Optimal Unemployment Insurance with Endogenous Search Effort

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In the framework of a search and matching model, when search effort enters the labor market matching function, search effort by one worker generates a negative externality on other workers searching for jobs. The solution to the social planner’s problem may not be decentralized in a competitive market. Calibration shows that the current US unemployment insurance (UI) system generates an 8.07% welfare loss relative to the socially optimal allocation. An alternative scheme with higher replacement rate and lower wage, which achieves the highest welfare level among all competitive equilibria with unemployment insurance, leads to a welfare loss of only 1.18%.

Key Words: Matching function; Search effort; Unemployment insurance.
JEL Classification Numbers: J38, J30.

1. INTRODUCTION

This paper studies the optimal unemployment insurance (UI) with endogenous search. A worker’s chances of getting employed depend on the number of job seekers and the average search effort of all these job seekers. By exerting more search effort, the worker increases her probability of getting a job, yet this decreases other job seekers’ chances of getting jobs. The worker ignores this negative externality on others and thus the decentralized equilibrium is not socially efficient.

A large literature on unemployment insurance has been focused on its risk-sharing role and disincentive effect. Gruber (1997) demonstrates that consumption following job loss drops less with more generous unemploy-

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ment benefits. Meyer (1989) finds a ten percent increase in UI benefits raises unemployment duration by about a week. Other studies deal with the moral hazard problem that is common to most social insurance systems. Hopenhayn and Nicolini (1997), for example, study a principle-agent problem and show that the optimal replacement ratio decreases over the unemployment spell.

Standard analysis of search and wage bargaining makes search effort in an explicit matching function. The probability that an unemployed worker gets employed depend on her choice of search effort, and a set of other aggregate variables, including the unemployment rate, the vacancy rate, and the average search effort of all unemployed workers. Pissarides (2000) uses a general equilibrium model to show how unemployment rate and search effort are endogenously determined. He also proves that if workers’ bargaining power satisfies a particular rule, the decentralized equilibrium coincides with the social optimum. The uniqueness and efficiency of equilibrium relies heavily on the assumption that the job-matching technology has constant returns to scale.

As to the efficiency of search and bargaining model, Hosios (1990) summarizes three versions of externalities that may make the decentralized equilibrium not socially efficient, one of them being entry and exit externalities, also described by Diamond (1982). Specifically, the presence of an additional worker (firm) makes it easier (harder) for vacancies to find workers but harder (easier) for workers to find jobs. Also relying on the assumption of constant returns to scale matching function, he finds a sharing rule to internalize these externalities.

Teulings and Gautier (2004) describe a search model with a continuum of workers and job types, free entry and transferable utility. Using an increasing return to scale matching function, they demonstrate that the decentralized equilibrium is not socially efficient. Unemployment benefits can reduce the loss by serving as a search subsidy. Coles and Masters (2000) argue that unemployment insurance put unemployed workers in a better bargaining position, which directly affects equilibrium wage determination.

This paper embeds unemployment insurance in a context of endogenous job search, and explores the welfare implications of the current US system. The efficiency argument in Pissarides (2000) does not apply in my model because I drop two of his assumptions: transferable utility and risk-neutral workers. My paper assumes risk-averse workers\(^1\), so that unemployment insurance is not just a subsidy to search; it is a way to undo uninsured risks. Indeed, my static model implies that only under very strict restrictions can

\(^1\)Studies on risk-averse workers do not provide closed-form conditions for the competitive equilibrium to coincide with the socially optimal allocation, see Acemoglu and Shimer (1999) as an example. Pissarides (2000a) and Rogerson et al. (2004, 2005) give closed-form conditions, but assuming risk-neutral workers.
Most literature on optimal unemployment insurance focuses on the trade-off between risk sharing and output. Yashiv (2000) claims a rise in the replacement rate lowers search effort and leads to higher unemployment. So at high levels of unemployment, reducing UI is effective. In contrast, Acemoglu and Shimer (1999) shows that with unemployment insurance, workers are more able to endure the risk of unemployment, enabling them to apply for high-wage/high-productivity jobs, which is more difficult to get. Thus moderate unemployment insurance not only increases risk sharing but also increases output. Acemoglu and Shimer (2000) quantify this idea in a dynamic model. They find that moderate increases in UI generosity starting from current US levels increase output, productivity, and welfare. In their model, worker’s probability of getting a job is simply a linear function of their search effort. Thus, they ignore the externality generated by search. My paper, however, models this probability as a function of other endogenous aggregate variables that are determined in equilibrium, such as vacancy rate and average search effort. In contrast to Acemoglu and Shimer (1999) and Acemoglu and Shimer (2000), I suggest a more generous unemployment insurance, which increases welfare but lowers output.

Section 2 lays out the model setup. In Section 3, I solve the social planner’s problem. Section 4 analyzes the competitive equilibrium, and explores the conditions for the competitive equilibrium to be socially optimal. The model is calibrated to the US economy in Section 5. The current system generates an 8.07% welfare loss relative to the socially optimal allocation. The optimal competitive equilibrium occurs under a more generous UI scheme and a higher wage. Section 6 characterizes the competitive equilibrium in an economy with heterogeneous workers. Three unemployment insurance schemes with different financing methods are analyzed. Conclusions and extensions are presented in Section 7.

2. A ONE-SHOT MODEL

2.1. Preferences and Technology

The setup of the model is similar to that of Acemoglu and Shimer (1999). There is a continuum 1 of identical workers. Workers’ utility function over final consumption, $U(\cdot)$, is twice continuously differentiable, strictly increasing, and strictly concave, with $\lim_{c \to 0} U'(c) = \infty$. There is a larger continuum of potential risk-neutral firms. Each firm can create a job by purchasing capital $k$. At the beginning of the period, all workers are unemployed. Firms decide whether to create a job or not. In the next stage workers and firms search in the labor market. Each filled job produces out-
put \( y \), pays a gross wage \( w \) to the worker, and yields a profit \( y - k - w \) to the firm. An unfulfilled job produces nothing, and its capital remains idle. An unemployment insurance system (UI), fully financed by income tax, pays unemployed workers \( z \), and imposes an income tax \( \tau \) on employed workers. So workers' consumption is \( w - \tau \) if employed, and \( z \) if unemployed.

Empirical estimates have found individual characteristics play an important role in the hazard rates across different individuals. In this model, search effort is a choice variable of workers in search of jobs. A worker can increase her chances of getting a job by increasing her search effort, at the expense of the disutility associated with search (Pissarides 2000, ch.5). To derive the matching function, let \( s \) be the average search effort of all workers, then the total number of search effort by the unemployed is \( su \). The aggregate matching function is

\[
m = m(su, v)
\]

where \( m \) is the total number of new job matches in a given period of time, \( u \) is the number of unemployed workers, and \( v \) is the number of job vacancies. Assume the population in the labor force is constant and normalize it to one, then \( u \) is also the unemployment rate. Assume \( m(0, \cdot) = m(\cdot, 0) = 0 \), with non-decreasing first-order partial differentials. Assume also \( m(su, v) \leq \min\{u, v\} \), i.e., the number of new matches cannot exceed the number unemployment workers, nor can it exceed the number of vacancies (Mortensen and Pissarides 1999). In particular, I assume the Cobb-Douglas matching function

\[
m(su, v) = \alpha (su)^{\lambda} v^{1-\lambda} \quad \text{with} \quad \alpha > 0, 0 < \lambda < 1
\]

Since all workers are unemployed when they begin to search, \( u = 1 \). Equation (2) becomes

\[
m(su, v) = \alpha s^\lambda v^{1-\lambda} \quad (2')
\]

Let \( s^i \) be the search effort of worker \( i \). Worker \( i \)'s probability of finding a job is \( \mu^i = \frac{s^i m(su, v)}{su} \). So a worker’s probability of getting employed depends on the average search effort, number of vacancies, and her own search effort. The number of vacancies has a positive externality on this probability, while other workers’ search effort poses a negative externality on this probability. \( (2') \) implies

\[
\mu^i = \frac{s^i m(su, v)}{su} = s^i \alpha s^{\lambda-1} v^{1-\lambda}
\]

\[2\]Petrongolo and Pissarides (2001) survey papers on the matching function.
Since all workers are identical, $s^i = s$. The above probability is the same across all workers. Let $s^i = s$ in the above equation and define a worker’s probability of getting employed by

$$\mu \equiv \mu^i = \alpha s^\lambda v^{1-\lambda}$$  \hspace{1cm} (4)

Let $g(s)$ be the disutility from search. Assume $g(0) = 0$, $g'(0) = 0$, $g(s) > 0$, $g'(s) > 0$, $g''(s) > 0$ for all $s > 0$, and $g(s) \to \infty$ as $s \to \infty$.

Firms’ probability of finding a worker is

$$\eta = \frac{m(su,v)}{v} = \alpha s^\lambda v^{-\lambda}$$  \hspace{1cm} (5)

Firms post vacancies until the free entry condition

$$\eta(y - w) - k = 0$$

This implies job’s probability of getting filled ($\eta$) is constant. Together with (5) this gives

$$v = s \left[ \alpha(y - w) \right]^{\frac{1}{\lambda}}$$  \hspace{1cm} (6)

Equation (6) is firms’ zero-profit condition. Since $y, w, k$ are all exogenous variables, (6) implies a linear relationship between search effort and number of vacancies. In a graph with $s$ being the horizontal axis and $v$ being the vertical axis, (6) implies a straight zero-profit line.

### 2.2. Definition of Equilibrium

**Definition 2.1.** An equilibrium is an allocation $\{w, z, v, s, \mu, \eta, W\}$ with the following properties:

1. Firms’ zero-profit: $v = s \left[ \alpha(y - w) \right]^{\frac{1}{\lambda}}$

2. Utility maximization:

$$W = \sup \mu U(w - \tau) + (1 - \mu)U(z) - g(s)$$

3. Government budget constraint: $\mu \tau = (1 - \mu) z$

### 3. THE SOCIAL PLANNER’S PROBLEM

The social planner chooses the consumption of the employed ($c$), the consumption of the unemployed ($b$), the amount given to filled jobs ($x$),
search effort \((s)\), number of job vacancies \((v)\), and solves the following problem:

\[
\max_{c,b,x,s,y} \mu U(c) + (1 - \mu)U(b) - g(s)
\]

\[
s.t \quad \eta x - k \geq 0 \quad \text{(firms’ non-negative profit)}
\]

\[
(1 - u)x + (1 - u)c + ub = (1 - u)y \quad \text{(resource constraint)}
\]

\[
\mu = \alpha s^\lambda v^{1-\lambda}
\]

\[
\eta = \alpha s^\lambda v^{-\lambda}
\]

Specifically, the social planner maximizes the ex ante utility such that firms’ profit is non-negative, and subject to resource constraint and matching technology. Because \(U(\cdot)\) is a strictly increasing function, firms’ non-negative profit condition should bind. That is, \(\eta = \frac{1}{\lambda}\). Together with (5) and the resource constraint, this implies

\[
v = s \left[ \frac{\alpha(y - c - \frac{ub}{1-u})}{k} \right]^{\frac{1}{\lambda}}
\]

The end-of-the-period unemployment rate \(u\) is expressed as

\[
u = 1 - \mu = 1 - \alpha s^\lambda v^{1-\lambda}
\]

Plug it into the expression for job vacancies to obtain

\[
s = \left[ \frac{k + \frac{b}{v}}{\alpha(y - c - b)} \right]^{\frac{1}{\lambda}} v
\]

I denote (7) “firms’ zero-profit condition”. (7) implies \(\lim_{v \to 0} s = \infty\) and \(\lim_{v \to \infty} s = 0\). Using the Implicit Function Theorem, I get

\[
\frac{ds}{dv} = \left[ \frac{1}{\alpha(y - c + b)} \right] \left( k + \frac{b}{v} \right)^{\frac{1}{\lambda}-1} \left[ k + (1 - \frac{1}{\lambda}) \frac{b}{v} \right]
\]

\[
\frac{d^2s}{dv^2} = \left[ \frac{1}{\alpha(y - c + b)} \right] \left( k + \frac{b}{v} \right)^{\frac{1}{\lambda}-2} \left( \frac{1}{\lambda} - 1 \right) \frac{b^2}{v^{2\lambda}}
\]

with \(\frac{dv}{ds} < 0, \frac{dv}{ds} > 0\) if \(v > \left( \frac{1}{\lambda} - 1 \right) \frac{k}{b}\). And \(\frac{d^2v}{ds^2} > 0, \frac{dv}{ds} < 0\) if \(v < \left( \frac{1}{\lambda} - 1 \right) \frac{k}{b}\). Moreover,

\[
\frac{d^2v}{ds^2} = 0, \frac{dv}{ds} = 0 \quad \text{if} \quad v = \left( \frac{1}{\lambda} - 1 \right) \frac{b}{k}.
\]
To illustrate, I draw workers’ indifference curve and firms’ zero-profit condition in a graph, with search effort \( s \) being the horizontal axis and number of vacancies \( v \) being the vertical axis. Workers’ utility has the following expression

\[
W = \alpha s^\lambda v^{1-\lambda} U(c) + (1 - \alpha s^\lambda v^{1-\lambda}) U(b) - g(s)
\]

The social planner’s problem is to maximize workers’ ex ante utility subject to firms’ zero-profit condition. The socially efficient allocation is obtained where workers’ indifference curve is tangent to firms’ zero-profit curve. Using the Implicit Function Theorem, the above expression gives

\[
\frac{dv}{ds} = \left( \frac{v}{s} \right)^{\lambda - 1} \frac{\alpha U(c) - U(b)}{\alpha U(c) - U(b)} - \frac{\lambda}{1 - \lambda} \frac{v}{s}
\]

\[
\frac{d^2v}{ds^2} = \frac{\lambda}{(1 - \lambda)^2} \frac{v}{s^2} \left[ \left( \frac{v}{s} \right)^{\lambda - 1} \frac{\alpha U(c) - U(b)}{\alpha U(c) - U(b)} - 1 \right]^2 + \frac{\alpha}{(1 - \lambda)\alpha[U(c) - U(b)]}
\]

Since \( g''(s) > 0 \) for all \( s > 0 \), \( \frac{d^2v}{ds^2} > 0 \). For any \( v > 0 \), \( \frac{dv}{ds} \to -\infty \) as \( s \to 0 \).

From (8), I can obtain \( \frac{dv}{ds} \). And (7) gives

\[
\frac{v}{s} = \left[ \frac{\alpha(y - c - b)}{k + \frac{b}{v}} \right]^{\frac{1}{\lambda}}
\]

Plug \( \frac{dv}{ds} \) into the left-hand side of (9) and plug \( (7') \) into the right-hand side of (9) to obtain

\[
g'(s) = [U(c) - U(b)]^{\frac{1}{\lambda}} \left( \frac{y - c + b}{k + \frac{b}{v}} \right)^{\frac{1 - \lambda}{\lambda}} \frac{k}{k + (1 - \frac{1}{\lambda})\frac{b}{v}}
\]

Thus (7) and (11) characterize the search effort \( s \) and number of vacancies \( v \) for the social planner’s problem, given consumption pair \( (c, b) \). Once \( v \) and \( s \) are determined, workers’ probability of finding a job \( (\mu) \), job’s probability of getting filled \( (\eta) \), and the amount given to firms with filled jobs \( (x) \), can be easily obtained. Therefore, the social planner chooses among different consumption pairs \( (c, b) \), to maximize workers’ ex ante utility. Because I assume marginal utility is infinity at zero, and the matching function \( m(sv, v) \) to be zero when \( s \) equals zero, the social planner will never choose an allocation such that \( s = 0 \), for which every worker is unemployed at the end of the period, and nothing is produced.
FIG. 1. The social planner’s problem for given \((c,b)\).

\[ s = \left[ \frac{k}{\alpha(y - w)} \right]^\frac{1}{\lambda} v \]  
(12)

Worker \(i\)'s utility is the following expression

\[ W = \mu^i U(w - \tau) + (1 - \mu^i)U(z) - g(s^i) \]
\[ = s^i \alpha s^{1-\lambda} v^{1-\lambda} U(w - \tau) + (1 - s^i \alpha s^{1-\lambda} v^{1-\lambda})U(z) - g(s^i) \]

where \(s^i\) is the search effort of worker \(i\), and \(s\) is the average search effort of all workers, which worker \(i\) takes as given. Using the Implicit Function Theorem to obtain

\[ \frac{dv}{ds^i} = \frac{1}{(1 - \lambda)s^i} \left[ \frac{g'(s^i)s^{1-\lambda} v^\lambda}{\alpha[U(w - \tau) - U(z)]} - v \right] \]

\(^3\)This argument is made in Acemoglu and Shimer (1999).
In a symmetric pure strategy equilibrium where \( s_i = s \) for all workers, the above equations become

\[
\frac{d^2v}{ds^2} = \frac{1}{(1 - \lambda)s^\lambda} \left\{ \frac{\lambda}{(1 - \lambda)v} \left[ \frac{g'(s)s^{1-\lambda}v^\lambda}{\alpha[U(w - \tau) - U(z)]} \right]^2 - \frac{2}{1 - \lambda} \frac{g'(s)s^{1-\lambda}v^\lambda}{\alpha[U(w - \tau) - U(z)]} + \frac{2 - \lambda}{1 - \lambda} \right\} + \frac{1}{(1 - \lambda)s^\lambda} \frac{g''(s)s^{1-\lambda}v^\lambda}{\alpha[U(w - \tau) - U(z)]}
\]

The competitive equilibrium occurs at the tangency between workers’ indifference curve and firms’ zero profit line. Using (6) to obtain \( \frac{dv}{ds} \) and \( \frac{v}{s} \) and plugging them into the left-hand side and right-hand side of (13) respectively, I get

\[
g'(s) = \frac{[U(w - \tau) - U(z)]\alpha^\frac{1}{k} \left( \frac{y - w}{k} \right)^{1 - \lambda}}{(2 - \lambda)} \quad (15)
\]

Denote consumption of employed workers by

\[ q \equiv w - \tau \]

The government budget constraint implies

\[
\tau = \frac{u}{1 - u} \frac{z}{\alpha s^\lambda v^{1-\lambda}}
\]

Plug it into firms’ zero profit condition to obtain

\[
\tau = \frac{y - w - ks}{ks} \frac{\left( \frac{\alpha(y-w)}{k} \right)^{\frac{1}{\lambda}}}{\alpha} - z \quad (16)
\]
Thus the competitive equilibrium is characterized by the following system of equations

\[ g'(s) = [U(q) - U(z)] \alpha^\lambda \left( \frac{y - w}{k} \right)^{\frac{1}{1-\lambda}} (2 - \lambda) \]  \hspace{1cm} (15)

\[ s = \left[ \frac{k}{\alpha(y - w)} \right]^{\frac{1}{\lambda}} v \]  \hspace{1cm} (12)

where

\[ q = w - \tau = w - y - w - ks \left[ \frac{\alpha(y - w)}{k} \right]^{\frac{1}{\lambda}} z \]

Therefore, once \((w, z)\) is determined, \(v\) and \(s\) will be determined. And workers' probability of finding a job (\(\mu\)), job's probability of getting filled (\(\eta\)) are also determined. This is illustrated in Figure 2. Since the indifference curve has the property that \(d^2v/ds^2 > 0\), and since the slope of firms’ zero profit line is constant, tangency occurs only once. This proves that if equilibrium corresponding to a pair \((w, z)\) exists, the equilibrium is unique. However, this does not prove the existence of equilibrium. For some \((w, z)\), tangency may not exist. The optimal wage/unemployment insurance scheme is a pair \((w, z)\) that gives the highest level of ex ante utility among all combinations of before-tax wage \(w\) and UI benefit \(z\) for which a competitive equilibrium exists.

**Definition 4.1.** An optimal wage/unemployment insurance scheme is one that induces a competitive equilibrium \(\{w, z, q, v, s, \mu, \eta, W\}\), with

\[ W = \sup \mu U(w - \tau) + (1 - \mu)U(z) - g(s) \]

### 4.2. Decentralization of the Socially Optimal Allocation

This subsection explores the conditions under which the socially optimal allocation is decentralized via some wage/unemployment insurance scheme. The social planner’s problem is characterized by (11) and (7) (denote as system I). The competitive equilibrium is characterized by (15) and (12) (denote as system II). Once \(s, v\) are determined through these equations, all other endogenous variables, \(\mu, \eta, u\), will be uniquely determined. So I can simply compare these two systems of equations.

Given workers’ preferences and the matching technology, suppose the planner chooses consumption bundle \((c^P, b^P)\) that induces \(s^P, v^P\), and
these in turn induces $\mu^P, \eta^P, u^P, W^P$. Suppose a competitive equilibrium \{w, z, q, v, s, \mu, \eta, W\} satisfies $q = c^P$, $z = b^P$, $s = s^P$, and $v = v^P$, then the socially optimal allocation can be decentralized by this competitive equilibrium. For (7) and (12) to be equivalent, I need

$$w = \frac{b^P y + k(c^P - b^P)}{\frac{b^P}{v^P} + k}$$

(17)

Together with the fact that (11) and (15) coincide with each other, I get

$$v^P = \frac{b^P 2 - \lambda}{k \lambda}$$

(18)

Plug (18) into (17) to obtain

$$w = \frac{\lambda y + (2 - \lambda)(c^P - b^P)}{2}$$

(19)

Thus, if by coincidence, the socially optimal allocation satisfies (18), the following wage/unemployment insurance scheme induces an equilibrium that is socially optimal

$$w = \frac{\lambda y + (2 - \lambda)(c^P - b^P)}{2}$$
\[
\tau = \frac{\lambda y + (2 - \lambda)(c^P - b^P)}{2} - c^P
\]

\[
z = b^P
\]

Note that (18) is a strong restriction that is generally not satisfied. Thus, I conclude that the socially optimal allocation cannot generally be decentralized. In fact, according to the calibration of the next section, (18) is not satisfied by the US economy.

Next I study the conditions under which the competitive equilibrium generates \( s = s^P \) and \( v = v^P \). But \( q = c^P \) and \( z = b^P \) are not satisfied. I study this case because it is easy to tell how much welfare loss this equilibrium generates relative to the social planner’s allocation, and this welfare loss is due solely to the degree of risk-sharing. To begin, note that equation (17) still holds. The wage/unemployment insurance scheme satisfy

\[
U(q) - U(z) = \frac{k}{k + (1 - \frac{1}{\lambda}) b^P y^P} \left( U(c^P) - U(b^P) \right) \frac{2 - \lambda}{2 - \lambda}
\]

\[
w = \frac{b^P y^P}{v^P} + k(c^P - b^P)
\]

\[
\tau = w - q = \frac{y - w - kv^P}{kv^P} z
\]

Unfortunately, given the parameterization in the next section, this wage/unemployment insurance scheme does not generate a competitive equilibrium.

5. CALIBRATION

5.1. Parameterization

In this section I calibrate the model to US data before the global economic downturn. I take one period to be a month. My choice of the production parameters is based on the FRED data on the website of the Federal Reserve Bank of St. Louis. On April 1, 2005, data for the first quarter of the year 2005 were released. The gross domestic product (GDP) is 12373.1, gross private domestic investment is 2054.2, government consumption expenditure and gross investment is 2338.2, compensation paid to employees is 7105.2, all in terms of billions of dollars. FRED also has monthly data for unemployment rate. On April 1, 2005, unemployment rate is 5.2 percent. Using the fact that \( \mu = 1 - u \), I set the probability of being employed to be \( \mu = 0.948 \). Since \( \mu y \) is the gross product, I use...
12373/(\mu \ast 3) to get monthly data for the product of a filled job. I take the cost of opening a job \( (k) \), to be the monthly sum of private and government investments. Wage is set to be monthly compensation paid to employees. Normalizing one unit of dollars to be one thousand billions of dollars, I obtain \( y = 4.3506, k = 1.4641, w = 2.3684 \). The share of capital in production \( (k/y) \) is about one third, which corresponds to standard macroeconomic models.

From the free-entry condition, I obtain job’s probability of being filled \( \eta = k/(y-w) = 0.7386 \). In view of the fact that \( \mu = v \eta \), I compute \( v = 1.2835 \) for the number of vacancies. Blanchard and Diamond (1989) use monthly aggregate data to estimate a Cobb-Douglas matching function for the US labor market. With new hires defined as the number of matches, they obtain an elasticity of 0.4 for unemployment rate \( (\lambda) \). They choose the scale parameter \( (\alpha) \) to range between 0.95 and 1.3. In my calibration, I set \( \alpha = 1.2 \). Using this matching function and firms’ zero-profit condition (12), average search effort \( (s) \) is 0.3123.

Gruber (1998) finds that the replacement rate is slightly less than 50%. Using a replacement rate equal to 50% and assuming the government budget balances, I obtain the after-tax payment to employed workers \( (q) \) and unemployment benefit to unemployed workers \( (z) \), with \( q = 2.3052, z = 1.1526 \).

I assume the utility function takes a constant relative risk aversion (CRRA) form, \( U(C) = C^{1-\sigma}/(1-\sigma) \). Hopenhayn and Nicolini (1997) set the degree of risk aversion to be 0.5 when using weekly data. They argue that the elasticity of substitution on weekly consumption is most probably larger than that corresponding to longer periods. In view of this argument, I set \( \sigma = 2 \) in my calibration on monthly data. I assume the disutility from search takes the form \( g(s) = As^2 \), using the fact that equation (15) is satisfied in equilibrium, I get \( A = 2.2606 \).

5.2. Welfare Implications

Table 1 gives the calibration result for a benchmark economy and three alternative wage/unemployment insurance schemes. The benchmark economy is the solution of the social planner’s problem using the parameters in the last subsection. Column I shows that the social planner provides almost full insurance against unemployment shocks. Unemployment rate is almost zero. Number of vacancies relative to number of workers is 1.78, which is the highest among all four schemes. The current system is represented in Column II. Column III provides an alternative unemployment scheme, with the same before-tax wage as the current US economy. Indeed, Column III describes an economy with the highest ex ante utility achievable under the current wage. Compared with the current equilibrium, it provides better risk-sharing, workers search less, and unemployment rate
is 10%. The optimal wage/UI scheme among all competitive equilibria is in Column IV, with a higher replacement rate and a lower wage than the current system. It induces a 0.06% unemployment rate, number of vacancies is higher, and workers exert less search effort.

I provide three measures of welfare in the last three rows of Table 1. Net output is defined by

\[ Y(v, s) = \mu y - vk \]

The number of filled jobs is equal to the measure of workers who find a job (\(\mu\)). The measure of jobs is \(v\). The first term on the right-hand side of the above equation is total output produced, and the second term is investment expenditures. Looking at Table 1, although the socially optimal allocation has the lowest level of net output among all four schemes, it generates the highest ex ante utility for workers. Welfare loss, the last row, is defined as the percentage loss of consumption in the benchmark economy, for a worker to be indifferent between being in the decentralized equilibrium, and being in the benchmark economy if this percentage of consumption is taken away. The current system generates a large welfare loss that equals 8.07%. With the current wage, the economy can do no better than generating a welfare loss of 7.93%. Switching to the optimal wage/UI scheme improves a lot on ex ante utility, yet induces a slightly lower level of net output than the current system.

6. WORKER HETEROGENEITY

As pointed out by Karni (1999), to attain efficient allocation, UI system should reflect the different unemployment risk faced by workers due to variations in personal characteristics and actions. In this section, I consider one form of worker heterogeneity: heterogeneity in disutility from search. Assume two types of workers exist. Let \(g(s)\) and \(h(s)\) denote the disutility for Type I and Type II workers, respectively. Assume these two functions are continuous, strictly increasing and continuously twice differentiable, with \(g(0) = h(0) = 0\), \(g'(s) < h'(s)\) and \(g''(s) < h''(s)\) for all \(s > 0\). The proportion of Type I workers is \(\Psi\). This proportion is publicly known. Another way of interpreting the search effort is the time devoted to search. Suppose the total endowment of time is fixed for an unemployed worker. Workers divide up this time endowment into search and leisure. The two types of workers differ in their preferences for leisure. Indeed, some individuals may have changing attitude toward leisure. For example, when a child is born, one of the parents may prefer staying at home with the child. My model assumes consumption and leisure are separable in workers’ utility function, which makes the analysis easier.
TABLE 1.
Results for the benchmark economy and three alternative schemes

<table>
<thead>
<tr>
<th>Variables</th>
<th>Column I</th>
<th>Column II</th>
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<tbody>
<tr>
<td></td>
<td>Benchmark</td>
<td>Current</td>
<td>Same wage</td>
<td>Optimal</td>
</tr>
<tr>
<td>Consumption of the employed</td>
<td>1.740239</td>
<td>2.3052</td>
<td>2.2373</td>
<td>2.2024</td>
</tr>
<tr>
<td>Consumption of the unemployed</td>
<td>1.740238</td>
<td>1.1526</td>
<td>1.1651</td>
<td>1.2289</td>
</tr>
<tr>
<td>Before-tax wage $w$</td>
<td>-</td>
<td>2.3684</td>
<td>2.3684</td>
<td>2.2032</td>
</tr>
<tr>
<td>Replacement rate $z/q$</td>
<td>-</td>
<td>0.5000</td>
<td>0.5208</td>
<td>0.5580</td>
</tr>
<tr>
<td>Search effort $s$</td>
<td>0.2663</td>
<td>0.3815</td>
<td>0.3617</td>
<td>0.3567</td>
</tr>
<tr>
<td>Vacancies $v$</td>
<td>1.7829</td>
<td>1.2835</td>
<td>1.2169</td>
<td>1.4658</td>
</tr>
<tr>
<td>Unemployment rate $u$</td>
<td>3.0144e-07</td>
<td>0.0520</td>
<td>0.1011</td>
<td>6.1190e-04</td>
</tr>
<tr>
<td>Workers’ probability of getting employed $\mu$</td>
<td>0.9999997</td>
<td>0.9480</td>
<td>0.8989</td>
<td>0.9994</td>
</tr>
<tr>
<td>Job’s probability of getting filled $\eta$</td>
<td>0.5609</td>
<td>0.7386</td>
<td>0.7386</td>
<td>0.6818</td>
</tr>
<tr>
<td>Ex ante utility $\tilde{\mu}$</td>
<td>-0.7349</td>
<td>-0.7854</td>
<td>-0.7844</td>
<td>-0.7418</td>
</tr>
<tr>
<td>Net output $\tilde{\eta}$</td>
<td>1.7403</td>
<td>2.2452</td>
<td>2.1291</td>
<td>2.2019</td>
</tr>
<tr>
<td>Welfare loss</td>
<td>-</td>
<td>8.07%</td>
<td>7.93%</td>
<td>1.18%</td>
</tr>
</tbody>
</table>

All pecuniary terms are in thousands of billions of dollars.

For now I assume each individual worker’s type is known by the government or the UI provider. Also, assume workers’ search effort cannot be observed or monitored. Let $s_1, s_2$ to be the search effort by Type I and Type II workers respectively. Then the average search effort ($\bar{s}$) of all workers is

$$\bar{s} = \Psi s_1 + (1 - \Psi) s_2$$  \hspace{1cm} (20)

The matching function is

$$m(su, v) = \alpha^{v^{1-\lambda}}$$  \hspace{1cm} (21)

Assume jobs are identical. Each job produces output $y$ and pays a gross wage $w$ to the worker, regardless of the worker’s type. In equilibrium, firms’ zero-profit condition again yield a linear relation ship between number of vacancies and average search effort (see equation (12))

$$\bar{s} = \left[ \frac{k}{\alpha(y - w)} \right]^{\frac{1}{\lambda}} v$$  \hspace{1cm} (22)

Assume $(q_1, z_1)$ is the net-wage and UI benefit received by Type I employed workers and unemployed workers respectively, and $(q_2, z_2)$ is the net-wage and UI benefit received by Type II employed workers and unemployed
workers respectively. Then search effort in equilibrium is characterized by (see Appendix B for proof)

\[ g'(s_1) = (U(q_1) - U(z_1)) \alpha^{1/\lambda} \left( \frac{y - w}{k} \right)^{1/\lambda - 1} \frac{\Psi(2 - \lambda) + (1 - \Psi) \frac{s_1}{\Psi}}{\Psi + (1 - \Psi) \frac{s_1}{\Psi}} \] (Type I) \tag{23}

\[ h'(s_2) = (U(q_2) - U(z_2)) \alpha^{1/\lambda} \left( \frac{y - w}{k} \right)^{1/\lambda - 1} \frac{\Psi \frac{s_2}{\Psi} + (1 - \Psi)(2 - \lambda)}{\Psi \frac{s_2}{\Psi} + (1 - \Psi)} \] (Type II) \tag{24}

Note if \( \Psi = 1 \), then only Type I workers are present. Equation (23) coincides with equation (15) (the case with homogeneous workers). On the other hand, if \( \Psi = 0 \), with only Type II workers, equation (24) is also consistent with equation (15).

Next I consider three different schemes, differing only in the method of financing UI. In all three schemes, UI is financed by income tax on employed workers.

**Scheme 1.** Uniform insurance policy for all workers, i.e., \( q_1 = q_2 = q \), \( z_1 = z_2 = z \).

The budget constraint of the government is

\[ uz = (1 - u)\tau \] with \( q = w - \tau \), \( u = \Psi u_1 + (1 - \Psi)u_2 \) \tag{25}

\( u_1 \) and \( u_2 \) are the unemployment rate among Type I and Type II workers respectively.

Specifically,

\[ u_1 = 1 - s_1 \alpha^\lambda (1 - \lambda) \tag{26} \]

\[ u_2 = 1 - s_2 \alpha^\lambda (1 - \lambda) \tag{27} \]

Plug firms’ zero-profit condition into equation (25) to obtain

\[ \tau = \frac{y - w - k\frac{\alpha(y - w)}{k}}{k\frac{\alpha(y - w)}{k}} \] \tag{28}

which resembles equation (16) for the economy with homogeneous workers.

**Scheme 2.** Separate insurance policies for the two types, both being actuarially fair.

The budget constraint of the government is

\[ u_1 z_1 = (1 - u_1)\tau_1 \] with \( q_1 = w - \tau_1 \n\]

\[ u_2 z_2 = (1 - u_2)\tau_2 \] with \( q_2 = w - \tau_2 \) \tag{29}
Equation (29) together with firms’ zero-profit condition implies

\[
\tau_1 = 1 - \frac{\alpha_1}{\lambda} s_1 \left( \frac{y - w}{k} \right)^{1/\lambda - 1} z_1^{1/\lambda - 1}
\]

\[
\tau_2 = 1 - \frac{\alpha_2}{\lambda} s_2 \left( \frac{y - w}{k} \right)^{1/\lambda - 1} z_2^{1/\lambda - 1}
\]

(30)

(30’)

**Scheme 3.** Separate insurance policies for the two types. Neither of the policies is necessarily actuarially fair, but there is cross-subsidization between the two policies.

The budget constraint of the government is

\[
u_1 z_1 + u_2 z_2 = (1 - u_1)\tau_1 + (1 - u_2)\tau_2\text{ with } q_1 = w - \tau_1, q_2 = w - \tau_2
\]

(31)

Because doing welfare comparisons of the above three UI schemes is analytically intractable, I resort to calibration. Recall in the parameterization of Section 2.5, I assume the disutility from search takes the form \(g(s) = As^2\), and set \(A = 2.2606\). Now for the case of heterogeneous workers, for simplicity, I assume \(g(s) = A_1 s^2, g(s) = A_2 s^2\) with \(A_2 > A_1 > 0\).

The current US UI system is a system of Scheme 1 with mandated pooling insurance, in which workers have no choice over UI benefit or tax/premium. Keeping all deep parameters in Section 2.5 except \(A\) and \(q\), I calibrate \(\Psi, A_1\) and \(A_2\) such that the competitive outcome of Scheme 1 is in accordance with the data in the following aspects: the unemployment rate \((u)\) is about 5.2%; the replacement rate is about 50%. I also let \(\Psi A_1 + (1 - \Psi) A_2 = 2.2606\).

The set of parameters \(A_1 = 2.2, A_2 = 4, \Psi = 0.9663\) results in a competitive equilibrium with \(u = 0.0511\) and a replacement rate of 49.98%. Column I of Table 2 illustrates the market equilibrium of the current UI system with this set of parameters. UI benefit, average search effort, number of vacancies, and job’s probability of getting filled are all resemble those in Column II of Table 1.

I define social welfare by the weighted average of the ex ante utility of the two types of workers, with weight being their respective proportion in the whole population. Under each of the three UI schemes, I compute the wage/UI benefit that maximizes the social welfare and the corresponding market equilibrium. Column II through Column IV of Table 2 show these equilibrium outcomes. Compared with Column I (current system), Column II (optimal wage/UI benefit under Scheme 1) leads to a lower unemployment rate that equals 2.2%. Replacement rate is higher (61.43%), and wage is lower. Both types of workers search less and are better-off.
Thus the allocation in Column II constitutes a Pareto improvement upon the current system. Using the economy in Column II as the benchmark economy, the welfare loss of the current system in term of consumption is 10.29% (see a detailed definition of welfare loss in Section 2.5.2). Note that in the economy of Column II, tax revenue minus UI payment is positive for Type I workers, and negative for Type II workers. So the low-risk workers (Type I) subsidize the high-risk workers (Type II).

Column III is an economy without cross-subsidization between the two types of workers. Switching from Column II to Column III improves social welfare by 0.36% in terms of consumption. Without having to subsidize Type II workers, Type I workers are better-off. They enjoy a replacement rate of 61.59%. While Type II workers suffer a utility loss, with a replacement rate of only 36.33%. In contrast with the benchmark economy (Column II) where Type II workers exert only one third of the search effort by Type I workers, both types of workers now exert almost the same amount of effort. Consequently, unemployment rate of Type II workers plummets from 65.21% to 0.2%.

Indeed, Scheme 1 is a special case of Scheme 3. Thus the optimal market outcome of Scheme 3 is no worse than that of Scheme 1. The economy of Column IV generates a welfare gain of 1.34% relative to the benchmark economy. Compared with the benchmark economy, Type I workers now search less; their replacement rate is higher (66.31%); they are better-off. However, Type II workers are worse-off. They search more and their replacement is now lower, only 41.38%. Type II workers are subsidized by Type I workers. Moreover, the income tax paid by employed Type II workers is negative. In other words, they get a subsidy from finding a job.

Given that the unemployment insurance provider knows each worker’s type, the above argument also applies to privately-provided UI. With a slight change of the algebraic expression, income tax can be interpreted as insurance premium. When individual worker’s type is private information, problems of adverse selection arise. Chiu and Karni (1998) demonstrate that a self-financing pooling UI policy provided by the private sector, rather than mandated by the government, does not exist. The reason is that, the low-risk workers (Type I workers in my paper) prefer no insurance to paying the extra premium to subsidize the high-risk workers (Type II workers in my paper) under a pooling policy. According to their assumption about utility function, consumption need not be positive. But in my paper, UI is essential for unemployed workers because consumption must always be positive given my assumptions for workers’ utility function. Crocker and Snow (1985) show that a competitive equilibrium with adverse selection is not necessarily Pareto optimal, and separate policies to high-risk and low-risk workers with cross-subsidization may constitute a Pareto improvement. This result inspires me to make comparisons of the three schemes in
this section. But workers may be free to choose among different UI policies, and this implies that the UI policies must be incentive-compatible. Actually, neither of the UI policies in Column III and IV of Table 2 is incentive-compatible. Both types of workers have an incentive to pretend to be of the other type. In fact, the welfare gain of these two schemes relative to the benchmark economy with pooling policy is not large (0.36% and 1.34%), and they offer upper bounds for the welfare gains of any incentive-compatible UI policies under the two schemes. Considering the small magnitude of the welfare gains and considering the impossibility or difficulty of inferring workers' type, the economy may just stay with the benchmark economy with a global pooling UI policy. It does not matter whether this UI is provided by the private sector or by the government.

Karni (1999) argues that since individuals have observable characteristics, such as occupation, education, age, family situation, and employment history, it may be possible to sort them into different risk groups. In my model, if search effort is observable, it also constitutes an indicator of workers' risk type. Indeed, when perfect experience rating of individual's type is present, the design of unemployment insurance policy can circumvent the issue of incentive compatibility.

7. CONCLUSION AND EXTENSION

Models on search behavior in the labor market have focused on the efficiency of decentralized equilibrium, as well as the mechanism determining wage and unemployment flow. Little is done on how unemployment insurance affects workers' search behavior and thus equilibrium unemployment rate. On the other hand, the literature on unemployment insurance pays a lot of attention to its disincentive effect, besides its risk-sharing aspect. Yet search is not costless. Simply encouraging search can be misleading: too much search effort may generate welfare loss. This paper studies optimal wage/unemployment insurance in a model with endogenous search effort. With risk-averse workers, search effort in the decentralized market is not socially efficient. My calibration of the US economy suggests that it is welfare-improving to switch to a scheme with lower wage and higher replacement rate than the current system. The welfare gains of this change are large, but this change requires an exogenous decrease of current wages. The mechanism of wage determination is not studied here. Future research may address this issue.

My argument does not rely on the assumption of constant returns to scale of the matching function. Therefore, extensions to other Cobb-Douglas matching functions are possible. Moreover, the unemployment insurance scheme in this paper is that workers get a constant level of unemployment benefit as long as they stay unemployed. So the optimal unemployment
Table 2.
Results for the current scheme and three alternative schemes

<table>
<thead>
<tr>
<th>Variables</th>
<th>Column I</th>
<th>Column II</th>
<th>Column III</th>
<th>Column IV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Current</td>
<td>Scheme 1</td>
<td>Scheme 2</td>
<td>Scheme 3</td>
</tr>
<tr>
<td>Before-tax wage $w$</td>
<td>2.3684</td>
<td>2.0956</td>
<td>2.0797</td>
<td>1.9684</td>
</tr>
<tr>
<td>Consumption of the unemployed $z_1$ (type I)</td>
<td>1.1526</td>
<td>1.2698</td>
<td>1.2809</td>
<td>1.2983</td>
</tr>
<tr>
<td>Consumption of the unemployed $z_1$ (type II)</td>
<td>1.1526</td>
<td>1.2698</td>
<td>0.7551</td>
<td>0.9057</td>
</tr>
<tr>
<td>Consumption of the employed $q_1$ (type I)</td>
<td>2.3063</td>
<td>2.0671</td>
<td>2.0797</td>
<td>1.9579</td>
</tr>
<tr>
<td>Consumption of the employed $q_1$ (type II)</td>
<td>2.3063</td>
<td>2.0671</td>
<td>2.0782</td>
<td>2.1888</td>
</tr>
<tr>
<td>Replacement rate $z_1/q_1$ (type I)</td>
<td>0.4998</td>
<td>0.6143</td>
<td>0.6159</td>
<td>0.6631</td>
</tr>
<tr>
<td>Replacement rate $z_2/q_2$ (type II)</td>
<td>0.4998</td>
<td>0.6143</td>
<td>0.3633</td>
<td>0.4138</td>
</tr>
<tr>
<td>Average unemployment rate $u$</td>
<td>0.0511</td>
<td>0.0220</td>
<td>7.7993e-05</td>
<td>0.0039</td>
</tr>
<tr>
<td>Unemployment rate $u_1$ (type I)</td>
<td>0.0298</td>
<td>1.4890e-06</td>
<td>1.1613e-05</td>
<td>3.3566e-08</td>
</tr>
<tr>
<td>Unemployment rate $u_2$ (type II)</td>
<td>0.6626</td>
<td>0.6521</td>
<td>0.0020</td>
<td>0.1173</td>
</tr>
<tr>
<td>Average search effort $\tau$</td>
<td>0.3818</td>
<td>0.3244</td>
<td>0.3281</td>
<td>0.3042</td>
</tr>
<tr>
<td>Average search effort $s_1$ (type I)</td>
<td>0.3904</td>
<td>0.3316</td>
<td>0.3282</td>
<td>0.3054</td>
</tr>
<tr>
<td>Average search effort $s_2$ (type II)</td>
<td>0.1358</td>
<td>0.1154</td>
<td>0.3275</td>
<td>0.2696</td>
</tr>
<tr>
<td>Vacancies $v$</td>
<td>1.2846</td>
<td>1.5064</td>
<td>1.5509</td>
<td>1.6206</td>
</tr>
<tr>
<td>Job’s probability of getting filled $\eta$</td>
<td>0.7386</td>
<td>0.6493</td>
<td>0.6447</td>
<td>0.6146</td>
</tr>
<tr>
<td>Average ex ante utility</td>
<td>-0.7823</td>
<td>-0.7261</td>
<td>-0.7243</td>
<td>-0.7196</td>
</tr>
<tr>
<td>Ex ante utility (type I)</td>
<td>-0.7819</td>
<td>-0.7258</td>
<td>-0.7178</td>
<td>-0.7160</td>
</tr>
<tr>
<td>Ex ante utility (type II)</td>
<td>-0.7949</td>
<td>-0.7351</td>
<td>-0.9119</td>
<td>-0.8236</td>
</tr>
<tr>
<td>Net output</td>
<td>2.2473</td>
<td>2.0496</td>
<td>2.0795</td>
<td>1.9606</td>
</tr>
<tr>
<td>Welfare loss</td>
<td>10.29%</td>
<td>-</td>
<td>-0.36%</td>
<td>-1.34%</td>
</tr>
</tbody>
</table>

All pecuniary terms are in thousands of billions of dollars.


insurance (UI) in my model is indeed a “constrained” optimal policy. Other UI payment schemes may be also considered. This paper assumes identical job vacancies and workers in terms of skill and skill requirement. However, when heterogeneity of jobs and workers are considered, one worker’s higher search effort may result in better matching quality and attract more entry of jobs, which in turn may improve the matching quality and job-finding probability of other job seekers. It would be interesting to examine how this positive externality may affect the design of optimal UI policy, when compounded by the negative externality demonstrated in this paper.\footnote{I thank an anonymous referee for this suggestion.}
APPENDIX

Proof of equations (23) and (24)

For an individual worker of Type I, if her search effort is $s_1^i$, given the average search effort of the two types of workers, $s_1$ and $s_2$, her probability of finding a job is

$$\mu_1^i = s_1^i \alpha \lambda^{-1} v^{1-\lambda} \quad (A.1)$$

In a graph with number of vacancies ($v$) being the vertical axis and search effort of Type I workers ($s_1$) being the horizontal axis, apply the Implicit Function Theorem to obtain

$$\frac{dv}{ds_1^i} = \frac{1}{(1-\lambda)s_1^i} \left[ \frac{g'(s_1)}{\alpha(U(q_1) - U(z_1))} - 1 \right] v \quad (A.2)$$

Setting $s_1^i = s_1$, (A.2) becomes

$$\frac{dv}{ds_1} = \frac{1}{(1-\lambda)} \left[ \frac{g'(s_1)}{\alpha(U(q_1) - U(z_1))} - 1 \right] v \quad (A.3)$$

Firms’ zero-profit line is given by

$$v = \left[ \frac{\alpha(y-w)}{k} \right]^{1/\lambda} \left[ \Psi s_1 + (1-\Psi)s_2 \right] \quad (A.4)$$

And (A.4) implies

$$\frac{dv}{ds_1} = \Psi \left[ \frac{\alpha(y-w)}{k} \right]^{1/\lambda} \quad (A.5)$$

$$\frac{v}{s_1} = \Psi \left[ \frac{\alpha(y-w)}{k} \right]^{1/\lambda} + (1-\Psi) \left[ \frac{\alpha(y-w)}{k} \right]^{1/\lambda} \frac{s_2}{s_1} \quad (A.6)$$

A necessary condition for the market equilibrium to occur is that Type I worker’s indifference curve is tangent to firms’ zero-profit line given by (A.4). Plug (A.5) into the left-hand side of (A.3) and plug (A.6) into the right-hand side of (A.3), this gives

$$g'(s_1) = \frac{U(q_1) - U(z_1)}{\alpha^{1/\lambda}} \left[ \frac{y-w}{k} \right]^{1/\lambda-1} \Psi(2-\lambda) + (1-\Psi) \frac{z_2}{s_1} \quad (Type \ I) \quad (23)$$

Applying similar argument to Type II workers, equation (24) can be obtained. The competitive equilibrium is a fixed point $(s_1, s_2, v)$ in a three-dimensional space, with (20), (22) and government budget constraint being satisfied.
REFERENCES


