The Spirit of Capitalism and the Equity Premium*

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This paper evaluates whether the spirit of capitalism can explain the equity premium puzzle. The spirit of capitalism implies that investors acquire wealth not just for consumption, but also to improve their social status. We set up a consumption-based capital asset pricing model incorporating this component. The simulated results from our calibrated model match the mean and the volatility of the equity premium observed in the data.

Key Words: Spirit of capitalism; Consumption-based capital asset pricing model.
JEL Classification Numbers: E21, G12.

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1. INTRODUCTION

The notion of the spirit of capitalism is proposed as capturing the drivers for wealth accumulation, consumption and social status. In recent studies, the spirit of capitalism is regarded as an important way to explain consumption behavior, for example, Bakshi and Chen (1996), Zou (1998), Gong and Zou (2002), Kenc and Dibooglu (2007), Luo, Smith, and Zou (2009) and Karnizova (2010). In addition, how the spirit of capitalism affects asset returns has been discussed in Smith (2001) and Boileau and Braeu (2007). However, the literature does not provide a direct answer to the question as to whether the spirit of capitalism can quantitatively explain the equity premium puzzle. This paper tries to answer this by following the analytical method in Bansal and Yaron (2004) and investigating whether the results from a consumption-based capital asset pricing (CCAPM) model, incorporating the spirit of capitalism, explain the equity premium puzzle.

In the literature, to solve the equity premium puzzle, the habit-formation model of Campbell and Cochrane (1999), building on work by Abel (1990) and Constantinides (1990), considers the importance of a positive effect from today’s consumption on tomorrow’s marginal utility of consumption. A small but persistent unobservable common component in the time-series processes of aggregate consumption and dividend growth, put forward by Bansal and Yaron (2004) as the long-run consumption risk (LRR), characterizes a specific cashflow dynamic, see Bansal et al. (2007a), Bansal et al. (2007b), Hansen et al. (2008) and Yang (2011). They combine the latent LRR component with recursive preference of Epstein and Zin (1989, 1991) and Weil (1989). This is a notable generalization of the power utility which separates the coefficient of relative risk aversion from the elasticity of intertemporal substitution in consumption. More recently, the role of rare disasters has been intensively discussed, e.g., Barro (2006), Wachter (2013), Barro (2009), Gabaix (2010) and Barro and Jin (2011). Dreyer et al. (2013) discuss the saving-based asset-pricing, where the growth rate of aggregate wealth is included in utility function.

In this paper, before we create a model of the spirit of capitalism to explain the equity premium, we study the empirical relationship between the wealth growth rate and the equity premium. If we regress the equity premium on the consumption growth rate, the dividend growth rate and the wealth growth rate, then we find that the wealth growth rate significantly affects the equity premium. Moreover, the results show that high wealth growth requires a high premium. This can be explained as the effect of the spirit of capitalism on the equity premium, as when investors care about their level of wealth and anticipate high wealth levels in the next period, they need a high premium to participate in a risky investment.
Next, we build our model according to Bakshi and Chen (1996), where wealth is directly included in the utility function. This implies that agents care about social status in addition to consumption. Following Bansal and Yaron (2004), we specify the processes for calculating consumption growth rate, dividend growth rate and conditional volatility, and solve the model in terms of state variables. Using real data, we calibrate the model and generate equity returns, risk-free rates and dividends. We find that our simulated data matches the means and the volatilities of the equity premia, equity returns and risk-free rates observed in the real data.

Numerically, in our base model, we set the risk aversion coefficient, \( \gamma \), at 10 and the social status sensitivity coefficient, \( \lambda \), to 15. Our model generates an equity premium of 6.03% annually with 19.35% volatility. In addition, we adjust the value of \( \lambda \) to analyze the effects of social status on the equity premium. We find that the equity premium increases with \( \lambda \).

The rest of this paper is organized as follows: Section 2 describes the data. Section 3 displays the empirical relationship between wealth and the equity premium. Section 4 introduces our model with the numerical results presented in Section 5. Section 6 concludes.

2. DATA

We collect annual data from 1950 to 2013. Consumption data is the sum of nondurable goods and services in real terms from the US Department of Commerce, Bureau of Economic Analysis, NIPA Table 2.3.3. The data source for US population is the US Department of Commerce, Bureau of Economic Analysis, NIPA Table 2.1, line 40. The data are transformed to be per capita and in log-difference form to capture the growth rate. The stock market returns are the value-weighted annual returns from CRSP. The deflator is inflation index from CRSP. Risk-free rates are the 90-day treasury bill rate. The stock market returns and the risk-free rates are inflation-adjusted.

Wealth is defined as asset wealth plus financial wealth. Asset wealth is the net worth of households and nonprofit organizations while financial wealth is total financial assets minus total liabilities of households and nonprofit organizations. All data are collected from the Board of Governors of the Federal Reserve System, Financial Accounts of the United States (Z.1), Table B.100. The net worth values of households and nonprofit organizations are from Series FL152090005.A. The total financial assets are from Series FL154090005.A. The total liabilities values of households and nonprofit organizations are from Series FL154190005.A.

Table 1 summarizes the means the standard deviations of the equity premium, the consumption growth rate and the wealth growth rate.
TABLE 1.
Statistic Properties

<table>
<thead>
<tr>
<th>Premium Consumption Growth</th>
<th>Dividend Growth</th>
<th>Wealth Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (%)</td>
<td>6.03</td>
<td>1.83</td>
</tr>
<tr>
<td>Std. Dev. (%)</td>
<td>18.02</td>
<td>1.17</td>
</tr>
</tbody>
</table>

3. EFFECTS OF WEALTH ON PREMIUM

Since the spirit of capitalism says that wealth can affect consumption and hence asset returns, we try to find empirical evidence of this. We regress the equity premium on the consumption growth rate, the dividend growth rate and the wealth growth rate. As shown in Figure 1, the time series of the four factors do not exhibit an obvious trend, so the ordinary least squares regression can be applied here.

Table 2 summarizes the results of the regression. The adjusted $R^2$ value is 0.65. The estimated coefficient of the wealth growth rate is 1.8088 with a $p$-value of 0, so the wealth growth rate has a positive effect on the equity premium.

A positive $\beta_3$ value implies that when investors anticipate a high wealth level in the next period, they need a higher risk premium to induce them to participate in a risky investment. This is reasonable since involvement in a risky investment may induce loss of wealth, which they care about.

TABLE 2.
Effects of wealth on premium

<table>
<thead>
<tr>
<th>estimate</th>
<th>Std.Dev.</th>
<th>$t$-stat</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>-0.0643</td>
<td>0.0263</td>
<td>-2.4414</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-0.5948</td>
<td>1.0937</td>
<td>-0.5439</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.5395</td>
<td>0.1301</td>
<td>4.1474</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>1.8088</td>
<td>0.3123</td>
<td>5.7912</td>
</tr>
</tbody>
</table>

This table provides the estimation results for the regression: $r_m - r_f = \beta_0 + \beta_1 g + \beta_2 g_d + \beta_3 g_w + \epsilon$, where $r_m - r_f$ is the equity premium series, $g$, $g_d$, $g_w$ are the consumption growth rate, dividend growth rate and wealth growth rate, respectively. The adjusted $R^2$ value is 0.65.

4. MODEL

4.1. Pricing Kernel

We consider a representative agent model, where the agent has a preference incorporating the spirit of capitalism. Following Bakshi and Chen

\[ U(C_t, W_t) = \frac{C_t^{1-\gamma}}{1-\gamma} W_t^{-\lambda}, \]  

(1)

where \( C_t \) and \( W_t \) are the agent’s consumption and wealth levels and \( \gamma > 0 \). 
\( \lambda > 0 \) when \( \gamma \geq 1 \) and \( \lambda < 0 \) otherwise. \( |\lambda| \) measures the extent to which the investor cares about social status.
Given the budget constraint, the agent maximizes

$$\max_{C_t, W_t} \sum_{t=1}^{\infty} \delta^t U(C_t, W_t)$$

(2)

$$W_t = P_{B,t} B_t + C_t + P_{S,t} S_t,$$

(3)

$$W_{t+1} = L_{t+1} + (P_{S,t+1} + D_{S,t+1}) S_t + B_t.$$  

(4)

Each period, the representative agent derives utility from consumption $C_t$, buys $S_t$ stocks at price $P_S$, and holds $B_t$ bonds at price $P_B$ which will return one dollar next period. Then, in the next period, the agent receives back $B_t$ and wage $L_{t+1}$, possesses $S_t$ stocks at price $P_{S,t+1}$ with its corresponding dividend $D_{S,t+1}$; the sum of the agent’s wealth.

The stochastic discount factor, $M_t$, implied in this model is

$$M_{t+1} = \delta \frac{\partial U(C_{t+1}, W_{t+1})/\partial C_{t+1}}{\partial U(C_t, W_t)/\partial C_t}.$$  

(5)

The derivation of the pricing kernel is in Appendix B. The logarithm, $m_{t+1}$, can be expressed as

$$m_{t+1} = \log \delta - \gamma g_{t+1} - \lambda r_{w,t+1},$$  

(6)

where $g_{t+1}$ is the growth rate of consumption and $r_{w,t+1}$ is the return of the portfolio, of which the dividend is equal to consumption.

Following Campbell and Shiller (1988) and Bansal and Yaron (2004), the market portfolio return, $r_{m,t+1}$, is:

$$r_{m,t+1} = k_{0,m} + k_{1,m} z_{m,t+1} - z_{m,t} + g_{d,t+1}$$  

(7)

where $d_t$ and $p_t$ represent the log-value of dividend and price, $z_{m,t} = \log \frac{D_t}{P_t}$, $d_t$ is the log price-dividend ratio, and $g_{d,t+1} = d_{t+1} - d_t$ is the dividend growth rate. $k_{0,m}$ and $k_{1,m}$ are constant.

Similarly, the log-wealth return, $r_{w,t+1}$, can be expressed as:

$$r_{w,t+1} = k_0 + k_1 z_{t+1} - z_t + g_{t+1},$$  

(8)

where $z_t = p_t - c_t$ is the log price-consumption ratio and $g_{t+1} = c_{t+1} - c_t$ is the consumption growth rate. $c_t$ is the log-value of consumption. $k_0$ and $k_1$ are constant.
4.2. Solving the Model

We specify the processes of the consumption growth rates and the dividend growth rates as

\[
\begin{align*}
g_{t+1} &= \mu + \phi_c g_t + \sigma_t \eta_{t+1} \\
g_{d,t+1} &= \mu_d + \phi_d g_{t+1} + \varphi_d \sigma_t u_{t+1} \\
\sigma^2_{t+1} &= \sigma^2 + \nu_1 (\sigma^2_t - \sigma^2) + \sigma_w w_{t+1} \\
\eta_{t+1}, u_{t+1}, w_{t+1} &\sim i.i.d. N(0, 1).
\end{align*}
\]  

(9)

Fluctuating economic uncertainty is represented by the process of $\sigma^2_t$ with a mean of $\sigma^2$. The three shocks in Equation (9), $\eta_{t+1}$, $u_{t+1}$ and $w_{t+1}$, are mutually independent. To ensure the stationarity of the process, we restrict $0 < \phi_c < 1$. We introduce two additional parameters, $\phi_d > 0$ and $\varphi_d > 0$, to calibrate the volatility of dividends and its correlation with consumption.

Following Bansal and Yaron (2004), we conjecture that the approximate solutions for the log price-consumption ratio $z_t$ and log price-dividend ratio $z_{m,t}$ are:

\[
\begin{align*}
z_t &= A_0 + A_1 g_t + A_2 \sigma^2_t \\
z_{m,t} &= A_{0,m} + A_{1,m} g_t + A_{2,m} \sigma^2_t.
\end{align*}
\]

(10)

Here, $g_t$ and $\sigma_t$ are two state variables, and $A_0$, $A_1$, $A_2$, $A_{0,m}$, $A_{1,m}$ and $A_{2,m}$ are constants. Their values are obtained by considering

\[
\begin{align*}
E_t[e^{m_{t+1}+r_{m,t+1}}] &= 1, \\
E_t[e^{m_{t+1}+r_{w,t+1}}] &= 1.
\end{align*}
\]

(11)

Details of the derivations are presented in Appendix E and H.

We see that $A_1$ and $A_2$ are

\[
\begin{align*}
A_1 &= \frac{(1-\lambda-\gamma)\phi_c}{(1-\lambda)(1-\lambda_1)}, \\
A_2 &= \frac{\lambda(1-\gamma+k_1 A_1 - k_1 A_2)^2}{(1-\lambda_1)(1-k_1 \lambda_1)}.
\end{align*}
\]

(12)

Given the values of $A_1$ and $A_2$, and that $m_{t+1} = \log \delta - \gamma g_{t+1} - \lambda r_{w,t+1}$, we derive the innovation of $m_{t+1}$ as

\[
m_{t+1} - E_t m_{t+1} = -\lambda_{m,\eta} \sigma_t \eta_{t+1} - \lambda_{m,w} \sigma_w w_{t+1},
\]

(13)

where $\lambda_{m,\eta} = \gamma + \lambda + \lambda k_1 A_1$ and $\lambda_{m,w} = \lambda k_1 A_2$. The $\lambda_{m,\eta}$ and $\lambda_{m,w}$ values capture the pricing kernel’s exposure to independent consumption shock and fluctuating economic uncertainty. A salient feature of the expression
is that both $\lambda_{m,n}$ and $\lambda_{m,w}$ increase as $\lambda$ increases, which implies that the more agents care about their social status, the larger is the magnitude of the innovation of the pricing kernel.

Similarly, $A_{1,m}$ and $A_{2,m}$ can be obtained as

$$
A_{1,m} = \frac{(-\lambda - \gamma - \lambda k_1 A_1 + \phi_d)\phi_c + \lambda A_1}{1 - k_{1,m} \phi_d},
$$

$$
A_{2,m} = \frac{1 - \lambda A_2 - \lambda k_1 A_2 \nu_1}{1 - \nu_1 k_{1,m}},
$$

where $H_m = (-\lambda - \gamma - \lambda k_1 A_1 + k_{1,m} A_{1,m} + \phi_d)^2 + \phi_d^2.$

### 4.3. Equity Premium and Market Volatility

After we obtain $A_0$, $A_1$, $A_2$, $A_{0,m}$, $A_{1,m}$, and $A_{2,m}$, the conditional equity premium is:

$$
E_t[r_{m,t+1} - r_{f,t+1}] = \beta_{m,n} \lambda_{m,n} \sigma_t^2 + \beta_{m,w} \lambda_{m,w} \sigma_w^2 - \frac{1}{2} (\beta_{m,n}^2 + \phi_d^2) \sigma_t^2 + \beta_{m,w}^2 \sigma_w^2.
$$

where

$$
\beta_{m,n} = k_{1,m} A_{1,m} + \phi_d \tag{16}
$$

$$
\beta_{m,w} = k_{1,m} A_{2,m} \tag{17}
$$

$$
\text{Var}_t(r_{m,t+1}) = E_t[(r_{m,t+1} - E_t r_{m,t+1})^2] = (\beta_{m,n}^2 + \phi_d^2) \sigma_t^2 + \beta_{m,w}^2 \sigma_w^2. \tag{18}
$$

The equity premium now has two sources of risk, $\sigma_t^2$ and $\sigma_w^2$, the first being from fluctuations in consumption growth, and the second from economic uncertainty.

The unconditional variance of market return is

$$
\text{Var}(r_m) = (k_{1,m} A_{1,m} + \phi_d)^2 E(\sigma_t^2) + [(k_{1,m} A_{1,m} + \phi_d) \phi_c - A_{1,m}]^2 \text{Var}(g_t) + (k_{1,m} \nu_1 - 1)^2 A_{2,m}^2 \text{Var}(\sigma_t^2) + \phi_d^2 E(\sigma_t^2) + k_{1,m}^2 A_{2,m}^2 \sigma_w^2. \tag{19}
$$

The unconditional expectation of the risk-free rate\(^1\) is

$$
E r_{f,t+1} = -\log \delta + \gamma E g_{t+1} + \lambda E r_{w,t+1} - \frac{1}{2} (\lambda_{m,n}^2 E(\sigma_t^2) + \lambda_{m,w}^2 \sigma_w^2)
$$

$$
+ \gamma \lambda (k_1 A_1 + 1) E(\sigma_t^2). \tag{20}
$$

The unconditional variance of the risk-free rate is

$$
\text{Var}(r_{f,t+1}) = [(\lambda + \lambda k_1 A_1) \phi_c - \lambda A_1]^2 \text{Var}(g_t)
$$

$$
+ [\lambda k_1 A_2 \nu_1 - \lambda A_2 - \frac{1}{2} \lambda_{m,n}^2 + \gamma \lambda (k_1 A_1 + 1)]^2 \text{Var}(\sigma_t^2). \tag{21}
$$

\(^1\)Here, $E(r_{w,t+1}) = k_0 + k_1 (A_0 + A_1 E g_{t+1} + A_2 E \sigma_{t+1}^2) - (A_0 + A_1 E g_t + A_2 E \sigma_t^2) + E g_{t+1}$, the details of which are shown in the appendix.
All the details are in Appendix D.

5. NUMERICAL RESULTS

We calibrate the model for annual frequency. The parameters are calibrated to match the first and the second moments of the real data. The calibrated parameters are listed in Table 3. Among these parameters, we notice that risk aversion coefficient $\gamma$ is set at 10, which is a reasonable value according to the literature.

<table>
<thead>
<tr>
<th>Table 3. Calibrated Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
</tr>
<tr>
<td>0.0062</td>
</tr>
</tbody>
</table>

We use the model to simulate the data, and compare with the real data. Table 4 reports the results. We can see that the sample means of the simulated data are very close to their counterparts in the real data. The risk premium generated by the model is 6.02% annually with a volatility of 19.35%. The risk premium using the real data is 6.03% with 18.02% volatility. In addition, most of the observed data fall into the 5th and 95th quantile intervals of the simulated counterparts, except for the expectation and volatility of the consumption growth rate.

5.1. Effect of the Spirit of Capitalism

To highlight the role played by the spirit of capitalism, we adjust the value of $\lambda$ and keep the other parameters fixed. In the baseline model, $\lambda$ is set at 15. Here, we set it at 20 and then 30, and see how $\lambda$ affects the simulated data. Table 5 summarizes the results.

When $\lambda$ increases from 15 to 20, and then 30, we see that the risk premium increases dramatically. When $\lambda = 20$, the risk premium increases to 9.64% and its volatility increases to 21.49%. This is not surprising because when investors care more about their social status, they require a higher premium to induce them to participate in a risky investment. Additionally, when they care more about social status, they choose to put more assets into the equity with a higher return, rather than in the risk-free asset, so as to accumulate wealth more quickly. So, we observe that both the equity return and the risk-free rate increase along with $\lambda$. 

![Table 3](image-url)
TABLE 4. Model-implied Numerical Results

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>5%</td>
</tr>
<tr>
<td>(E(g))[%]</td>
<td>1.27</td>
<td>1.20</td>
</tr>
<tr>
<td>(\sigma(g))[%]</td>
<td>0.86</td>
<td>0.73</td>
</tr>
<tr>
<td>(AC1(g))</td>
<td>0.51</td>
<td>0.44</td>
</tr>
<tr>
<td>(E(g_d))[%]</td>
<td>2.08</td>
<td>1.31</td>
</tr>
<tr>
<td>(\sigma(g_d))[%]</td>
<td>11.78</td>
<td>10.08</td>
</tr>
<tr>
<td>(AC1(g_d))</td>
<td>0.15</td>
<td>0.08</td>
</tr>
<tr>
<td>(corr(g, g_d))</td>
<td>0.54</td>
<td>0.49</td>
</tr>
<tr>
<td>(E(r_m - r_f))[%]</td>
<td>6.02</td>
<td>5.20</td>
</tr>
<tr>
<td>(E(r_m))[%]</td>
<td>6.58</td>
<td>5.54</td>
</tr>
<tr>
<td>(E(r_f))[%]</td>
<td>0.53</td>
<td>0</td>
</tr>
<tr>
<td>(\sigma(r_m - r_f))[%]</td>
<td>19.35</td>
<td>16.83</td>
</tr>
<tr>
<td>(\sigma(r_m))[%]</td>
<td>19.62</td>
<td>17.05</td>
</tr>
<tr>
<td>(\sigma(r_f))[%]</td>
<td>1.70</td>
<td>1.40</td>
</tr>
<tr>
<td>(E(\log \frac{P_D}{P}))</td>
<td>35.95</td>
<td>33.11</td>
</tr>
<tr>
<td>(\sigma(\log \frac{P_D}{P}))</td>
<td>0.282</td>
<td>0.239</td>
</tr>
<tr>
<td>(AC1(\log \frac{P_D}{P}))</td>
<td>0.91</td>
<td>0.88</td>
</tr>
</tbody>
</table>

This table summarizes the means and the 95% and 5% quantiles of the simulated data. The model-implied results are based on 1000 simulations. In each simulation, 1000 observations are simulated. The last column shows the empirical annual results, based on the data from 1950 to 2013.

Moreover, the volatilities of both the equity return and the risk-free rate increase to 21.85% and 2.06%, respectively, when \(\lambda = 20\). This is due to, as shown in Appendix F, the volatility of the SDF increasing when \(\lambda\) becomes large. This leads to more volatile PD ratio, equity return and risk-free rate values.

### 5.2. Effect of Risk Aversion

We also explore the effect of risk aversion coefficient \(\gamma\). Obviously, when \(\gamma\) is larger, the investor is more risk averse which generates a high premium. Our simulated data displays this property clearly, as shown in Table 6. When \(\gamma = 10\), the premium is 6.01%, while it is 14.33% when we increase \(\gamma\) to 20. We also observe that both equity return and the risk-free rate increase when \(\gamma\) increases. This is because the intertemporal elasticity of substitution is \(1/\gamma\), which decreases with an increase in \(\gamma\). So the high \(\gamma\) value decreases the substitution effect. When the intertemporal elasticity of substitution is low, investors prefer to consume, instead of save. Therefore,
TABLE 5.
Model Implications for $\lambda$

<table>
<thead>
<tr>
<th>$\lambda = 15$</th>
<th>$\lambda = 20$</th>
<th>$\lambda = 30$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean 5% 95%</td>
<td>Mean 5% 95%</td>
</tr>
<tr>
<td>$E(r_m - r_f) [%]$</td>
<td>6.03 5.08 6.91</td>
<td>9.64 8.74 10.56</td>
</tr>
<tr>
<td>$E(r_m) [%]$</td>
<td>6.57 5.47 7.69</td>
<td>10.62 9.43 11.87</td>
</tr>
<tr>
<td>$E(r_f) [%]$</td>
<td>0.51 0 1.06</td>
<td>0.98 0.35 1.65</td>
</tr>
<tr>
<td>$\sigma(r_m - r_f) [%]$</td>
<td>19.23 16.84 21.97</td>
<td>21.49 18.82 23.98</td>
</tr>
<tr>
<td>$\sigma(r_m) [%]$</td>
<td>19.58 17.09 22.27</td>
<td>21.85 19.16 24.60</td>
</tr>
<tr>
<td>$\sigma(r_f) [%]$</td>
<td>1.67 1.38 1.95</td>
<td>2.06 1.76 2.40</td>
</tr>
<tr>
<td>$E(\frac{P_D}{P})$</td>
<td>35.90 33.07 38.77</td>
<td>14.42 12.97 15.82</td>
</tr>
<tr>
<td>$\sigma(\log P_D) [%]$</td>
<td>0.283 0.238 0.325</td>
<td>0.349 0.299 0.404</td>
</tr>
<tr>
<td>AC1(\log P_D)</td>
<td>0.91 0.89 0.94</td>
<td>0.92 0.90 0.94</td>
</tr>
</tbody>
</table>

This table reports the results of 1000 simulations for different values of $\lambda$.

The risk-free rate increases with $\gamma$. Since the risk premium increases, the equity return increases as well.

Similarly, the volatilities of both equity return and the risk-free rate increase with $\gamma$. When $\lambda$ becomes large, the volatility of the SDF increases, which leads to more volatile PD ratio, equity return and risk-free rate values.

TABLE 6.
Model Implications by $\gamma$

<table>
<thead>
<tr>
<th>$\gamma = 10$</th>
<th>$\gamma = 15$</th>
<th>$\gamma = 20$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean 5% 95%</td>
<td>Mean 5% 95%</td>
</tr>
<tr>
<td>$E(r_m - r_f) [%]$</td>
<td>6.01 5.10 6.91</td>
<td>9.25 8.34 10.18</td>
</tr>
<tr>
<td>$E(r_m) [%]$</td>
<td>6.54 5.45 7.66</td>
<td>10.27 9.12 11.67</td>
</tr>
<tr>
<td>$E(r_f) [%]$</td>
<td>0.52 0 1.05</td>
<td>1.06 0.17 2.08</td>
</tr>
<tr>
<td>$\sigma(r_m - r_f) [%]$</td>
<td>19.21 16.82 21.95</td>
<td>22.06 19.42 24.78</td>
</tr>
<tr>
<td>$\sigma(r_m) [%]$</td>
<td>19.47 17.07 22.14</td>
<td>22.52 19.96 25.21</td>
</tr>
<tr>
<td>$\sigma(r_f) [%]$</td>
<td>1.66 1.41 1.95</td>
<td>2.95 2.50 3.46</td>
</tr>
<tr>
<td>$E(\frac{P_D}{P})$</td>
<td>36.03 33.08 38.77</td>
<td>16.31 14.60 18.05</td>
</tr>
<tr>
<td>$\sigma(\log P_D) [%]$</td>
<td>0.281 0.240 0.327</td>
<td>0.365 0.311 0.423</td>
</tr>
<tr>
<td>AC1(\log P_D)</td>
<td>0.91 0.89 0.94</td>
<td>0.92 0.90 0.94</td>
</tr>
</tbody>
</table>

This table reports the results of 1000 simulations for different values of $\gamma$.

6. CONCLUSION

This paper explores whether the spirit of capitalism can explain the equity premium puzzle. First, we regress the equity premium on the con-
consumption growth rate, dividend growth rate and wealth growth rate. We find that the estimated coefficient of the wealth growth rate is significant and positive, which means that wealth does affect the equity premium.

Next, we set up a CCAPM which includes the spirit of capitalism, and follow the method of Bansal and Yaron (2004) to solve the model. We find that the simulated data from our calibrated model matches the mean and the volatility of the equity premia, the equity returns and the risk-free rates in the real data. This means that the spirit of capitalism can explain the equity premium. Moreover, when \( \lambda \), the extent to which the investor cares about social status, is adjusted, we find that the equity premium increases with \( \lambda \).

Our paper uses the framework of a representative agent model. While, it is quite reasonable to assume that investors are heterogeneous in terms of \( \lambda \) and explore the extent of the effect that the framework has on the equity premium puzzle. This is left for future research.

APPENDIX: A. GENERAL EXPRESSION FOR THE EQUITY PREMIUM

Starting from Euler equation \( E_t[(1 + R_{i,t+1})M_{t+1}] = 1 \), in which \( M_{t+1} = \delta u'(C_{t+1}) \) is the stochastic discount factor, by defining \( r_{i,t+1} = \log(1 + R_{i,t+1}) \) and \( m_{t+1} = \log M_{t+1} \), we have this alternative form of the Euler equation:

\[
E_t[e^{r_{i,t+1} + m_{t+1}}] = 1.
\]

By taking the log of \( E_t[(1 + R_{i,t+1})M_{t+1}] = 1 \), we have\(^2\)

\[
E_tr_{i,t+1} + Em_{t+1} + \frac{1}{2}(\sigma_i^2 + \sigma_m^2 + 2\sigma_{im}) = 0
\]

where \( r_{i,t+1}, m_{t+1} \) are assumed to be jointly log-normal and homoskedastic.

For the risk-free rate, since \( \sigma_f^2 = \sigma_f m = 0 \), we have:

\[
E_tr_{f,t+1} + Em_{t+1} + \frac{1}{2}\sigma_m^2 = 0.
\]

So we have the equity premium:

\[
E_t[r_{i,t+1} - r_{f,t+1}] + \frac{1}{2}\sigma_i^2 = -\sigma_{im}.
\]

\(^2\)If \( X \) is conditionally log-normal, we have log \( X \) is conditionally normal. Also from the moment generating function of normal distribution, we know that \( E(e^{tY}) = e^{t\mu + \frac{1}{2}t^2\sigma^2} \) if \( Y \sim N(\mu, \sigma^2) \). So \( E_t[X] = E_t[e^{t \log X}] = e^{E_t[\log X] + \frac{1}{2}V_{ar}t[\log X]} \Rightarrow \log E_t[X] = E_t[\log X] + \frac{1}{2}V_{ar}t[\log X] \), where \( V_{ar}t[\log X] = E_t[(\log X - E_t \log X)^2] \).
or alternatively:

\[ E_t[r_{i,t+1} - r_{f,t+1}] + \frac{1}{2} Var_t(r_{i,t+1}) = -Cov_t[r_{i,t+1} - E_t r_{i,t+1}, m_{t+1} - E_t m_{t+1}] . \]

APPENDIX: B. THE PRICING KERNEL

The pricing kernel under the utility of Spirit of Capitalism can be derived as follows:

\[ m_{t+1} = \log M_{t+1} = \log(\delta \partial U(C_{t+1}, W_{t+1})/\partial C_{t+1}) \]
\[ = \log(\delta \partial U(C_t, W_t)/\partial C_t) \]
\[ = \log(\delta C_t^{-\gamma} W_t^{-\lambda}) \]
\[ = \log \delta - \gamma \log(C_{t+1}/C_t) - \lambda \log(W_{t+1}/W_t) \]
\[ = \log \delta - \gamma g_{t+1} - \lambda r_{w,t+1} \]

APPENDIX: C. APPROXIMATION

The log-market return

Here \( P_t \) is the price level and \( D_t \) is the dividend,

\[ r_{m,t+1} = \log(1 + R_{m,t+1}) = \log \frac{P_{t+1} + D_{t+1}}{P_t} \]
\[ = \log \frac{P_{t+1} + D_{t+1}}{P_t + D_t} + \log \frac{P_t + D_t}{P_t} \]
\[ = \log(1 + \frac{P_{t+1} + D_{t+1} - P_t - D_t}{P_t + D_t}) - \log \frac{P_t}{P_t + D_t} \]
\[ \approx \frac{P_{t+1} + D_{t+1} - P_t - D_t}{P_t + D_t} = \log \frac{1}{1 + e^{d_t-p_t}} \]

Define \( d_t = \log D_t, p_t = \log P_t \), also assume that the ratio of price to the sum of price and dividend to be approximately constant over time\(^3\).

\(^3\)This assumption is from Campbell & Shiller (1988), which fits the real data since such a ratio is highly stable (especially in monthly data).
The log-wealth return

\[ r_{m,t+1} = \frac{k_{1,m}(P_{t+1} - P_t)}{P_t} + \frac{(1 - k_{1,m})(D_{t+1} - D_t)}{D_t} - \log k_{1,m} \]

\[ = k_{1,m}(p_{t+1} - p_t) + (1 - k_{1,m})(d_{t+1} - d_t) - \log k_{1,m} \]

\[ = k_{1,m}p_t + (1 - k_{1,m})d_t + (1 - k_{1,m})(d_t - p_t) - p_t - \log k_{1,m} \]

\[ = k_{0,m} + k_{1,m}p_t + (1 - k_{1,m})d_t - p_t \]

Here we define \( k_{0,m} = -\log k_{1,m} - (1 - k_{1,m})(d_t - p_t) = -\log k_{1,m} - (1 - k_{1,m}) \log(\frac{1}{k_{1,m}} - 1) \).

\[ r_{m,t+1} = k_{0,m} + k_{1,m}p_t + (1 - k_{1,m})d_t + d_{t+1} - d_t \]

\[ = k_{0,m} + k_{1,m}z_{m,t+1} - z_{m,t} + g_{d,t+1} \]

Here we define \( z_{m,t} = \log \frac{P_t}{P_{t-1}} = p_t - d_t \) and \( g_{d,t+1} = d_{t+1} - d_t \).

The log-wealth return

\( C_t \) is the consumption.

\[ r_{w,t+1} = \log(1 + R_{w,t+1}) = \log \frac{P_{t+1} + C_{t+1}}{P_t} \]

\[ = \log \frac{P_{t+1} + C_{t+1}}{P_t + C_t} + \log \frac{P_t + C_t}{P_t} \]

\[ = \log(1 + \frac{P_{t+1} + C_{t+1} - P_t - C_t}{P_t + C_t}) - \log \frac{P_t}{P_t + C_t} \]

\[ \approx \frac{P_{t+1} + C_{t+1} - P_t - C_t}{P_t + C_t} - \log \frac{1}{1 + e^{c_t - p_t}} \]

We suppose the ratio of wealth invested in the assets is a constant \( k_1 \), i.e. \( k_1 = \frac{P_t}{c_t + k_t} = \frac{1}{1 + e^{c_t - p_t}} \), in which \( c_t = \log C_t, p_t = \log P_t \).

\[ r_{w,t+1} = k_1(P_{t+1} - P_t) + \frac{C_{t+1} - C_t}{C_t} - \log k_1 \]

\[ = k_1(p_{t+1} - p_t) + (1 - k_1)(c_{t+1} - c_t) - \log k_1 \]

\[ = k_1p_t + (1 - k_1)c_{t+1} - (1 - k_1)(c_t - p_t) - p_t - \log k_1 \]

\[ = k_0 + k_1p_t + (1 - k_1)c_{t+1} - p_t \]

Here we define \( k_0 = -\log k_1 - (1 - k_1)(c_t - p_t) = -\log k_1 - (1 - k_1) \log(\frac{1}{k_1} - 1) \)

\(^4 k_1 \) is approximately 0.997 according to Bansal & Yaron (2004)
\[ r_{w,t+1} = k_0 + k_1 p_{t+1} - k_1 c_{t+1} - p_t + c_{t+1} - c_t \]
\[ = k_0 + k_1 z_{t+1} - z_t + g_{t+1} \]
in which \( z_t = p_t - c_t, g_{t+1} = c_{t+1} - c_t \).

**APPENDIX: D. THE UNCONDITIONAL MOMENTS**

If \( g_{t+1} = \mu + \phi g_t + \sigma \eta_{t+1} \) and \( \sigma^2_{t+1} = \sigma^2 + \nu_1 (\sigma^2_t - \sigma^2) + \sigma_w w_{t+1} \) are stationary processes, then the unconditional moments of the process can be calculated as:

\[ E(g) = \frac{\mu}{1 - \phi} \quad SD(g) = \frac{\sigma}{\sqrt{1 - \phi^2}} \quad AC1(g) = \phi \]

\[ E(g_d) = \mu_d + \phi_d \frac{\mu}{1 - \phi} \quad SD(g_d) = \sqrt{\frac{\phi_d^2}{1 - \phi_d^2} + \phi_d^2} \quad AC1(g_d) = \frac{\phi_d^2 \phi_c}{\phi_d + \phi_d^2 (1 - \phi^2)} \]

\[ corr(g, g_d) = \frac{\phi_d}{\sqrt{\phi_d^2 + \phi_d^2 (1 - \phi^2)}} \]

\[ E(\log P^D) = A_{0,m} + A_{1,m} \frac{\mu}{1 - \phi} + A_{2,m} \sigma^2 \quad sd(\log P^D) = \sqrt{A_{1,m}^2 \frac{\sigma^2}{1 - \phi^2} + A_{2,m}^2 \frac{\sigma^2}{1 - \nu_1^2}} \]

\[ AC1(\log P^D) = \frac{A_{1,m}^2 \phi_c \frac{\sigma^2}{1 - \phi^2} + A_{2,m}^2 \nu_1 \frac{\sigma^2}{1 - \nu_1^2}}{A_{1,m}^2 \frac{\sigma^2}{1 - \phi^2} + A_{2,m}^2 \frac{\sigma^2}{1 - \nu_1^2}} \]

**APPENDIX: E. SOLVE A_1 AND A_2**

Let \( r_{t,t+1} = r_{w,t+1} \) and solve \( A_1, A_2 \).
Now we have the Euler equation as:

\[ E_t \{ \exp[\log \delta - \gamma g_{t+1} + (1 - \lambda) r_{w,t+1}] \} = 1 \]

In all the following steps, we omit the constant terms (terms that have nothing to do with \( g_t \) and \( \sigma^2_t \)), because we can let them to be 1 by setting a proper value for \( A_0 \).

5The conditions for them to be stationary are \(|\phi_c| < 1\) and \(|\nu_1| < 1\), which will be embodied in our calibration.

6In every step followed, we will drop some constants. Our goal is to let all the constants dropped in all steps combined to be 1, so we don’t equate the following expressions to 1.
$E_t \{ \exp[-\gamma g_{t+1} + (1 - \lambda)(k_1 z_{t+1} - z_t + g_{t+1})] \}$

$E_t \{ \exp[(1 - \lambda - \gamma)g_{t+1} + (1 - \lambda)k_1(A_1 g_{t+1} + A_2 \sigma^2_{t+1}) - (1 - \lambda)(A_1 g_t + A_2 \sigma^2_t)] \}$

$E_t \{ \exp[(1 - \lambda - \gamma + k_1 A_1 - \lambda k_1 A_1)(\sigma_2^t)_{t+1} + \phi_c g_t] + (1 - \lambda)k_1 A_2(\nu_1 \sigma^2_t + \sigma_w w_{t+1} - (1 - \lambda)A_1 g_t - (1 - \lambda)A_2 \sigma^2_t) \}$

$\exp[\frac{1}{2}(1 - \lambda - \gamma + k_1 A_1 - \lambda k_1 A_1)^2 \phi_c - (1 - \lambda)A_1 g_t] \cdot \exp\{[(1 - \lambda - \gamma + k_1 A_1 - \lambda k_1 A_1)\phi_c - (1 - \lambda)A_2 \nu_1 - (1 - \lambda)A_2 \sigma^2_t] \cdot \exp[\frac{1}{2}(1 - \lambda - 2)^2 k^2_1 A^2_2 \sigma^2_w] \}$

The last term $\exp[\frac{1}{2}(1 - \lambda - 2)^2 k^2_1 A^2_2 \sigma^2_w]$ is a constant, so we drop it. The remaining three terms should equal 1 for all values of $g_t$ and $\sigma^2_t$. So we have:

$$(1 - \lambda - \gamma + k_1 A_1 - \lambda k_1 A_1)\phi_c - (1 - \lambda)A_1 = 0$$

$$\frac{1}{2}(1 - \lambda - \gamma + k_1 A_1 - \lambda k_1 A_1)^2 + (1 - \lambda)k_1 A_2 \nu_1 - (1 - \lambda)A_2 = 0$$

Solve them shows:

$$A_1 = \frac{(1 - \lambda - \gamma)\phi_c}{(1 - \lambda)(1 - k_1 \phi_c)}$$

$$A_2 = \frac{\frac{1}{2}(1 - \lambda - \gamma + k_1 A_1 - \lambda k_1 A_1)^2}{(1 - \lambda)(1 - k_1 \nu_1)}$$

All the constant terms we omit are:

$$\log \delta + (1 - \lambda - \gamma + (1 - \lambda)k_1 A_1)\mu + (1 - \lambda)k_0 + (1 - \lambda)k_1 A_0 + (1 - \lambda)k_1 A_2(1 - \nu_1)\sigma^2 + \frac{1}{2}(1 - \lambda)^2 k^2_1 A^2_2 \sigma^2_w = 0$$

So we can calculate $A_0$ as:

$$A_0 = \frac{\log \delta + (1 - \lambda - \gamma + (1 - \lambda)k_1 A_1)\mu + (1 - \lambda)k_0 + (1 - \lambda)k_1 A_2(1 - \nu_1)\sigma^2 + \frac{1}{2}(1 - \lambda)^2 k^2_1 A^2_2 \sigma^2_w}{(1 - \lambda)(1 - k_1)}$$

**APPENDIX: F. THE INNOVATION OF PRICING KERNEL**

Since $E_t g_{t+1} = \mu + \phi_c g_t$, $E_t z_{t+1} = A_0 + A_1 E_t g_{t+1} + A_2 E_t \sigma^2_{t+1}$, $E_t \sigma^2_{t+1} = \sigma^2 + \nu_1(\sigma^2_t - \sigma^2)$, The innovation of pricing kernel ($m_{t+1} = \log \delta - \gamma g_{t+1} - \lambda r_{w,t+1}$) can be calculated as:

$$m_{t+1} - E_t m_{t+1} = -\gamma (g_{t+1} - E_t g_{t+1}) - \lambda (r_{w,t+1} - E_t r_{w,t+1})$$

$$= -\gamma \sigma_2^t \eta_{t+1} - \lambda [k_1 (z_{t+1} - E_t z_{t+1}) + (g_{t+1} - E_t g_{t+1})]$$

$$= (-\gamma - \lambda) \sigma_2^t \eta_{t+1} - \lambda k_1 [A_1 (g_{t+1} - E_t g_{t+1}) + A_2 (\sigma^2_{t+1} - E_t \sigma^2_{t+1})]$$

$$= (-\gamma - \lambda) \sigma_2^t \eta_{t+1} - \lambda k_1 A_2 \sigma_w w_{t+1}$$

$$= -\lambda m_n \sigma_2^t \eta_{t+1} - \lambda m_w \sigma_w w_{t+1} + 1$$
in which \( \lambda_{m,n} = \gamma + \lambda k_1 A_1 \), \( \lambda_{m,w} = \lambda k_1 A_2 \).

The conditional variance of pricing kernel:

\[
Var_t(m_{t+1}) = E_t^2(m_{t+1} - E_t m_{t+1}) = \lambda_{m,y}^2 \sigma_t^2 + \lambda_{m,w}^2 \sigma_w^2
\]

**APPENDIX: G. THE INNOVATION AND THE EQUITY PREMIUM OF \( R_{W,T+1} \)**

The innovation of \( r_{w,t+1} \):

\[
r_{w,t+1} - E_t(r_{w,t+1}) = k_1[z_{t+1} - E_t(z_{t+1})] + g_{t+1} - E_t(g_{t+1})
= k_1[A_1(g_{t+1} - E_t(g_{t+1})) + A_2(\sigma_{t+1}^2 - E_t(\sigma_{t+1}^2))] + g_{t+1} - E_t(g_{t+1})
= (1 + k_1 A_1) \sigma_{t+1} + k_1 A_2 \sigma_w + g_{t+1}
\]

So the conditional variance is:

\[
Var_t(r_{w,t+1}) = (1 + k_1 A_1) \sigma_t^2 + k_1^2 A_2^2 \sigma_w^2
\]

The conditional premium of \( r_{w,t+1} \):

\[
E_t[R_{w,t+1} - r_{t+1}] = -Cov_t[m_{t+1} - E_t m_{t+1}, r_{w,t+1} - E_t(r_{w,t+1})] - \frac{1}{2} Var_t(r_{w,t+1})
= \lambda_{m,y}(1 + k_1 A_1) \sigma_t^2 + \lambda_{m,w} k_1 A_2 \sigma_w^2 - \frac{1}{2}(1 + k_1 A_1)^2 \sigma_t^2 + k_1^2 A_2^2 \sigma_w^2
\]

**APPENDIX: H. SOLVE \( A_{1,M} \) AND \( A_{2,M} \)**

Let \( r_{t+1} = r_{m,t+1} \) and solve \( A_{1,m} \), \( A_{2,m} \). We have the Euler equation as:

\[
E_t[e^{\log \delta - \gamma g_{t+1} - \lambda r_{w,t+1} + r_{m,t+1}}] = 1
\]

For the same reason, we omit the constant terms every step.

\[
E_t\{\exp[-\gamma g_{t+1} - \lambda(k_{1,m} z_{t+1} - z_t + g_{t+1}) + (k_{1,m} z_{m,t+1} - z_{m,t} + g_{d,t+1})]\}
= E_t\{\exp[-\lambda - \gamma) g_{t+1} - \lambda k_1(A_1 g_{t+1} + A_2 \sigma_{t+1}^2) + \lambda(A_1 g_{t+1} + A_2 \sigma_{t+1}^2) + k_1 A_2(A_{1,m} g_{t+1} + A_{2,m} \sigma_{t+1}^2)]
= (1 + k_1 A_1) \sigma_t + k_1 A_2 \sigma_w + g_{t+1}
\]

\[
E_t\{\exp[-(\lambda - \gamma - \lambda k_1 A_1 + k_{1,m} A_{1,m} + \phi d)(\sigma_{t+1}^2 + \sigma_w u_{t+1}) + (\lambda A_2 - A_{2,m}) \sigma_t^2 + \psi d u_{t+1}^2]\}
= (1 + k_1 A_1 + k_{1,m} A_{2,m} + \phi d) \sigma_t + \psi d u_{t+1}^2
\]

\[
e^\frac{1}{2}(-\lambda - \gamma - \lambda k_1 A_1 + k_{1,m} A_{1,m} + \phi d)^2 \sigma_t^2 \cdot \exp\{(-\lambda - \gamma - \lambda k_1 A_1 + k_{1,m} A_{1,m} + \phi d)^2 \sigma_t^2 \}
\]

\[
\cdot \exp\{\lambda k_1 A_2 [k_{1,m} A_{2,m} - \lambda k_1 A_2] \nu_1 + \lambda A_2 - A_{2,m} \sigma_t^2 \}
\]

\[
\cdot \exp\{k_{1,m} A_{2,m} - \lambda k_1 A_2 \nu_1 + \lambda A_2 - A_{2,m} \sigma_t^2 \}
\]
The last term \( \exp[\frac{1}{2}(k_{1,m}A_{2,m} - \lambda k_1 A_2)^2 \sigma_w^2] \) is a constant, so we drop it. The remaining four terms should equal to 1 for all values of \( g_t \) and \( \sigma_t^2 \). So we have:

\[
\left(-\lambda - \gamma - \lambda k_1 A_1 + k_{1,m} A_{1,m} + \phi_d\right) \phi_c + \lambda A_1 - A_{1,m} = 0 \\
\frac{1}{2}(-\lambda - \gamma - \lambda k_1 A_1 + k_{1,m} A_{1,m} + \phi_d)^2 + (k_{1,m} A_{2,m} - \lambda k_1 A_2) \nu_1 + \lambda A_2 - A_{2,m} + \frac{1}{2} \phi_d^2 = 0
\]

Solve them shows:

\[
A_{1,m} = \frac{(-\lambda - \gamma - \lambda k_1 A_1 + \phi_d) \phi_c + \lambda A_1}{1 - k_{1,m} \phi_c} \\
A_{2,m} = \frac{\frac{1}{2} H_m + \lambda A_2 - \lambda k_1 A_2 \nu_1}{1 - \nu_k k_{1,m}}
\]

where \( H_m = (-\lambda - \gamma - \lambda k_1 A_1 + k_{1,m} A_{1,m} + \phi_d)^2 + \phi_d^2 \).

All the constant terms we omit are:

\[
\log \delta - \lambda k_0 - \lambda k_1 A_0 + \lambda A_0 + k_{0,m} + k_{1,m} A_{0,m} - A_0,m + \mu_d - (\gamma + \lambda + \lambda k_1 A_1 - k_{1,m} A_{1,m} - \phi_d) \mu - (\lambda k_1 A_2 - k_{1,m} A_{2,m})(1 - \nu_1) \sigma^2 + \frac{1}{2}(\lambda k_1 A_2 - k_{1,m} A_{2,m})^2 \sigma_w^2 = 0
\]

So we can calculate \( A_{0,m} \) as:

\[
A_{0,m} = \left(1 - k_{1,m} A_{1,m} + \phi_d \right) \phi_c + \lambda A_1 - A_{1,m} \]

APPENDIX: I. THE EQUITY PREMIUM AND THE MARKET VOLATILITY

Since \( E_t z_{m,t+1} = A_{0,m} + A_{1,m} E_t g_{t+1} + A_{2,m} E_t \sigma_t^2 + 1, E_t g_{d,t+1} = \mu_d + \phi_d E_t g_{t+1} \).

The innovation of \( r_{m,t+1} \) can be calculated as:

\[
r_{m,t+1} - E_t r_{m,t+1} = k_{1,m}(z_{m,t+1} - E_t z_{m,t+1}) + g_{d,t+1} - E_t g_{d,t+1} \\
= k_{1,m}[A_{1,m}(g_{t+1} - E_t g_{t+1}) + A_{2,m}(\sigma_t^2 - E_t \sigma_t^2 + 1)] \\
+ \phi_d \sigma_t u_{t+1} + \phi_d (g_{t+1} - E_t g_{t+1}) \\
= (k_{1,m} A_{1,m} + \phi_d) \sigma_t \eta_{t+1} + k_{1,m} A_{2,m} \sigma_w u_{t+1} + \phi_d \sigma_t u_{t+1} \\
+ \beta_{m,n} \sigma_t \eta_{t+1} + \beta_{m,w} \sigma_w u_{t+1} + \phi_d \sigma_t u_{t+1}
\]

in which \( \beta_{m,n} = k_{1,m} A_{1,m} + \phi_d, \beta_{m,w} = k_{1,m} A_{2,m} \).

So the conditional variance can be calculated as:

\[
Var_t(r_{m,t+1}) = E_t(r_{m,t+1} - E_t r_{m,t+1})^2 = (\beta_{m,n}^2 + \phi_d^2) \sigma_t^2 + \beta_{m,w}^2 \sigma_w^2
\]
By the same logic, we also have

\[ \text{Var}(\varepsilon_r) = \beta_{m,\eta} \lambda_m \sigma^2 + \beta_{m,w} \lambda_{m,w} \sigma^2_w - \frac{1}{2} \left[ (\beta_{m,\eta}^2 + \phi_d^2) \sigma^2 + \beta_{m,w}^2 \sigma^2_w \right] \]

So the unconditional equity premium is:

\[ E[r_{m,t+1} - r_{f,t+1}] = \beta_{m,\eta} \lambda_m \sigma^2 + \beta_{m,w} \lambda_{m,w} \sigma^2_w - \frac{1}{2} \left[ (\beta_{m,\eta}^2 + \phi_d^2) \sigma^2 + \beta_{m,w}^2 \sigma^2_w \right] \]

Then the unconditional variance of market return can be derived like this:

\[ r_{m,t+1} - E r_{m,t+1} = k_{1,m} (z_{m,t+1} - E z_{m,t+1}) - (z_{m,t} - E z_{m,t}) + \phi_d (g_{t+1} - E g_{t+1}) \]

\[ = k_{1,m} [A_{1,m} (g_t - \frac{\mu}{1 - \phi_d}) + A_{2,m} (\sigma^2_{t+1} - \sigma^2)] - [A_{1,m} (g_t - \frac{\mu}{1 - \phi_d}) + A_{2,m} (\sigma^2_t - \sigma^2)] + \phi_d \sigma_t u_{t+1} + \phi_d (g_{t+1} - \frac{\mu}{1 - \phi_d}) \]

\[ = (k_{1,m} A_{1,m} + \phi_d) (\sigma^2_t + \phi_d \sigma_t u_{t+1} + \phi_d (g_{t+1} - \frac{\mu}{1 - \phi_d})) + k_{1,m} A_{2,m} \sigma_t (\sigma^2_t - \sigma^2) \]

\[ = (k_{1,m} A_{1,m} + \phi_d) (\sigma^2_t + \phi_d \sigma_t u_{t+1} + \phi_d (g_{t+1} - \frac{\mu}{1 - \phi_d})) + k_{1,m} A_{2,m} \sigma_t (\sigma^2_t - \sigma^2) \]

\[ So \ Var(r_m) = E(r_{m,t+1} - E r_{m,t+1})^2 \text{ can be calculated as:} \]

\[ Var(r_m) = (k_{1,m} A_{1,m} + \phi_d) (\sigma^2_t + \phi_d \sigma_t u_{t+1} + \phi_d (g_{t+1} - \frac{\mu}{1 - \phi_d})) + k_{1,m} A_{2,m} \sigma_t (\sigma^2_t - \sigma^2) \]

**APPENDIX: J. RISK-FREE RATE AND ITS VOLATILITY**

For the risk-free rate, we have the Euler Equation as:

\[ E \{ \exp(\log \delta - \gamma g_{t+1} - \lambda r_{w,t+1} + r_{f,t+1}) \} = 1 \]

\[ \exp \log \delta \cdot \exp r_{f,t+1} \cdot \exp[-\gamma E g_{t+1} + \frac{1}{2} \text{var}(\gamma g_{t+1})] \cdot \exp[-\lambda E r_{w,t+1} + \frac{1}{2} \text{var}(\lambda r_{w,t+1})] = 1 \]

\[ r_{f,t+1} = -\log \delta + \gamma E g_{t+1} + \lambda E r_{w,t+1} = -\frac{1}{2} \text{var}(\gamma g_{t+1} + \lambda r_{w,t+1}) \]

\[ = -\frac{1}{2} \text{var}(\sigma^2_t + \phi_d \sigma_t u_{t+1} + \phi_d (g_{t+1} - \frac{\mu}{1 - \phi_d})) \]

\[ = -\frac{1}{2} \lambda^2 \sigma^2_t + \phi_d^2 \sigma_t^2_w + \lambda^2 \sigma^2_w \]

\[ \text{Var}(\sigma_{\eta_{t+1}}) = E(\sigma^2_{\eta_{t+1}}) - E^2(\sigma_{\eta_{t+1}}) = E(\sigma^2_{t})E(\eta^2_{t+1}) - E^2(\sigma_{t})E^2(\eta_{t+1}) = E(\sigma^2_{t}) \]

By the same logic, we also have \( \text{Var}(\sigma_{u_{t+1}}) = E(\sigma^2_{t}) \).

\( r_{w,t+1} \) depends on \( z_t, z_{t+1} \) and \( g_{t+1} \). \( g_{t+1} \) depends on \( \eta_{t+1} \). \( z_t \) depends on \( g_{t+1} \) and \( \sigma^2_t \). \( \sigma^2_t \) depends on \( w_t \). So, ultimately, both \( r_{w,t+1} \) and \( \eta_{t+1} \) are normal processes.
So the unconditional expectation of risk-free rate is:

\[ Er_{f,t+1} = -\log \delta + \gamma E g_{t+1} + \lambda E r_{w,t+1} - \frac{1}{2} (\lambda_{m,\eta}^2 E(\sigma_t^2) + \lambda_{m,w}^2 \sigma_w^2) + \gamma \lambda (k_1 A_1 + 1) E(\sigma_t^2) \]

in which \( E r_{w,t+1} = k_0 + k_1 (A_0 + A_1 E g_{t+1} + A_2 E(\sigma_t^2)) - (A_0 + A_1 E g_t + A_2 E(\sigma_t^2)) + E g_{t+1} \).

For the unconditional variance of risk-free rate, we have:

\[ \tau_{f,t+1} = \gamma (E g_{t+1} - E g_{t+1}) + \lambda (E r_{w,t+1} - E r_{w,t+1}) - \frac{1}{2} \lambda_{m,\eta}^2 (\sigma_t^2 - E(\sigma_t^2)) + \gamma \lambda (k_1 A_1 + 1) (\sigma_t^2 - E(\sigma_t^2)) + \lambda (\gamma + \lambda k_1 A_1) (\mu - \phi_c g_t - \frac{\mu}{1 - \phi_c}) + \lambda (\gamma + \lambda k_1 A_1) (\sigma_t^2 - E(\sigma_t^2) - \lambda A_1 (g_t - \mu) - \frac{\mu}{1 - \phi_c}) \]

So \( Var(\tau_{f,t+1}) = [(\lambda + \gamma + \lambda k_1 A_1) \phi_c - \lambda A_1]^2 Var(g_t) + [\lambda k_1 A_1 \mu - \lambda A_2 - \frac{1}{2} \lambda_{m,\eta}^2 + \gamma \lambda (k_1 A_1 + 1)]^2 Var(\sigma_t^2). \]

REFERENCES


\[ \gamma_{cov}(g_{t+1}, z_{t+1}) = \gamma_{cov}(g_{t+1}, k_0 + k_1 z_{t+1} - z_t + g_{t+1}) = \gamma_{cov}(g_{t+1}, k_1 z_{t+1}) + \gamma_{cov}(g_{t+1}, g_{t+1}) = \gamma \lambda k_1 \gamma_{cov}(g_{t+1}, A_0 + A_1 g_{t+1} + A_2 \sigma_{t+1}^2) = \gamma \lambda \gamma_{var}(g_{t+1}) = \gamma \lambda (k_1 A_1 + 1) \gamma_{var}(g_{t+1}) = \gamma \lambda (k_1 A_1 + 1) E \{g_{t+1} - E(g_{t+1})\}^2 = \gamma \lambda (k_1 A_1 + 1) E \{\sigma_{t+1}^2\} = \gamma \lambda (k_1 A_1 + 1) \sigma_t^2 \]

\[ \gamma_{cov}(g_{t+1}, z_{t+1}) \] and \( \sigma_t^2 \) are independent since they depend on \( \eta_t \) and \( w_{t,1} \), respectively, which are independent.


