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Nash Bargaining, Money Creation, and Currency Union

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This paper is an attempt to combine global macroeconomic objectives with an explicit analysis of resource allocation efficiency. It determines how money creation must be shared between Monetary Union members, given national particularities in the monetary transmission mechanisms. In a two-country "New Open Macroeconomics" model, we outline the optimality of an unequal treatment of nations. To this end, the original Nash bargaining concept is modified to allow a differentiated treatment of countries. By favoring the more flexible country and relying on international money flows to provide liquidity to the more rigid nation, all Union members register efficiency gains which compensate an unfavorable intertemporal inflation activity arbitrage in the Union Central Bank objective.

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1. INTRODUCTION

Since January 1st 1999, the European Central Bank (ECB) conducts the European Monetary policy. As defined in the second article of the European System of Central Banks (ESCB) statutes, the ECB must maintain price stability with a view to contributing to the realization of the European Community objectives, with particular emphasis on a high level of employment and the achievement of a balanced and sustainable development. In pursuing these objectives the ECB shall favor an efficient allocation of resources in the Union.

Though the ECB task is institutionally clearly defined, in practice it faces many challenges in the definition of what a "European" monetary policy should be, given the heterogeneity of European countries. Reviewing this issue in 1998, Dornbusch et al. outlined two critical aspects that are still discussed in the academic literature: (1) the focus on Europe-wide averages rather than on each local situation within member countries, and (2) the neglect of national asymmetries in the monetary transmission mechanism between participating nations, as regards of financial and wage-price processes.

At a first glance, before the introduction of the Euro, discrepancies between the official rules implemented by the ECB and the conclusions of a wide range of academic research emphasizing the weight of such national particularities (see Cecchetti (1999) or Guiso et al. (1999), to name just a few) constituted something of a puzzle. As a further example, De Grauwe (2000, 2003) analyzed the integration of national idiosyncrasies in the definition of the objective of the European monetary Policy in a simplified macroeconomic setting, where union members differ according to their short term Phillips curve slope. Comparing a "national aggregation" scheme — where regional particularities are taken into account — with a "Euro aggregation" procedure, De Grauwe outlines the suboptimality of the latter (See also De Grauwe-Mongelli, 2005, for a review of the literature).

The homogeneous treatment of the European Monetary Union (EMU) members by the ECB was then justified by many reasons. First, an excessive focus on local conditions may paralyze decision makers, as each of them will tend to lend a greater weight to the economic conditions of his or her country. Furthermore, a regional bias in the ECB's policy could degenerate into a beggar-thy-neighbor situation, as a short term tightening of interest rates produces an uneven distribution of output losses between Union members. Finally, the monetary transmission mechanism evolves, since both financial structure and wage-price practices will converge. Current asymmetries may be viewed as temporary, thus justifying their absence in the definition of a medium term European monetary policy.

Nevertheless, as outlined by Ciccarelli and Rebucci (2006), these differences might decrease only if they were due to factors such as bad trade and financial integration — making cycles badly synchronized between member countries — but they could also persist for a long time if due to differences in the financial structures rooted in the national legal frameworks. Focusing on the four major European economies (France, Germany, Italy and Spain) Ciccarelli and Rebucci show that the transmission mechanism has changed in the round up of the EMU but they also find that the cross country differences in the long run effects of a common homoskedastic monetary shock have not significantly decreased in Europe since the adoption of the Euro. They interpret these results as an evolution of the transmission under the EMU that is rather slow and synchronized in all these countries.

Taking for granted that the objectives of the ECB are defined according to European wide aggregates, this paper, on the basis of efficiency, exploits asymmetries regarding the transmission mechanism of the ECB policy. Our conclusion is that more money creation rights should be allocated to the country with the more efficient banking system. To this end, more money creation rights are given to the countries with the more efficient banking system. In this case, an unequal treatment of the member countries can be linked to the objective of the ECB to favor an efficient allocation of resources in the Union, and makes all participants better off, since the less efficient country benefits from a positive externality through a net money inflow from the more flexible country.

To study this question we extend the seminal Obstfeld and Rogoff Redux model (1995) to the case of a monetary union. The model is deliberately stylized to use a modification of the Nash bargaining, which simplifies the demonstrations. We assume that the two nations differ with respect to both their relative size and their sensitivity of bank lending to changes in the instrument of monetary policy of the common Central Bank. The average per capita level of money creation in the Union is a combination of the national level of money creation, weighted by their relative population. This assumption replicates the rules implicitly adopted by the ECB. The sharing of money creation is not directly taken into account in the ECB status. However, this feature is linked to the distribution of seigniorage revenues among members in accordance with the rule defined by Article 32 of the Protocol No. 18 (ex No. 3) on the Statute of the European System of Central Banks and of the European Central Bank (ECB) of the Maastricht Treaty. The revenue of seigniorage depends on the country capital share in the ECB, which is determined according to an index that combines the average of that country's share in EU average population and GDP. This lead to an equal treatment of countries, once corrected by the size of their population and the value of the GDP.

Since our aim is to focus on how money creation should be shared, we propose to borrow a convenient concept from game theory proposed by Nash (1950, 1953). The Nash scheme is an arbitrage procedure which is an attempt to formalize the outcome of some bargaining process. The original concept is slightly modified to allow a differentiated treatment of countries. Different sharing rules of the money creation lead to different levels of welfare for both countries. In some sense, a transfer of money creation rights between countries is equivalent to a transfer of utility. The modified Nash scheme selects a pair of utility levels which corresponds to a particular way to share money creation rights. This process makes it possible to introduce a non standard component in the loss function of the monetary authorities. This indicator, based upon consumption growth differential, takes into account intra-union transfers between the participating members, that give rise to a better resource allocation. This paper can thus be considered as an attempt to combine global macroeconomic objectives with an explicit analysis of resource allocation efficiency.

We outline the Pareto improving properties of a small regional bias in the ECB's strategy, as long as the relative flexibility of the monetary transmission mechanism differs across countries. Indeed, by favoring the more flexible country, the ECB can improve resource allocation by reducing the weight lent to the most rigid mechanism in the transmission of the ECB's decision. More emphasis is thus given to international payment adjustment — here, the net flow of money between countries — as the transmission variable of monetary policy in the Union. If the ECB is faced with an intertemporal loss due to an unfavorable output-inflation arbitrage, it can compensate it with a net gain from a better resource allocation in the Union. More importantly, we show that this regional bias is not a beggar-thy-neighbor policy. Without it, the optimal monetary policy would be to create no money at all in the Union, leading the economy to stick to its initial steady state, which constitute a disagreement point between the EMU members.

The paper is organized as follows: the second section presents a "New Open Macroeconomics" model of a monetary union assuming idiosyncrasies in the monetary transmission mechanism. The third section focuses in a compact way on the interregional adjustment following a reduction of the ECB fund rate. Noting that this theoretic framework offers an intuitive relation to efficiency issues, the fourth section assesses the optimality of a global utility founded regional bias in the ECB's strategy. Section 5 combines a measure of reallocation gains with the more standard measure of authorities preferences regarding output gains and inflation costs, to define an optimal value for the money creation bias in the monetary union. All the computations can be found in appendices.

2. THE MACROECONOMIC SETTING

This first section adapts the two-country world setting of Obstfed and Rogoff (1995, 1996) to the case of a Monetary Union. We assume that the two nations differ in their relative size and in the sensitivity of domestic credit with respect to the Union Central Bank fund rate. Indeed, as shown by Angeloni et al (2002), the bank lending channel is quite different between union countries, implying different reaction of the money supply in these nations to a common variation of ECB rate. To keep things as simple as possible, neither the external dimension of the Union monetary policy nor fiscal and public debt questions are addressed.

2.1. The Private Sector

The world is inhabited by a continuum of immortal consumer/producer individuals indexed by z, $z \in [0, 1]$, each being specialized in the production of a single differentiated product, and consuming the whole range of goods. The home country consists of individuals on the $[0, n]$ interval, and the remainder $(n, 1]$ live in the foreign country. Thus, n represents the size of the domestic economy (with $n \to 0$ for a small open economy) whereas $(1 - n)$ stands for its openness degree. For expositional reasons, we will keep the domestic country as a benchmark and assume that it is relatively smaller than the other nation of the Union (i.e., $n < \frac{1}{2}$).

Individual preferences depend on consumption, money holdings (which yield liquidity services) and leisure. They are independent of the part of the world agents belong to.

Each producer has access to the same Cobb-Douglas function with fixed technology parameter (A) and capital (K) (such that $Y_t(z){=}AK^{\frac{1}{2}}L_t^{\frac{1}{2}}$). As a consequence, labour depends on production according to $L_t = \frac{\kappa}{2}Y_t(z)^2$ with $\kappa = 2\,(A^2K)^{-1}$.

A one period composite real bond is traded on a fully integrated capital market, so that people face the same real rate of interest in both parts of the world.

The behavior of the representative domestic agent "z" can thus be described by the maximization of a welfare index U_t with respect to consumption, money holdings and — through work effort — the production of the good of type "z" given a budget constraint, as follows:

$$\begin{cases} \max U_t = \sum_{s=t}^{\infty} (\frac{1}{1+\delta})^{s-t} \left[\,\log C_s + \chi \log \frac{M_s^d}{P_s} - \frac{\kappa}{2}\, Y_s(z)^2 \right], \\ \text{subject to:} \\ P_s R_s B_s + M_{s-1}^d + P_s T_s + P_s(z) Y_s(z) = P_s C_s + M_s^d + P_s B_{s+1} \quad \forall s \geq t. \end{cases} \tag{1}$$

In these equations, M_s^d denotes money holdings, $Y_s(z)$ the supply of goods from producer z, R_s the gross real rate of interest between periods $(s - 1)$

and s, B_{s+1} the stock of bonds held at the end of period s and T_s the real value of money transfers received from the domestic central bank. Coefficient δ represents the rate of time preference, χ a positive preference parameter related to real money holdings.

$C_s(z)$ stands for the consumption of the representative agent and $P_s(z)$ is the individual price of good z. We define C_s, the aggregate consumption level of the representative agent, and P_s, the consumption-based money price index according to,

$$C_s = \left[\int_0^1 C_s(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}} \quad \text{and} \quad P_s = \left[\int_0^1 P_s(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}},$$

where θ stands for the intratemporal elasticity of substitution across goods.

The solution to (1) must satisfy three first order conditions which insure both the internal and external equilibria of the economy at period t. Therefore, we are provided with a consumption based bond Euler equation, a real money demand function depending on consumption and the nominal interest rate, and a supply function for each differentiated good "z":

$$\begin{cases} C_{t+1} = \frac{R_{t+1}}{1+\delta} C_t, \\ \frac{M_t}{P_t} = \chi \frac{I_{t+1}}{I_{t+1}-1} C_t, \\ Y_t^s(z)^{1+\frac{1}{\theta}} = \frac{1}{\kappa} \frac{\theta-1}{\theta} (n C_t + (1-n) C_t^*)^{\frac{1}{\theta}} C_t^{-1}, \end{cases} \quad \text{for all } t \tag{2}$$

where $I_{t+1} = R_{t+1} \frac{P_{t+1}}{P_t}$ features the nominal gross rate of interest between periods t and $t+1$, according to Fischer decomposition. The problem is symmetric for the foreign country, except that variables are presented with a "$*$" exponent.

2.2. The General Equilibrium of the Model

In this simplified setting, public sector decisions affect private sector behavior through money creation. We assume that the Union System of Central Banks combines a Union Central Bank (UCB) and two National Central Banks (NCB). Noting $V_t(i_{ft})$ (resp. $V_t^*(i_{ft})$) the domestic (resp. foreign) per capita money supply increase following a reduction in the UCB fund rate i_{ft}, and assuming no public bonds, we can write the budget constraint of the NCBs according to,

$$T_t = V_t(i_{ft}) \quad \text{and} \quad T_t^* = V_t^*(i_{ft}). \tag{3}$$

We assume that domestic credit is more responsive to fund rate variations in the domestic country so that, $v_t > v_t^* > 0$, where small letters represent the per capita growth rate of domestic credit. The average per capita level of money creation in the Union (V_t^w) is a linear combination of the national level of money creation, weighted by their relative population, i.e.,

$$V_t^w = n\, V_t(i_{ft}) + (1-n)\, V_t^*(i_{ft}). \tag{4}$$

Applying Walras law, the equilibrium conditions of the Monetary Union can be set up ignoring the labour market. The goods market, the unified money market and the financial market clear at any period $s = t$, so that,

$$\begin{cases} n\, C_t + (1-n)\, C_t^* = n\, Y_t(h) + (1-n)\, Y_t^*(f), \\ n\, M_t^d + (1-n)\, M_t^{d*} = M_t^w, \\ n\, B_{t+1} + (1-n)\, B_{t+1}^* = 0, \end{cases} \quad \text{for all } t \tag{5}$$

where the notations "$z = h$" and "$z = f$" have been introduced to define representative home and foreign variables, respectively.

The intertemporal equilibrium condition is given for all t by:

$$\begin{cases} B_{t+1} - B_t = \dfrac{P_t(h)}{P_t}\, Y_t(h) - C_t + (R_t - 1)B_t - \dfrac{(M_t^d - M_{t-1}^d) - V_t}{P_t}, \\ B_{t+1}^* - B_t^* = \dfrac{P_t(f)}{P_t}\, Y_t^*(f) - C_t^* + (R_t - 1)B_t^* - \dfrac{(M_t^{d^*} - M_{t-1}^{d^*}) - V_t^*}{P_t}. \end{cases} \tag{6}$$

It indicates that a country accumulation of net financial claims with respect to the rest of the Monetary Union depends (i) on the difference between the revenue of both activity and claims it already held and the sum of consumption spending and (ii) on the real value of the net money inflow with respect to the level of money creation in this region. Expressed in nominal terms, this net money inflow defines the nominal adjustment variable that balances international payments between Union members. Indeed, a domestic increase of money demand with respect to the NCB money creation induces a net money inflow in this economy, which in turn reduces the private sector revenue available for subscribing net foreign private bonds.

3. THE MACROECONOMIC CONSEQUENCES OF A REDUCTION IN THE UCB FUND RATE

This section focusses on the macroeconomic consequences of a permanent reduction in the UCB fund rate on the general equilibrium of the economy, given regional asymmetries in the monetary mechanism. We assume that prior to this decision, the Monetary Union is in a steady state in which all Union members have already converged towards the same per capita value of their macroeconomic aggregates.

3.1. A Log Linear Framework

The initial steady state is characterized by $C_t = C_0$ and $C_t^* = C_0^*$ for all $t \leq 0$, furthermore we have $C_0 = C_0^*$ which means equal per capita consumption levels. Under this flat consumption profile, the real interest rate equals the rate of time preference, so that $R_1 - 1 = \delta$. Thus at the initial steady state, $C_0 = Y_0(h) + \delta B_1$ and $C_0^* = Y_0^*(f) + \delta B_1^*$. On the other hand, $Y_0(h) = Y_0^*(f) = (\frac{1}{\kappa}(1 - \frac{1}{\theta}))^{\frac{1}{2}}$, $B_1 = B_1^* = 0$. Eventually, $\frac{M_0}{P_0} = \frac{M_0^*}{P_0} = \chi \frac{1+\delta}{\delta} C_0$. In what follows, we define $k_0 = \frac{M_0}{P_0 Y_0} = \chi \frac{1+\delta}{\delta}$ as the inverse of velocity in the steady state. As in this steady state, per capita output is equal in the two countries, the difference between gross domestic products only depends on the size of the populations.

The linearization of the model around its initial steady state is implemented by defining the log deviation of an "X" variable from its steady state value as $x_t = \frac{X_t - X_0}{X_0}$ when $X_0 \neq 0$, so that $X_t \approx X_0 e^{x_t}$ for small values of x_t. When $X_0 = 0$, we choose another variable, say $Z_0 \neq 0$, to be the reference variable. We construct the deviation expression in the following way, $x_t = \frac{X_t - X_0}{Z_0}$ so that $X_t = Z_0 \, x_t$.

We distinguish between the short and long term values of the different variables. In the short run (period $t = 1$), both individual prices and the Union price index levels are fixed (i.e. $p_1(h) = p_1(f) = p_1 = 0$) and activity is determined according to aggregate demand. Prices adjust after just one period, so that activity is supply determined and the economy reaches a new steady state at the beginning of period $t = 2$. Thus for each variable "X", $X_t = X_2 = X_0 e^{x_2}$ for $t \geq 2$. In what follows, we keep the index $t = 1$ to characterize the short term motion of a variable and the index $t = 2$ to define its long term adjustment.

The structural form of the model presented in log deviations from the initial steady state is summarized in Table 1.

These expressions can be combined to express the main aggregates of the model (current account, terms of trade, consumption, activity, inflation and the real component of utility) in terms of the short term value of both average Union consumption growth rate (c_1^w) and in terms of the consumption growth rate differential across Union members ($c_1 - c_1^*$). Results are summarized in Table 2. The solution of the model eventually turns out as finding the values of both c_1^w and $(c_1 - c_1^*)$ in terms of v, v^* and v^w. In the rest of the paper, we assume that the value given to $(v - v^*)$ does not affect that of v^w. Before computing the reduced form of the model, we outline some of its main characteristics by distinguishing between global, international and national consequences of a permanent reduction in the UCB fund rate.

3.2. Global Consequences

As outlined in Table 2, world consumption level determines the global value of the model variables. The value of per capita world consumption is computed, in the IS-LM tradition, by combining a monetary relation — that is obtained by adding short term national money demand equations,

TABLE 1.

The model in terms of log deviation from the steady state

Global relations	
$p_t = n\,p_t(h) + (1-n)\,p_t(f)$	
$n\,c_t + (1-n)\,c_t^* \equiv c_t^w = y_t^w \equiv n\,y_t(h) + (1-n)\,y_t^*(f)$	
$n\,m_t^d + (1-n)\,m_t^{d*} \equiv m_t^{dw} = v_t^w \equiv n\,v_t + (1-n)\,v_t^*$	
$n\,b_t + (1-n)\,b_t^* \equiv b_t^w = 0$	
Domestic relations	**Foreign relations**
$c_2 = c_1 + \frac{\delta}{1-\delta} r_2$	$c_2^* = c_1^* + \frac{\delta}{1-\delta} r_2$
$m_1^d - p_1 = c_1 - \frac{1}{1+\delta}\, r_2 - \frac{1}{\delta}(p_2 - p_1)$	$m_1^{d*} - p_1 = c_1^* - \frac{1}{1+\delta}\, r_2 - \frac{1}{\delta}(p_2 - p_1)$
$m_2^d - p_2 = c_2$	$m_2^{d*} - p_2 = c_2^*$
$(\frac{\theta+1}{\theta}) y_t^s(h) = -c_t + \frac{1}{\theta} c_t^w$	$(\frac{\theta+1}{\theta}) y_t^{s*}(f) = -c_t^* + \frac{1}{\theta} c_t^w$
$y_t^d(h) = \theta[p_t - p_t(h)] + c_t^w$	$y_t^{d*}(h) = \theta[p_t - p_t(f)] + c_t^w$
$c_1 = y_1(h) - b_2 - k_0(m_1^d - v_1)$	$c_1^* = y_1^*(f) - b_2^* - k_0(m_1^{d*} - v_1^*)$
$c_2 = p_2(h) - p_2 + y_2(h) - \delta b_2 -$	$c_2^* = p_2(f) - p_2 + y_2^*(f) - \delta b_2^* -$
$-k_0[m_2^d - m_1^d - v_2]$	$-k_0[m_2^{d*} - m_1^{d*} - v_2^*]$
$u^R = (c_1 - \frac{\theta-1}{\theta} y_1(h)) + \frac{1}{\delta}(c_2 - \frac{\theta-1}{\theta} y_2(h))$	$u^{R*} = (c_1^* - \frac{\theta-1}{\theta} y_1^*(f)) + \frac{1}{\delta}(c_2^* - \frac{\theta-1}{\theta} y_2^*(f))$

TABLE 2.

Semi reduced-expression of the model

Global variables	
$y_1^w = c_1^w$	$p_1 = 0$
$y_2^w = c_2^w = 0$	$p_2 = v_2^w$
International variables	
$b_2 = -\frac{1-n}{n}\, b_2^* = -(1-n)(c_1 - c_1^*) - k_0(m_1^d - v_1)$	
$p_1(h) - p_1(f) = 0$	$p_2(h) - p_2(f) = \frac{1}{\theta+1}(c_1 - c_1^*)$
Domestic variables	**Foreign variables**
$c_1 = c_1^w + (1-n)(c_1 - c_1^*)$	$c_1^* = c_1^w - n(c_1 - c_1^*)$
$c_2 = (1-n)(c_1 - c_1^*)$	$c_2^* = -n(c_1 - c_1^*)$
$y_1 = c_1^w$	$y_1^* = c_1^w$
$y_2 = -(1-n)\frac{\theta}{\theta+1}(c_1 - c_1^*)$	$y_2^* = n\frac{\theta}{\theta+1}(c_1 - c_1^*)$
$u^R = \frac{1}{\theta} c_1^w + (1-n)\big[\frac{\delta(1+\theta)+2\theta}{\delta(1+\theta)}\big](c_1 - c_1^*)$	
	$u^{R*} = \frac{1}{\theta} c_1^w - n\big[\frac{\delta(1+\theta)+2\theta}{\delta(1+\theta)}\big](c_1 - c_1^*)$

after weighting each relation by its country size — with a real sector relation — coming from an accordingly weighted sum of the Euler equations. Both relations link the Union consumption growth rate to that of the real interest rate, given money creation:

$$\begin{cases} c_1^w - \dfrac{1}{1+\delta}\, r_2 = v_1^w + \dfrac{1}{\delta}\, v_2^w, \\ c_1^w + \dfrac{\delta}{1+\delta}\, r_2 = 0. \end{cases} \tag{7}$$

Solving (7) under a permanent reduction in the UCB fund rate (i.e. assuming $v_1^w = v_2^w = v^w$), the solution turns out as,

$$c_1^w = v^w > 0 \qquad \text{and} \qquad r_2 = -\frac{1+\delta}{\delta}\, v^w < 0, \tag{8}$$

introducing this value in Table 2 gives the reduced value of global variables of Table 3.

We can thus remark that the inflation activity trade-off exists on an intertemporal ground at the Union level. As prices are fixed in the short run and because output lies below its competitive level, the entire money creation translates, through an increase in aggregate demand, into activity gains (i.e., $y_1^w = v^w > 0$ and $p_1 = 0$). As soon as prices have become flexible, the economy adjusts to the permanent increase of the growth rate of money supply through a positive inflation rate equal to the permanent growth rate of money supply. Thus, once all adjustments have occurred, there is no gain from money creation as, $p_2 = v^w > 0$ and $y_2^w = 0$. Since θ is the same in both countries, asymmetry in the nations' monetary transmission mechanism is only due to differences in their domestic credit responsiveness to UCB fund rate variations.

3.3. International Consequences

Given the asymmetry in the monetary mechanism, a modification in the UCB fund rate induces international adjustment between countries, which can be assessed according to three main variables: the net money flow across Union members, the short term adjustment of the current account and the long term evolution of the terms of trade. As shown in Table 2, the last two real indicators depend on the consumption growth differential.

To document these issues we construct two relations. They link the net growth rate of money inflow in the benchmark economy to the consumption growth rate differential across Union members in order to keep the intertemporal equilibrium (i) on the goods market (GG schedule) and (ii) on the money market (MM schedule). They are set as,

$$(m_1^d - v_1) \;=\; -(1-n)\,\frac{\delta(\theta+1)+2\theta}{k_0\,\delta\,(\theta+1)}\,(c_1 - c_1^*) + (1-n)\,\frac{1}{\delta}\,(v_2 - v_2^*), \tag{9a}$$

$$(m_1^d - v_1) \;=\; (1-n)\,(c_1 - c_1^*) + (1-n)\,(v_1 - v_1^*). \tag{9b}$$

TABLE 3.

Reduced form of the model

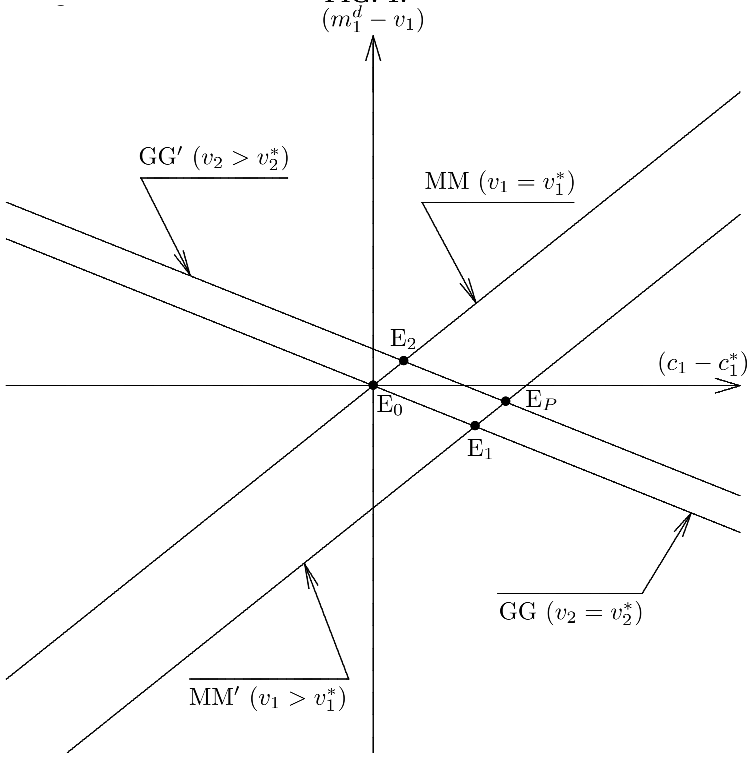
Global variables	
$y_1^w = c_1^w = v^w$	$p_1 = 0$
$y_2^w = c_2^w = 0$	$p_2 = v^w$
International and national variables (transitory asymmetries)	
$b_2 = -\frac{1-n}{n}\, b_2^* = (1-n)\, k_0 \left[\frac{2\theta k_0}{\delta(1+\theta)(1+k_0)+2\theta}\right](v_1 - v_1^*)$	
$p_1(h) - p_1(f) = 0$	$p_2(h) - p_2(f) = \left[\frac{\delta k_0}{\delta(1+\theta)(1+k_0)+2\theta}\right](v_1 - v_1^*)$
$c_1 = v^w + (1-n)\left[\frac{\delta(1+\theta)k_0}{\delta(1+\theta)(1+k_0)+2\theta}\right](v_1 - v_1^*)$	
	$c_1^* = v_1^w - n\left[\frac{\delta(1+\theta)k_0}{\delta(1+\theta)(1+k_0)+2\theta}\right](v_1 - v_1^*)$
$c_2 = (1-n)\left[\frac{\delta(1+\theta)k_0}{\delta(1+\theta)(1+k_0)+2\theta}\right](v_1 - v_1^*)$	$c_2^* = -n\left[\frac{\delta(1+\theta)k_0}{\delta(1+\theta)(1+k_0)+2\theta}\right](v_1 - v_1^*)$
$y_1 = v^w$	$y_1^* = v^w$
$y_2 = -(1-n)\left[\frac{\theta\delta k_0}{\delta(1+\theta)(1+k_0)+2\theta}\right](v_1 - v_1^*)$	$y_2^* = n\left[\frac{\theta\delta k_0}{\delta(1+\theta)(1+k_0)+2\theta}\right](v_1 - v_1^*)$
$u^R = \frac{v^w}{\theta} + (1-n)\left[\frac{k_0[\delta(1+\theta)+2\theta]}{\delta(1+\theta)(1+k_0)+2\theta}\right](v_1 - v_1^*)$	
	$u^{R*} = \frac{v^w}{\theta} - n\left[\frac{k_0[\delta(1+\theta)+2\theta]}{\delta(1+\theta)(1+k_0)+2\theta}\right](v_1 - v_1^*)$
International and national variables (permanent asymmetries)	
$b_2 = -\frac{1-n}{n}\, b_2^* = (1-n)\, k_0 \left[\frac{(\theta-1)-k_0(1+\theta)}{\delta(1+\theta)(1+k_0)+2\theta}\right](v - v^*)$	
$p_1(h) - p_1(f) = 0$	$p_2(h) - p_2(f) = \left[\frac{(1+\delta)k_0}{\delta(1+\theta)(1+k_0)+2\theta}\right](v - v^*)$
$c_1 = v^w + (1-n)\left[\frac{(1+\delta)(1+\theta)k_0}{\delta(1+\theta)(1+k_0)+2\theta}\right](v - v^*)$	
	$c_1^* = v_1^w - n\left[\frac{(1+\delta)(1+\theta)k_0}{\delta(1+\theta)(1+k_0)+2\theta}\right](v - v^*)$
$c_2 = (1-n)\left[\frac{(1+\delta)(1+\theta)k_0}{\delta(1+\theta)(1+k_0)+2\theta}\right](v - v^*)$	$c_2^* = -n\left[\frac{(1+\delta)(1+\theta)k_0}{\delta(1+\theta)(1+k_0)+2\theta}\right](v - v^*)$
$y_1 = v^w$	$y_1^* = v^w$
$y_2 = -(1-n)\left[\frac{\theta(1+\delta)k_0}{\delta(1+\theta)(1+k_0)+2\theta}\right](v - v^*)$	$y_2^* = n\left[\frac{\theta(1+\delta)k_0}{\delta(1+\theta)(1+k_0)+2\theta}\right](v - v^*)$
$u^R = \frac{v^w}{\theta} + \frac{(1-n)(1+\delta)}{\delta}\left[\frac{k_0[\delta(1+\theta)+2\theta]}{\delta(1+\theta)(1+k_0)+2\theta}\right](v - v^*)$	
	$u^{R*} = \frac{v^w}{\theta} - \frac{n(1+\delta)}{\delta}\left[\frac{k_0[\delta(1+\theta)+2\theta]}{\delta(1+\theta)(1+k_0)+2\theta}\right](v - v^*)$

Figure 1 provides a graphical interpretation of the international equilibrium of the Union as defined by Eqs. (9).

The GG schedule (9a) is obtained by combining the long and short term expressions of the current accounts given financial market equilibrium. It indicates that a negative relation is required between $(m_1^d - v_1)$ and $(c_1 - c_1^*)$ to keep the intertemporal equilibrium of the real part of the model. Indeed, an increase in the domestic consumption growth rate with respect to the rest of the Monetary Union deteriorates the current account of the domestic economy. This intertemporally requires a net money outflow from this economy, to balance international payments in the Union.

The MM schedule (9b) is obtained by subtracting short term money demand relations. This partial equilibrium requires a positive relation between $(m_1^d - v_1)$ and $(c_1 - c_1^*)$. Indeed, an increase in domestic relative

FIG. 1.



consumption growth leads, given the structure of money demand equations, to an extra money demand in the domestic economy with respect to the rest of the Union. For a given money supply, the monetary equilibrium of the model requires a net money inflow.

Each relation takes into account the degree of openness $(1 - n)$ of the benchmark economy and is affected by the asymmetry in the monetary mechanism, as long as $v_t \neq v_t^*$. Note that temporary asymmetries affect the MM schedule while long term asymmetries affect the real component of the model (GG schedule). This last feature is original since, in the flexible exchange rate situation studied by Obstfeld and Rogoff (1995, 1996), the real schedule is independent of monetary factors.

First, assuming transitory asymmetries in the monetary mechanism, (i.e. imposing $v_2 = v_2^*$ in (9a)), a permanent UCB fund rate decrease moves the monetary equilibrium schedule MM rightwards in Fig. 1, given the current assumption $v_t > v_t^*$. As GG is unaffected, the new equilibrium, E_1, is defined according to,

$$\begin{cases} (c_1 - c_1^*) = \frac{\delta\,(\theta+1)\,k_0}{\delta\,(\theta+1)\,(1+k_0)+2\,\theta}(v_1 - v_1^*) > 0, \\ (m_1^d - v_1) = -\,\frac{(1-n)\,[\delta\,(1+\theta)\,+2\,\theta]}{\delta\,(\theta+1)\,(1+k_0)+2\,\theta}(v_1 - v_1^*) < 0. \end{cases} \tag{10}$$

It is characterized both by a net domestic money outflow and by an increase in relative consumption. Because of short term price rigidity, the asymmetry in the monetary mechanism leads in turn to higher domestic short term per capita domestic revenue and consumption. Because of consumption smoothing, this last increase is less than proportional to the rate of domestic money increase (solving (9) with $v_1 > v_1^*$ and $v_2 = v_2^* = 0$, we get $\frac{d(c_1 - c_1^*)}{d(v_1 - v_1^*)} < 1$). Thus relative money demand increases less than relative money supply and there is a net money outflow. On the other hand, the relative domestic consumption increase implies a short term domestic current account deficit and a long term improvement in the domestic terms of trade (Table 2).

Second, when asymmetries are permanent, the international consequences of a permanent reduction of the UCB fund rate are obtained by imposing $v_1 = v_2 = v$ and $v_1^* = v_2^* = v^*$ in (9) with $v = v^*$. This moves both GG and MM rightwards in Fig. 1 and the Union reaches a new equilibrium E_P defined according to,

$$\begin{cases} (c_1 - c_1^*) = \frac{(1+\delta)(\theta+1)k_0}{\delta(\theta+1)(1+k_0)+2\theta}(v - v^*) > 0, \\ (m_1^d - v_1) = (1-n)\,\frac{(1+\theta)k_0 - [2\theta+(1+\theta)\delta]}{\delta(\theta+1)(1+k_0)+2\theta}(v - v^*) < 0. \end{cases} \tag{11}$$

E_P is characterized by a greater increase in relative domestic consumption and a smaller net money outflow comparatively to E_1. As consumption differential is reinforced, so are both the short term current account deficit and the long term improvement in the terms of trade. The influence of long term asymmetries on this result can be understood by imposing $v_1 = v_1^*$ in (9b). The GG schedule moves rightwards, while the MM line is unaffected in Fig. 1. The domestic economy experiences a short term money inflow and a relative consumption increase at point E_2, given by,

$$\begin{cases} (c_1 - c_1^*) = \frac{(\theta+1)k_0}{\delta(\theta+1)(1+k_0)+2\theta}(v_2 - v_2^*) > 0, \\ (m_1^d - v_1) = \frac{(1-n)\,(\theta+1)k_0}{\delta(\theta+1)(1+k_0)+2\theta}(v_2 - v_2^*) > 0. \end{cases}$$

In the long run, as more per capita revenue is given to the domestic economy, both relative domestic consumption and terms of trade rise. Because of intertemporal consumption smoothing, part of this extra future revenue is affected to short term consumption, thus creating a current account deficit. On the other hand, this positive consumption differential raises relative domestic money demand. As money supply is fixed in the short run, this induces a net money inflow in the economy, which compensates for part of the short term outflow. Comparing (10) and (11), it should be noted that as asymmetries last, the weight of the real component grows while that of the monetary component decreases in the international adjustment.

Combining (10) (resp. (11)) with the semi reduced expressions of the endogenous variable (Table 2) gives the general intertemporal equilibrium of the Monetary Union under a permanent money supply growth rate according to Table 3, depending upon the duration of asymmetries in national transmission mechanisms.

Efficiency consequences of a reduction in the UCB fund rate are directly measured through the value of the utility deviations u^R and u^{R*} in Table 3. They combine a global component proportional to the average per capita money supply growth rate in the Union — according to the flexible rate situation analyzed in Obstfeld and Rogoff (1995, 1996) — with an international transfer towards the economy that is characterized by the more responsive domestic credit with respect to the ease of the UCB monetary conditions.

4. THE SCOPE FOR REGIONAL CONCERNS IN THE UCB POLICY

This section determines whether this utility transfer between regions is optimal for the Union as a whole (which would establish the superiority of a uniform monetary policy) or whether the UCB must correct it. In what follows, we carry out this analysis in terms of regional per capita money supply increases, without going into the institutional reforms that would be necessary to achieve these values.

4.1. A Global Utility Founded Distribution Scheme

We introduce a mechanism that provides the basis for an agreement between the Union members to access a mutually advantageous redistribution of monetary creation. We assume that the two members of the Monetary Union have to reach an agreement, on some feasible point in the space of utilities, that corresponds to a Pareto situation. Given a monetary expansion v^w, the related feasible set in the utility space is defined as the pairs of utility deviations attainable with some allocation of the considered money expansion (v, v^*). The Pareto frontier corresponds to allocations verifying $n\,v+(1-n)\,v^* = v^w$. Each Pareto situation has an underlying allocation of the given monetary expansion. Given the reduced form of utility deviations presented in Table 3, the Pareto frontier is defined in the (u^R, u^{R*}) space for a given value v^w as,

$$u^{R*} = -\frac{n}{1-n}\,u^R + \frac{1}{1-n}\,\frac{1}{\theta}\,\frac{1}{1+\delta}\,v^w. \qquad (12)$$

According to the terms of this social choice problem, the UCB allocates money so as to fulfill the rules determined in the negotiation process. Although a wide range of solution concepts are available in the literature, we adopt a modified version of the Nash bargaining process (Nash, 1950, 1953). The Nash scheme is an attempt to formalize the outcome of a bargaining process. This choice is motivated by the relative simplicity of the problem and the possibility of introducing — in a very natural way — a single parameter describing the decision of the UCB.

The two members of the Monetary Union have to reach an agreement on some point of the set of Pareto situations described by Equation (12). If the nations do not reach any agreement, the UCB does not create any new money and the Union members stick to their initial stationary utility levels. As a consequence no welfare improvement occurs. The point $(u^R, u^{R*}) = (0, 0)$ in the utility space can be considered as the disagreement point.

In its original form, the Nash scheme proposes a sharing rule for all the situations supporting the relevant formalization. The Nash solution (NS), for a feasible set $\mathcal{D}$ and a disagreement point $d = (d_1, d_2)$ is,

$$NS(\mathcal{D}, d) = \operatorname{argmax} (x_1 - d_1) \cdot (x_2 - d_2) \quad \text{for} \quad (x_1, x_2) \in \mathcal{D}.$$

Expressed in words, the Nash solution of the problem $(\mathcal{D}, d)$ is the feasible vector (x_1, x_2) which maximizes the objective function $(x_1 - d_1)(x_2 - d_2)$. The Nash solution is proved to be the unique solution satisfying a set of axioms. In this paper, we shall skip all the axiomatic aspects of this theory. For an extensive survey on models of bargaining please refer to Thomson (1994).

Here, we are interested in a modified version of the Nash solution in which we introduce some degree of freedom concerning an asymmetric treatment of countries. This is done by assigning weights to the two countries so that our solution solves the following maximization problem,

$$\begin{array}{c} \max\ \left(u^R\right)^{\alpha} \cdot \left(u^{R*}\right)^{1-\alpha} \\ \text{s.t.}\ \ \text{Eq. (12)}. \end{array} \tag{13}$$

The introduction of the parameter α allows an unequal per capita treatment of the two countries.

4.2. The Definition of an Optimal Distribution Rule for the Increase in the Union Money Supply

The unique solution to problem (13) solves,

$$\begin{cases} \frac{\alpha}{1-\alpha} \frac{u^{R*}}{u^R} = \frac{n}{1-n}, \\ u^{R*} + \frac{n}{1-n} u^R = \frac{1}{1-n} \frac{v^w}{\theta}, \end{cases}$$

which implies,

$$u^R = \frac{\alpha}{n}\frac{v^w}{\theta} \qquad \text{and} \qquad u^{R*} = \frac{1-\alpha}{1-n}\frac{v^w}{\theta}\,. \tag{14}$$

An intuitive interpretation of the modified Nash solution can be presented in terms of utility acquisition. Let us suppose that the countries agree to preserve a constant ratio, τ, between their respective utility variations,

$$\frac{u^{R*}}{u^R} = \tau. \tag{15}$$

Given this procedure, and assuming efficiency, the respective utility variations solve,

$$\begin{cases} \dfrac{u^{R*}}{u^R} = \tau, \\ u^{R*} = -\frac{n}{1-n}\,u^R + \frac{1}{1-n}\,\frac{v^w}{\theta}, \end{cases}$$

which implies,

$$u^R = \frac{1}{(1-n)\,\tau + n}\frac{v^w}{\theta} \qquad \text{and} \qquad u^{R*} = \frac{\tau}{(1-n)\,\tau + n}\frac{v^w}{\theta}\,. \tag{16}$$

This solution coincides with that of our modified Nash bargaining problem if,

$$\tau = \frac{n}{1-n}\frac{1-\alpha}{\alpha}. \tag{17}$$

It is worth noting that the choice $\alpha = n$ in the modified Nash bargaining problem leads to $\tau = 1$, which means the adoption of an egalitarian per capita policy in terms of welfare improvement. The choice of $\alpha = 0$ or $\alpha = 1$ would lead to corner solutions (namely, $u^R = 0$ or $u^{R*} = 0$ respectively) independently of the relative size of the countries.

The problem of the UCB is to allocate the monetary expansion between NCBs in a consistent manner with the result of this utility improvement bargaining process. First, when asymmetries are transitory, combining the semi reduced form of u^R in Table 2 with the expressions of u^R in (14), of c_1^w in (8) and of $(c_1 - c_1^*)$ in (10), we have,

$$\frac{\alpha - n}{n} = (1-n)\,\frac{[\delta(\theta+1) + 2\theta]\,k_0\,\theta}{[\delta(\theta+1)(1+k_0) + 2\theta}\frac{v_1 - v_1^*}{v^w}. \tag{18a}$$

Second, when asymmetries are permanent, the same computation with the expression of $(c_1 - c_1^*)$ given by (5) instead of (4), leads to

$$\frac{\alpha - n}{n} = (1-n)\,\frac{1+\delta}{\delta}\,\frac{[\delta(\theta+1) + 2\theta]\,k_0\,\theta}{[\delta(\theta+1)(1+k_0) + 2\theta}\frac{v - v^*}{v^w}. \tag{18b}$$

Following (18a) and (18b), discrepancies in Union members' domestic credit responsiveness with respect to UCB fund rate variations implies an unequal per capita treatment of nations (i.e., $\alpha \neq n$). Regional concerns improve the outcome of the Union monetary policy when the monetary mechanism is asymmetric between member nations. Since the signs of $(\alpha - n)$, $(v_1 - v_1^*)$ and $(v - v^*)$ are equal, the UCB favors the more flexible country, thus compensating the other member's relative inertia. The international adjustment process between Union members allows the less efficient country to benefit from this policy through a net short term money inflow. This increases the global efficiency of the Union monetary mechanism. The regional bias is emphasized if asymmetries are permanent.

5. THE OPTIMAL SHARING RULE

Though the utility based distribution scheme establishes the optimality of a small regional bias favoring the economy with the more sensitive credit reaction to the UCB fund rate, it leaves undetermined the money creation share distributed to each NCB. This section assesses the effective repartition key with respect to the optimization of the UCB objective function, once macroeconomic objectives are combined with resource allocation concerns.

5.1. The Union Central Bank Objective Function

The UCB loss function proves to be a critical aspect of our analysis, since the standard structure encountered in the monetary policy literature (see for example Walsh (1999) or Clarida et al. (1999)) must be adapted to cope with our intertemporal optimizing model and the problem under study. According to the ECB statutes, we assume that the UCB has preferences on activity gains, on inflation losses, and also on resource allocation efficiency. To be consistent with the rest of our analysis, the UCB loss function is a discounted infinite sum of period loss functions that adopts the initial per capita egalitarian steady state of the model as a benchmark.

Furthermore, the nominal anchor of the system is exogenously given by the medium term inflation rate targeted by the authorities. Using the fact that $p_2 = v^w$ (Table 3), we assume, according to the ECB official announcement (European Central Bank (1999)), that the permanent value of the money supply growth rate is moderately positive. As shown in Section 3, this policy induces both short term activity gains and long term inflation. Since the UCB accords relatively more importance to price stability than to output gains, we express the cost Φ_{1t} induced at time t by an increase in the Union money supply according to,

$$\Phi_{1t} = -(Y_s - Y_0) + \lambda_1 \frac{P_s - P_0}{P_0},$$

where $\lambda_1 > 1$ is a policy parameter. Since we have proved that $(Y_s - Y_0)$ and $(P_s - P_0)$ are non-negative for an increase in money supply, we do not need the usual quadratic form here. This structure is also consistent with the log-linearization method, in which quadratic terms are neglected.

As shown above (Table 2 and Section 4), information related to resource allocation among representative agents depends on the difference between each country's per capita consumption level, $C_t - C_t^*$. As this last expression is only related to representative agents, it cannot directly be used as a reliable indicator for international resource reallocation across two unequally populated countries. Indeed, a transfer of one unit of consumption from the foreign country to the domestic country represents a per capita decrease of $1/(1-n)$ unit of consumption in the first country and a per capita increase of $1/n$ unit of consumption in the other one. As a consequence this flow introduces a net per capita bias of

$$\frac{1}{n} - \frac{1}{1-n} = \frac{1-2n}{n(1-n)},$$

between representative agents of the two countries of the Union. Since the UCB can only distinguish between the two countries, a natural way to translate $(C_t - C_t^*)$ in international terms is to divide this expression by the previous bias so that the following term,

$$\Phi_{2t} = \frac{n(1-n)}{(1-2n)}(C_t - C_t^*),$$

can be interpreted as the number of consumption units transferred from the foreign country to the domestic one. This indicator is compatible with the dimensions of $(Y_s - Y_0)$ and $(P_s - P_0)/P_0$.

Φ_{2t} must be introduced in the UCB loss function in a way that features an improvement in the Union allocation of resources. As its sign in the loss function cannot be a priori determined, we premultiply Φ_{2t} by a parameter $\varepsilon \in \{-1, +1\}$. The effective value of this parameter will be fixed below on efficiency grounds to be consistent with (18a) and (18b).

Assuming that the authorities lend a relative weight λ_2 to efficiency gains, we can write the general structure of their loss function for a given period t as a discounted infinite sum of period loss functions combining the two arguments according to,

$$\Lambda_t = \sum_{s=t}^{\infty} R_t \Pi_{\nu=t}^{s} R_{\nu}^{-1} \Big[-(Y_s - Y_0) + \lambda_1 \frac{P_s - P_0}{P_0} + \varepsilon \lambda_2 \frac{n(1-n)}{(1-2n)} (C_s - C_s^*) \Big]. \tag{19}$$

5.2. The Optimal Share of Money Creation

Expanding (19) from period $t = 1$ onwards and linearizing it in the neighborhood of the symmetric steady state, we can write the intertemporal authorities' loss function as,

$$\Lambda_1 = -Y_0 y_1^w + \lambda_1 \frac{1}{\delta} p_2 + \varepsilon \lambda_2 \frac{1+\delta}{\delta} \frac{n(1-n)}{(1-2n)} C_0 (c_1 - c_1^*). \tag{20}$$

Taking into account the reduced form of the global variables presented in Table 3 and the optimal per capita money supply growth rate differential defined by (18a) and (18b), we can rewrite (20) as,

$$\Lambda_1 = [-Y_0 + \lambda_1 \frac{1}{\delta} + \varepsilon\, \lambda_2 \frac{1}{\theta} \frac{(\alpha - n)}{(1-2n)} \frac{(1+\delta)(\theta+1)}{\delta(\theta+1)+2\theta} C_0 \,]\, v^w, \tag{21a}$$

if asymmetries are transitory or as,

$$\Lambda_1 = [-Y_0 + \lambda_1 \frac{1}{\delta} + \varepsilon\, \lambda_2 \frac{1+\delta}{\delta\,\theta} \frac{(\alpha - n)}{(1-2n)} \frac{(1+\delta)(\theta+1)}{\delta(\theta+1)+2\theta} C_0 \,]\, v^w, \tag{21b}$$

if they are permanent.

The minimization of (21a) and (21b) must take into account the fact that the terms between brackets and the money supply shock cannot be treated separately. Although v^w is exogenous, the possibility of having a positive moderate value for this variable clearly depends on the value taken by the terms in brackets. Two cases must be distinguished. First if $\alpha = n$, Λ_1 reduces to $[-Y_0 + \lambda_1 \frac{1}{\delta}]v^w$ and a positive moderate growth rate of the Union money supply cannot be considered as an optimal outcome. Indeed, if the terms in brackets are positive, the optimal policy is to set $v^w = 0$, which implies $u^R = u^{R*} = 0$. Inversely, if the terms in brackets are negative, (21) is minimized for $v^w \to -\infty$, which is unbearable since this would incur utility losses. Once again the optimal policy would be to stick to $v^w = 0$. Finally, if the terms in brackets equal zero, v^w could be moderately positive, but as $\alpha = n$ corresponds to the disagreement point this again induces $v^w = 0$, as shown in Section 4. Second, if $\alpha \neq n$, the only possibility to minimize Λ_1 for a positive moderate rate of money creation in the Union is to set the terms in brackets equal to zero in (21) since, as before, a positive value induces $v^w = 0$, while a negative one should lead to $v^w \to -\infty$, which is unbearable.

The implicit relation between v^w, and $(\alpha - n)$ is such that $v^w = 0$ when $\alpha = n$ and $v^w > 0$ when $\alpha \neq n$. The $\alpha \neq n$ outcome (corresponding to $\Lambda_1 = 0$) Pareto dominates the $\alpha = n$ situation (corresponding to the steady state value $\Lambda_0 = 0$ as $v^w = 0$) since it implies that $u^R > u^{R*} > 0$. Under an inegalitarian money creation distribution, both countries benefit from a welfare improvement. The optimal money creation share is thus defined according to,

$$\alpha = n - \varepsilon(1 - 2n)\, \frac{\theta[\delta(\theta + 1) + 2\theta]}{(1 + \delta)\,(\theta + 1)}\, \frac{\frac{1}{\delta}\lambda_1 - Y_0}{\lambda_2 Y_0}, \tag{22a}$$

if asymmetries are temporary, or by,

$$\alpha = n - \varepsilon(1 - 2n)\, \frac{\delta}{1 + \delta}\, \frac{\theta[\delta(\theta + 1) + 2\theta]}{(1 + \delta)\,(\theta + 1)}\, \frac{\frac{1}{\delta}\lambda_1 - Y_0}{\lambda_2 Y_0}, \tag{22}$$

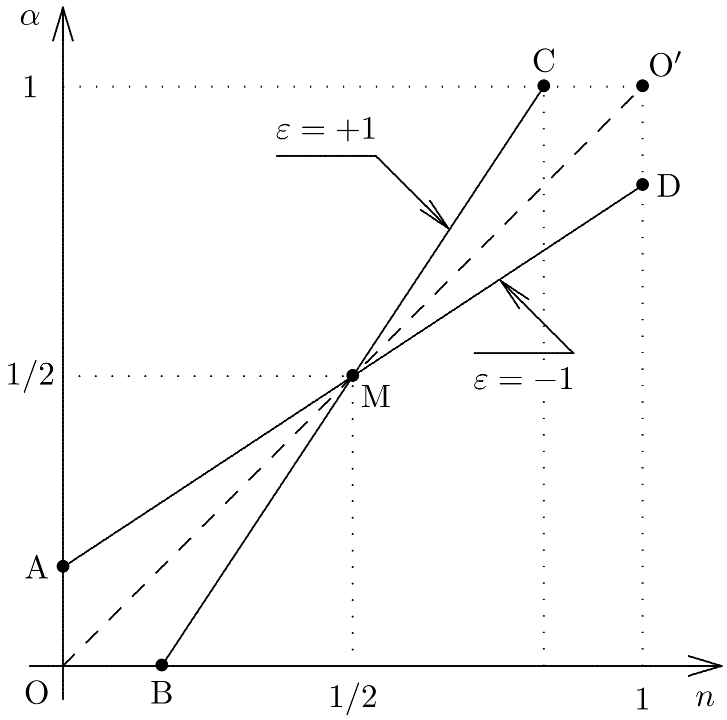
if they are permanent.

To feature an improvement in resource allocation among Union members, the rule (22a) (resp. (22b)) must be consistent with (18a) (resp. (18b)). This requires that $\varepsilon = -1$ in the authorities' loss function.

Figure 2, below, summarizes the model outcomes with respect to the benchmark country's relative size and relative monetary transmission mechanism flexibility. The case that has been extensively studied in this article corresponds to the AM segment. The BM segment describes the situation where the benchmark economy is relatively small and its monetary transmission process is relatively rigid. Inversely, when the benchmark economy is relatively big, the optimal sharing rule is depicted by MC (when its monetary transmission mechanism is relatively flexible) or by MD (when it is relatively rigid). The distance between the relevant segment and the straight dotted line OMO' is an indicator of the weight lent to the net flow of money between Union members as a substitute for the most rigid national mechanism in the transmission of the Union monetary policy. It is related to the macroeconomic output-inflation trade-off in the Monetary Union. One should note that there exists a critical size for which it becomes optimal for the UCB to clearly favor one member for money creation. Note that for transitory asymmetries, the relevant straight lines AD and BC slide towards the broken line OO'.

Analyzing the principles organizing the monetary policy in the European Union with the conclusions of this general intertemporal equilibrium model allows us to outline some interesting features. If the UCB is able to distinguish among member countries with regard to the monetary transmission process, it must favor the most flexible country as the Union benefits as a whole through the international adjustment mechanism in the

FIG. 2.



Union. Otherwise, authorities must implement measures to correct idiosyncrasies in the national transmission mechanisms, thus making a homogeneous monetary policy optimal at the Union level.

6. CONCLUSION

In this paper, we have established a sharing rule for money creation in a two-country Monetary Union. The proposed solution takes into account the fact that the Union Central Bank follows objectives in terms of activity and inflation but also in terms of resource allocation between the participating countries.

As a particularity, the present work integrates a bargaining process in a "New Open Macroeconomics" model that reinforces the link between the positive and the normative dimensions of the analysis. This allows us to introduce a non standard component in the loss function of the monetary authorities. This indicator, which is based upon consumption growth differential, takes into account intra-union transfers between the participating members, which give rise to better resource allocation.

Our result outlines the importance of a regional bias in the definition of the optimal rate of money creation in a Monetary Union, as the possibility of having a moderate positive rate of money creation is only compatible with an unequal per capita distribution scheme between participating countries. We show that efficiency gains arise when more weight is given to the more flexible national monetary transmission mechanism and to international money flows between nations. By allowing the more flexible country to create more money, the Union Central Bank can thus compensate an intertemporal unfavorable macroeconomic arbitrage between inflation and activity, making a moderate increase of the Union money supply - and of Union prices in the medium run - an optimal outcome of monetary policy. This solution Pareto dominates the institutional solution adopted in the European Monetary Union.

To simplify the demonstrations, we have chosen to develop a minimal model, so the present work should be extended to take into account features such as budgetary policies and exchanges with the rest of the world. These elements can be used to define the optimal value of money creation, which is impossible with our framework.

APPENDIX A

The first order conditions

$$U_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+\delta} \right)^{s-t} \left[\ln\, C_s + \chi\, \ln \frac{M_s^d}{P_s} - \frac{\kappa}{2} Y_s(z)^2 \right],$$

subject to the intertemporal budget constraint:

$$P_s R_s B_s + M_{s-1}^d + P_s T_s + P_s(z) Y_s(z) = P_s C_s + M_s^d + P_s B_{s+1}.$$

Let us write down the Lagrangian of this optimization problem,

$$\mathcal{L} = \sum_{s=t}^{\infty} \left(\frac{1}{1+\delta} \right)^{s-t} \left[\ln C_s + \chi\, \ln \frac{M_s^d}{P_s} - \frac{\kappa}{2} Y_s(z)^2 \right] + \sum_{s=t}^{\infty} \lambda_s\, R_t \left[\prod_{\nu=t}^{s} R_\nu^{-1} \right] \cdot \left[P_s C_s + M_s^d + P_s B_{s+1} - \left[P_s R_s B_s + M_{s-1}^d + P_s T_s + P_s(z) Y_s(z) \right] \right].$$

With the transversality condition,

$$\lim_{s \to \infty} \prod_{\nu=t}^{s} R_\nu^{-1} B_{\nu+1} = 0.$$

The control variables are: C_s, B_{s+1}, M_s^d, and $Y_s(z)$ for all $s \geq t$.

The Euler condition.

Let us first differentiate the Lagrangian with respect to the consumption choice C_s versus B_{s+1}.

$$\frac{\partial \mathcal{L}}{\partial C_s} = 0 \Leftrightarrow \left(\frac{1}{1+\delta}\right)^{s-t} \frac{1}{C_s} + \lambda_s R_t \left[\prod_{\nu=t}^{s} R_\nu^{-1}\right] P_s = 0, \quad \text{(A.1)}$$

$$\frac{\partial \mathcal{L}}{\partial C_{s+1}} = 0 \Leftrightarrow \left(\frac{1}{1+\delta}\right)^{s-t+1} \frac{1}{C_{s+1}} + \lambda_{s+1} R_t \left[\prod_{\nu=t}^{s+1} R_\nu^{-1}\right] P_{s+1} = 0, \quad \text{(A.2)}$$

$$\frac{\partial \mathcal{L}}{\partial B_{s+1}} = 0 \Leftrightarrow \lambda_s R_t \left[\prod_{\nu=t}^{s} R_\nu^{-1}\right] P_s - \lambda_{s+1} R_t \left[\prod_{\nu=t}^{s+1} R_\nu^{-1}\right] P_{s+1} R_{s+1} = 0. \quad \text{(A.3)}$$

Dividing Eq. (A.1) by Eq. (A.2),

$$(1+\delta)\frac{C_{s+1}}{C_s} = \frac{\lambda_s}{\lambda_{s+1}} \frac{P_s}{P_{s+1}} R_{s+1}. \quad \text{(A.4)}$$

Equation (A.3) gives,

$$\frac{\lambda_s}{\lambda_{s+1}} = \frac{P_s + 1}{P_s}. \quad \text{(A.5)}$$

Equations (A.4) and (A.5) together give the standard Euler condition,

$$C_{s+1} = \frac{1}{1+\delta} R_{s+1} C_s. \quad \text{(A.6)}$$

Optimal choice of M_s^d/P_s.

Let us differentiate the Lagrangian with respect to M_s^d,

$$\frac{\partial \mathcal{L}}{\partial M_s^d} = \left(\frac{1}{1+\delta}\right)^{s-t} \chi \frac{1/P_s}{M_s^d/P_s} + \lambda_s R_t \Big[\prod_{\nu=t}^{s} R_\nu^{-1}\Big] - \lambda_{s+1} R_t \Big[\prod_{\nu=t}^{s+1} R_\nu^{-1}\Big] = 0. \quad \text{(A.7)}$$

Equations (A.1) and (A.2) give,

$$\begin{cases} \lambda_s = -\left(\frac{1}{1+\delta}\right)^{s-t} \frac{1}{C_s}\frac{1}{P_s} R_t^{-1}\Big[\prod_{\nu=t}^{s} R_\nu\Big], \\ \lambda_{s+1} = -\left(\frac{1}{1+\delta}\right)^{s-t+1} \frac{1}{C_{s+1}}\frac{1}{P_{s+1}} R_t^{-1}\Big[\prod_{\nu=t}^{s+1} R_\nu\Big]. \end{cases} \quad \text{(A.8)}$$

We can thus rewrite Eq. (A.7),

$$\Big(\frac{1}{1+\delta}\Big)^{s-t}\chi\frac{1}{M_s^d/P_s}-\Big(\frac{1}{1+\delta}\Big)^{s-t}\frac{1}{C_s}+\Big(\frac{1}{1+\delta}\Big)^{s-t+1}\frac{1}{C_{s+1}}\frac{P_s}{P_{s+1}}=0,$$

after relevant simplifications and reorganizing the expression,

$$\chi\frac{1}{M_s^d/P_s}=\frac{1}{C_s}-\frac{1}{1+\delta}\frac{1}{C_{s+1}}\frac{P_s}{P_{s+1}}=\frac{1}{C_s}\Big[1-\frac{1}{1+\delta}\frac{C_s}{C_{s+1}}\frac{P_s}{P_{s+1}}\Big].$$

Using the standard Euler condition as developed in Eq. (A.6),

$$\chi\frac{1}{M_s^d/P_s}=\frac{1}{C_s}\Big[1-\frac{1}{R_{s+1}}\frac{P_s}{P_{s+1}}\Big]=\frac{1}{C_s}\Big[1-\frac{1}{R_{s+1}}\frac{1}{\frac{P_{s+1}}{P_s}}\Big]. \tag{A.9}$$

Using the Fischer decomposition, namely $I_{s+1}=(1+i_{s+1})=R_{s+1}(P_{s+1}/P_s)$,

$$\chi\frac{C_s}{M_s^d/P_s}=1-\frac{1}{I_{s+1}}=\frac{I_{s+1}-1}{I_{s+1}}=\frac{i_{s+1}}{1+i_{s+1}}.$$

And we have a functional relation between consumption and M_s/P_s,

$$\frac{M_s^d}{P_s}=\chi\,\frac{I_{s+1}}{I_{s+1}-1}\,C_s. \tag{A.10}$$

Optimal choice of $Y_s(z)$

Let us differentiate the Lagrangian with respect to $Y_s(z)$,

$$\frac{\partial\mathcal{L}}{\partial Y_s(z)}=-\Big(\frac{1}{1+\delta}\Big)^{s-t}\kappa\,Y_s(z)-\lambda_s\,R_t\Big[\prod_{\nu=t}^{s}R_\nu^{-1}\Big]\Big[P_s(z)+Y_s(z)\frac{\partial P_s(z)}{\partial Y_s(z)})\Big]=0. \tag{A.11}$$

Replacing in Eq. (A.11) λ_s as given in System (A.8),

$$\frac{\partial\mathcal{L}}{\partial Y_s(z)}=-\Big(\frac{1}{1+\delta}\Big)^{s-t}\kappa\,Y_s(z)+\Big(\frac{1}{1+\delta}\Big)^{s-t}\frac{1}{C_s}\frac{1}{P_s}\Big[P_s(z)+Y_s(z)\frac{\partial P_s(z)}{\partial Y_s(z)})\Big]=0.$$

After simplifications, and factorizing $P_s(z)$ in the expression between brackets,

$$\kappa\,Y_s(z)=\frac{1}{C_s}\frac{P_s(z)}{P_s}\Big[1+\frac{Y_s(z)}{P_s(z)}\frac{\partial P_s(z)}{\partial Y_s(z)})\Big].$$

In the term between brackets we recognize the elasticity θ_z. For symmetry reasons $\theta_z=\theta$,

$$\kappa\,Y_s(z)=\frac{1}{C_s}\frac{P_s(z)}{P_s}\,(1-\frac{1}{\theta}). \tag{A.12}$$

Using the relation, (cf. Obstfeld Rogoff (1996) Eq. 10 p. 665)

$$\frac{P_s(z)}{P_s} = Y_s(z)^{-1/\theta} \, (C_s^w)^{1/\theta}. \tag{A.13}$$

By substitution of Eq. (A.13) in Eq. (A.12),

$$\kappa \, Y_s(z) = \frac{1}{C_s} \frac{\theta - 1}{\theta} \, Y_s(z)^{-1/\theta} \, (C_s^w)^{1/\theta}.$$

After simplifications,

$$Y_s(z)^{\frac{1+\theta}{\theta}} = \frac{1}{\kappa} \frac{1}{C_s} \frac{\theta - 1}{\theta} \, (C_s^w)^{1/\theta}. \tag{A.14}$$

Let us rewrite Eq. (A.12) for $s = 0$ using the fact that $C_0 = Y_0(z)$ and $P_0(z) = P_0$,

$$\kappa \, Y_0(z) = \frac{1}{Y_0(z)} \frac{P_0(z)}{P_0} \, (1 - \frac{1}{\theta}).$$

Solving for $Y_0(z)$,

$$Y_0(z) = \Big(\frac{1}{\kappa} \frac{\theta - 1}{\theta}\Big)^{\frac{1}{2}}. \tag{A.15}$$

APPENDIX B

Linearization around the steady state

Let us rewrite Eq. (A.6) as,

$$\frac{C_{t+1}}{C_t} = \frac{R_{t+1}}{1+\delta},$$

so that,

$$\frac{C_0 e^{c_{t+1}}}{C_0 e^{c_t}} = \frac{R_1 e^{\frac{dR_{t+1}}{R_1}}}{1+\delta}.$$

Using elementary results of calculus ($e^x = 1 + x + o(x^2)$ for x small enough),

$$1 + c_{t+1} - c_t = \Big(1 + \frac{dR_{t+1}}{R_1}\Big) \frac{R_1}{1+\delta}. \tag{B.1}$$

We denote by ρ_{t+1} the net real rate of interest at time t, (i.e. between t and $t+1$), so that $R_{t+1} = 1 + \rho_{t+1}$. At the initial steady state we have thus, $\rho_1 = \delta$, and $R_1 = 1+\delta$. We denote the deviation of ρ_{t+1} with respect to its steady state value, ρ_1, by r_{t+1}, (i.e. $r_{t+1} = \frac{d\rho_{t+1}}{\rho_1}$). Let us compute the deviation of R_{t+1}, with respect to its steady state value, as a function of the deviation of ρ_{t+1} (the real rate of interest between t and $t+1$),

$$\frac{d\,R_{t+1}}{R_1} = \frac{d\,\rho_{t+1}}{1+\rho_1} = \frac{d\,\rho_{t+1}}{\rho_1}\,\frac{\rho_1}{1+\rho_1} = r_{t+1}\,\frac{\delta}{1+\delta}. \tag{B.2}$$

We substitute the result of Eq. (B.2) in Eq. (B.1) and we obtain the linear expression of the Euler relation,

$$c_{t+1} = \frac{\delta}{1+\delta} r_{t+1} + c_t,$$

and for $t = 1$,

$$c_2 = \frac{\delta}{1+\delta}\, r_2 + c_1. \tag{B.3}$$

Log-linearization of the real money demand function

Using Eq. (A.9), at the initial steady state,

$$\chi\,\frac{P_0\,C_0}{M_0^d} = 1 - \frac{P_0}{R_1\,P_0} = 1 - \frac{1}{1+\delta} = \frac{\delta}{1+\delta}. \tag{B.4}$$

By Eq. (A.9) and using the exponential form, for period s,

$$\chi\,\frac{P_0\,C_0}{M_0^d}\,e^{(p_s+c_s-m_s^d)} = 1 - \frac{1}{R_1}\,e^{(p_s-p_{s+1}-\frac{dR_{s+1}}{R_1})}.$$

Using Eq. (B.4) and applying the linear approximation,

$$\frac{\delta}{1+\delta}(1+p_s+c_s-m_s^d) = 1 - \frac{1}{1+\delta}(1+p_s-p_{s+1}-\frac{dR_{s+1}}{R_1}).$$

Using Eq. (B.2) and after simplifications,

$$\delta\,[\,p_s + c_s - m_s^d] = p_{s+1} - p_s + \frac{\delta}{1+\delta}\,r_{t+1}.$$

Solving for the real deviation of money holdings from the steady state,

$$m_s^d - p_s = c_s - \frac{1}{1+\delta} r_{s+1} - \frac{1}{\delta}(p_{s+1} - p_s),$$

for period 1, this comes as,

$$m_1^d - p_1 = c_1 - \frac{1}{1+\delta} r_2 - \frac{1}{\delta}(p_2 - p_1), \tag{B.5}$$

and for period 2 onwards, as the money demand is insensitive to the nominal rate of interest,

$$m_2^d - p_2 = c_2. \tag{B.6}$$

Log-linearization of the supply function

Let us first rewrite Eq. (A.14) in steady state as,

$$Y_0^s(z) = \Big[\frac{1}{\kappa} \frac{1}{C_0} \frac{\theta - 1}{\theta} (C_0^w)^{\frac{1}{\theta}} \Big]^{\frac{\theta}{1+\theta}}. \tag{B.7}$$

By Eq. (A.14) once more and using the exponential form, for period t,

$$Y_0^s(z) \exp\Big(y_t^s(z) \Big) = \Big[\frac{1}{\kappa} \frac{1}{C_0} \frac{\theta - 1}{\theta} (C_0^w)^{\frac{1}{\theta}} \Big]^{\frac{\theta}{1+\theta}} \exp\Big(\frac{\theta}{1+\theta} (\frac{1}{\theta} c_t^w - c_t) \Big).$$

Using Eq. (B.7), we simplify,

$$\exp\Big(y_t^s(z) \Big) = \exp\Big(\frac{\theta}{1+\theta} (\frac{1}{\theta} c_t^w - c_t) \Big),$$

which is equivalent to,

$$\frac{1+\theta}{\theta} y_t^s(z) = \frac{1}{\theta} c_t^w - c_t. \tag{B.8}$$

Log-linearization of the demand function

In our analysis we concentrate on a representative agent of each economy. Therefore, we shall denote "h" the domestic representative agent and "f" the foreign representative agent. From Eq. (A.13) we have,

$$Y_s^d(h) = \Big(\frac{P_s(h)}{P_s} \Big)^{-\theta} C_s^w, \tag{B.9}$$

and in steady state,

$$Y_0^d(h) = \Big(\frac{P_0(h)}{P_0} \Big)^{-\theta} C_0^w.$$

Equation (B.9) can be rewritten as,

$$Y_0^d(h)\,\exp\Big(y_s^d(h)\Big) = \Big(\frac{P_0(h)}{P_0}\Big)^{-\theta} C_0^w\,\exp\Big(\theta(p_s - p_s(h)) + c_s^w\Big).$$

Using the steady state condition and after simplifications,

$$\exp\Big(y_s^d(h)\Big) = \exp\Big(\theta\,(\,p_s - p_s(h)) + c_s^w\Big),$$

which is equivalent to,

$$y_s^d(h) = \theta\,\big(\,p_s - p_s(h)\big) + c_s^w. \tag{B.10}$$

Log-linearization of the balance of payments

In the text we have,

$$B_{s+1} - B_s = \frac{P_s(h)\,Y_s(h)}{P_s} - C_s + (R_s - 1)\,B_s - \frac{(M_s^d - M_{s-1}^d) - V_s}{P_s}. \tag{B.11}$$

Since $B_0 = V_0 = 0$, $V_s = M_0^d\,v_s$, and $B_{s+1} = C_0\,b_{s+1}$, we can write Eq. (B.11) as,

$$\begin{aligned} C_0[\,b_{s+1} - b_s] \;&=\; \frac{P_0(h)\,Y_0(h)}{P_0} e^{(p_s(h)+y_s(h)-p_s)} - C_0 e^{c_s} \\ &\quad + (R_1 e^{r_s} - 1)\,C_0\,b_s - \frac{M_0^d}{P_0}(e^{m_s^d} - e^{m_{s-1}^d} - v_s)e^{-p_s}. \end{aligned} \tag{B.12}$$

Since $Y_0 = C_0$, and $P_0 = P_0(h)$,

$$\begin{aligned} b_{s+1} - b_s \;&=\; e^{(p_s(h)+y_s(h)-p_s)} - e^{c_s} \\ &\quad + (R_1 e^{r_s} - 1)\,b_s - \frac{M_0^d}{P_0\,C_0}(e^{m_s^d} - e^{m_{s-1}^d} - v_s)e^{-p_s}. \end{aligned}$$

Denoting by k_0 the inverse of the money demand velocity,

$$k_0 = \frac{M_0^d}{P_0\,Y_0} = \frac{M_0^d}{P_0\,C_0}.$$

Taking the Taylor expansion of e^x in the neighborhood of zero,

$$\begin{aligned} b_{s+1} - b_s \;&=\; p_s(h) + y_s(h) - p_s - c_s \\ &\quad + (R_1(1 + r_s) - 1)\,b_s - k_0(m_s^d - m_{s-1}^d - v_s)(1 - p_s). \end{aligned}$$

After relevant cancellations, using the fact that the product of two small quantities can be neglected, in particular, $r_s\, b_s \approx 0$, $v_s\, p_s \approx 0$, and $m_s^d\, p_s = m_{s-1}^d\, p_s \approx 0$, and that $\delta = R_1 - 1$, we can express Eq. (B11) in Log-deviation form,

$$b_{s+1} - b_s = p_s(h) + y_s(h) - p_s - c_s + \delta\, b_s - k_0(m_s^d - m_{s-1}^d - v_s). \quad \text{(B.13)}$$

Thus for the first period of the analysis, as $B_1 = C_0\, b_1$ and $B_1 = 0$, we have $b_1 = 0$,

$$b_2 = y_1(h) - c_1 - k_0(m_1^d - v_1) \Leftrightarrow c_1 = y_1(h) - b_2 - k_0(m_1^d - v_1). \quad \text{(B.14)}$$

Since the economy reaches its new steady state at the beginning of the second period, we have $b_2 = b_s$ for all $s \geq 2$, and in particular $b_2 = b_3$,

$$c_2 = [\,p_2(h) - p_2] + y_2(h) + \delta\, b_2 - k_0(m_2^d - m_1^d - v_2). \quad \text{(B.15)}$$

Log-linearization of the utility function

Following Obstfeld and Rogoff analysis, we restrict the discussion to the real component of utility. Let us first compute the utility level that would be available without the monetary expansion, the economy being at its initial steady state,

$$U_0^R = \sum_{s=0}^{\infty}(\frac{1}{1+\delta})^s[\;\ln\, C_0 - \frac{\kappa}{2}Y_0(h)^2] = \frac{1+\delta}{\delta}\,[\;\ln\, C_0 - \frac{\kappa}{2}Y_0(h)^2]. \quad \text{(B.16)}$$

Let us compute now utility at time $t = 1$, with the monetary shock.

$$U_1^R = [\;\ln\, C_1 - \frac{\kappa}{2}Y_1(h)^2] + \sum_{s=2}^{\infty}(\frac{1}{1+\delta})^{s-1}[\;\ln\, C_2 - \frac{\kappa}{2}Y_2(h)^2].$$

After computation of the summation,

$$U_1^R = [\;\ln\, C_1 - \frac{\kappa}{2}Y_1(h)^2] + \frac{1}{\delta}\,[\;\ln\, C_2 - \frac{\kappa}{2}Y_2(h)^2]. \quad \text{(B.17)}$$

Using the identity $\ln(C_0 e^{c_1}) = \ln(C_0) + c_1$, and the usual approximation $e^{2y} \approx 1 + 2y$,

$$U_1^R = [\;\ln\,(C_0\, e^{c_1}) - \frac{\kappa}{2}\, Y_0(h)^2\, e^{2y_1(h)}] + \frac{1}{\delta}\,[\;\ln\,(C_0\, e^{c_2}) - \frac{\kappa}{2}Y_0(h)^2\, e^{2y_2(h)}].$$

Expanding,

$$\begin{aligned} U_1^R \;&=\; [\;\ln\, C_0 + c_1 - \frac{\kappa}{2}\, Y_0(h)^2\,(1 + 2\, y_1(h))] \\ &\quad + \frac{1}{\delta}[\;\ln\, C_0 + c_2 - \frac{\kappa}{2}Y_0(h)^2\,(1 + 2\, y_2(h))]. \end{aligned}$$

We recognize in the last equation the expression of U_0^R,

$$u_1^R = d\,U_1^R = U_1^R - U_0^R = [\,c_1 - \kappa Y_0(h)^2\ y_1(h)] + \frac{1}{\delta}[\,c_2 - \kappa Y_0(h)^2\ y_2(h)].$$

By Eq. (A.15),

$$u_1^R = [\,c_1 - \frac{\theta-1}{\theta}\ y_1(h)] + \frac{1}{\delta}\,[\,c_2 - \frac{\theta-1}{\theta}\ y_2(h)]. \tag{B.18}$$

Log-linearization of the price index

Since $P_t(z) = P_t(h)$ for all $z \in [0,\, n]$ and $P_t(z) = P_t(f)$ for all $z \in (n,\, 1]$, we have,

$$P_t = \Big[\,n\,P_t(h)^{1-\theta} + (1-n)\,(P_t(f))^{1-\theta}\Big]^{\frac{1}{1-\theta}}. \tag{B.19}$$

At the steady state,

$$P_0 = \Big[\,n\,P_0(h)^{1-\theta} + (1-n)\,(P_0(f))^{1-\theta}\Big]^{\frac{1}{1-\theta}}.$$

We write Eq. (B.19) as,

$$P_0^{(1-\theta)}e^{(1-\theta)p_t} = \Big[\,n\,P_0(h)^{1-\theta}e^{(1-\theta)p_t(h)} + (1-n)\,(P_0(f))^{1-\theta}e^{(1-\theta)p_t(f)}\Big].$$

Using the fact that $P_0 = P_0(h) = P_0(f)$, we simplify,

$$e^{(1-\theta)p_t} = \Big[\,n\,e^{(1-\theta)p_t(h)} + (1-n)\,e^{(1-\theta)p_t(f)}\Big].$$

Applying Taylor expansion formula,

$$1 + (1-\theta)\,p_t = n\,\big[1 + (1-\theta)\,p_t(h)\big] + (1-n)\,\big[1 + (1-\theta)\,p_t(f)\big],$$

and simplifying, leads to,

$$p_t = n\,p_t(h) + (1-n)\,p_t(f). \tag{B.20}$$

APPENDIX C

The variables semi-reduced and reduced forms

C.1 The semi-reduced form

Let us subtract the log-linearized national Euler relations (B.3),

$$c_2 - c_2^* = c_1 - c_1^*. \tag{C.1}$$

C.1.1 International variables

Let us rewrite Eq. (B.14) and its equivalent for the foreign country,

$$\begin{aligned} c_1 &= y_1(h) - b_2 - k_0(m_1^d - v_1), \\ c_1^* &= y_1^*(h) - b_2^* - k_0(m_1^{d*} - v_1^*). \end{aligned}$$

Therefore, as $n\, b_2 + (1-n)\, b_2^* = 0$,

$$(c_1 - c_1^*) = \big[y_1(h) - y_1^*(f)\big] - \frac{1}{1-n}\, b_2 - k_0\big[(m_1^d - v_1) - (m_1^{d*} - v_1^*)\big].$$

Since output is demand determined in the short run, and given the expressions of $y_1^d(h)$ and $y_1^{d*}(f)$ in Table 1, we have,

$$y_1(h) - y_1^*(f) = y_1^d(h) - y_1^{d*}(f) = 0,$$

and,

$$m_1^{dw} = n\, m_1^d + (1-n)\, m_1^{d*} = n\, v_1 + (1-n)\, v_1^* = v_1^w,$$

or,

$$(m_1^{d*} - v_1^*) = -\frac{n}{1-n}(m_1^d - v_1). \tag{C.2}$$

Therefore,

$$(c_1 - c_1^*) = -\frac{1}{1-n}\, b_2 - k_0 \frac{1}{1-n}\, (m_1^d - v_1),$$

solving for b_2 leads to,

$$b_2 = -(1-n)(c_1 - c_1^*) - k_0(m_1^d - v_1). \tag{C.3}$$

C.1.2 Global variables

As prices are flexible in the long run, the level of activity is determined by the supply side. Let us add up the supply functions of the two countries as expressed by Eq. (B.8),

$$\begin{aligned} \frac{1+\theta}{\theta}\big[n\, y_2^s(h) + (1-n)\, y_2^{s*}(f)\big] &= -\big[n\, c_2 + (1-n)\, c_2^*\big] + \frac{1}{\theta}\, c_2^w \\ &= -c_2^w + \frac{1}{\theta}\, c_2^w = \frac{1-\theta}{\theta}\, c_2^w. \end{aligned}$$

Therefore,

$$\big[\, n\, y_2^s(h) + (1-n)\, y_2^{s*}(f)\big] = y_2^w = \frac{1-\theta}{1+\theta}\, c_2^w.$$

Since at the equilibrium $y_2^w = c_2^w$,

$$y_2^w = c_2^w = 0. \tag{C.4}$$

In the long term, following the Classical model, the price level is determined on the money market. Let us add up the equilibrium conditions of the countries as expressed by Eqs. (B.6),

$$\left[n\, m_2^d + (1-n)\, m_2^{d*}\right] - p_2 = \left[n\, c_2 + (1-n)\, c_2^*\right] = c_2^w = 0.$$

Thus,

$$p_2 = \left[n\, m_2^d + (1-n)\, m_2^{d*}\right] = v_2^w, \tag{C.5}$$

where v_2^w denotes the money supply growth rate at time $t = 2$ at the world level.

In the short run, as prices are fixed ($p_1 = 0$), the level of activity is demand determined, i.e. $y_1^w = c_1^w$.

C.1.3 National variables

Let us consider first the Aoki Decomposition,

$$\begin{aligned} & x^w = n\, x + (1-n)\, x^* = n\, x + x^* - n\, x^* \\ \Leftrightarrow\ & x^* = x^w - n\,(x - x^*) \\ \Leftrightarrow\ & x^* + x - x = x^w - n\,(x - x^*) \\ \Leftrightarrow\ & x - (x - x^*) = x^w - n\,(x - x^*) \\ \Leftrightarrow\ & x = x^w + (1-n)\,(x - x^*). \end{aligned} \tag{C.6}$$

The second line of Eq. (C.6) will be used for the foreign country, while the last one will be kept for the domestic country.

Let us apply the Aoki decomposition to the long run analysis,

$$\begin{cases} c_2 = c_2^w + (1-n)\,(c_2 - c_2^*) = 0 + (1-n)\,(c_1 - c_1^*), \\ c_2^* = c_2^w - n\,(c_2 - c_2^*) = 0 - n\,(c_1 - c_1^*), \end{cases} \tag{C.7}$$

and,

$$\begin{cases} y_2 = y_2^w + (1-n)\,(y_2 - y_2^*) = (1-n)\,(y_2 - y_2^*), \\ y_2^* = y_2^w - n\,(y_2 - y_2^*) = -n\,(y_2 - y_2^*). \end{cases}$$

As in the long run, the level of activity is determined by the supply side,

$$\frac{1+\theta}{\theta}\,(y_2 - y_2^*) = -(c_2 - c_2^*) \quad \Leftrightarrow \quad (y_2 - y_2^*) = -\frac{\theta}{1+\theta}\,(c_2 - c_2^*).$$

We have thus,

$$\begin{cases} y_2 = -(1-n)\,\frac{\theta}{1+\theta}\,(c_1 - c_1^*), \\ y_2^* = n\,\frac{\theta}{1+\theta}\,(c_1 - c_1^*). \end{cases} \tag{C.8}$$

In the short run,

$$\begin{cases} c_1 = c_1^w + (1-n)\,(c_1 - c_1^*), \\ c_1^* = c_1^w - n\,(c_1 - c_1^*). \end{cases} \tag{C.9}$$

and,

$$\begin{cases} y_1 = y_1^w + (1-n)\,(y_1 - y_1^*), \\ y_1^* = y_1^w - n\,(y_1 - y_1^*). \end{cases} \tag{C.10}$$

Since $(y_1^* - y_1) = 0$,

$$\begin{cases} y_1 = y_1^w = c_1^w, \\ y_1^* = y_1^w = c_1^w. \end{cases} \tag{C.11}$$

Combining Eqs. (C.10) and (C.11) with Eq. (B.18),

$$\begin{aligned} u_1^R \;&=\; [c_1^w + (1-n)\,(c_1 - c_1^*) - \frac{\theta - 1}{\theta}\,c_1^w] \\ &\quad + \frac{1}{\delta}\,[(1-n)\,(c_1 - c_1^*) + \frac{\theta - 1}{\theta + 1}\,(1-n)\,(c_1 - c_1^*)]. \end{aligned}$$

Gathering relevant terms,

$$u_1^R = [1 - \frac{\theta - 1}{\theta}\,]\,c_1^w + (1-n)\left[1 + \frac{1}{\delta} + \frac{1}{\delta}\,\frac{\theta - 1}{\theta + 1}\right](c_1 - c_1^*).$$

And finally,

$$u_1^R = \frac{1}{\theta}\,c_1^w + (1 - n\,)\,\frac{\delta\,(1+\theta) + 2\,\theta}{\delta\,(1+\theta)}\,(c_1 - c_1^*). \tag{C.12}$$

C.2 The reduced form

C.2.1 Global variables

Summing up the two Euler conditions, and using the fact that $c_2^w = 0$,

$$0 = c_2^w = c_1^w + \frac{\delta}{1+\delta}\,r_2. \tag{C.13}$$

In the short run $m_1^{dw} = v_1^w$. We sum up the demand function of money, Eq. (B.5), and its equivalent for the foreign country,

$$m_1^{dw} - p_1 = v_1^w = c_1^w - \frac{1}{1+\delta}\,r_2 - \frac{1}{\delta}\,(p_2 - p_1).$$

The price is constant in the first period, $p_1 = 0$, thus,

$$m_1^{dw} = v_1^w = c_1^w - \frac{1}{1+\delta}\, r_2 - \frac{1}{\delta}\, p_2.$$

Assuming that the monetary expansion is permanent, that is $v^w = v_1^w = v_2^w$, and using $p_2 = v_2^w$,

$$c_1^w - \frac{1}{1+\delta}\, r_2 = v_1^w + \frac{1}{\delta}\, v_2^w = \frac{1+\delta}{\delta}\, v^w. \tag{C.14}$$

Equations (C.13) and (C.14) form a system of equations the unknown variables of which are c_1^w and r_2,

$$\begin{pmatrix} 1 & \frac{\delta}{1+\delta} \\ 1 & -\frac{1}{1+\delta} \end{pmatrix} \begin{pmatrix} c_1^w \\ r_2 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1+\delta}{\delta}\, v^w \end{pmatrix},$$

we solve,

$$\begin{pmatrix} c_1^w \\ r_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{1+\delta} & \frac{\delta}{1+\delta} \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{1+\delta}{\delta}\, v^w \end{pmatrix} = \begin{pmatrix} v^w \\ -\frac{1+\delta}{\delta}\, v^w \end{pmatrix}. \tag{C.15}$$

C.2.2 International variables

— Computation of the GG equation.

Let us write Eq. (B.15) and its equivalent for the foreign country,

$$c_2 = p_2(h) - p_2 + y_2(h) + \delta\, b_2 - k_0 \left[m_2^d - m_1^d - v_2\right],$$

$$c_2^* = p_2(f) - p_2 + y_2^*(f) + \delta\, b_2^* - k_0 \left[m^{d*} - m_1^{d*} - v_2^*\right].$$

The two last equations lead to,

$$\begin{aligned} c_2 - c_2^* \;=\; & \left[p_2(h) - p_2(f)\right] + \left[y_2(h) - y_2^*(f)\right] + \delta \left[b_2 - b_2^*\right] \\ & -k_0\left[m_2^d - m_2^{d*}\right] + k_0\left[m_1^d - m_1^{d*}\right] + k_0\left[v_2 - v_2^*\right]. \end{aligned}$$

Using the relations

$$\begin{cases} n\, b_2 + (1-n)\, b_2^* = 0 \quad \Leftrightarrow \quad b_2^* = -\frac{n}{1-n}\, b_2, \\ (m^d - m_2^{d*}) = (c_2 - c_2^*), \\ (m_1^d - m_1^{d*}) = (c_1 - c_1^*). \end{cases}$$

$$\begin{aligned} c_2 - c_2^* \;=\; & \left[p_2(h) - p_2(f)\right] + \left[y_2(h) - y_2^*(f)\right] + \frac{1}{1-n}\, \delta\, b_2 \\ & -k_0(c_2 - c_2^*) + k_0\left[c_1 - c_1^*\right] + k_0\left[v_2 - v_2^*\right]. \end{aligned}$$

By Eq. (C.1),

$$c_1 - c_1^* = \big[p_2(h) - p_2(f)\big] + \big[y_2(h) - y_2^*(f)\big] + \frac{1}{1-n}\,\delta\, b_2 + k_0(v_2 - v_2^*). \quad \text{(C.16)}$$

Equation (B.8) gives,

$$y_2^s(h) - y_2^{s*}(f) = -\frac{\theta}{1+\theta}\,(c_2 - c_2^*) = -\frac{\theta}{1+\theta}\,(c_1 - c_1^*),$$

and Eq. (B.10),

$$y_2^d(h) - y_2^{d*}(f) = -\theta\,[\,p_2(h) - p_2(f)].$$

Since in the long run, $y_2^d(h) = y_2^s(f)$ and $y_2^{d*}(h) = y_2^{s*}(f)$, we have,

$$\begin{cases} p_2(h) - p_2(f) = \frac{1}{1+\theta}\,(c_1 - c_1^*), \\ y_2(h) - y_2^*(f) = -\frac{\theta}{1+\theta}\,(c_1 - c_1^*). \end{cases} \quad \text{(C.17)}$$

Introducing these results in Eq. (C.16),

$$c_1 - c_1^* = \big[\frac{1}{1+\theta}\,(c_1 - c_1^*)\big] + \big[-\frac{\theta}{1+\theta}\,(c_1 - c_1^*)\big] + \frac{1}{1-n}\,\delta\, b_2 + k_0(v_2 - v_2^*).$$

Gathering the terms related to $(c_1 - c_1^*)$ and replacing the expression of b_2 by the short run expression found above Eq. (C.3), we can write,

$$\begin{aligned}(c_1 - c_1^*) \; &= \; -\frac{k_0}{1-n}\,(m_1^d - v_1) \\ &\quad -\frac{1}{1-n}\Big[(1-n)\,\frac{1}{\delta}\,\frac{2\,\theta}{1+\theta}\,(c_1 - c_1^*) - (1-n)\,\frac{k_0}{\delta}\,(v_2 - v_2^*)\Big].\end{aligned}$$

After simplifications,

$$(c_1 - c_1^*)\big[1 + \frac{1}{\delta}\,\frac{2\,\theta}{1+\theta}\big] = \frac{k_0}{\delta}\,(v_2 - v_2^*) - \frac{k_0}{1-n}\,(m_1^d - v_1).$$

Isolating $(m_1^d - v_1)$ we obtain the GG equation,

$$(m_1^d - v_1) = -(1-n)\,\Big[\frac{\delta\,(1+\theta) + 2\,\theta}{\delta(1+\theta)\,k_0}\Big](c_1 - c_1^*) + (1-n)\,\frac{1}{\delta}\,(v_2 - v_2^*). \quad \text{(C.18)}$$

— Computation of the MM equation Rewriting Eq. (B.5) and its foreign counterpart:

$$\begin{cases} m_1^d - p_1 = c_1 - \frac{1}{1+\delta}\left[r_2 + \frac{1+\delta}{\delta}(p_2 - p_1)\right], \\ m_1^{d*} - p_1 = c_1^* - \frac{1}{1+\delta}\left[r_2 + \frac{1+\delta}{\delta}(p_2 - p_1)\right]. \end{cases}$$

subtracting,

$$m_1^d - m_1^{d*} = c_1 - c_1^*. \tag{C.19}$$

We rewrite Eq. (C.19) in the following form,

$$(m_1^d - v_1) - (m_1^{d*} - v_1^*) + (v_1 - v_1^*) = (c_1 - c_1^*), \tag{C.20}$$

and substitute Eq. (C.16) in Eq. (C.20),

$$\frac{1}{1-n}(m_1^d - v_1) = (c_1 - c_1^*) - (v_1 - v_1^*).$$

And finally the MM equation comes as,

$$(m_1^d - v_1) = (1-n)(c_1 - c_1^*) - (1-n)(v_1 - v_1^*). \tag{C.21}$$

— The GG–MM System

The system of Eqs. (C.18) and (C.21) is a system of two equations the unknown variables of which are $(m_1^d - v_1)$ and $(c_1 - c_1^*)$. Let us write this system under its matrix form,

$$\begin{pmatrix} 1 & (1-n)\left[\frac{\delta(1+\theta)+2\theta}{\delta(1+\theta)k_0}\right] \\ 1 & -(1-n) \end{pmatrix} \begin{pmatrix} m_1^d - v_1 \\ c_1 - c_1^* \end{pmatrix} = \begin{pmatrix} (1-n)\frac{1}{\delta}(v_2 - v_2^*) \\ -(1-n)(v_1 - v_1^*) \end{pmatrix}.$$

After a matrix inversion,

$$\begin{aligned} \begin{pmatrix} m_1^d - v_1 \\ c_1 - c_1^* \end{pmatrix} &= \frac{-\delta(1+\theta)k_0}{(1-n)\left[\delta(1+\theta)(1+k_0)+2\theta\right]} \\ &\quad \cdot \begin{pmatrix} -(1-n) & -(1-n)\left[\frac{\delta(1+\theta)+2\theta}{\delta(1+\theta)k_0}\right] \\ -1 & 1 \end{pmatrix} \begin{pmatrix} (1-n)\frac{1}{\delta}(v_2 - v_2^*) \\ -(1-n)(v_1 - v_1^*) \end{pmatrix}. \end{aligned}$$

So,

$$\begin{cases} (m_1^d - v_1) = & \frac{\delta(1+\theta)k_0}{\left[\delta(1+\theta)(1+k_0)+2\theta\right]}(1-n) \\ & \left[\frac{1}{\delta}(v_2 - v_2^*) - \left[\frac{\delta(1+\theta)+2\theta}{\delta(1+\theta)k_0}\right](v_1 - v_1^*)\right], \\ (c_1 - c_1^*) = & \frac{\delta(1+\theta)k_0}{\left[\delta(1+\theta)(1+k_0)+2\theta\right]}\left[\frac{1}{\delta}(v_2 - v_2^*) + (v_1 - v_1^*)\right]. \end{cases}$$

For permanent asymmetries, $v_1 = v_2 = v$ and $v_1^* = v_2^* = v^*$, the last equation becomes,

$$(c_1 - c_1^*) = \frac{(1+\delta)\,(1+\theta)\,k_0}{\delta\,(1+\theta)\,(1+k_0) + 2\,\theta}\,(v - v^*). \tag{C.22}$$

For transitory asymmetries, since $v_2 = v_2^*$, we have,

$$(c_1 - c_1^*) = \frac{\delta\,(1+\theta)\,k_0}{\delta\,(1+\theta)\,(1+k_0) + 2\,\theta}\,(v_1 - v_1^*). \tag{$C.22'$}$$

In the following we concentrate on the computations related to a permanent asymmetry situation. In the case of temporary asymmetries, we only need to use Eq. (C.22′) instead of Eq. (C.22).

Thus, combining Eqs. (C.22) and (C.3) gives after factorization,

$$p_2(h) - p_2(f) = \frac{(1+\delta)\,k_0}{\delta(1+\theta)(1+k_0) + 2\theta}\,(v - v^*). \tag{C.23}$$

Combining Eqs. (C.22) and (C.4) gives,

$$b_2 = (1-n)\,\frac{1}{\delta}\,\frac{2\,\theta}{1+\theta}\,\frac{(1+\delta)(1+\theta)k_0}{\delta(1+\theta)(1+k_0) + 2\theta}\,(v - v^*) - (1-n)\,\frac{1}{\delta}\,k_0\,(v - v^*).$$

We factorize,

$$b_2 = (1-n)\,\frac{1}{\delta}\,k_0\left[\frac{2\,\theta\,(1+\delta)}{\delta(1+\theta)(1+k_0) + 2\theta} - 1\right](v - v^*),$$

which finally gives,

$$b_2 = (1-n)\,k_0\left[\frac{(\theta-1) - k_0(1+\theta)}{\delta(1+\theta)(1+k_0) + 2\theta}\right](v - v^*). \tag{C.24}$$

C.2.3 The national variables

Combining Eqs. (C.7) and (C.22),

$$\begin{cases} c_2 = (1-n)\,\frac{(1+\delta)\,(1+\theta)\,k_0}{\delta(1+\theta)(1+k_0)+2\theta}\,(v - v^*), \\ c_2^* = -n\,\frac{(1+\delta)\,(1+\theta)\,k_0}{\delta(1+\theta)(1+k_0)+2\theta}\,(v - v^*). \end{cases} \tag{C.25}$$

Combining Eqs. (C.8) and (C.22),

$$\begin{cases} y_2 = -(1-n)\,\frac{\theta(1+\delta)\,k_0}{\delta(1+\theta)(1+k_0)+2\theta}\,(v - v^*), \\ y_2^* = n\,\frac{\theta(1+\delta)\,k_0}{\delta(1+\theta)(1+k_0)+2\theta}\,(v - v^*). \end{cases} \tag{C.26}$$

Combining Eqs. (C.9) and (C.22),

$$\begin{cases} c_1 = v^w + (1-n)\,\frac{(1+\delta)\,(1+\theta)\,k_0}{\delta(1+\theta)(1+k_0)+2\theta}\,(v-v^*), \\ c_1^* = v^w - n\,\frac{(1+\delta)\,(1+\theta)\,k_0}{\delta(1+\theta)(1+k_0)+2\theta}\,(v-v^*). \end{cases} \tag{C.27}$$

Combining Eqs. (C.12), (C.15), and (C.22),

$$\begin{cases} u^R = \frac{1}{\theta}\,v^w + (1-n)\,\frac{1+\delta}{\delta}\,k_0\left[\frac{\delta(1+\theta)+2\theta}{\delta\,(1+\theta)\,(1+k_0)+2\,\theta}\right](v-v^*), \\ u^{R*} = \frac{1}{\theta}\,v^w - n\,\frac{1+\delta}{\delta}\,k_0\left[\frac{\delta\,(1+\theta)+2\,\theta}{\delta\,(1+\theta)\,(1+k_0)+2\,\theta}\right](v-v^*). \end{cases} \tag{C.28}$$

APPENDIX D

The loss function

The general form of the loss function is:

$$\Lambda_t = \sum_{s=t}^{\infty} R_t \Pi_{v=t}^{s} R_v^{-1}\Big[-(Y_s - Y_0) + \lambda_1 \frac{P_s - P_0}{P_0} + \varepsilon\lambda_2 \frac{n(1-n)}{(1-2n)}(C_s - C_s^*)\Big],$$

Developing this expression leads to,

$$\begin{aligned} \Lambda_1 \;=\; & -(Y_1 - Y_0) + \lambda_1 \frac{P_1 - P_0}{P_0} + \varepsilon\,\lambda_2 \frac{n(1-n)}{(1-2n)}(C_1 - C_1^*) \\ & + \frac{1}{(1+\rho_2)}[-(Y_2 - Y_0) + \lambda_1 \frac{P_2 - P_0}{P_0} + \varepsilon\,\lambda_2 \frac{n(1-n)}{(1-2n)}(C_2 - C_2^*)] \\ & + \frac{1}{(1+\rho_2)(1+\rho_3)}[-(Y_3 - Y_0) + \lambda_1 \frac{P_3 - P_0}{P_0} + \varepsilon\,\lambda_2 \frac{n(1-n)}{(1-2n)}(C_3 - C_3^*)] \\ & + \frac{1}{(1+\rho_2)(1+\rho_3)(1+\rho_4)} \\ & \times[-(Y_4 - Y_0) + \lambda_1 \frac{P_4 - P_0}{P_0} + \varepsilon\,\lambda_2 \frac{n(1-n)}{(1-2n)}(C_4 - C_4^*)] \\ & + \cdots \end{aligned}$$

we use the fact that : $Y_t = Y_0 e^{y_t^w} = Y_0(1 + y_t^w)$ with $y_t^w = 0$, for $t > 1$, $P_t = P_0 e^{p_t} = P_0(1 + p_t)$ with $p_1 = 0$ and $p_t = p_2$, for $t \geq 2$, $C_t - C_t^* = C_0(c_1 - c_1^*)$ for all t. Similarly, $(1+\rho_t)^{-1} = R_1^{-1} e^{-r_t} = R_1^{-1}(1 - r_t)$. We found previously that $r_t = 0$ for $t > 2$, thus we can write that $\frac{1}{(1+\rho_2)} = R_1^{-1}(1 - r_2)$, $\frac{1}{(1+\rho_2)(1+\rho_3)} = R_1^{-2}(1 - r_2)$, $\frac{1}{(1+\rho_2)(1+\rho_3)(1+\rho_4)} = R_1^{-3}(1 - r_2)$ etc.

Taking into account all these elements and gathering relevant terms, we can write the previous expression as,

$$\begin{aligned}\Lambda_1 &= -Y_0\, y_1^w + \lambda_1\, p_2\, R_1^{-1}[1 + R_1^{-1} + R_1^{-2} + R_1^{-3} + \cdots] \\ &\quad + \varepsilon\, \lambda_2\, \frac{n\,(1-n)}{(1-2n)}\, C_0\,(c_1 - c_1^*)[1 + R_1^{-1} + R_1^{-2} + R_1^{-3} + \cdots]\end{aligned}$$

as $r_2\, p_2 \approx 0$, $r_2\, c_1 \approx 0$ and $r_2\, c_1 \approx 0$. It thus comes,

$$\Lambda_1 = -Y_0\, y_1^w + \lambda_1\, \frac{1}{\delta}\, p_2 + \varepsilon\, \lambda_2\, \frac{1+\delta}{\delta}\, \frac{n(1-n)}{(1-2n)} C_0\,(c_1 - c_1^*),$$

which proves Eq. (20).

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