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The Right to Bear Arms, Private Property, and Economic Growth

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This paper provides an examination of the crucial role played by the right to keep and bear arms in protecting individuals' life, liberty, and property. Through our analysis, we demonstrate that the accumulation of firearms, serving as a means of safeguarding life, liberty, and property rights, leads to advancements in physical capital accumulation, output production, and overall welfare. Utilizing a robust mathematical model, we offer theoretical support for the natural right to bear arms, a principle deeply ingrained in ancient wisdom and enshrined in modern constitutional frameworks, notably exemplified by the Second and Fourth Amendments of the US Constitution.

Key Words: Arms spending; Private property; Capital accumulation; Economic growth.

JEL Classification Numbers: E20, E22, H56, O10, O40, P16, P48

1. INTRODUCTION

Private property holds significant importance in a free society, serving two crucial functions that intertwine with each other. Firstly, stable property rights incentivize the creation of wealth and prosperity. Secondly, property rights are closely tied to individual liberty. Respect for private ownership helps decentralize power and strengthen individual autonomy from government interference. Property ownership acts as a safeguard of liberty by limiting the reach of legitimate government and empowering individuals to participate in the political process. In the absence of property rights, the enjoyment of other individual liberties becomes precarious, making self-government unlikely, and a truly free government must uphold

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the sacredness of personal liberty and private property, underscoring the indispensable role of property rights in maintaining a free society.

The possession of firearms has been regarded as one of the most crucial methods for safeguarding life and private property throughout millennia of human history. From ancient times to the modern era, individuals and societies have recognized the inherent need for self-defense against various threats, including aggression, invasion, and criminal activities. The ability to bear arms provides a sense of security and empowerment, allowing individuals to protect themselves, their loved ones, and their possessions from harm or infringement. Across different civilizations and cultures, the possession and use of weapons have often been synonymous with autonomy, independence, and the preservation of individual freedoms. While the specific laws and regulations surrounding firearm ownership have evolved over time, the fundamental principle of self-defense and the right to bear arms has remained deeply ingrained in human societies.

The nexus between firearms and liberty frequently arises in discussions concerning individual rights, personal autonomy, and self-preservation. Across various nations, such as the United States, the right to bear arms is legally upheld, considered a cornerstone of personal freedom, granting individuals the ability to protect themselves, their families, and their belongings from threats posed by criminals or authoritarian regimes. For many, firearms symbolize freedom and defiance against oppression, rooted in historical events like the American Revolution. Additionally, some view firearms as essential for self-defense, providing a means of protection in dangerous situations where law enforcement may be unavailable. This debate intersects with broader conversations about civil liberties and government authority, as supporters of gun rights argue that firearm ownership safeguards individual freedoms and acts as a check against potential abuses of state power.

In Switzerland, the militia system is deeply embedded within the nation's military and political framework. Referred to as the "militia army" or "citizen army", it revolves around universal conscription and active citizen involvement in national defense. Key elements of the Swiss militia system include universal conscription, requiring Swiss men to serve in the military or civil defense forces upon reaching adulthood, typically at 18 years old, with women also able to volunteer. These citizen soldiers undergo military training while maintaining civilian lives and careers, participating part-time with periodic refresher courses. Alongside the regular army, Switzerland maintains a robust home guard system of trained reservists for border defense and internal security, with civil defense forces integrated for emergency response. The system's decentralized structure empowers each canton to organize and manage its militia units, ensuring close ties to local communities and responsiveness to regional needs. Re-

flecting Switzerland's commitment to direct democracy, citizens influence military decisions through referendums on defense policies and spending. With roots dating back to medieval times, the Swiss militia tradition symbolizes national identity, sovereignty, and a unique approach to military organization characterized by self-defense, civic engagement, and decentralized governance.

This paper begins by offering a concise review of the right to bear arms, liberties, and property rights in Section 2. Following this, Sections 3 and 4 introduce an optimal economic growth model incorporating both physical capital accumulation and investments in firearms for the protection of life, liberty, and property rights. Finally, our conclusions are presented in Section 5.

2. A BRIEF, SELECTED HISTORICAL REVIEW OF THE RIGHT TO BEAR ARMS, PROPERTY RIGHTS, AND LIBERTIES

Let us recall that emperor Qin Shi Huang's (259 – 210 BC) consolidation of power during the unification of the six states serves as a poignant example of despotism, particularly evident in the confiscation of weapons by governments. Understanding the symbolic significance of weapons in asserting authority, Emperor Qin Shi Huang ordered the nationwide seizure of arms to strengthen control over the Qin Dynasty. Motivated by a failed assassination attempt on his life, he underscored the necessity of stringent weapon control. This led to the systematic collection and destruction of weapons across the realm, with the materials repurposed to craft twelve bronze statues, symbolizing his dominance. This extensive effort aimed to suppress potential challenges to his reign, particularly from the former aristocracy. The confiscation extended to privately owned weapons, aiming to prevent uprisings fueled by widespread access to arms, effectively curtailing life, liberties, and property rights. Subsequent to Emperor Qin Shi Huang's reign, the Han Dynasty reinforced these policies, further restricting households from possessing weapons and prohibiting the sale of arms in markets. These measures, recorded in historical archives, underscore the enduring legacy of Emperor Qin Shi Huang's weapon confiscation as emblematic of centralized authority, totalitarian control, and the suppression of liberties and property rights.[1]

Oppressive regimes have often disarmed their populations prior to committing mass atrocities, disregarding fundamental rights such as life, liberty, and property. In 1911, within the Ottoman Empire, the disarmament

[1] "All political power comes from the barrel of a gun. The Communist Party must command all the guns; that way, no guns can ever be used to command the party." The quote was from Mao Zedong, founder of China.

of Armenians foreshadowed the subsequent genocide. Ottoman authorities enacted policies to disarm Armenian communities, depriving them of the ability to defend themselves and rendering them susceptible to persecution. This disarmament campaign was a component of a broader strategy aimed at suppressing and controlling the Armenian populace. As tensions heightened, particularly during World War I, the absence of Armenian arms provided an opportunity for Ottoman authorities to carry out systematic violence and mass killings. This ultimately led to the Armenian Genocide, during which approximately 1.5 million Armenians endured brutal massacres, deportation, and forced labor. The disarmament of Armenians significantly facilitated the Ottoman government's ability to perpetrate these atrocities with minimal resistance, underscoring the dire consequences of weapon confiscation amidst state-sponsored violence and genocide.

During Stalin's rule in the Soviet Union from around 1929, there was a concerted effort to disarm the population, primarily aimed at political opponents and potential dissidents. This campaign was part of Stalin's strategy to consolidate power and quell dissent. Methods included confiscating firearms through raids, searches, and surrender programs, backed by state propaganda portraying private gun ownership as a threat. Strict gun control laws were imposed, enforced by secret police like the NKVD, leading to widespread surveillance and crackdowns on dissenters. The disarmament left millions vulnerable to state oppression, with opposition swiftly suppressed through imprisonment, forced labor, or execution. Overall, this disarmament bolstered Stalin's authoritarian control and stifled dissent in the Soviet Union.

The disarmament of German Jews starting in 1933, escalating with prohibitions on firearms ownership and extensive raids. Legal frameworks were established during the Weimar Republic, facilitating government control over weapons. Following Nazi ascension, existing laws were used to disarm Jews, leading to further restrictions and confiscations. The 1938 weapons law overhaul intensified regulations, culminating in the complete prohibition of Jewish firearm possession after Kristallnacht. This disarmament was enforced through raids, home searches, and the revocation of permits. In the end, gun control measures were implemented before the Holocaust, resulting in the slaughter of approximately 6 million Jews and millions of others.

Before assuming power in Cuba in 1959, Fidel Castro advocated for widespread gun ownership, viewing it as essential for democracy and the defense of just causes. He distributed weapons to various groups, including soldiers, militiamen, and civilians, totaling hundreds of thousands of firearms. However, upon assuming power, Castro swiftly reversed his stance, echoing Mao's belief that firearms should be monopolized by the state. A mandate was issued for all citizens to surrender their combat

weapons, citing the need for strict control due to the perceived threat from enemies. Failure to comply was met with severe consequences, including punishment by Revolutionary Tribunals known for their harsh sentences, including death.

The decline of Venezuela into economic chaos and authoritarianism during Chavez's regime from 2012 onwards serves as a cautionary tale, with the nation's gun control policies playing a pivotal role. Chavez initiated stringent gun control measures upon assuming power, banning all firearms and imposing severe penalties for violations. Exclusive control over firearms was vested in the Venezuelan Armed Forces, empowering them with authority over registration and potential confiscation. Presently, many citizens lament the oppressive legislation, acknowledging its contribution to the government's tyrannical actions. Despite widespread discontent and flawed electoral processes marred by corruption, Venezuelans find themselves unable to challenge their oppressive regime due to strict gun control laws. This situation underscores the significance of the Second Amendment, as conceived by America's Founding Fathers, not only for personal safety and property protection but also as a bulwark against tyranny — a historical lesson that should not be overlooked.

When a government initiates the confiscation of firearms, it signals a notable advancement toward despotism. Such actions imply that the government is failing to uphold the rights to life, liberty, and property of its citizens.

In ancient Greece and Rome, the right to bear arms was deeply intertwined with concepts of civic duty, personal defense, and property protection. Citizens were expected to defend their city-states or republics, forming militias and serving in the military when needed. Beyond collective defense, the right to bear arms also facilitated individual self-defense and safeguarding property. Possession of arms symbolized citizenship and membership in the political community, with regulations governing their use and citizens undergoing military training. Additionally, owning arms could confer social status and prestige, particularly among wealthy citizens.

In ancient Greece, particularly in Sparta and Athens, the concept and organization of militias differed significantly.

Sparta:

The Spartan military system was renowned for its emphasis on military training, discipline, and readiness for warfare. The Spartan society was highly militarized, with all male citizens being trained as warriors from a young age. This training was rigorous and began in childhood, focusing on physical fitness, combat skills, and obedience to authority. Spartans were part of a professional standing army, known as the Spartiate class, which formed the backbone of Spartan military power. The military organization in Sparta was highly centralized and controlled by the state. The state,

through its institutions such as the Council of Elders (Gerousia) and the Ephors, exercised strict control over the military and society as a whole. Spartans were known for their disciplined and cohesive phalanx formation in battle, which contributed to their military successes and dominance in ancient Greece.

Athens:

In Athens, the concept of militia was different from Sparta. Athenian society was more democratic and open, with a focus on trade, commerce, and culture in addition to military matters. The Athenian military consisted of citizen-soldiers who served part-time in the military when needed, rather than being professional warriors like the Spartans. The Athenian militia was organized into a citizen army called the hoplite phalanx. Hoplites were citizens who provided their own armor and weapons and were expected to defend Athens in times of conflict. Unlike Sparta, where military training was the primary focus of education, Athenian citizens received a more well-rounded education that included arts, philosophy, and politics in addition to military training. The Athenian military system was more decentralized compared to Sparta, with decisions about military matters being made by democratic institutions such as the Assembly and the Council of 500.

Roman Republic:

The Roman republican militia played a vital role in military organization, state defense, and safeguarding the life, liberty, and property of citizens. Comprised of citizen-soldiers, it formed the core of the Roman military, evolving over time to adapt to changing needs. The Roman militia encompassed several key features: comprised of citizen-soldiers, mainly male citizens obligated to serve when called upon, military duty was considered a civic duty essential to citizenship, with soldiers drawn from the land-owning class and responsible for their equipment. Organized into legions, which included infantry, cavalry, and support personnel, each legion consisted of Roman citizens divided into centuries and cohorts, with officers selected from the aristocracy or elected by soldiers. Military service was determined by age and property qualifications, with citizens typically serving for a specified period, and rigorous training emphasized discipline and obedience, with severe penalties for misconduct. The militia played a crucial role in defending the Republic and expanding its territory through conquest, contributing to Rome's dominance across the Mediterranean region, while military service conferred social status and political influence, especially on successful commanders and veterans. Towards the Republic's end, the militia system evolved into a professional army to adapt to changing military needs, a transition that accelerated under the Roman Empire, leading to a standing professional army recruited from various provinces.

The shift from a traditional militia system to a professional army represented a significant transformation in the Roman military during the late

Republic and into the Empire. Factors such as expansion, warfare professionalization, recruitment changes, military reforms, and imperial influence played crucial roles in this transition. However, this shift also contributed to the downfall of the Roman Republic, leading to the establishment of authoritarian rule under the emperors. Dependency on military leaders, erosion of republican values, civil conflicts, power struggles, centralization of power, and the diminishing of republican virtues were pivotal dynamics. Overall, the transition weakened republican institutions, undermined liberties and property rights, fueled instability, and facilitated the ascent of autocratic governance in ancient Rome.

Renowned for his pragmatic approach to statecraft, Niccolò Machiavelli emphasized the pivotal role of militias in governance and national defense. He championed citizen-soldiers, recognizing them as vital defenders of their communities and proponents of civic virtue and patriotism, essential for safeguarding life, liberty, and property. Machiavelli esteemed militias for their cost-effectiveness and reliability in upholding state sovereignty, advocating for defensive strategies to deter aggression. However, he warned against excessive military empowerment, acknowledging the potential risks of armed forces challenging civilian authority. Drawing inspiration from ancient Rome, Machiavelli revered Roman virtues, military prowess, republican institutions, and adaptability, considering them models of effective governance and leadership.

In his analysis of historical figures such as Moses, Theseus, Romulus, Cyrus the Great, Agathocles of Syracuse, Caesar Borgia, and King Ferdinand of Spain, Machiavelli extolled them as exemplary armed princes who adeptly wielded power. He lauded Moses for his adept use of military force and divine authority to establish a new nation, commended Theseus for his military prowess and role in unifying Greek city-states, and praised Romulus for his audacity and cunning in building Rome. Additionally, Machiavelli admired Cyrus the Great's conquests and administrative skills, Agathocles of Syracuse's boldness, Caesar Borgia's strategic brilliance, and King Ferdinand of Spain's successful unification efforts. These examples underscore Machiavelli's belief in the pivotal importance of military strength and decisive leadership in governance. Furthermore, Machiavelli juxtaposed these armed princes with Girolamo Savonarola, critiquing the latter for his inability to defend himself against adversaries, highlighting the vulnerability of unarmed rulers in the political realm. Through historical anecdotes and cautionary tales, Machiavelli advocated for the pragmatic use of power, including arms, to safeguard property, life, liberty, and territory, thereby ensuring stability and authority in governance.

John Locke's labor theory of private property, articulated in his work "Two Treatises of Government" (1689), asserts that individuals have a natural right to acquire property through their labor. According to Locke,

this right stems from the inherent ownership individuals have over themselves and their labor. Property rights are established when individuals mix their labor with unowned natural resources, thereby legitimizing their ownership claim. However, Locke imposes limits on property acquisition, stating that individuals may only appropriate as much property as they can use without waste, ensuring equitable opportunities for others. Locke justifies private property as essential for preserving individual rights, liberty, and happiness, providing security, productivity incentives, and means for improving well-being. He emphasizes the government's role in protecting property rights and upholding laws to safeguard individual rights. Locke's labor theory of private property is rooted in his broader social contract theory, where individuals enter civil society to protect their natural rights, consenting to government authority in exchange for the common good. Additionally, Locke's philosophy on the right to revolution asserts that individuals retain the right to resist or overthrow tyrannical governments that violate their natural rights. This right, grounded in self-preservation and defense of liberty and property, may include armed resistance as a last resort against oppressive regimes. Locke's ideas on revolution and the right to bear arms highlight the inherent natural rights of individuals and the importance of protecting these rights against governmental encroachment and tyranny.

The Second Amendment of the United States Constitution affirms the right of individuals to keep and bear arms, highlighting the importance of a well-regulated militia for the security of a free state. It emphasizes the pivotal role of armed citizens in defending against tyranny and external threats, while also protecting individual rights to possess firearms for personal defense and property protection. The constitutional framework grants Congress and states the authority to deploy the militia to maintain domestic order and uphold federal laws, underscoring its significance in safeguarding the nation's security.

The Fourth Amendment protects individuals' rights to privacy and property against unreasonable searches and seizures, stemming from colonial opposition to British Writs of Assistance. This historical context, marked by intrusive searches authorized by British authorities, fueled discontent among colonists like James Otis, who championed individual liberties. His advocacy laid the groundwork for the Fourth Amendment's ratification, which ensures that warrants are issued only upon probable cause and with specific descriptions of the items to be seized. Rooted in the defense of individual rights and privacy, the Fourth Amendment serves as a safeguard against government overreach, reflecting a commitment to fundamental principles of liberty and justice.

In summary, the absence of the right to bear arms would have posed significant obstacles to American economic development from 1600 to 2000.

Firearms played a pivotal role in various facets of American growth and societal evolution during this period. In the early stages of American history, firearms were indispensable for frontier life and westward expansion, enabling settlers to defend against indigenous peoples and secure territory for settlement. The right to bear arms was perceived as essential for protecting property rights, particularly in areas with limited law enforcement. It became ingrained in American culture, symbolizing individual freedom and resistance to tyranny. This sense of security facilitated economic stability, encouraged entrepreneurship, and stimulated investment. Firearms were crucial for defense and security against external threats, facilitating westward expansion and economic activities like agriculture and trade. They were essential tools for homesteaders, pioneers, and entrepreneurs, contributing to agricultural productivity and land development. The firearms industry itself played a significant role in American industrialization, creating jobs and driving technological advancements. Ultimately, the right to bear arms safeguarded individual liberties, protected property rights, and maintained political stability, all of which were integral to America's economic prosperity and growth throughout its history.

3. THE DYNAMIC MODEL

Nozick (1974), in his book "Anarchy, State, and Utopia," presents the idea of protective agencies competing in the state of nature as part of his argument for a minimal state or "night-watchman" state. Nozick's theory explores how governance and protection could function in a society without a centralized government.

In the state of nature, according to Nozick, individuals have natural rights to life, liberty, and property. These rights imply that individuals have the freedom to pursue their interests and goals without interference from others. However, in the absence of a governing authority, conflicts may arise, and individuals may seek protection and enforcement of their rights. Nozick proposes that in such a scenario, individuals would voluntarily contract with protective agencies to safeguard their rights and interests. These protective agencies would function similarly to private security firms or insurance companies in today's society. Individuals would pay fees or premiums to these agencies in exchange for protection and enforcement of their rights.

The key concept in Nozick's theory is the idea of competitive enforcement. In the absence of a monopolistic government, multiple protective agencies would coexist and compete for customers. Competition among these agencies would ensure efficiency, innovation, and accountability in the provision of protective services. If one protective agency violates the rights of its clients or fails to deliver satisfactory services, individuals could

withdraw their patronage and switch to a different agency. This competitive dynamic would incentivize protective agencies to respect individual rights, provide quality service, and resolve disputes peacefully. Nozick argues that in a society where protective agencies compete freely, individuals would have greater autonomy and control over their lives, while the risk of tyranny or abuse of power by a centralized government would be minimized.

In our dynamic model, we build upon Nozick's concepts by suggesting that individuals fulfill dual roles. Firstly, they act as producers, leveraging capital and a fixed unit of labor (assumed to be one and embodied in capital) to produce goods and services. Secondly, they function as self-protectors, wielding arms to defend the fruits of their labor, their possessions, and their lives as they navigate daily life. Consider m as the inventory of firearms, i_m as the total expenditure on acquiring new firearms along with the expenses associated with regular firearms training[2], c as consumption, and k as capital. The utility function of the representative agents is $u(c, i_m, m)$, and the production function is $f(k, i_m, m)$. An increase in i_m and m leads to more utility because i_m and m provide security for individuals. That is to say, the presence of firearms boosts individuals' feelings of safety and security, thereby positively influencing their overall well-being and satisfaction. Simultaneously, firearms' security measures aid in safeguarding capital stock, k, and output by diminishing the risk of loss or damage resulting from theft, vandalism, appropriation, and other acts of violence. These assumptions can be summarized mathematically: $u_1 > 0$, $u_2 > 0$, $u_3 > 0$; $f_1 > 0$, $f_2 > 0$, and $f_3 > 0$. It is further assumed that $u_{11} < 0$, $u_{22} < 0$, $u_{33} < 0$, $f_{11} < 0$, $f_{22} < 0$, and $f_{33} < 0$.

The agents accumulate firearms and capital, respectively, as follows,

$$\frac{dm}{dt} = i_m - \delta_m m, \tag{1}$$

$$\frac{dk}{dt} = i_k - \delta_k k = f(k, i_m, m) - c - p i_m - \delta_k k, \tag{2}$$

where δ_m and δ_k are the depreciation rates of weapon stock and capital stock, respectively; p is the price of firearms in terms of consumption goods. The initial stocks of firearms and capital are given by $m(0) = m_0$ and $k(0) = k_0$, respectively. The agent maximizes his discounted utility at the time discount rate, ρ:

$$\max \int_0^\infty u(c, i_m, m) e^{-\rho t} dt. \tag{3}$$

[2]For example, routine training of firearms is viewed by many Americans as a practical and proactive measure to enhance personal safety, protect property, and promote responsible citizenship.

The present-value Hamiltonian is defined as:

$$\mathcal{H} = u(c, i_m, m) + \lambda_1(f(k, i_m, m) - c - pi_m - \delta_k k) + \lambda_2(i_m - \delta_m m), \quad (4)$$

where λ_1 and λ_2 are costate variables that measure the shadow prices of capital and arms, respectively. The first-order conditions for the optimization are

$$u_1 - \lambda_1 = 0, \quad (5)$$
$$u_2 + \lambda_1(f_2 - p) + \lambda_2 = 0, \quad (6)$$
$$\lambda_1(f_1 - \delta_k) = \rho\lambda_1 - \dot{\lambda}_1, \quad (7)$$
$$u_3 + \lambda_1 f_3 - \lambda_2\delta_m = \rho\lambda_2 - \dot{\lambda}_2. \quad (8)$$

The transversality conditions are

$$\lim_{t\to\infty} k\lambda_1 e^{-\rho t} = 0, \quad (9)$$
$$\lim_{t\to\infty} m\lambda_2 e^{-\rho t} = 0. \quad (10)$$

To derive the complete dynamic system of c, i_m, k, and m, we differentiate (5) with respect to t:

$$u_{11}\dot{c} + u_{12}\dot{i}_m + u_{13}\dot{m} = \dot{\lambda}_1. \quad (11)$$

Substitute (5) and (11) into (7):

$$\dot{c} = -\frac{u_{12}}{u_{11}}\dot{i}_m - \frac{u_{13}}{u_{11}}\dot{m} - \frac{u_1}{u_{11}}(f_1 - \delta_k - \rho). \quad (12)$$

Next, from (6) we obtain

$$-\lambda_2 = u_2 + u_1(f_2 - p) \equiv D. \quad (13)$$

Differentiate (13) with respect to t:

$$-\dot{\lambda}_2 = D_c\dot{c} + D_{im}\dot{i}_m + D_k\dot{k} + D_m\dot{m}, \quad (14)$$

where

$$D_c = u_{21} + u_{11}(f_2 - p),$$
$$D_{im} = u_{22} + u_{12}(f_2 - p) + u_1 f_{22},$$
$$D_k = u_1 f_{21},$$
$$D_m = u_{23} + u_{13}(f_2 - p) + u_1 f_{23}.$$

Furthermore, substitute (5) and (13) into (8):

$$-\dot{\lambda}_2 = u_3 + u_1 f_3 + (u_2 + u_1(f_2 - p))(\delta_m + \rho) \equiv \Phi. \tag{15}$$

Combine (12), (14) and (15):

$$\dot{i}_m = \Omega_k \dot{k} + \Omega_m \dot{m} + \Omega_c(f_1 - \delta_k - \rho) + \Omega_\Phi \Phi, \tag{16}$$

where

$$\Omega_k = \frac{D_k u_{11}}{D_c u_{12} - D_{im} u_{11}}, \qquad \Omega_m = -\frac{D_c u_{13} - D_m u_{11}}{D_c u_{12} - D_{im} u_{11}},$$
$$\Omega_c = -\frac{D_c u_1}{D_c u_{12} - D_{im} u_{11}}, \quad \Omega_\Phi = -\frac{u_{11}}{D_c u_{12} - D_{im} u_{11}}.$$

Therefore, the dynamic system is given by

$$\dot{c} = -\frac{u_{12}}{u_{11}} \dot{i}_m - \frac{u_{13}}{u_{11}} \dot{m} - \frac{u_1}{u_{11}}(f_1 - \delta_k - \rho), \tag{12}$$
$$\dot{i}_m = \Omega_k \dot{k} + \Omega_m \dot{m} + \Omega_c(f_1 - \delta_k - \rho) + \Omega_\Phi \Phi, \tag{16}$$
$$\dot{k} = f(k, i_m, m) - \delta_k k - c - p i_m, \tag{17}$$
$$\dot{m} = i_m - \delta_m m. \tag{18}$$

For a generic variable x we denote its value at the steady state as x^*. To examine the stability of the steady state, we linearize the system (12), (16), (17), and (18) around the steady state:

$$\begin{bmatrix} \dot{c} \\ \dot{i}_m \\ \dot{k} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} J_{11}^* & J_{12}^* & J_{13}^* & J_{14}^* \\ J_{21}^* & J_{22}^* & J_{23}^* & J_{24}^* \\ -1 & f_2^* - p & f_1^* - \delta_k & f_3^* \\ 0 & 1 & 0 & -\delta_m \end{bmatrix} \begin{bmatrix} c - c^* \\ i_m - i_m^* \\ k - k^* \\ m - m^* \end{bmatrix}, \tag{19}$$

where

$$
\begin{aligned}
J_{11}^* =& -\frac{u_{12}}{u_{11}} J_{21}^*, \\
J_{12}^* =& -\frac{u_{12}}{u_{11}} J_{22}^* - \frac{u_{12}}{u_{11}} - \frac{u_1}{u_{11}} f_{12}, \\
J_{13}^* =& -\frac{u_{12}}{u_{11}} J_{23}^* - \frac{u_1}{u_{11}} f_{11}, \\
J_{14}^* =& -\frac{u_{12}}{u_{11}} J_{24}^* + \frac{u_{12}}{u_{11}} \delta_m - \frac{u_1}{u_{11}} f_{13}, \\
J_{21}^* =& -\Omega_k + \Omega_\Phi [u_{13} + u_{11} f_3 + (u_{12} + u_{11}(f_2 - p))(\delta_m + \rho)], \\
J_{22}^* =& \Omega_k (f_2 - p) + \Omega_m + \Omega_c f_{12} + \Omega_\Phi [u_{23} + u_{12} f_3 + u_1 f_{23} \\
& + (u_{22} + u_{12}(f_2 - p) + u_1 f_{22})(\delta_m + \rho)], \\
J_{23}^* =& \Omega_k (f_1 - \delta_k) + \Omega_c f_{11} + \Omega_\Phi [u_1 f_{13} + u_1 f_{12} (\delta_m + \rho)], \\
J_{24}^* =& \Omega_k f_3 - \Omega_m \delta_m + \Omega_c f_{13} + \Omega_\Phi [u_{33} + u_{13} f_3 + u_1 f_{33} \\
& + (u_{23} + u_{13}(f_2 - p) + u_1 f_{23})(\delta_m + \rho)].
\end{aligned}
$$

Since this dynamic system has two pre-determined state variables, the steady-state equilibrium is locally saddle-point stable if and only if the Jacobian matrix has two negative eigenvalues and two positive eigenvalues.

4. AN EXAMPLE WITH A SEPARABLE CRRA UTILITY FUNCTION AND THE COBB-DOUGLAS PRODUCTION FUNCTION

4.1. Steady state

Let the utility function be

$$
u(c, i_m, m) = \frac{c^{1-\sigma}}{1-\sigma} + \theta \frac{i_m^{1-\tau}}{1-\tau} + \chi \frac{m^{1-\phi}}{1-\phi}, \tag{20}
$$

where σ, τ, and ϕ are the coefficients of (relative) risk aversion of c, i_m, and m, respectively. θ and χ capture the preferences for i_m and m, respectively. Furthermore, we assume the production function has the form

$$
f(k, i_m, m) = A k^\alpha i_m^\beta m^\gamma, \tag{21}
$$

where $A > 0$ represents the total factor productivity, α, β, and γ measure the productivity of k, i_m, and m, respectively. We assume $\alpha > 0$, $\beta > 0$,

$\gamma > 0$, and $\alpha + \beta + \gamma < 1$. Rewrite the dynamic system

$$\dot{c} = \frac{c}{\sigma}(f_1 - \delta_k - \rho), \tag{22}$$

$$\dot{i}_m = -\frac{u_1 f_{21}}{u_{22} + u_1 f_{22}}\dot{k} - \frac{u_1 f_{23}}{u_{22} + u_1 f_{22}}\dot{m} + \frac{u_1(f_2 - p)}{u_{22} + u_1 f_{22}}(f_1 - \delta_k - \rho) + \frac{1}{u_{22} + u_1 f_{22}}\Phi, \tag{23}$$

$$\dot{k} = Ak^{\alpha} i_m^{\beta} m^{\gamma} - \delta_k k - c - p i_m, \tag{24}$$

$$\dot{m} = i_m - \delta_m m. \tag{25}$$

The steady state value (c^*, i_m^*, k^*, m^*) is given by

$$f_1^* - \rho - \delta_k = 0, \tag{26}$$

$$u_3^* + u_1^* f_3^* + (u_2^* + u_1^*(f_2^* - p))(\delta_m + \rho) = 0. \tag{27}$$

$$f^* - \delta_k k^* - c^* - p i_m^* = 0, \tag{28}$$

$$i_m^* - \delta_m m^* = 0. \tag{29}$$

To solve for the steady state, we start with (21),(26)-(29). Firstly, from (29):

$$i_m^* = \delta_m m^*, \tag{30}$$

and combine (21) with (26):

$$f_1^* = \rho + \delta_k = A\alpha (k^*)^{\alpha - 1} i_m^{*\beta} m^{*\gamma} = \alpha \frac{f^*}{k^*}, \tag{31}$$

or $f^* = \dfrac{\rho + \delta_k}{\alpha} k^*$. Therefore, (28) implies

$$\left(\frac{\rho + \delta_k}{\alpha} - \delta_k\right) k^* = c^* + p\delta_m m^*. \tag{32}$$

Next, substitute (30) into (31):

$$\rho + \delta_k = A\alpha \delta_m^{\beta} (k^*)^{\alpha - 1} (m^*)^{\beta + \gamma}. \tag{33}$$

Thus, we can express m^* as a function of k^*:

$$m^* = \left(\frac{\rho + \delta_k}{A\alpha \delta_m^{\beta}}\right)^{\frac{1}{\beta + \gamma}} (k^*)^{\frac{1 - \alpha}{\beta + \gamma}} \equiv \Theta (k^*)^{\frac{1 - \alpha}{\beta + \gamma}}. \tag{34}$$

Note that $u_1^* = (c^*)^{-\sigma}$, $u_2^* = \theta(i_m^*)^{-\tau}$, $u_3^* = \chi(m^*)^{-\phi}$, $f_2^* = \beta\dfrac{f^*}{i_m^*}$, and $f_3^* = \gamma\dfrac{f^*}{m^*}$, (27) implies:

$$\chi(m^*)^{-\phi} + (c^*)^{-\sigma}\gamma\frac{f^*}{m^*} + \left[\theta(i_m^*)^{-\tau} + (c^*)^{-\sigma}\left(\beta\frac{f^*}{i_m^*} - p\right)\right](\delta_m + \rho) = 0. \tag{35}$$

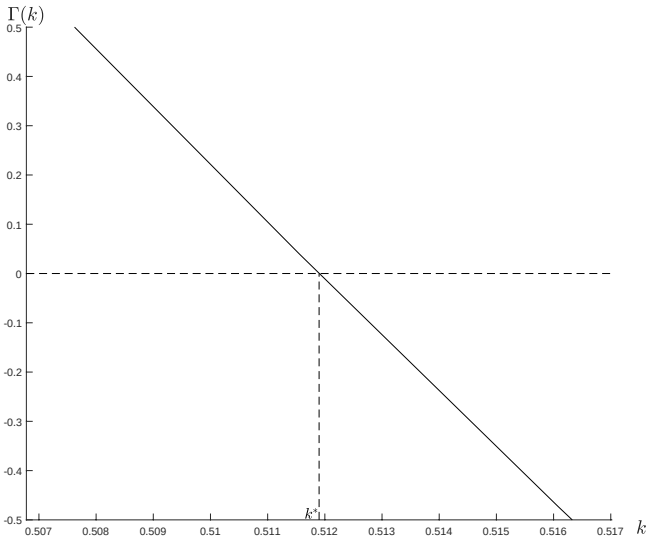
Combine (30), (32), (34), and (35) gives

$$\chi\left(\Theta {k^*}^{\frac{1-\alpha}{\beta+\gamma}}\right)^{-\phi} + \left(\left(\frac{\rho+\delta_k}{\alpha} - \delta_k\right)k^* - p\delta_m\Theta {k^*}^{\frac{1-\alpha}{\beta+\gamma}}\right)^{-\sigma}\left(\gamma\frac{\rho+\delta_k}{\Theta\alpha}(k^*)^{1-\frac{1-\alpha}{\beta+\gamma}}\right)$$

$$+\left[\theta\left(\delta_m\Theta {k^*}^{\frac{1-\alpha}{\beta+\gamma}}\right)^{-\gamma} + \left(\left(\frac{\rho+\delta_k}{\alpha} - \delta_k\right)k^* - p\delta_m\Theta {k^*}^{\frac{1-\alpha}{\beta+\gamma}}\right)^{-\sigma}\left(\beta\frac{\rho+\delta_k}{\delta_m\Theta\alpha}(k^*)^{1-\frac{1-\alpha}{\beta+\gamma}} - p\right)\right](\delta_m+\rho) = 0. \tag{36}$$

The left-hand-side of (36) is a function of physical capital, k^*, only, which we denote as $\Gamma(k)$, hence the optimal k^* solves the equation $\Gamma(k) = 0$.

FIG. 1. The graph of function $\Gamma(k)$



To examine the stability of steady state, we rewrite (19) as

$$\begin{bmatrix} \dot{c} \\ \dot{i}_m \\ \dot{k} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} 0 & \frac{c^*}{\sigma} f_{12}^* & \frac{c^*}{\sigma} f_{11}^* & \frac{c^*}{\sigma} f_{13}^* \\ J_{21}^* & \delta_m + \rho & J_{23}^* & J_{24}^* \\ -1 & f_2^* - p & f_1^* - \delta_k & f_3^* \\ 0 & 1 & 0 & -\delta_m \end{bmatrix} \begin{bmatrix} c - c^* \\ i_m - i_m^* \\ k - k^* \\ m - m^* \end{bmatrix}, \tag{37}$$

where

$$J_{21}^* = \frac{u_1^* f_{12}^*}{u_{22}^* + u_1^* f_{22}^*} + \frac{u_{11}^*}{u_{22}^* + u_1^* f_{22}^*}(f_3^* + (f_2^* - p)(\delta_m + \rho)), \tag{38}$$

$$\begin{aligned} J_{23}^* = &-\frac{u_1^* f_{12}^*}{u_{22}^* + u_1^* f_{22}^*}(f_1^* - \delta_k) + \frac{u_1^* f_{11}^*}{u_{22}^* + u_1^* f_{22}^*}(f_2^* - p) \\ &+ \frac{u_1^*}{u_{22}^* + u_1^* f_{22}^*}(f_{13}^* + f_{12}^*(\delta_m + \rho)), \end{aligned} \tag{39}$$

$$\begin{aligned} J_{24}^* = &-\frac{u_1^* f_{21}^*}{u_{22}^* + u_1^* f_{22}^*} f_3^* + \frac{u_1^* f_{23}^*}{u_{22}^* + u_1^* f_{22}^*}\delta_m \\ &+ \frac{u_1^* f_{13}^*}{u_{22}^* + u_1^* f_{22}^*}(f_2^* - p) + \frac{u_{33}^* + u_1^*(f_{33}^* + f_{23}^*(\delta_m + \rho))}{u_{22}^* + u_1^* f_{22}^*}, \end{aligned} \tag{40}$$

4.2. Simulation

For a better understanding of this model, we assign the following parameter values in Table 1

TABLE 1.

Parameter values in the benchmark case

Parameter	Description	Value
σ	The coefficient of relative risk aversion of ordinary consumption, c	2
τ	The coefficient of relative risk aversion of expenditure on arms, i_m	0.8
ϕ	The coefficient of relative risk aversion of accumulation on arms, m	0.8
θ	The preference for expenditure on arms	0.75
χ	The preference for accumulation on arms	0.75
ρ	The subjective discount rate	0.05
A	The total factor productivity	0.4
α	The productivity of physical capital, k	0.35
β	The productivity of expenditure on arms, i_m	0.05
γ	The productivity of arms capital, m	0.1
p	The price of firearms in terms of consumption goods	2
δ_k	The depreciation rate for capital	0.1
δ_m	The depreciation rate for arms	0.1

With these parameter values, the steady-state values of our dynamic system are

$$c^* = 0.1307,\ \ i_m^* = 0.0187,\ \ k^* = 0.5119,\ \ m^* = 0.1875.$$

The eigenvalues of the Jacobian matrix are (37) are

$$\mu_1 = -0.2049, \quad \mu_2 = -0.0617, \quad \mu_3 = 0.1117, \quad \mu_4 = 0.2549.$$

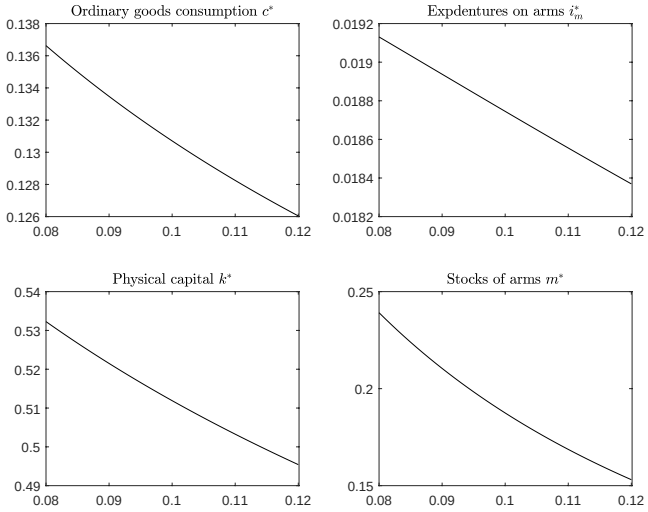
Therefore, the steady state is saddle-point stable.

4.3. Comparative dynamic

To explore the impact of parameter changes on steady-state values of the four endogenous variables, we vary a few key parameter values and closely examine the resulting changes in steady-state outcomes.

Firstly, let the value of δ_m change from 0.08 to 0.12, and the results are shown in Fig. 2.

FIG. 2. The impact of the depreciation rate for arms on long-run steady state



A higher value of δ_m, representing a higher rate of depreciation of firearms stock, exacerbates the vulnerability of firearms to wear and tear, hastening the degradation of an individual's capacity for self-protection and the maintenance of property-rights capital. A very high depreciation rate of firearms also represents the cases when governments confiscate firearms from citizens. As a consequence, the safeguarding of production activities,

property rights, personal consumption, and the accumulation of physical capital is compromised.

In recent times, there has been a discernible global trend towards constraining the right to keep and bear arms, particularly noticeable in nations with welfare-oriented governance structures and authoritarian regimes. It has become commonplace for authorities in these countries to confiscate firearms from citizens, a practice increasingly normalized within their legal frameworks. Consequently, a multitude of firearms have been designated as illegal, significantly reducing the arsenal available to private citizens. Moreover, this trend has had the effect of hastening the depreciation rate of private citizens' weapon accumulation, further curbing their ability to arm themselves in order to protect their life, liberty and property. Such developments raise concerns about the erosion of individual freedoms and the potential for government overreach. Many see parallels between these restrictions and the encroachment on personal liberties outlined in Friedrich Hayek's "The Road to Serfdom", highlighting the potential dangers of a society moving towards government tyranny on a global scale. These observations underscore ongoing debates regarding the balance between public safety measures and the preservation of fundamental rights, with implications for democratic values and governance structures worldwide.

Even in the United States, there has been a growing trend to restrict the right to keep and bear arms in the United States since the 1950s. This trend has resulted in the classification of numerous firearms as illegal, effectively diminishing private citizens' stocks of arms or increasing the depreciation rate of private citizens' weapon accumulation. Throughout this period, various laws and regulations have been implemented at both federal and state levels, aimed at controlling the possession, sale, and use of firearms. These measures include the Gun Control Act of 1968, which prohibited certain categories of individuals from purchasing firearms and established licensing requirements for gun dealers, as well as subsequent legislation such as the Brady Handgun Violence Prevention Act of 1993 and the Assault Weapons Ban of 1994. Additionally, court decisions and interpretations of the Second Amendment have played a significant role in shaping gun laws. All this has compromised the right to keep and bear arms.

In particular, the recent regulation prohibiting bump-stock devices is an executive-directed measure by a federal agency, effectively criminalizing previously widespread and lawful private possessions.[3]

This is a significant way to depreciate firearms stocks for the protection of life, liberty and property. The Second Amendment aptly speaks of "the

[3]Trump's bump stock ban mandated that owners dispose of or surrender their devices by March 2019. Despite challenges to block the ban in other lawsuits, which were appealed to the Supreme Court after three appeals courts rejected the challenges to ATF's final rule, the justices declined to hear any of those cases.

right to keep and bear arms", encompassing firearms, ammunition, and their accessories, rather than solely focusing on firearms. This broader terminology underscores that the natural right to self-defense extends beyond just firearms to include essential components like ammunition and accessories like bump-stock devices. It is worth recalling that the American Revolutionary War was sparked when the British army seized gunpowder from civilian colonists—highlighting that the tipping point of British oppression was the confiscation of non-firearm arms.

The regulation banning bump-stock devices was not passed by Congress but rather implemented under a broad interpretation of executive authority. Both the 45th President Donald Trump and the 46th President Joe Biden support this approach, asserting the executive's right to enact regulations without congressional approval, even if it potentially conflicts with the Constitution. The justification for such bans, including the bump-stock ban, is framed around the concept of ensuring safety rather than upholding constitutional or natural rights.

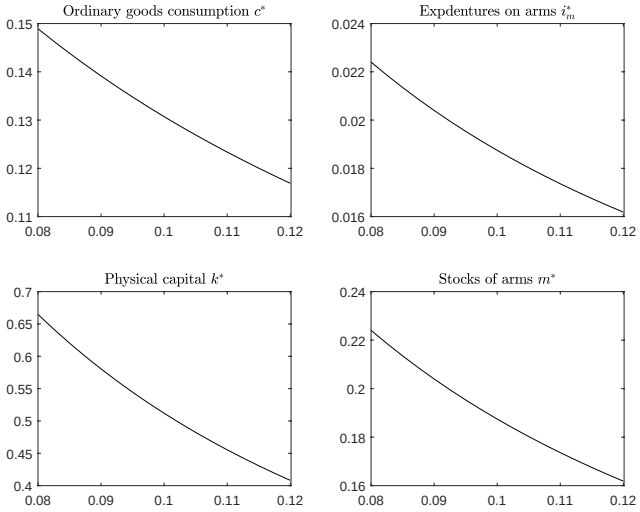
"The bump-stock ban directly and unequivocally limits the right to keep and bear arms that are already commonly owned and in use. In other words, those who own bump-stock devices are required either to turn in their private property to the government or else to destroy them without any recompense. This is a gross and indefensible use of executive power. To reiterate: the bump-stock device ban is a regulation created by a federal agency under the direction of the executive that renders previously common and lawful private property illegal and makes it a felony (up to 10 years imprisonment and a $250,000 fine) to continue to own it even after the same agency stated it was legal two decades before." (Brendan Patrick Purdy: The Right to Keep and Bear Property. In Law & Liberty, April 2, 2024)

Secondly, we vary the value of δ_k from 0.08 to 0.12

A rise in the depreciation rate of physical capital, δ_k, results in diminished levels of steady-state consumption, firearms investment, physical capital accumulation, and firearms accumulation. This outcome is straightforward: rapid depreciation of physical capital necessitates increased resources for investment to replace existing capital, thereby reducing both consumption and firearms investments. Consequently, over the long term, individuals find themselves possessing diminished levels of physical capital, firearms stock (essential for property-right protection), output, and consumption.

Similar outcomes can be derived from an increase in the subjective time discount rate, ρ, and a decrease in total factor productivity, A. In both cases, steady-state consumption, firearms investment, physical capital accumulation, and firearms accumulation experience declines. This is due to the fact that a higher time discount rate diminishes the value of future returns, prompting reduced investments in both physical and firearms-related

FIG. 3. The impact of the depreciation rate for physical capital on long-run steady state



capital. Likewise, a decrease in total factor productivity cuts overall output, leading to reduced resources available for consumption, firearms investment, and capital accumulation. As a result, individuals find themselves with diminished levels of physical capital, firearms stock crucial for property rights protection, as well as reduced output and consumption in the long run.

Thirdly, when the preference for expenditure on arms, θ, rises from 0.3 to 1.5, the results are shown in Fig. 4:

Preferences for purchasing weapons, firearms, firearms training, and firearms accumulation would lead to increases in physical capital and weapon accumulation. This has an "inverted U-shaped" effect on ordinary consumption.

Fourthly, we let the preference for firearms capital, χ, increase from 0.3 to 1.5.

Preferences for purchasing weapons, firearms, military training, and military capital accumulation would foster increases in physical capital and military levels. This has a "inverted U-shaped" effect on ordinary consumption, where smaller military preferences promote consumption, while larger military preferences crowd out consumption.

An increase in preference for weapons accumulation, χ, exhibits similar effects as preferences for expenditure on them. This is because individuals with a higher preference for firearms accumulation allocate more resources towards military expenditure to satisfy their preferences. While

FIG. 4. The impact of the preference for expenditure on arms on long-run steady state

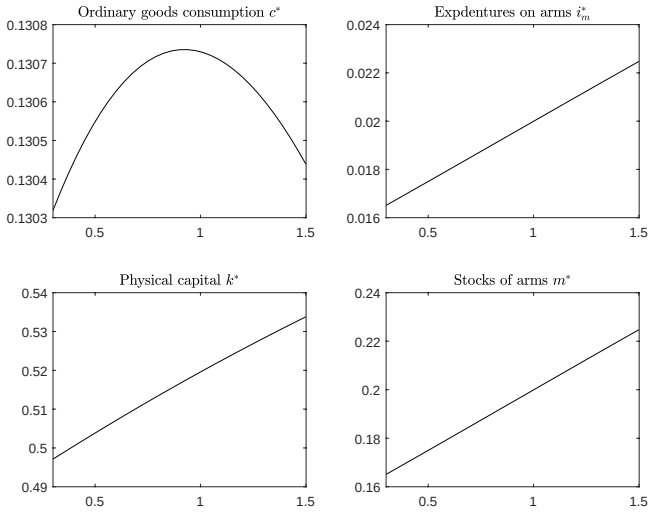
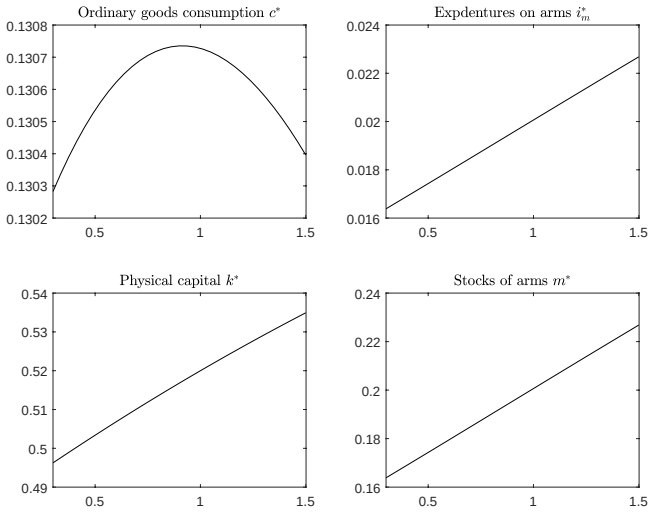


FIG. 5. The impact of the preference for accumulation on arms on long-run steady state



increased military capital promotes and safeguards total output, it may reduce demand for ordinary consumption. Nations such as Sparta, Athens, Rome, Switzerland, and the United States, which prioritize the right to keep and bear arms, exhibit a profound commitment to safeguarding life, liberty, and property. This dedication stems from a recognition of the crucial role firearms ownership plays in ensuring individual and collective security. These nations often possess robust legal frameworks and cultural norms that uphold the right to self-defense and emphasize the importance of firearms in protecting citizens' rights and freedoms. Consequently, they tend to experience greater levels of societal resilience and civic engagement, as well as a heightened sense of individual empowerment and responsibility for ensuring the safety and well-being of their communities.

The right to keep and bear arms has been integral to the history of the United States, spanning from the colonial era through the westward expansion into the Midwest and California in the 19th and early 20th centuries. Firearms held multifaceted importance throughout this period:

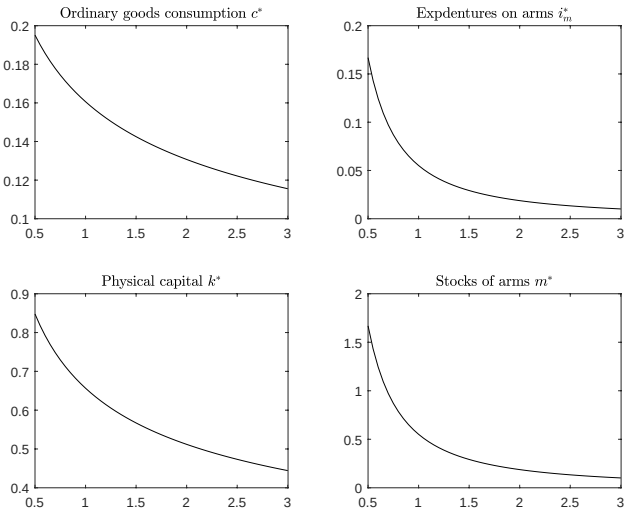
During the colonial era (1600s-1770s), firearms served as essential tools for hunting, self-defense, and protection against indigenous peoples and rival European colonial powers. They were crucial for safeguarding settlements and asserting independence from British rule. In the American Revolution (1775-1783), firearms were pivotal in resisting British forces, with citizen-soldiers utilizing them to secure victories at key battles like Lexington and Concord, Bunker Hill, and Saratoga. This right was fundamental in securing American independence and establishing the nation.

As settlers expanded westward into territories like the Ohio Valley, Great Plains, and California, firearms remained vital for survival and security. Settlers faced various challenges, including conflicts with indigenous peoples, wildlife, and lawlessness. Firearms provided protection, enabling them to defend themselves, their families, and their property. Manifest Destiny ideology fueled western expansion in the 19th century, with firearms playing a crucial role in establishing new settlements, safeguarding against threats, and asserting American sovereignty. The California Gold Rush further accelerated migration westward, with firearms being essential for mining and settlement endeavors.

Beyond security, firearms were indispensable tools for homesteading, farming, and ranching activities in the Midwest and California. They were used for hunting game, controlling predators, and protecting livestock, crops, and property. Overall, the right to keep and bear arms was vital for securing American independence, protecting property rights, and ensuring the safety and liberties of settlers during the nation's expansion from the colonial period to the early 20th century.

Finally, when the price of expenditure on arms, p, rises from 0.5 to 3, the results are shown in Fig. 6

FIG. 6. The impact of the exogenous price of new firearms investment on long-run steady state



The rise in the costs associated with acquiring weapons, p, exerts downward pressure on individuals' willingness to invest in their firearms expenditures. This, in turn, leads to a reduction in the accumulation of arms capital, physical capital, output protection, and ultimately, consumption. In essence, elevated firearm costs undermine everyone's capacity to safeguard their own life, liberty, property, production, and overall welfare. Unfortunately, this trend is not confined to a few isolated instances but is observed across various countries worldwide. Aggressive governments, in particular, have persistently raised the barriers for citizens to access arms and weapons, exacerbating the challenges faced by individuals in maintaining their fundamental rights and security. This pattern underscores the ongoing struggle between governmental control and individual autonomy, with significant implications for the broader societal fabric and the balance of power between citizens and their rulers.

Since the founding of the United States in the late 18th century, the right to keep and bear arms has undergone significant changes, reflecting shifts in societal norms, legal interpretations, and public policy. While the Second Amendment to the U.S. Constitution, ratified in 1791, enshrined the right of individuals to possess firearms, the extent and application of this right have evolved over time.

During the early years of the republic, gun ownership was widespread and deeply ingrained in American culture, with firearms playing a crucial role

in self-defense, hunting, and militia service. However, various factors have contributed to the gradual diminishment of this right over the centuries.

One key development in the erosion of gun rights was the enactment of firearm regulations at the state and federal levels, beginning in the 19th century. These regulations sought to address concerns related to public safety, crime prevention, and the proliferation of firearms among certain groups. For example, the National Firearms Act of 1934 imposed restrictions on the ownership and transfer of certain types of firearms, such as machine guns and sawed-off shotguns, in response to concerns about organized crime and gang violence.

Throughout the 20th century, the issue of gun control became increasingly politicized, leading to a series of legislative battles and policy debates. High-profile incidents of gun violence, such as the assassinations of political figures and mass shootings, galvanized public opinion and spurred calls for stricter firearm regulations. In response, lawmakers enacted measures aimed at enhancing background checks, restricting access to certain types of firearms, and imposing waiting periods for gun purchases.

Legal interpretations of the Second Amendment have also evolved over time, influencing the scope and application of gun rights in the U.S. While early court rulings tended to emphasize the collective right of states to maintain militias, the landmark Supreme Court decision in District of Columbia v. Heller (2008) affirmed an individual's right to possess firearms for self-defense within the home. Subsequent rulings, such as McDonald v. Chicago (2010), extended this right to the states, further shaping the landscape of gun rights in America.

4.4. The optimal path

Given the Jacobian matrix and eigenvalues in (37) and (39), the corresponding eigenvectors are

$$\mathbf{V}_1 = \begin{bmatrix} -0.0566 \\ -0.0599 \\ -0.8167 \\ 0.5712 \end{bmatrix}, \quad \mathbf{V}_2 = \begin{bmatrix} -0.1270 \\ -0.0183 \\ -0.8687 \\ -0.4784 \end{bmatrix}, \quad \mathbf{V}_3 = \begin{bmatrix} 0.0923 \\ -0.0371 \\ -0.9795 \\ -0.1751 \end{bmatrix}, \quad \mathbf{V}_4 = \begin{bmatrix} 0.0644 \\ 0.1171 \\ -0.9345 \\ 0.3299 \end{bmatrix}.$$

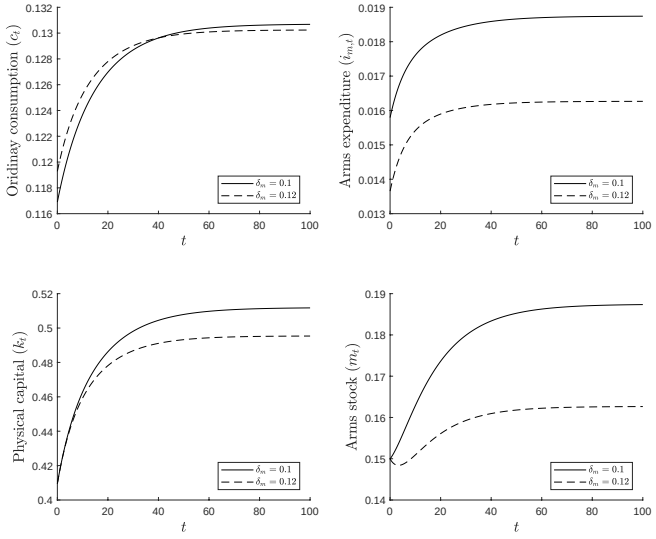
The optimal paths around the steady state are

$$\mathbf{X} = \mathbf{X}^* + C_1\mathbf{V}_1 e^{\mu_1 t} + C_2\mathbf{V}_2 e^{\mu_2 t} + C_3\mathbf{V}_3 e^{\mu_3 t} + C_4\mathbf{V}_4 e^{\mu_4 t},$$

where $\mathbf{X} = [c, i_m, k, m]'$ and $\mathbf{X}^*$ denotes the steady state values. C_1, C_2, C_3, and C_4 are undetermined coefficients. The transversality conditions (9) and (10) require the undetermined coefficients of the positive eigenvalues, C_3 and C_4, to be zero. At the same time, C_1 and C_2 are determined by initial conditions, k_0 and m_0. For example, when the economy starts at the

initial position of $k_0 = 0.8k^*$ and $m_0 = 0.8m^*$, this implies $C_1 = 0.0185$ and $C_2 = 0.1005$. The transition path is given by Fig. 7.

FIG. 7. The evolution of variables over time along the saddle-path convergence trajectory near the steady state.

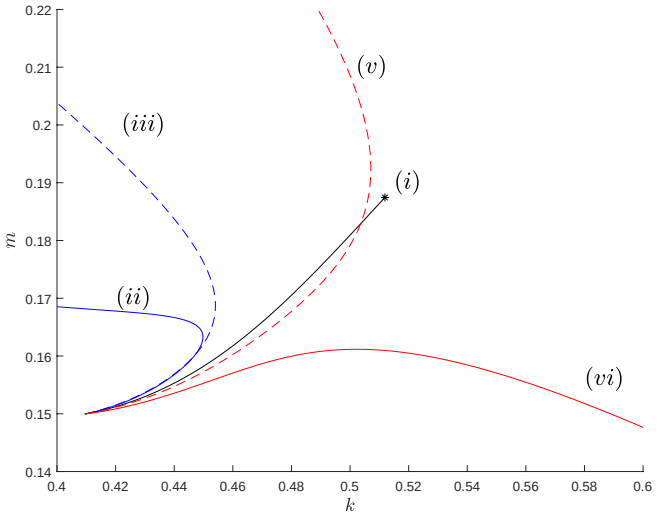


Starting from a position that both capital stock and arms stock are below the steady state, agents choose a corresponding optimal consumption and firearms purchase. Over time, individuals accumulate more physical capital and firearms stock for safeguarding production and personal utility, thereby enhancing their long-run output and welfare. Another case is illustrated by the dashed lines in Fig. 7, which shows the transitional dynamics with a higher depreciation rate of firearms. In the long run, the economy ends up with lower consumption, lower capital accumulation, and lower firearms stock. The economy converges to the steady state more rapidly as a higher depreciation rate of firearms increases the absolute value of the two negative eigenvalues.

It is challenging to draw a phase diagram for the four-dimension system. However, we can see the evolution of physical capital and firearms accumulation over time in a two-dimension m-k phase portrait. As illustrated in Fig. 8, the economy starts from the initial state at the bottom left corner, then

(i) the solid black line depicts the trajectory wherein individuals optimize their initial consumption and firearms investment choices, proceeding along the saddle-path trajectory toward the long-term steady state.

FIG. 8. The optimal path with different initial consumption decision.



(ii) the solid blue line depicts a trajectory where individuals exhibit excessive consumption and insufficient investment in physical capital accumulation;

(iii) the dashed blue line illustrates a trajectory where individuals allocate an excessive amount of resources to firearms purchases, resulting in an over-accumulation of military capital and a deficiency in physical capital accumulation;

(iv) the solid red line illustrates a trajectory where individuals reduce their consumption and firearms investments excessively, leading to an over-accumulation of physical capital and an insufficient accumulation of firearms;

(v) the dashed red line illustrates a trajectory where individuals decrease their ordinary consumption but increase military spending. Initially, driven by savings, the economy undergoes overinvestment in physical capital compared to the saddle-path convergence trajectory. However, as time elapses, individuals continue to boost consumption expenditure, leading to excessive accumulation of military capital and insufficient physical capital.

5. CONCLUSIONS

This paper presents a robust demonstration of the pivotal role of the right to keep and bear arms in safeguarding individuals' life, liberty, and property. Our analysis reveals that firearms accumulation, serving as a

form of life, liberty, and property rights protection, leads to increases in physical capital accumulation, output production, and overall welfare. By establishing a solid mathematical model, we provide theoretical backing for the natural right to bear arms, a principle deeply rooted in ancient wisdom and enshrined in modern constitutional frameworks, particularly exemplified by the Second and Fourth Amendments of the US Constitution. Furthermore, our examination extends to historical precedents such as the Bill of Rights 1689, which granted English Protestant citizens the right to possess arms for defense, thereby limiting the Crown's authority over arms regulation.

Additionally, we draw attention to concerning global trends, notably the rising costs and constraints on bearing arms observed across various countries. This trend, compounded by aggressive government actions, including the confiscation of firearms, poses significant challenges to individuals' ability to uphold their fundamental rights and security. Such developments underscore the delicate balance between governmental control and individual autonomy, with profound implications for societal dynamics and the distribution of power.

The parallels drawn between these restrictions and the encroachments on personal liberties outlined in Friedrich Hayek's "The Road to Serfdom" emphasize the potential dangers of societal shifts towards government tyranny.

As we conclude this paper, it's imperative to analyze contemporary government restrictions and bans on citizens' right to bear arms and random searches worldwide, drawing parallels to the historical context of 18th-century colonial America. During this period, colonial subjects endured arbitrary and invasive searches authorized by the British king through the contentious "Writs of Assistance." These writs granted British troops and officials unchecked authority to search homes and private properties without warrants, primarily to uncover goods imported illegally or on which taxes had not been paid.

Central to the resistance against such intrusions was James Otis, a prominent Massachusetts lawyer and political activist often revered as "the Founding Father of the 4th Amendment." In a notable 1761 speech condemning the Writs of Assistance, Otis vividly illustrated the threat posed by unchecked government search powers to the liberty and tranquility of the people. He passionately argued for the sanctity of a person's home, likening it to a castle where individuals should be shielded from unwarranted intrusion.

Otis's fervent advocacy against government overreach laid the groundwork for the principles enshrined in the 4th Amendment of the Bill of Rights. His efforts were instrumental in ensuring that violations of privacy and property rights would not be tolerated following America's independence. The profound impact of Otis's principles was acknowledged by a

young John Adams, who famously remarked that "the child independence was born" during Otis's speech. Ultimately, Otis's principles became the cornerstone of individual liberty, private property protection, and privacy law in the United States.

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