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Inattentive Capital Investment with Nonconvex Costs

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This paper extends Sargent's adaptation of Lucas and Prescott's model of investment under uncertainty within a competitive industrial equilibrium. In this framework, firms incur capital costs proportional to adjustment size and make investment decisions amid rational inattention. Equilibrium analysis reveals that aggregate investment then exhibits partial adjustment, with individual firms adjusting investments infrequently based on an optimal probability and with fixed costs. Additionally, in a general equilibrium setting, marginal q increases with the degree of rational inattention, while the relationship between Tobin's q and the optimal investment rate under state uncertainty remains ambiguous.

Key Words: Rational Inattention; Uncertainty; Investment; Adjustment Cost; Asset Pricing.

JEL Classification Numbers: D83, E22, E23, G12.

1. INTRODUCTION

The study of investment is crucial as firm's investment decisions determine future output, which is a major measurement of both firm and country performance. Current investment theory primarily focuses on two categories of variables: those measuring past investment decisions, such as lagged capital stock, and those measuring current market opportunities, including factor prices, interest rate, and profits. Prices, influenced by demand, are also affected by uncertain factors such as weather, politics, interest rates, etc. Lucas and Prescott (1971) pioneer the integration of uncertain future conditions into adjustment-cost models of firms, exploring optimal responses to current conditions and future uncertain states. They introduce a model assuming stochastic shifts in industry demand and solve for the firm's objective function under rational expectations. In equilibrium, capital evolvement, output and prices are jointly determined as functions of current observable explanatory variables.

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Meanwhile, the literature on adjustment cost has sparked extensive debate regarding its empirical implications. Caballero (1999) states that the quadratic adjustment cost assumption deals with the frequent adjustment, aiming to smooth aggregate investment dynamics and make infrequent adjustment unimportant. However, empirical studies reveal that investment often exhibits lumpiness, especially at the plant level. Studies by Doms and Dunne (1993) and Bloom, Bond, and Breenen (2007) indicate that capital accumulation patterns are characterized by discrete episodes of intense investment interspersed with periods of lower activity. Pyndick (1988) examines irreversible investment in micro models, illustrating how firms adjust their investments in response to changes in capital stock, aligning marginal costs with optimal levels of capacity. So far, most literature has assumed nonconvex adjustment cost in micro models and convex adjustment cost in macro models, which are inconsistent.

A relevant body of literature on investment under uncertainty includes the work of Sargent (1987). The author adopts the basic assumptions of the rational expectations environment and further studies investment under uncertainty with robustness. In his model, firms maximizes their discounted future profits given the discount rate, factor prices and initial capital stock. The optimal investment decision rule is obtained in terms of aggregate capital dynamics. When the demand shock follows an AR(1) process, the resulting investment decision rule produces a capital accumulation function that responds to current price shock rather than to anticipations of all future price shocks. Consequently, aggregate output becomes a function of observable explanatory variables, reducing reliance on unobservable factors that would otherwise need to be replaced by proxies. Furthermore, this type of uncertainty often leads over responsiveness or precautionary behaviors, aptly described as "*making hay while the sun shines*".

This paper adopts Sargent's linear quadratic framework, where firms solve their profit maximization problem, and capital adjustment cost(CAC) is assumed nonconvex. At the individual level, the existence of fixed cost induces firms to adjust their actions infrequently. Specifically, in the presence of AR(1) demand shocks, firms optimize adjustment probabilities to minimize welfare losses from deviating from the optimal path. Nonconvex costs capture increasing returns and nonlinear microeconomic adjustments, so it breaks the smooth and continuous adjustment of investment under convex adjustment cost, and reconciles lumpy, infrequent plant-level investment with smooth aggregate investment. In our model, since agents are subject to state uncertainty or rational inattention(RI), the idiosyncratic endogenous shocks cancel out and aggregate investment is smoothed like implied by a convex adjustment cost model. As a result, at the aggregate level, state uncertainty smooths aggregate investment responses to the demand shocks, producing a sluggish, distributed lag pattern of investment. Above all, state uncertainty results in a slow-moving, hump shaped investment process that is consistent with empirical evidence. A close paper by Wang and Wen (2012) assumes that individual firms entails an idiosyncratic shock to the rate of investment return, and the occasionally binding financial constraint related to this investment-specific shocks helps produce sluggish aggregate investment. Another relevant paper is Flori (2012), which uses a two-sector model with nonconvex capital adjustment cost in the investment sector to smooth the shift in investment supply following shocks, hindering the increase of production.

Finally, a general equilibrium model which analyzes the effect of rational inattention on asset price is also introduced in the last section. When firms are inattentive, consumers choose lower level of consumption due to decreased excess return from the claim of the risky stock issued by firms. Also, as firms become more inattentive, asset price is more attributed to the installed capital stock level and less to the future profitability, making the installed capital more valuable than the prospect to increase capital and output. Rational inattention thus sufficiently implies a higher Tobin's q, or higher market to book value. This result is consistent with Hayashi (1982), that investment theory with installation cost and the q theory are equivalent, and greater concavity of installation function leads to higher Tobin's marginal q. In contrast to the evidence that Tobin's q lacks empirical accountability, my result show that the investment rate does not necessarily have a positive correlation with marginal q.

The remainder of the paper will be organized as follows. Section 2 presents the empirical evidence on aggregate investment dynamics. Section 3 develops a benchmark model with capital adjustment costs. Section 4 explores the RI model in an induced signaling extraction problem, discussing investment dynamics and firm behavior. Section 5 introduces a general equilibrium framework of a consumer's consumption and investment decisions, where the claim that delivers the aggregate dividends is priced and optimal consumption is derived. The concluding section summarizes the main findings and discusses avenues for future research.

2. EMPIRICAL EVIDENCE

This section introduces the empirical evidence. Aggregate investment is smooth, while firm-level investment is lumpy and infrequent. Table 1 lists the percentage of observations with zero investment in firms and plants in different sizes. This table, taken from Bloom, Bond, and Reenen (2007), presents data from a sample of U.K. manufacturing companies and establishments that contain one or two plants at the same location. It shows that firms of different sizes, across different types of capital(e.g., buildings, equipment, vehicles), exhibit zero investment rates. When these different types of capital are aggregated within a single plant or firm, the percentage of zero investment rates decreases significantly. The table also indicates that aggregating plants within a firm or establishment on a specific capital gives the same result. From the table, we can infer that firm-level or plant-level investment in multiple types of capital goods can be inactive periodically.

TABLE 1.

Episodes of Zero Investment in Different Types of Data

	% of observations with zero investment			
	Buildings	Equipment	Vehicles	Total
Firms	5.9	0.1	*n.a*	0.1
Establishments	46.8	3.2	21.2	1.8
Single Plants	53.0	4.3	23.6	2.4
Small Single Plants	57.6	5.6	24.4	3.2

TABLE 2.

Correlation Between investment Rate and Tobin' q

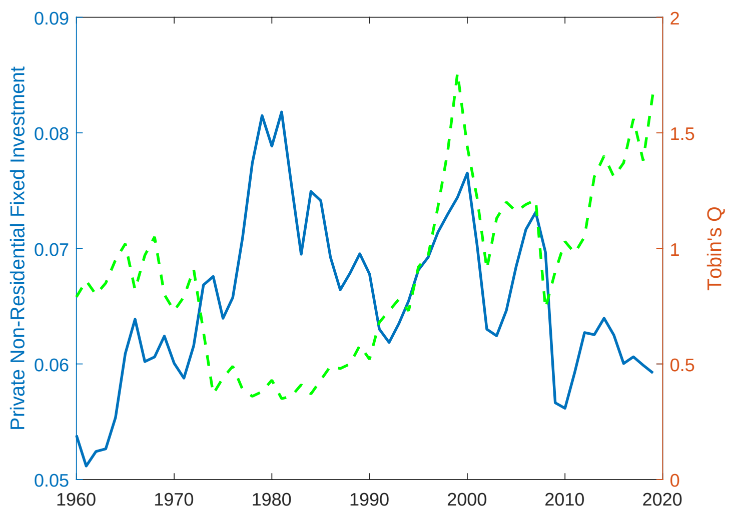
	$1960-1996$	$1960-2019$
$corr(\frac{I}{K}, q)$	-0.74	-0.28

In Table 2, I report the correlation between the investment rate and Tobin's q for the periods of $1960-1996$ and $1960-2019$. The data is from U.S. Bureau of Economic Analysis (BEA), with the investment data from NIPA tables and Tobin's q constructed from flow of funds Z1 release data. The investment rate is constructed as the private nonresidential fixed investment divided by the nonresidential fixed assets(Equipment, software and structures) at their historical cost. Tobin's q is from Harper and Retus (2022) and is calculated as the market value of outstanding equity divided by the net stock of produced assets.[1] Figure 1 compares to a similar graph in Hassett and Hubbard (1996). The solid line plots the investment rate, while the dashed line shows the Tobin's q as defined above. The data reveals that the investment rate and Tobin's q are not positively correlated during the period of 1960-1996, with a correlation coefficient of -0.74. The pattern is same as in Hassett and Hubbard (1996), who as well plot the correlation between business fixed investment and Tobin's q. For the period 1960-2019, the investment rate appears less negatively correlated with Tobin's q after 1997, yielding a correlation coefficient of -0.28. Overall, there is no positive correlation between the business fixed investment rate and Tobin's

[1]Produced assets refer to the net stock of capital plus inventories valued at current(replacement) cost.

q, which contradicts the theoretical predictions of Hayashi (1982), Tobin (1969), Lucas and Prescott (1971), and others.

FIG. 1. Business Fixed investment Rate and Tobin's q in the Period of 1960-2019



3. A FULL-INFORMATION RATIONAL EXPECTATIONS PRODUCTION MODEL

First, I consider a competitive partial equilibrium with adjustment cost and discuss the implied investment dynamics. There are n firms in the industry, each of which produces a single homogenous good. n is a large number so that the industry is competitive. Demand for the single good is governed by an inverse demand function:

$$p_t = A_o - A_1 \overline{q}_t + v_t. \tag{1}$$

The demand shock v_t conveys other factors such as the aggregate conditions or consumer's taste, and it follows an AR(1) process:

$$v_{t+1} = \rho_v v_t + C_v \epsilon_{t+1}, \tag{2}$$

where $\epsilon_{t+1} \sim N(0,1)$ is *iid*. A presentative firm entails a one-period quadratic cost function with capital adjustment cost:

$$\sigma(q_t, q_{t+1}) = \frac{d}{2}(q_{t+1} - q_t + c)^2, \tag{3}$$

where I assume the price of capital goods, $\{J_{t+1}\}_{j=0}^{\infty}$ to be a constant c for tractability. With quadratic adjustment cost, the law of motion is given by $k_{t+1} = (1-\delta)k_t + I_t$. Net profit each period includes an additional quadratic installation cost, so capital adjustment is more costly at high levels and this specification allows for a variant Tobin's q. Since I assume that production function takes the simple linear form $q_t = f_0 k_t$, $f_0 = 1$, the firm's one-period profits are $\pi_t = p_t q_t - \sigma(q_t, q_{t+1})$. The representative firm is a price taker and operates over an infinite periods of time. The firm maximizes its discounted net profits with respect to sequences $\{q_{t+1}\}$, with the uncertainty originating from the path of v_t, which is unknown until realized at each period. The firm believes that the law of motion for aggregate output follows a linear function:

$$\overline{q}_{t+1} = l_q(\overline{q}_t, v_t). \tag{4}$$

This linear specification facilitates the aggregation of future output and transforms the individual firm's maximization problem into one that maximizes the social surplus.

The timing is as follows. At the beginning of period t, the demand shock v_t is realized, firms collect last period's investment and undepreciated capital, and production takes place. At the end of the period, firms form expectations about next period's demand and determine the investment for the next period, so the current capital stock and demand jointly determine next period's production. I assume firms are risk neutral and maximize their discounted future expected profit stream. The problem can be formulated as follows:[2]

$$\max_{\{q_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \{p_t q_t - \frac{d}{2}(q_{t+1} - q_t + c)^2\}, \tag{5}$$

subject to q_0 given.

The specifications make the problem naturally Linear-Quadratic-Gussian and easily tractable. The equilibrium solution of the two-player zero-sum game should take the form:

$$q_{t+1} = \phi_q(q_t, v_t, \overline{q}_t), \tag{6}$$

where the last two arguments determine the price level. Since the firm perceives the motion of capital to be linear as specified in (4), and for the belief to be ex post correct, applying homogeneity, the actual law of motion

[2] I assume constant cost of capital in firm's maximizing problem, as investment and cost of capital are weakly correlated or uncorrelated both in my empricial study and in the literature. See Shapiro and Lovell (1986) for a discussion.

can be obtained as:

$$\overline{q}_{t+1} = \phi_q(\overline{q}_t, v_t, \overline{q}_t). \tag{7}$$

Thus, a competitive equilibrium is a fixed point of the mapping from $l_q(\overline{q}, v)$ to $\phi_q(\overline{q}, v, \overline{q})$. Extending the argument of Lucas and Prescott (1972) and Sargent (1987), the solution can be computed directly by solving the central planner's maximization problem:

$$\max_{\{q_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \{S(\overline{q}_t, v_t) - \frac{d}{2}(\overline{q}_{t+1} - \overline{q}_t + c)^2\}, \tag{8}$$

where the firm maximizes the consumer surplus net cost function, $S(\overline{q}_t, v_t)$, which is defined as:

$$S(\overline{q}_t, v_t) = \int_0^{\overline{q}_t} (A_0 - A_1 x + v_t) dx = A_0 \overline{q}_t - \frac{A_1}{2} \overline{q}_t^2 + \overline{q}_t v_t. \tag{9}$$

Thus, the competitive problem can be implicitly rationalized as a maximization problem of social welfare instead of being solved as a complex fixed-point problem, and we can conveniently find the equilibrium sequence of $\{p_{t+j}\}, \{k_{t+j}\}$. A solution to this problem would be:

$$\overline{q}_{t+1} = l_q(\overline{q}_t, v_t). \tag{10}$$

I use the optimal linear regulator methods to solve this problem. Namely, the problem can be written as:

$$\max_{\{\overline{q}_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{bmatrix} u_t' & s_t' \end{bmatrix} \begin{bmatrix} R & W' \\ W & Q \end{bmatrix} \begin{bmatrix} u_t \\ s_t \end{bmatrix} \right\}, \tag{11}$$

subject to

$$s_{t+1} = A s_t + B u_t + C \epsilon_{t+1}, \tag{12}$$

where $s_t' = (\ \overline{q}_t \ \ v_t \ \ 1\ )$, $u_t' = (\overline{q}_{t+1})$, and the other matrices elements are functions of model parameters. The solution for the optimal linear regulator problem has the form:

$$u_t' = -F s_t. \tag{13}$$

The above equation gives $\overline{q}_{t+1}$ as a function of current period's production $\overline{q}_t$ and the demand shock v_t.

Next I solve the model using optimal linear regulator. The following proposition holds:

PROPOSITION 1. *Under rational expectations, the solution to the central planning problem takes the following form:*

$$\overline{k}_{t+1} = \lambda_1 \overline{k}_t + \frac{\lambda_1 f_0 \rho_v v_t}{d(1-\rho_v \beta \lambda_1)} - \frac{\lambda_1 \left[(1-\beta)c - A_0 f_0\right]}{d(1-\rho_v \beta \lambda_1)},$$

where $(F^2 + \frac{\phi}{\beta}F + \frac{1}{\beta}) = (F-\lambda_1)(F-\lambda_2)$, $\lambda_1 < 1 < \frac{1}{\beta} < \lambda_2$, *and* $\phi = -((1+\beta) + \frac{A_1 f_0^2}{2d})$.

Proof. See Appendix A.1. ▌

The above equation is the typical solution of partial adjustment, under which capital stock(or output) is a weighted average of last period's capital stock level and realization of the price shock. In Appendix A.1, I derived the rational expectations solution with no information frictions, showing that capital stock is a weighted average of past stock and target stock level, the latter capturing the expectations of all future demand shocks that affecting the marginal profitability of capital. Furthermore, let $\overline{I}_t = \overline{k}_{t+1} - \overline{k}_t$, where the subscript t on the left-hand side of the equation denotes the time the investment cost is incurred. Assuming no depreciation, the aggregate investment can be written as:

$$\overline{I}_t = \lambda_1 \overline{I}_{t-1} + \frac{\lambda_1 \rho_v(\rho_v - 1)}{d(1-\rho_v \beta \lambda_1)} v_{t-1} + u_t, \tag{14}$$

where $u_t = \frac{\lambda_1 \rho_v C_v}{d(1-\rho_v \beta \lambda_1)} \epsilon_t$. Aggregate investment adjusts partially to last period's investment, and responds positively to increases in the demand shocks. Moreover, with convex adjustment cost, investment is a weighted average of last period's investment stock and past profitability news, thus is sluggish.

Capital adjustment cost assumption is key to the introduction of autocorrelations of investment. Previously, convex adjustment costs have been used to address the slow and smooth adjustment in aggregate capital formation, yet there are also criticisms: (i) CACs are not consistent with firm-level data, as individual firm's investments are often lumpy and infrequent; (ii) To generate a realistic equity premium, the coefficient on CAC function is often too large; (iii) Empirical analysis based on micro data does not find CACs play a role in explaining firm-level dynamics. In contrast, McDonald and Siegel (1986) and subsequent research have assumed an option approach in micro models. They find that irreversibility and the value of waiting widen the region of inaction for two subsequent periods of investment, so firms are more cautious in investment and plant-level data are lumpy. Bloom, Bond and Breenen (2007) demonstrate that the effects of uncertainty and irreversibility on short-run investment dynamics can be detected in their firm-level investment analysis, finding evidence of more cautious behavior for firms facing greater uncertainty.

In the next section, I would introduce a rational inattention model without the CAC assumption, and analyze individual firm's investment dynamics as well as aggregate dynamics.

4. THE RI MODEL

4.1. Incorporating State Uncertainty

Following Sims (2003) and Luo et. al. (2008, 2010), in this section I incorporate state uncertainty into the LQG model without adjustment cost. Agents have finite capacity to update states given the current information set, and I analyze how rational inattention due to imperfect observations impacts the joint dynamics of the demand shocks and aggregate investment.

The assumptions of rationally inattentive agents are specified as follows. Usually, we do not know what the world's true state is. With finite capacity $\kappa \in (0, \infty)$, the true state s_{t+1} is observed with error with the time $t+1$ information set I_{t+1}. I_{t+1} is generated by the entire history of perceived states $\{s_j^*\}$. From time t to $t+1$, the uncertainty reduced from updating information is constrained by the finite capacity κ. I define uncertainty as entropy, which measures the dispersion of the distribution of an unknown state. The information processing constraint can be described as:

$$H(s_{t+1}|I_t) - H(s_{t+1}|I_{t+1}) \leq \kappa, \tag{15}$$

where $H(s_{t+1}|I_t)$ denotes the entropy of the state prior to observing the new signal at $t+1$, and $H(s_{t+1}|I_{t+1})$ is the entropy after observing the new signal. κ denotes the upper bound of the amount of entropy that can be reduced at each period, which is invariant with time in our setting.[3]

Furthermore, I assume that in the steady state, the true state is Gaussian after observing the noisy signal: $s_t|I_t \sim N(E[s_t|I_t], \Sigma_t)$, where Σ_t is the steady state conditional variance of the state: $\Sigma_t = E_t[(s_t - \widehat{s}_t)(s_t - \widehat{s}_t)^T]$. Denote $\Psi_t = var_t(s_{t+1})$. When the conditional distribution of the state is Gaussian, in our case, the above inequality implies that:

$$\ln |\Psi_t| - \ln |\Sigma_{t+1}| \leq 2\kappa. \tag{16}$$

We need another constraint to ensure that information flow is increasing

$$\Psi_t \succeq \Sigma_t, \tag{17}$$

[3] An elastic attention case can be refered in Li, Luo and Nie (2017).

where $\Psi_t - \Sigma_t$ is positive semi-definite. This is called the 'no-forgetting' constraint since the variance of the states cannot be improved by forgetting about the information previously received.

I can show that under the above assumptions, a rational inattention problem is observationally equivalent to a signal extraction problem. Here I solve the problem assuming a noisy signal form. The noisy signal takes the additive form $s^*_{t+1} = s_{t+1} + \xi_{t+1}$, where ξ_{t+1} is the endogenous noise caused by finite capacity. I assume that ξ_{t+1} is *iid* idiosyncratic from the inattentive agent's information processing constraint and is independent of the fundamental shock, $\xi_{t+1} \sim N(0, \Lambda_{t+1})$.

In this case, imperfect observations of the true state lead to welfare losses, and agents use noisy signal to estimate the true state.[4] Specifically, I assume here that agents use a Kalman filter to update the perceived state $\widehat{s}_{t+1} = E[s_{t+1}|I_{t+1}]$ in the steady state so that

$$\widehat{s}_{t+1} = (1-\theta)(A\widehat{s}_t + Bu_t) + \theta s^*_{t+1}, \tag{18}$$

where θ is the Kalman gain to be derived. This weighted average approach can be interpreted as follows. The conditional variance-covariance Σ_t is the mean squared deviation of the signal vector from its mean, so it can be considered as a measure of the information quality. The Bayesian updating procedure adopted by firms gives the precision of the estimation of the true state. Over time, with updating, the variance-covariance matrix converges to a positive steady state Σ independent of the initial condition Σ_t. Then the learning process becomes stabilized. At this moment, the Kalman gain is a fixed weight, and beliefs only respond to observations and are characterized by changes in the conditional mean.[5]

Suppose now firms cannot observe perfectly the demand shock v_t when they make investment decisions. By aggregating, idiosyncratic endogenous shocks will later be canceled out, yet we will see that an agent who only knows the distribution of the noise will choose an optimal probability to invest with the existence of fixed cost. This is key to explaining the infrequent and volatile firm-level investment and smooth aggregate investment. v_t conveys information like macroeconomic conditions and consumer taste. Since these factors and total quantity produced together determine the current price, it should be taken into account when firms make current production decision at the beginning of the period. From Section 3 aggregate output can be seen as an MA(1) process of v_t, so the history of v_t and the initial $\overline{q}_0$ gives all the information about $\overline{q}_t$ and hence investment I_t.

[4]Note that in a rational inattention problem, the steady variance-covariance matrix is endogenously determined by allocating different capacity to elements, while in a SE problem, it's exogenously specified first. But the two are observationally equivalent in a univariant case.

[5]See Detemple (1986) for an univariable example.

So even if firms cannot observe $\overline{q}_t$ perfectly, allocating all the attention to v_t gives the firm maximum benefit.

Let perceived price shocks under RI be $\widehat{v}_{t+1} = (1-\theta)(\rho\widehat{v}_t)+\theta v^*_{t+1}$, where $v^*_{t+1} = v_{t+1} + \xi_{t+1}$. For the conditional variance of v_t, σ_t^2, condition (16) reduces to the following condition:

$$\ln \sigma_t^2 - \ln(\rho^2\sigma_{t-1}^2 + C_v^2) \leq 2\kappa. \tag{19}$$

Then we can get the full evolvement equation of the perceived state for the RI model of aggregate investment:

$$\widehat{v}_{t+1} = \rho_v \widehat{v}_t + \eta_{t+1}, \tag{20}$$

where $\eta_{t+1} = -\theta\rho(\widehat{v}_t - v_t) + \theta(C_v\epsilon_{t+1} + \xi_{t+1})$, $v_t - \widehat{v}_t = \frac{(1-\theta)C_v\epsilon_t}{1-(1-\theta)\rho\cdot L} - \frac{\theta\xi_t}{1-(1-\theta)\rho\cdot L}$, L is the lag operator, and θ is the Kalman gain.

Furthermore, using the standard techniques to solve the Kalman filter problem, we get $\theta = \Sigma\Lambda^{-1}$, where Σ and Λ are the steady-sate values of the conditional variance of the state and noise, respectively. Combining with agent's information processing constraint, we get the Kalman gain $\theta = 1 - 1/\exp(2\kappa)$.

Now perceived price shock v_t is a weighted average of contemporaneous shocks and all past history of shocks, with the weight becoming smaller and smaller as we date back to earliest times. Apparently, price is also a synthesis of current output and all past demand shocks. This is consistent with the result in Mackowiak and Wiederholt (2009) where they solve the optimal price level in a log-linearized problem as a function of aggregate conditions and idiosyncratic shocks. Since their problem is a static one, the attention allocation between the two arguments influences total profit. In fact, the framework in this paper can also be seen as a log-linearized version of a standard profit-maximizing problem like in their case, where variables are all Gaussian with zero mean.

Let's return to the firm's maximization problem (5), but this time assume a linear capital cost function instead, that is, $\sigma(q_t, q_{t+1}) = d(q_{t+1} - q_t + c)$. Since we are focused solely on the aggregate consumer surplus, the idiosyncratic noise for each individual cancels out, leaving us with essentially the same maximization problem. However, the key difference is that the perceived state variables are now unobservable and subject to limited learning capacity. Firms solve a similar optimal linear regulator problem, and the policy functions are the same:

$$u_t = -F'\widehat{s}_t, \tag{21}$$

where $\widehat{s}_t = \left[\widehat{v}_t \;\; 1 \right]$. Now next period's production can be written as:

$$\overline{q}_{t+1} = av_t - \frac{a(1-\theta)C_v\epsilon_t}{1-(1-\theta)\rho_v \cdot L} + b, \tag{22}$$

where a and b are some constants, $a > 0$, or alternatively,

$$\overline{q}_{t+1} = (1-\theta)\rho_v\overline{q}_t + a\theta v_t. \tag{23}$$

The above equation reduces to the form:

$$\frac{I_t}{K_t} = (1-\theta)\rho_v - 1 + \frac{a\theta v_t}{K_t}. \tag{24}$$

When $\theta = 1$, Equation (23) reduces to the solution without capital adjustment cost assumption. Using the above results, I can write down the aggregate investment process similar to the form of Equation (14):

$$I_{t+1} = (1-\theta)\rho_v I_t + a\theta(v_{t+1} - v_t). \tag{25}$$

The first term in Equation (25) recovers the role for lagged term in the current investment decision, the second term measures the response to increase in demand. The above equation shows that investment evolvement equation in a RI model with a proportional cost function resembles that under capital adjustment cost assumption. When $\theta < 1$, the information flow between periods is finite and determined by capacity parameter κ, so investment adjustment in each period is incomplete and learning with finite capacity produces a delayed, hump-shaped response of aggregate investment to the demand shock as generally observed in business cycle literature.

4.2. Discussion on Infrequent Capital Adjustment

While aggregate investment is smooth and highly correlated, empirical evidence has found that investment at the firm level is rarely zero but instead characterized by discrete and lumpy adjustments, especially in plant-level data. Most literature has assumed nonconvex cost for individual firms and convex adjustment cost for aggregate investment. As discussed by Luo, Nie and Young (2014), when the agent's decision choice is infrequent, introducing fixed adjustment cost can endogenize the probability of re-adjusting. Thus, inaction results even without introducing an investment option. Caballero (1996) sketches the lumpy investment behavior when the cost of adjustment is proportional to the size of adjustment, and marginal cost rises sharply in the neighborhood of no adjustment due to fixed cost. In his model, fixed cost introduces additional increasing returns to investment, causing investment to occur in discrete, lumpy fashion.

In our previous example, each firm is assumed to incur investment every period. In fact, since firms are observing demands with different endogenous noise, when fixed costs are large(c are large enough), a fraction of firms may choose to delay investment in the current period until more favorable demand shocks arrive. The rationale is that each agent doesn't know the endogenous noise that is realized on her but only the distribution of noise, and will therefore choose a probability to adjust at each period. At the aggregate level, the idiosyncratic endogenous noise cancels out, eliminating nonlinearities. Hence, rational inattention at the micro level with fixed cost can generate lumpiness at the firm level while maintaining smooth quantities at the aggregate level. In contrast, Wang and Wen (2012) demonstrate that collateral borrowing constraint can simultaneously generate lumpiness in plant-level investment and convex adjustment cost at the aggregate level, regardless of the presence of irreversibility. In our model, when endogenous noise is realized each period, if the large fixed cost of investment outweighs the benefit, investment would not take place and a fraction of the total number of firms will remain inactive.

For simplicity, let's assume that on average a fraction of π of all firms is inactive at each period. In other words, the probability of a firm adjusting at each period is given by a constant π. π is determined endogenously from an agent's optimizing choice, which will be demonstrated later. Following Bar-Ilan and Blinder (1992), I show that in a FI-RE model, a consumer with full information about the state chooses to adjust when the welfare improvements from adjusting exceed the fixed cost incurred. Here we compare the welfare loss of infrequent adjustment against the FI-RE model. In this context, welfare loss arises from two resources: the first being the incomplete adjustment due to rational inattention, and second, the infrequent adjustment because of the existence of fixed cost.

First, let's derive the welfare loss due to deviations from the optimizing path of perfect observations of true states. Let the optimal production scheme of an individual firm under imperfect state observation be $\widehat{k}_{t+1} = H_k\widehat{v}_t$. In this case, each agent's expected capital stock plan, given noisy observations, is the same at period t. The discounted stream of the social planner's profit can be written as:

$$\begin{aligned}
\widehat{\Pi}_t &= \max E_t \sum_{j=t}^{\infty} \beta^{j-t} \Pi_t(\widehat{\widehat{k}}_t, \widehat{v}_t) \\
&= E_t \sum_{j=t}^{\infty} \beta^{j-t} \{(A_0 - \frac{A_1}{2} f_0 \widehat{\widehat{k}}_t + v_t)\widehat{\widehat{k}}_t - d(\widehat{\widehat{k}}_{t+1} - \widehat{\widehat{k}}_t + nc)\} \\
&= E_t \sum_{j=t}^{\infty} \beta^{j-t} n(A_0 H_k \widehat{v}_{t-1} - \frac{1}{2}(H_k \widehat{v}_{t-1})^2 - dH_k\widehat{v}_t + dH_k\widehat{v}_{t-1} - dc), \quad (26)
\end{aligned}$$

where we have used the fact that $A_1 \approx 10^{(-3)} = \frac{1}{n}$. Since $\widehat{v}_t = E[v_t|I_t]$, $E_t(v_j - \widehat{v}_t) = 0$, other deviations from optimal path except the second and third terms vanish in period t expectation, so only the quadratic terms remain. Hence, each firm's welfare loss due to incomplete adjustment can be written as:

$$v^1 = \frac{1}{2} \min E_t \sum_{j=t}^{\infty} \beta^{t-j}(k_j - kH_j^*)^2 = \frac{H_k^2 \sigma^2}{1-\beta}, \tag{27}$$

where $j \geq t$ and σ^2 is the steady state conditional variance of the state which can be derived from the Kalman filter. Since the fixed cost at each period is dc, their discounted present value is $\frac{dc}{1-\beta}$, conditional on that investment already being adjusted at period t.

We can then consider the model of infrequent adjustment of investment. Assume that a typical firm minimizes the quadratic loss function, which depends on the difference between the prevailing capital stock k_t at period t and the instantaneously-adjusted optimal plan k_t^*. It would choose the optimal capital stock to minimize:

$$\frac{1}{2} E_t \sum_{j=t}^{\infty} \beta^{t-j}(k_j - k_j^*)^2. \tag{28}$$

The subsequent argument goes the same as in Luo and Young (2014). Assuming the demand shock follows an AR(1) process so that $v_{t+1} = v_t + C_v \epsilon_{t+1}$, I can calculate the welfare loss due to infrequent adjustment after adjusting at period t:

$$v^2 = \frac{1}{2} \frac{H_k^2}{1-\beta} \left[\frac{\beta(1-\pi)}{1-\beta(1-\pi)} C_v^2 + \sigma^2 \right] + \frac{\pi\beta}{1-\beta} c. \tag{29}$$

Taking first order condition with respect to π gives the optimal endogenous probability:

$$\pi^* = \frac{H_k C_v^2}{\beta\sqrt{2c}} + \frac{\beta - 1}{\beta}. \tag{30}$$

So, at any period, each firm adjusts with the optimal probability π to minimize the welfare loss due to infrequent adjustment, and the welfare loss is increasing in the steady-state conditional variance of the state variable.

5. ASSET PRICING IMPLICATIONS

In this section I introduce a general equilibrium setting of PIH and analyze how information frictions in the production sector impact asset prices issued by the firm, as well as investor's consumption and asset allocation choices.

5.1. RI Model with Precautionary Savings and Risky Asset

Following Wang (2003), I introduce a Bewley-type heterogenous-agents equilibrium model, in which a representative consumer receives an exogenous process of labor income:

$$y_t = \Phi_0 + \Phi_1 y_{t-1} + \chi w_t, \tag{31}$$

where $\sigma > 0$, the initial level y_0 of income is given, and $\{w_1, w_2 ...\}$ are independent innovations with a distribution having zero mean and unit variance. $|\Phi_1|$ is assume to be smaller than 1 to preserve stationarity.

Suppose that a representative firm issues a single stock contingent on the aggregate product they receive from each period's production, net of retained earnings to finance investment in that period:

$$D_t = \overline{q}_t - I_t(\overline{q}_t, v_t). \tag{32}$$

Following Peng (2004) and Wang (2003), I analyze a discrete version of an investor's consumption and portfolio choice when stock price and dividends distributed are functions of the fundamentals, which, in this case, are defined by $s_t = (k_t, v_t, 1)$. I assume there are two financial assets in the economy: a risky asset which is the stock defined above, and a risk-free storage technique with return rate r. Assume that the consumer adopts a CRRA preference with a risk averse parameter γ. Consumers don't own the firm; instead they are speculative traders of the shares of the firms. The investor's problem can thus be specified as:

$$V(W_t, s_t, y_t) = \max_{c_t, \alpha} E_t \left\{ \sum_{k=0}^{\infty} \beta^k \left[\frac{\exp(-\gamma c_{t+k})}{-\gamma} \right] \right\}, \tag{33}$$

subject to the dynamic budget constraint:

$$W_{t+1} = (1 + r)W_t + y_t - c_t + \alpha(P_{t+1} - P_t + D_{t+1} - rP_t), \tag{34}$$

and

$$W_t \geq 0 \tag{35}$$

is the beginning-of-period financial wealth. Following Campbell and Kyle (1993), Wang (1993) and Peng (2003), the investment opportunity is described as an undiscounted cumulative cash flow from a zero-wealth portfolio long one share of stock, and the position is fully financed by borrowing at the riskless rate of interest r. I assume the risk free rate r is determined in the consumption loan market, and investors take the rate as given when investing in the risky claim. Note that the expected gains consumers get from investing in the risky stock will influence on consumption at current period.

In the equilibrium, the demand for the risky asset equals the total amount of asset supplied in the financial market: $\alpha = 1$. Solving the problem gives the following proposition

PROPOSITION 2. *Denote the evolvement of aggregate capital with rational inattention in Section (4.1) as* $\overline{k}_{t+1} = (1-\theta)\rho_v\overline{k}_t + a\theta v_t + b(1-(1-\theta)\rho_v)$, *and suppose the Laplace transform* $\zeta(\centerdot)$ *of the income innovation* w_t *is finite over the range from* 0 *through* $-\gamma\sigma ra$, *the consumer's value function* $V(W_t)$ *can be written as:*

$$V(W_t) = \frac{1}{r\gamma}\exp(-r\gamma(W_t + a_0 y_t + b_0)). \tag{36}$$

The optimal consumption the investor decides to consume at each period is:

$$c_t = r(W_t + a_0 y_t + b_0), \tag{37}$$

where $m(k) = \log\zeta(k)$, $\zeta(k) = \int_R e^{kz} dv(z)$,

$$a_0 = \frac{1}{1 + r - \Phi_1}, \tag{38}$$

$$b_0 = a_0 - \frac{1}{\theta\gamma}\log(1 + r), \tag{39}$$

$$\Pi(r) = m(-\gamma r\sigma a) - \frac{r^2\gamma^2}{2} C_v^2 \left[M^{(2)}\right]^2, \tag{40}$$

$$\Psi(r) = \log\frac{1}{\beta(1 + r)} \tag{41}$$

captures the saving demand of relative patience, and

$$\Gamma(r) = \frac{1}{\gamma r^2}\left[\Pi(r) - \Psi(r)\right] \tag{42}$$

is the demand for precautionary savings due to the interaction of income uncertainty and risk aversion. The equilibrium stock price is jointly determined as:

$$P_t = Ms_t + m, \tag{43}$$

where P_t *is the asset price under RI production,* $s_t = \left[\overline{k}_t,\, v_t,\, 1\right]$, $M = \left[\frac{(1-\theta)\rho_v - 2}{(1-\theta)\rho_v-(1+r)},\ \frac{a\theta(1-r)}{[(1+r)-(1-\theta)\rho_v][(1+r)-\rho_v]},\ \frac{b(1-(1-\theta)\rho_v)(1-r)}{r(1+r)-(1-\theta)\rho_v}\right]$, $m = -\gamma C_v^2[M^{(2)}]^2$, $\partial M^{(1)}/\partial\theta < 0$, $\partial M^{(2)}/\partial\theta > 0$.

Proof. See Appendix A.3. ▮

The solution has several features. First, because the investor has constant relative risk aversion preference, the demand for risky asset is independent of current level of wealth, and so is the equilibrium asset price. Second, since levels of dividends and prices have constant variances and are normally distributed, they allow for a linear stationary presentation of solutions. Also, the CRRA preference implies that the equilibrium stock prices and expected dividends are discounted by the riskless rate of interest, subtracting a term from the price as the risk premium.

5.2. Impact of RI on Asset Price

On the other hand, the degree of rational inattention influences asset prices. From Equation (43), I decompose the market value of the firm into two subcomponents: the value of assets in place and the value of growth options. Given $\partial M^{(1)}/\partial\theta < 0$ and $\partial M^{(2)}/\partial\theta > 0$, the market value of the firm in a RI model consists of a larger proportion attributed to assets in place and a smaller proportion to the value of growth options. This is because RI makes it more challenging to effectively capitalize on promising growth options. Consequently, installed capital now has a larger market value than under RE model, implying a higher average q, or a higher $M^{(1)}$ by definition. Assuming a linear representation, the average q is equivalent to the marginal q. Since q and θ share a one-to-one mapping, I can express θ as follows:

$$\theta = 1 - \frac{1+r}{\rho_v} + \frac{1-r}{\rho_v(q-1)}. \tag{44}$$

From this equation, it is evident that marginal q decreases as θ decreases, indicating that more inattentiveness leads to a lower marginal q. Substituting the above equation into Equation (24) yields:

$$\frac{I_t}{K_t} = (\frac{1+r}{\rho_v} - \frac{1-r}{\rho_v(q-1)})\rho_v - 1 + a(1 - \frac{1+r}{\rho_v} + \frac{1-r}{\rho_v(q-1)})\frac{v_t}{K_t}. \tag{45}$$

This equation derives the optimal rate of investment as a function of marginal q, thereby recovering Tobin's q theory under the assumption of rational inattention. The work by Abel (1979) and Hayashi (1982) connects q theory with existing partial theories, particularly adjustment cost assumption. They show that convex adjustment costs model are equivalent to a q model. However, as noted in much of the literature, such as Hassett and Hubbard (1996b), the q model has lacked its success in empirical evidence. The two authors use U.S. aggregate data and show that unconditional correlation between average q and investment is low.

Here, I consider two scenarios regarding the model-implied correlation of the investment rate and Tobin's q.

Scenario 1: Independent Attention. First, we assume $cov\left(v_t, \theta\right) = 0$, i.e. the degree of RI is independent of the state variable v_t. In this case, when the amount of attention allocated to the signal extraction problem, $\kappa = -\frac{1}{2}\log\left(1-\theta\right)$, does not vary with v_t, Tobin's q is also uncorrelated with v_t. Given that $E\left[\frac{v_t}{K_t}|v_{t-1}\right] < 0$ and $cov\left(\frac{1}{q-1}, q\right) < 0$, by employing the law of total covariance, the second term in the investment rate equation (45) increases with q. This implies a positive correlation between the investment rate and Tobin's q.

Scenario 2: Positive Covariance Between Demand Shock and Attention. Second, suppose $cov\left(v_t, \theta\right) > 0$. It suggests that a higher demand shock encourages more attention allocated to the filtering problem. This could occur when attention is elastic and the marginal benefit of more attention outweighs the cost of acquiring more information. As a result, the correlation between the second term on the right-hand side of Equation (45) with Tobin's q becomes undetermined, leading to an ambiguous correlation between the investment rate and Tobin's q.

To summarize, my results differ from those implied by the typical q theory. Equation (45) indicates that, while an increased q raises the investment rate by making the value of installed capital more valuable, it at the same time reduces the stochastic component of investment that responds to fluctuations of the aggregate demand. The latter is because higher q values suggest that it's now harder to efficiently transform increased demands into installed capital due to investment stagnation. It may thus result in an ambiguous role of Tobin's q as a sufficient statistic of investment. Note that when q is held constant, an increase in demand still leads to an increase in investment, making proxies like sales growth indicative of the investment rate.

I have shown that with linear technology and a competitive industry facing a linear, stochastic demand curve, Tobin's q theory is recovered. However, when considering rationally inattentive firms in a general equilibrium framework with asset markets and an exogenous income process, the role of Tobin's q as a sufficient statistic for investment becomes ambiguous.

6. CONCLUSION

This paper explores the optimal investment rule for firms facing stochastic demand shocks within a competitive industry equilibrium under rational inattention, without assuming capital adjustment cost. When demand shocks follow an AR(1) process, the presence of nonconvex capital adjustment costs for individual firms leads to infrequent investment adjustments with an optimal probability, aligning with empirical evidence for plant-level investment. On the other hand, information frictions smooth aggregate investment, resulting in a distributed lag pattern. The serial correlation of aggregate investment arises from the effects of rational inattention. In essence, rational inattention reintroduces the dynamics of partial adjustment models, such as those involving convex adjustment costs. Importantly, a linear capital cost function under rational inattention reconciles plant-level investment behavior with aggregate dynamics.

Additionally, I introduce a general equilibrium framework where consumers make consumption and asset allocation decisions based on an exogenous income process. Assuming a zero net supply for the riskless asset, investment in risky assets determines their prices and, consequently, the market value of firms. The endogenously determined asset prices indicate that installed capital gains more market value when information processing constraint is tighter, or θ is smaller. The relationship between marginal q and θ recovers the optimal investment as a function of Tobin's q, although the correlation between q and the investment rate remains ambiguous.

APPENDIX A

A.1. DERIVING FIRM'S MAXIMIZATION PROBLEM IN A RATIONAL EXPECTATIONS MODEL

In the benchmark model, there are n identical competitive firms in the industry, each using a single input to produce a homogenous good. The industry demand curve for output is given by:

$$p_t = A_0 - A_1\overline{q}_t + v_t,\ A_0, A_1 > 0, \tag{A.1}$$

where p_t is the price of output, $\overline{q}_t$ is industry output, and v_t is a shock to the demand. The individual firm's output is $f_0 k_t$ and k_t is firm's capital stock. Then industry output is $\overline{q}_t = nf_0k_t$.

In the fictitious central planner's problem, the representative firm is competitive in the output and factor markets and thus is a price taker with respect to the output prices$\{p_{t+j}\}_{j=0}^{\infty}$. The sequences are taken as exogenous stochastic processes. The firm's time t information set consists of

$\{p_t, p_{t-1}, ..., J_t, J_{t-1}, ..., \overline{k}_t, \overline{k}_{t-1}, ..., v_t, v_{t-1}, ...\}$. A representative firm lives for infinite time periods and chooses the sequence of $\{\overline{k}_{t+j}\}_{j=0}^{\infty}$ to maximize

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ S(\overline{q}_{t+j}, v_{t+j}) - J_{t+j}(\overline{k}_{t+j} - \overline{k}_{t+j-1}) - \frac{d}{2}(\overline{k}_{t+j} - \overline{k}_{t+j-1})^2 \right\}, \tag{A.2}$$

subject to $\overline{q}_{t-1}$ given.

We can obtain an equilibrium pair of sequences $\{p_{t+j}\}_{j=0}^{\infty}$ and $\{\overline{k}_{t+j}\}_{j=0}^{\infty}$ that constitutes a rational expectations equilibrium:

(i) Given the representative firm's plan for setting $\{\overline{k}_{t+j}\}_{j=0}^{\infty}$, $\{p_{t+j}\}_{j=0}^{\infty}$ clears the output market.

(ii) When the representative firm faces the prices $\{p_{t+j}\}_{j=0}^{\infty}$, it chooses the sequence $\{\overline{k}_{t+j}\}_{j=0}^{\infty}$ to maximize expected present value.

If we take the factor prices to be constant, that is, $J_{t+j} = c$, $j = 0, 1, ...,$ then the problem can be solved as

$$\overline{k}_{t+j+1} = \lambda_1 \overline{k}_{t+j} - \frac{\lambda_1}{d} \sum_{i=0}^{\infty} (\frac{1}{\lambda_2})^i E_{t+j}\{(1-\beta)c - f_0 v_{t+j+1+i} - A_0 f_0\}, \tag{A.3}$$

and

$$p_{t+j} = A_0 - A_1 f_0 \overline{k}_{t+j} + v_{t+j}, \tag{A.4}$$

where

$$(F^2 + \frac{\phi}{\beta} F + \frac{1}{\beta}) = (F - \lambda_1)(F - \lambda_2) \text{ and } \lambda_1 < 1 < \frac{1}{\beta} < \lambda_2,\ \phi = -\left(1 + \beta + \frac{A_1 f_0^2}{d}\right). \tag{A.5}$$

Given the demand shock process

$$v_{t+1} = \rho_v v_t + C_v \epsilon_{t+1}, \tag{A.6}$$

the capital accumulation process in time t can be written as

$$\overline{k}_{t+1} = \lambda_1 \overline{k}_t + \frac{\lambda_1}{d}\{\frac{f_0 \rho_v v_t}{1 - \rho_v \beta \lambda_1} - \frac{(1-\beta)c - A_0 f_0}{1 - \beta \lambda_1}\}. \tag{A.7}$$

To estimate the parameters, we first set $f_0 = 1$. The time discount β is set to be 0.98. We use quarterly GNP deflators and aggregate GDP from NIPA tables to estimate the demand function (A.4) and obtain A_0, A_1. Both the GNP deflators and GDP are linearly detrended by regressing the series on a constant, a linear trend and the trend squared. The residuals are the demand shock series $\{v_t\}$. To estimate the capital accumulation function (A.7), we take first differences of both left-hand and right-hand side of the equation, and use linearly detrended real equipment investment from FRED to estimate:

$$\overline{I}_t = \lambda_1 \overline{I}_{t-1} + \frac{\lambda_1 \rho_v (\rho_v - 1)}{d(1 - \rho_v \beta \lambda_1)} v_{t-1} + u_t,$$

where $u_t = \frac{\lambda_1 \rho_v C_v}{d(1-\rho_v \beta \lambda_1)} \epsilon_t$. Estimates of the free parameters $\Theta = \{d, \rho_v, C_v\}$ are then obtained using the OLS regression of the investment on lagged investment and the demand shocks, subject to A.5. Results are reported in Table 3.

TABLE 3.

Model Parameters

Parameters		
$A_1 = 0.0014$	$d = 0.39$	$\rho_v = 0.95$
$A_0 = 0$	$\beta = 0.98$	$C_v = 0.73$
$f_0 = 1$		

A.2. DERIVING FIRM'S MAXIMIZATION PROBLEM IN A LINEAR REGULATOR PROBLEM

The representative firm's maximizing problem is:

$$\max_{\{q_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ S(\overline{q}_t, v_t) - \frac{d}{2}(\overline{q}_{t+1} - \overline{q}_t + c)^2 \right\}. \quad \text{(A.8)}$$

Since

$$\begin{aligned} & S(\overline{q}_t, v_t) - \frac{d}{2}(\overline{q}_{t+1} - \overline{q}_t + c)^2 \\ = \; & A_0 \overline{q}_t - \frac{A_1}{2} \overline{q}_t^2 + \overline{q}_t v_t - \frac{d}{2}(\overline{q}_{t+1} - \overline{q}_t + c)^2 \\ = \; & -\frac{d}{2}\overline{q}_{t+1}^2 - (\frac{A_1}{2} + \frac{d}{2})\overline{q}_t^2 + d\overline{q}_{t+1}\overline{q}_t - cd\overline{q}_{t+1} + (cd + A_0)\overline{q}_t + \overline{q}_t v_t, \quad \text{(A.9)} \end{aligned}$$

I can formulate the problem as a linear regulator problem:

$$\max_{\{\overline{q}_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left[\begin{array}{cc} u_t' & s_t' \end{array} \right] \left[\begin{array}{cc} R & W' \\ W & Q \end{array} \right] \left[\begin{array}{c} u_t \\ s_t \end{array} \right] \right\}, \quad \text{(A.10)}$$

subject to

$$s_{t+1} = A s_t + B u_t + C \epsilon_{t+1}, \quad \text{(A.11)}$$

where

$$s_t' = (\overline{q}_t \ v_t \ 1),\ u_t' = (\overline{q}_{t+1}), \tag{A.12}$$

$$R = \left[-\tfrac{d}{2}\right],\ Q = \begin{bmatrix} -\frac{A_1+d}{2} & \frac{1}{2} & \frac{A_0+cd}{2} \\ \frac{1}{2} & 0 & 0 \\ \frac{A_0+cd}{2} & 0 & 0 \end{bmatrix},\ W = \begin{bmatrix} \frac{d}{2} \\ 0 \\ -\frac{cd}{2} \end{bmatrix},\ A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \rho_v & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\ C = \begin{bmatrix} 0 \\ C_v \\ 0 \end{bmatrix}.$$

This problem can be conveniently solved numerically.

A.3. DERIVING THE CONSUMER'S PROBLEM

Conjecture the risky claim's price takes the form $P_t = Ms_t$, and let the dividend process $D_t = (1 - (1-\theta)\rho_v)\overline{k}_t - a\theta v_t - b$. The excess return obtained from investing in the risky asset can be written as:

$$P_{t+1} + D_t - (1+r)P_t = Ms_{t+1} - (1+r)Ms_t + Ns_t \equiv A^0 s_t + B^0 \epsilon_{t+1}. \tag{A.13}$$

Then consumer's budget constraint can be written as:

$$W_{t+1} = (1+r)W_t + y_t - c_t + \alpha(A^0 s_t + B^0 \epsilon_{t+1}). \tag{A.14}$$

The Bellman equation associated with the optimization problem (33) is:

$$V(W_t, s_t, y_t) = \max u(c_t) + \beta E_t V(W_{t+1}, s_{t+1}, y_{t+1}). \tag{A.15}$$

First, conjecture the value function for problem (33) takes the following form:

$$V(W, s, y) = -\frac{1}{r\gamma}\exp(-r\gamma(W + \overline{A}s + \overline{B}y + \overline{b})). \tag{A.16}$$

In Equation (A.15), the first-order conditions with respect to c_t and W_t can jointly imply:

$$u'(c) = \frac{1}{1+r} V_1(W, s, y). \tag{A.17}$$

From (A.17) and (A.16), the optimal consumption rule can be written as:

$$c_t^* = r(W_t + \overline{A}s_t + \overline{B}y_t + a_0), \tag{A.18}$$

where

$$a_0 = \overline{b} + \frac{1}{r\gamma}\log(1+r). \tag{A.19}$$

Using (A.16) and (A.18), the Bellman equation (A.15) can be written as

$$V(W_t, s_t, y_t) = \frac{r}{1+r} V(W_t, s_t, y_t) - \frac{\beta}{r\gamma} E_t \exp[-r\gamma(W_{t+1} + \overline{A} s_{t+1} + \overline{B} y_{t+1} + \bar{b})]. \tag{A.20}$$

Plugging in the income process and the evolvement of the state space from (A.14) and matching coefficients on state variables, I first obtain that

$$\overline{A} = 0;\ \overline{B} = \frac{1}{1 + r - \Phi_1}. \tag{A.21}$$

The above results show that the value function is invariant to the current period's dividend. Substituting this into the value function, taking the derivative with respect to α, and using the fact that consumer demands a unit of risky asset at equilibrium gives

$$A^0 s_t = r\gamma(B^0)H^2. \tag{A.22}$$

With Equations (A.13) and (A.22), we can derive the equilibrium asset price. Since the consumer will demand a unit of asset regardless of the current state, the expected excess return must be constant for demand to meet supply. In equilibrium, the asset price are determined so that excess return $A^0 s_t + B^0 \epsilon_{t+1}$ is endogenously determined and only depends on the pricing matrix, M and the noise term matrix, B. Matching the constants gives the optimal consumption rule as in proposition 2.

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