’s expected value being dominated.

PROPOSITION 2. *The effects of λ_o on labor market tightnesses are ambiguous. However, under some conditions, we may observe:*

$$\frac{\partial \theta_j}{\partial \lambda_o} < 0, \quad \frac{\partial \theta_s}{\partial \lambda_o} > 0.$$

Proof. See appendix. ■

The effects of retirement delay on market tightnesses in both sectors are ambiguous. If we assume that $J_j^o - J_j^y \Big|_{\lambda_y=0} > 0$, meaning that hiring an old worker is more beneficial, a decrease in λ_o increases the probability of matching with an old worker, benefiting the firm. However, changes in λ_o also affect the firm’s value by altering wages for both young and old workers. Therefore, if the negative effect on the firm’s value dominates when λ_o decreases, we have $\frac{\partial \theta_j}{\partial \lambda_o} < 0$. The proof for $\frac{\partial \theta_s}{\partial \lambda_o}$ follows a similar logic, except that the positive effect on the firm’s value dominates.

5. QUANTITATIVE ANALYSIS

5.1. Calibration

We calibrate our model to using annual Chinese data for the 1990–1999 period. A Cobb-Douglas matching function is employed, given by $M = M_0 u_i^\xi v_i^{1-\epsilon}$, where M_0 represents the efficiency of the matching process, which we normalize to one. The model economy is characterized by 26 parameters: the unemployment elasticity of the matching function ϵ , the interest rate r , the average separation rates s_j and s_s , the rate of aging λ_y ,

the retirement rate λ_o , the mortality rate of the old d , the prices of intermediate goods in both sectors p_j and p_s , productivity differences between young and old workers in both sectors ξ_j and ξ_s , the minimal payment rate for unskilled-intensive firms ε_r , the average annual wage from the previous year \bar{w} , the pension insurance contribution rate for by unskilled-intensive firms ε_f , the unemployment insurance contribution rate for firms ε_u^f , job creation costs for both markets c_j and c_s , the minimum wage \underline{w} , the pension insurance contribution rate for employed workers ε_w , the unemployment insurance contribution rate for workers ε_u^w , the pension insurance contribution rate for unemployed workers ε_I , the unemployment benefit rate ε_b , wage and production tax rates τ_w and τ_p , the number of young workers in the senior labor market L , and the worker's bargaining power β .

Using data from the China Statistical Yearbook, we first calculate the annual real interest rate as 3.9%, based on the average 30-year treasury bond rate. Second, we set the rate of aging λ_y equal to the birth rate in the model, resulting in $\lambda_y = 0.122$, and calculate the mortality rate for old workers as $d = 0.068$. Third, we normalize the minimum wage to 1,000 yuan, with the average minimum wage \underline{w} for major cities set at is 0.5067 for major cities. Fourth, we set the worker's bargaining power β to 0.5, aligning $\varepsilon = \beta$ to satisfy the Hosios condition, which states that the unemployment elasticity of the matching function should equal the bargaining power (see Hosios, 1990). Fifth, following Dai et al. (2022), we set $s_j = 0.01$, $c_j = 0.1$, $\xi_j = 1.2$ and $\xi_s = 1.5$. We assume both sectors have identical separation rates and job posting costs. Sensitivity tests will be conducted to examine the impact of this assumption, as well as productivity differences.

Each province's Department of Human Resources and Social Security determines the specific pension and unemployment insurance rates, leading to slight variations across provinces. For this study, we adopt the most commonly used rates: $\varepsilon_r = 0.6$, $\varepsilon_f = 0.16$, $\varepsilon_u^f = 0.02$, $\varepsilon_w = 0.08$, $\varepsilon_u^w = 0.01$, $\varepsilon_b = 0.9$, $\varepsilon_I = 0.2$. Tax rates on production price and wage income are set at 0.25 and 0.15, respectively, which fall within the range established by the State Taxation Administration.

The remaining parameters are calibrated jointly to match two targets. First, the old-age dependence ratio is set at 0.11, based on data from the China Population and Employment Statistical Yearbook (CPESY). Second, the aggregate vacancy-to-unemployment ratio is 0.948, as reported in the China Labour Statistical Yearbook. The full calibration results are summarized in Table 1.

TABLE 1.
Calibration results

Parameters	Values	Interpretation
ϵ	0.5	The unemployment elasticity of the matching function.
β	0.5	The worker's bargaining power.
ϵ_r	0.6	The minimal payment rate required of unskilled-intensive firms.
ϵ_f	0.16	The pension insurance rate paid by unskilled-intensive firms.
ϵ_u^f	0.02	The rate of unemployment insurance that firms must contribute.
ϵ_w	0.08	The share of pension insurance that employees must contribute.
ϵ_u^w	0.01	The required share of unemployment insurance paid by employees.
ϵ_b	0.9	The rate used to calculate unemployment benefits.
ϵ_I	0.2	The share of insurance that unemployed workers must contribute.
τ_p	0.25	The tax rate on production price.
τ_w	0.15	The tax rate on wage income.
<i>Calculated from China Statistical Yearbook:</i>		
r	0.039	The annual interest rate.
λ_y	0.122	The rate of aging.
d	0.068	The mortality rate of the old.
\underline{w}	0.5067	The minimum wage (in thousands).
<i>Jointly Calibrated to Match data from CPESY:</i>		
λ_o	1.02	The average vacancy-to-unemployment ratio in the senior labor market.
L	0.78	The aggregate vacancy-to-unemployment ratio.

5.2. Counterfactual Experiments

We will conduct six counterfactual experiments to analyze the effects of delayed retirement on the labor market and evaluate how changes in associated fiscal policies influence these outcomes.

First, delaying retirement corresponds to a decrease in λ_o , the retirement rate. We examine the percentage changes in labor market variables by reducing the probability of retirement by 5% while keeping other parameters constant. Table 2 presents the results, showing that a reduced retirement probability leads to a 0.78% increase in market tightness θ_j for the unskilled-intensive sector and a 0.1% decrease in market tightness θ_s for the skilled-intensive sector. Wages increase across all groups except for young workers in the unskilled-intensive labor market. Unemployment for

young workers declines by 0.23%, while unemployment for old workers rises by 0.42%.

As λ_o decreases, the total number of old unemployed workers increases, raising firms' meeting rates with the old and incentivizing firms to post more vacancies, positively affecting market tightness in both markets. However, the larger pool of unemployed old workers exerts downward pressure on market tightnesses, particularly in the skilled-intensive sector. Moreover, the inability of firms to discriminate based on age creates a positive spillover effect across both labor markets. Overall, the positive effect dominates in the unskilled-intensive sector, while the negative effect prevails in the skilled-intensive sector. Consequently, market tightness θ_j rises, θ_s falls, and the unemployment rate for young workers decreases while that of the old increases. Furthermore, as delayed retirement expands the tax base, we find that the government could reduce contribution rates or taxes to maintain optimal levels of fiscal balance.

TABLE 2.

The effects of delayed retirement (λ_o): percentage changes

	Unskilled-Intensive		Skilled-Intensive				
w_j^y	-0.09	w_j^o	0.40	w_s^y	0.004	w_s^o	0.01
u_j^y	-0.34	u_j^o	-0.06	u_s^y	0.04	u_s^o	0.51
θ_j	0.78	UR^y	-0.23	θ_s	-0.10	UR^o	0.42
Average wage	0.12	Total UR	-0.63	τ_p^*	-3.10	τ_w^*	-3.23
ε_I^*	-2.62	ε_f^*	-2.22	ε_w^*	-2.30		

Second, we examine the effect of reducing the required share of pension insurance contributions paid by unemployed workers. This rate, which is also applicable to self-employed workers, is set at 20% in the baseline model, a level considered high in China. Table 3 shows that a 5% decrease in ε_I result in a 13.10% decline in market tightness θ_j in the unskilled-intensive sector and a 5.62% decline in market tightness θ_s in the skilled-intensive sector. The wages of young workers in the unskilled-intensive labor market rise by 0.32%, while wages for old workers in the skilled-intensive labor market increase by a modest 0.02%. For workers who span both markets, their wages increase by 0.37% and 0.07%, respectively.

The number of unemployed individuals rises between 2.55% and 9.92% depending on the worker type, leading to a 12.46% overall increase in the unemployment rate. Specifically, the unemployment rate for young workers increases by 5.24%, and for old workers, it rises by 8.07%. The decrease in ε_I boosts the flow value for unemployed workers and their wages, nega-

tively impacting market tightness. Conversely, retirees' flow value increases through a tax-benefit channel, positively affecting market tightness. However, the negative effect dominates, resulting in a decline in market tightness in both markets. The impact is smaller in the skilled-intensive sector because pension contributions are based on real wages rather than the average wages. Consequently, while a reduction in ε_I leads to higher wages, it also increases pension payments for workers and firms, limiting wage growth in this sector.

Overall, the decrease in market tightness causes equilibrium wages to rise and unemployment rates to increase across all worker types. The government must offset these effects by raising other contribution rates or taxes to balance the budget constraints.

TABLE 3.

The effects of required share paid by unemployed workers (ε_I): percentage changes

	Unskilled-Intensive		Skilled-Intensive				
w_j^y	0.32	w_j^o	0.37	w_s^y	0.07	w_s^o	0.02
u_j^y	6.38	u_j^o	9.92	u_s^y	2.54	u_s^o	3.99
θ_j	-13.10	UR^y	5.24	θ_s	-5.62	UR^o	8.07
Average wage	0.23	Total UR	5.78	τ_p^*	0.62	τ_w^*	0.62
ε_f^*	0.46	ε_w^*	0.64				

Third, we examine the impact of reducing firms' required share of pension insurance. In China, firms are required to contribute at least 16% of the minimum pension insurance base, which is considered high relative to other countries. Table 4 shows that a 5% reduction in ε_f significantly affects the unskilled-intensive labor market. Specifically, market tightness θ_j increases by 29.36%, while θ_s in the skilled-intensive market rises by 2.96%. The wages of young workers in the unskilled-intensive sector increase by 10.83%, and the wages of old workers in the skilled-intensive sector rise by 10.52%. For workers who are active across both markets, their wages increase by 24.24% and 0.5%, respectively.

Unemployment decreases by 1.26% to 15.84% across different groups, with the unemployment rates of both young and old workers declining. However, despite these improvements, the total unemployment rate increases by 8.66%. A reduction in ε_f lowers hiring costs, which in turn boosts market tightness by encouraging firms to post more vacancies. On the downside, the flow value of retirees declines as firms contribute less to pensions, exerting downward pressure on market tightness in both sectors. Nonetheless, the positive effect of lowering hiring costs dominates, espe-

cially in the unskilled-intensive sector, because the payment base does not change, which further helps the firm save costs. Finally, we show that the government would need to increase other contribution rates or raise the wage tax to satisfy budget constraints.

TABLE 4.
The effects of required share paid by firms (ε_f): percentage changes

	Unskilled-Intensive		Skilled-Intensive				
w_j^y	10.83	w_j^o	24.24	w_s^y	0.50	w_s^o	0.52
u_j^y	-10.84	u_j^o	-15.84	u_s^y	-1.26	u_s^o	-1.95
θ_j	29.39	UR^y	-7.99	θ_s	2.96	UR^o	-11.52
Average wage	12.75	Total UR	-8.66	τ_p^*	-7.31	τ_w^*	2.53
ε_f^*	20.70	ε_w^*	2.21				

Fourth, we analyze the effect of increasing the required share of pension insurance paid by employed workers. In the baseline model, this rate is set at 8%, which is considered low compared to international standards. Table 5 shows that a 5% increase in ε_w lead to a 16.41% decline in market tightness θ_j in the unskilled-intensive sector and a 1.9% decreases in θ_s in the skilled-intensive sector. Wages increase across all groups except for the old in the unskilled-intensive sector. Unemployment rises by 0.84% to 12.83% across different worker types, resulting in a 6.63% increase in the total unemployment rate.

An increase in ε_w reduces the flow value of employment, negatively affecting market tightness in both sectors. Although higher contributions also increase the flow value of retirees, which positively affects market tightness, the overall negative impact dominates. Consequently, unemployment rates increase for all worker types, with a more pronounced effect in the unskilled-intensive labor market. This is because pension contributions constitute a larger share of wages in the unskilled-intensive market compared to the skilled-intensive market, making the unskilled-intensive market more sensitive to changes in ε_w . Holding other parameters constant, the results indicate that to balance the government’s budget constraint, raising τ_w^* , and ε_f^* while reducing τ_p^* and ε_I^* are necessary.

Fifth, we examine the impact of increasing the production tax on the labor market. Table 6 shows that a 5% rise in τ_p leads to a 1.99% reduction in market tightness θ_j in the unskilled-intensive sector and an 8% reduction in θ_s in the skilled-intensive sector. Wages decline by 0.53% to 2.82% across different worker groups, while the number of unemployed increases by 0.89% to 5.80%. Overall, the total unemployment rate rises by 1.91%.

TABLE 5.

The effects of required share paid by employed workers (ε_w): percentage changes

	Unskilled-Intensive			Skilled-Intensive			
w_j^y	3.52	w_j^o	-2.74	w_s^y	0.02	w_s^o	0.01
u_j^y	8.20	u_j^o	12.83	u_s^y	0.84	u_s^o	1.31
θ_j	-16.41	UR^y	6.01	θ_s	-1.90	UR^o	9.25
Average wage	-0.02	Total UR	6.63	τ_p^*	-0.17	τ_w^*	0.03
ε_I^*	-10.92	ε_f^*	0.03				

An increase in production tax raises the cost for firms, resulting in fewer posted vacancies in both labor markets. Consequently, market tightness decreases in both sectors, leading to lower wages and higher unemployment across all worker types. The impact is more pronounced in the skilled-intensive sector, where firms face higher tax burdens due to greater productivity, making the intensive-skilled labor market more sensitive to changes in the production tax. To balance the government’s budget, holding other parameters constant, the results suggest that the government should increase τ_w and ε_f^* while reducing ε_I^* and ε_w^* .

TABLE 6.

Production tax (τ_p): percentage changes

	Unskilled-Intensive			Skilled-Intensive			
w_j^y	-0.53	w_j^o	-2.82	w_s^y	-1.33	w_s^o	-1.50
u_j^y	0.89	u_j^o	1.36	u_s^y	3.68	u_s^o	5.80
θ_j	-1.99	UR^y	1.72	θ_s	-8.00	UR^o	2.74
Average wage	-1.72	Total UR	1.91	τ_w^*	1.45	ε_I^*	-1.82
ε_f^*	0.79	ε_w^*	-0.32				

Finally, we analyze the impact of an increase in the wage tax on the labor market by raising τ_w by 5%. Table 7 shows that this results in a modest 0.02% increase in market tightness θ_j in the unskilled-intensive sector, while θ_s in the skilled-intensive sector declines by 3.91%. The unemployment rate for the young increases by 0.51%, and for old workers by 0.84%, leading to a total unemployment rate increase of 0.57%.

The impact on the skilled-intensive sector is more pronounced than in the unskilled-intensive sector, as the wage tax primarily affects workers in the skilled-intensive market. As a result, the skilled-intensive labor market is more responsive to changes in τ_w . Additionally, the findings suggest a higher optimal production tax rate τ_p^* and lower optimal contribution rates would be necessary to maintain fiscal balance under these conditions.

TABLE 7.

Wage tax (τ_w): percentage changes

	Unskilled-Intensive		Skilled-Intensive				
w_j^y	-0.003	w_j^o	0.005	w_s^y	0.05	w_s^o	0.01
u_j^y	-0.01	u_j^o	-0.01	u_s^y	1.75	u_s^o	2.73
θ_j	0.02	UR^y	0.51	θ_s	-3.91	UR^o	0.84
Average wage	-0.0003	Total UR	0.57	τ_p^*	0.01	ε_I^*	-0.19
ε_f^*	-0.02	ε_w^*	-0.05				

5.3. Sensitivity Tests

Given the productivity difference between young and old workers in both sectors (ξ_j and ξ_s) may be either greater or less than one, we conduct sensitivity tests by varying these parameters while holding all others constant. The results, summarized in Table 8, corresponding to Table 2, reveals that θ_j and the unemployment rates in the unskilled-intensive labor market are the most responsive to changes in ξ_j . As the productivity difference between young and old workers increases, old workers become more productive in the unskilled-intensive labor market, leading to a higher employment value for firms hiring old workers. Because firms cannot pre-select the type of workers they match with, they must post more vacancies in the unskilled-intensive market to match with more old workers. Consequently, θ_j rises and unemployment declines in this sector.

Similarly, θ_s and unemployment rates in the skilled-intensive labor market are the most responsive to ξ_s changes. As old workers become more productive in the skilled-intensive sector, θ_s increases and unemployment decreases. As a result, the optimal production tax, wage tax, ε_f^* and ε_w^* all decreases with higher values of ξ_j and ξ_s . This is because a greater production differential leads to higher output and a broader tax base, allowing for a reduction in these tax rates and contribution requirements to balance the government budget constraint. However, the model suggests that it may be optimal for the government to increase ε_I^* to encourage unemployed workers to find jobs. A higher contribution rate for unemployed workers would incentivize job-seeking behavior, thereby reducing unemployment and supporting overall labor market efficiency.

Finally, we conduct sensitivity tests on the parameters s_s and c_s for the skilled-intensive sector. In the baseline model, we assume that $s_s = s_j$ and $c_s = c_j$. This analysis tests scenarios where s_s and c_s are half, double, and four times their corresponding values in the unskilled-intensive labor market while holding other parameters constant. Table 9 shows the results for

TABLE 8.

Sensitivity with respect to ξ_j and ξ_s : The effects of λ_o (percentage changes)

%changes	$\xi_j = 1$	$\xi_j = 1.5$	$\xi_j = 2$	$\xi_s = 1$	$\xi_s = 2$	$\xi_s = 3$	Baseline
Unskilled-Intensive							
w_j^y	3.10	-7.49	-15.29	-0.10	-0.10	-1.0	-0.09
w_j^o	-4.26	10.48	20.25	0.4	0.4	0.4	0.40
u_j^y	8.71	-19.42	-37.21	-0.35	-0.35	-0.35	-0.34
u_j^o	14.16	-27.13	-48.92	-0.07	-0.07	-0.07	-0.06
θ_j	-17.30	61.91	178.95	0.79	0.79	0.79	0.78
UR^y	6.09	-13.67	-26.17	36.63	-6.51	-11.72	-0.23
Skilled-Intensive							
w_s^y	0.002	0.002	-0.002	0.97	-0.42	-0.86	0.004
w_s^o	0.02	0.02	0.02	-3.62	0.72	1.33	0.01
u_s^y	-0.10	-0.10	-0.10	123.94	-21.06	-38.57	0.04
u_s^o	0.29	0.29	0.29	287.75	-29.86	-51.23	0.51
θ_s	0.22	0.22	0.22	-86.76	70.75	196.43	-0.10
UR^o	10.18	-18.36	-33.42	89.98	-9.05	-15.71	0.42
Aggregates							
Average wage	-0.75	1.79	3.12	-1.14	0.33	0.50	0.12
Total UR	6.31	-15.01	-27.93	45.97	-7.48	-12.93	-0.63
τ_p^*	-2.61	-4.12	-5.01	1.56	-3.86	-4.47	-3.10
τ_w^*	-3.02	-3.74	-4.19	15.66	-5.89	-7.99	-3.23
ε_I^*	-13.60	28.89	74.97	-12.67	-0.97	0.37	-2.62
ε_f^*	-1.65	-3.36	-4.37	-1.42	-2.35	-2.45	-2.22
ε_w^*	-1.57	-3.76	-5.07	-1.90	-2.36	-2.41	-2.30

the unskilled-intensive labor market are relatively insensitive to changes in s_s and c_s , whereas the skilled-intensive market exhibits significant sensitivity. A higher s_s and c_s directly affects firms' propensity to post vacancies in the skilled-intensive sector, leading to substantial fluctuations in market tightness θ_s , the number of unemployed workers, and unemployment rates. Optimal taxes, ε_f^* , and ε_w^* increase with rising c_s , as a higher job creation cost reduces the tax base. On the other hand, optimal taxes and ε_I^* decrease, while ε_f^* and ε_w^* increase with higher s_s .

6. CONCLUSION

We utilize a labor search model with two different skill-intensive markets, where both the young and old can participate, to examine how various components of pension systems and fiscal policies affect labor markets,

TABLE 9.

Sensitivity with respect to s_s and c_s : The effects of λ_o (percentage changes)

%changes	$c_s = 0.5c_j$	$c_s = 2c_j$	$c_s = 4c_j$	$s_s = 0.5s_j$	$s_s = 2s_j$	$s_s = 4s_j$	Baseline
Unskilled-Intensive							
w_j^y	-0.10	-0.10	-0.10	-0.31	0.33	1.14	-0.09
w_j^o	0.40	0.40	0.40	0.71	-0.21	-1.38	0.40
u_j^y	-0.35	-0.35	-0.35	-0.94	0.84	3.09	-0.34
u_j^o	-0.07	-0.07	-0.07	-0.96	1.74	5.23	-0.06
θ_j	0.79	0.79	0.79	2.15	-1.87	-6.67	0.78
UR^y	-7.42	8.99	20.90	0.06	-0.51	-0.11	-0.23
Skilled-Intensive							
w_s^y	-0.50	0.47	0.82	0.05	-0.06	-0.14	0.004
w_s^o	0.82	-0.97	-2.17	-0.07	0.14	0.27	0.01
u_s^y	-24.14	31.03	71.08	2.43	-3.69	-7.68	0.04
u_s^o	-33.86	54.48	141.48	4.27	-5.24	-11.19	0.51
θ_s	86.53	-47.03	-72.71	-5.38	9.01	20.04	-0.10
UR^o	-10.30	17.25	44.37	0.97	-0.12	0.43	0.42
Aggregates							
Average wage	0.36	-0.20	-0.61	0.15	0.04	-0.13	0.12
Total UR	-8.45	9.97	24.69	-0.29	-0.96	-0.53	-0.63
τ_p^*	-3.96	-1.98	-0.48	-3.05	-3.17	-3.19	-3.10
τ_w^*	-6.27	0.94	6.85	-2.94	-3.68	-4.13	-3.23
ε_I^*	-0.73	-5.08	-8.34	-2.01	-3.88	-6.48	-2.62
ε_f^*	-2.36	-2.02	-1.77	-2.24	-2.17	-2.05	-2.22
ε_w^*	-2.37	-2.21	-2.08	-2.34	-2.22	-2.05	-2.30

particularly unemployment rates. In the context of an aging population, these reforms can alter the tax base by influencing the extensive margin of the labor market. Additionally, we investigate the implications of delayed retirement policies and optimal contribution rates and taxes.

Our findings identify three critical channels through which pension reforms and fiscal policies operate in both labor markets. First, by modifying the meeting rate between firms and workers, these reforms affect a firm's flow value of employment. Second, they have the opposite effect on the flow value of retirees. Third, there is a spillover effect across different worker groups in the labor market, as firms cannot discriminate based on age during the hiring process. Taken together, these dynamics lead to ambiguous effects on market tightness and unemployment rates in both markets and for both types of workers.

Lastly, we calibrate the model to the Chinese data and conduct several counterfactual experiments to simulate the effects of the proposed delayed retirement policy and potential pension reforms on the labor market. Our results suggest that a delayed retirement policy decreases unemployment among the old while increasing unemployment among young workers. Furthermore, reductions in the statutory contribution rate for unemployed workers, coupled with increases in the contribution rate for employed workers, production tax, and wage tax, tend to increase unemployment for both groups. However, reducing firms' contribution rate lowers unemployment among the old and the young. The magnitude of the labor market's response is contingent on the interaction between specific parameters and market conditions.

APPENDIX A

Proof of Lemma 1

We begin by analyzing the derivatives of key labor market variables with respect to market tightness θ_j .

$$\frac{\partial w_j^o}{\partial \theta_j} = \frac{\beta(1-\beta)(r+s_j+\lambda_o+d)\frac{\partial m(\theta_j)}{\partial \theta_j}}{(r+s_j+\beta m(\theta_j)+\lambda_o+d)^2} [\xi_j p_j(1-\tau_p) - \Phi_j^o - (\varphi_j^o - \eta_j^o)] > 0,$$

This expression indicates that the wage of old workers w_j^o increases with market tightness θ_j .

Next, the derivative of the unemployment rate for young workers with respect to market tightness:

$$\frac{\partial u_j^y}{\partial \theta_j} = -\frac{\lambda_y + s_j}{(\lambda_y + s_j + m(\theta_j))^2} \frac{\partial m(\theta_j)}{\partial \theta_j} < 0,$$

shows that as θ_j increases, the unemployment rate of young workers decreases.

For the unemployment rate of older workers as a proportion of the total population of older workers:

$$\frac{\partial u_j^o/P_j^o}{\partial \theta_j} = -\frac{[(\lambda_o + d + s_j)(\lambda_y + s_j)(\lambda_o + d + \lambda_y + s_j + 2m(\theta_j)) + s_j m(\theta_j)^2] \frac{\partial m(\theta_j)}{\partial \theta_j}}{(\lambda_o + d + s_j + m(\theta_j))^2 (\lambda_y + s_j + m(\theta_j))^2} < 0.$$

For the derivative of the meeting probability ϕ_j with respect to θ_j :

$$\frac{\partial \phi_j}{\partial \theta_j} = \frac{(\lambda_o + d)\lambda_y^2(s_j + \lambda_o + d)\frac{\partial m(\theta_j)}{\partial \theta_j}(\lambda_y + s_j)}{\{(d + \lambda_o + \lambda_y)(\lambda_y + s_j)(d + \lambda_o + s_j) + m(\theta_j)[\lambda_y s_j + d(\lambda_y + s_j) + \lambda_o(\lambda_y + s_j)]\}^2} > 0.$$

Lastly, the derivative of w_j^y with respect to θ_j is more complex:

$$\frac{\partial w_j^y(\theta_j, w_j^o)}{\partial \theta_j} = \frac{\partial w_j^y}{\partial \theta_j} + \frac{\partial w_j^y}{\partial w_j^o} \frac{\partial w_j^o}{\partial \theta_j} \leq 0,$$

where $\frac{\partial w_j^y}{\partial w_j^o} < 0$ and $\frac{\partial w_j^o}{\partial \theta_j} > 0$.

Thus, the sign of $\frac{\partial w_j^y}{\partial \theta_j}$ depends on whether the direct effect of θ_j on w_j^y is sufficiently large and positive to offset the negative indirect effect of w_j^o on w_j^y . If the direct effect dominates, $\frac{\partial w_j^y}{\partial \theta_j} > 0$; otherwise, $\frac{\partial w_j^y}{\partial \theta_j} < 0$.

Proof of Lemma 2

Since the unemployment and employment rates in the skilled-intensive sector are symmetric to those in the unskilled-intensive sector, we omit the full proof here. However, the key derivative expressions are as follows:

$$\frac{\partial w_s^o}{\partial \theta_s} = \frac{(1-\beta)\beta(r+s_s+\lambda_o+d)\frac{\partial m(\theta_s)}{\partial \theta_s}\{\xi_s p_s(1-\tau_p)[\varepsilon_I+(1-\tau_w)(1-\varepsilon_w^w-\varepsilon_w)]-b_s^o[1+(1-\tau_p)(\varepsilon_f+\varepsilon_u^f)]\}}{\left\{ \begin{aligned} &(r+s_s+\lambda_o+d)(1-\tau_w)+(1-\beta)(r+s_s+\lambda_o+d)[\varepsilon_I-(1-\tau_w)(\varepsilon_w^w+\varepsilon_w)] \\ &+\beta[(r+s_s+\lambda_o+d)\tau_w+m(\theta_s)+(\varepsilon_f+\varepsilon_u^f)(r+s_s+m(\theta_s)+\lambda_o+d)] \end{aligned} \right\}^2} \leq 0,$$

Thus, if the inequality $(1-\tau_p)\xi_s p_s[\varepsilon_I+(1-\tau_w)(1-\varepsilon_w^w-\varepsilon_w)] > b_s^o[1+(1-\tau_p)(\varepsilon_f+\varepsilon_u^f)]$ holds, we have $\frac{\partial w_s^o}{\partial \theta_s} > 0$, this is because the firm gains more from employing old workers, which motivates them to raise wage levels.

For the case where $\lambda_y = 0$, the wage for young workers in the skilled-intensive sector can be expressed as:

$$w_s^y|_{\lambda_y=0} = \frac{(1-\beta)\frac{b_s^y}{r+s_s+m(\theta_s)+\lambda_y} + \beta\frac{(1-\tau_p)p_s}{r+s_s}}{(1-\beta)\frac{(1-\tau_w)(1-\varepsilon_w^w-\varepsilon_w^u)+\varepsilon_I}{r+s_s+m(\theta_s)} + \beta\frac{(1-\tau_p)(\varepsilon_f+\varepsilon_u^f)+1}{r+s_s}},$$

which follows similar conditions for parameters as in the case of $\frac{\partial w_s^o}{\partial \theta_s}$ to ensure $\frac{\partial w_s^y|_{\lambda_y=0}}{\partial \theta_s} > 0$.

Inspired by this, we decompose the derivatives of w_s^y as:

$$\frac{\partial w_s^y}{\partial \theta_s} = \frac{\partial w_s^y|_{\lambda_y=0}}{\partial \theta_s} + \frac{\partial A}{\partial \theta_s},$$

where A is given by:

$$A = \frac{-(1-\beta)\lambda_y\frac{(1-\tau_w)(1-\varepsilon_w^w-\varepsilon_w^u)w_s^o-b_s^o+\varepsilon_I w_s^o}{(r+s_s+m(\theta_s)+\lambda_o+d)(r+s_s+m(\theta_s)+\lambda_y)} + \beta\lambda_y\frac{(1-\tau_p)(\xi_s p_s-(\varepsilon_f+\varepsilon_u^f)w_s^o)-w_s^o}{(r+s_s+\lambda_o+d)(r+s_s+\lambda_y)}}{(1-\beta)\frac{(1-\tau_w)(1-\varepsilon_w^w-\varepsilon_w^u)+\varepsilon_I}{r+s_s+m(\theta_s)+\lambda_y} + \beta\frac{(1-\tau_p)(\varepsilon_f+\varepsilon_u^f)+1}{r+s_s+\lambda_y}}.$$

The second term, $\frac{\partial A}{\partial \theta_s}$, reflects two opposing effects related to aging. On one hand, young workers may resign as they age, leading to lower wages. On the other hand, wages could rise as workers become more productive with age. Thus, the sign of $\frac{\partial w_s^y}{\partial \theta_s}$ depends on which of these two effects dominates.

Proof of Proposition 1

We begin by rewriting equations (48) and (49):

$$\begin{aligned}
 F_j &\equiv \frac{c_j(r + s_j + \lambda_o + d)}{q(\theta_j)} \\
 &- \phi_j \left\{ \frac{r + s_j + \lambda_o + d}{r + s_j + \lambda_y} [(1 - \tau_p)(p_j - \Phi_j^y) - w_j^y] + \frac{\lambda_y}{r + s_j + \lambda_y} [(1 - \tau_p)(\xi_j p_j - \Phi_j^o) - w_j^o] \right\} \\
 &- (1 - \phi_j) [(1 - \tau_p)(\xi_j p_j - \Phi_j^o) - w_j^o], \tag{A1}
 \end{aligned}$$

$$\begin{aligned}
 F_s &\equiv \frac{c_s(r + s_s + \lambda_o + d)}{q(\theta_s)} \\
 &- \phi_s \left\{ \frac{r + s_s + \lambda_o + d}{r + s_s + \lambda_y} [(1 - \tau_p)(p_s - \Phi_s^y) - w_s^y] + \frac{\lambda_y}{r + s_s + \lambda_y} [(1 - \tau_p)(\xi_s p_s - \Phi_s^o) - w_s^o] \right\} \\
 &- (1 - \phi_s) [(1 - \tau_p)(\xi_s p_s - \Phi_s^o) - w_s^o]. \tag{A2}
 \end{aligned}$$

From equation (A1), we derive:

$$\begin{aligned}
 \lim_{\theta_j \rightarrow 0} F_j &= -\lim_{\theta_j \rightarrow 0} \phi_j \left\{ \frac{r + s_j + \lambda_o + d}{r + s_j + \lambda_y} [(1 - \tau_p)(p_j - \Phi_j^y) - \underline{w}_j^y] + \frac{\lambda_y}{r + s_j + \lambda_y} [(1 - \tau_p)(\xi_j p_j - \Phi_j^o) - \underline{w}_j^o] \right\} \\
 &- \lim_{\theta_j \rightarrow 0} (1 - \phi_j) [(1 - \tau_p)(\xi_j p_j - \Phi_j^o) - \underline{w}_j^o] \\
 &= -\frac{\lambda_o + d}{\lambda_o + d + \lambda_y} \frac{r + s_j + \lambda_o + d}{r + s_j + \lambda_y} (1 - \beta) [(1 - \tau_p)(p_j - \Phi_j^y) - \varphi_j^y] \\
 &- (1 - \beta) [(1 - \tau_p)(\xi_j p_j - \Phi_j^o) - (\varphi_j^y - \eta_j^o)] \frac{\lambda_y(r + s_j + \lambda_y + \lambda_o + d)}{(\lambda_o + d + \lambda_y)(r + s_j + \lambda_y)} < 0,
 \end{aligned}$$

where, $\underline{w}_j^o \equiv \lim_{\theta_j \rightarrow 0} w_j^o = \beta(1 - \tau_p)(\xi_j p_j - \Phi_j^o) + (1 - \beta)(\varphi_j^o - \eta_j^o)$, $\underline{w}_j^y \equiv \lim_{\theta_j \rightarrow 0} w_j^y = \beta(1 - \tau_p)(p_j - \Phi_j^y) + (1 - \beta)\varphi_j^y$, and $\lim_{\theta_j \rightarrow 0} \phi_j = \frac{\lambda_o + d}{\lambda_o + d + \lambda_y}$. Additionally,

$$\lim_{\theta_j \rightarrow \infty} F_j = \infty,$$

where, $\overline{w}_j^o \equiv \lim_{\theta_j \rightarrow \infty} w_j^o = (1 - \tau_p)(\xi_j p_j - \Phi_j^o)$, $\overline{w}_j^y \equiv \lim_{\theta_j \rightarrow \infty} w_j^y = \beta(1 - \tau_p)(p_j - \Phi_j^y)$, and $\lim_{\theta_j \rightarrow \infty} \phi_j = \frac{(\lambda_o + d)(s_j + \lambda_y)}{\lambda_y s_j + (\lambda_y + s_j)(\lambda_o + d)}$.

Taking the derivative of F_j with respect to θ_j :

$$\begin{aligned} \frac{\partial F_j}{\partial \theta_j} &\equiv \frac{\partial [c_j(r + s_j + \lambda_o + d)/q(\theta_j)]}{\partial \theta_j} \\ &+ \frac{\partial \phi_j}{\partial \theta_j} \left\{ [(1 - \tau_p)(\xi_j p_j - \Phi_j^o) - w_j^o] \frac{r + s_j}{r + s_j + \lambda_y} - \frac{r + s_j + \lambda_o + d}{r + s_j + \lambda_y} [(1 - \tau_p)(p_j - \Phi_j^y) - w_j^y] \right\} \\ &+ \left\{ \frac{r + s_j + \lambda_o + d}{r + s_j + \lambda_y} \frac{\partial w_j^y}{\partial \theta_j} + \frac{\lambda_y}{r + s_j + \lambda_y} \frac{\partial w_j^o}{\partial \theta_j} \right\} \phi_j + \frac{\partial w_j^o}{\partial \theta_j} (1 - \phi_j). \end{aligned}$$

For a unique θ_j , we require $\frac{\partial F_j}{\partial \theta_j} > 0$.

Assuming that: $\frac{(1 - \tau_p)(\xi_j p_j - \Phi_j^o) - w_j^o}{r + s_j + \lambda_o + d} > \frac{(1 - \tau_p)(p_j - \Phi_j^y) - w_j^y}{r + s_j}$, i.e., $J_j^o > J_j^y \Big|_{\lambda_y=0}$,

implies that hiring an old worker is more beneficial than hiring a young worker. When θ_j increases, the probability of meeting a young worker ϕ_j also increases, potentially reducing the firm's expected value. However, as analyzed in Lemma 1 and Lemma 2, an increased θ_j may also reduce wage levels, which increases the firm's expected value. The positive wage effect must dominate to ensure a unique θ_j .

Similarly, for a unique θ_s , we require $\frac{(1 - \tau_p)(\xi_s p_s - \Phi_s^o) - w_s^o}{r + s_s + \lambda_o + d} > \frac{(1 - \tau_p)(p_s - \Phi_s^y) - w_s^y}{r + s_s}$, i.e., $J_s^o > J_s^y \Big|_{\lambda_y=0}$, with the condition that the negative effect on the firm's expected value dominates to ensure a unique θ_s .

Proof of Proposition 2

According to the implicit function theorem, we have $\frac{\partial \theta}{\partial \lambda_o} = -\frac{\partial F / \partial \lambda_o}{\partial F / \partial \theta}$, where F is derived from equation (A1) or (A2). For the unskilled-intensive sector, we can express $\frac{\partial F_j}{\partial \lambda_o}$ as:

$$\frac{\partial F_j}{\partial \lambda_o} = -\frac{\partial \phi_j}{\partial \lambda_o} B - \frac{\partial B}{\partial \lambda_o} \phi_j - \left[\frac{-(1 - \tau_p) \frac{\partial w_j^o}{\partial \lambda_o} (r + s_j + \lambda_o + d) - [(1 - \tau_p)(\xi_j p_j - \Phi_j^o) - w_j^o]}{(r + s_j + \lambda_o + d)^2} \right],$$

where,

$$B \equiv \frac{(1 - \tau_p)(p_j - \Phi_j^y) - w_j^y}{r + s_j} - \frac{(1 - \tau_p)(\xi_j p_j - \Phi_j^o) - w_j^o}{r + s_j + \lambda_o + d} = J_j^y - J_j^o \Big|_{\lambda_y=0}.$$

Additionally,

$$\begin{aligned} \frac{\partial \phi_j}{\partial \lambda_o} &= \frac{\lambda_y(\lambda_y + s_j)[(s_j + \lambda_o + d)^2(\lambda_y + s_j) + s_j(2d + 2\lambda_o + \lambda_y + 2s_j)m(\theta_j) + s_j m(\theta_j)^2]}{\{(\lambda_y + \lambda_o + d)(s_j + \lambda_o + d)(\lambda_y + s_j) + [\lambda_y s_j + d(s_j + \lambda_y) + \lambda_o(s_j + \lambda_y)] m(\theta_j)\}^2} > 0, \\ \frac{\partial w_j^o}{\partial \lambda_o} &= -\frac{(1 - \beta)\beta m(\theta_j)[(1 - \tau_p)(\xi_j p_j - \Phi_j^o) - \varphi_j^o + \eta_j^o]}{(r + s_j + \beta m(\theta_j) + \lambda_o + d)^2} < 0, \\ \frac{\partial w_j^y}{\partial \lambda_o} &= \frac{(r + s_j + \lambda_y)(r + s_j + \lambda_y + m(\theta_j)) \left[\frac{(1 - \beta)\lambda_y(w_j^o - \varphi_j^o + \eta_j^o)}{(r + s_j + m(\theta_j) + \lambda_o + d)^2(r + s_j + \lambda_y + m(\theta_j))} - \frac{\beta\lambda_y[(1 - \tau_p)(\xi_j p_j - \Phi_j^o) - \varphi_j^o]}{(r + s_j + \lambda_o + d)^2(r + s_j + \lambda_y)} \right]}{r + s_j + \lambda_y + \beta m(\theta_j)} \leq 0, \\ \frac{\partial A}{\partial \lambda_o} &= -\frac{\partial w_j^y / \partial \lambda_o}{r + s_j + \lambda_y} - \frac{r + s_j}{r + s_j + \lambda_y} \left\{ \frac{-\frac{\partial w_j^o}{\partial \lambda_o}(r + s_j + \lambda_o + d) - [(1 - \tau_p)(\xi_j p_j - \Phi_j^o) - w_j^o]}{(r + s_j + \lambda_o + d)^2} \right\} \leq 0. \end{aligned}$$

Thus, to ensure $\frac{\partial F_j}{\partial \lambda_o}$ has an unambiguous sign, we need further restrictions on the parameters. For example, if we assume $B < 0$ as in Proposition 1, which implies that hiring old workers is more beneficial, a decrease in λ_o will increase the probability of meeting the old, which benefits the firm. However, changes in λ_o also affect the firm’s value by altering the wages of both young and old workers. If the negative wage effect dominates, we can obtain $\frac{\partial F_j}{\partial \lambda_o} > 0$.

According to Proposition 1, $\frac{\partial F_j}{\partial \theta_j} > 0$. Therefore, using the implicit function theorem, we have:

$$\frac{\partial \theta_j}{\partial \lambda_o} = -\frac{\partial F_j / \partial \lambda_o}{\partial F_j / \partial \theta_j} < 0.$$

The proof for $\frac{\partial \theta_s}{\partial \lambda_o}$ follows similarly, except that the positive effect on the firm’s value dominates in this case.

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