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Price Momentum and Reversal: An Information Cascade Rationale

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I develop a model in which price momentum builds up as a result of investors' rational learning. Investors make sequential buy or sell decisions based on the past history of price movements and a private signal. The private signal has a stronger impact in the early stage, but beyond certain point the influence gradually dies out and subsequent investors tend to follow the trend. In the presence of upward momentum, early buyers impose a negative externality on later buyers by increasing the incidence of large losses. A self-fulfilling reversal occurs once a correction factor is added to investors' valuation function.

Key Words: Information cascade; Price momentum; Price reversal; Private signal
JEL Classification Numbers: G14, L10, L22.

1. INTRODUCTION

Over the past two decades, the literature has uncovered two principle ways to model the amply documented phenomena of stock price momentum and reversal. One class of behavioral finance models posits that some cognitive biases among investors are sufficient to generate both short-horizon momentum and long-horizon reversals. Examples along this vein include Barberis et al. (1998) and Daniel et al. (1998). Another type of model, led chiefly by Hong and Stein (1999), abandons this representative agent- and psychology-based narrative, while focusing on the interaction between boundedly rational heterogeneous agents; see also DeLong et al. (1990) and Cutler et al. (1991) for models based on positive-feedback trading with irrational investors. In these models, investors either have some cognitive

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bias (e.g., overconfidence) or can only use a small portion of information to predict returns. So the question of how rational investors' perception of the market can influence the buildup of price momentum is still not well understood.

On balance, later authors have found evidence that supports the predictions of these behavioral models in the US market, although the significance of the result appears to be more pronounced in the upward direction and tends to vary with different sample periods; see Cooper et al. (2004). It is generally accepted that momentum profits are due to delayed overreactions that are eventually reversed; see Chan et al. (1996), Hong et al. (2000), Lee and Swaminathan (2000), Jegadeesh and Titman (1993, 2001) and Badrinath and Wahal (2002). The pattern of price momentum followed by a reversal is not limited to the world's most developed markets — a growing body of empirical evidence has unveiled that momentum trading and the profits thereof are a pervasive feature of less developed markets as well. To name a few, Naughton et al. (2008) investigate various momentum trading strategies for equities listed on the Shanghai Stock Exchange and find evidence of substantial momentum profits during the period 1995 to 2005. Kang et al. (2002) examine data (1993-2000) on "A" shares accessible only to mainland China investors and find statistically significant abnormal profits for some short-horizon contrarian and intermediate-horizon momentum strategies. On average, authors have found that the momentum effect is more pronounced for value-weighted portfolios compared to equal-weighed ones. Most recently, Wu (2011) finds that a strategy based on the rolling-regression parameter estimates of the model combining mean reversion and momentum generates both statistically and economically significant excess returns. The combined strategy outperforms both pure momentum and pure contrarian strategies; see also McInish et al. (2008).

The current paper models the same phenomenon from a learning perspective. In doing so, my goal is to shed some light on how the momentum-reversal cycle can be shaped by investors' short-term beliefs updated over time. Rather than relying on the mean-variance portfolio- and linear regression-based methods, I take a probabilistic approach by explicitly characterizing the learning behavior of buyers and sellers. As it turns out, the model is able to deliver a full range of dynamics with flexible constraints on investors' rationality. In particular, I highlight the tension between buyers who entered the market in an early phase when price was relatively low and buyers who entered in a later phase when the risk of the asset being overvalued has increased by a wide margin. Consequently, rational investors will modify their valuation (a measure of reservation payoff) as a function of past price movements to reflect the heightened risk of holding an overbought asset. At some point, the momentum reverses itself and price gradually falls back to normal.

To keep the model tractable yet rich enough to capture the stylized facts, the information set is modeled as the past history of price movements plus a private signal. The private signal relates to whether the true state is good or bad and it is assumed that this signal also picks up the residual effect of macroeconomic news and shocks. In the next two sections, I introduce the key elements of the model and solve for the trajectory of investors' decision rules. Section 4 concludes. All proofs are given in the appendix.

2. THE MODEL

Consider n investors who make sequential buy orsell decisions based on past price movements and a private signal. At each point in time, the stock price will respond according to whether a buyer-or seller-initiated trade has been closed: A unit of upward movement corresponds to a previous buy and a unit of downward movement to a previous sell. Denote these two movements by $\{U, D\}$, the history of which is observed by everyone. Each investor also receives a private signal, $s = \{H, L\}$, whereby H means the high return (good) state and L means the low return (bad) state. The signal is correct with probability p, which we postulate to be greater than 0.5 so that it satisfies the informativeness condition. The value of the stock is normalized to $V = 1$ in the good state and $V = 0$ in the bad state. $Pr(V = 1) = Pr(V = 0) = 0.5$. In addition, let investors be heterogeneous in terms of their perception of the overdervaluedness of the stock, and call this parameter c ("cost") which lies in $[0, 1]$. Assume that investors' types are distributed as $c \sim U(0, 1)$. For an average investor i, she knows her own type c_i (taken to be 0.5 in the analysis) and the overall distribution, but not the specific value of others. Given the above assumptions, the high return state pays $1 - 0.5 = 0.5$, while the low return state pays $0 - 0.5 = -0.5$.

Some discussions of the above assumptions are in place. First, the focus on an average investor ($c = 0.5$) may seem a bit restrictive, but this is actually inconsequential and is imposed mainly to ease exposition. The crucial part is that one wants to have $1 - p < c < p \leq 1$. To see this and recall that $p > 0.5$, suppose $c \leq 1 - p$, then the expected payoff of a buy order is $p - c > 0$ when receiving $s = H$ and $1 - p - c > 0$ when receiving $S = L$, in which case investors will always buy regardless of the private signal. Now suppose $c > p$ then the expected payoff of a buy order is $p - c < 0$ when receiving $s = H$ and $1 - p - c < 0$ when receiving $s = L$, in which case investors will never buy. These are the uninteresting cases, so I rule them out from the beginning. Second, the uniform distribution of c is not critical either and other distributions such as the normal can be entertained. It is used to minimize the distraction of nonessential elements of the model.

Investors base their decisions on the expected payoffs conditional on information up to the current time period. To be more specific, investor i's expected payoff is

$$0 \times Pr(V = 0|I_i) + 1 \times P(V = 1|I_i) - c = Pr(V = 1|I_i) - c, \qquad (1)$$

where I_i is the information set of investor i, an example of which can be $I_i = \{U_1U_2, \ldots, U_{i-1}H_i\}$. It says that investor i has observed upward price movements for $i - 1$ time periods and has just received a good state signal ($s = H$). The general rule is summarized in Proposition 1:

PROPOSITION 1. *For $i \in \{1, 2, \ldots, n\}$, investor i buys the stock if $Pr(V = 1|I_i) - c > 0$ and sells if $Pr(V = 1|I_i) - c < 0$.*

Price momentum can be viewed as a chain reaction starting from the first investor whose information set I_1 consists of only her private signal. By Proposition 1, the first investor's decision rule is straightforward: she will buy upon receiving H since $Pr(V = 1|H_1) - c = p - c > 0$ by assumption and will sell upon receiving L since $Pr(V = 1|L) - c = 1 - p - c < 0$. I model a sequence of buys ($\{U_1U_2, \ldots\}$) as the upward momentum and that of sells ($\{D_1D_2 \ldots\}$) as the downward momentum. I will focus on the upward trend for the rest of the paper; the opposite direction can be analyzed in a symmetric way. Now that the first investor has purchased the stock driving the market price slightly higher, the second investor observes the price change and receives another private signal before she makes a move. Thus the second investor buys if $Pr(V = 1|I_2 = \{U_1s\}) - c > 0$ and sells if $Pr(V = 1|I_2 = \{U_1s\}) - c < 0$. For the downward momentum, one is interested in $Pr(V = 0|I_i = \{D_1D_2 \ldots D_{i-1}L_i\})$, etc.

In the next section, I show four key results. First, $Pr(V = 1|I_i = \{U_1U_2 \ldots U_{i-1}H_i\})$ is always greater than $Pr(V = 1|I_i = \{U_1U_2 \ldots U_{i-1}L\})$. This is quite intuitive because by definition a signal of H is more indicative of the high reture state. Second, the private signal has a stronger impact on investors' decisions in the early stage, but beyond certain point it ceases to have any influence and subsequent investors will follow suit and appear to be "led away" by the past history of price movements. This is when the real "momentum" starts: the upward pattern of price has become so strong that investors can effectively ignore their private signals. Third, I introduce aprobabilistic correction factor to c (the overvaluedness parameter) and demonstrate that at the end the momentum will reverse itself and investors will start to sell. Fourth, I show that the reversal is self-fulfilling in the sense that once it begins a cascade of sells will follow until price falls back to normal. To be more specific, for $i \geq 3$ and $j > i$ it is true that

$$\begin{aligned} & Pr(V = 1 | I_i = \{U_1 U_2 \dots U_{i-2} D_{i-1} L_i\}) \\ < \; & Pr(V = 1 | I_i = \{U_1 U_2 \dots U_{i-2} D_{i-1} H_i\}), \\ & Pr(V = 1 | \{U_1 U_2 \dots U_{i-2} D_{i-1} \dots D_{j-1} L_j\}) \\ < \; & Pr(V = 1 | \{U_1 U_2 \dots U_{i-2} D_{i-1} \dots D_{j-2} H_{j-1}\}), \end{aligned}$$

and

$$\begin{aligned} & Pr(V = 1 | \{U_1 U_2 \dots U_{i-2} D_{i-1} \dots D_{j-1} L_j\}) \\ < \; & Pr(V = 1 | \{U_1 U_2 \dots U_{i-2} D_{i-1} \dots D_{j-2} L_{j-1}\}). \end{aligned}$$

3. PRICE MOMENTUM AND REVERSAL

I derive the main results in this section. From $Pr(V = 1) = Pr(V = 0) = 0.5$, $Pr(1|H) = Pr(0|L) = p$ and Proposition 1, one can easily show that $Pr(U) = Pr(D) = 0.5$, $Pr(H) = Pr(L) = 0.5$, $Pr(H|1) = Pr(L|0) = p$ and $Pr(U|1) = Pr(D|1) = Pr(D|0) = p^2 + (1-p)^2$. Now the second investor has observed $\{U_1\}$, so she infers that the probability of a good state ($V = 1$) conditional on this information is

$$Pr(1|U_1) = \frac{Pr(U_1|1)}{Pr(U_1|1) + Pr(U_1|0)} = p^2 + (1-p)^2. \tag{2}$$

After she receives the private signal, the above probability is revised to reflect the additional information:

$$Pr(1|U_1 H_2) = \frac{Pr(1, U_1 H_2)}{Pr(U_1 H_2)} = \frac{Pr(U_1 H_2|1)}{Pr(U_1 H_2|1) + Pr(U_1 H_2|0)}, \tag{3}$$

and

$$Pr(1|U_1 L_2) = \frac{Pr(1, U_1 L_2)}{Pr(U_1 L_2)} = \frac{Pr(U_1 L_2|1)}{Pr(U_1 L_2|1) + Pr(U_1 L_2|0)}. \tag{4}$$

LEMMA 1. *The updated probabilities for the second investor are*

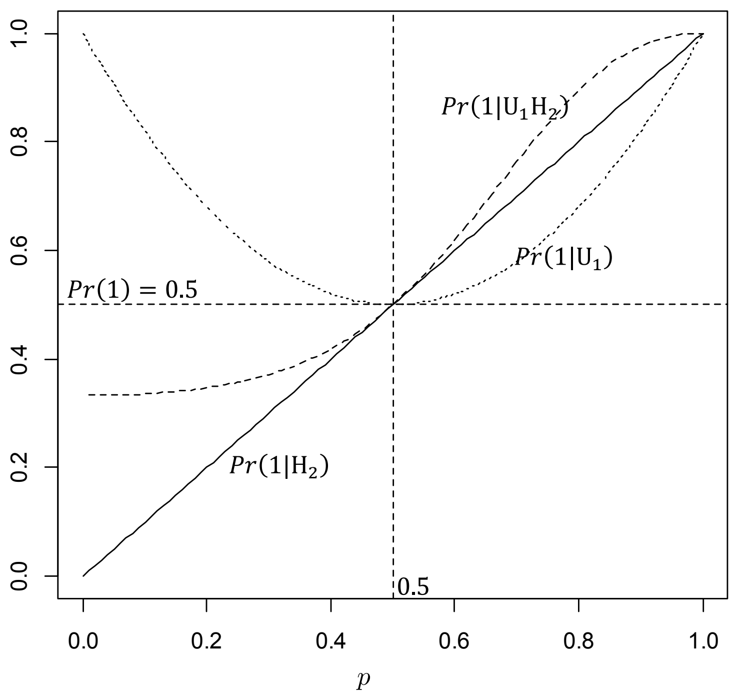
$$Pr(1|U_1 H_2) = \frac{p^2 + (1-p)^2}{p^2 + 3(1-p)^2} > 0.5, \tag{5}$$

and

$$Pr(1|U_1 L_2) = \frac{p^2 + (1-p)^2}{3p^2 + (1-p)^2} < 0.5. \tag{6}$$

Combing Proposition 1 and Lemma 1, the second investor will buy if she receives H and sell if she receives L. In Figure 1, I illustrate the contribution of extra information to the revised conditional probabilities by plotting $Pr(1|U_1H_2), Pr(1|U_1)$ and $Pr(1|H_2)$ relative to the unconditional probability $Pr(V=1)=0.5$. Focusing on the upper-right quadrant where $p>0.5$, the ordering of posterior probabilities is given by $Pr(1|U_1H_2) > Pr(1|H_2) > Pr(1|U_1) > Pr(1)$.

FIG. 1. The second investor's probabilities conditional on different information sets as p varies.



3.1. General Case

Now suppose the third investor has observed $\{U_1U_2\}$ plus a private signal, her updated probabilities become $Pr(1|U_1U_2H_3)$ and $Pr(1|U_1U_2L_3)$.

Lemma 2. *Under the assumptions of the model, the third investor will update her probabilities according to*

$$Pr(1|U_1U_2H_3) = \frac{(p^2+(1-p)^2)^2}{(p^2+(1-p)^2)^2+4p(1-p)^3} > Pr(1|U_1H_2) > 0.5, \quad (7)$$

and

$$Pr(1|U_1U_2L_3) = \frac{(p^2 + (1-p)^2)^2}{(p^2 + (1-p)^2)^2 + 4p^3(1-p)} > Pr(1|U_1L_2). \quad (8)$$

Lemma 2 shows that although $Pr(1|U_1U_2L_3)$ is greater than $Pr(1|U_1L_2)$, it is not necessarily larger than 0.5. This point is made clear by looking at Figure 2. While $Pr(1|U_1)$ is greater than 0.5 so that observing an upward price movement increases the odds of the good state, a negative private signal L is bad enough to overturn the conditional probability to below 0.5, i.e., $Pr(1|U_1L_2) < Pr(1)$. The effect of a bad private signal is dampened as the signal gets more accurate (p increases) and as one observes more and more upward momentumover time. Unlike the two investors case in which $Pr(1|U_1L_2)$ is decreasing monotonically towards zero, the third investor's belief is strengthened by observing two consecutive upward price movements. After first dipping below 0.5 for a while, $Pr(1|U_1U_2L_3)$ rears up and goes on to increase to one. It holds that $Pr(1|U_1U_2H_3) > Pr(1|U_1H_2) > Pr(1|U_1U_2L_3) > Pr(1|U_1L_2)$. Later I show that this is not an exception: Investors' judgment is heavily influenced by the persistence of the ongoing price trend.

THEOREM 1. *Under the assumption of the model, for $i \in \{3, \ldots, n\}$, the ith investor will update her probabilities according to*

$$Pr(1|U_1 \ldots U_{i-1}H_i) = \frac{(p^2 + (1-p)^2)^{i-1}}{(p^2 + (1-p)^2)^{i-1} + (2p(1-p))^{i-1}(1-p)/p} > 0.5, \quad (9)$$

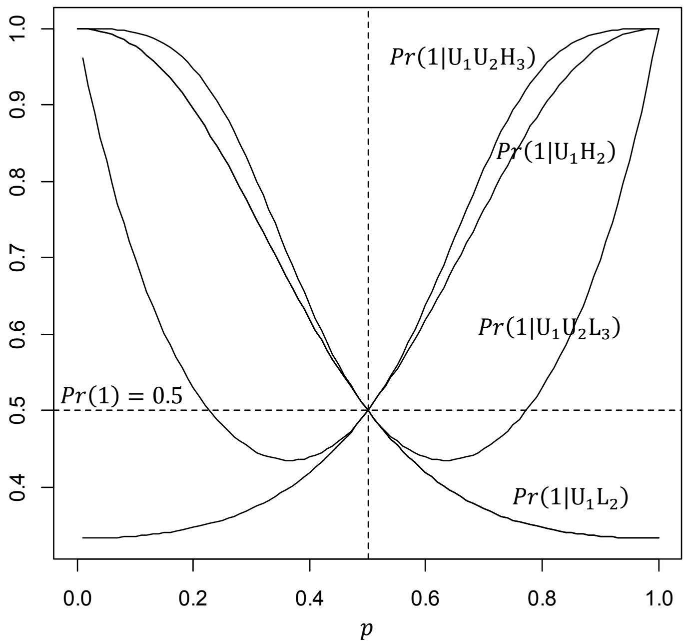
and

$$\begin{aligned} Pr(1|U_1 \ldots U_{i-1}L_i) &= \frac{(p^2 + (1-p)^2)^{i-1}}{(p^2 + (1-p)^2)^{i-1} + (2p(1-p))^{i-1}p/(1-p)} \\ &> Pr(1|U_1 \ldots U_{i-2}L_{i-1}). \quad (10) \end{aligned}$$

Further, for a given $p > 0.5$, $Pr(1|U_1 \ldots U_{i-1}L_i)$ becomes larger than 0.5 as i increases above certain level. When this happens, subsequent investors will always buy regardless of their private signals.

In Figure 3, I plot $Pr(1|U_1 \ldots U_{i-1}L_i)$ and $Pr(1|U_1 \ldots U_{i-1}H_i)$ as functions of i. It can be seen that for a given p, these probabilities are monotonically increasing in i which implies that the accumulated impact of observing protracted period of price momentum can be so large that even a private signal of bad state will not change investor's buy decision. It is worthwhile to note that while $Pr(1|U_1 \ldots U_{i-1}H_i)$ is always greater than 0.5, $Pr(1|U_1 \ldots U_{i-1}L_i)$ always has a segment below 0.5 for a shrinking

FIG. 2. Conditional probabilities of the second and third investors as p varies.



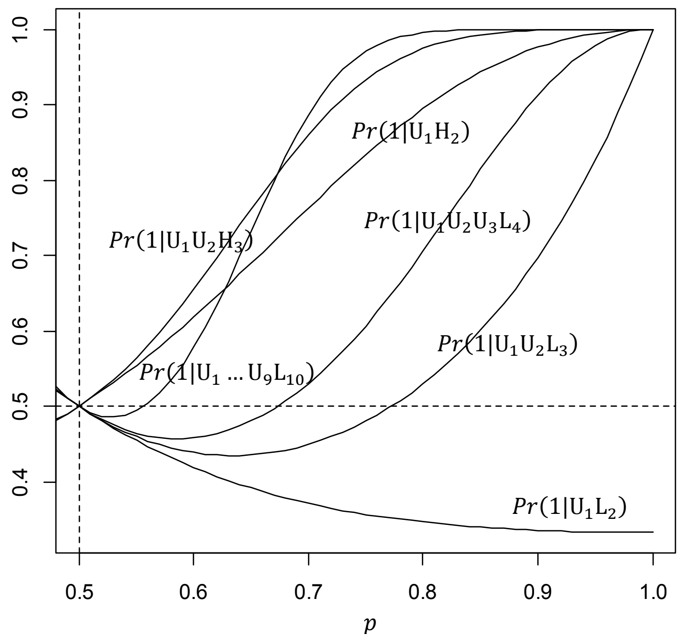
range of p. In the limit, the length of this segment drops to when $i \to +\infty$. As long as p is not too small, $Pr(1|U_1 \dots U_{i-1}L_i)$ will overshoot 0.5 after a relatively short period of price momentum, beyond which the upward pressure on prices becomes irresistible. This result highlights the fact that momentum trading can occur not because investors are nave or biased; rather, it is mostly due to the dominant influence of information cascading in the price trend. When investors get to the point, it is hard to stop it.

In Figure 4, I plot the two conditional probabilitiesas functions of p and i For a given p, it always holds that $Pr(1|U_1 \dots U_{i-1}L_i) < Pr(1|U_1 \dots U_{i-1}H_i)$. As p gets larger, $Pr(1|U_1 \dots U_{i-1}L_i)$ breaks the 0.5 threshold very fast and both functions converge to probability one.

3.2. Reversal

Up to this point, I have shown that price momentum can be generated endogenously from investors' learning behavior. The more accurate a signal gets the more likely that momentum pressure will develop under reasonable assumptions. There is an important omission from the above discussions though. Recall that c is used to measure the overvaluedness of a stock. As themarket price increases, it is appropriate to introduce a correction

FIG. 3. Information cascading as a function of p and the number of investors.

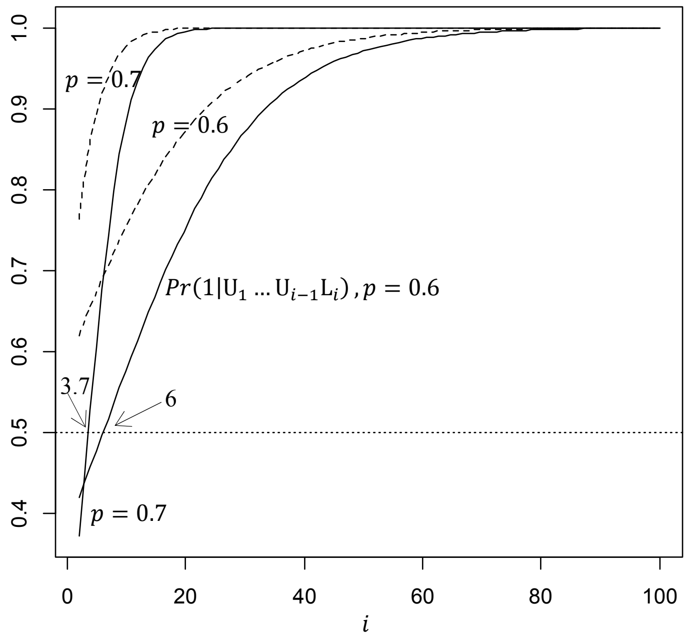


factor which reflects the heightened risk of the stock being overvalued or overbought:

$$c_i = 0.5 + \sum_{j=1}^{i-1} I_{U_j}\Delta \text{ or } 0.5 + \sum_{j=1}^{i-1} I_{U_j}\Delta I_{i\geq K}, \tag{11}$$

where Δ is a small positive number, say, 0.01 and $I_{i\geq K}$ is an indicator function which equals one when i is greater than some cutoff point K. I_{U_j} is another indicator function which equals one if the jth investor buys and price notches up and zero otherwise. I have adhered to a linear form of Eq. (11), which could be determined endogenously in more elaborate models. This specification captures investors' risk aversion and the externalities imposed by early buyers on later buyers: While the first few buyers can comfortably hold the stock and see the price rise, late comers face a higher risk of buying into an asset that is being increasingly overvalued thus limiting the potential gain. Another way to look at Eq. (11) is to view it as an increasing reservation payoff. The effect of this modification is best illustrated in Figure 5, where I plot $Pr(1|U_1 \ldots U_{i-1}L_i)$ and $Pr(1|U_1 \ldots U_{i-1}H_i)$ for $i \in \{2, \ldots, 100\}$ and $p = 0.6$. As an example, I have used $c_i = 0.5 + 0.01(i-1)$.

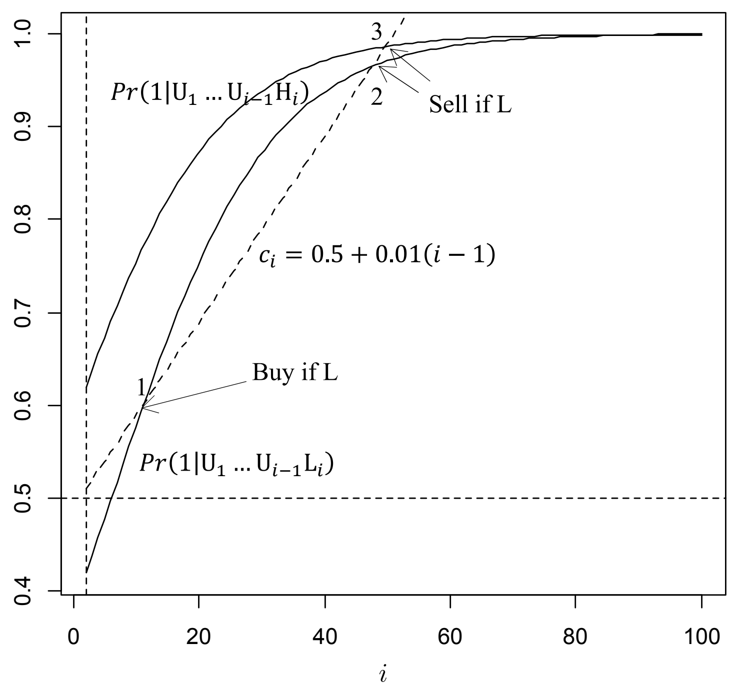
FIG. 4. $Pr(1|U_1 \ldots U_{i-1}L_i)$ (solid line) and $Pr(1|U_1 \ldots U_{i-1}H_i)$ (dashed line) as functions of p and i.



Three intersections (labeled 1,2 and 3) are of particular interest. Before the market reaches point 1, momentum can only build up when investors receive a signal of H because the expected payoff is not high enough to compensate for the extra risk of holding an overvalued asset. Although at this point both probabilities are higher than the unmodified 0.5 benchmark, investors will only buy when they receive a good signal. If there are so many buyers that the upward price momentum passes point 1, then investors will ignore their private signal and buy until they reach point 2. At point 2, the investor will sell if she receives Lbut will keep buying if she receives H. Finally, when the market hits point 3, the investor at this point will sell even if she receives H. Between point 2 and point 3, the market prepares for a reversal.

An interesting question to ask is whether the reversal itself cannot be reversed or interrupted, i.e., whether it is sustainable until price falls back to normal levels. The answer is affirmative. Suppose investor $i-1$, is just on the right of point 2 and receives L, she then sells because $0.5+(i-1)\Delta > Pr(1|U_1 \ldots I_{i-1})$. The next investor, i, observes $\{U_1U_2 \ldots U_{i-2}D_{i-1}\}$ plus a private signal. The market may fluctuate between point 2 and 3 if investors receive H and L in turn, but once it passes point 3, sell orders will dominate

FIG. 5. An illustration of the effect of c_i, $\Delta = 0.01$, $p = 0.6$.



regardless of whether H or L is received. The next theorem shows that the updated probabilities get smaller as more downward price movements are observed.

Theorem 2. *Under the model assumptions, for some positive $j > i$ we have*

$$\begin{aligned} & Pr(1|U_1 \dots U_{i-2} D_{i-1} \dots D_{j-1} H_j) \\ = \ & \frac{(p^2 + (1-p)^2)^{i-2}}{(p^2 + (1-p)^2)^{i-2} + \lambda(2p(1-p))^{i-2}(1-p)/p}, \end{aligned} \tag{12}$$

$$\begin{aligned} & Pr(1|U_1 \dots U_{i-2} D_{i-1} \dots D_{j-1} L_j) \\ = \ & \frac{(p^2 + (1-p)^2)^{i-2}}{(p^2 + (1-p)^2)^{i-2} + \lambda(2p(1-p))^{i-2} p/(1-p)}, \end{aligned} \tag{13}$$

where $\lambda = (p^2 + (1-p)^2/p(1-p))^{j-i+1}$.

Theorem 3. *When the market is between point 2 and 3, investor i sells after receiving L and observing investor $i-1$ sells; investor $i+1$ sells after receiving L and observing investor i sells, and so on. The process continues until c drops to below the continuously updated probability $Pr(1|U_1 \dots U_{i-2}D_{i-1}D_i \dots L_n)$ for some model-determined n. The following pairs of inequalities hold, with one for rolling window and the other for fixed window:*

$$\begin{aligned} c_{i-1} &> Pr(1|U_1 \dots U_{i-2}L_{i-1}) > Pr(1|U_1 \dots U_{i-2}D_{i-1}L_i) \\ &> Pr(1|U_1 \dots U_{i-2}D_{i-1}D_iL_{i+1}) > \cdots, \qquad (14) \\ c_j &> Pr(1|U_1 \dots U_{j-1}L_j) > Pr(1|U_1 \dots U_{j-2}D_{j-1}L_j) \\ &> Pr(1|U_1 \dots U_{j-3}D_{j-2}D_{j-1}L_j) > \cdots. \qquad (15) \end{aligned}$$

When the market is beyond point 3, investor i sells after observing investor $i-1$ sells; investor $i+1$ sells after observing investor i sells, and so on. The process continues until c drops to below the continuously updated probability $Pr(1|U_1 \dots U_{i-2}D_{i-1}D_i \dots H_n)$ for some model-determined n.

$$\begin{aligned} c_{i-1} &> Pr(1|U_1 \dots U_{i-2}H_{i-1}) > Pr(1|U_1 \dots U_{i-2}D_{i-1}H_i) \\ &> Pr(1|U_1 \dots U_{i-2}D_{i-1}D_iH_{i+1}) > \cdots, \qquad (16) \\ c_j &> Pr(1|U_1 \dots U_{j-1}H_j) > Pr(1|U_1 \dots U_{j-2}D_{j-1}H_j) \\ &> Pr(1|U_1 \dots U_{j-3}D_{j-2}D_{j-1}H_j) > \cdots, \qquad (17) \end{aligned}$$

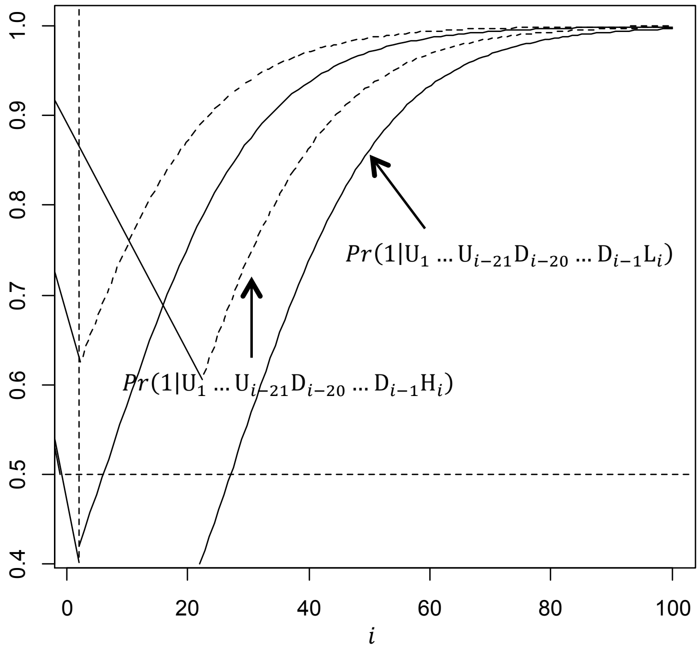
and for any $3 \le i < j$,

$$Pr(1|U_1 \dots U_{i-2}D_{i-1}D_i \dots L_j) < Pr(1|U_1 \dots U_{i-2}D_{i-1}D_i \dots H_n). \qquad (18)$$

Theorem 2 and 3 are illustrated in Figure 6 and 7 for $i \in \{2, \dots, 100\}$. Figure 6 is the same as Figure 5 except that I have added $Pr(1|U_1 \dots U_{i-21}D_{i-20} \dots D_{i-1}L_i)$ and $Pr(1|U_1 \dots U_{i-3}D_{i-20} \dots D_{i-1}H_i)$ with twenty downward price movements (starting from $i = 22$). Compared to the original curves, the two added ones are shifted down by a great deal due to the recent sequence downward price movements. For a given i, the larger the number of past sells observed, the more likelyan upward trend is reversed. This is true for both signals.

In Figure 7, I plot the rolling window conditional probabilities beyond point 2 and 3. Point 2 corresponds to 47 consecutive upward price movements and point 3 corresponds to 49 such movements. Investors adjust their expected payoffs immediately when the upward momentum looks too good to be true. The self-fulfilling mechanism is also obvious here: Investors will keep selling until price falls back to the fundamental level where c_i is

FIG. 6. Reversal probabilities conditional on the number of sells as i varies, $p = 0.6$.

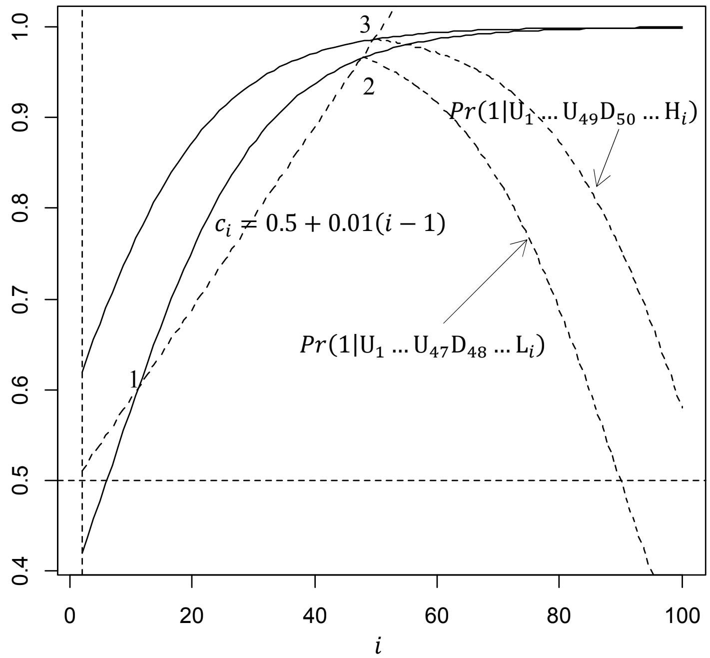


reset to 0.5 and the cycle ends. A new cycle starts afterward. Of course, Δ for an upward price trend need not be the same as for a downward trend. When it is very large (e.g., 0.1), the market can be said to be highly resistant to price momentum, in which case investors cease to be aggressive price chasers and become conservative very quickly. Contrariwise, when Δ is very small, the investor is so reluctant to modify her posterior that the momentum may drag on for a long time. In the latter situation, there will be substantial momentum profits for early buyers. In reality, Δ is not common knowledge and each investor may have a different correction factor depending on some external shocks, so the exact timing of the unwinding of the upward momentum will be stochastic. This, however, does not change the qualitative result of the model.

3.3. Sophisticated Investors

Institutional investors are sophisticated participants in the capital market and they act as momentum traders when entering the market; see Badrinath and Wahal (2002). In this section, I examine the information contents of these investors' purchase decisions. Let the pool of sophisticated investors be such that they receive aprivate signal that is correct

FIG. 7. An illustration of reversal, $\Delta = 0.01$, $p = 0.6$.



with probability $p_s > p > 0.5$, i.e., more accurate than ordinary ones. Assume that the ranking is common knowledge. Also use U_s and D_s to denote their impact on prices. On average, institutions are able to lead the rest of the market primarily because of their ability to receive and process relevant information much faster. Thus the working assumption in this section is that sophisticated investors are the "first investor" in the model and are followed by their unsophisticated peers. As before on the upward trend so that the first investor buys. Consider first the two investors case and the following lemma:

Lemma 3. *Under the model assumptions, the second investor updates her probabilities according to*

$$Pr(1|U_sH_2) = \frac{p_s^2 + (1-p_s)^2}{p_s^2 + (1-p_s)^2 + 2p_s(1-p_s)(1-p)/p} > Pr(1|U_1H_2), \quad (19)$$

$$Pr(1|U_sL_2) = \frac{p_s^2 + (1-p_s)^2}{p_s^2 + (1-p_s)^2 + 2p_s(1-p_s)p/(1-p)} > Pr(1|U_1L_2) \quad (20)$$

Lemma 3 can be used as a springboard to prove the general result:

Theorem 4. *For $i \geq 3$, investors update their probabilities according to*

$$\begin{aligned} & Pr(1|U_s \ldots U_{i-1}H_i) \qquad (21) \\ = & \frac{(p^2+(1-p)^2)^{i-2}}{(p^2+(1-p)^2)^{i-2}+(2p(1-p))^{i-2}2p_s(1-p_s)(1-p)/(pp_s^2+p(1-p_s)^2)} \\ > & Pr(1|U_1 \ldots U_{i-1}H_i) \end{aligned}$$

and

$$\begin{aligned} & Pr(1|U_s \ldots U_{i-1}L_i) \qquad (22) \\ = & \frac{(p^2+(1-p)^2)^{i-2}}{(p^2+(1-p)^2)^{i-2}+(2p(1-p))^{i-2}2pp_s(1-p_s)/((1-p)(p_s^2+(1-p_s)^2))} \\ > & Pr(1|U_1 \ldots U_{i-1}L_i) \end{aligned}$$

The brief conclusion is that sophisticated investors make the upward price movement more informative about the true state. When they buy, it is more likely that the true state is good; when they sell, chances are that the true state is bad. Results analogous to Theorem 2 and 3 can be easily derived and are omitted in the interest of brevity. Essentially, the cascading of information impounded in the past history of price changes becomes stronger to the degree that sophisticated investors are acting as guideposts for their followers. Price reversal occurs in a later stage when c_i rises to above the expected payoffs. The model also predicts that, as Δ gets larger (the slope parameter), price reversal will take place sooner (Figure 7). This is not surprising since more risk-averse investors no longer chase prices blindly; instead, they modify their perception of the market state and exit the market in a timely manner.

4. CONCLUDING REMARKS

I have proposed a parsimonious learning mechanism for price momentum and reversal. By varying a small set of parameters, the model is able to deliver a wide range of dynamics to accommodate the fact that the strength and direction of empirical evidence in favor of price momentum often varies by markets. Some countries do not experience significant momentum effects and some have a stronger upward than downward in general, price reversal after a long stretch of one-sided movements is a prevalent characteristic of all markets. The exact timing of a reversal is stochastic and hinges upon the extent to which investors modify their reservation payoffs according to the past price movements.

It is often argued that the observed price momentum and abrupt reversals reflect a super-speculative environment of capital markets. Prevailing explanations include the lack of faithful information disclosure by firms, the absolute dominance of unsophisticated retail investors and market manipulation by syndicate speculators. However, the net effect of these causes may be exaggerated. Price momentum can be a natural consequence of investors rationally revising their posterior beliefs about the market state which need not be affected by animal spirit or cognitive biases.

APPENDIX

The proof of Proposition 1 is trivial and is therefore omitted.

Proof of Lemma 1

The conditional probabilities can be calculated as follows:

$$
\begin{aligned}
Pr(U_1H_2|1) &= Pr(H_1U_1H_2|1) + Pr(L_1U_1H_2|) && \text{(A.1)}\\
&= ppp + (1-p)(1-p)p = p^3 + (1-p)^2 p, \\
Pr(U_1H_2|0) &= Pr(H_1U_1L_2|1) + Pr(L_1U_1H_2|0) && \text{(A.2)}\\
&= (1-p)p(1-p) + p(1-p)^2 = 2p(1-p)^2 \\
Pr(U_1L_2|1) &= Pr(H_1U_1L_2|1) + Pr(L_1U_1L_2|1) && \text{(A.3)}\\
&= pp(1-p) + (1-p)^2 = p^2(1-p) + (1-p)^3 \\
Pr(U_1L_2|0) &= Pr(H_1U_1L_2|0) + Pr(L_1U_1L_2|0) && \text{(A.4)}\\
&= (1-p)pp + p(1-p)p = 2p^2(1-p).
\end{aligned}
$$

$\blacksquare$

In the proof, I have used the assumption that an investor views other investors' types distributed as $c \sim U[0,1]$. Based on Proposition 1, when the previous investor receives H, a buy is made if her type falls in $[0,p]$; when she receives L, a buy is made if her type falls in $[0,1-p]$. I have probably belabored the obvious in the hope that one can notice the pattern and ordering of these terms and apply them to proofs of the other results. Note the flipping between p and $1-p$ by comparing Eq. (A.1) with Eq. (A.3) and Eq. (A.2) with Eq. (A.4). This trick makes the derivation of more general results possible. Substituting these quantities into Eq. (3) and (4) yields the desired results. To see that $Pr(1|U_1H_2) > 0.5$ and $Pr(1|U_1L_2) < 0.5$, one only needs to use $p^2 > (1-p)^2$ for $p > 0.5$.

Proof of Lemma 2

I focus on $Pr(1|U_1U_2L_3)$ and the case of $Pr(1|U_1U_2H_3)$ can be done in a similar fashion. First note that

$$\begin{aligned} & Pr(1|U_1U_2L_3) \\ = \; & \frac{Pr(U_1U_2L_3|1)}{Pr(U_1U_2L_3|1) + Pr(U_1U_2L_3|0)} \\ = \; & \frac{Pr(H_1U_1U_2L_3|1) + Pr(L_1U_1U_2L_3|1)}{Pr(H_1U_1U_2L_3|1) + Pr(L_1U_1U_2L_3|1) + Pr(H_1U_1U_2L_3|0) + Pr(L_1U_1U_2L_3|0)} \end{aligned} \tag{A.5}$$

These conditional probabilities can be further decomposed using another round of the total probability rule:

$$\begin{aligned} Pr(H_1U_1U_2L_3|1) &= Pr(H_1U_1H_2U_2L_3|1) + Pr(H_1U_1L_2U_2L_3|1) \\ &= pppp(1-p) + pp(1-p)^3 \\ &= (p^3 + (1-p)^2p)p(1-p) = Pr(U_1L_2|1)p^2, \\ Pr(L_1U_1U_2L_3|1) &= Pr(L_1U_1H_2U_2L_3|1) + Pr(L_1U_1L_2U_2L_3|1) \\ &= (1-p)^2p^2(1-p) + (1-p)^5 \\ &= (p^2 + (1-p)^2)(1-p)^3 = Pr(U_1L_2|1)(1-p)^2, \\ Pr(H_1U_1U_2L_3|0) &= Pr(H_1U_1H_2U_2L_3|0) + Pr(H_1U_1L_2U_2L_3|0) \\ &= 2(p(1-p)p^2 - p^2(1-p)p^2) \\ &= 2p(1-p)p^2(1-p) = Pr(U_1L_2|0)p(1-p), \\ Pr(L_1U_1U_2L_3|0) &= Pr(L_1U_1H_2U_2L_3|0) + Pr(L_1U_1L_2U_2L_3|0) \\ &= 2((1-p)p^3 - p(1-p)p^3) \\ &= 2p(1-p)p^2(1-p) = Pr(U_1L_2|0)p(1-p). \end{aligned}$$

Substituting these into Eq. (A.5), one gets

$$Pr(1|U_1U_2L_3) = \frac{(p^2 + (1-p)^2)^2}{(p^2 + (1-p)^2)^2 + 4p^3(1-p)}.$$

The proof of $Pr(1|U_1U_2H_3)$ proceeds in the same manner through multiple decompositions:

$$\begin{aligned} & Pr(1|U_1U_2H_3) \\ = \; & \frac{Pr(U_1U_2H_3|1)}{Pr(U_1U_2H_3|1) + Pr(U_1U_2H_3|0)} \\ = \; & \frac{Pr(H_1U_1U_2H_3|1) + Pr(L_1U_1U_2H_3|1)}{Pr(H_1U_1U_2H_3|1) + Pr(L_1U_1U_2H_3|1) + Pr(H_1U_1U_2H_3|0) + Pr(L_1U_1U_2H_3|0)}. \end{aligned}$$

The details are omitted to save space. ▌

Proof of Theorem 1

The proof of this theorem is based on an iterative argument. We have

$$\begin{aligned}
Pr(U_1H_2|1) &= Pr(H_1U_1H_2|1) + Pr(L_!U_1H_2|1) = (p^2 + (1-p)^2)Pr(H_2|1),\\
Pr(U_1H_2|0) &= Pr(H_1U_1H_2|0) + Pr(L_1U_1H_2|0) = 2p(1-p)Pr(H_2|0),\\
Pr(U_1L_2|1) &= Pr(H_1U_1L_2|1) + Pr(L_!U_1L_2|1) = (p^2 + (1-p)^2)Pr(L_2|1),\\
Pr(U_1L_2|0) &= Pr(H_!U_1L_2|0) + Pr(L_1U_1L_2|0) = 2p(1-p)Pr(L_2|0)
\end{aligned}$$

Next,

$$\begin{aligned}
Pr(U_1U_2H_3|1) &= Pr(U_1H_2U_2H_3|1) + Pr(U_1L_2U_2H_3|1)\\
&= (p^2 + (1-p)^2)Pr(U_1H_2|1),\\
Pr(U_1U_2H_3|0) &= Pr(U_1H_2U_2H_3|0) + Pr(U_1L_2U_2H_3|0)\\
&= 2p(1-p)Pr(U_1H_2|0),\\
Pr(U_1U_2L_3|1) &= Pr(U_1H_2U_2L_3|1) + Pr(U_!L_2U_2L_3|1)\\
&= (p^2 + (1-p)^2)Pr(U_1L_2|1),\\
Pr(U_1U_2L_3|0) &= Pr(U_1H_2U_2L_3|0) + Pr(U_1L_2U_2L_3|0)\\
&= 2p(1-p)Pr(U_1L_2|0).
\end{aligned}$$

Finally,

$$\begin{aligned}
Pr(U_1U_2U_3H_4|1) &= Pr(U_1U_2H_3U_3H_4|1) + Pr(U_1U_2L_3U_3H_4|1)\\
&= (p^2 + (1-p)^2)Pr(U_1U_2H_3|1),\\
Pr(U_1U_2U_3H_4|0) &= Pr(U_1U_2H_3U_3H_4|0) + Pr(U_1U_2L_3U_3H_4|0)\\
&= 2p(1-p)Pr(U_1U_2H_3|0),\\
Pr(U_1U_2U_3L_4|1) &= Pr(U_!U_2H_3U_3L_4|1) + Pr(U_1U_2L_3U_3L_4|1)\\
&= (p^2 + (1-p)^2)Pr(U_1U_2L_3|1),\\
Pr(U_1U_2U_3L_4|0) &= Pr(U_1U_2H_3U_3L_4|0) + Pr(U_1U_2L_3U_3L_4|0)\\
&= 2p(1-p)Pr(U_1U_2L_3|0).
\end{aligned}$$

It is clear that a pattern has emerged and by continuous substitution we get Eq. (9) and (10). ▮

Proof of Theorem 2 and 3
Starting from the basic case of $i = 3$ we have

$$\begin{aligned}
Pr(U_1D_2L_3|1) &= Pr(U_1H_2D_2L_3|1) + Pr(U_1L_2D_2L_3|1) \\
&= 2p(1-p)Pr(U_1L_2|1), \\
Pr(U_1D_2L_3|0) &= Pr(U_1H_2D_2L_3|0) + Pr(U_1L_2D_2L_3|0) \\
&= (p^2 + (1-p)^2)Pr(U_1L_2|0), \\
Pr(U_1D_2H_3|1) &= Pr(U_1H_2D_2H_3|1) + Pr(U_1L_2D_2H_3|1) \\
&= 2p(1-p)Pr(U_1H_2|1), \\
Pr(U_1D_2H_3|0) &= Pr(U_1H_2D_2H_3|0) + Pr(U_1L_2D_2H_3|0) \\
&= (p^2 + (1-p)^2)Pr(U_1H_2|0).
\end{aligned}$$

For $i = 4$,

$$\begin{aligned}
Pr(U_1D_2D_3L_4|1) &= Pr(U_1D_2H_3D_3L_4|1) + Pr(U_1D_2L_3D_3L_4|1) \\
&= 2p(1-p)Pr(U_1D_2L_3|1), \\
Pr(U_1D_2D_3L_4|0) &= Pr(U_1D_2H_3D_3L_4|0) + Pr(U_1D_2L_3D_3L_4|0) \\
&= (p^2 + (1-p)^2)Pr(U_1D_2L_3|0), \\
Pr(U_1D_2D_3H_4|1) &= Pr(U_1D_2H_3D_3H_4|1) + Pr(U_1D_2L_3D_3H_4|1) \\
&= 2p(1-p)Pr(U_1D_2H_3|1), \\
Pr(U_1D_2D_3H_4|0) &= Pr(U_1D_2H_3D_3H_4|0) + Pr(U_1D_2L_3D_3H_4|0) \\
&= (p^2 + (1-p)^2)Pr(U_1D_2H_3|0)
\end{aligned}$$

The same pattern applies to the general case. One can derive

$$\begin{aligned}
Pr(1|U_1D_2L_3) &= \frac{Pr(U_1L_2|1)}{Pr(U_1L_2|1) + Pr(U_1L_2|0)(p^2 + (1-p)^2)/2p(1-p)} \\
&= \frac{p^2 + (1-p)^2}{p^2 + (1-p)^2 + 2p^2\theta},
\end{aligned}$$

and

$$\begin{aligned}
Pr(1|U_1D_dH_3) &= \frac{Pr(U_1H_2|1)}{Pr(U_1H_2|1) + Pr(U_1H_2|0)(p^2 + (1-p)^2)/2p(1-p)} \\
&= \frac{p^2 + (1-p)^2}{p^2 + (1-p)^2 + 2p^2\theta},
\end{aligned}$$

where $\theta = (p^2 + (1-p)^2)/2p(1-p) > 1$ for $p > 0.5$. It is easily shown that $Pr(1|U_1D_2L_3) < Pr(1|U_1L_2)$, $Pr(1|U_1D_2H_3) < Pr(1|U_1H_2)$ and $Pr(1|U_1D_2H_3) < Pr(1|U_1D_2H_3)$.

By continuous substitution we can derive the general results:

$$\begin{aligned}
& Pr(1|U_1\ldots U_{i-2}D_{i-1}H_i) \\
= \;& \frac{(p^2+(1-p)^2)^{i-2}}{(p^2+(1-p)^2)^{i-2}+(2p(1-p))^{i-2}(p^2+(1-p)^2)/2p^2} \\
< \;& Pr(1|U_1\ldots U_{i-2}H_{i-1}), \\
& Pr(1|U_1\ldots U_{i-2}D_{i-1}L_i) \\
= \;& \frac{(p^2+(1-p)^2)^{i-2}}{(p^2+(1-p)^2)^{i-2}+(2p(1-p))^{i-2}(p^2+(1-p)^2)/2(1-p)^2} \\
< \;& Pr(1|U_1\ldots U_{i-2}L_{i-1}),
\end{aligned}$$

and

$$Pr(1|U_1\ldots U_{i-2}D_{i-1}H_i) > Pr(1|U_1\ldots U_{i-2}D_{i-1}L_i).$$

Further, for $j > i$

$$\begin{aligned}
& Pr(1|U_1\ldots U_{i-2}D_{i-1}\ldots D_{j-1}H_j) \\
= \;& \frac{(p^2+(1-p)^2)^{i-2}}{(p^2+(1-p)^2)^{i-2}+\lambda(2p(1-p))^{i-2}(1-p)/p} \\
> \;& Pr(1|U_1\ldots U_{i-2}D_{i-1}\ldots D_jH_{j+1}) > \cdots, \\
& Pr(1|U_1\ldots U_{i-2}D_{i-1}\ldots D_{j-1}L_j) \\
= \;& \frac{(p^2+(1-p)^2)^{i-2}}{(p^2+(1-p^2))^{i-2}+\lambda(2p(1-p))^{i-2}p/(1-p)} \\
> \;& Pr(1|U_1\ldots U_{i-2}D_{i-1}\ldots D_jL_{j+1}) > \cdots,
\end{aligned}$$

where $\lambda = (p^2+(1-p)^2/p(1-p))^{j-i+1}$. Dividing the numerator and denominator by $(p^2+(1-p)^2)^{i-2}$ and varying i, one can show the fixed window inequalities:

$$\begin{aligned}
Pr(1|U_1\ldots U_{j-1}L_j) &> Pr(1|U_1\ldots U_{j-2}D_{j-1}L_j) \\
&> Pr(1|U_1\ldots U_{j-3}D_{j-2}D_{j-1}L_j) > \cdots, \\
Pr(1|U_1\ldots U_{j-1}H_j) &> Pr(1|U_1\ldots U_{j-2}D_{j-1}H_j) \\
&> Pr(1|U_1\ldots U_{j-3}D_{j-2}D_{j-1}H_j) > \cdots.
\end{aligned}$$

Fix the number of buys (U) and vary the number of sells (D), we arrive at the rolling window inequalities:

$$\begin{aligned}
Pr(1|U_1\ldots U_{i-2}L_{i-1}) &> Pr(1|U_1\ldots U_{i-2}D_{i-1}L_i) \\
&> Pr(1|U_1\ldots U_{i-2}D_{i-1}D_iL_{i+1}) > \cdots, \\
Pr(1|U_1\ldots U_{i-2}H_{i-1}) &> Pr(1|U_1\ldots U_{i-2}D_{i-1}H_i) \\
&> Pr(1|U_1\ldots U_{i-2}D_{i-1}D_iH_{i+1}) > \cdots.
\end{aligned}$$

■

Proof of Lemma 3

The argument proceeds as in the proof of Lemma 1.

$$\begin{aligned}
Pr(U_sH_2|1) &= Pr(H_sU_sH_2|1) + Pr(L_sU_sH_2|1) = p_s^2p + (1-p_s)^2p, \\
Pr(U_sH_2|0) &= Pr(H_sU_sH_2|0) + Pr(L_sU_sH_2|0) \\
&= (1-p_s)p_s(1-p) + p_s(1-p_s)(1-p), \\
Pr(U_sL_2|1) &= Pr(H_sU_sL_2|1) + Pr(L_sU_sL_2|1) \\
&= p_s^2(1-p) + (1-p_s)^2(1-p), \\
Pr(U_sL_2|0) &= Pr(H_sU_sL_2|0) + Pr(L_sU_sL_2|0) \\
&= (1-p_s)p_sp + p_s(1-p_s)p.
\end{aligned}$$

Note that

$$Pr(1|U_sH_2) = \frac{Pr(U_sH_2|1)}{Pr(U_sH_2|1) + Pr(U_sH_2|0)},$$

and a similar decomposition applies to $Pr(1|U_sL_2)$. Substitution yields Eq. (19) and (20). ■

Proof of Theorem 4

Comparing Lemma 3 with Lemma 1 and 2 reveals that the difference is twofold. First, use $p_s^2 + (1-p_s)^2$ to replace the first of all multiples of $p^2 + (1-p)^2$ in the numerator and that of the corresponding term in the denominator. Second, use $p_s(1-p_s)$ to replace the first of all multiples of $p(1-p)$ in the corresponding term in the denominator. The proof is complete by noting the same pattern in the proof of Theorem 1. ■

REFERENCES

Badrinath, Swaminathan G., and Sunil Wahal, 2002. Momentum trading by institutions. *Journal of Finance* **57(6)**, 2449-2478.

Barberis, Nicholas, Andrei Shleifer, and Robert Vishny, 1998. A model of investor sentiment. *Journal of Financial Economics* **49(3)**, 307-343.

Chan, Louis K., Narasimhan Jegadeesh, and Josef Lakonishok, 1996. Momentum strategies. *Journal of Finance* **51(2)**, 1681-1713.

Cooper, Michael J., Roberto C. Gutierrez, and Allaudeen Hameed, 2004. Market states and momentum. *Journal of Finance* **59(3)**, 1345-1365.

Cutler, David M., James M. Poterba, and Lawrence H. Summers, 1991. Speculative dynamics and the role of feedback traders. *American Economic Review Papers and Proceedings* **80**, 63-68.

Daniel, Kent D., David Hirshleifer, and Avanidhar Subrahmanyam, 1998. Investor psychology and security market under-and over-reactions. *Journal of Finance* **53(6)**, 1839-1885.

DeLong, Bradford J., Andrei Shleifer, Lawrence H. Summers, and Robert Waldmann, 1990. Positive feedback investment strategies and destabilizing rational speculation. *Journal of Finance* **45(2)**, 379-395.

Hong, Harrison, Terence Lim, and Jeremy C. Stein, 2000. Bad news travels slowly: Size, analyst coverage, and the profitability of momentum strategies. *Journal of Finance* **55(1)**, 265-295.

Hong, Harrison, and Jeremy C. Stein, 1999. A unified theory of underreaction, momentum trading and overreaction in asset markets. *Journal of Finance* **54(6)**, 2143-2184.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993. Returns to buying winners and selling losers: Implications for stock market efficiency. *Journal of Finance* **48(1)**, 65-91.

Jegadeesh, Narasimhan, and Sheridan Titman, 2001. Profitability of momentum strategies: An evaluation of alternative explanations. *Journal of Finance* **56(2)**, 699-720.

Kang, Joseph., Ming-Hua Liu, and Sophie X. Ni, 2002. Contrarian and momentum strategies in the China stock market: 1993-2000. *Pacific-Basin Finance Journal* **10(3)**, 243-265.

Lee, Charles, and Swaminathan, Badrinath, 2000. Price momentum and trading volume. *Journal of Finance* **55(5)**, 2017-2069.

McInish, Thomas H., David K. Ding, Chong S. Pyun, and Udomsak Wongchoti, 2008. Short-horizon contrarian and momentum strategies in Asian markets: An integrated analysis. *International Review of Financial Analysis* **17(2)**, 312-329.

Naughton, Tony, Cameron Truong, and Madhu Veeraraghavan, 2008. Momentum strategies and stock returns: Chinese evidence. *Pacific-Basin Finance Journal* **16(4)**, 476-492.

Wu, Yangru, 2011. Momentum trading, mean reversal and overreaction in Chinese stock market. *Review of Quantitative Finance and Accounting* **37(3)**, 301-323.