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Optimal Fiscal Policy for Protecting Future Generations*

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We study the design of fiscal policy within a two-period model involving a central government and multiple regions that differ in the privately observable durability of their local intergenerational public goods (IPG). The IPG is financed through local debt and fiscal transfers, and the regions are less patient than the central government. We address the joint frictions of asymmetric information and present bias. We find that regions with greater durability should be allocated higher levels of debt issuance, regardless of the presence of informational friction. Regarding the endogenous interaction between these two fiscal policies, our analysis indicates that, with a power utility function, debt issuance and fiscal transfers function as complementary tools in the first-best optimum. However, when accounting for these two frictions, resources should be transferred from regions with higher durability to those with lower durability, suggesting that debt issuance and fiscal transfers act as substitutes in restoring social optimality.

Key Words: Present bias; Intergenerational public good; Endogenous policy interaction; Asymmetric information; Mechanism design.
JEL Classification Numbers: D82, H74.

1. INTRODUCTION

There appears to be a global consensus that fiscal policy should be carefully designed to prevent or mitigate adverse intergenerational consequences. The fiscal policies implemented today can have both negative and positive impacts on the welfare of future generations. Notably, issuing a larger amount of public debt now tends to generate greater negative intergenerational externalities, as future generations will face higher debt and

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interest repayment obligations. Conversely, if current fiscal revenues are allocated toward funding intergenerational public goods—such as environmental protection, natural resource conservation, and investments in basic science, new technologies, and human capital—this can generate significant positive externalities for future generations. Therefore, an effective fiscal policy should strike a balance by avoiding excessive government borrowing while ensuring adequate provision of intergenerational public goods.

We examine the optimal design of fiscal policy within a federal system of governments, where the central government exhibits patience while the regional governments are impatient. The regional authorities are responsible for local borrowing and the provision of intergenerational public goods at the local level, whereas the central government manages interregional fiscal transfers.[1] To highlight the potential informational frictions between higher and lower levels of government, we consider regions that differ in the durability of their local intergenerational public goods. Specifically, some regions are more sustainable than others, in the sense that they allocate a larger proportion of current fiscal revenue toward funding public goods that remain usable for future generations. These regions are thus designated as H-regions, while the opposing regions are labeled as L-regions. The durability is considered private information for each region, and only the distribution of this variable is publicly available.[2] We thus employ the mechanism design approach to determine the socially optimal fiscal policies, treating the central government as the mechanism designer or principal, and the regional governments as heterogeneous agents.

Before addressing the mechanism design problem faced by the central government, we first establish the first-best optimum. This involves determining the allocation of debt issuance levels and interregional fiscal transfers for the two types of regions, with the goal of maximizing the combined welfare of both current and future generations. Compared to this first-best benchmark, two frictions are present: asymmetric information and present-biased preferences of regional governments. Consequently, we explore the following two questions: Given these frictions, which type of regions should be allocated higher debt issuance—H-regions with higher durability or L-

[1]For example, fiscal transfer payments from provincial and central governments constitute a significant portion of the funding for basic education in rural China. Data indicates that in 2002, transfer payments from the central government allocated for rural compulsory education totaled 593 billion yuan, representing approximately 44.7% of the total national education budget appropriations (Zhang and Zou 2023).

[2]Other sources of informational asymmetry between higher and lower levels of government, or between relevant principals and agents in terms of incentive design, are examined by numerous existing studies, such as Gilbert and Picard (1996), Cremer and Pestieau (1997), Bucovetsky et al. (1998), Lockwood (1999), Bordignon et al. (2001), Cornes and Silva (2000, 2002), Huber and Runkel (2006), Breuillé and Gary-Bobo (2007), Kıbrıs and Tapkı (2014), Klibanoff and Poitevin (2022), and Wang and Xiao (2024).

regions with lower durability? Additionally, from the perspective of societal optimality, should interregional fiscal transfers serve as a complement or a substitute to the allocation scheme of local debt issuance in addressing these frictions?

Regarding the first question, we find that H-regions should be allocated a higher level of debt issuance than L-regions, a conclusion that holds in both the first-best benchmark and the asymmetric information optimum. The underlying intuition is that the intergenerational public good provided by H-regions exhibits greater durability and generates a larger positive intergenerational externality compared to L-regions. The positive intergenerational externality can partially offset the negative intergenerational externality resulting from debt issuance. Consequently, it is socially optimal for H-regions to issue more debt than L-regions.

With respect to the second question, our finding indicates that, under asymmetric information, these two fiscal policies function as substitutes for maximizing the combined welfare of both present and future generations. That is, H-regions are allocated a higher level of debt issuance while contributing to fiscal transfers, and L-regions are assigned a lower level of debt issuance while benefitting from interregional transfers. This contrasts with the results observed in the full information benchmark. In the full information setting, these policies act as complements under a power utility function: H-regions are allocated higher debt issuance and also receive interregional fiscal transfers, whereas L-regions are assigned lower debt issuance and also contribute to transfers. In particular, if the log preference is adopted under complete information, then interregional fiscal transfers should not be utilized at all. This indicates that the characteristics of interregional fiscal transfers cannot be uniquely identified without specifying the utility functional forms. This finding contrasts with the results observed under the asymmetric information optimum, where the identification of interregional fiscal transfers is independent of the specific utility functional form.

The underlying rationale for these differences is as follows. In the full information benchmark, the primary driver for allocating debt issuance between the two types of regions is balancing the negative intergenerational externality caused by public borrowing against the positive externality generated by durable public good provision. To maximize overall welfare across these regions, it is optimal to transfer resources from L-regions—where the positive intergenerational externality is smaller—to H-regions, which have a larger positive externality. Since the negative externality per unit of debt issuance is the same across both types, these policies tend to act as complements to mutually enhance their effectiveness from an efficiency perspective.

However, under asymmetric information, this arrangement must be revised due to the self-selection issue. Resources should now be transferred from H-regions to L-regions to prevent misreporting or cheating. Consequently, these two fiscal policies serve as substitutes, simultaneously addressing the self-selection problem and balancing the negative and positive intergenerational externalities associated with public borrowing and public good provision, respectively.

The paper primarily relates to two branches of existing research. First, the paper relates to the literature that examines the joint design of optimal debt issuance allocation and fiscal transfers across heterogeneous regions, such as Huber and Runkel (2008), Dai et al. (2019a,b), Dai et al. (2022), Dai and Tian (2023), Dai et al. (2026), and Dai (2026). These studies examine the informational frictions stemming from asymmetric information between higher and lower levels of government, and they highlight the horizontal fiscal externalities associated with interregional migration, spillovers of interregional public goods, or the adoption of alternative regional criteria. However, none of these works account for the present-bias friction stemming from the observation that the central government or mechanism designer is often more patient than regional subnational governments or politicians. This is particularly relevant in the context of China, where local politicians compete for promotion within the governmental hierarchy.[3] Compared to these existing studies, our work highlights the combined impact of asymmetric information and differences in patience levels between higher and lower levels of government, with the aim of informing fiscal policy design.

Second, this paper relates to the existing literature examining how present-biased preferences and time inconsistency influence the design of public debt policy. Relevant studies include Amador et al. (2006), Halac and Yared (2014, 2018, 2022), Bisin et al. (2015), and Arawatari and Ono (2021, 2023). Our analysis diverges from these works in a key aspect: none of these papers consider the fiscal policy of interregional fiscal transfers. Consequently, they do not address the endogenous interaction or the joint functioning of fiscal transfer policies and debt issuance allocation across heterogeneous regions. Additionally, they account for informational frictions that differ from that considered in the current model. To our knowledge, none of these studies examine the optimal provision of intergenerational public goods. Consequently, they are unable to identify the potential informational frictions arising from the durability inherent in these public goods.

The article is organized as follows: Section 2 introduces the economic environment. Section 3 establishes the first-best allocation under complete

[3]See, e.g., Chen et al. (2005), Xu (2011), Lin (2022), and Qu et al. (2023).

information. Section 4 derives the asymmetric information optimum. Section 5 offers concluding remarks. The proofs of the results are included in the Appendix.

2. THE MODEL

2.1. Set-up

We study a two-period simple model of fiscal policy within a federation consisting of a central government and a unit mass of regional governments located in different regions. The regions vary in the parameter that measures the durability—or quality—of the local intergenerational public good, denoted by θ, which is private information for each region.[4] We refer to θ as the *type* of these regions. It is drawn from the set $\{\theta_L, \theta_H\}$, satisfying $0 < \theta_L < \theta_H < \infty$. The proportion of regions with type θ_L is p_L, and the proportion with type θ_H is p_H, where $p_L, p_H \in (0, 1)$, and $p_L + p_H = 1$.[5]

The present welfare value for regions of type θ is expressed as follows:[6]

$$g(G_1) + \beta \cdot g(\theta G_1 + G_2), \tag{1}$$

where the parameter $\beta \in (0, 1)$ represents the region's discount factor, and

$$\begin{aligned} G_1 &= b + z, \\ G_2 &= \tau - (1 + r)b \end{aligned}$$

denote the levels of public good supply in periods 1 and 2, respectively. In period 1, regions receive federal transfers amounting to z, which can be negative, and incur public debt b. If $z < 0$, these regions pay a lump-sum tax to the central government. The repayment of debt and interest is scheduled for period 2 at an established interest rate $r > 0$. The positive tax revenue $\tau > 0$ is collected to finance the provision of local public goods

[4]The specification of the intergenerational public good aligns with that presented in Rangel (2005) and Conley et al. (2019). Given that a non-depreciated portion of the public good is carried over from the first to the second period, this specification aligns with the concept of productive public goods, which is frequently discussed in the macroeconomic public finance literature (Bassetto and Sargent 2006; Chatterjee and Ghosh 2011). In a broader context, liberal ideas, social norms, property rights capital, and the institutional frameworks that safeguard private property—recently emphasized by Sun and Zou (2025), Zhan and Zou (2025), and Zou (2025)—can be interpreted as forms of intergenerational public goods. These elements have the potential to generate positive spillovers across generations.

[5]Throughout the analysis, the term "L-regions" is interchangeable with "type-θ_L regions," and "H-regions" with "type-θ_H regions."

[6]Building on the theoretical studies of fiscal policy initiated by Halac and Yared (2014, 2018, 2022), we also assume away private consumption to focus on the frictions arising from asymmetric information and present bias, which influence the optimal design of fiscal policy.

G_2. For our purposes, we assume that r and τ are exogenously specified parameters. The utility function $g(\cdot)$ is continuously differentiable, strictly increasing, strictly concave, and satisfies the Inada conditions.

In addition to private information, we assume that the degree of patience varies between the central government and the regions. Specifically, the central government is assumed to be perfectly patient, treating the present and future generations equally in policy decisions. Formally, by utilizing the previously specified budget constraints, the value functions of the regions and the central government are denoted by V and W, respectively, as follows:

$$\begin{aligned} V(b,z;\theta,\beta) &\equiv g(b+z)+\beta\cdot g(\theta(b+z)+\tau-(1+r)b), \\ W(b,z;\theta) &\equiv g(b+z)+g(\theta(b+z)+\tau-(1+r)b). \end{aligned} \tag{2}$$

We employ a mechanism design approach to determine the optimal fiscal policy concerning local debt issuance and interregional fiscal transfers. To simplify the notation, we denote the policy menu prepared for these two types by

$$b(\theta_L)\equiv b_L,\ z(\theta_L)\equiv z_L,\ b(\theta_H)\equiv b_H,\ \text{and}\ z(\theta_H)\equiv z_H.$$

Assuming a scenario of pure interregional redistribution, the fiscal budget constraint for the central government is

$$p_L\cdot z_L+p_H\cdot z_H\leq 0. \tag{3}$$

Therefore, by using (2), the optimal fiscal policy mix is determined by solving the following program:

$$\max_{b_L,z_L,b_H,z_H}\ p_L W(b_L,z_L;\theta_L)+p_H W(b_H,z_H;\theta_H) \tag{4}$$

subject to the budget constraint (3) and the following incentive-compatibility (IC) constraints:

$$\begin{aligned} V(b_L,z_L;\theta_L,\beta) &\geq V(b_H,z_H;\theta_L,\beta)\ (IC_L), \\ V(b_H,z_H;\theta_H,\beta) &\geq V(b_L,z_L;\theta_H,\beta)\ (IC_H). \end{aligned} \tag{5}$$

The IC_L constraint ensures that L-regions (weakly) prefer the menu associated with (b_L,z_L) over that of (b_H,z_H). In other words, when this condition is satisfied, L-regions have no incentive to mimic H-regions. The IC_H constraint can be interpreted in a similar manner.

2.2. Single-crossing property

We now characterize regional indifference curves by interpreting z as a differentiable function of b within the (b,z)-space. By exploiting the value function V defined by equation (2), we derive the slope of an indifference curve in the (b,z)-space as follows:

$$\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{\mathrm{d}V=0} = \frac{\beta\left(1+r-\theta\right)g'\left(\theta G_1+G_2\right)-g'\left(G_1\right)}{g'\left(G_1\right)+\beta\theta g'\left(\theta G_1+G_2\right)}. \tag{6}$$

The curvature can be given by[7]

$$\left.\frac{\mathrm{d}^2 z}{\mathrm{d}b^2}\right|_{\mathrm{d}V=0} > 0. \tag{7}$$

Consequently, using (6) and (7) yields that the indifference curve is U-shaped with its minimum at the point where the intertemporal rate of substitution (IRS) equals the intertemporal rate of transformation (IRT), namely,

$$\underbrace{\frac{g'\left(G_1\right)}{\beta g'\left(\theta G_1+G_2\right)}}_{\text{IRS}} = \underbrace{1+r-\theta}_{\text{IRT}}. \tag{8}$$

As is evident, the IRT also reflects the opportunity cost of borrowing, which is jointly determined by the negative intergenerational externality of local borrowing, measured by $1+r$, and the positive intergenerational externality of durable public provision, measured by θ.

Moreover, differentiating (6) with respect to θ yields[8]

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{\mathrm{d}V=0}\right) < 0. \tag{9}$$

The assumption $\theta_H > \theta_L$ then implies that, at every point in the (b,z)-space, the indifference curve of an H-region has a smaller slope than that of an L-region. Consequently, equation (9) establishes the single-crossing property or the Spence-Mirrlees condition (Laffont and Martimort 2002). Note that the single-crossing condition is crucial for guaranteeing that only one incentive constraint will bind at the optimum.

[7] The proof is relegated to the Appendix.
[8] The proof is relegated to the Appendix.

3. THE FIRST-BEST BENCHMARK

In the presence of complete information, we disregard the incentive compatibility constraints (5). Consequently, the central government addresses the following simplified program:

$$\max_{b_L, z_L, b_H, z_H} \quad p_L W(b_L, z_L; \theta_L) + p_H W(b_H, z_H; \theta_H) \tag{10}$$

subject to the budget constraint (3).

By solving the problem (10), we present the results in the following proposition. The welfare optimum under complete information is highlighted by the superscript ^{FB}.

PROPOSITION 1. *The welfare optimum under full information satisfies:*
(i) The optimal intertemporal decision-making satisfies

$$\underbrace{\frac{g'\left(G_{1L}^{FB}\right)}{g'\left(\theta_L G_{1L}^{FB} + G_{2L}^{FB}\right)}}_{IRS_L^{FB}} = \underbrace{1 + r - \theta_L}_{IRT_L^{FB}}, \quad \underbrace{\frac{g'\left(G_{1H}^{FB}\right)}{g'\left(\theta_H G_{1H}^{FB} + G_{2H}^{FB}\right)}}_{IRS_H^{FB}} = \underbrace{1 + r - \theta_H}_{IRT_H^{FB}},$$

and $G_{1L}^{FB} < G_{1H}^{FB}$, $G_{2L}^{FB} > G_{2H}^{FB}$, *and* $b_L^{FB} < b_H^{FB}$.

(ii) The identification of the optimal interregional fiscal transfers depends on the functional forms of the utility function. Specifically, we have:

(a) If $g(\cdot) = \ln(\cdot)$, *then* $z_L^{FB} = z_H^{FB} = 0$, *indicating that interregional fiscal transfers should not be implemented at all.*

(b) If $g(\cdot) = (\cdot)^\alpha$, *for a constant* $\alpha \in (0,1)$, *then we have* $z_L^{FB} < 0 < z_H^{FB}$, *implying that resources should be transferred from L-regions to H-regions.*

(c) Suppose $g(x) = -\frac{1}{\gamma} \cdot e^{-\gamma x}$ *for a constant* $\gamma > 0$. *If* μ, θ_L *and* θ_H *satisfy*

$$\frac{1+r}{e \cdot (1 + r - \theta_H)} < \mu < \frac{1+r}{1 + r - \theta_L},$$
$$\frac{1 + r - \theta_L}{1 + r - \theta_H} < e,$$

then we have $z_L^{FB} < 0 < z_H^{FB}$, *where* $e \approx 2.71828$ *is the well-known natural constant and* μ *denotes the Lagrange multiplier associated with the government budget constraint. However, if* μ *satisfies*

$$\mu < \min\left\{\frac{1+r}{e \cdot (1 + r - \theta_L)}, \frac{1+r}{1 + r - \theta_L}\right\} = \frac{1+r}{e \cdot (1 + r - \theta_L)},$$

then we have $z_H^{FB} < 0 < z_L^{FB}$.

Proof. See the Appendix. ▮

The first-best benchmark eliminates asymmetric information frictions and the divergence in patience levels between the central government and the regions. Consequently, in terms of optimal intertemporal decision-making in public policy, the IRS equals the IRT for each type. The negative intergenerational externality resulting from debt issuance increases the IRT, whereas the positive externality from the provision of intergenerational public goods decreases the IRT. The negative externality dominates, ensuring that the IRT remains positive across all regions.

Regions classified as H-regions, which supply public goods with greater durability or sustainability—meaning a higher proportion of the public good produced in period one can still be utilized in period two—should provide more of such public goods than L-regions. Failing to do so would hinder the realization of social efficiency. Regarding optimal fiscal policy, H-regions should be allocated a higher level of debt issuance and should also receive interregional fiscal transfers. This arrangement guarantees that the first-period public good supplied by H-regions exceeds that provided by L-regions. We thus show that H-regions can borrow more and benefit from interregional fiscal transfers, whereas L-regions can borrow less and contribute to interregional income redistribution. Therefore, to achieve social optimality without the distortions caused by informational and present bias frictions, the first-best optimal policy entails that debt issuance should complement interregional fiscal transfer policies.

Nevertheless, we can only guarantee the validity of this finding when a power utility function is employed, or when the negative exponential utility function is adopted, provided that certain additional restrictions are imposed on the shadow price of the government budget constraint and the relevant model parameters. For example, our analysis demonstrates that interregional fiscal transfers are unnecessary when a logarithmic (ln) utility function is adopted. Moreover, under the negative exponential utility function, if the shadow price of the government budget constraint is relatively small, then the optimal fiscal transfer system involves transferring resources from H-regions to L-regions under complete information. Consequently, if this condition is satisfied, we establish that the interregional fiscal transfer scheme and the debt issuance allocation between these two types of regions exhibit substitutability, rather than the complementarity observed under the power utility function.

4. THE ASYMMETRIC INFORMATION OPTIMUM

To emphasize the difference in levels of patience between the center and the regional governments, we exploit Proposition 1 and introduce the following assumption.

ASSUMPTION 1. *The discount factor β is sufficiently small such that it satisfies*

$$\beta < \min\left\{1, \frac{g\left(G_{1H}^{FB}\right) - g\left(G_{1L}^{FB}\right)}{g\left(\theta_H G_{1H}^{FB} + G_{2H}^{FB}\right) - g\left(\theta_L G_{1H}^{FB} + G_{2H}^{FB}\right)}\right\},$$

where

$$\frac{g\left(G_{1H}^{FB}\right) - g\left(G_{1L}^{FB}\right)}{g\left(\theta_H G_{1H}^{FB} + G_{2H}^{FB}\right) - g\left(\theta_L G_{1H}^{FB} + G_{2H}^{FB}\right)} > 0$$

is independent of β.

By solving the problem characterized by equations (4) and (5), we present the results in the following proposition. The welfare optimum under asymmetric information is highlighted by the superscript *.

PROPOSITION 2. *Suppose Assumption 1 holds true. Then, the asymmetric information optimum satisfies:*
(i) If $\lambda_H^* = 0$ and $\lambda_L^* \in (0, p_H)$, then we have

$$\underbrace{\frac{g'\left(G_{1L}^*\right)}{\beta g'\left(\theta_L G_{1L}^* + G_{2L}^*\right)}}_{IRS_L^*} = \underbrace{(1 + r - \theta_L)}_{IRT_L^*} \cdot \underbrace{\left(\frac{\frac{p_L}{\beta} + \lambda_L^*}{p_L + \lambda_L^*}\right)}_{>1},$$

$$\underbrace{1 + r - \theta_H}_{IRT_H^*} < \underbrace{\frac{g'\left(G_{1H}^*\right)}{\beta g'\left(\theta_H G_{1H}^* + G_{2H}^*\right)}}_{IRS_H^*} < \underbrace{(1 + r - \theta_H)}_{IRT_H^*} \cdot \underbrace{\left(\frac{\frac{p_H}{\beta} - \lambda_L^*}{p_H - \lambda_L^*}\right)}_{>1},$$

and $G_{1L}^* < G_{1H}^*$, $G_{2L}^* > G_{2H}^*$, $b_L^* < b_H^*$, and $z_H^* < 0 < z_L^*$. Here, λ_L^* and λ_H^* denote the optimal values of the Lagrange multipliers associated to the IC_L and IC_H constraints, respectively.

(ii) If $\lambda_L^* = 0$ *and* $\lambda_H^* \in (0, p_L)$*, then we have*

$$\underbrace{\frac{g'\left(G_{1L}^*\right)}{\beta g'\left(\theta_L G_{1L}^* + G_{2L}^*\right)}}_{IRS_L^*} > \underbrace{(1 + r - \theta_L)}_{IRT_L^*} \cdot \underbrace{\left(\frac{\frac{p_L}{\beta} - \lambda_H^*}{p_L - \lambda_H^*}\right)}_{>1},$$

$$\underbrace{\frac{g'\left(G_{1H}^*\right)}{\beta g'\left(\theta_H G_{1H}^* + G_{2H}^*\right)}}_{IRS_H^*} = \underbrace{(1 + r - \theta_H)}_{IRT_H^*} \cdot \underbrace{\left(\frac{\frac{p_H}{\beta} + \lambda_H^*}{p_H + \lambda_H^*}\right)}_{>1},$$

and $G_{1L}^* < G_{1H}^*$, $G_{2L}^* > G_{2H}^*$, $b_L^* < b_H^*$, *and* $z_H^* < 0 < z_L^*$.

Proof. See the Appendix. ▌

The presence of asymmetric information introduces additional friction, as the degree of patience of the central government (mechanism designer) differs from that of the regions (agents). Due to these frictions, it is reasonable to expect that the asymmetric information optimum may exhibit features distinct from those of the first-best optimum. As demonstrated in Proposition 2, regardless of whether the incentive to mimic the opponent type resides with the H-regions or the L-regions—resulting in the corresponding incentive compatibility constraint being binding and the associated Lagrange multiplier being positive—the H-regions should always be allocated a higher level of debt issuance than the L-regions. The underlying intuition is that H-regions generate a greater positive intergenerational externality compared to L-regions, while both types of regions impose the same level of negative intergenerational externality per unit of debt issued in the first period.

A particularly interesting aspect concerns the optimal interregional fiscal transfer policy, which diverges from the policy under complete information. In the first-best scenario, L-regions contribute to interregional fiscal transfers, while H-regions benefit from them. Conversely, under asymmetric information, H-regions contribute to fiscal transfers, and L-regions benefit from them. Consequently, in the first-best, debt issuance and fiscal transfers function as complements to achieve social optimality. However, in the presence of informational and present-bias frictions, they act as substitutes. This divergence constitutes a key contribution of this study.

To intuitively highlight the key differences and similarities in the features of welfare optimum between the cases with and without informational friction, we present Table 1. In the absence of private information, L-regions—characterized by lower sustainability in terms of local intergenerational public good provision—tend to issue less debt, resulting in a smaller borrowing capacity. Additionally, they face a form of "penalty" by transferring a portion of their resources to opposing regions. Conversely, when private information is present, L-regions can still issue less debt; however, the "penalty" is now imposed on the more sustainable H-regions. This adjustment helps address the self-selection problem caused by asymmetric information.

TABLE 1.

Optimal intertemporal and debt allocation, and fiscal transfers

	intertemporal allocation	debt allocation	fiscal transfers
first best	IRS = IRT for L-regions, IRS = IRT for H-regions	$b_L^{FB} < b_H^{FB}$	power utility: $z_L^{FB} < 0 < z_H^{FB}$
constrained optimum	IRS > IRT for L-regions, IRS > IRT for H-regions	$b_L^* < b_H^*$	$z_H^* < 0 < z_L^*$

5. CONCLUDING REMARKS

The major findings of the paper can be summarized in two points. First, regarding the allocation of debt issuance across heterogeneous regions, we find that if a region issues debt today to finance public goods that generate greater positive spillovers for future generations, then this region should be permitted to borrow more than other regions that primarily use debt to benefit the current generation. This approach ensures the maximization of positive spillovers for future generations. Second, from a central regulation perspective, the fiscal scheme of interregional fiscal transfers (or intergovernmental grants) and the mechanism for allocating debt issuance across heterogeneous regions should serve as substitutes for the regulator. Such mechanisms aim to maximize the combined welfare of both present and future generations while satisfying the balanced budget constraint and the truth-telling constraints in the presence of private information.

These two results provide the following insights and policy implications regarding the design of fiscal rules in real-world economies. First, in a federal system of government, achieving an optimal allocation of debt issuance across heterogeneous regions requires that more sustainable regions are encouraged to borrow more than less sustainable ones. Sustainability, in this context, is defined by the extent of positive intergenerational spillovers resulting from their current public goods provision. In other words, public debt should primarily be issued to fund investments in environmental protection, natural resources, basic science, free education, and human capital development, rather than being used solely for current consumption. All else being equal, the decision to issue more or less debt depends on whether the debt can generate greater or smaller positive intergenerational spillovers. The underlying intuition is straightforward: since debt and interest repayment obligations are passed on to future generations, fiscal policy design aimed at protecting those future generations must balance the associated negative and positive intergenerational spillovers to sustain an appropriate level of efficiency.

For instance, there is a growing concern and recognition in Germany that maintaining a fiscal surplus may hinder effective investment in key areas such as digitalization, infrastructure, and education (*The Economist* 2019, 2021). These sectors are critical for driving the country's green and sustainable transformation, which will benefit future generations. As a result, based on current analysis, the German government could consider running a moderate fiscal deficit. This approach could help mitigate the adverse consequences of underinvestment in these critical areas by relaxing the government debt constraint.

Second, since the central government or regulator generally lacks complete knowledge of which regions are sustainable and which are not in terms of accounting for the welfare of future generations, the implementation of the mechanism must include provisions to compensate less sustainable regions that face more stringent borrowing constraints. This approach creates incentives for these heterogeneous regions to truthfully reveal their types rather than engage in strategic misreporting. This is critical because acquiring accurate information is typically costly in real-world economies.

The substitutive relationship between the fiscal scheme of interregional transfers and the debt issuance allocation mechanism enhances the policy flexibility of the regulator. Specifically, the fiscal transfer scheme addresses the self-selection problem, while the debt issuance mechanism is employed to finance sufficient intergenerational public goods, thereby promoting social optimality. In other words, fiscally compensating less sustainable regions through smaller debt issuance not only helps prevent misreporting or cheating during regulation but also ensures that the debt issuance allocation—where more sustainable regions are allocated higher debt issuance—is socially optimal for safeguarding future generations. If fiscal transfers across regions are not feasible, the regulator must incur additional costs to obtain information about regional types. Without such efforts, the distribution of debt issuance among heterogeneous regions could become inefficient.

APPENDIX A

Proof of Equation (7)

Differentiating (6) with respect to b yields:

$$\begin{aligned}\left.\frac{\mathrm{d}^2 z}{\mathrm{d}b^2}\right|_{\mathrm{d}V=0} = & \left(\frac{1}{g_1' + \beta\theta g_2'}\right) \cdot \left[\beta\left(1+r-\theta\right) g_2''\right] \cdot \left[-\left(1+r-\theta\right) + \theta \left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{\mathrm{d}V=0}\right] \\ & - \left(\frac{1}{g_1' + \beta\theta g_2'}\right) \cdot g_1'' \cdot \left(1 + \left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{\mathrm{d}V=0}\right) \\ & - \left[\frac{\beta\left(1+r-\theta\right) g_2' - g_1'}{\left(g_1' + \beta\theta g_2'\right)^2}\right] \cdot g_1'' \cdot \left(1 + \left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{\mathrm{d}V=0}\right) \\ & - \left[\frac{\beta\left(1+r-\theta\right) g_2' - g_1'}{\left(g_1' + \beta\theta g_2'\right)^2}\right] \cdot \beta\theta g_2'' \cdot \left[-\left(1+r-\theta\right) + \theta \left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{\mathrm{d}V=0}\right],\end{aligned} \tag{A.1}$$

where $g_1' \equiv g'(G_1)$ and $g_2' \equiv g'(\theta G_1 + G_2)$ are used to simplify the notation. Rearranging (A.1) gives

$$\begin{aligned}& \left(g_1' + \beta\theta g_2'\right)^2 \cdot \left.\frac{\mathrm{d}^2 z}{\mathrm{d}b^2}\right|_{\mathrm{d}V=0} \\ = \ & \left(g_1' + \beta\theta g_2'\right)\left\{\beta\left(1+r-\theta\right) g_2'' \cdot \left[\theta \cdot \left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{\mathrm{d}V=0} - \left(1+r-\theta\right)\right]\right\} \\ & - \left(g_1' + \beta\theta g_2'\right) \cdot g_1'' \cdot \left(1 + \left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{\mathrm{d}V=0}\right) \\ & + \left[g_1' - \beta\left(1+r-\theta\right) g_2'\right]\left\{\beta\theta g_2'' \cdot \left[\theta \cdot \left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{\mathrm{d}V=0} - \left(1+r-\theta\right)\right]\right\} \\ & + \left[g_1' - \beta\left(1+r-\theta\right) g_2'\right] \cdot g_1'' \cdot \left(1 + \left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{\mathrm{d}V=0}\right) \\ = \ & \beta\left(1+r\right) g_1' \cdot g_2'' \cdot \left[\theta \cdot \left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{\mathrm{d}V=0} - \left(1+r-\theta\right)\right] \\ & - \beta\left(1+r\right) g_2' \cdot g_1'' \cdot \left(1 + \left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{\mathrm{d}V=0}\right).\end{aligned} \tag{A.2}$$

By substituting (6) into (A.2) and then collecting the terms, we obtain

$$\underbrace{\frac{\left(g_1' + \beta\theta g_2'\right)^2}{\beta\left(1+r\right)}}_{+} \cdot \left.\frac{\mathrm{d}^2 z}{\mathrm{d}b^2}\right|_{\mathrm{d}V=0} = -\frac{\left(1+r\right)\left(g_1'\right)^2 g_2''}{g_1' + \beta\theta g_2'} - \frac{\beta\left(1+r\right)\left(g_2'\right)^2 g_1''}{g_1' + \beta\theta g_2'} > 0,$$

which thus confirms the desired assertion. ▮

Proof of Equation (9)
Differentiating (6) with respect to θ yields:

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{\mathrm{d}V=0}\right) = \left(\frac{1}{g_1' + \beta\theta g_2'}\right) \cdot \left\{-\beta g_2' + \beta\left(1+r-\theta\right)\left(b+z\right) g_2''\right\}$$
$$- \left[\frac{\beta\left(1+r-\theta\right) g_2' - g_1'}{\left(g_1' + \beta\theta g_2'\right)^2}\right] \cdot \left\{\beta g_2' + \beta\theta\left(b+z\right) g_2''\right\}.$$

Then, we get

$$\begin{aligned}
&\underbrace{\left(g_1' + \beta\theta g_2'\right)^2}_{+} \cdot \frac{\mathrm{d}}{\mathrm{d}\theta}\left(\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{\mathrm{d}V=0}\right) \\
&= \left(g_1' + \beta\theta g_2'\right)\left\{-\beta g_2' + \beta\left(1+r-\theta\right)\left(b+z\right) g_2''\right\} \\
&\quad + \left[g_1' - \beta\left(1+r-\theta\right) g_2'\right]\left\{\beta g_2' + \beta\theta\left(b+z\right) g_2''\right\} \\
&= -\left(g_1' + \beta\theta g_2'\right)\left[\beta g_2' + \beta\theta\left(b+z\right) g_2''\right] \\
&\quad + \left(g_1' + \beta\theta g_2'\right)\beta\left(1+r\right)\left(b+z\right) g_2'' \\
&\quad + \left(g_1' + \beta\theta g_2'\right) \cdot \left[\beta g_2' + \beta\theta\left(b+z\right) g_2''\right] \\
&\quad - \beta\left(1+r\right) g_2' \cdot \left[\beta g_2' + \beta\theta\left(b+z\right) g_2''\right] \\
&= \beta\left(1+r\right)\left(b+z\right) g_1' g_2'' - \beta^2\left(1+r\right)\left(g_2'\right)^2 < 0,
\end{aligned}$$

as desired. ∎

Proof of Proposition 1
We shall complete the proof in 4 steps.
Step 1: The Lagrange function of problem (10) is

$$\mathcal{L}_0 = p_L \cdot W\left(b_L, z_L; \theta_L\right) + p_H \cdot W\left(b_H, z_H; \theta_H\right) - \mu \cdot \left(p_L z_L + p_H z_H\right),$$

where μ denotes the nonnegative Lagrange multiplier associated to the fiscal budget constraint. Assuming an interior solution, then the first-order conditions (FOCs) are

$$\begin{aligned}
\frac{\partial \mathcal{L}_0}{\partial b_L} &= p_L \cdot W_b\left(b_L, z_L; \theta_L\right) = 0, \\
\frac{\partial \mathcal{L}_0}{\partial z_L} &= p_L \cdot W_z\left(b_L, z_L; \theta_L\right) - \mu p_L = 0, \\
\frac{\partial \mathcal{L}_0}{\partial b_H} &= p_H \cdot W_b\left(b_H, z_H; \theta_H\right) = 0, \\
\frac{\partial \mathcal{L}_0}{\partial z_H} &= p_H \cdot W_z\left(b_H, z_H; \theta_H\right) - \mu p_H = 0.
\end{aligned} \tag{A.3}$$

By substituting the definition of W into (A.3), we get

$$b_L^{FB}: \quad g'(G_{1L}) - (1+r-\theta_L) \cdot g'(\theta_L G_{1L} + G_{2L}) = 0, \tag{A.4}$$
$$z_L^{FB}: \quad g'(G_{1L}) + \theta_L g'(\theta_L G_{1L} + G_{2L}) - \mu = 0, \tag{A.5}$$
$$b_H^{FB}: \quad g'(G_{1H}) - (1+r-\theta_H) g'(\theta_H G_{1H} + G_{2H}) = 0, \tag{A.6}$$

and

$$z_H^{FB}: \quad g'(G_{1H}) + \theta_H g'(\theta_H G_{1H} + G_{2H}) - \mu = 0. \tag{A.7}$$

By plugging (A.5) in (A.4) and (A.7) in (A.6), we get

$$\begin{aligned} g'(\theta_L G_{1L} + G_{2L}) &= \frac{\mu}{1+r}, \\ g'(\theta_H G_{1H} + G_{2H}) &= \frac{\mu}{1+r}. \end{aligned} \tag{A.8}$$

Using (A.8) yields $\theta_L G_{1L} + G_{2L} = \theta_H G_{1H} + G_{2H}$. Then, combining (A.4) with (A.6) gives

$$\frac{g'(G_{1L})}{g'(G_{1H})} = \frac{1+r-\theta_L}{1+r-\theta_H} > 1,$$

which implies that $G_{1L} < G_{1H}$. This result, combined with the relation

$$\theta_L G_{1L} + G_{2L} = \theta_H G_{1H} + G_{2H},$$

and the inequality $0 < \theta_L < \theta_H$, implies that

$$G_{2L} > G_{2H}.$$

Consequently, we have

$$b_H > b_L.$$

Step 2: Suppose $g(\cdot) = \ln(\cdot)$, then using (A.4) yields the following:

$$\begin{aligned} & \frac{1}{G_{1L}} = \frac{1+r-\theta_L}{\theta_L G_{1L} + G_{2L}} \\ \Longleftrightarrow \quad & \tau = 2(1+r-\theta_L) b_L + (1+r-2\theta_L) z_L. \end{aligned} \tag{A.9}$$

It follows from (A.8) that

$$\begin{aligned} & \theta_L G_{1L} + G_{2L} = \frac{1+r}{\mu} \\ \Longleftrightarrow \quad & \theta_L (b_L + z_L) + \tau - (1+r) b_L = \frac{1+r}{\mu} \\ \Longleftrightarrow \quad & \tau + \theta_L z_L - \frac{1+r}{\mu} = (1+r-\theta_L) b_L. \end{aligned} \tag{A.10}$$

Applying (A.10) to (A.9) yields

$$\begin{aligned} \tau &= 2\tau + 2\theta_L z_L - 2\left(\frac{1+r}{\mu}\right) + (1+r-2\theta_L)\, z_L \\ \iff \quad 2\left(\frac{1+r}{\mu}\right) - \tau &= (1+r)\cdot z_L. \end{aligned} \tag{A.11}$$

By symmetry, we have

$$2\left(\frac{1+r}{\mu}\right) - \tau = (1+r)\cdot z_H. \tag{A.12}$$

Thus, combining (A.11) with (A.12) reveals that $z_L = z_H$. Given $p_L z_L + p_H z_H = 0$, we must have $z_L = z_H = 0$, as desired.

Step 3: Suppose $g(\cdot) = (\cdot)^\alpha$ for the parameter $\alpha \in (0,1)$. Then using (A.4) again gives

$$\theta_L G_{1L} + G_{2L} = (1+r-\theta_L)^{\frac{1}{1-\alpha}} \cdot G_{1L}. \tag{A.13}$$

Applying this utility function to (A.8) gives

$$\theta_L G_{1L} + G_{2L} = \left[\frac{\alpha\,(1+r)}{\mu}\right]^{\frac{1}{1-\alpha}}. \tag{A.14}$$

Combining (A.13) with (A.14) shows

$$\begin{aligned} G_{1L} &= \left(\frac{1}{1+r-\theta_L}\right)^{\frac{1}{1-\alpha}} \cdot \left[\frac{\alpha\,(1+r)}{\mu}\right]^{\frac{1}{1-\alpha}}, \\ G_{2L} &= \left[\frac{\alpha\,(1+r)}{\mu}\right]^{\frac{1}{1-\alpha}} - \theta_L \left(\frac{1}{1+r-\theta_L}\right)^{\frac{1}{1-\alpha}} \left[\frac{\alpha\,(1+r)}{\mu}\right]^{\frac{1}{1-\alpha}}. \end{aligned} \tag{A.15}$$

By symmetry, we obtain

$$\begin{aligned} G_{1H} &= \left(\frac{1}{1+r-\theta_H}\right)^{\frac{1}{1-\alpha}} \cdot \left[\frac{\alpha\,(1+r)}{\mu}\right]^{\frac{1}{1-\alpha}}, \\ G_{2H} &= \left[\frac{\alpha\,(1+r)}{\mu}\right]^{\frac{1}{1-\alpha}} - \theta_H \left(\frac{1}{1+r-\theta_H}\right)^{\frac{1}{1-\alpha}} \left[\frac{\alpha\,(1+r)}{\mu}\right]^{\frac{1}{1-\alpha}}. \end{aligned} \tag{A.16}$$

Thus, combining (A.15) with (A.16) yields

$$G_{1L} - G_{1H} = \underbrace{\left[\frac{\alpha\,(1+r)}{\mu}\right]^{\frac{1}{1-\alpha}}}_{+} \cdot \underbrace{\left[\left(\frac{1}{1+r-\theta_L}\right)^{\frac{1}{1-\alpha}} - \left(\frac{1}{1+r-\theta_H}\right)^{\frac{1}{1-\alpha}}\right]}_{-}. \tag{A.17}$$

Additionally, using (A.15)-(A.16) yields the following:

$$b_H - b_L = \left(\frac{1}{1+r}\right)(G_{2L} - G_{2H}), \tag{A.18}$$

where

$$\begin{aligned} G_{2L} - G_{2H} &= \left[\frac{\alpha(1+r)}{\mu}\right]^{\frac{1}{1-\alpha}} \cdot \left[1 - \theta_L \left(\frac{1}{1+r-\theta_L}\right)^{\frac{1}{1-\alpha}}\right] \\ &\quad - \left[\frac{\alpha(1+r)}{\mu}\right]^{\frac{1}{1-\alpha}} \cdot \left[1 - \theta_H \left(\frac{1}{1+r-\theta_H}\right)^{\frac{1}{1-\alpha}}\right] \\ &= \left[\frac{\alpha(1+r)}{\mu}\right]^{\frac{1}{1-\alpha}} \cdot \left[\theta_H \left(\frac{1}{1+r-\theta_H}\right)^{\frac{1}{1-\alpha}} - \theta_L \left(\frac{1}{1+r-\theta_L}\right)^{\frac{1}{1-\alpha}}\right]. \end{aligned} \tag{A.19}$$

Consequently, combining (A.17) with (A.18)-(A.19) yields:

$$\begin{aligned} z_L - z_H &= G_{1L} - G_{1H} + b_H - b_L \\ &= \left[\frac{\alpha(1+r)}{\mu}\right]^{\frac{1}{1-\alpha}} \cdot \left[\left(\frac{1}{1+r-\theta_L}\right)^{\frac{1}{1-\alpha}} - \left(\frac{1}{1+r-\theta_H}\right)^{\frac{1}{1-\alpha}}\right] \\ &\quad + \left[\frac{\alpha(1+r)}{\mu}\right]^{\frac{1}{1-\alpha}} \left[\left(\frac{\theta_H}{1+r}\right)\left(\frac{1}{1+r-\theta_H}\right)^{\frac{1}{1-\alpha}} - \left(\frac{\theta_L}{1+r}\right)\left(\frac{1}{1+r-\theta_L}\right)^{\frac{1}{1-\alpha}}\right] \\ &= \left[\frac{\alpha(1+r)}{\mu}\right]^{\frac{1}{1-\alpha}} \cdot \left(\frac{1+r-\theta_L}{1+r}\right) \cdot \left(\frac{1}{1+r-\theta_L}\right)^{\frac{1}{1-\alpha}} \\ &\quad - \left[\frac{\alpha(1+r)}{\mu}\right]^{\frac{1}{1-\alpha}} \cdot \left(\frac{1+r-\theta_H}{1+r}\right) \cdot \left(\frac{1}{1+r-\theta_H}\right)^{\frac{1}{1-\alpha}} \\ &= \underbrace{\left(\frac{1}{1+r}\right)\left[\frac{\alpha(1+r)}{\mu}\right]^{\frac{1}{1-\alpha}}}_{+} \cdot \underbrace{\left[\left(\frac{1}{1+r-\theta_L}\right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{1}{1+r-\theta_H}\right)^{\frac{\alpha}{1-\alpha}}\right]}_{-}. \end{aligned}$$

We thus have $z_L < 0 < z_H$, as desired.

Step 4: Suppose $g(x) = -\frac{1}{\gamma} \cdot e^{-\gamma x}$ for the constant $\gamma > 0$. Then, applying this functional form to equation (A.4) yields:

$$\theta_L G_{1L} + G_{2L} - G_{1L} = \frac{1}{\gamma} \ln(1 + r - \theta_L). \tag{A.20}$$

Similarly, using equation (A.8) yields:

$$\theta_L G_{1L} + G_{2L} = \frac{1}{\gamma} \cdot \ln\left(\frac{1+r}{\mu}\right). \tag{A.21}$$

Solving the equation system of (A.20)-(A.21) gives

$$\begin{aligned} G_{1L} &= \frac{1}{\gamma} \cdot \ln\left(\frac{1+r}{(1+r-\theta_L)\cdot\mu}\right), \\ G_{2L} &= \frac{1}{\gamma} \cdot \ln\left(\frac{1+r}{\mu}\right) - \left(\frac{\theta_L}{\gamma}\right) \cdot \ln\left(\frac{1+r}{(1+r-\theta_L)\cdot\mu}\right). \end{aligned} \tag{A.22}$$

Symmetrically, we can obtain

$$\begin{aligned} G_{1H} &= \frac{1}{\gamma} \cdot \ln\left(\frac{1+r}{(1+r-\theta_H)\cdot\mu}\right), \\ G_{2H} &= \frac{1}{\gamma} \ln\left(\frac{1+r}{\mu}\right) - \left(\frac{\theta_H}{\gamma}\right) \cdot \ln\left(\frac{1+r}{(1+r-\theta_H)\cdot\mu}\right). \end{aligned} \tag{A.23}$$

By substituting equations (A.22) and (A.23) into the public budget constraints faced by the regional governments across the two periods, we obtain

$$\begin{aligned} z_L - z_H &= G_{1L} - G_{1H} + \left(\frac{1}{1+r}\right) \cdot (G_{2L} - G_{2H}) \\ &= \frac{1}{\gamma} \ln\left(\frac{1+r}{(1+r-\theta_L)\cdot\mu}\right) - \frac{1}{\gamma} \ln\left(\frac{1+r}{(1+r-\theta_H)\cdot\mu}\right) \\ &\quad + \left(\frac{1}{1+r}\right) \cdot \frac{1}{\gamma} \cdot \left[\theta_H \ln\left(\frac{1+r}{(1+r-\theta_H)\cdot\mu}\right) - \theta_L \cdot \ln\left(\frac{1+r}{(1+r-\theta_L)\cdot\mu}\right)\right] \\ &= \left[\frac{1+r-\theta_L}{\gamma(1+r)}\right] \cdot \ln\left(\frac{1+r}{(1+r-\theta_L)\cdot\mu}\right) - \left[\frac{1+r-\theta_H}{\gamma(1+r)}\right] \cdot \ln\left(\frac{1+r}{(1+r-\theta_H)\cdot\mu}\right) \\ &= \frac{1}{\gamma(1+r)} \left[\Phi(\theta_L) - \Phi(\theta_H)\right], \end{aligned} \tag{A.24}$$

where

$$\Phi(\theta) \equiv (1+r-\theta) \cdot \ln\left(\frac{1+r}{(1+r-\theta)\,\mu}\right). \tag{A.25}$$

By differentiating (A.25) with respect to θ, we obtain

$$\Phi'(\theta) > 0 \iff \mu > \frac{1+r}{e\,(1+r-\theta)} \tag{A.26}$$

and

$$\Phi'(\theta) < 0 \iff \mu < \frac{1+r}{e\,(1+r-\theta)}, \tag{A.27}$$

where $e \approx 2.71828$ represents the well-known natural constant. Based on equations (A.22) and (A.23), the following condition must be satisfied:

$$\mu < \min\left\{\frac{1+r}{1+r-\theta_H}, \frac{1+r}{1+r-\theta_L}\right\} = \frac{1+r}{1+r-\theta_L}. \tag{A.28}$$

Consequently, by applying equations (A.26)-(A.28) to (A.24), the desired results can be established. ∎

Proof of Proposition2

We shall complete the proof in 3 steps.

Step 1: The Lagrange function is now written as

$$\begin{aligned}\mathcal{L} = \;& p_L \cdot W\left(b_L, z_L; \theta_L\right) + p_H W\left(b_H, z_H; \theta_H\right) - \mu\left(p_L z_L + p_H z_H\right) \\ & + \lambda_L\left[V\left(b_L, z_L; \theta_L, \beta\right) - V\left(b_H, z_H; \theta_L, \beta\right)\right] \\ & + \lambda_H\left[V\left(b_H, z_H; \theta_H, \beta\right) - V\left(b_L, z_L; \theta_H, \beta\right)\right],\end{aligned} \tag{A.29}$$

where $\lambda_L \geq 0$ and $\lambda_H \geq 0$ denote Lagrange multipliers associated to the constraints IC_L and IC_H, respectively.

We first consider the special case where both IC_L and IC_H are not binding at the optimal solution. Then, the complementary slackness conditions imply that $\lambda_L = \lambda_H = 0$. Consequently, the Lagrangian (A.29) reduces to the Lagrangian $\mathcal{L}_0$ introduced in the proof of Proposition 1. In other words, the corresponding solution is characterized by the features described in Proposition 1.

Next, we need to verify whether the full-information optimum violates the IC constraint(s). To do so, we evaluate IC_L at the full-information optimum, which yields:

$$\begin{aligned}& V\left(b_L^{FB}, z_L^{FB}; \theta_L, \beta\right) - V\left(b_H^{FB}, z_H^{FB}; \theta_L, \beta\right) \\ =& g\left(G_{1L}^{FB}\right) + \beta g\left(\theta_L G_{1L}^{FB} + G_{2L}^{FB}\right) \\ & - \left[g\left(G_{1H}^{FB}\right) + \beta g\left(\theta_L G_{1H}^{FB} + G_{2H}^{FB}\right)\right] \\ =& \underbrace{g\left(G_{1L}^{FB}\right) - g\left(G_{1H}^{FB}\right)}_{-} + \beta \underbrace{\left[g\left(\theta_H G_{1H}^{FB} + G_{2H}^{FB}\right) - g\left(\theta_L G_{1H}^{FB} + G_{2H}^{FB}\right)\right]}_{+},\end{aligned} \tag{A.30}$$

where we have used the fact that $\theta_L G_{1L}^{FB} + G_{2L}^{FB} = \theta_H G_{1H}^{FB} + G_{2H}^{FB}$. Further exploiting (A.30) shows

$$\begin{aligned}
& V\left(b_L^{FB}, z_L^{FB}; \theta_L, \beta\right) < V\left(b_H^{FB}, z_H^{FB}; \theta_L, \beta\right) \\
\Longleftrightarrow \quad & \beta \cdot \underbrace{\left[g\left(\theta_H G_{1H}^{FB} + G_{2H}^{FB}\right) - g\left(\theta_L G_{1H}^{FB} + G_{2H}^{FB}\right)\right]}_{+} < \underbrace{g\left(G_{1H}^{FB}\right) - g\left(G_{1L}^{FB}\right)}_{+} \\
\Longleftrightarrow \quad & \beta < \frac{g\left(G_{1H}^{FB}\right) - g\left(G_{1L}^{FB}\right)}{g\left(\theta_H G_{1H}^{FB} + G_{2H}^{FB}\right) - g\left(\theta_L G_{1H}^{FB} + G_{2H}^{FB}\right)},
\end{aligned}$$

which is guaranteed by Assumption 1. Thus, the IC_L constraint is violated. Therefore, either IC_L or IC_H must be binding at the optimum in the context of asymmetric information.

Step 2: Suppose IC_L is binding while IC_H is not. Then, using the complementary slackness conditions yields $\lambda_L > 0$ and $\lambda_H = 0$. The Lagrange function (A.29) is written as

$$\begin{aligned}
\mathcal{L}_L = {} & p_L \cdot W\left(b_L, z_L; \theta_L\right) + p_H W\left(b_H, z_H; \theta_H\right) - \mu\left(p_L z_L + p_H z_H\right) \\
& + \lambda_L\left[V\left(b_L, z_L; \theta_L, \beta\right) - V\left(b_H, z_H; \theta_L, \beta\right)\right].
\end{aligned}$$

Assuming the existence of an interior solution, the FOCs are

$$\begin{aligned}
\frac{\partial \mathcal{L}_L}{\partial b_L} &= p_L W_b\left(b_L, z_L; \theta_L\right) + \lambda_L V_b\left(b_L, z_L; \theta_L, \beta\right) = 0, \\
\frac{\partial \mathcal{L}_L}{\partial z_L} &= p_L W_z\left(b_L, z_L; \theta_L\right) + \lambda_L V_z\left(b_L, z_L; \theta_L, \beta\right) - \mu p_L = 0, \\
\frac{\partial \mathcal{L}_L}{\partial b_H} &= p_H W_b\left(b_H, z_H; \theta_H\right) - \lambda_L V_b\left(b_H, z_H; \theta_L, \beta\right) = 0, \\
\frac{\partial \mathcal{L}_L}{\partial z_H} &= p_H W_z\left(b_H, z_H; \theta_H\right) - \lambda_L V_z\left(b_H, z_H; \theta_L, \beta\right) - \mu p_H = 0.
\end{aligned} \tag{A.31}$$

Applying the definitions of W and V to (A.31) gives:

$$b_L^* : \quad \left(p_L + \lambda_L\right) g'\left(G_{1L}\right) - \left(p_L + \beta\lambda_L\right)\left(1 + r - \theta_L\right) g'\left(\theta_L G_{1L} + G_{2L}\right) = 0, \tag{A.32}$$

$$z_L^* : \quad \left(p_L + \lambda_L\right) g'\left(G_{1L}\right) + \left(p_L + \beta\lambda_L\right)\theta_L g'\left(\theta_L G_{1L} + G_{2L}\right) = \mu p_L, \tag{A.33}$$

$$\begin{aligned}
b_H^* : \quad & \left(p_H - \lambda_L\right) g'\left(G_{1H}\right) - p_H\left(1 + r - \theta_H\right) g'\left(\theta_H G_{1H} + G_{2H}\right) \\
& + \beta\lambda_L\left(1 + r - \theta_L\right) g'\left(\theta_L G_{1H} + G_{2H}\right) = 0,
\end{aligned} \tag{A.34}$$

and

$$\begin{aligned}
z_H^* : \quad & \left(p_H - \lambda_L\right) g'\left(G_{1H}\right) + p_H \theta_H g'\left(\theta_H G_{1H} + G_{2H}\right) \\
& - \beta\lambda_L \theta_L g'\left(\theta_L G_{1H} + G_{2H}\right) = \mu p_H.
\end{aligned} \tag{A.35}$$

By substituting (A.33) into (A.32), we obtain

$$(p_L + \beta\lambda_L)(1+r)\, g'(\theta_L G_{1L} + G_{2L}) = \mu p_L. \tag{A.36}$$

Similarly, by substituting (A.35) into (A.34), we have

$$\begin{aligned} & p_H (1+r)\, g'(\theta_H G_{1H} + G_{2H}) \\ = \;& \beta\lambda_L (1+r)\, g'(\theta_L G_{1H} + G_{2H}) + \mu p_H \\ > \;& \beta\lambda_L (1+r)\, g'(\theta_H G_{1H} + G_{2H}) + \mu p_H, \end{aligned}$$

which implies that

$$(p_H - \beta\lambda_L)(1+r)\, g'(\theta_H G_{1H} + G_{2H}) > \mu p_H. \tag{A.37}$$

Then, we must impose the following condition:

$$p_H > \beta\lambda_L. \tag{A.38}$$

Given (A.38), then dividing (A.37) by (A.36) leads to

$$\begin{aligned} & \left(\frac{p_H - \beta\lambda_L}{p_L + \beta\lambda_L}\right) \cdot \left[\frac{g'(\theta_H G_{1H} + G_{2H})}{g'(\theta_L G_{1L} + G_{2L})}\right] > \frac{p_H}{p_L} \\ \Longleftrightarrow \;& \frac{g'(\theta_H G_{1H} + G_{2H})}{g'(\theta_L G_{1L} + G_{2L})} > \frac{p_L p_H + \beta\lambda_L p_H}{p_L p_H - \beta\lambda_L p_L} > 1, \end{aligned}$$

which yields

$$\theta_H G_{1H}^* + G_{2H}^* < \theta_L G_{1L}^* + G_{2L}^*. \tag{A.39}$$

Given that IC_L is binding, then using (A.39) gives

$$\begin{aligned} 0 &= V(b_L^*, z_L^*; \theta_L, \beta) - V(b_H^*, z_H^*; \theta_L, \beta) \\ &= g(G_{1L}^*) - g(G_{1H}^*) + \beta\underbrace{[g(\theta_L G_{1L}^* + G_{2L}^*) - g(\theta_L G_{1H}^* + G_{2H}^*)]}_{+}, \end{aligned}$$

which thus implies that $G_{1L}^* < G_{1H}^*$. By combining this result with (A.39), we have $G_{2L}^* > G_{2H}^*$. Thus, $b_H^* > b_L^*$ follows accordingly.

Furthermore, it follows from (A.34) that

$$\begin{aligned} & (p_H - \lambda_L)\, g'(G_{1H}) - p_H (1 + r - \theta_H)\, g'(\theta_H G_{1H} + G_{2H}) \\ & = -\beta\lambda_L (1 + r - \theta_L)\, g'(\theta_L G_{1H} + G_{2H}) \\ & < -\beta\lambda_L (1 + r - \theta_H)\, g'(\theta_H G_{1H} + G_{2H}), \end{aligned}$$

which then yields

$$\left(p_H - \lambda_L\right) g'\left(G_{1H}\right) < \left(p_H - \beta\lambda_L\right)\left(1 + r - \theta_H\right) g'\left(\theta_H G_{1H} + G_{2H}\right). \tag{A.40}$$

Put

$$p_H > \lambda_L, \tag{A.41}$$

then using (A.40) gives

$$\frac{g'\left(G^*_{1H}\right)}{\beta\left(1 + r - \theta_H\right) g'\left(\theta_H G^*_{1H} + G^*_{2H}\right)} < \underbrace{\frac{\frac{p_H}{\beta} - \lambda^*_L}{p_H - \lambda^*_L}}_{>1}. \tag{A.42}$$

Similarly, rearranging (A.32) gives

$$\frac{g'\left(G^*_{1L}\right)}{\beta\left(1 + r - \theta_L\right) g'\left(\theta_L G^*_{1L} + G^*_{2L}\right)} = \frac{\frac{p_L}{\beta} + \lambda^*_L}{p_L + \lambda^*_L} > 1, \tag{A.43}$$

which, combined with the slope of the indifference curve of L-regions, implies that (b^*_L, z^*_L) lies on the decreasing part of the indifference curve of L-regions.

FIG. 1. The case with $g'(G^*_{1H}) < \beta(1 + r - \theta_H)g'\left(\theta_H G^*_{1H} + G^*_{2H}\right)$.

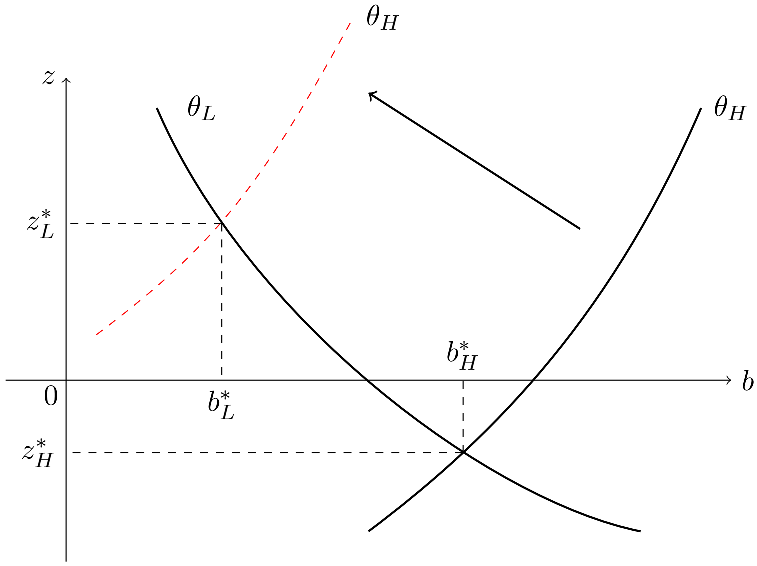
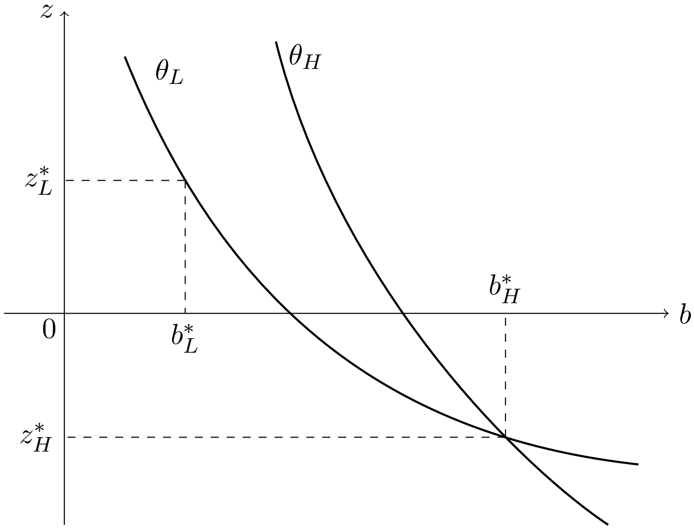


FIG. 2. The case with binding IC_L.



If we put

$$\frac{g'\left(G_{1H}^*\right)}{\beta\left(1+r-\theta_H\right)g'\left(\theta_H G_{1H}^*+G_{2H}^*\right)}<1$$

in (A.42), then (b_H^*, z_H^*) lies on the increasing part of the indifference curve of H-regions. Given the single-crossing property specified by equation (9), the relationship between a typical L-type indifference curve and a typical H-type indifference curve is illustrated in Figure 1. It is obvious that the IC_H constraint is violated in this case. That is, the case shown by Figure 1 is invalid.

Therefore, we have

$$1\leq\frac{g'\left(G_{1H}^*\right)}{\beta\left(1+r-\theta_H\right)g'\left(\theta_H G_{1H}^*+G_{2H}^*\right)}<\frac{\frac{p_H}{\beta}-\lambda_L^*}{p_H-\lambda_L^*}. \tag{A.44}$$

Then, by combining (A.43) and (A.44), the relationship between (b_L^*, z_L^*) and (b_H^*, z_H^*) can be illustrated using Figure 2.

Step 3: Suppose IC_H is binding while IC_L is not, then we have $\lambda_H>0$ and $\lambda_L=0$. The Lagrange function (A.29) is rewritten as

$$\begin{aligned}\mathcal{L}_H=&\ p_L W\left(b_L,z_L;\theta_L\right)+p_H W\left(b_H,z_H;\theta_H\right)-\mu\left(p_L z_L+p_H z_H\right)\\&+\lambda_H\left[V\left(b_H,z_H;\theta_H,\beta\right)-V\left(b_L,z_L;\theta_H,\beta\right)\right].\end{aligned}$$

Assuming the existence of an interior solution, the FOCs are given by

$$\begin{aligned}\frac{\partial \mathcal{L}_H}{\partial b_L} &= p_L W_b\left(b_L, z_L; \theta_L\right) - \lambda_H V_b\left(b_L, z_L; \theta_H, \beta\right) = 0,\\ \frac{\partial \mathcal{L}_H}{\partial z_L} &= p_L W_z\left(b_L, z_L; \theta_L\right) - \lambda_H V_z\left(b_L, z_L; \theta_H, \beta\right) - \mu p_L = 0, \qquad \text{(A.45)}\\ \frac{\partial \mathcal{L}_H}{\partial b_H} &= p_H W_b\left(b_H, z_H; \theta_H\right) + \lambda_H V_b\left(b_H, z_H; \theta_H, \beta\right) = 0,\end{aligned}$$

and

$$\frac{\partial \mathcal{L}_H}{\partial z_H} = p_H W_z\left(b_H, z_H; \theta_H\right) + \lambda_H V_z\left(b_H, z_H; \theta_H, \beta\right) - \mu p_H = 0. \qquad \text{(A.46)}$$

Applying the definitions of W and V to (A.45) and (A.46) gives

$$\begin{aligned}b_L^*: \quad & \left(p_L - \lambda_H\right) g'\left(G_{1L}\right) - p_L \cdot \left(1 + r - \theta_L\right) g'\left(\theta_L G_{1L} + G_{2L}\right)\\ & + \lambda_H \cdot \beta\left(1 + r - \theta_H\right) g'\left(\theta_H G_{1L} + G_{2L}\right) = 0, \qquad \text{(A.47)}\\ z_L^*: \quad & \left(p_L - \lambda_H\right) g'\left(G_{1L}\right) + p_L \theta_L g'\left(\theta_L G_{1L} + G_{2L}\right)\\ & - \lambda_H \beta \theta_H g'\left(\theta_H G_{1L} + G_{2L}\right) = \mu p_L, \qquad \text{(A.48)}\\ b_H^*: \quad & \left(p_H + \lambda_H\right) g'\left(G_{1H}\right) - \left(p_H + \lambda_H \beta\right)\left(1 + r - \theta_H\right) g'\left(\theta_H G_{1H} + G_{2H}\right) = 0, \qquad \text{(A.49)}\end{aligned}$$

and

$$z_H^*: \quad \left(p_H + \lambda_H\right) g'\left(G_{1H}\right) + \left(p_H + \lambda_H \beta\right) \theta_H g'\left(\theta_H G_{1H} + G_{2H}\right) = \mu p_H. \qquad \text{(A.50)}$$

Assuming $p_L > \lambda_H$, then rearranging (A.47) yields

$$\begin{aligned}\left(p_L - \lambda_H\right) g'\left(G_{1L}\right) - p_L\left(1 + r - \theta_L\right) g'\left(\theta_L G_{1L} + G_{2L}\right)\\ = -\lambda_H \beta\left(1 + r - \theta_H\right) g'\left(\theta_H G_{1L} + G_{2L}\right)\\ > -\lambda_H \beta\left(1 + r - \theta_L\right) g'\left(\theta_L G_{1L} + G_{2L}\right),\end{aligned}$$

which thus leads to

$$\frac{g'\left(G_{1L}^*\right)}{\beta\left(1 + r - \theta_L\right) g'\left(\theta_L G_{1L}^* + G_{2L}^*\right)} > \frac{\frac{p_L}{\beta} - \lambda_H^*}{p_L - \lambda_H^*} > 1. \qquad \text{(A.51)}$$

Then, (A.51) implies that (b_L^*, z_L^*) lies on the decreasing part of the indifference curve of L-regions.

By substituting (A.48) into (A.47), we obtain

$$(p_L - \lambda_H \beta)(1+r)\, g'(\theta_L G_{1L} + G_{2L}) < \mu p_L. \tag{A.52}$$

Rearranging (A.49) gives

$$\frac{g'(G_{1H}^*)}{\beta(1+r-\theta_H)\, g'(\theta_H G_{1H}^* + G_{2H}^*)} = \frac{\frac{p_H}{\beta} + \lambda_H^*}{p_H + \lambda_H^*} > 1, \tag{A.53}$$

which implies that (b_H^*, z_H^*) lies on the decreasing part of the indifference curve of H-regions.

Furthermore, by plugging (A.50) in (A.49), we get

$$(p_H + \lambda_H \beta)(1+r)\, g'(\theta_H G_{1H} + G_{2H}) = \mu p_H. \tag{A.54}$$

Dividing (A.52) by (A.34) gives

$$\frac{g'(\theta_L G_{1L}^* + G_{2L}^*)}{g'(\theta_H G_{1H}^* + G_{2H}^*)} < \underbrace{\left(\frac{p_L}{p_H}\right) \cdot \left(\frac{p_H + \lambda_H \beta}{p_L - \lambda_H \beta}\right)}_{>1}. \tag{A.55}$$

Dividing (A.51) by (A.53) gives

$$\left[\frac{g'(G_{1L}^*)}{g'(G_{1H}^*)}\right] \cdot \left(\frac{1+r-\theta_H}{1+r-\theta_L}\right) \cdot \left[\frac{g'(\theta_H G_{1H}^* + G_{2H}^*)}{g'(\theta_L G_{1L}^* + G_{2L}^*)}\right] > \left(\frac{\frac{p_L}{\beta} - \lambda_H}{p_L - \lambda_H}\right) \cdot \left(\frac{p_H + \lambda_H}{\frac{p_H}{\beta} + \lambda_H}\right)$$

$$\iff \frac{g'(G_{1L}^*)}{g'(G_{1H}^*)} > \underbrace{\frac{(p_H + \lambda_H)(p_L - \beta\lambda_H)}{(p_H + \beta\lambda_H)(p_L - \lambda_H)}}_{>1} \cdot \underbrace{\left(\frac{1+r-\theta_L}{1+r-\theta_H}\right)}_{>1} \cdot \underbrace{\frac{g'(\theta_L G_{1L}^* + G_{2L}^*)}{g'(\theta_H G_{1H}^* + G_{2H}^*)}}_{?}. \tag{A.56}$$

If we put

$$\frac{g'(\theta_L G_{1L}^* + G_{2L}^*)}{g'(\theta_H G_{1H}^* + G_{2H}^*)} \geq 1,$$

which is equivalent to

$$\theta_L G_{1L}^* + G_{2L}^* \leq \theta_H G_{1H}^* + G_{2H}^*,$$

then using (A.56) yields $g'(G_{1L}^*) > g'(G_{1H}^*)$, which is equivalent to

$$G_{1L}^* < G_{1H}^*. \tag{A.57}$$

Noting that

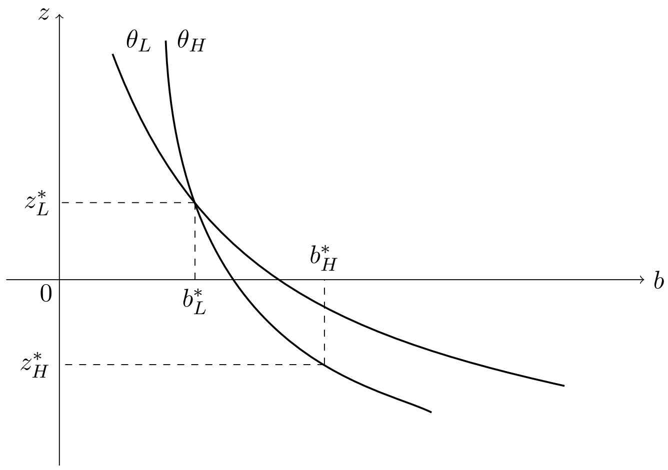
$$\begin{aligned} 0 &= V(b_H^*, z_H^*; \theta_H, \beta) - V(b_L^*, z_L^*; \theta_H, \beta) \\ &= \underbrace{g(G_{1H}^*) - g(G_{1L}^*)}_{+} + \beta\left[g(\theta_H G_{1H}^* + G_{2H}^*) - g(\theta_H G_{1L}^* + G_{2L}^*)\right], \end{aligned}$$

it follows that $\theta_H G^*_{1H} + G^*_{2H} < \theta_H G^*_{1L} + G^*_{2L}$, which is equivalent to

$$\theta_H \left(G^*_{1H} - G^*_{1L}\right) < G^*_{2L} - G^*_{2H}.$$

Then, using again (A.57) yields $G^*_{2L} > G^*_{2H}$, which immediately implies that $b^*_L < b^*_H$.

FIG. 3. The case with binding IC_H.



On the other hand, if we put

$$\frac{g'\left(\theta_L G^*_{1L} + G^*_{2L}\right)}{g'\left(\theta_H G^*_{1H} + G^*_{2H}\right)} < 1,$$

which is equivalent to

$$\theta_L G^*_{1L} + G^*_{2L} > \theta_H G^*_{1H} + G^*_{2H},$$

then we get

$$\theta_H G^*_{1L} + G^*_{2L} > \theta_L G^*_{1L} + G^*_{2L} > \theta_H G^*_{1H} + G^*_{2H}. \tag{A.58}$$

Noting that

$$\begin{aligned} 0 &= V\left(b^*_H, z^*_H; \theta_H, \beta\right) - V\left(b^*_L, z^*_L; \theta_H, \beta\right) \\ &= g\left(G^*_{1H}\right) - g\left(G^*_{1L}\right) + \beta\underbrace{\left[g\left(\theta_H G^*_{1H} + G^*_{2H}\right) - g\left(\theta_H G^*_{1L} + G^*_{2L}\right)\right]}_{-}, \end{aligned}$$

it follows that $G_{1H}^* > G_{1L}^*$, applying which to (A.58) gives

$$G_{2L}^* - G_{2H}^* > \theta_H \left(G_{1H}^* - G_{1L}^*\right) > 0.$$

Thus, we have $b_L^* < b_H^*$.

Therefore, by utilizing (A.51) and (A.53), the relationship between a typical L-type indifference curve and a typical H-type indifference curve can be illustrated using Figure 3. ▮

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