ANNALS OF ECONOMICS AND FINANCE **4**, 491–507 (2003)

The Tradeoff Between Inequality and Growth

Jess Benhabib

Department of Economics, New York University
269 Mercer Street, 7th floor, New York, NY 10003, USA
E-mail: jess.benhabib@nyu.edu

Is there a trade-off between inequality and economic growth? The theory and the evidence are so far inconclusive. So far the theory and the evidence are inconclusive. We want to construct a political economy model of growth to demonstrate that excessive inequality can disrupt the economy by inviting political interference through rent-seeking behavior and appropriation, but that policies supporting some modest inequality to take advantage of productivity differences can lead to the best growth rates. Thus we show that the relation between inequality and growth may be mildly hump-shaped: growth may rise modestly at first, as we move away from complete equality, and then drop again as inequality increases further.

Key Words: Politics; Inequality; Growth; Conflict; Distribution
JEL Classification Numbers: O10, C73

1. INTRODUCTION

Is there a trade-off between inequality and economic growth? The theory and the evidence are inconclusive. In the 1950es and 60es a prevalent view was that inequality leads to higher savings because the rich save proportionately more than the poor, and that this leads to increases the rate of investment and growth (see Kaldor (1957), Kuznets (1955)). More recently, it has been argued that inequality hurts growth because it leads to redistributive pressures, either through the median voter who enacts redistributive taxes (Tabellini and Persson(1993)), or through generating social conflict, expropriation, and rent seeking behavior (see Alesina and Rodrik (1994), Alesina (1994), Benhabib and Rustichini (1996), Benabou (1996), Perotti (1996), Acemoglu and Robinson (2000)). All such activities dilute

1529-7373/2002

the return on investment and reduce the rate of growth[1]. Another view is that inequality coupled with borrowing constraints and financial market imperfections prevents the talented poor to undertake profitable investments in physical and human capital, thereby limiting the full potential for the growth of the economy (Galor and Zeira (1993), Banerjee and Newman (1993). By contrast, there is also a recent literature which views inequality as the result of growth spurts that are associated with skill-biased technical change (Aghion (2002)).

On the empirical side, the evidence on the trade-off between inequality and growth, despite a large number of recent studies, remains inconclusive. In a recent paper Deininger and Squire (1996) document the fragility and the non-robustness of the results obtained by using cross-country regressions (see also Levine and Renelt (1992), Benhabib and Spiegel (1994), Islam (1995) and Easterly (2001)). In addition, and to complicate matters further, Forbes (2000) now finds a significant positive association between inequality and growth in the short and medium run.

Redistributive pressures, insecure property rights and social conflict that stem from significant inequality may well discourage investment and hinder growth. On the other hand, excessive interference with economic inequality that follows from differences in effort, productivity, enterprise and initiative is also likely to reduce growth and investment. In this paper we present a theoretical political economy model of inequality and growth. We want to demonstrate that while excessive inequality can disrupt the economy by inviting political interference through rent-seeking behavior and appropriation, policies which support some modest inequality to take advantage of productivity differences will lead to the best growth rates. Thus we show that the relation between inequality and growth may be mildly hump-shaped: growth may rise modestly at first, as we move away from complete equality, and then drop again as inequality increases further.

2. THE SOCIAL PLANNER AND COOPERATIVE SOLUTIONS

We start by considering the policies of a social planner, or a government, in an economy with two classes of agents. For simplicity, the utilities of the two sets of agents are logarithmic, they are increasing in consumption, and they are quadratic and decreasing in labor. Output is produced with a Cobb-Douglas production function, $Ak^{\alpha}(l_1)^{\mu}(l_2)^{\sigma}$, using capital k, and there are two types of labor, l_1 and l_2, with different productivity levels. We will not necessarily restrict the production function to constant returns

[1]But see St. Paul and Verdier(1993), who argue that redistributive taxes can enhance growth if they are channeled to puiblic goods or education that makes the economy more productive.

to scale in order to allow the possibility of endogenous growth with $\alpha = 1$. Given the logarithmic utility of consumption and the quadratic disutility of work, the problem is well-defined, with the case of constant returns, $\alpha + \mu + \sigma = 1$, as a special case. Let us first assume that the government assigns weights a and $(1-a)$, to the two agents and can choose their consumptions and labor supplies to maximize the discounted utilities of the agents. If we denote the value function by $V(k)$, the problem is given by:

$$V(k) = \max_{c_1,c_2,l_1,l_2} \left\{ \begin{array}{c} a\ \ell n\ c_1 + (1-a)\,\ell n\ c_2 - 0.5B\left(a\,(l_1)^2 + (1-a)\,(l_2)^2\right) \\ +\beta V\left(Ak^\alpha\,(l_1)^\mu\,(l_2)^\sigma - c_1 - c_2\right) \end{array} \right\}$$

where we will set $B = 1$.[2] To solve the problem let $y = Ak^\alpha\,(l_1)^\mu\,(l_2)^\sigma$. The first order conditions for consumption are:

$$\begin{aligned} a\,(c_1)^{-1} &= (1-a)\,(c_2)^{-1} = \beta V'\left(Ak^\alpha\,(l_1)^\mu\,(l_2)^\sigma - c_1 - c_2\right) \\ (c_i)^{-1} &= \beta\alpha\,(c_i')^{-1}\left(\frac{y'}{k'}\right) \qquad i = 1,2 \end{aligned}$$

where primes are next period values. Let $c_i = \lambda_i y$. Then, the solution for the consumption of the first agent must satisfy:

$$\begin{aligned} (\lambda_1 y)^{-1} &= \beta\alpha\,(\lambda_1 y')^{-1}\left(\frac{y'}{(1-\lambda_1-\lambda_2)\,y}\right) \\ (1-\lambda_1-\lambda_2) &= \beta\alpha \end{aligned}$$

$$\lambda_1 = a\,(1-\beta\alpha)\,, \qquad \lambda_2 = (1-a)\,(1-\beta\alpha)$$

We can now solve for the optimal labor supplies l_i that maximize the utility function[3]:

$$\begin{aligned} al_1 &= \beta V' Ak^\alpha\,(l_1)^{\mu-1}\,(l_2)^\sigma\,\mu && (1) \\ &= a\,(c_1)^{-1}\,Ak^\alpha\,(l_1)^{\mu-1}\,(l_2)^\sigma\,\mu && (2) \\ l_1 &= (a\,(1-(\beta\alpha)))^{-1}\,y^{-1}y\,(l_1)^{-1}\,\mu \\ (l_1)^2 &= (a\,(1-(\beta\alpha)))^{-1}\,\mu \\ (l_1)^2 &= \frac{\mu}{a\,(1-\beta\alpha)};\ (l_2)^2 = \frac{\sigma}{(1-a)\,(1-\beta\alpha)} \end{aligned}$$

[2]In such cases where we require labor supply to be inelastic we can take $B = 0$, and $l_1 = l_2 = 1$.

[3]Equation (2) is simply the the standard labor market condition which requires the marginal disutility of labor to equal the marginal product of capital times the marinal utility of consumption.

We are now in a position to obtain the value function for the government:

$$\begin{aligned}
V(k) &= s\,\ell n\, k + I = a\,\ell n\,\left[a\left(1-(\beta\alpha)\right)Ak^{\alpha}\left(l_1\right)^{\mu}\left(l_2\right)^{\sigma}\right] \\
&\quad +(1-a)\,\ell n\,\left[(1-a)\left(1-(\beta\alpha)\right)Ak^{\alpha}\left(l_1\right)^{\mu}\left(l_2\right)^{\sigma}\right]-0.5\left(\frac{\mu}{a\left(1-\beta\alpha\right)}\right) \\
&\quad -0.5\left(\frac{\sigma}{(1-a)\left(1-\beta\alpha\right)}\right)+\beta s\,\ell n\,\left[(\beta\alpha)\,Ak^{\alpha}\left(l_1\right)^{\mu}\left(l_2\right)^{\sigma}\right]+\beta I \\
&= \alpha a\,\ell n\,k\ +\alpha(1-a)\,\ell n\,k\ +\alpha\beta s\,\ell n\,k \\
&\quad +a\,\ell n\,a\left(1-(\beta\alpha)\right)A\left(l_1\right)^{\mu}\left(l_2\right)^{\sigma}+(1-a)\,\ell n\,(1-a)\left(1-(\beta\alpha)\right)A\left(l_1\right)^{\mu}\left(l_2\right)^{\sigma} \\
&\quad +\beta s\,\ell n\,(\beta\alpha)\,A\left(l_1\right)^{\mu}\left(l_2\right)^{\sigma}-0.5\left(1-\beta\alpha\right)^{-1}\left(\frac{\mu}{a}+\frac{\sigma}{1-a}\right)+\beta I
\end{aligned}$$

Note that the value function is concave in k. We can now solve for s and I by equating coefficients of $\ell n\, k$ and of the constant terms

$$s=\left(1-\beta\alpha\right)^{-1}\alpha$$

$$\begin{aligned}
I &= \left(1-\beta\right)^{-1}\left[a\,\ell n\,a+(1-a)\,\ell n\,(1-a)\right] \\
&\quad +\left(1-\beta\right)^{-1}\left[\begin{array}{c}\ell n\,\left(1-(\beta\alpha)\right)A\left(l_1\right)^{\mu}\left(l_2\right)^{\sigma} \\ +\beta s\,\ell n\,(\beta\alpha)\,A\left(l_1\right)^{\mu}\left(l_2\right)^{\sigma}-0.5\left(1-\beta\alpha\right)^{-1}\left(\frac{\mu}{a}+\frac{\sigma}{1-a}\right)\end{array}\right]
\end{aligned}$$

We note that this allocation of consumptions and labor supplies can be achieved in a decentralized setting by the social planner through a simple redistribution of the initial capital stocks, so that

$$k_1(0)=ak(0)\,,\quad k_2(0)=(1-a)\,k(0)\,,$$

provided we have constant returns to scale in production. For example, if labor supply is inelastic ($B=0$), $\left(l_1\right)^{\mu}\left(l_2\right)^{\sigma}=1$ and we set $\alpha=1$ to achieve balanced growth, the decentralized setting described above would implement the planner's solution after the initial capital was allocated according to the specified weights to the two agents.[4] Then if each agent consumed and saved optimally, expecting a return on capital given by the marginal product of capital, the first order conditions of each agent with respect to c_i (that is their Euler equations), would be equivalent to the

[4]For $\mu,\sigma\to 0$, $\alpha\to 1$, we approach the standard endogenouis growth model. The Hessian of the utility function $\ell n(Ak_t\left(l_1\right)^{\mu}\left(l_2\right)^{\sigma}-k_{t+1})-0.5\left(a\left(l_1\right)^2+(1-a)\left(l_2\right)^2\right)$ will have a root approaching zero, associated with the balanced growth path, with the other roots negative, as the utilty of leisure is strictly concave.This can be checked by computation or deduced from the continuity of roots in parameters.

first order conditions of the social planner. In this case each agent's capital would grow at the rate $(\beta A - 1)$. With increasing returns to scale, as is well-known, the planner's allocation in the context of a decentralized framework would require a system of taxes and subsidies. It may not be possible to design taxes and subsidies to identify or differentially target the segments of the population with different productivity levels however.

Note that if we abandon balanced growth by having $\alpha < 1$, and focus on steady state levels of income and consumption, we could reintroduce differential labor productivities and still maintain constant returns to scale. This structure would preserve the incentive issues associated with labor supply in a context where a fully decentralized implementation of the social planner's solution is feasible by a simple reallocating initial stocks.

We can now also compute, for future use, the value function for the agent consuming a share $\lambda_1 = a(1-\beta\alpha)$ of the output:

$$\begin{aligned}
V_1^a(k) &= s_a \ell n\ k + I_a = \ell n\ a(1-\beta\alpha) A k^{\alpha} (l_1)^{\mu} (l_2)^{\sigma} \\
&\quad -0.5\left(\frac{\mu}{a(1-\beta\alpha)}\right) + \beta s_a \ell n\ (\beta\alpha_1) A k^{\alpha} (l_1)^{\mu} (l_2)^{\sigma} + \beta I_1^a \\
&= \alpha\ \ell n\ k\ + \alpha\beta s_a\ \ell n\ k - 0.5\left(\frac{\mu}{a(1-\beta\alpha)}\right) \\
&\quad +\ell n\ a(1-\beta\alpha) A (l_1)^{\mu} (l_2)^{\sigma} + \beta s_a\ \ell n\ (\beta\alpha) A (l_1)^{\mu} (l_2)^{\sigma} + \beta I_1^a
\end{aligned}$$

$$s_a = \alpha(1-\beta\alpha)^{-1}$$

$$I_1^a = (1-\beta)^{-1}\begin{pmatrix} \ell n\ a(1-\beta\alpha) A (l_1)^{\mu} (l_2)^{\sigma} \\ -0.5\left(\frac{\mu}{a}\right)(1-\beta\alpha)^{-1} + \beta s\ \ell n\ (\beta\alpha) A (l_1)^{\mu} (l_2)^{\sigma} \end{pmatrix}$$

3. THE NON-COOPERATIVE SOLUTION: MARKOV STRATEGIES

We now consider a situation where each agent can choose to appropriate some consumption from the output, with the residual output, if any, becoming the capital stock for the next period. This reflects a situation where property rights, and in particular the returns to investment, are insecure, ill-defined or manipulable. We envisage a political system where pressure groups have power to implement redistributive policies. Such policies can range from outright expropriation to redistributive taxation, inflationary finance that sustains powerful groups, exchange rate policies that favor certain constituencies over others, the provision of government employment targeted to specific sections of the population, price controls, monopolistic marketing boards created to advantage particular classes, or other legislative measures that can affect and alter the bargaining power of labor and/or capital. It is important to note that while policies favoring many diverse groups can coexist, they my no means wash out, or cancel each other out. Each one represents a dilution of returns to investment that tends to diminish and discourage growth. These redistributive policies however may be the unavoidable result of the political process, and it must be the job of a good government and purpose of effectively designed institutions to achieve the least costly implementation of the political consensus.

Sustaining a productive political consensus will require a specification of the equilibrium for the case where the consensus breaks down. We will model this as a non-cooperative (Markov-Nash) equilibrium where each group appropriates consumption as a best response to the competing group's appropriation. For simplicity we model the political power of the two groups as symmetric, without specifying limits to appropriation. The model could easily be modified to reflect unequal political power, and upper different bounds on the appropriability of output[5].

The value function of the first agent is given by:

$$V^s(k) = Max_{c_1} \left\{ \ell n \; c_1 - 0.5 \left(l_1^s\right)^2 + \beta V \left(\left(1-\lambda_2\right) A k^\alpha \left(l_1^s\right)^\mu \left(l_2^s\right)^\sigma - c_1 \right) \right\}$$

where the superscript s in $l_i^s,\ i = 1, 2$, is used to indicate the labor supplies chosen in the non-cooperative case, and which are different than the labor supplies $l_i,\ i = 1, 2$, chosen in the cooperative case. The first order conditions are given by:

$$\left(c_1\right)^{-1} = \beta\alpha \left(c_1'\right)^{-1} \left(\frac{y'}{k'}\right) \left(1-\lambda_2\right)$$

Let $c_i = \lambda_i y$. Then the above simplifies to:

$$1 - \lambda_1 - \lambda_2 = \beta\alpha \left(1-\lambda_2\right),$$

A symmetric equilibrium then implies consumption shares, identical for the two agents, of:

$$\lambda = \frac{1-\beta\alpha}{2-\beta\alpha}$$

[5]To fully define the game, outcomes must be explicitly specified for situations where the agents attempt to appropriate amounts that sum to more than the total output available. We will overlook such considerations here, but for an explicit treatment of such issues see Benhabib and Rustichini (1996).

We can also solve for the labor supplies of the first and second agents using their first order conditions for labor supply:

$$\begin{aligned} l_1^s &= \beta V' \mu \left(\frac{y}{l_1^s}\right)(1-\lambda_2) = (c_1)^{-1} \mu \left(\frac{y}{l_1^s}\right)(1-\lambda_2) \\ c_1 l_1^s &= \left(\frac{1-\beta\alpha}{2-\beta\alpha}\right) y l_1^s = \mu \left(\frac{y}{l_1^s}\right)(1-\lambda_2) = \mu \left(\frac{y}{l_1^s}\right)(2-\beta\alpha)^{-1} \end{aligned}$$

Solving the above, the labor supplies for the two agents are given by:

$$\begin{aligned} (l_1^s)^2 &= \mu (1-\beta\alpha)^{-1} = a (l_1)^2 \\ (l_2^s)^2 &= \sigma (1-\beta\alpha)^{-1} = (1-a)(l_2)^2 \end{aligned}$$

Given consumption shares and labor supplies we can evaluate the value functions associated with this non-cooperative equilibrium:

$$\begin{aligned} V^s(k) &= s_s \, \ell n \, k + I_s = \ell n \; \lambda A k^\alpha (l_1^s)^\mu (l_2^s)^\sigma \\ &\quad -0.5\left(\mu(1-\beta\alpha)^{-1}\right) + \beta s_s \, \ell n \; (1-2\lambda) A k^\alpha (l_1^s)^\mu (l_2^s)^\sigma + \beta I_s \end{aligned}$$

where

$$s_s = \alpha (1-\alpha\beta)^{-1} = s$$

and

$$I_s = \frac{\left(\ell n \; \lambda A (l_1^s)^\mu (l_2^s)^\sigma + \beta s_s \, \ell n \; \left(\frac{\beta\alpha}{2-\beta\alpha}\right) A (l_1^s)^\mu (l_2^s)^\sigma - 0.5\mu(1-\beta\alpha)^{-1}\right)}{(1-\beta)}$$

The capital accumulation equation for the social planner's solution is:

$$k_{t+1} = (\alpha\beta) A k_t^\alpha \left(\frac{\mu}{(1-\beta\alpha)}\right)^{0.5\mu} \left(\frac{\sigma}{(1-\beta\alpha)}\right)^{0.5\sigma} \left(a^{-0.5\mu}(1-a)^{-0.5\sigma}\right)$$

while for the non-cooperative solution it is:

$$k_{t+1} = \left(\frac{\alpha\beta}{2-\alpha\beta}\right) A (k_t)^\alpha (l_1)^\mu (l_2)^\sigma \left(a^{0.5\mu}(1-a)^{0.5\sigma}\right)$$

Note that these accumulation equations lead to steady states if $\alpha < 1$. They lead to sustained balanced growth paths only if $\alpha = 1$, assuming that parameter configurations under logarithmic preferences also satisfy the sufficiency conditions.

4. SUSTAINABLE EQUILIBRIA

Sustainable equilibria are consumption and labor paths that yield higher utilities to the agents than they could achieve unilaterally by breaking the agreement and reverting to the non-cooperative equilibrium. One simple way to model this is to assume that defection does not yield an initial period advantage to the defector during which his opponent continues to consume and supply labor according to the agreed upon cooperative path. We may assume that the defection is instantly detected, so that reversion to the non-cooperative equilibrium is immediate. By construction, a symmetric allocation that treats both agents identically will dominate the symmetric non-cooperative equilibrium, but given the dispersion in productivities, a pertinent question is the degree of inequality, parametrized by a, that can be sustained by the social planner, and whether there is an optimal degree of inequality that attains the highest level of growth.

Before trying to answer this question, let us note that an alternative specification of the model, discussed in the appendix, would allow the defector a one period advantage before both agents revert to the non-cooperative equilibrium, and make defection more attractive. In such a case the degree of enforceable inequality may in fact shrink. Under such circumstances, the social planner's preferred solution, even under complete equality, may not be enforceable and other second-best solutions must be found (Benhabib and Rustichini (1996)). However, the non-cooperative equilibrium described above may not be the worst equilibrium to which the players can defect: the threat or fear of reverting to an even worse equilibrium (anarchy?) may be able to sustain more cooperation, even if it entails more inequality.

To evaluate the degree of sustainable inequality, we must compare the values of the cooperative and the non-cooperative allocations. Since $s_s = s$, the comparison of the discounted sum of utilities hinge on I_a and I_s. We have, since $a^{0.5}l_1 = l_1^s$ and $(1-a)^{0.5}\, l_2 = l_2^s$:

$$I_s = \frac{\left(\begin{array}{c} \ell n\ \left(\frac{1-\beta\alpha}{2-\beta\alpha}\right) A\left(a^{0.5}l_1\right)^{\mu}\left((1-a)^{0.5}\,l_2\right)^{\sigma} \\ +\beta s_s\ \ell n\ \left(\frac{\beta\alpha}{2-\beta\alpha}\right) A\left(a^{0.5}l_1\right)^{\mu}\left((1-a)^{0.5}\,l_2\right)^{\sigma} - 0.5\frac{\mu}{(1-\beta\alpha)} \end{array}\right)}{(1-\beta)}$$

$$I_1^a = \frac{\left(\begin{array}{c} \ell n\, a\,(1-(\beta\alpha))\,A\,(l_1)^{\mu}\,(l_2)^{\sigma} \\ +\beta s\ \ell n\ (\beta\alpha)\,A\,(l_1)^{\mu}\,(l_2)^{\sigma} - 0.5\,(1-\beta\alpha)^{-1}\left(\frac{\mu}{a}\right) \end{array}\right)}{(1-\beta)}$$

Then the comparison of discounted utilities between the cooperative and non-cooperative regimes for the first agent reduces to:

$$\begin{aligned}&\ell n\ a\,(1-\beta\alpha)+\beta s\ \ell n\ (\beta\alpha)-0.5\,(1-\beta\alpha)^{-1}\left(\frac{\mu}{a}\right)\\ \lesseqgtr\ &\ell n\ \left(\frac{1-\beta\alpha}{2-\beta\alpha}\right)+\beta s_s\ \ell n\ \left(\frac{\beta\alpha}{2-\beta\alpha}\right)\\ &+(1+\beta s)\,\ell n\left(a^{0.5\mu}\,(1-a)^{0.5\sigma}\right)-0.5\,(1-\beta\alpha)^{-1}\,\mu\end{aligned}$$

or,

$$\begin{aligned}&\ell n\ a+0.5\,(1-\beta\alpha)^{-1}\,\mu\left(\frac{a-1}{a}\right)\\ \lesseqgtr\ &(1-\beta\alpha)^{-1}\left(\ell n\left(a^{0.5\mu}\,(1-a)^{0.5\sigma}\right)-\ell n\,(2-\beta\alpha)\right)\end{aligned}$$

Similarly for second agent, we obtain:

$$\begin{aligned}&\ell n\ (1-a)+0.5\,(1-\beta\alpha)^{-1}\,\sigma\left(\frac{a}{a-1}\right)\\ \lesseqgtr\ &(1-\beta\alpha)^{-1}\left(\ell n\left(a^{0.5\mu}\,(1-a)^{0.5\sigma}\right)-\ell n\ (2-\beta\alpha)\right)\end{aligned}$$

Figure 1 illustrates the growth rates when $\alpha = 1$, as well as the growth rates as a function of a, for parameter values $\beta\alpha = 0.98, \mu = 0.15,\ \sigma = 0.3$[6] However, the sustainable growth rates are those corresponding to values of a for which both lines, representing the difference between the values of cooperation and non-cooperation of each agent, are non-negative. Note that this sustainable interval in Figure 1 is skewed to the left because $\sigma > \mu$, and μ is the productivity of the agent assigned weight a. The agent with the higher productivity is less willing to cooperate if he is given a low utility weight, because he can do better in the non-cooperative situation. However, note that the growth rate is maximized if the productive agent is given a low weight (in fact a zero weight, the lowest possible value for $1 - a$. The figure is provided only for $a \in [0.3, 0.7]$ for better visibility. This is because the optimal labor assignments of the agents are decreasing

[6] Of course these growth rates would be transitory if $\alpha < 1$. The level of the attainable steady state when $\alpha < 1$ would be positively related to the value of the function attained in the lower figures depicting the growth rate. Paralell results hold if we assume that the government aims at maximizing the steady state level of output instead of the growth rate: the government would then implement a moderate level of inequality at the boundaries of the sustainable interval. None of the qualitative features of the Figures change if we pick, assuming constant returns, $\beta\alpha = \beta\,(1-\sigma-\mu)$. This would imply, for $\mu+\sigma = 0.55$, a value of $\beta\alpha$ around 0.5. (See Figure 3.)

in their utility weight. To maximize growth, it is best to assign a low utility weight to productive agents that leaves them with low initial stocks of capital so as to get more work out of them. If the only mechanism available to the social planner to elicit work, or more appropriately, to elicit effective effort is a decentralized system of taxes and subsidies, then the wealthier agents who receive higher utility and higher consumption will also supply less effort. This is an artifact of the preference specification where leisure and consumption are complements, so that higher consumption is accompanied by a lower labor supply. Of course appropriate non-wage subsides or the prestige associated with higher position may reverse this feature of the model. Alternatively, if each agent were operating his or her own stock of capital in combination with his or her labor rather than with one aggregate capital and a single production function, then there would be an additional tendency to allocate the capital towards the group that had the more productive labor. This would tend to partially offset the labor supply effect of wealth and consumption, and may lead to an optimal allocation of capital, in the sense of maximizing the growth of output, that would favor the more productive groups.

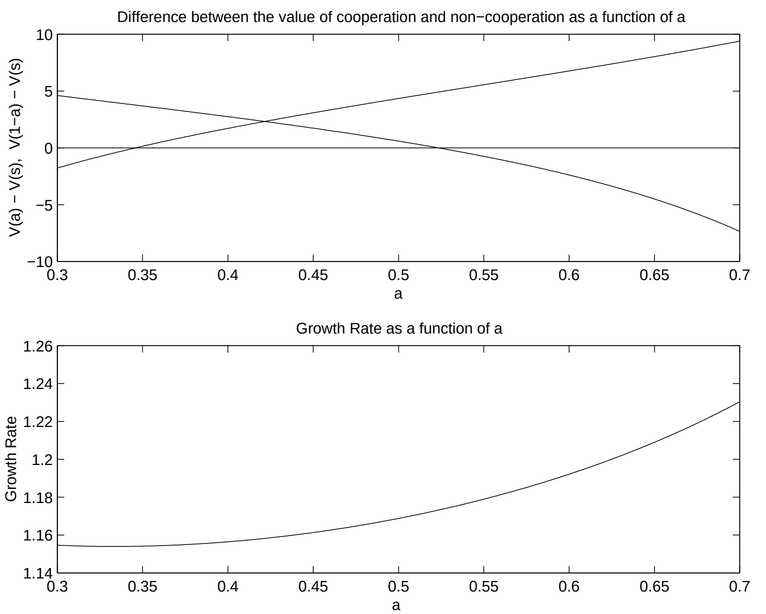

FIG. 1.

Figure 2 illustrates the case $\mu = 0.3$, $\sigma = 0.15$. In this case the sustainable interval is skewed to the right since the first agent is the more productive one, and the growth is higher as a becomes lower. These results are not sensitive to the choices of α and β. Under an alternative specification where $\beta\alpha = 0.5$ and for $\mu = 0.3, \sigma = 0.15$, very similar results obtain, as is clear from the Figure 3.

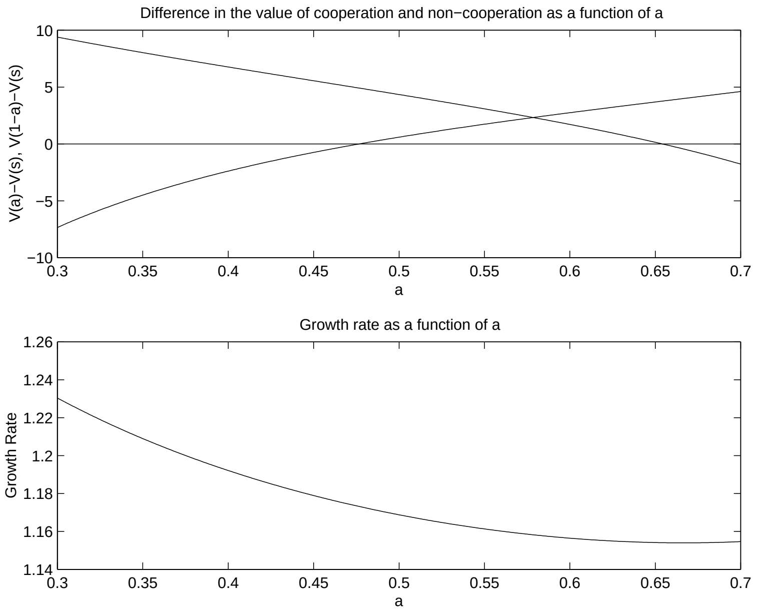

FIG. 2.

5. THE TRADE-OFF BETWEEN GROWTH AND INEQUALITY

The main point of this analysis is to study the optimal redistribution of wealth that maximizes the growth rate. Redistribution in terms of utility can be implemented as a reallocation of the productive initial capital stock. As we see from the figures presented above, when agents cannot interfere with the political distribution mechanism, the growth maximizing allocation is to impoverish the most productive agents, thereby forcing them to supply more effort. However, such a policy will not work because of political constraints: agents can withhold effort, and they can resort to political rent-seeking activities to obtain higher consumption if the distribution of wealth and income is too unfavorable to them. We see that outside the sustainable interval for the parameter a, the economy will revert to a non-cooperative equilibrium with a low growth rate (assuming $\alpha = 1$) given

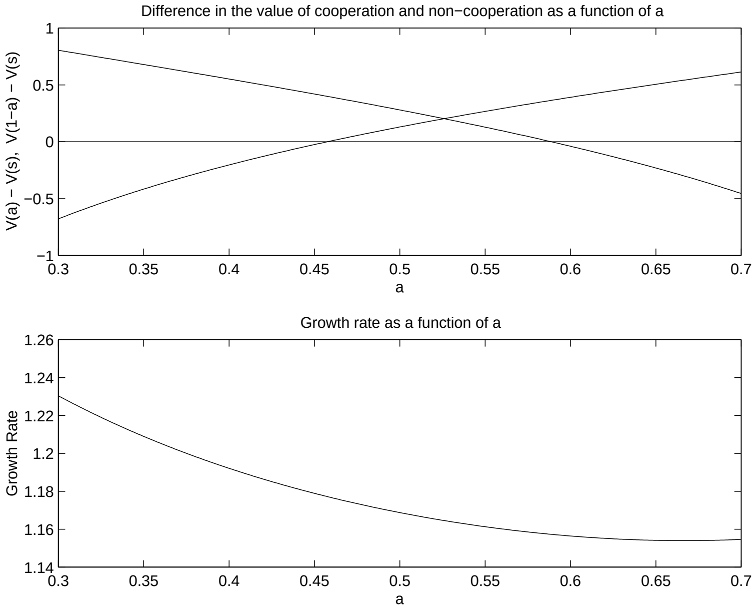

FIG. 3.

by

$$g_s = \left(\frac{\alpha\beta}{2-\alpha\beta}\right) A \left(\mu\,(1-\beta\alpha)^{-1}\right)^{0.5\mu} \left(\sigma\,(1-\beta\alpha)^{-1}\right)^{0.5\sigma}.$$

Note that this growth rate is independent of a. The value of a that maximizes steady state level of output or the growth rate of the economy must be chosen to maximize

$$g = \left((\alpha\beta)\,Ak_t^{\alpha}\left(\frac{\mu}{(1-\beta\alpha)}\right)^{0.5\mu}\left(\frac{\sigma}{(1-\beta\alpha)}\right)^{0.5\sigma} a^{-0.5\mu}\,(1-a)^{-0.5\sigma}\right)$$

and must be at the boundary of the sustainable interval in the figures above. Whether the optimal a is the upper or lower boundary depends on the relative productivities of the two agents. If $\mu > \sigma$, then we choose the lower bound for a, and if $\mu < \sigma$, we choose the higher bound (unless of course productive agents can be offered non-wage compensation outside the market that gives them additional utility associated with their jobs, working conditions, and the prestige of their positions.)

We should note that our logarithmic specification for the preferences assures that agents save a constant fraction of their income. The total savings rate remains constant as we vary a. In a decentralized economy, the supply of labor or effort then becomes the primary mechanism through which incentives affect steady state output levels.

If we plot growth as a function of inequality, we will observe growth rates that are low and constant for high degrees of inequality, corresponding to non-cooperative outcomes. As inequality decreases we may observe a jump in the growth rate as we enter the range of sustainable cooperation. Thereafter, growth rates will decline as we move towards perfect equality, and as we diminish the incentives for the more productive agents to supply effort. (see Figure 4.) We may conclude therefore that the relation between growth and inequality is hump-shaped, and that a growth maximizing social planner should aim at a moderate level of inequality designed to elicit effort from the most productive agents in the economy.

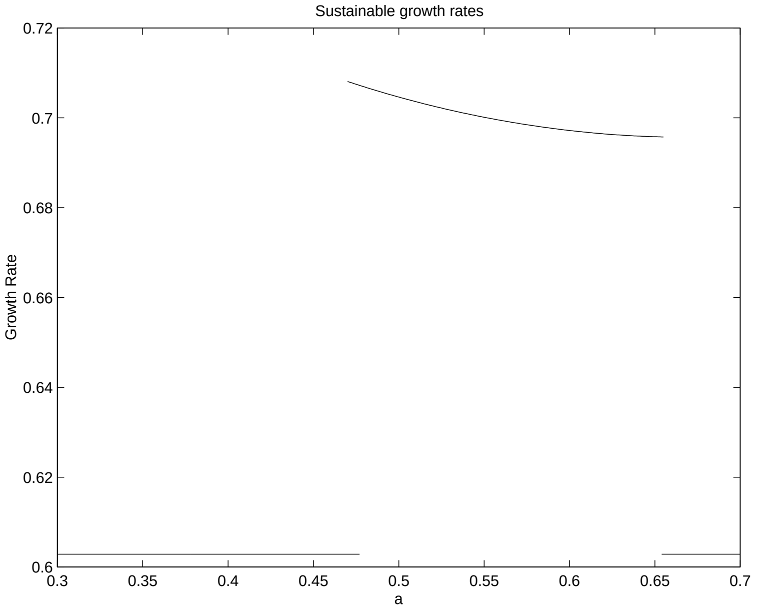

FIG. 4.

It may be possible for the government to improve the growth rate of the economy by enlarging the range of sustainable cooperative outcomes. This can be achieve by making deviation to the non-cooperative equilibria more costly for the agents. If punitive costs for deviating from the cooperative outcome could be imposed, a greater degree of inequality could be sustained. In Figure 1 for example, increasing the costs of deviation could raise the two curves representing the difference between the value of cooperation and the value of deviation, enlarging the range of a for which the cooperative equilibrium is sustainable. In the case depicted in Figure

1, this would permit the choice of a higher value of a, and therefore of a higher growth rate for the economy. Imposing such costs on deviation in order to improve the growth rate however, may not always be politically feasible or desirable.

APPENDIX: SUSTAINABLE EQUILIBRIA WITH A ONE PERIOD ADVANTAGE OF DEVIATION

In this case, the defecting agent (the first agent in this case) considers his optimal consumption and labor supply against the cooperating agent (the second agent) consuming a fraction $\lambda = (1-a)(1-\beta\alpha)$ of the output and supplying labor $l_2 = \frac{\sigma}{(1-a)(1-\beta\alpha)}$, with the understanding that both agents will revert to non-cooperative behavior from the next period on[1]. The maximization problem for this agent is given as:

$$\max_c \ \ell n c_1^d - 0.5(l_1^d)^2 + \beta\alpha(1-\alpha\beta)^{-1}\ell n(Ak^\alpha (l_1^d)^\mu (l_2)^\sigma (1-(1-a)(1-\beta\alpha)) - c_1^d)$$
$$+ \quad \beta(1-\beta)^{-1}\left(\begin{array}{c} \ell n \left(\frac{1-\beta\alpha}{2-\beta\alpha}\right) A\left(a^{0.5}l_1\right)^\mu \left((1-a)^{0.5}l_2\right)^\sigma \\ +\beta s_s \ \ell n \left(\frac{\beta\alpha}{2-\beta\alpha}\right) A\left(a^{0.5}l_1\right)^\mu \left((1-a)^{0.5}l_2\right)^\sigma - 0.5\frac{\mu}{(1-\beta\alpha)} \end{array}\right)$$

The first order conditions are:

$$\left(c_1^d\right)^{-1} = \beta\alpha\left(1-\alpha\beta\right)^{-1}\left(Ak^\alpha \left(l_1^d\right)^\mu \left(l_2\right)^\sigma \left(1-\left(1-a\right)\left(1-\beta\alpha\right)\right) - c_1^d\right)^{-1}$$

and they can be reduced to:

$$\begin{aligned} c_1^d &= \left[\left(1-\left(1-a\right)\left(1-\beta\alpha\right)\right)\left(1-\beta\alpha\right)\right] Ak^\alpha \left(l_1^d\right)^\mu \left(l_2\right)^\sigma \\ &\equiv \lambda_d^1 Ak^\alpha \left(l_1^d\right)^\mu \left(l_2\right)^\sigma \end{aligned}$$

It is also straightforward to compute the optimal defection consumption of the second agent. It is given by:

$$\begin{aligned} c_2^d &= \left[\left(1-a\left(1-\beta\alpha\right)\right)\left(1-\beta\alpha\right)\right] Ak^\alpha \left(l_2^d\right)^\mu \left(l_1\right)^\sigma \\ &\equiv \lambda_d^2 Ak^\alpha \left(l_2^d\right)^\mu \left(l_1\right)^\sigma \end{aligned}$$

Similarly, labor supply of the defecting first agent is the solution to the first order conditions:

$$\begin{aligned} l_1^d &= \beta\alpha\left(1-\alpha\beta\right)^{-1}\left(Ak^\alpha \left(l_1^d\right)^\mu \left(l_2^s\right)^\sigma \left(1-\left(1-a\right)\left(1-\beta\alpha\right)\right) - c\right)^{-1} \\ &\quad \cdot Ak^\alpha \left(l_1^d\right)^{\mu-1} \left(l_2^s\right)^\sigma \left(1-\left(1-a\right)\left(1-\beta\alpha\right)\right)\mu \\ &= c^{-1} Ak^\alpha \left(l_1^d\right)^{\mu-1} \left(l_2\right)^\sigma \left(1-\left(1-a\right)\left(1-\beta\alpha\right)\right)\mu \end{aligned}$$

[1] See also Kaitala and Pohjola (1990), and Benhabib and Rustichini (1996).

They can be simplified to:

$$\begin{aligned}
\left(l_1^d\right)^2 &= \left[\left(1-\left(1-a\right)\left(1-\beta\alpha\right)\right)\left(1-\beta\alpha\right)\right]^{-1}\left(1-\left(1-a\right)\left(1-\beta\alpha\right)\right)\mu \\
&= \frac{\left(1-\left(1-a\right)\left(1-\beta\alpha\right)\right)}{\left(1-\left(1-a\right)\left(1-\beta\alpha\right)\right)\left(1-\beta\alpha\right)}\mu \\
&= \left(1-\beta\alpha\right)^{-1}\mu
\end{aligned}$$

The optimal labor supply of the defecting second agent against the first is, by symmetry,

$$\left(l_2^s\right)^2 = \left(1-\beta\alpha\right)^{-1}\sigma$$

We can also compute the value function for the defection of the first agent:

$$\begin{aligned}
V_1^D &= s_D \,\ell n\, k + I_1^D \\
&= \ell n \left(\left[\left(1-\left(1-a\right)\left(1-\beta\alpha\right)\right)\left(1-\beta\alpha\right)\right] A \left(l_1^d\right)^{\mu}\left(l_2\right)^{\sigma} k^{\alpha}\right) - 0.5\left(l_1^d\right)^2 \\
&+ \beta\alpha\left(1-\alpha\beta\right)^{-1} \ell n \,\left(Ak^{\alpha}\left(l_1^d\right)^{\mu}\left(l_2\right)^{\sigma}\left\{\left(1-\left(1-a\right)\left(1-\beta\alpha\right)\right)\left(1-\beta\alpha\right)\right\}\right) \\
&+ \beta\left(1-\beta\right)^{-1}\left(\begin{array}{c} \ell n\,\left(\frac{1-\beta\alpha}{2-\beta\alpha}\right) A\left(l_1^s\right)^{\mu}\left(l_2^s\right)^{\sigma} \\ +\beta s_s\,\ell n\,\left(\frac{\beta\alpha}{2-\beta\alpha}\right) A\left(l_1^s\right)^{\mu}\left(l_2^s\right)^{\sigma} - 0.5\frac{\mu}{(1-\beta\alpha)} \end{array}\right)
\end{aligned}$$

where

$$s_D = \alpha\left(1+\beta\alpha\left(1-\alpha\beta\right)^{-1}\right) = \alpha\left(1-\alpha\beta\right)^{-1} = s_s = s$$

and

$$\begin{aligned}
I_1^D &= \ell n \left(\left[\left(1-\left(1-a\right)\left(1-\beta\alpha\right)\right)\left(1-\beta\alpha\right)\right] A\left(l_1^d\right)^{\mu}\left(l_2\right)^{\sigma}\right) - 0.5\left(l_1^d\right)^2 \\
&+ \beta\alpha\frac{\ell n \left(\begin{array}{c} A\left(l_1^d\right)^{\mu}\left(l_2\right)^{\sigma} \\ \cdot\left\{1-\left(1-a\right)\left(1-\beta\alpha\right)-\left[\left(1-\left(1-a\right)\left(1-\beta\alpha\right)\right)\left(1-\beta\alpha\right)\right]\right\} \end{array}\right)}{\left(1-\alpha\beta\right)} \\
&+ \beta\left(1-\beta\right)^{-1}\left(\begin{array}{c} \ell n\,\left(\frac{1-\beta\alpha}{2-\beta\alpha}\right) A\left(l_1^s\right)^{\mu}\left(l_2^s\right)^{\sigma} \\ +\beta s_s\,\ell n\,\left(\frac{\beta\alpha}{2-\beta\alpha}\right) A\left(l_1^s\right)^{\mu}\left(l_2^s\right)^{\sigma} - 0.5\frac{\mu}{(1-\beta\alpha)} \end{array}\right)
\end{aligned}$$

For the second agent, the value of defection is given by $V_1^D = s_D k + I_2^D$, where

$$
\begin{aligned}
I_2^D &= \ell n\left(\left[\left(1-a\left(1-\beta\alpha\right)\right)\left(1-\beta\alpha\right)\right] A\left(l_1\right)^{\mu}\left(l_2^d\right)^{\sigma}\right) - 0.5\left(l_2^d\right)^2 \\
&+ \beta\alpha\left(1-\alpha\beta\right)^{-1}\ell n\begin{pmatrix} A\left(l_1\right)^{\mu}\left(l_2^d\right)^{\sigma} \\ \cdot\left\{1-a\left(1-\beta\alpha\right)-\left[\left(1-a\left(1-\beta\alpha\right)\right)\left(1-\beta\alpha\right)\right]\right\}\end{pmatrix} \\
&+ \beta\left(1-\beta\right)^{-1}\begin{pmatrix} \ell n\left(\frac{1-\beta\alpha}{2-\beta\alpha}\right) A\left(l_1^s\right)^{\mu}\left(l_2^s\right)^{\sigma} \\ +\beta s_s\ \ell n\left(\frac{\beta\alpha}{2-\beta\alpha}\right) A\left(l_1^s\right)^{\mu}\left(l_2^s\right)^{\sigma} - 0.5\frac{\mu}{(1-\beta\alpha)}\end{pmatrix}
\end{aligned}
$$

We can, at this point, inquire whether cooperation can be sustained for some special cases. When $\beta \to 0$, we get:

$$
\begin{aligned}
V_1^D - V_1^a &= \mu\,\ell n\left(\frac{l_1^d}{l_1}\right) - 0.5\mu\left(1-\frac{1}{a}\right) \\
&= 0.5\mu\left(\ell n\left(a\right) - \frac{a-1}{a}\right)
\end{aligned}
$$

This quantity is positive if $a > e^{\frac{a-1}{a}}$. This will be the case for $a \in (0,1)$.[2] Thus there can be no cooperation as $\beta \to 0$. Furthermore one could show that when $a = 0.5$ and $\mu = \sigma$, then it is always better to cooperate as $\beta \to 1$.

REFERENCES

Acemoglu, Daron and James A., Robinson, 2002, Why did the west extend the franchise? Democracy, inequality, and growth in historical perspective. *Quartely Journal of Economics* **115(4)**, 1167-1099.

Alesina, Alberto, 1994, Political models of macroeconomic policy and fiscal reforms. In: Stephan Haggard and Steven Webb, *Voting Reform: Democracy, Political Liberalization, and Economic Adjustment*. Oxford University Press, New York, New York.

Aghion, Philippe, 2000, Schumpeterian growth theory and the dynamics of income inequality. *Econometrica* **70(3)**, 855-882.

Alesina, Alberto and Dani, Rodrik, 1994, Distributive politics and economic growth. *Quarterly Journal of Economics* **109(2)**, 465-490.

Banerjee, A. and A. Newman, 1993, Occupational choice and the process of development. *Journal of Political Economy* **101**, 274-298.

Benabou, Roland, 1996, Inequality and growth. In: Ben S. Bernanke and Julio Rotemberg. *NBER Macroeconomics Annual 1996*. Cambridge, MA: MIT Press, 11-74.

Benhabib, Jess and Spiegel, Mark, 1994, The role of human capital and political instability in economic development. *Journal of Monetary Economics* **34**, 143-173.

[2] Again by symmetry, $V_2^D - V_2^a$ is negative for the second agent if $(1-a) > e^{\frac{-a}{1-a}}$.

Benhabib, Jess and Aldo, Rustichini, 1996, Social conflict and growth. *Journal of Economic Growth* **1**, 125-142.

Deininger, Klaus and Lyn, Squire, 1996, A new data set measuring inequality. *World Bank Economic Review* **10(3)**, 565-591.

Easterly, William, The middle class consensus and economic development. *Journal of Economic Growth* **6**, 317-335.

Forbes, K., 2000, A reassessment of the relationship between inequality and growth. *American Economic Review* **90(4)**, 869-887.

Galor, O. and J. Zeira, 1993, Income distribution and macroeconomics. *Review of Economic Studies* **60**, 35-52.

Islam, Nazrul, 1995, Growth empirics: A panel data approach. *Quarterly Journal of Economics* **110(4)**, 1127-1170.

Kaitala, Veijo and Matti, Pohjola, 1990, Economic development and agreeable redistribution in capitalism: Efficient game equilibria in a two-class neoclassical growth model. *International Economic Review* **31, No.2**, 421-438.

Kaldor, N., 1957, A model of economic growth. *Economic Journal* **67**, 591-624.

Kuznets, S., 1955, Economic growth and income inequality. *American Economic Review* **45(1)**, 1-28.

Levine, Ross E. and David, Renelt, 1992, A sensitivity analysis of cross country regressions. *American Economic Review* **82(4)**, 942-963.

Perotti, R., 1996, Growth, income distribution, and democracy. *Journal of Economic Growth* **1**, 149-187.

Persson, Torsten and Guido, Tabellini, 1994, Is inequality harmful for growth. *American Economic Review* **84(3)**, 600-621.

St. Paul, Gilles and Thierry, Verdier, 1993, Education, democracy, and growth. *Journal of Development Economics* **42(2)**, 339-407.