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Macroeconomic Policies and Foreign Asset Accumulation in a Finite-Horizon Model*

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This paper considers foreign asset holdings and macroeconomic policies in a finite-horizon model with real balances and foreign asset holdings in a small open economy. Both the long- and short-run effects of these macroeconomic policies on the economy are reexamined. The main results stand in striking contrast to those of Obstfeld (1981), who used an endogenous time preference. (1) Foreign exchange intervention leads to more foreign asset holdings and more consumption in the long run. However, it affects foreign asset accumulation ambiguously. (2) Inflation results in more foreign asset accumulation and consumption, but the effect of inflation on real balance holdings is ambiguous. (3) Government spending affects foreign asset accumulation ambiguously, and it always reduces real balances and crowds out private consumption.

Key Words: Foreign asset; Finite-horizon model; Macroeconomic policies.
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1. INTRODUCTION

This paper reexamines the effects of macroeconomic policies on foreign asset accumulation in a small open economy. It obtains policy implications

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that are different from those in many existing studies, such as Turnovsky (1985, 1987) and, in particular, Obstfeld's (1981) model.

In an often-cited paper, Obstfeld (1981) presents three interesting results regarding the effects of government policies on foreign asset holdings: (1) foreign exchange intervention is found to have no real effects when official foreign reserves earn interest that is distributed to the public; (2) inflation leads to higher long-run consumption and foreign claims; and (3) an increase in government spending induces a surplus on current accounts in the short-run and larger foreign asset accumulation in the long-run. The intertemporal optimization framework used by Obstfeld (in this study and in related studies [Obstfeld, 1982, 1990]) has also influenced open-economy macroeconomics in the past decade.

It is well known that Obstfeld's use of an endogenous time preference in a framework that includes money stocks was erroneous. In particular, Obstfeld uses the dimensionality reduction suggested by Uzawa (1968). However, because Obstfeld's problem was non-autonomous, the conditions for the dimensionality reduction do not hold. Kompas and Francis (2001) and Mendoza (1995) provide a solution to this problem and find similar effects to Obstfeld (1981).

However, many economists have raised doubts about Uzawa's assumption. For example, Blanchard and Fischer (1989) state that, for Uzawa's specification, "in steady state, a higher level of consumption implies a higher rate of time preference. The assumption is difficult to defend a priori; indeed, we usually think it is the rich who are more likely to be patient. ... The Uzawa function ... is not particularly attractive as a description of preferences and is not recommended for general use." (pp. 72-75).

In this paper, we reexamine the policy implications of Obstfeld's model hinging on the special assumption of Uzawa's (1968) time preference. The analysis pursued here is based on the extension of an infinite-horizon framework to a finite-horizon framework. In sharp contrast to Obstfeld (1981), we find that the finite-horizon generates results that are more consistent with the Mundell-Flemming framework.

The well-known continuous-time finite-horizon model belongs to Yarri (1965) and Blanchard (1985), whose studies examine the dynamics of consumption and capital accumulation in using this finite-horizon model. Since these studies were published, many economic topics have been reexamined under this framework. For example, Weil (1989, 1993) examines the effects of monetary policy on the economy and presents the non-neutrality of monetary growth in the finite-horizon framework. Buiter (1988) also reexamines debt neutrality under this framework. Saint-Paul (1992) discusses the effects of macroeconomic policies on growth in an endogenous growth model with a finite-horizon framework. Heijdra and Ward (2005) construct an overlapping generations model for a small open economy that incorporates a realistic description of the mortality process to study analytically a number of typical shocks that affect such an economy.

This paper reexamines macroeconomic policies and exchange rate dynamics under the finite-horizon framework. The main results derived from our model stand in striking contrast to those of Obstfeld (1981): (1) foreign exchange intervention leads to more foreign asset holdings and more consumption in the long run, although, it affects foreign asset accumulation ambiguously; (2) inflation results in more foreign asset accumulation and consumption, but the effect of inflation on real balance holdings is ambiguous; and (3) government spending affects foreign asset accumulation ambiguously, and it always reduces real balances and crowds out private consumption.

We also utilize the approach developed by Judd (1982) and Cui and Gong (2006) to quantify the short-run effects of government policy shocks on real variables. Although many studies have relied on phase diagrams or long-run equilibrium to derive certain qualitative, short-run analysis, with the help of the Laplace transform, we can provide an exact quantitative expression for the short-run effects on the current account of different (temporary or permanent, present or future) shocks. Using this approach, we derive many interesting findings regarding the short-run impact of different shocks on the current account in the Yaari-Blanchard model: (1) a permanent increase in the future monetary growth rate will increase current consumption and decrease current real balance holdings, but will have a positive effect on current foreign asset accumulation; (2) a permanent increase in government spending will increase current real balance holdings and decrease current consumption, but will have an ambiguous effect on foreign asset accumulation; and (3) a permanent increase in the central bank's reserves will decrease current real balance holdings and increase current consumption, but will have an ambiguous effect on foreign asset accumulation.

The rest of this paper is organized as follows. Section 2 sets up the basic finite-horizon model with real balance and foreign asset holdings in a small open economy. The dynamics for per capita consumption, assets, and real balance holdings are also derived in this section. Section 3 presents the macroeconomic equilibrium and derives the full dynamics of the economy. Section 4 presents a detailed comparative study of the effects of macroeconomic policies on the economy. In Section 5, we demonstrate the short-run responses of consumption, asset holding, and real balance holdings to different macroeconomic policy shocks. We conclude our paper in Section 6.

2. THE MODEL

In this paper, we follow Blanchard (1985), Weil (1989, 1991), Obstfeld (1981), and Saint-Paul (1992) to set up our model. First, we describe the model briefly.

2.1. Consumers

In an open economy, at any time, there is a continuum of generations indexed by the date on which they were born, s. People have an infinite horizon, but die with a constant probability per unit of time, λ. If the probability of death is constant, then the expected remaining life for an agent of any age is given by $\int_0^\infty t\lambda e^{-\lambda t}dt = 1/\lambda$. Therefore, when λ goes to zero, $1/\lambda$ goes to infinity, and agents have infinite horizons.

Following Blanchard (1985), we denote by $c(s,t), y(s,t), w(s,t), h(s,t)$, and $m(s,t)$ the consumption, noninterest income, nonhuman wealth, human wealth, and real balance holdings of an agent born at time s, as of time t. Thus, the agent maximizes

$$E_t \int_t^\infty u(c(s,v), m(s,v))e^{\theta(t-v)}dv, \tag{1}$$

where $\theta > 0$ is the discounted rate, and the expectations are taken over the random life length of the individual. $u(c,m)$ is the instantaneous utility function, and we specify it as a logarithmic function, namely,

$$u(c,m) = \log c + \mu \log m,$$

where $\mu > 0$ is a constant.

Given the constant probability of death λ, optimization problem (1) can be deduced as

$$\max \int_t^\infty u(c(s,v), m(s,v))e^{(\theta+\lambda)(t-v)}dv. \tag{2}$$

Therefore, the finite-horizon optimization problem is equivalent to an intertemporal optimization problem in which the effective discount rate is $\theta + \lambda$. Thus, even if $\theta = 0$, the agent will discount the future utility if λ is positive.

At each time, the agent's real output is $y(s,t)$, and it is assumed to be fixed and exogenous. The agent's other income comes from his/her interest payments from holdings of foreign asset $w(s,t)$ at time t, the sum of $rw(s,t)$ in interest and $\lambda w(s,t)$ from an insurance company[1], expected

[1]The term $\lambda w(s,t)$ is the premium payments from an insurance company. Following Yaari (1965) and Blanchard (1985), individuals are assumed to make a contract with

real transfers from the government, $\tau(s,t)$, and the expected capital gains on the real balance holdings, and is equal to $-\pi(t)m(s,t)$. Thus, the agent's dynamic budget constraint is

$$\frac{da(s,t)}{dt} = (r+\lambda)w(s,t) - \pi(t)m(s,t) + y(s,t) + \tau(s,t) - c(s,t), \tag{3}$$

where

$$a(s,t) = w(s,t) + m(s,t) \tag{4}$$

is his/her total asset holdings. $\tau(s,t)$ is the real transfers from the government, $\pi(t)$ is the expected inflation rate, and r is the return on foreign assets, which is given as a constant in world capital markets.

An additional transversality condition is needed to prevent agents from going infinitely into debt and protecting themselves by buying life insurance. Following Blanchard (1985), we impose the condition

$$\lim_{v\to\infty} a(s,v)e^{-(r+\lambda)(v-s)} = 0. \tag{5}$$

We define the Hamiltonian associated with the above optimization problem (1) and (3) as

$$\begin{aligned} H \;=\; & \log c(s,t) + \mu \log m(s,t) \\ & +\omega((r+\lambda)w(s,t) - \pi m(s,t) + y(s,t) + \tau(s,t) - c(s,t)) \\ & +\gamma(w(s,t) + m(s,t) - a(s,t)), \end{aligned} \tag{6}$$

where ω is the co-state variable associated with the state variable $a(s,t)$ and represents the marginal utility of wealth, and γ is the Lagrange multiplier associated with the wealth constraint (5).

The first-order conditions are

$$1/c = \omega, \tag{7a}$$

$$\mu/m = \omega\pi - \gamma, \tag{7b}$$

$$-\gamma = \omega(r+\lambda), \tag{7c}$$

and

$$\frac{d\omega}{dt} = (\theta+\lambda)\omega + \gamma, \tag{7d}$$

an insurance company to avoid unintended bequests: the insurance company makes premium payments to the living in return for the receipt of their estate in the event that they die. A competitive insurance industry with free entry ensures that the premium must be λ per unit of time.

and the transversality condition is

$$\lim_{v\to\infty} \omega a(s,v)e^{-(\theta+\lambda)(v-s)} = 0. \tag{7e}$$

From equations (7a), (7b), and (7c), we have

$$m(s,t) = \mu\frac{c(s,t)}{i(t)}, \tag{8}$$

where $i(t) = r+\lambda+\pi(t)$. Therefore, the marginal utility from real balance holdings and consumption is equal.

Equations (7a), (7d), and (7c) determine the dynamics for the individual's consumption,

$$\frac{dc(s,t)}{dt} = xc(s,t), \tag{9}$$

where $x = r-\theta$.

Therefore, we derive the dynamic accumulation equation for assets:

$$\frac{da(s,t)}{dt} = (r+\lambda)a(s,t) + y(s,t) + \tau(s,t) - c(s,t)(1+\mu). \tag{10}$$

For simplicity, we suppose that income and government transfers for all individuals are equal:

$$y(s,t) = y(t), \tau(s,t) = \tau(t). \tag{11}$$

Combining (11) with equations (8) and (10), and noting condition (5), we have

$$c(s,t) = \frac{\Delta}{1+\mu}(a(s,t)+h(s,t)), \tag{12}$$

where Δ is the marginal propensity for consumption, defined as

$$\Delta = (\int_t^\infty e^{\int_t^v (x-r-\lambda)dv'}dv)^{-1} = \theta+\lambda, \tag{13}$$

and $h(s,t)$ is the agent's human wealth, which can be expressed as

$$h(s,t) = \int_t^\infty (y(v)+\tau(s,v))e^{-(r+\lambda)(v-t)}dv. \tag{14}$$

2.2. Dynamics for Aggregate Variables

To derive the full dynamics for the economy, we first consider the aggregate economy. Similar to Buiter (1988) and Weil (1993), we denote $N(t)$ population at time t, with a constant birth rate n, and the size of the cohort born at time t is $nN(t)$. The size of the surviving cohort at time t born at time $s \leq t$ is $nN(s)e^{-\lambda(t-s)} = ne^{-\lambda t}e^{ns}$. Therefore, the total population can be derived as

$$N(t) = e^{\beta t} = ne^{-\lambda t}\int_{-\infty}^{t} e^{ns}ds = e^{(n-\lambda)t},$$

and $\beta = n - \lambda$ can be defined as the effective population growth rate.

Therefore, the relationship between any aggregate variable $X(t)$ and an individual counterpart $x(s,t)$ is

$$X(t) = ne^{-\lambda t}\int_{-\infty}^{t} x(s,t)e^{ns}ds,$$

where we denote the aggregate variables by the associated uppercase letters.

Let $C(t), Y(t), A(t)$, and $H(t)$ denote aggregate consumption, noninterest income, nonhuman wealth, and human wealth at time t, respectively. Then, the dynamic equations for aggregate nonhuman and human wealth can be derived as

$$\dot{A} = rA + Y + T - (1+\mu)C, \tag{15}$$

$$\dot{H} = (r+n)H - Y - T, \tag{16}$$

where T is aggregate real transfers from the government.

From equation (12), aggregate consumption can be derived similarly as

$$C = \frac{\Delta}{1+\mu}(A+H). \tag{17}$$

Equations (15), (16), and (17) determine the aggregate dynamics for the economy, and we use them to derive the dynamics for the per capita economy.

2.3. Dynamics of per capita variables

The associated per capita variable $x(t)$ is defined as $X(t)/N(t) = X(t)e^{-\beta t}$. Therefore, the dynamics of per capita assets and human income can be derived as

$$\dot{a} = (r-\beta)a + y + \tau - (1+\mu)c, \tag{15'}$$

and

$$\dot{h} = (r+n-\beta)h - y - \tau, \tag{16'}$$

and equation (8) can also be reduced to

$$m = \mu c / i. \tag{8'}$$

Equation (17) also tells us that per capita consumption is a linear function of per capita wealth

$$c = \frac{\Delta}{1+\mu}(a+h), \tag{17'}$$

where $x = r - \theta$, and $\Delta = \theta + \lambda$.

Therefore, from equations (8), (15'), (16'), and (17'), we can derive the dynamic equation for per capita consumption

$$\dot{c} = (r-\theta)c - n\frac{\theta+\lambda}{1+\mu}a. \tag{18}$$

3. MACROECONOMIC EQUILIBRIUM

To derive macroeconomic equilibrium, we must consider the exchange market. Suppose the home price of goods is p, and the corresponding world price is p^*. From purchasing power parity (PPP), we have

$$p = Ep^*, \tag{19}$$

where E is the exchange rate. With proper normalization, p^* can be set to one.

To fully spell out the dynamics, we need to specify the government sector. The government's revenue comes from money creation and interest earnings from the central bank's reserves, i.e., rR, and R denotes the amount of the reserves. However, the government consumes goods, g, and makes transfers, τ, to a representative agent. Therefore, the budget constraint for the government can be expressed as

$$g + \tau = \dot{M}/p + rR. \tag{20}$$

Let the monetary growth rate be a positive constant σ, namely,

$$\dot{M}/M = \sigma.$$

Then, we can rewrite equation (20) as

$$g + \tau = \sigma m + rR. \tag{21}$$

By definition, the dynamics for per capita real balance holdings $m = M/(pe^{\beta t})$ can be derived as

$$\dot{m} = (\dot{M}/M - \beta - \dot{p}/p)m. \tag{22}$$

On the perfect foresight path, the expected inflation rate is equal to the actual inflation rate

$$\dot{p}/p = \dot{e}/e = \pi(t),$$

where e is the expected rate of exchange rate depreciation. Therefore, we have

$$\dot{m} = (\sigma - \beta - \pi(t))m. \tag{23}$$

Now, the macroeconomic equilibrium of the economy is summarized by equations (8'), (15'), (16'), (17'), (18), (21), and (23) and the transversality condition, from which we have

$$\begin{aligned}
\dot{m} &= (\sigma - \beta - \pi)m, \\
\dot{b} &= (r - \beta)b + (r + \pi)m + y + rR - g - (1 + \mu)c, \\
\dot{c} &= (r - \theta)c - n\frac{\Delta}{1 + \mu}a,
\end{aligned}$$

and

$$m = \mu\frac{c}{r + \lambda + \pi},$$

with $\Delta = \theta + \lambda$.

From the above equations, we can determine per capita real balances, m, per capita consumption, c, per capita foreign asset holdings, b, and inflation rate π.

4. LONG-RUN EFFECTS OF MACROECONOMIC POLICIES

Similar to Obstfeld (1981), we examine the effects of macroeconomic policies on the economy. First, we consider the steady state.

4.1. Steady state

When steady-state per capita real balances m^*, per capita consumption c^*, and per capita foreign asset holdings b^* reach $\dot{m} = \dot{b} = \dot{c} = 0$, they are characterized as

$$\sigma - \beta - \pi = 0, \tag{24a}$$

$$(r - \beta)b + (r + \pi)m + y + rR - g - (1 + \mu)c = 0, \tag{24b}$$

$$(r - \theta)c - \frac{n}{1 + \mu}(\theta + \lambda)(b + m) = 0, \tag{24c}$$

and

$$m = \mu \frac{c}{r + \lambda + \pi}, \tag{24d}$$

where $\beta = n - \lambda$.

On the perfect foresight path, the steady state is saddle-point stable. The Appendix presents the condition for saddle-point stability. Next, we analyze the effects of macroeconomic policies on the economy.

4.2. Long-run Effects of Macroeconomic Policies

To discuss the effects of macroeconomic policies on the economy, we take the total differentiation on equations (24a)-(24c):

$$\begin{pmatrix} r+\lambda+\pi & 0 & -\mu \\ -\lambda & r-\beta & -1 \\ -\frac{n}{1+\mu}(\theta+\lambda) & -\frac{n}{1+\mu}(\theta+\lambda) & r-\theta \end{pmatrix} \begin{pmatrix} dm^* \\ db^* \\ dc^* \end{pmatrix}$$
$$= \begin{pmatrix} -m^* d\sigma + m^* dn - 2m^* d\lambda \\ -dy + dg - rdR + b^* dn + (m^* - b^*) d\lambda \\ \frac{(\theta+\lambda)a^*}{1+\mu} dn + \frac{na^*}{1+\mu} d\lambda \end{pmatrix}. \tag{25}$$

From equation (25), we can derive the effects of monetary growth, foreign exchange intervention, and government spending on the economy.

Effects of monetary growth

First, we consider the effects of the monetary growth rate on the economy. From equation (25), we have

$$\begin{aligned} \frac{dm^*}{d\sigma} &= \frac{-m^*((r-\beta)(r-\theta) - \frac{n}{1+\mu}(\theta+\lambda))}{D}, \\ \frac{db^*}{d\sigma} &= \frac{m^*(-\lambda(r-\theta) - \frac{n}{1+\mu}(\theta+\lambda))}{D} > 0, \text{ and} \\ \frac{dc^*}{d\sigma} &= \frac{-m^* \frac{n}{1+\mu}(\theta+\lambda)(\lambda+r-\beta)}{D} > 0, \end{aligned}$$

where $D = \sigma((r-\beta)(r-\theta) - \frac{n}{1+\mu}(\theta+\lambda)) + [(r-\beta)(r-\theta) - n(\theta+\lambda)](\lambda + r - \beta) < 0$ is the saddle-point stability condition.

Therefore, we have the following.

PROPOSITION 1. *An increase in the inflation rate increases foreign asset accumulation and the consumption level, however; the effect of inflation on real balance holdings is ambiguous.*

Proposition 1 presents the positive effects of monetary growth on foreign asset holdings and long-run consumption, which are consistent with those in Tobin (1965) in a closed economy. This can be explained as follows. An increase in the cost of real balances as a result of a higher inflation rate leads people to lower their real balance holdings, and thus, from equation (24d), the marginal utility of consumption will be smaller. In this case, people will invest more in foreign assets and consume less in the short run, which will lead to a short-run surplus in the current account. Therefore, in the long run, there will be more foreign assets, more interest income, and hence more consumption. As for real balances, a higher cost of inflation tends to reduce them, and higher income tends to raise them, and the sign is ambiguous.

Effects of government spending

As mentioned earlier, Obstfeld (1981) shows shown that the government has no effect on asset holdings if its spending has no effect on private preference and production. He also shows that an increase in government spending leads to greater asset accumulation if that pending affects private preference. However, in our finite-horizon model, we have

$$\begin{aligned}
\frac{dm^*}{dg} &= \frac{\frac{n\mu}{1+\mu}(\theta+\lambda)}{D} < 0,\\
\frac{db^*}{dg} &= \frac{((r+\lambda+\pi)(r-\theta) - \frac{n\mu}{1+\mu}(\theta+\lambda))}{D}, \text{ and}\\
\frac{dc^*}{dg} &= \frac{\frac{n}{1+\mu}(\theta+\lambda)(\lambda+r+\pi)}{D} < 0.
\end{aligned}$$

Therefore, we have the following.

PROPOSITION 2. *Government spending always decreases long-run real balances and private consumption, but the effects of such spending on foreign asset holdings is ambiguous.*

The economic reason for this result can be explained as follows. An increase in government spending entails a current-account deficit at the initial level, b, of foreign assets. With an increase in foreign asset holdings, however, private consumption decreases. In turn, real balance holdings decrease because of equation (24d). However, a decrease in real balances stimulates an increase in foreign asset holdings. Therefore, we get the ambiguous effects of government expenditure on foreign asset holdings, and the negative effects of government expenditure on real balances and private consumption.

Effects of consumer income

As for the effects of consumer income on the economy, from equation (25), we also have

$$\begin{aligned}
\frac{dm^*}{dy} &= \frac{-\frac{n\mu}{1+\mu}(\theta+\lambda)}{D} > 0, \\
\frac{db^*}{dy} &= \frac{-((r+\lambda+\pi)(r-\theta) - \frac{n\mu}{1+\mu}(\theta+\lambda))}{D}, \text{ and} \\
\frac{dc^*}{dy} &= \frac{-\frac{n}{1+\mu}(\theta+\lambda)(\lambda+r+\pi)}{D} > 0.
\end{aligned}$$

Therefore, the effects of consumer income are just opposite to the effects of government spending.

PROPOSITION 3. *With an increase in output, long-run real balances and consumption also increase; however, the effects of output on foreign asset holdings are ambiguous.*

Effects of foreign exchange intervention

Another interesting comparison between Obstfeld's model and ours is that of the result of the central bank's foreign exchange intervention. In Obstfeld's model, if the central bank intervenes in the foreign exchange market by purchasing foreign bonds from the public with domestic currency, then the total real assets in the economy are not affected, and, as the central bank's reserves also earn interest income, which is distributed to the public in lump-sum form, the representative agent's real income and wealth remain the same. Therefore, intervention by the central bank has no real effects on foreign asset holdings, consumption, or real balances. It only occasions a rise in the price level that is exactly proportional to the increase in the money supply.

In our finite-horizon model, even though the interest income earned by the central bank's reserves is still redistributed to the public, the returns to foreign assets must be plus a return from the insurance company; therefore, the budget constraint changes, and the symmetry of the foreign bonds and central bank's reserves in Obstfeld's model disappears. Shortly after an increase in intervention by the central bank, the real balances issued by the government and the foreign bonds held by the private sector will decrease, and this decrease in real balance holdings increases the private consumption level. To restore equilibrium, the representative agent increases consumption and then increase real balance holdings and foreign asset holdings because of equation (24c). In the new equilibrium, the effects on total assets (the sum of private assets and the central bank's assets) are ambiguous, but private consumption and real balances increase.

Namely, from equation (25), we have

$$\begin{aligned}
\frac{dm^*}{dR} &= \frac{-r\frac{n\mu}{1+\mu}(\theta+\lambda)}{D} > 0, \\
\frac{db^*}{dR} &= \frac{-r((r+\lambda+\pi)(r-\theta) - \frac{n\mu}{1+\mu}(\theta+\lambda))}{D}, \text{ and} \\
\frac{dc^*}{dR} &= \frac{-r\frac{n}{1+\mu}(\theta+\lambda)(\lambda+r+\pi)}{D} > 0.
\end{aligned}$$

Therefore, we have the following.

PROPOSITION 4. *The central bank's purchase of foreign claims from the public with domestic currency leads to more real balances and consumption. However, it affects foreign asset accumulation ambiguously.*

5. SHORT-RUN EFFECTS OF MACROECONOMIC POLICIES

To study the policy implications in more detail, in this section, we concentrate on analyzing the short-run effects of macroeconomic policies on the economy. Similar to Judd (1982) and Cui and Gong (2006), we suppose that at $t = 0$, the economy has reached the steady state (c^*, m^*, b^*) with labor income y, monetary growth rate σ, government expenditure g, central bank's reserves R, and government transfers τ. Now, the government announces that at $t \geq 0$, σ, g, and R will be $\epsilon z_\sigma(t)$, $\epsilon z_g(t)$, and $\epsilon z_R(t)$ greater, respectively, where z_σ, z_g, and z_R are eventually constant functions of time; ϵ is a parameter and represents the scale of the policy change.

Associated with this policy change, the dynamics of the economy are determined by

$$\begin{aligned}
\dot{m} &= (\sigma + \epsilon z_\sigma(t) - \beta - \pi)m, \\
\dot{b} &= (r-\beta)b + (r+\pi)m + y + r(R + \epsilon z_R(t)) - g - \epsilon z_g(t) - (1+\mu)c, \\
\dot{c} &= (r-\theta)c - n\frac{\Delta}{1+\mu}a, \text{ and} \\
m &= \mu\frac{c}{r+\lambda+\pi},
\end{aligned}$$

with boundary conditions $|\lim_{t\to\infty} b(t)| < \infty$, $b(0) = b_0$.

Denote $q(t,\epsilon)$, $c(t,\epsilon)$, and $k(t,\epsilon)$ as the solutions to the above differential equations. We want to derive the impact of the policy change on the critical variables at future times; that is, we need to find the values of

$$\frac{\partial k}{\partial \epsilon}(t,0)=k_\epsilon(t), \frac{\partial}{\partial \epsilon}\left(\frac{\partial k}{\partial t}\right)(t,0)=\dot{k}_\epsilon(t), k=c,m,b.$$

Taking the differential on the above different system around $\epsilon=0$ yields

$$\begin{pmatrix} \dot{m}_\epsilon(t) \\ \dot{b}_\epsilon(t) \\ \dot{c}_\epsilon(t) \end{pmatrix} = J \begin{pmatrix} m_\epsilon(t) \\ b_\epsilon(t) \\ c_\epsilon(t) \end{pmatrix} + \begin{pmatrix} m^* z_\sigma(t) \\ r z_R(t) - z_g(t) \\ 0 \end{pmatrix}, \tag{26}$$

where coeffcient matrix J is defined as

$$J = \begin{pmatrix} r+\lambda+\pi & 0 & -\mu \\ -\lambda & r-\beta & -1 \\ -\frac{n}{1+\mu}(\theta+\lambda) & -\frac{n}{1+\mu}(\theta+\lambda) & r-\theta \end{pmatrix}.$$

Because the economy is initially in the steady state, the system is linear with constant coefficient, and we can solve it with the Laplace transform. [2] Let $\mathcal{M}_\epsilon(s), \mathcal{C}_\epsilon(s), \mathcal{B}_\epsilon(s), Z_\sigma(s), Z_R(s)$, and $Z_g(s)$ be the Laplace transforms of $m_\epsilon(t)$, $c_\epsilon(t)$, $b_\epsilon(t)$, $z_\sigma(t), z_R(t)$, and $z_g(t)$, respectively.

Taking the Laplace transform on equation (26), we obtain

$$\begin{pmatrix} s\mathcal{M}_\epsilon(s) \\ s\mathcal{B}_\epsilon(s) \\ s\mathcal{C}_\epsilon(s) \end{pmatrix} = J \begin{pmatrix} \mathcal{M}_\epsilon(s) \\ \mathcal{B}_\epsilon(s) \\ \mathcal{C}_\epsilon(s) \end{pmatrix} + \begin{pmatrix} m^* Z_\sigma(s) + m_\epsilon(0) \\ r Z_R(s) - Z_g(s) + b_\epsilon(0) \\ c_\epsilon(0) \end{pmatrix}. \tag{27}$$

Because the state variable b cannot jump initially, we have $b_\epsilon(0)=0$. As for the effects of the policy change on initial consumption, $c_\epsilon(0)$, and the initial real balance holding, $m_\epsilon(0)$, we follow Cui and Gong (2006) to determine them.

As the Jacobian J is invertible, it can be diagonalized by a transform P:

$$J = P^{-1}\Phi P, \tag{28}$$

where Φ is a diagonal matrix whose diagonal elements are the characteristic roots of J, which are the solutions for equation

$$\phi^3 - tr(J)\phi^2 + b(J)\phi - \det(J) = 0,$$

where $\det(J)$ is the determinant of the Jacobian,

$$\det(J) = \sigma((r-\beta)(r-\theta) - \frac{n}{1+\mu}(\theta+\lambda)) + [(r-\beta)(r-\theta) - n(\theta+\lambda)](\lambda+r-\beta);$$

[2]The Laplace transform of a function $f(t)$ with parameter s is defined as $F(s) = \int_0^\infty e^{-st} f(t)dt$.

$b(J)$ is the sum of all 2×2 principal minors of matrix J,

$$b(J) = (r-\beta)(r-\theta) - n(\theta+\lambda) + (r-\beta+r-\theta)(\sigma+\lambda+r-\beta);$$

$tr(J)$ is the trace of matrix J,

$$tr(J) = r+\lambda+\sigma-\beta+r-\beta+r-\theta > 0;$$

and P is a 3×3 matrix whose rows are linearly independent left-eigenvectors of J, which is given by

$$(P_{i1}, P_{i2}, P_{i3}) = \left(\frac{n(\theta+\lambda)}{\mu(1+\mu)(\phi_i-(r-\beta))} - \frac{\phi_i-(r-\beta)}{\mu}, -\frac{n(\theta+\lambda)}{(1+\mu)(\phi_i-(r-\beta))}, 1\right), \quad i = 1,2,3.$$

Under the assumption of $\det(J) < 0$ and $tr(J) > 0$, it is easy to show that the Jacobian J has one negative characteristic root and two characteristic roots with positive real parts. Denote ϕ_1 and ϕ_2 as the two positive characteristic roots, and we assume that $\phi_1 < \phi_2$. It is easy to prove that $r-\beta < \phi_1 < r-\beta+\sqrt{\frac{n(\theta+\lambda)}{1+\mu}} < \phi_2$. To derive $m_\epsilon(0)$ and $c_\epsilon(0)$, substituting equation (28) into (27) and left multiplying by P, we have

$$(sI-\Phi)P\begin{pmatrix}\mathcal{M}_\epsilon(s)\\ \mathcal{B}_\epsilon(s)\\ \mathcal{C}_\epsilon(s)\end{pmatrix} = P\begin{pmatrix}m^* Z_\sigma(s)+m_\epsilon(0)\\ rZ_R(s)-Z_g(s)+b_\epsilon(0)\\ c_\epsilon(0)\end{pmatrix}. \tag{29}$$

For the eigenvalues with positive real part $\phi_i, i = 1,2$, the Laplace transforms $\mathcal{M}_\epsilon(\phi_i)$, $\mathcal{C}_\epsilon(\phi_i)$, and $\mathcal{B}_\epsilon(\phi_i), i = 1,2$ must be bounded. However, when $s = \phi_1$(or ϕ_2), the first (second) equation in the left-hand side of linear system (29) is zero, which requires that the corresponding equation in the right-hand side equals zero. Thus, the initial effects of the policy change on consumption and real balance holdings are obtained as follows.

$$c_\epsilon(0) = \left(\frac{1}{P_{11}-P_{21}}\right)\left\{\begin{array}{c} P_{11}P_{21}\left[Z_\sigma(\phi_1)-Z_\sigma(\phi_2)\right] + P_{12}P_{21}rZ_R(\phi_1)\\ -P_{11}P_{22}rZ_R(\phi_2) - \left[P_{12}P_{21}Z_g(\phi_1) - P_{11}P_{22}Z_g(\phi_2)\right]\end{array}\right\}, \tag{30}$$

and

$$m_\epsilon(0) = \left(\frac{1}{P_{11}-P_{21}}\right)\left\{\begin{array}{c} P_{21}Z_\sigma(\phi_2) - P_{11}Z_\sigma(\phi_1) + P_{22}rZ_R(\phi_2)\\ -P_{12}rZ_R(\phi_1) + P_{12}Z_g(\phi_1) - P_{22}Z_g(\phi_2)\end{array}\right\}. \tag{31}$$

Substituting equations (30) and (31) back into equation (26), we obtain the initial impact of the policy change on investment:

$$\dot{b}_\epsilon(0) = -\lambda m_\epsilon(0) - c_\epsilon(0) + rz_R(0) - z_g(0). \tag{32}$$

Equation (32) provides the impact of the policy change on investment, from which we find certain aspects of the relationship between current government policies and short-run foreign asset accumulation. First, an increase in government expenditure at $t = 0$, $z_g(0)$, initially leads to a dollar for dollar decrease in current foreign asset accumulation. In a life-cycle model such as this one, a consumer endeavors to have a steady-state level of consumption; hence, a momentary spurt in government expenditure of $z_g(0)$ at $t = 0$ will be satisfied by a decrease in asset investment. Second, if the government increases its current foreign exchange intervention by $z_R(0)$, then the current foreign asset accumulation will increase $rz_R(0)$. As the central bank's reserves earn interest income, which is allocated to the public in lump-sum form, the representative agent's real income and wealth increase. In this case, a consumption smoothing motion still leads to a dollar for dollar increase in current foreign asset accumulation. In the following, we focus on the short-run impact of future permanent government policy change on current consumption, real balance holdings, and the investment rate.

A permanent increase in government policies can be defined by

$$z_i(t) = 1,\ i = \sigma, R, g,\ \text{and}\ t \geq 0.$$

The Laplace transform of $z_i(t)$ with parameter $\phi_j, j = 1, 2$ is

$$Z_i(\phi_j) = 1/\phi_j. \tag{33}$$

According to this definition, equations (30), (31), and (32) indicate the following propositions.

Effects of permanent monetary growth rate change

Let $i = \sigma$ in equation (33), and we have the following.

PROPOSITION 5. *A permanent increase in the future monetary growth rate increases current consumption and current real balance holdings, but has a positive effect on current foreign asset accumulation, that is,*

$$\begin{aligned}
c_\epsilon(0) &= \left(\frac{1}{P_{11} - P_{21}}\right) P_{11}P_{21}\left[\frac{1}{\phi_1} - \frac{1}{\phi_2}\right] < 0, \\
m_\epsilon(0) &= \left(\frac{1}{P_{11} - P_{21}}\right)\left(\frac{-P_{11}}{\phi_1} + \frac{P_{21}}{\phi_2}\right) < 0,\ \textit{and} \\
\dot{b}_\epsilon(0) &= -\lambda m_\epsilon(0) - c_\epsilon(0) > 0
\end{aligned}$$

Proof. Substituting equation (33) into (31) and (32), we obtain the impact of a future permanent monetary growth rate change on current consumption and real balance holdings. Moreover, by the definition of the linearly independent left-eigenvectors of J, equation (29), and $0 < r - \beta <$

$\phi_1 < r - \beta + \sqrt{\frac{n(\theta+\lambda)}{1+\mu}} < \phi_2$, we know that $P_{11} > 0$ and $P_{21} < 0$. Thus, we prove the proposition. ▮

Here we give a formal explanation for why inflation has a positive effect on asset accumulation, as described by the Mundell-Tobin effect. An increase in the monetary growth rate raises the cost of holding real balances as a result of a higher inflation rate. The representative consumer, therefore, decreases his/her real balance demand, provided that the real balance is a normal good. In this case, people consume less in the short run and convert to investing more in foreign assets, which brings about a current-account surplus and higher asset stocks in the long run.

Effects of government spending

Let $i = g$ in equation (33), and we have the following.

PROPOSITION 6. *A permanent increase in government spending increases current real balance holdings, and decreases current consumption, but has an ambiguous effect on foreign asset accumulation:*

$$\begin{aligned} c_\epsilon(0) &= -\left(\frac{1}{P_{11} - P_{21}}\right)\left(\frac{P_{12}P_{21}}{\phi_1} - \frac{P_{11}P_{22}}{\phi_2}\right) < 0, \textit{ and} \\ m_\epsilon(0) &= -\left(\frac{1}{P_{11} - P_{21}}\right)\left(\frac{P_{22}}{\phi_2} - \frac{P_{12}}{\phi_1}\right) > 0. \end{aligned}$$

This proposition implies that an increase in future government spending reduces today's consumption but increase today's real balance demand. The economic intuition for this result is as follows. From the consumer's budget constraint, equation (3), an increase in g in the future without a rise in the monetary growth rate will reduce both the consumer's future disposable income and the current price of holding real balances. If they expect a decrease in future consumption and an increase in the current relative price, then people will reduce their consumption today and hold more real balances. Therefore, current consumption is reduced as a result of a future increase in government spending. A similar result has been obtained by Judd (1985) and Zou (1994). In the framework of the neoclassical growth model with income taxation and government borrowing, Judd shows that an increase in future government spending reduces consumption today and, consequently, encourages investment today. Zou (1994) shows that in a finite-horizon open economy model, an increase in future government expenditure also reduces consumption today and improves today's current account. In this paper, however, the trade-off between current consumption and real balance holdings produces an ambiguous effect for government spending on foreign asset accumulation.

With the same intuition, we can exam the effect of future central bank's reserves on today's real variables.

Effects of foreign exchange intervention

Let $i = R$ in equation (33), and we have the following.

PROPOSITION 7. *A permanent increase in the central bank's reserves decreases current real balance holdings, increases current consumption, and has an ambiguous effect on foreign asset accumulation:*

$$\begin{aligned} c_\epsilon(0) &= \left(\frac{r}{P_{11} - P_{21}}\right)\left(\frac{P_{12}P_{21}}{\phi_1} - \frac{P_{11}P_{22}}{\phi_2}\right) > 0, \text{ and} \\ m_\epsilon(0) &= \left(\frac{r}{P_{11} - P_{21}}\right)\left(\frac{P_{22}}{\phi_2} - \frac{P_{12}}{\phi_1}\right) < 0. \end{aligned}$$

Proposition 7 presents the mechanism by which the central bank's reserves affects the long-run economy. With an increase in the central bank's reserves, the real balances issued by the government decrease, and initial consumption increases. In the long run, the representative agent increases consumption and real balance holdings because of equation (24c). In the new equilibrium, the effects on private consumption and real balances increases.

6. CONCLUSION

This paper examines the effects of macroeconomic policies on foreign asset accumulation in the finite-horizon model used by Blanchard (1985). Different from Obstfeld's (1981) model, which turned the conventional Mundell-Fleming model on its head, the finite-horizon framework generates results that are more consistent with the Mundell-Flemming framework.

First, it is shown that an increase in government spending decreases real balance holdings and the consumption level, but affects foreign asset accumulation ambiguously. This is consistent with the result presented in the conventional Mundell-Fleming model.

Second, different from Obstfeld's model, which finds no effect for the central bank's purchase of foreign claims on total asset holdings, we find that such a purchase by the central bank with domestic currency leads to more real balances and consumption and affects foreign asset accumulation ambiguously.

We also examine the effects of the inflation rate on the economy, and find that inflation increases foreign asset accumulation and the consumption level, but its effects on real balance holdings are ambiguous.

Short-run analysis shows the mechanism by which macroeconomic policies affect the economy. (1) A permanent increase in the future monetary growth rate increases current consumption and current real balance holdings and has a positive effect on current foreign asset accumulation. (2) A permanent increase in government spending increases current real balance holdings and decreases current consumption, but has an ambiguous effect on foreign asset accumulation. (3) A permanent increase in the central bank's reserves decreases current real balance holdings and increases current consumption, but has an ambiguous effect on foreign asset accumulation.

Evaluating the consequences of macroeconomic policies is complicated, and the results are often very sensitive to the optimization framework we have utilized. The finite-horizon model provides a different perspective on these problems, but it should only be taken as complementary to the many existing models.

APPENDIX: SADDLE-POINT STABILITY FOR THE STEADY STATE

First, to study the stability of the steady state, we linearize the system around the steady state,

$$\begin{pmatrix} \dot{m} \\ \dot{b} \\ \dot{c} \end{pmatrix} = \begin{pmatrix} r+\lambda+\pi & 0 & -\mu \\ -\lambda & r-\beta & -1 \\ -\frac{n}{1+\mu}(\theta+\lambda) & -\frac{n}{1+\mu}(\theta+\lambda) & r-\theta \end{pmatrix} \begin{pmatrix} m-m^* \\ b-b^* \\ c-c^* \end{pmatrix}.$$

We denote J as the coefficient matrix of the above linear system. To ensure saddle-point stability, the dynamic system must have two positive and one negative eigenvalues.

First, the trace for the coefficient matrix is

$$tr(J) = r+\lambda+\sigma-\beta+r-\beta+r-\theta > 0,$$

which is positive because $r-\theta > 0$ and $r-\beta > 0$.

Second, the determinant for the coefficient matrix can de derived as

$$\begin{aligned} D &= \det(J) = \sigma((r-\beta)(r-\theta) - \frac{n}{1+\mu}(\theta+\lambda)) \\ &\quad +[(r-\beta)(r-\theta) - n(\theta+\lambda)](\lambda+r-\beta), \end{aligned}$$

which must be negative because of saddle-point stability.

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