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The Republican Spirit of Innovism and Long-Run Growth*

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This paper embeds the republican spirit of innovism — dignity for ordinary people, liberty of entry and speech, and fair reward — into a modern ideas-based growth model to explain the Great Enrichment. Innovism is treated as a produced, nonrival social technology that multiplies production and raises the effectiveness of research, reconciling economic history with a semi-endogenous Jones framework. Small, persistent gains in civic permission and legal predictability sustain the frontier's one-to-two percent per-capita growth as demographics slow, while backsliding lowers the slope. The model yields transparent balanced-growth conditions, ties adjudication and entry reforms to capital deepening, and shows how open science enhances research productivity.

Key Words: Republican spirit of innovism; Dignity, liberty, and reward; Great Enrichment; Ideas-based growth; Nonrival social technology; Civic capital; Open discourse; Long-run prosperity.

JEL Classification Numbers: O10, O31, O33, O40, O43, D02, N10, N30, P16.

"Almost everything to be explained by growth theory lies in the Solow residual." If so, then the residual must be given a face and a forum: dignity for the non-elite, liberty to enter and speak, and reliable reward — the republican spirit of innovism.

1. INTRODUCTION

This paper takes economic historians such as David Landes, Douglass North, Joel Mokyr, Deirdre McCloskey, Paul David, and others seriously, and begins from a historical fact that no mathematical trick can explain away: over two centuries, ordinary people in a few societies were newly permitted and encouraged to try things — and the world's economic slope

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changed. The change cannot be reduced to piling up machines, thickening markets, or to ideas production in isolation as in canonical R&D models (Romer 1990; Grossman and Helpman 1991; Aghion and Howitt 1992; Jones 2005, 2022). Rather, as McCloskey argues, it was a moral and rhetorical revolution that dignified the "middling sort," widened liberty of entry and speech, and secured fair reward for successful betterment (McCloskey 2006, 2010, 2016, 2018). We call this produced and nonrival social technology the republican spirit of innovism. In McCloskey's idiom, national product is the fruit of an innovation function that multiplies a familiar neoclassical aggregator — permission and esteem first, then gears and bricks (McCloskey 2010, 2018).

What distinguished the modern rise was not the brilliance of a narrow elite but the social licensing of ordinary people to try, err, share, and try again. In McCloskey's emphasis, the "middling sort" acquired dignity and "equality of permission," so that innovation ceased to be a courtly curiosity and became a civic practice (McCloskey 2006; 2010; 2016). Edmund Phelps (2013) makes the same point from a different angle: mass, grassroots innovation — shop-floor tweaks, new retail formats, local process improvements — drove dynamism once people outside guilds and patronage networks had permission and prospect (Phelps 2013). The institutions historians chronicle mattered precisely because they multiplied audiences and participants: printing and cheap pamphlets widened conversation (Eisenstein 1979), coffeehouses and journals rehearsed public reason (Habermas 1962), and constitutional commitments protected small investors and traders as much as grandees (North and Weingast 1989; Brewer 1989; Neal 1990). The republican spirit of innovism is therefore literal: esteem, entry, and fair reward available to the many, not privileges dispensed to the few.

Once upon a time, economists believed that growth depended mainly on piling up machines — physical capital — and later the fashion shifted to various versions of human capital and to the creative destruction of technical ideas represented by blueprints, patents, and quality ladders. Yet, as William Easterly (2001) criticized in his attack on "capital fundamentalism," neither capital deepening nor schooling alone can explain the long-run divergence in prosperity. The historical record assembled in McCloskey's Bourgeois trilogy points in a different direction (McCloskey 2006, 2010, 2016): sustained growth emerged only where dignity, liberty, and open rewards for innovation were generalized to ordinary people. Peter Howitt's (2006) review of modern cross-country evidence reinforces this view. He finds that more than sixty percent of the variation in income per worker is due to productivity rather than to the accumulation of physical or human capital, and over ninety percent of differences in growth rates likewise trace to productivity. In short, as Howitt concludes, "almost everything to be explained by the theory lies in the Solow residual." For our purposes,

that residual is not a mysterious ether but the lived climate of innovism, $Z_t(D_t, B_t, R_t)$: dignity (D_t) for ordinary tinkerers, liberty (B_t) to enter and speak, and reliable reward (R_t) under law. The conclusion is Smithian as well as modern. In The Theory of Moral Sentiments Smith treated the dignity variable; in The Wealth of Nations he defended the system of natural liberty — together a moral and legal order sufficient, in his time, for modest growth, and in ours, once generalized and defended, for sustained enrichment (Smith 1759/1790; 1776).

Economic historians have told this story in granular detail. Elizabeth Eisenstein showed how printing standardized the genres of technical knowledge — manual, diagram, table — so that workshop improvements traveled and stuck (Eisenstein 1979). Jürgen Habermas described how coffeehouses, pamphlets, and periodicals formed publics capable of "testing" ideas in common (Habermas 1962). Steven Shapin and Robert Merton traced how credibility, replication, and the virtues of inquiry moved beyond courts and cloisters into broader practice (Merton 1938/1973; Shapin 1996). Joel Mokyr documented the marriage of "useful knowledge" with artisanal craft and the Republic of Letters that carried insights from bench to bench (Mokyr 2002; 2009; 2017). Douglass North and Barry Weingast explained how constitutional settlements constrained predation and made commitments credible (North and Weingast 1989; North 1990; 2005), while John Brewer and Larry Neal showed how public finance and law knit together investors and entrepreneurs (Brewer 1989; Neal 1990). David Skeel's history of bankruptcy made plain that "forgiveness technologies" accelerate selection rather than indulge failure (Skeel 2001; Sandgren and Skeel 2001). Broad reconstructions by Stephen Broadberry and coauthors and by Angus Maddison chart the frontier's one-to-two percent per-capita slope; Eric L. Jones and David Landes emphasized how Europe's particular bundle of institutions, mores, and ideas unlocked sustained change (Broadberry et al. 2015; Maddison 2001; E. L. Jones 1981/2003; Landes 1998). More microevidence links reading and religious competition to human capital, urban dynamism, and growth (Baten and van Zanden 2008; Cantoni, Dittmar, and Yuchtman 2018), and legal origins to external finance and investor protection (La Porta et al. 1997; 1998). The convergence is striking: where esteem for non-elite innovators, liberty of entry and speech, and predictable adjudication were produced and defended, the slope endured.

We organize these episodes with McCloskey's "Four R's plus Revaluation," treating them as historically dated ratchets that raised both the level and the productivity of the innovation climate for the many. Reading, born of Gutenberg, made publics (Eisenstein 1979; Baten and van Zanden 2008). The Reformation habituated lay argument and conscience (Cantoni, Dittmar, and Yuchtman 2018; Merton 1938/1973). Revolts — paradigmatically the Dutch — installed commercial republican practices

and plural toleration (de Vries and van der Woude 1997). Revolutions — English, then American and French — constitutionalized predictable law and finance (North and Weingast 1989; Brewer 1989; Neal 1990). Revaluation dignified bourgeois pursuits, turning tinkering and trade from suspect to honorable (McCloskey 2006; 2010; 2016). Each step widened participation: reading lowered the cost of joining the conversation; dissent trained lay reasoning; constitutional settlements protected small traders and savers; revolutions broadened representation; and revaluation made everyday improvement honorable. The effect was to push innovation from courts and academies into shops, docks, offices, and households — precisely the dispersion a nonrival, mass climate captures.

By "republican," we mean both a moral stance and a breadth condition. The stance dignifies common callings and commercially tested betterment; the breadth condition insists that dignity, liberty, and reward be generalized. Where speech is free only for the well-connected, where incorporation is cheap only for insiders, or where bankruptcy forgiveness is selective, the climate collapses back into patronage. The enrichment required the opposite: low fixed costs of participation — cheap press and postal routes, general incorporation, examinable patents, small-claims enforcement — so that millions could experiment (Brewer 1989; Neal 1990; La Porta et al. 1997, 1998; Skeel 2001). This mass character is the mechanism that turns a trickle of discoveries into the persistent one-to-two percent slope we measure at the frontier (Broadberry et al. 2015; Maddison 2001).

The contribution of this paper is to place that historical mechanism cleanly inside a modern ideas-and-growth discipline without blunting its force. We emphasize that Z_t is not merely a relabeled total factor productivity residual or a static institutional dummy variable. Unlike generic productivity terms, the republican spirit of innovism is treated here as a produced stock requiring identifiable civic effort (s_Z), subject to erosion (δ_Z) under neglect, and constrained by balanced-growth accounting. This structure allows us to move beyond the claim that "institutions matter" to a quantitative framework where the maintenance of dignity and liberty competes for resources, and where their neglect leads to quantifiable backsliding. We adopt the semi-endogenous perspective associated with Charles I. Jones: ideas are nonrival, but their production faces diminishing dynamic returns, so explosive scale effects are ruled out (Jones 2005, 2016, 2022). In such a world, once demographic tailwinds ebb, per-capita growth cannot be kept aloft by blueprints alone. McCloskey's innovation function — dignity, liberty, reward — supplies the missing state: a produced, nonrival, mass climate that both multiplies the productivity of rival inputs and makes research more effective. A modest but persistent drift in this climate explains why frontier economies held to one-to-two percent per year even as population growth fell; it also provides a framework for interpreting

thresholds, takeoffs, and hysteresis, suggesting why backsliding lowers the slope so painfully when permission is withdrawn (Broadberry et al. 2015; Maddison 2001; De Long 2022).

2. ECONOMIC ENVIRONMENT

Output is produced under a mass innovation climate. Following McCloskey and Phelps, we treat the republican spirit of innovism not as elite genius but as the permission and esteem extended to ordinary people — "letting people have a go" — together with predictable reward that anyone can count on, not a privilege for guilds or patrons (McCloskey 2006, 2010, 2016, 2018; Phelps 2013). In reduced form, the final good Y_t is produced as

$$Y_t \;=\; Z_t\, A_t^{\theta-\alpha}\, K_t^{\alpha}\left(L_t s_{Y,t}\right)^{1-\alpha}, \qquad 0<\alpha<1,\ \alpha<\theta\le 1. \tag{1}$$

Here A_t is the stock of useful ideas in the Romer-Jones tradition, Z_t is a produced, broadly shared social technology capturing the republican spirit of innovism (RSI), K_t and $L_t s_{Y,t}$ are rival inputs, and $s_{Y,t}$ is the time share in final goods production. The multiplier Z_t reflects the breadth of participation: the more citizens who can publish cheaply, form firms easily, enforce contracts predictably, and restart after failure, the larger the nonrival boost on the same bricks and gears.

The dynamics of Z_t embody both continuous civic effort and the great historical ratchets that McCloskey termed the Four R's plus Revaluation. We specify the RSI drift as

$$\frac{\dot{Z}_t}{Z_t} = g_Z^{\text{exo}}(t) + \kappa\, s_{Z,t}^{\xi} - \delta_Z \equiv g_{Z,t}, \qquad \kappa>0,\ \xi\in(0,1),\ \delta_Z\ge 0. \tag{2}$$

This per-capita-effort specification is disciplined by considerations that respect both balanced-growth accounting and the historical record. First, regarding the theoretical status of this specification: by allowing the drift $g_{Z,t}$ to depend on the allocation $s_{Z,t}$, the model introduces a mechanism similar to fully endogenous growth frameworks like Peretto (1998), where policy choices can permanently alter growth rates. However, we retain the semi-endogenous perspective because $s_{Z,t}$ is bounded by the rival time constraint and subject to diminishing returns ($\xi<1$). In our framework, Z_t acts to relax the diminishing returns to ideas accumulation characteristic of Jones (2005), but it does not eliminate them entirely without a continuous, historically rare expansion in the intensity of civic permission.

Second, regarding the depreciation term δ_Z, we interpret this not as the decay of an abstract spirit, but as the necessity of maintenance. History is replete with backsliding — from the closing of the Chinese mind after the

Song dynasty to the re-imposition of censorship in early modern Europe. Liberty, dignity, and fair reward require continual legal, discursive, and institutional upkeep. Similarly, the time allocation constraint (s_Z) represents the opportunity cost of this maintenance: courts must be staffed, patents examined, and open discourse defended. These activities compete for scarce attention and resources even as they energize all sectors indirectly. This aligns perfectly with the insights of North, Weingast, and McCloskey: the "good" institutions are not fixed endowments but costly achievements.

We want to emphasize that, to many economic historians, the correct unit of analysis for the nonrival innovation climate is the density of civic maintenance, not the number of bodies. In McCloskey's language, the rise of modern prosperity was propelled by the Four R's — Reading, Reformation, Revolts, Revolutions — and then a broad Revaluation of bourgeois virtues. Each R was a per-capita shock to attention and practice. Reading after Gutenberg lowered the cost of joining public conversation, multiplying the hours that ordinary people could devote to pamphlets, manuals, newspapers, and scientific journals. Reformation trained laity in conscience and disputation, moving argumentative competence from cloister to kitchen table. Revolts — exemplified by the Dutch — installed commercial republicanism and toleration in city institutions, embedding everyday adjudication and registries. Revolutions — English, American, French — constitutionalized predictable finance and law. Finally, Revaluation dignified the pursuits of shopkeepers, mechanics, and clerks. None of these depended on population size; all of them raised the share of time a typical person could allocate to discourse, reform, and adjudication. That is why the law for RSI's drift belongs in per-capita form, $\dot{\ln Z} = g_Z^{\text{exo}} + \kappa s_Z^{\xi} - \delta_Z$ so that doubling headcount without changing civic time use does not magically double the percentage improvement in liberty, dignity, and fair reward.

Mokyr's (2017) Culture of Growth explains the same shift through the emergence of a Republic of Letters and a liberal Enlightenment. What mattered was a networked culture that normalized contestation, replication, and publicity — "credit by disclosure" rather than secrecy. Coffeehouses, salons, journals, and learned societies created thick audiences for useful knowledge, so that a marginal hour spent reading, writing, or testing traveled farther and stuck longer. This, too, is a per-capita phenomenon: the salience of s_Z rises with the density of communicative institutions. In such a republic, a single pamphlet can alter many minds; the same hour of civic effort has social scaling because it moves on rails laid by cheap print, postal routes, and periodicals. Hence tying $\dot{\ln Z}$ to s_Z matches the Mokyrian mechanism: it is the intensity of participation, not the size of the crowd, that builds the climate.

Phelps (2013) adds the humanist root of modern dynamism: ordinary people, not just elites, seek "lives of agency" and will flourish when given

permission to imagine and try. The Renaissance recovered the dignity of individual making and judging; workplaces and streets became sites of discovery. In our mapping, that humanist spark raises the return to each hour spent on open discourse, institutional upkeep, and fair-dealing enforcement — again a rise in s_Z and a fall in δ_Z rather than an appeal to scale. When the mass of citizens expect fair reward and social esteem for useful novelty, a modest but persistent drift in Z_t follows, and the Jones-style idea engine translates that drift into sustained growth.

Landes's (1998) argument in The Wealth and Poverty of Nations locates the decisive margins in culture — habits of punctuality, trust, probity, curiosity, and responsibility — embedded in small, self-governing units that made commerce and experiment respectable. These are everyday allocations of attention: showing up to court, honoring contracts, auditing accounts, and publishing results. Cultural change is thus a reallocation of time shares toward maintaining dignity, liberty, and reward. Measured in our model, that is a rise in s_Z and a structural shift that keeps $\dot{\ln Z}$ positive even when demographic tailwinds abate. Landes's thesis becomes operational when we treat culture as the systematic choice to spend hours on civic maintenance rather than on rent-seeking, flattery, or avoidance.

Viewed through this lens, the small republics — Italian city-states, Geneva, the Dutch Republic, post-1689 England, the early United States — were not "tiny miracles" because they had many people. They were intense because each person's typical week contained more civic content: reading in vernacular, disputation outside courtly patronage, cheap incorporation, routine bankruptcy forgiveness, accessible petty courts, and pluralist media. The cumulative effect of millions of such hours is a rising Z_t. Their example vindicates the homogeneity claim: it is not L_t but s_Z that moves the national climate. A city of 50,000 with a literate, disputatious public and predictable law can outpace an empire of 50 million that suppresses speech and entangles enterprise in patronage.

The agrarian empires of Eurasia — imperial China, tsarist Russia, the Ottoman and Byzantine realms, Mughal India — show the counterfactual. Their bureaucratic and clerical hierarchies taxed speech, channeled talent into orthodoxy, and treated law as the sovereign's instrument. Printing in the vernacular was delayed or discouraged; municipal autonomy was stunted; incorporation was privilege, not right; bankruptcy was disgrace, not restart. In our terms: s_Z was near zero and δ_Z persistently high. More people did not imply more liberty or dignity; it often meant more surveillance per capita. The Four R's stalled, the Republic of Letters lacked rails, the humanist quest for agency was confined to courts, and institutions secured predation rather than promises. The result was stagnation in Z_t, regardless of resource endowments or population.

Ideas accumulate semi-endogenously with diminishing dynamic returns (Jones 2005, 2022), modified here to recognize the decisive role of the innovism climate Z_t:

$$\dot{A}_t \;=\; \nu\big(Z_t^{\eta}\, L_t s_{A,t}\big)^{\lambda}\, A_t^{\phi}, \qquad \nu>0,\ \lambda>0,\ 0<\phi<1,\ \eta\geq 0. \tag{3}$$

With $\phi < 1$, the growth of ideas alone cannot sustain frontier per-capita income once population growth fades. The missing impulse must come from the civic and moral environment that makes research effective — captured here by Z_t. The term Z_t^{η} represents the power of dignity, liberty, and fair reward to magnify the productivity of research labor. It measures how freely ideas circulate, how securely innovators expect recognition, and how open the public sphere is to criticism and imitation.

This formulation integrates the central insight of McCloskey (2006, 2010, 2016, 2018): that the modern rise of prosperity was driven not by capital accumulation or clever machines but by a revolution in attitudes — a widening of esteem and permission that turned innovation into a civic virtue. Landes (1998) similarly argues that Europe's escape from stagnation rested on a culture that honored improvement and tolerated experiment, while Mokyr (2002, 2017) documents how the "Republic of Letters" linked knowledge to production through open exchange and trust. Phelps (2013) adds that modern dynamism stems from ordinary people finding meaning in discovery itself, not merely from incentives or scale.

Interpreted in this light, Z_t is not a passive background condition but the chief determinant of whether science, technology, and enterprise advance. When liberty is curtailed or dignity denied, the same research effort yields little cumulative knowledge; when civic trust, open discourse, and fair reward prevail, every unit of effort multiplies itself through replication and recombination. Equation (3) therefore locates the real engine of the Great Enrichment in the moral and institutional climate — the republican spirit of innovism — that transformed curiosity into progress and ordinary work into enduring growth.

Goods accounting and capital follow the standard rival logic:

$$Y_t \;=\; C_t + I_t, \qquad \dot{K}_t \;=\; I_t - \delta K_t, \quad \delta>0. \tag{4}$$

3. PREFERENCES, OPTIMAL ALLOCATION, AND EQUILIBRIUM

The representative household chooses consumption, investment, and time allocation across final-goods production, research, and RSI maintenance. Let $c_t \equiv C_t/L_t$ and population grow at rate n. Preferences value both

material consumption and the innovist climate:

$$\max_{\{c_t,\,K_t,\,s_{Y,t},\,s_{A,t},\,s_{Z,t}\}} \int_0^\infty e^{-\rho t}\, L_t\, Z_t^{\chi}\, \frac{c_t^{1-\zeta}-1}{1-\zeta}\, dt, \qquad \zeta>0,\ \chi\geq 0,\ \rho>n+\chi\, g_Z. \tag{5}$$

Time shares satisfy $s_{Y,t}+s_{A,t}+s_{Z,t}=1$. We include Z_t in the representative household's utility because dignity, liberty, and fair reward are goods in themselves, not just instruments for producing more output. We acknowledge that allowing Z_t to enter production, ideas creation, and utility simultaneously loads significant weight onto a single variable. We treat this as a deliberate joint hypothesis derived from the historical narrative: the same shift in dignity and liberty that lowered entry barriers (production) also encouraged disclosure and recombination (ideas) and was valued as a form of human flourishing (utility). While the model's qualitative predictions regarding growth would survive restricting Z_t to the production or ideas blocks alone, the historical argument insists that these channels moved in tandem. By letting Z_t enter utility, we align welfare with this republican spirit of innovism: a broad social climate that fosters grassroots creativity, experimentation, and the everyday experience of agency.

The current-value Hamiltonian for this problem can be written as:

$$\begin{aligned}\mathcal{H}_t = {} & L_t Z_t^{\chi}\,\frac{c_t^{1-\zeta}-1}{1-\zeta} + \mu_t\Big[Z_t A_t^{\theta-\alpha} K_t^{\alpha}(L_t(1-s_{A,t}-s_{Z,t}))^{1-\alpha} - L_t c_t - \delta K_t\Big] \\ & + q_t\Big[\nu(Z_t^{\eta} L_t s_{A,t})^{\lambda} A_t^{\phi}\Big] + \pi_t\Big[\big(g_Z^{\text{exo}}(t) + \kappa s_{Z,t}^{\xi} - \delta_Z\big) Z_t\Big]\end{aligned}$$

where μ_t, q_t, π_t are the co-state variables (shadow prices) associated with physical capital K_t, knowledge capital A_t, and the innovist climate Z_t, respectively.

The first term captures instantaneous welfare — utility from per-capita consumption weighted by population L_t and amplified by the civic climate Z_t^{χ}. The next three terms internalize the shadow value of accumulating the three productive stocks. Because the representative household owns all assets, these shadow prices represent internal transfer values, not external market prices.

From (1) the marginal product of capital is

$$r_t \;=\; \alpha\,\frac{Y_t}{K_t}. \tag{6}$$

This is the familiar neoclassical result: each unit of capital earns its marginal product.

The consumption Euler equation is

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\zeta}\Big(r_t - \delta - \rho + \chi\, g_{Z,t}\Big) = \frac{1}{\zeta}\Big(\alpha\,\frac{Y_t}{K_t} - \delta - \rho + \chi\, g_{Z,t}\Big). \tag{7}$$

The innovation here is the patience wedge $\chi g_{Z,t}$: as the civic climate improves, effective impatience falls, inducing deeper capital accumulation and smoother consumption paths. This embodies McCloskey's and Phelps's insight that secure liberty and dignity expand time horizons and strengthen the propensity to invest in the future.

The intratemporal allocation conditions equate the marginal product of labor across activities. Because all wages are equalized in equilibrium, the household's time allocation satisfies:

$$(1-\alpha)\,\frac{Y_t}{L_t s_{Y,t}} = \frac{q_t}{\mu_t}\,\lambda\,\nu\,Z_t^{\eta\lambda}\,(L_t s_{A,t})^{\lambda-1}\,A_t^{\phi}, \tag{8a}$$

$$(1-\alpha)\,\frac{Y_t}{L_t s_{Y,t}} = \frac{\pi_t}{\mu_t}\,\kappa\,\xi\,s_{Z,t}^{\xi-1}\,\frac{Z_t}{L_t}. \tag{8b}$$

Equation (8a) balances the foregone marginal product of labor in final-good production with its shadow return in idea creation. Equation (8b) does the same for RSI maintenance. The equal wage condition across the three sectors follows immediately: the left-hand side represents the marginal revenue product of labor in production, and the right-hand sides convert the shadow value of increasing $A_t$or Z_t into equivalent consumption units. Note that here, q_t/μ_t is the current-value shadow price of ideas A_t and π_t/μ_t the current-value shadow price of Z_t, both in units of consumption goods.

Together, these optimality conditions show how the representative household internalizes all nonrival externalities. We adopt this representative-agent planner environment to focus on the accounting and measurement of these nonrival spillovers, rather than on the political economy of their provision. Introducing heterogeneous agents or institutional frictions would likely make the high-Zequilibrium harder to sustain, reinforcing our argument about the fragility of the Great Enrichment, but we leave that extension for future work.

4. BALANCED GROWTH, EXISTENCE, AND HISTORICAL MAGNITUDES

Along a balanced-growth path (BGP), the shares of labor devoted to goods production, research, and RSI-building (s_Y, s_A, s_Z) remain constant, so the economy expands at constant proportional rates. The growth rates of all key aggregates — ideas (A_t), the republican spirit of innovism (Z_t),

output (Y_t), capital (K_t), and consumption (C_t) — settle into constant values. Co-state variables (μ_t, q_t, π_t), representing shadow prices, may also grow at constant rates.

From the ideas law (3), with constant shares, the growth rate of ideas is:

$$\frac{\dot{A}_t}{A_t} = g_A = \nu \left[Z_t^{\eta} L_t s_A\right]^{\lambda} A_t^{\phi-1} \implies g_A = \frac{\lambda}{1-\phi}\left(\eta g_Z + n\right). \tag{9}$$

Equation (9) is the semi-endogenous Jones result, modified by the climate of innovism. The rate of idea growth depends on population growth n and on how the civic climate Z_t amplifies the productivity of research (ηg_Z). When the innovist environment improves — when liberty of inquiry broadens, rewards are more secure, and dignity is extended to new participants — each research hour becomes more effective, sustaining frontier growth even as demographic tailwinds fade. Historically, this mechanism captures the transition from Enlightenment and early industrial Europe to the modern age of continuous improvement: open discourse, examinable patents, and credible courts made knowledge production more efficient per capita.

Next, growth accounting from (1) for per-capita output $y_t = Y_t/L_t$ yields:

$$(1-\alpha)\, g_y \;=\; g_Z + (\theta-\alpha)\, g_A, \qquad \Rightarrow \quad g_y \;=\; \frac{g_Z}{1-\alpha} + \frac{\theta-\alpha}{1-\alpha}\, g_A. \tag{10}$$

This expression shows that growth in per-capita output arises from two engines: the drift in the republican spirit of innovism (g_Z) and the accumulation of useful ideas (g_A). If $\phi < 1$, ideas alone cannot maintain sustained growth without demographic expansion; therefore, the persistent drift in Z_t — the civic and moral climate that dignifies innovation — is essential. This is precisely McCloskey's historical claim: the Great Enrichment was not just technological but moral and rhetorical, rooted in social permission and esteem for ordinary creativity.

Combining (9) and (10), per-capita growth is a function of the innovism drift and population growth:

$$g_y \;=\; \frac{g_Z}{1-\alpha} + \frac{\theta-\alpha}{1-\alpha} \cdot \frac{\lambda}{1-\phi}\left(\eta g_Z + n\right). \tag{11}$$

Equation (11) demonstrates that sustained growth in material living standards requires a continuing, positive drift in Z_t. In the absence of such a moral–institutional driver ($g_Z = 0$), per-capita growth collapses as $n \to 0$, reproducing the classical stagnation of pre-Enlightenment civilizations. McCloskey's historical narrative fits precisely here: before the Five R's (Reading, Reformation, Revolts, Revolutions, and Revaluation), most societies lacked the civic dynamism to sustain progress; afterward, Western

Europe developed a self-reinforcing climate of open entry, critical discourse, and secure reward that generated permanent slope.

Capital deepening on a BGP follows from (7). With $g_c = g_y$ and $r = \alpha Y/K$, constancy of K/Y requires $g_K = g_Y = g_y + n$. Hence the steady capital–output ratio satisfies:

$$\frac{K}{Y} = \frac{\alpha}{\delta + \rho - \chi g_Z + \zeta g_y}, \qquad \frac{I}{Y} = (\delta + n + g_y)\frac{K}{Y}. \tag{12}$$

The first relation shows how a higher innovism drift (g_Z) deepens capital: by lowering effective impatience (the term $-\chi g_Z$), it raises the steady-state capital–output ratio. This mechanism unites the historical experience of liberal modernization with macroeconomic behavior: secure property, impartial adjudication, and reliable credit institutions encouraged long-horizon investment and accumulation (North and Weingast 1989; La Porta et al. 1997, 1998).

Two comparative-static results link the model directly to history. First, institutional reforms that accelerate justice, broaden entry, and liberalize media raise κ and reduce δ_Z, boosting g_Z and hence both growth and welfare. This matches the observed surges following nineteenth-century legal codifications (Skeel 2001) and the spread of free press and open science. Second, policies that increase transparency, disclosure, and replicability — open science, examinable patents, and academic freedom — raise η, improving the conversion of effort into usable knowledge (Mokyr 2017; David 1990; Shapin 1996). In both cases, the model explains sustained progress as a byproduct of republican institutions, not an unexplained "Solow residual."

Finally, for completeness, the balanced-growth rates of all dynamic variables are:

$$g_A = \frac{\lambda}{1-\phi}(\eta g_Z + n), \tag{BGP-1}$$

$$g_y = \frac{g_Z}{1-\alpha} + \frac{\theta - \alpha}{1-\alpha} g_A, \tag{BGP-2}$$

$$g_c = g_y, \qquad g_K = g_y + n, \tag{BGP-3}$$

$$g_\mu = \chi g_Z - \zeta g_y, \tag{BGP-4}$$

$$g_q = g_\mu + g_y - \phi g_A - \eta\lambda g_Z - (\lambda - 1)n, \tag{BGP-5}$$

$$g_\pi = g_\mu + g_y - g_Z + n. \tag{BGP-6}$$

The constancy of time shares (s_Y, s_A, s_Z) ensures that this system remains balanced over time. Historically, such balance reflects the post-Enlightenment social equilibrium: sustained civic maintenance of dignity, liberty, and reward (Z_t); continuous, decentralized idea production (A_t);

and steady capital accumulation (K_t) — together generating the Great Enrichment that lifted modern humanity into a permanently dynamic age.

5. MEASUREMENT, ELASTICITIES, AND HISTORICALLY FAITHFUL PARAMETER REGIONS

This section turns the structure developed above into a measurement device and a counterfactual engine, with the specific aim of showing that the model can reproduce the sustained one-to-two percent per-capita growth characteristic of the Great Enrichment in the developed economies from roughly 1750 to 2025. The logic proceeds in three layers. First, we state a minimal calibration routine that backs out the RSI drift from observables and checks the implied level ratios. Second, we assemble closed-form elasticities that map small, permanent changes in policy-relevant primitives — faster adjudication, broader entry, open publication — into growth, capital deepening, and welfare. Third, we locate a historically faithful parameter region — consistent with the evidence assembled by McCloskey, Mokyr, North, Landes, Broadberry, Maddison, and De Long — in which the model yields the observed long-run path and its epochal variation, and in which the oomph (McCloskey's word) of the republican spirit of innovism is quantitatively decisive rather than decorative.

The minimal calibration uses only per-capita growth, an ideas growth proxy, and standard national-accounts ratios. We emphasize that this procedure is essentially a form of disciplined growth accounting. The inferred g_Z is an accounting residual conditional on our maintained functional forms. Our contribution is not causal identification in the econometric sense, but rather to make explicit what is otherwise hidden in the Solow residual. By backing out the required drift in the innovism climate, we can ask whether its time path aligns with independent historical evidence on entry, speech, and adjudication — converting a "measure of our ignorance" into a testable historical quantity. Start from the growth-accounting identity derived from the production function,

$$(1-\alpha)\, g_y \;=\; g_Z + (\theta-\alpha)\, g_A, \tag{13}$$

which holds for any balanced-share path under (1). Given an empirically measured or inferred g_y (frontier slope near one to two percent per year), and a long-horizon proxy for g_A (patent-based, TFP-based, or idea-count-based, with due caution about measurement), equation (13) backs out the required RSI drift,

$$g_Z = (1-\alpha)\, g_y - (\theta-\alpha)\, g_A. \tag{14}$$

Because α is pinned by capital's income share (national accounts) and (g_y, g_A) are observed or proxy-measured, (14) provides a first diagnostic. In a second step, we check level ratios using the Euler condition. Combining the Keynes–Ramsey rule with balanced growth delivers the steady capital-output ratio,

$$\frac{K}{Y} = \frac{\alpha}{\delta + \rho - \chi g_Z + \zeta g_y}, \tag{15}$$

and the implied investment share,

$$\frac{I}{Y} = (\delta + n + g_y)\frac{K}{Y}. \tag{16}$$

Calibrating (δ, ρ, ζ) to standard macro ranges, inserting the measured g_y and the implied g_Z from (14), and setting n to the demographic data by epoch (rising in the nineteenth century, falling post-1950), (15)–(16) produce benchmark K/Y and I/Y at the frontier that can be read against national accounts. The same discipline ties down the consumption share; internal consistency with post-war data is a necessary condition for the plausibility of the historical g_Z sequence. In practice, we find that modest g_Z (tens of basis points) suffices to reconcile a stable frontier slope with falling n, while also raising K/Y through the effective-patience wedge χg_Z; this echoes the legal-financial consolidation narratives stressed by North–Weingast, La Porta et al., and Skeel.

The second layer derives elasticities that translate policy levers into macro outcomes. Two channels dominate: the direct RSI drift and the RSI–R&D complementarity. Differentiating (13) gives the growth semi-elasticity with respect to the RSI drift,

$$\frac{\partial g_y}{\partial g_Z} = \frac{1}{1-\alpha}. \tag{17}$$

A one-time permanent increase of g_Z by Δg_Z therefore raises the frontier slope by $\Delta g_y = \Delta g_Z/(1-\alpha)$; with $\alpha \approx 1/3$, the amplification is roughly 1.5. This is the pure "permission and esteem" channel. The second channel operates through ideas. Using the semi-endogenous ideas equation, balanced-growth ideas growth satisfies

$$g_A = \frac{\lambda}{1-\phi}(\eta g_Z + n), \tag{18}$$

so the RSI drift also tilts the efficacy of research. Substituting (18) into (13) yields per-capita growth as a function of primitives,

$$g_y \;=\; \frac{g_Z}{1-\alpha} + \frac{\theta - \alpha}{1-\alpha} \cdot \frac{\lambda}{1-\phi}\,(\eta g_Z + n). \tag{19}$$

Two comparative-statics are immediate:

$$\frac{\partial g_y}{\partial \eta} = \frac{\theta - \alpha}{1 - \alpha} \cdot \frac{\lambda}{1 - \phi} g_Z > 0, \qquad \frac{\partial g_y}{\partial n} = \frac{\theta - \alpha}{1 - \alpha} \cdot \frac{\lambda}{1 - \phi} > 0. \tag{20}$$

Open science, examinable patents, and credible disclosure (higher η) raise the return to research because dignified and free participation turns research hours into traveling knowledge. Conversely, when demographic tailwinds fade, g_Z becomes the decisive state for holding the slope.

Welfare elasticities follow transparently from preferences. Let lifetime utility be $U = \int_0^\infty e^{-(\rho - n)t} Z_t^\chi u(c_t) dt$ with CRRA $u(c) = (c^{1-\zeta} - 1)/(1-\zeta)$. Along balanced paths with $g_c = g_y$ and a permanent Δg_Z, a first-order welfare change relative to the consumption-equivalent metric is

$$\frac{\partial \ln U}{\partial g_Z} = \frac{\chi}{\rho - n - \chi g_Z} + \frac{1}{\zeta} \cdot \frac{1}{\rho - n - \chi g_Z - (1 - \zeta) g_y} \cdot \frac{\partial g_y}{\partial g_Z}. \tag{21}$$

where $\partial g_y / \partial g_Z$ is given by the full effect from (19). The first term is the intrinsic valuation of living in a dignified, liberal, fairly rewarded society; the second is the compounding effect through higher consumption growth. For reasonable (ζ, ρ, χ), the welfare multiplier with respect to Δg_Z is large even when Δg_Z itself is only a few basis points.

The third layer identifies a historically faithful parameter region consistent with both the long frontier slope and within-era variation. Setting $\alpha \in [0.30, 0.40]$, $\delta \in [0.04, 0.08]$, $\zeta \in [1, 3]$, $\rho \in [0.02, 0.05]$, and (λ, ϕ) in the standard Jones range (e.g., $\lambda \in [0.5, 1]$, $\phi \in [0.3, 0.8]$), equations (14)–(19) yield the following robust pattern. First, with nineteenth-century n between 0.7 and 1.2 percent and moderate g_Z between 0.2 and 0.5 percent, the model reproduces $g_y \approx 1.5 - 2$ percent and investment shares aligned with national accounts via (15)–(16). In particular, when $\alpha = 1/3, \theta = 0.70, \delta = 0.04, \zeta = 2.0, \rho = 0.03, \chi = 0.15, n = 0.01, \lambda = 0.75, \phi = 0.6, \eta = 0.5, \nu = 1.5, \kappa = 0.04, \xi = 0.8, \delta_Z = 0.01, g_Z^{\text{exo}} = 0.005$, we can solve for the balanced growth path. The numerical strategy is in Appendix F. The results are: $s_Z = 0.2597, s_Y = 0.6178, s_A = 0.1225, g_Z = 0.004, g_A = 0.0226, g_y = 0.0186, K/Y = 3.128, I/Y = 0.2146$, and the real interest rate r= 0.0666.

Second, as n declines toward zero after the demographic transition, maintaining g_y in the one-to-two percent band requires only modest increases in g_Z or η (the latter reflecting institutionalization of open science and higher-quality patent examination). Third, backsliding episodes — censorship, politicized courts, crony finance — map into temporary rises in δ_Z, visible as dips in measured g_Z; the identity (14) then explains subdued g_y even when measured g_A appears resilient.

The upshot is that the model makes McCloskey's rhetorical and historical thesis operational. The Great Enrichment's slope can be matched with small, persistent RSI drift and plausible ideas dynamics, and the same structure quantifies the gains from defending the republican, mass climate of innovism. Cheapening entry and publication, speeding adjudication, safeguarding bankruptcy discharge, raising patent-examination quality, and widening pluralist media do not merely bump levels; via (17)–(21) they tilt the entire slope of the frontier and compound in welfare.

6. CONCLUSION

This paper set out to make a central historical claim operational: the decisive engine of modern prosperity is neither capital deepening nor "raw" knowledge accumulation in isolation, but a produced, nonrival social technology — the republican spirit of innovism — that dignifies ordinary makers, keeps entry and discourse open, and secures fair reward by predictable law. We embedded that climate, Z, directly into a Jones-style ideas economy, so that innovism multiplies production possibilities, raises the efficacy with which research hours become traveling knowledge, and, through tastes, lengthens horizons and deepens capital. The resulting structure reconciles the discipline of semi-endogenous growth with the historical narratives emphasized by McCloskey, Mokyr, North, Landes, and others: permission and esteem are produced, defended, and sometimes lost; small, persistent gains in this climate cumulate into the Great Enrichment.

The economic environment makes precise what the historians describe. Ideas determine which machine varieties exist; raw capital is the aggregation of those varieties; and the innovism stock scales the whole technology because it lowers the fixed costs of widespread experimentation and disclosure. Treating Z as a produced stock — with historically dated ratchets, continuous civic effort, and depreciation under neglect — delivers threshold behavior and hysteresis that match episodes from printing and Reformation to constitutional settlement, codification, and modern open science, as well as reversals under censorship, media capture, or politicized adjudication. The same stock enters preferences, so that living in a dignified, liberal, fairly rewarded society "pays twice": directly, as part of what people value, and indirectly, by raising growth.

Analytically, the model yields transparent conditions for balanced growth and closes with familiar ratios. A modest, sustained drift in Z keeps per-capita growth in the one-to-two-percent band even when demography ceases to help; innovism–R&D complementarity lifts the slope further by improving the conversion of research time into useful, traveling ideas; and the Euler condition acquires a patience wedge that endogenously raises the capital–output ratio when the climate is credibly defended. These mech-

anisms are not adornments; they are the minimal additions needed for a Jones economy to match the frontier path from the late eighteenth century to the present without resorting to explosive scale effects.

Empirically, the framework becomes a measurement device and a counterfactual engine. A simple identity recovers an implied g_Z from observed per-capita growth and ideas proxies; the capital-output and investment ratios disciplined by the Euler condition then check internal consistency with national accounts. Elasticities map policy-relevant primitives into growth, deepening, and welfare: faster adjudication, broader entry, higher patent-examination quality, and cheaper, pluralist media tilt time toward maintaining Z, raise its drift, and — via disclosure — increase the efficacy of research. Because Z is nonrival, small, permanent improvements compound; because Z depreciates, backsliding near threshold regions can shrink the basin of attraction around high growth and leave long scars.

The broader lesson is straightforward. Long-run growth at the frontier is a civic achievement, not a mechanical consequence of piling brick on brick or patent on patent. Societies that allocate a steady share of educated time to open discourse, legal maintenance, and fair reward keep the slope of income growth alive; those that neglect it watch the slope fade. By placing McCloskey's republican, mass innovism inside a modern ideas model with clear accounting, this paper turns a persuasive historical thesis into a tractable tool — one that can be calibrated, stress-tested, and used to value reforms. The agenda ahead is equally clear: bring better measures of Z and its pillars to the data; distinguish channels (discourse, reform, adjudication) in time-use and institutional statistics; quantify thresholds and basins of attraction across countries and eras; and evaluate policy packages that advance all three pillars together. If the Great Enrichment was, as the historians argue, a matter of permission and esteem spreading to the many, then preserving and extending that climate is the central task of economic policy — and the surest guard for growth that endures.

A natural extension is to specify Z as an explicit social-production function of its pillars, for example a CES or multiplicative aggregator over dignity (D), liberty (B), and reward (R). Doing so would not alter the paper's qualitative conclusions, but it would sharpen identification and policy design in three ways. First, it would allow decomposition of the innovism drift into pillar-specific contributions, letting data on speech freedom, entry costs, and contract enforcement map onto parameters and deliver testable predictions about which reforms matter most in time and place. Second, curvature across pillars would reveal threshold and bottleneck effects: if the aggregator is complementary, the weakest pillar governs the slope, implying high returns to the most deficient margin and explaining historical cases where progress stalled despite advances elsewhere. Third, explicit D–B–R dynamics would let us study transitional paths under asymmetric shocks

(e.g., media capture with stable contract law), quantify hysteresis when one pillar depreciates, and evaluate coordinated versus piecemeal reforms. The empirical program then becomes estimating pillar elasticities and complementarities while preserving the central message: broad permission, open discourse, and predictable rewards jointly sustain modern growth.

APPENDIX

A.1. FIRST-ORDER CONDITIONS AND CO-STATE EQUATIONS

The representative household's problem is to maximize (5) subject to (1)–(4). We form the current-value Hamiltonian and record the conditions used in the text.

Euler equation (consumption), identical to (7) in the text:

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\zeta}\Big(\alpha\frac{Y_t}{K_t} - \delta - \rho + \chi g_{Z,t}\Big). \tag{A.1}$$

Intratemporal FOCs (labor allocation), identical to (8a)–(8b):

$$(1-\alpha)\frac{Y_t}{L_t s_{Y,t}} = \frac{q_t}{\mu_t}\lambda\nu Z_t^{\eta\lambda}(L_t s_{A,t})^{\lambda-1}A_t^{\phi}, \tag{A.2}$$

$$(1-\alpha)\frac{Y_t}{L_t s_{Y,t}} = \frac{\pi_t}{\mu_t}\kappa\xi s_{Z,t}^{\xi-1}\frac{Z_t}{L_t}. \tag{A.3}$$

Co-state (current value) ODEs:

$$\dot{\mu}_t = \rho\mu_t - \mu_t\Big(\alpha\frac{Y_t}{K_t} - \delta\Big), \tag{A.4}$$

$$\dot{q}_t = \rho q_t - \left(\mu_t(\theta-\alpha)\frac{Y_t}{A_t} + q_t\phi\frac{\dot{A}_t}{A_t}\right), \tag{A.5}$$

$$\dot{\pi}_t = (\rho - g_{Z,t})\pi_t - \chi L_t Z_t^{\chi-1}\frac{c_t^{1-\zeta}-1}{1-\zeta} - \mu_t\frac{Y_t}{Z_t} - q_t\eta\lambda\frac{\dot{A}_t}{Z_t}. \tag{A.6}$$

A.2. BALANCED-GROWTH PATH (BGP)

Assume constant shares (s_Y, s_A, s_Z) and constant growth rates (g_Z, g_A, g_y). Ideas:

$$g_A = \frac{\lambda}{1-\phi}(\eta g_Z + n). \tag{A.7}$$

Output per capita growth (g_y):

$$g_y = \frac{g_Z}{1-\alpha} + \frac{\theta-\alpha}{1-\alpha} g_A. \tag{A.8}$$

Capital-output ratio and investment share:

$$\frac{K}{Y} = \frac{\alpha}{\delta + \rho - \chi g_Z + \zeta g_y}, \tag{A.9}$$

$$\frac{I}{Y} = (\delta + n + g_y)\frac{K}{Y}. \tag{A.10}$$

Co-state growth rates (consistent with $\mu_t = Z_t^{\chi} c_t^{-\zeta}$):

$$g_\mu = \chi g_Z - \zeta g_y, \tag{A.11}$$

$$g_q = g_\mu + g_y - \phi g_A - \eta\lambda g_Z - (\lambda - 1)n, \tag{A.12}$$

$$g_\pi = g_\mu + g_y - g_Z + n. \tag{A.13}$$

A.3. SHARES IN LEVELS

From the level FOCs:

$$(1-\alpha)\frac{Y}{Ls_Y} = \frac{q}{\mu}\,\lambda\nu\, Z^{\eta\lambda}(Ls_A)^{\lambda-1}A^{\phi}, \tag{A.14}$$

$$(1-\alpha)\frac{Y}{Ls_Y} = \frac{\pi}{\mu}\,\kappa\xi\, s_Z^{\xi-1}\frac{Z}{L}. \tag{A.15}$$

Given primitives and (μ, q, π) at a reference date, these determine (s_Y, s_A, s_Z).

A.4. COMPARATIVE STATICS

Direct effects:

$$\frac{\partial g_y}{\partial g_Z} = \frac{1}{1-\alpha} + \frac{\theta-\alpha}{1-\alpha}\cdot\frac{\lambda}{1-\phi}\,\eta, \tag{A.16}$$

$$\frac{\partial g_y}{\partial n} = \frac{\theta-\alpha}{1-\alpha}\cdot\frac{\lambda}{1-\phi}. \tag{A.17}$$

Capital deepening:

$$\frac{\partial(K/Y)}{\partial g_Z} = -\frac{\alpha}{(\delta + \rho - \chi g_Z + \zeta g_y)^2}\Big(-\chi + \zeta\,\frac{\partial g_y}{\partial g_Z}\Big). \tag{A.18}$$

A.5. TRANSITIONAL DYNAMICS (FOR SIMULATIONS, NOT CONDUCTED IN THIS PAPER)

$$\dot{c}/c = \frac{1}{\zeta}\Big(\alpha Y/K - \delta - \rho + \chi g_Z\Big), \tag{A.19}$$

$$\dot{A} = \nu \left(Z^{\eta} L s_A\right)^{\lambda} A^{\phi}, \tag{A.20}$$

$$\dot{Z}/Z = g_Z^{\text{exo}}(t) + \kappa\, s_Z^{\xi} - \delta_Z, \tag{A.21}$$

$$\dot{q} = \Big(\rho - \phi\frac{\dot{A}}{A}\Big)\, q - \mu\,(\theta - \alpha)\, Y/A, \tag{A.22}$$

$$\dot{\pi} = \big(\rho - g_Z\big)\,\pi - \chi L Z^{\chi-1}\frac{c^{1-\zeta}-1}{1-\zeta} - \mu\, Y/Z - q\,\eta\lambda\,\dot{A}/Z, \tag{A.23}$$

$$\dot{\mu} = \rho\,\mu - \mu\,(\alpha Y/K - \delta)\,, \tag{A.24}$$

$$s_Y + s_A + s_Z = 1, \qquad \dot{K} = Y - Lc - \delta K. \tag{A.25}$$

A.6. NUMERICAL STRATEGY FOR THE BALANCED-GROWTH PATH

This subsection details a constructive algorithm to compute a BGP that respects the intratemporal FOCs, Euler condition, and the semi-endogenous ideas block.

A.6.1. Normalizations and inputs

Normalize $L_0 = 1$ and $K_0 = 1$. Fix the constant g_Z^{exo} and the primitives $\{\alpha, \theta, \delta, \rho, \zeta, \chi, \nu, \lambda, \phi, \eta, \kappa, \xi, \delta_Z, n\}$. On a BGP, (s_Y, s_A, s_Z) are constants with $s_Y + s_A + s_Z = 1$.

A.6.2. Outer loop variable

Bracket $s_Z \in [s_Z^{\min}, s_Z^{\max}] \subset (0,1)$ (e.g., 10^{-4} to 0.5).

A.6.3. Given s_Z, build growth rates and static ratios

Compute in the following sequence,

$$g_Z = g_Z^{\text{exo}} + \kappa s_Z^{\xi} - \delta_Z, \tag{A.26}$$

$$g_A = \frac{\lambda}{1-\phi}(\eta g_Z + n), \quad g_y = \frac{g_Z}{1-\alpha} + \frac{\theta-\alpha}{1-\alpha} g_A, \quad g_K = g_y + n, \tag{A.27}$$

$$g_\mu = \chi g_Z - \zeta g_y,\; g = g_\mu + g_y - \phi g_A - \eta\lambda g_Z - (\lambda - 1)n,\; g_\pi = g_\mu + g_y - g_Z + n. \tag{A.28}$$

Then

$$\frac{K}{Y} = \frac{\alpha}{\delta + \rho - \chi g_Z + \zeta g_y}, \quad Y_0 = \frac{1}{K/Y}, \quad I_0 = \delta + g_K, \quad C_0 = Y_0 - I_0. \tag{A.29}$$

Note that we made use of the normalization of $K_0 = 1$ in the calculations above.

A.6.4. First expression for $q_0 A_0/\mu_0$ from the q-ODE

From $\dot{q} = \rho q - \mu(\theta - \alpha)Y/A - q\,\phi g_A$ on the BGP:

$$\frac{q_0 A_0}{\mu_0} = \frac{(\theta - \alpha)\, Y_0}{\rho - \phi g_A - g_q}. \tag{A.30}$$

A.6.5. Obtain $\mu_0/(\pi_0 Z_0)$ from the $\dot{\pi}$ equation

Using $\mu_0 = Z_0^{\chi} c_0^{-\zeta} \Rightarrow Z_0^{\chi-1} = \mu_0 c_0^{\zeta}/Z_0$ and defining $x \equiv \mu_0/(\pi_0 Z_0)$, the $\dot{\pi}$ equation implies

$$g_\pi = \rho - g_Z - x \left[\chi\, c_0^{\zeta}\, \frac{c_0^{1-\zeta} - 1}{1-\zeta} + Y_0 + \left(\frac{q_0 A_0}{\mu_0} \right) \eta\lambda\, g_A \right]. \tag{A.31}$$

Solving for x and substituting (A.30) yields

$$x = \frac{\rho - g_Z - g_\pi}{\chi\, c_0^{\zeta}\, \frac{c_0^{1-\zeta}-1}{1-\zeta} + Y_0 \left[1 + \eta\lambda\, g_A\, \frac{\theta-\alpha}{\rho - \phi g_A - g_q} \right]}. \tag{A.32}$$

A.6.6. Recover s_Y from Y–Z wage equalization using x

From (8b) at $t = 0$ with $L_0 = 1$:

$$(1-\alpha)\frac{Y_0}{s_Y} = \frac{\pi_0}{\mu_0}\, \kappa\xi\, s_Z^{\xi-1} Z_0 \quad \Longrightarrow \quad s_Y = \frac{(1-\alpha)\, Y_0\, x}{\kappa\xi\, s_Z^{\xi-1}}. \tag{A.33}$$

Reject the current s_Z if $s_Y \notin (0,1)$.

F.7 Compute s_A and a second expression for $q_0 A_0/\mu_0$.

Set $s_A = 1 - s_Y - s_Z$; reject if $s_A \notin (0,1)$. Using (8a) and g_A,

$$\left[\frac{q_0 A_0}{\mu_0} \right]_{\text{YA-FOC}} = \frac{(1-\alpha) Y_0\, s_A}{\lambda\, g_A}. \tag{A.34}$$

A.6.7. Bisection to enforce consistency

Define the diagnostic from the two expressions of $q_0 A_0/\mu_0$:

$$\Psi(s_Z) \equiv \frac{(1-\alpha)Y_0\, s_A}{\lambda g_A} - \frac{(\theta-\alpha)\, Y_0}{\rho - \phi g_A - g_q}. \tag{A.35}$$

If $\Psi(s_Z) > 0$, increase s_Z; if $\Psi(s_Z) < 0$, decrease s_Z. Apply bisection on $[s_Z^{\min}, s_Z^{\max}]$ until $|\Psi(s_Z)| < \varepsilon$.

A.6.8. Recover levels and co-states at the solution

With the converged $s_Z^\star$:

1. Compute $g_Z^\star, g_A^\star, g_y^\star, g_K^\star$, Y_0, C_0, and I_0 via (A.26)–(A.29).
2. Compute $x^\star$ from (A.32) and $s_Y^\star$ from (A.33); set $s_A^\star = 1 - s_Y^\star - s_Z^\star$.
3. From $Y_0 = Z_0\, A_0^{\theta-\alpha}\, K_0^{\alpha} \left(L_0 s_Y^*\right)^{1-\alpha}$and $g_A = \nu\, (Z_0^{\eta} s_A^*)^{\lambda}\, A_0^{\phi-1}$, we can eliminate Z_0 and solve for A_0, and then solve forZ_0.
4. With Z_0 and c_0 in place, we can obtain $\mu_0 = Z_0^{\chi} c_0^{-\zeta}$
5. With A_0 and μ_0 in place, we can obtain q_0 from either identity:

$$q_0 = \frac{(1-\alpha)Y_0\, s_A^*}{\lambda g_A} \cdot \frac{\mu_0}{A_0}. \tag{A.36}$$

6. We can also obtain π_0 easily: $\pi_0 = \mu_0/(xZ_0)$.

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