Sectoral Interactions and Monetary Policy
under Costly Price Adjustments[*]

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This paper presents a state-dependent pricing model with a two-stage chain-of-production structure and serially correlated, idiosyncratic price adjustment cost process in each sector. The model can explain much of the observed volatility and persistence of inflation and output, and nonlinearity and asymmetry in the responses of prices and quantities to monetary shocks. We derive analytical solutions in a static version of the model to illustrate the main results and to gain insights. We solve the dynamic model using a modified nonlinear solution method that features indirect inference and self-validating inflation forecasts as key components.

Key Words: State-dependent pricing; Chain of production; Persistent price adjustment costs; Self-validating inflation forecasts; Asymmetry.

JEL Classification Numbers: E0, E3.

1. INTRODUCTION

The question of how monetary shocks affect inflation and output has held a central stage in macroeconomics. Attempts to meeting this challenge have led to a widespread use of models with frictional price adjustments. These models fall into two categories: time-dependent pricing models, where price

adjustments occur at exogenously specified time, and state-dependent pricing models, where the timing of price adjustments is endogenous. While time-dependent models are subject to the criticism of lacking micro foundations, state-dependent models have the desirable feature of modeling the source of the frictions explicitly.[1]

Early state-dependent pricing models were usually framed in specialized environments and made distributional assumptions in order to aggregate an economy, which were not amenable to quantitative business cycle analysis.[2] In the stochastic dynamic general equilibrium paradigm, time-dependent pricing models have become a workhorse in macroeconomic investigation of the volatility and persistence of inflation and output. Yet, in addition to their lack of micro foundations, time-dependent models face a difficulty in explaining the observed magnitude of the real effects of monetary shocks.[3] Time-dependent models also generate smaller price and larger output responses to positive shocks than to negative shocks, an asymmetry that is exactly opposite to what is observed from the data.[4]

It is not until recently models with state-dependent pricing have been made operational in a SDGE framework. Results from this recent line of research are, however, not more encouraging. Studies by Dotsey, King, and Wolman (1997, 1999), Golosov and Lucas (2004), and Gertler and Leahy (2004) suggest that quantitative state-dependent pricing models face the same difficulty as quantitative time-dependent pricing models in explaining

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[1]Several studies attempt to rationalize the behavior of time-dependent pricing. For example, Caballero (1989) finds that firms may follow time-dependent pricing rules if the main cost of a price adjustment is associated with information gathering and reaching a decision rather than making the actual adjustment. See, also, Bonomo and Carvalho (2005) and the references therein. Yet, empirical evidence provided by Levy, Bergen, Dutta, and Venable (1997) and Zbaracki, Ritson, Levy, Dutta, and Bergen (2004) suggests that the cost associated with making an actual adjustment, such as communicating and negotiating the intended price change with customers and physically issuing a new price, under which a state-dependent pricing rule is optimal, is far more significant than the cost of decision-making and information gathering. Empirical studies by Slade (1998), Aguirregabiria (1999), and Willis (2000a, 2000b) provide corroborating evidence. Further, Bonomo (1992) and Midrigan (2005a, 2005b) find that under some conditions prices can behave as in the time-dependent case even when firms actually follow state-dependent pricing rules.

[2]Early contributions to state-dependent approach include Barro (1972), Sheshinski and Weiss (1977, 1983), Caplin and Spulber (1987), Ball and Romer (1990), Caballero and Engle (1992), Caplin and Leahy (1991a, 1991b, 1997), Ball and Mankiw (1994), and Conlon and Liu (1997).

[3]See, for example, Chari, Kehoe, and McGrattan (2000), who conclude that "mechanisms to solve the persistence problem must be found elsewhere." Dotsey and King (2005) show that state-dependent pricing can have quite different implications than time-dependent pricing for dynamic macroeconomic models.

[4]Ball and Mankiw (1994) and Devereux and Siu (2004) use models with both time- and state-dependent pricing to analyze an asymmetry in price and output responses to monetary shocks. Ireland (1997) employs a hybrid model to study disinflation.

the observed magnitude of the real effects of monetary shocks. The problem is that, following a shock in these models, many firms choose to adjust their prices, and the magnitude of a price change by each adjusting firm is large compared to the size of the shock. As a result, the shock has a large effect on price and little effect on output.

We here enrich this recent line of research by incorporating a two-stage chain-of-production structure and serially correlated, idiosyncratic price adjustment cost processes into a SDGE state-dependent pricing framework. We show that the two added ingredients enhance the model significantly so that it can explain much of the observed volatility and persistence of inflation and output, along with the observed nonlinearity and asymmetry in the responses of prices and output to monetary shocks. The estimated model can account for almost the same volatility of CPI inflation, 94% of the volatility of PPI inflation, almost 20% of the volatility of real GDP, and the PPI inflation is more volatile than the CPI inflation in the model, as in the data. The model can also explain 89% of the persistence in the CPI inflation, 97% of the persistence in the PPI inflation, 93% of the persistence in real GDP, and the PPI inflation is less persistent than the CPI inflation in the model, as in the data. In addition to matching these aggregate features of the data quite well, our model can also account for the microeconomic features of the data concerning the frequencies of price changes in U.S. retail and manufacturing sectors: the model captures 88% of the average frequency of price changes in the retail sector and 95% of the average frequency of price changes in the manufacturing sector. The effects of monetary shocks in our model are nonlinear: shocks of smaller magnitude lead to less sensitive price responses and greater output multipliers than shocks of greater magnitude; and asymmetric: negative shocks lead to smaller disinflations and greater contractions than the inflations and expansions associated with positive shocks of the same size.

We identify four asymmetric interactions in our model that play crucial roles in generating the above results. Two of these interactions are among firms sitting at each of the two different stages of production, and two are between these two groups of firms across the two stages. We show that these interactions are endogenously interconnected through the input-output structure to dampen the responses of prices and magnify the responses of output to positive and negative monetary shocks in an asymmetric way to produce the results.

The cross-stage interactions investigated in this paper is related in spirit to those studied by Cooper (1990) and Chatterjee, Cooper, and Ravikumar (1993). In these two-sector models, interactions between the decisions of agents situated in the two sectors can produce a plausible form of strategic complementarity that can be an important source of nominal stickiness or lead to a high degree of persistence in the endogenous variables. The key

behind these results is that the output of one sector is valuable to producers in the other sector — it is this input-output connection that generates the strategic complementarity between the two sectors. The role of sectoral interactions in generating nominal rigidities is also studied by Blanchard (1983), Huang and Liu (2001), and Bhaskar (2002), and empirical investigations of price changes at different production stages are conducted by Means (1935), Clark (1999), and Hanes (1999).[5]

We show that serial correlations in the price adjustment cost processes in our model are essential to generating the observed price adjustment pattern in the data, characterized by many small price changes coupled with a few large spikes (e.g., Bils and Klenow, 2003 and Klenow and Krystov, 2003). We find that this heterogeneity in firms' pricing behaviors contributes significantly to generating the aforementioned results as well. Existence of serial correlations in price adjustment costs is supported by our own estimates, as well as by the empirical evidence of Willis (2000a, 2000b). With persistence in the adjustment cost process, a firm that realizes a higher adjustment cost and chooses to adjust may make a larger price change in order to reduce the frequency of future adjustments, while a firm that realizes a lower adjustment cost may make a smaller price change since its expected future adjustment costs conditional on the current one are also low. The decisions also depend on how long it has been since their last price adjustments, as well as on the current and expected future realizations of monetary shocks.

This creates a high degree of heterogeneity even among adjusting firms in each sector, which results in a highly complex price distribution at each stage of the production chain. The complexity of equilibrium price distributions is compounded by sector interactions, since firms have to condition their own pricing decisions not only on the current and expected future adjustment costs and pricing decisions of their peers in the same sector, but also on the current and expected future adjustment costs and pricing decisions of their input-suppliers or output-demanders in the other sector.[6]

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[5]Research along somewhat different avenues includes Ball and Romer (1989), who demonstrate that asynchronization in price setting may be an equilibrium outcome if there are firm-specific shocks that arrive at different time for different firms, and Ball and Cecchetti (1988), who show that, with imperfect information, firms may not distinguish between aggregate demand shocks and firm-specific shocks, and thus do not have an incentive to synchronize. Lau (2001) demonstrates a link between strategic complementarity and asynchronization.

[6]It is this interlock between persistent adjustment costs and sector interactions that, we believe, captures what Gordon (1990) pictures as the essence of the input-output matrix. In his words, "The gigantic matrix represents the real world, full of heterogeneous firms enmeshed in a web of intricate supplier-demander relationships." He provides a heuristic argument for why the complexity of the input-output table makes it difficult for firms to synchronize their pricing decisions. See, also, Gordon (1981) and Blanchard (1987).

Given the complexity of the equilibrium distributions of firms across prices, aggregate state vector in our model is a high dimension object, which renders standard nonlinear solution methods impractical. We approximate the aggregate state vector by a lower dimension object and we solve the model using a solution method closely related to those proposed by den Haan (1996, 1997) and Krusell and Smith (1997, 1998). We embody this modified method in an indirect inference procedure to produce a simulation-based estimation-computation algorithm. A key component of the modified solution method involves stochastic forecast rules for sector inflation rates and sector relative prices. These forecast rules capture the perceived laws of motion for some low-order moments of the price distributions that firms use in solving their dynamic optimization problems. In a rational expectations equilibrium, the forecast rules must be self-validating, in the sense that their predictions about the moments for the aggregate state vector must be consistent with the aggregations of individual firms' solutions to their optimization problems obtained under the perceived laws of motion for the moments.

In Section 2, we derive analytical solutions in a static model to show the main results and to gain sights. In Section 3, we extend the static model to a dynamic framework. In Section 4, we describe our data, estimation strategy, and solution method. In Section 5, we report our simulation results for the dynamic economy. In Section 6, we conduct sensitivity analysis to get a sense about the relative importance of each of the two new ingredients in our model, and of their interaction, for generating the results in our baseline framework.

2. A STATIC MODEL

In this section we present a one-period model that is a simplified version of the dynamic economy to be developed in the next section. The economy is featured with a consumer goods sector and a producer goods sector, with a continuum of monopolistically competitive firms in each sector, indexed on the unit interval $[0, 1]$, and each producing a differentiated goods. The production of a producer goods requires primary factors as inputs, while a consumer goods is produced from many differentiated producer goods. A representative distributor combines all individual consumer goods $\{Y_j\}_{j \in [0,1]}$ into a basket of consumer goods, corresponding to real GDP,

$$C = \left[\int_0^1 Y_j^{\frac{\theta_p - 1}{\theta_p}} \, dj \right]^{\frac{\theta_p}{\theta_p - 1}}, \tag{1}$$

where $\theta_p \in (1, \infty)$ denotes the elasticity of substitution between the individually differentiated consumer goods. The distributor takes the prices $\{P_j\}_{j \in [0,1]}$ of the individual consumer goods as given and chooses the bundle of these individual consumer goods to minimize the cost of fabricating a given basket of the consumer goods. It sells the composite consumer goods at the unit fabricating cost, corresponding to the consumer price index, the CPI,

$$P = \left[\int_0^1 P_j^{1-\theta_p} dj \right]^{\frac{1}{1-\theta_p}}, \tag{2}$$

while the demand for a type j consumer goods is given by

$$Y_j = \left(\frac{P_j}{P} \right)^{-\theta_p} C. \tag{3}$$

A type j consumer goods Y_j is produced from the individually differentiated producer goods $\{X_{i,j}\}_{i \in [0,1]}$ according to

$$Y_j = \left\{ A_p \left[\int_0^1 X_{i,j}^{\frac{\theta_o - 1}{\theta_o}} di \right]^{\frac{\theta_o}{\theta_o - 1}} \right\}^{\frac{1}{\eta_p}}, \tag{4}$$

where A_p is a technology level which may have a sector specific component and $\theta_o \in (1, \infty)$ denotes the elasticity of substitution between the individually differentiated producer goods. By allowing $\eta_p > 1$ we allow for the possibility of some fixed factor in the production of the consumer goods sector. Cost minimization gives rise to the producer price index, corresponding to the PPI, as a function of the prices $\{O_i\}_{i \in [0,1]}$ of the individual producer goods,

$$O = \left[\int_0^1 O_i^{1-\theta_o} di \right]^{\frac{1}{1-\theta_o}}, \tag{5}$$

and the demand for a type i producer goods,

$$X_i = \frac{1}{A_p} \left(\frac{O_i}{O} \right)^{-\theta_o} \left(\frac{Q}{P} \right)^{-\theta_p \eta_p} C^{\eta_p}, \tag{6}$$

where Q is an auxiliary consumer price index,

$$Q = \left[\int_0^1 P_j^{-\theta_p \eta_p} dj \right]^{-\frac{1}{\theta_p \eta_p}}, \tag{7}$$

in the spirit of Yun (1996).

At the earlier stage of the production process, a type i producer goods is produced using labor as input according to

$$X_i = (A_o L_i)^{\frac{1}{\eta_o}},$$

(8)

where A_o is a technology level which may have a sector specific component. By allowing $\eta_o > 1$ we allow for the possibility of some fixed factor in the production of the producer goods sector.

Real profit gross of any price adjustment cost is

$$\tilde{\Pi}_{p,j} = \frac{P_j}{P} Y_j - \frac{1}{A_p} \frac{O}{P} Y_j^{\eta_p},$$

(9)

for a firm j in the consumer goods sector and

$$\tilde{\Pi}_{o,i} = \frac{O_i}{P} X_i - \frac{1}{A_o} \frac{W}{P} X_i^{\eta_o},$$

(10)

for a firm i in the producer goods sector, where W denotes the nominal wage rate.

A representative household has a utility function, $u(C, L) = \log C - \Phi L$, for some $\Phi > 0$. The household purchases consumption C at P and supplies labor L at W. The first order conditions for the household's labor supply and consumption decisions imply

$$\Phi C = \frac{W}{P}.$$

(11)

We consider a money demand relation, $M = PC/\nu$ which we use to describe nominal aggregate demand conditions, where ν denotes the velocity of money. Using this relation and (11), we can write the two demand schedules (3) and (6) as

$$Y_j = \nu \left(\frac{P_j}{P} \right)^{-\theta_p} \left(\frac{M}{P} \right),$$

(12)

$$X_i = \frac{\nu^{\eta_p}}{A_p} \left(\frac{O_i}{O} \right)^{-\theta_o} \left(\frac{Q}{P} \right)^{-\theta_p \eta_p} \left(\frac{M}{P} \right)^{\eta_p}.$$

(13)

Using (12) and (13), we can rewrite the real profit functions (9) and (10) as

$$\tilde{\Pi}_{p,j} = \nu \left(\frac{P_j}{P} \right)^{1-\theta_p} \left(\frac{M}{P} \right) - \frac{\nu^{\eta_p}}{A_p} \left(\frac{O}{P} \right) \left(\frac{P_j}{P} \right)^{-\theta_p \eta_p} \left(\frac{M}{P} \right)^{\eta_p},$$

(14)

$$\tilde{\Pi}_{o,i} = \frac{\nu^{\eta_p}}{A_p} \left(\frac{O}{P}\right) \left(\frac{O_i}{O}\right)^{1-\theta_o} \left(\frac{Q}{P}\right)^{-\theta_p \eta_p} \left(\frac{M}{P}\right)^{\eta_p}$$

$$- \frac{\nu^{\eta_p \eta_o + 1} \Phi}{A_p^{\eta_o} A_o} \left(\frac{O_i}{O}\right)^{-\theta_o \eta_o} \left(\frac{Q}{P}\right)^{-\theta_p \eta_p \eta_o} \left(\frac{M}{P}\right)^{\eta_p \eta_o + 1} \tag{15}$$

The profit function in (14) involves an individual firm's price relative to the prices of its competitors, $p_j = P_j/P$, the level of the PPI relative to the level of the CPI, $op = O/P$, and real money balances, M/P. The profit function in (15) depends on an individual firm's price relative to the prices of its competitors, $o_i = O_i/O$, the level of the PPI relative to the level of the CPI, $op = O/P$, real money balances, M/P, as well as the level of the auxiliary CPI relative to the level of CPI, $qp = Q/P$.

The real profit functions in (14) and (15), and in (9) and (10), are measured in units of consumption. It is convenient to express the profit functions in units of utility,

$$\Pi_{p,j} = U_c \tilde{\Pi}_{p,j} = \frac{\tilde{\Pi}_{p,j}}{C} = \tilde{\Pi}_{p,j} \left(\frac{P}{\nu M}\right), \tag{16}$$

$$\Pi_{o,j} = U_c \tilde{\Pi}_{o,j} = \frac{\tilde{\Pi}_{o,j}}{C} = \tilde{\Pi}_{o,j} \left(\frac{P}{\nu M}\right), \tag{17}$$

or, more explicitly,

$$\Pi_{p,j} = \left(\frac{P_j}{P}\right)^{1-\theta_p} - \frac{\nu^{\eta_p - 1}}{A_p} \left(\frac{O}{P}\right) \left(\frac{P_j}{P}\right)^{-\theta_p \eta_p} \left(\frac{M}{P}\right)^{\eta_p - 1}, \tag{18}$$

$$\Pi_{o,i} = \frac{\nu^{\eta_p - 1}}{A_p} \left(\frac{O}{P}\right) \left(\frac{O_i}{O}\right)^{1-\theta_o} \left(\frac{Q}{P}\right)^{-\theta_p \eta_p} \left(\frac{M}{P}\right)^{\eta_p - 1}$$

$$- \frac{\nu^{\eta_p \eta_o} \Phi}{A_p^{\eta_o} A_o} \left(\frac{O_i}{O}\right)^{-\theta_o \eta_o} \left(\frac{Q}{P}\right)^{-\theta_p \eta_p \eta_o} \left(\frac{M}{P}\right)^{\eta_p \eta_o}. \tag{19}$$

Firms make two decisions to maximize their profits: a firm decides whether or not to adjust its price (the extensive margin), and if to adjust, by how much (the intensive margin). A firm that decides to adjust its price must pay a fixed cost in units of labor to do so. The cost is fixed in the sense that it is independent of the magnitude of the price adjustment. Given the above preferences specification, the fixed cost is effectively expressed in units of utility. Also, it is clear from (18) that the steady-state revenue for a firm in the retail sector is one unit of utility, so the fixed

cost facing a firm in this sector is expressed as a fraction of the firm's steady-state revenue as well. For the wholesale sector, by substituting (62) and (63) into (19), we can show that the steady-state revenue for a firm in this sector equals $(\theta_p - 1)/(\eta_p \theta_p)$ units of utility, and thus a one unit fixed utility cost facing a firm in the wholesale sector constitutes $(\eta_p \theta_p)/(\theta_p - 1)$ fraction of the firm's steady-state revenue.

The fixed cost is stochastic. For now we can assume that the distribution of fixed costs is uniform on $[0, \psi_{\text{pmax}}]$ for firms in the retail sector and uniform on $[0, \psi_{\text{omax}}]$ for firms in the wholesale sector. Ex post, firms in each sector are randomly assigned to the interval. Let $\psi_{p,j}$ denote firm j's fixed cost realization (in the retail sector) and $\psi_{o,i}$ denote firm i's fixed cost realization (in the wholesale sector), we have, without loss of generality,

$$\psi_{p,j} = j\psi_{\text{pmax}}, \quad j \in [0,1], \tag{20}$$

$$\psi_{o,i} = i\psi_{\text{omax}}, \quad i \in [0,1]. \tag{21}$$

We can now use this static framework to analyze several key issues in a transparent way. For this purpose, it is sufficient to set $A_p = A_o = \eta_p = \eta_o = \Phi = \nu = 1$, since these variables and parameters do not play any important role in gaining the insight of our analysis here.

We begin by considering a deterministic steady state in which all firms' prices satisfy the markup rule implied by profit-maximization,

$$\bar{O}_i = \bar{O} = \frac{\theta_o}{\theta_o - 1}\bar{M}, \quad i \in [0,1], \tag{22}$$

$$\bar{P}_j = \bar{P} = \frac{\theta_p}{\theta_p - 1}\bar{O} = \frac{\theta_p}{\theta_p - 1}\frac{\theta_o}{\theta_o - 1}\bar{M}, \quad j \in [0,1], \tag{23}$$

where note that there is no difference between individual prices and the price index in either sector in this steady state.

Suppose now there is a shock to money supply so that money stock changes from $\bar{M}$ to $\tilde{M}$. In response to the shock, there is a unique $z_p \in (0,1]$ and a unique $z_o \in (0,1]$ such that: (i) all firms $j \in [0, z_p]$ would choose to adjust their prices while all firms $j \in (z_p, 1]$ would choose to stay with $\bar{P}$; and (ii) all firms $i \in [0, z_o]$ would choose to adjust their prices while all firms $i \in (z_o, 1]$ would choose to stay with $\bar{O}$. The adjusting firms in each sector will choose the same prices, say, $\tilde{P}$ and $\tilde{O}$, given by the markup rule. Thus we have

$$P_j = \tilde{P} = \frac{\theta_p}{\theta_p - 1}O, \quad j \in [0, z_p] \qquad \text{and} \qquad P_j = \bar{P}, \quad j \in (z_p, 1], \tag{24}$$

$$O_i = \tilde{O} = \frac{\theta_o}{\theta_o - 1}\tilde{M}, \quad i \in [0, z_o] \qquad \text{and} \qquad O_i = \bar{O}, \quad i \in (z_o, 1]. \quad (25)$$

The CPI, the auxiliary CPI, and the PPI change correspondingly to

$$P = \left[z_p \tilde{P}^{1-\theta_p} + (1 - z_p)\bar{P}^{1-\theta_p} \right]^{\frac{1}{1-\theta_p}}, \quad (26)$$

$$Q = \left[z_p \tilde{P}^{-\theta_p} + (1 - z_p)\bar{P}^{-\theta_p} \right]^{-\frac{1}{\theta_p}}, \quad (27)$$

$$O = \left[z_o \tilde{O}^{1-\theta_o} + (1 - z_o)\bar{O}^{1-\theta_o} \right]^{\frac{1}{1-\theta_o}}. \quad (28)$$

Real profit for an adjusting firm in the retail sector, that is, for $j \in [0, z_p]$, is

$$\Pi_p^{\mathrm{a}} = \left(\frac{\tilde{P}}{P} \right)^{1-\theta_p} - \left(\frac{O}{P} \right) \left(\frac{\tilde{P}}{P} \right)^{-\theta_p}, \quad (29)$$

and for a non-adjusting firm in the retail sector, that is, for $j \in (z_p, 1]$, is

$$\Pi_p^{\mathrm{na}} = \left(\frac{\bar{P}}{P} \right)^{1-\theta_p} - \left(\frac{O}{P} \right) \left(\frac{\bar{P}}{P} \right)^{-\theta_p}. \quad (30)$$

Real profit for an adjusting firm in the wholesale sector, that is, for $i \in [0, z_o]$, is

$$\Pi_o^{\mathrm{a}} = \left(\frac{O}{P} \right) \left(\frac{\tilde{O}}{O} \right)^{1-\theta_o} \left(\frac{Q}{P} \right)^{-\theta_p} - \left(\frac{\tilde{O}}{O} \right)^{-\theta_o} \left(\frac{Q}{P} \right)^{-\theta_p} \left(\frac{\tilde{M}}{P} \right), \quad (31)$$

and for a non-adjusting firm in the wholesale sector, that is, for $i \in (z_o, 1]$, is

$$\Pi_o^{\mathrm{na}} = \left(\frac{O}{P} \right) \left(\frac{\bar{O}}{O} \right)^{1-\theta_o} \left(\frac{Q}{P} \right)^{-\theta_p} - \left(\frac{\bar{O}}{O} \right)^{-\theta_o} \left(\frac{Q}{P} \right)^{-\theta_p} \left(\frac{\tilde{M}}{P} \right). \quad (32)$$

The threshold values z_p and z_o must satisfy

$$
\begin{aligned}
z_p &= \frac{\Pi_p^{\mathrm{a}} - \Pi_p^{\mathrm{na}}}{\psi_{\mathrm{pmax}}}, &&\text{if } \Pi_p^{\mathrm{a}} - \Pi_p^{\mathrm{na}} < \psi_{\mathrm{pmax}}, \\
&= 1, &&\text{if } \Pi_p^{\mathrm{a}} - \Pi_p^{\mathrm{na}} \geq \psi_{\mathrm{pmax}};
\end{aligned}
\quad (33)
$$

$$z_o \;=\; \frac{\Pi_o^{\mathrm{a}} - \Pi_o^{\mathrm{na}}}{\psi_{\mathrm{omax}}}, \qquad \text{if } \Pi_o^{\mathrm{a}} - \Pi_o^{\mathrm{na}} < \psi_{\mathrm{omax}},$$

$$\;=\; 1, \qquad\qquad \text{if } \Pi_o^{\mathrm{a}} - \Pi_o^{\mathrm{na}} \geq \psi_{\mathrm{omax}}. \tag{34}$$

With conditions (24)-(33), we can proceed to analyzing the several key issues of interest.

2.1. Relative Volatilities and the Real Effects of Monetary Policy Shocks

A well-known empirical fact is that the rate of change in the consumer price index is less volatile than the rate of change in the producer price index. Figure 1 illustrates this contrast between the volatilities in sector price inflations for the U.S. economy.

FIG. 1. Percentage changes in CPI and PPI — quarter to quarter.
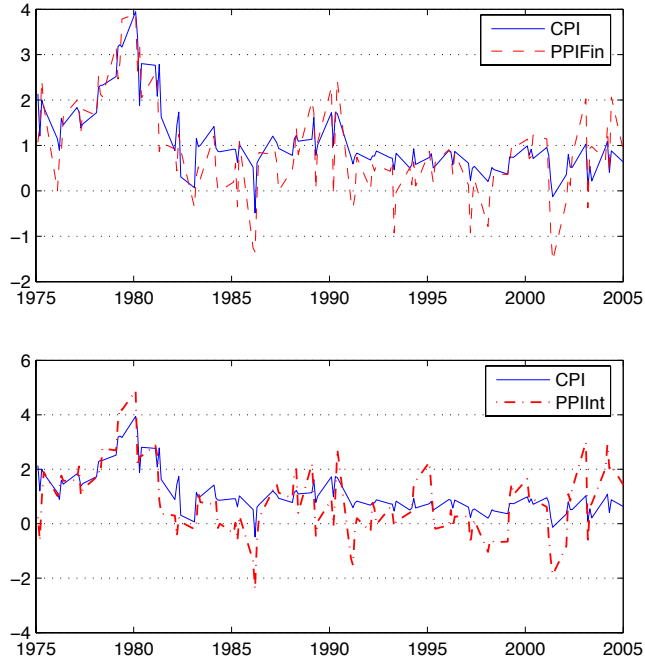
Source: Bureau of Labor Statistics/Haver Analytics.

Our simple static model can account for this empirical fact. To see this, let ΔM denote the gross rate of change in the money supply, ΔP denote the gross rate of change in the CPI, and ΔO denote the gross rate of change

in the PPI. We can manipulate (22)-(28) to get

$$(\Delta O)^{1-\theta_o} - 1 = z_o[(\Delta M)^{1-\theta_o} - 1], \tag{35}$$

$$(\Delta P)^{1-\theta_p} - 1 = z_p[(\Delta O)^{1-\theta_p} - 1]. \tag{36}$$

Given that $\theta_o > 1$ and $\theta_p > 1$, we can show from (35) and (36) that: (i) if $\Delta M > 1$, then $1 < \Delta P \leq \Delta O \leq \Delta M$, where the first weak inequality holds as an equality if and only if $z_p = 1$ and the second weak inequality holds as an equality if and only if $z_o = 1$; and (ii) if $\Delta M < 1$, then $1 > \Delta P \geq \Delta O \geq \Delta M$, where the first weak inequality holds as an equality if and only if $z_p = 1$ and the second weak inequality holds as an equality if and only if $z_o = 1$.

These findings can be summarized into the following points. First, if money supply increases (a positive shock), then both the CPI and the PPI increase and, as long as the shock is not too large so that some firms in the retail sector choose not to adjust their prices, the CPI increases less in percentage than the PPI does. Second, if money supply decreases (a negative shock), then both the CPI and the PPI decrease and, as long as the shock is not too large so that some firms in the retail sector choose not to adjust their prices, the CPI decreases less in percentage than the PPI does. In both cases, the rate of change in the CPI is less volatile than the rate of change in the PPI and the money supply shock has a real effect. Third, if the shock is large enough so that all firms in the retail sector choose to adjust their prices, then the CPI and the PPI responses are the same in percentage terms. Fourth, even in this third case, as long as some firms in the wholesale sector choose not to adjust their prices, the rate of change in the CPI and in the PPI is less in absolute value than the rate of change in the money supply, and it can be shown that the money supply shock has a real effect. Last, if the shock is so large that all firms choose to adjust their prices, then the CPI and the PPI change in the same percentage as does the money supply, and it can be shown that the money supply shock has no real effect.

These points together lead to another observation, that is, the chain of production structure may help to both generate and magnify the real effect of a monetary policy shock — the rate of change in real GDP is given by $\Delta C = \Delta M / \Delta P$. This has an important implication, especially for state-dependent pricing models. Standard one-sector state-dependent pricing models calibrated to match the average frequency of price changes in the retail sector of the United States, estimated by Bils and Klenow (2003) and Klenow and Krystov (2003), which in our static model corresponds to the fraction of adjusting firms in the consumer goods sector, generate very little output volatility from monetary policy shocks. In these models, firms

adjust prices frequently and the price change by an adjuster is large. These two margins work in tandem to make monetary shocks almost neutral.

Our above analysis shows that, adding a wholesale sector on top of a retail sector dampens the price change by an adjusting firm in the retail sector, as long as not all firms in the wholesale sector adjust their prices. The latter condition is supported by our empirical estimate of the average frequency of price changes by the Federal Reserve Bank of Kansas City's Tenth District Manufacturers, which in our static model corresponds to the fraction of adjusting firms in the wholesale sector. The fact that many wholesalers do not adjust their prices implies a smaller variation in the marginal cost facing a retailer compared to the case without the wholesale sector for a given size of money shocks. Thus, although many retailers may adjust their prices, the price change by an adjuster is smaller compared to the case without the wholesale sector. As a consequence, there is a smaller variation in the CPI and a larger variation in real GDP in our chain of production model than in the single sector model for a given size of monetary shock, provided that the size is not too big.

2.2. Non-Linear and Asymmetric Effects of Monetary Policy Shocks

Many studies find empirical evidence on non-linearities and asymmetries in the responses of prices and output to monetary policy shocks.[7] Shocks of smaller magnitude lead to less sensitive price responses and greater output multipliers than shocks of greater magnitude; that is, the effects of monetary policy shocks are non-linear. Further, negative shocks have smaller disinflationary effects on prices and greater contractionary effects on output than the inflations and expansions associated with positive shocks of the same magnitude; that is, the effects of monetary policy shocks are asymmetric.

Our model can account for such non-linearities and asymmetries. To see this in the current static model in a transparent way, we use (22)-(25) to rewrite gross profit differentials for a retailer and for a wholesaler between adjusting and non-adjusting, which measure incentives for the firms to adjust, as

$$\Pi_p^{\mathrm{a}} - \Pi_p^{\mathrm{na}} = \frac{1}{\theta_p} \left(\frac{\Delta O}{\Delta P} \right)^{1-\theta_p} - \left(1 - \frac{\theta_p - 1}{\theta_p} \Delta O \right) (\Delta P)^{\theta_p - 1}, \qquad (37)$$

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[7]See, among others, Cover (1992), Macklem, Paquet, and Phaneuf (1996), and Ravn and Sola (2004).

$$\Pi_o^{\mathrm{a}} - \Pi_o^{\mathrm{na}} = \frac{\theta_p - 1}{\theta_p} \left[\frac{(\Delta M)^{1-\theta_o}}{\theta_o} - 1 + \frac{\theta_o - 1}{\theta_o}(\Delta M) \right] \frac{(\Delta O)^{\theta_o}}{\Delta P} \left(\frac{\Delta Q}{\Delta P} \right)^{-\theta_p},$$
(38)

where ΔQ denote the gross rate of change in the auxiliary CPI.

Denote by o, p, q, and m the log deviations of O, P, Q, and $\tilde{M}$ from $\bar{O}$, $\bar{P}$, $\bar{Q}$, and $\bar{M}$. We take a third-order approximation of the right hand side of (37) and (38) around the steady state, which gives rise to

$$\Pi_p^{\mathrm{a}} - \Pi_p^{\mathrm{na}} = \frac{\theta_p - 1}{2} \left[o^2 - \frac{\theta_p - 2}{3} o^3 + (\theta_p - 1)o^2 p \right],$$
(39)

$$\Pi_o^{\mathrm{a}} - \Pi_o^{\mathrm{na}} = \frac{(\theta_o - 1)(\theta_p - 1)}{2\theta_p} \left[m^2 - \frac{\theta_o - 2}{3} m^3 + \theta_o m^2 o - m^2 p \right].$$
(40)

These expressions involve only second- and third-order terms.

Up to a second order, the incentive for a firm to adjust its price depends only on the deviation of its marginal cost from the steady state, o for a retailer and m for a wholesaler. The fact that the second-order term is greater for a larger deviation in the marginal cost from the steady state implies non-linearities in the incentive for price adjustment. This is true for both the retailer and the wholesaler. The only parameter that affects this incentive for the retailer is θ_p, the elasticity of substitution between the individually differentiated consumer goods. The smaller is θ_p, the smaller the gain of adjusting. In other words, a less competitive consumer goods market leads to a smaller incentive for the retailer to adjust. In contrast, two parameters affect this incentive for the wholesaler to adjust: these are θ_o, the elasticity of substitution between the individually differentiated producer goods, as well as θ_p. The smaller is θ_o or θ_p, the smaller the gain of adjusting. In other words, a less competitive consumer or producer goods market leads to a smaller incentive for the wholesaler to adjust.

The third-order terms imply asymmetries in the adjustment incentives. The importance of these effects depends on θ_p for retailers and on both θ_o and θ_p for wholesalers.

There are two third-order terms in (39). The first concerns a change in the marginal cost faced by a retailer that is of third-order magnitude. If $\theta_p > 2$, this term gives the retailer less incentive to adjust its price in response to a rise in the marginal cost than to a decline in the marginal cost of the same magnitude. The second captures the effect of an asymmetric strategic interaction between retailers' pricing decisions. To see this, recall that o and p always move in the same direction as m, as is demonstrated in the previous section. When $o > 0$, p is positive, and this term increases the retailer's incentive to adjust its price, while when $o < 0$, p is negative, and this term decreases the retailer's incentive to adjust its price. In other

words, this term renders the retailer a greater incentive to raise its price if other retailers raise theirs (with a rising marginal cost), but less incentive to cut its price if other retailers cut theirs (with a declining marginal cost). This term has a relatively large coefficient (more than three times greater than the coefficient of the first third-order term), giving rise to a relatively large third-order effect.

The interpretations of the first and the second third-order terms in (40) are parallel to the interpretations of the first and the second third-order terms in (39). In particular, the second third-order term in (40) renders a wholesaler a greater incentive to raise its price if other wholesalers raise theirs (following a positive shock), but less incentive to cut its price if other retailers cut theirs (following a negative shock), and this term has a relatively large coefficient. It is this asymmetric strategic interaction between wholesalers' pricing decisions that tends to give the wholesaler a greater incentive to adjust its price in response to a positive shock than to a negative shock.

FIG. 2. Incentives to adjust prices: gross profit differentials between adjusting and non-adjusting for a retailer (the upper panel) and for a wholesaler (the lower panel).
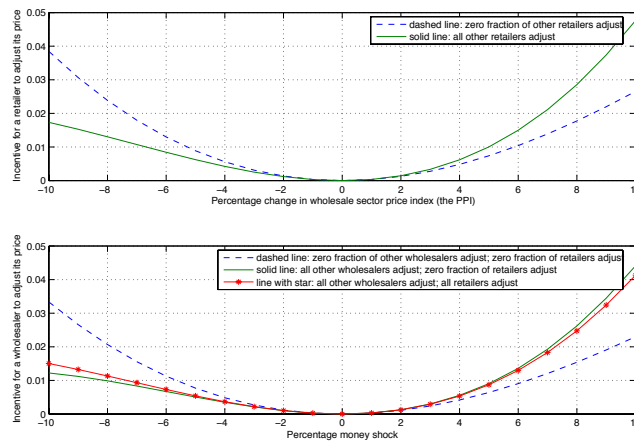
It is useful to recall that all terms in (39) are endogenous and determined, conditional on p, by the wholesale sector's price index o, or, the PPI. Thus the incentive for a retailer to adjust its price, conditional on the pricing decisions by other retailers, is determined by how much wholesalers adjust theirs. This impact of wholesalers' pricing decisions on a retailer's incentive to adjust its price is non-linear and asymmetric, as illustrated by

the upper panel of Figure 2,[8] plotted with $\theta_o = \theta_p = 7.5$.[9] In fact, this asymmetry tends to render the retailer a greater incentive to adjust its price in response to a decline in the PPI than to a rise in the PPI of the same magnitude, as revealed by the dashed line, plotted with the asymmetric strategic interactions between retailers' pricing decisions, captured by the second third-order term in (39), ignored. The solid line reveals that the effect of the asymmetric strategic interactions between retailers' pricing decisions is strong that the retailer actually ends up with less incentive to adjust its price in response to a decline in the PPI than to a rise in the PPI.

While a greater variation in the PPI gives a retailer a greater incentive to change its price via the endogenous and asymmetric marginal cost effect, the last third-order term in (40) captures the effect of another asymmetric strategic interaction between pricing decisions by retailers and by wholesalers. This term reveals that the nature of how a change in the retail sector's price index p, or, the CPI, affects the incentive for a wholesaler to adjust its price depends on the sign of the change. The wholesaler has a smaller incentive to raise its price if retailers raise theirs, but a greater incentive to reduce its price if retailers reduce theirs. This asymmetry arises from a sector relative price effect: a rise (decline) in the CPI, which is the price index used for deflating nominal profits to get their real counterparts for firms in both sectors, narrows (widens) the real profit differential (the gain of adjusting) for a wholesaler, rendering the wholesaler a smaller (larger) incentive to adjust its price.

The lower panel of Figure 1 decomposes the non-linear and asymmetric effects of the different terms in (40) on the incentive for a wholesaler to adjust its price. The dashed line captures an exogenous marginal cost effect: when both the asymmetric strategic interactions between wholesalers' pricing decisions and the sector relative price effect, reflected respectively by the second and the third third-order terms in (40), are ignored, the wholesaler would have a greater incentive to adjust its price in response to a negative marginal cost shock than to a positive marginal cost shock of the same magnitude. The effect of the asymmetric strategic interactions between wholesalers' pricing decisions is strong that, when it is taken into account, the wholesaler ends up with less incentive to adjust its price in response to a negative shock than to a positive shock, as the solid line

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[8]To plot the figure, we have made use of two first-order relationships that $o = z_o m$ and $p = z_p o = z_p z_o m$, which can be derived from (35) and (36).

[9]Equation (39) is similar to an equation derived by Devereux and Siu (2004) in a one-sector model with both time- and state-dependent pricing. The key difference between our equation here and the one in their paper is that the marginal cost facing firms in the retail sector in our model is endogenous, arising from the unique feature of the chain of production structure, while this counterpart in their model is an exogenous shock.

shows. The effect of this asymmetry is counter-forced somewhat by the asymmetric strategic interaction between pricing decisions by retailers and by wholesalers. Yet, even with this sector interaction taken into account, the incentive for the wholesaler to adjust its price remains greater in response to a positive shock than to a negative shock, as shown by the line with star.

2.3. Equilibrium Analysis

The analysis in the previous subsections helps to reach the following. First, interactions between pricing decisions by wholesalers and retailers can play an important role in generating the observed order of relative volatilities of sector prices and output and significant real effects of monetary policy shocks. Second, strategic interactions within each sector and cross the two sectors can all contribute to generating the observed non-linearity in the responses of prices and output to monetary shocks. Third, although cross-sector interactions tend to generate an opposite asymmetry in the incentives for firms to adjust prices in response to positive and negative shocks, the effects from within-sector interactions are generally stronger that the firms end up with an asymmetric incentive for price adjustment that is consistent with the observed direction of asymmetry in the responses of prices and output to positive and negative shocks.

We turn now to equilibrium analysis. Up to a first order, we can derive from (35) and (36) two approximate equilibrium relationships, $o = z_o m$ and $p = z_p o = z_p z_o m$. Substituting these relations into (39) and (40), we obtain

$$\text{netgain}_p = \frac{\theta_p - 1}{2} m^2 \left[z_o^2 - \frac{\theta_p - 2}{3} m z_o^3 + (\theta_p - 1) m z_o^3 z_p \right] - z_p \psi_{\text{pmax}}, \quad (41)$$

$$\text{netgain}_o = \frac{(\theta_o - 1)(\theta_p - 1)}{2\theta_p} m^2 \left[1 - \frac{\theta_o - 2}{3} m + \theta_o m z_o - m z_o z_p \right] - z_o \psi_{\text{omax}}. \quad (42)$$

The key is to find the equilibrium cutoff points z_o and z_p. Clearly, for any size of shocks, z_o and z_p must be strictly positive. We consider shocks that are not too large so that there is a marginal firm in each sector that is indifferent between adjusting and not adjusting its price in response to the shocks. We show, with some algebra, that z_o and z_p can be obtained by solving the following pair of equations

$$z_p = \left[\theta_o - \frac{2\theta_p \psi_{\text{omax}}}{(\theta_o - 1)(\theta_p - 1)m^3} \right] + \left(\frac{1}{m} - \frac{\theta_o - 2}{3} \right) \frac{1}{z_o}, \quad (43)$$

$$z_p = \frac{\theta_p - 1}{2\psi_{\text{pmax}}} \left\{ \left[\theta_p m^2 - \frac{(\theta_o - 2)(\theta_p - 1)}{3} m^3 \right] z_o^2 \right.$$
$$\left. + \left[\left(\theta_o(\theta_p - 1) - \frac{\theta_p - 2}{3} \right) m^3 - \frac{2\theta_p \psi_{\text{omax}}}{\theta_o - 1} \right] z_o^3 \right\}. \qquad (44)$$

Note that this system is highly non-linear. Our numerical examinations indicate that, when this system has a solution in $(0, 1]^2$, it has a unique such solution.

FIG. 3. Equilibrium fractions of wholesalers and retailers that choose to adjust prices in response to positive and negative money shocks.
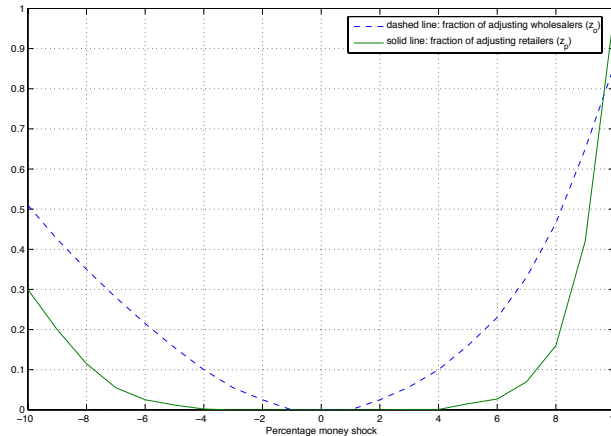

Figure 3 plots the equilibrium values of z_o and z_p for positive and negative money shocks of various sizes. To plot the figure, we need to set values for ψ_{pmax} and ψ_{omax}. Levy, et al. (1997) find, based on a store-level dataset for five large U.S. supermarket chains, that per-store menu costs of changing prices comprise of 0.7 percent of annual revenue. Thus we set $\psi_{\text{pmax}} = 0.028$ with a quarterly model in our mind. Zbaracki, et al. (2004) study a large U.S. manufacturing firm and find that the costs associated with changing a price fall into three categories: the cost of decision-making and information gathering in order to determine an optimal price, the cost of communicating and negotiating the intended price change with customers, and the cost of physically issuing a new price. They find that each of the three categories accounts for 0.28, 0.91, and 0.04 percent of annual revenue, respectively. We identify the latter two categories as corresponding to the fixed cost of price adjustment for a firm in the producer goods sector in our model. Thus, with a quarterly model in our mind, we set $\psi_{\text{omax}} = 0.0438$, where

we have made a necessary adjustment when converting labor and utility units into the unit of the firm's revenue.

The figure shows that changes in the equilibrium fractions of adjusting firms in response to money shocks are non-linear and asymmetric. Shocks of smaller magnitude lead to smaller changes than shocks of greater magnitude and negative shocks lead to smaller changes than positive shocks of the same magnitude. Further, with small or moderate shocks the changes tend to be more dramatic for wholesalers than for retailers, and this pattern tends to be reversed with large shocks.[10]

FIG. 4. Equilibrium responses of the PPI, the CPI, and real GDP to positive and negative money shocks of various sizes.
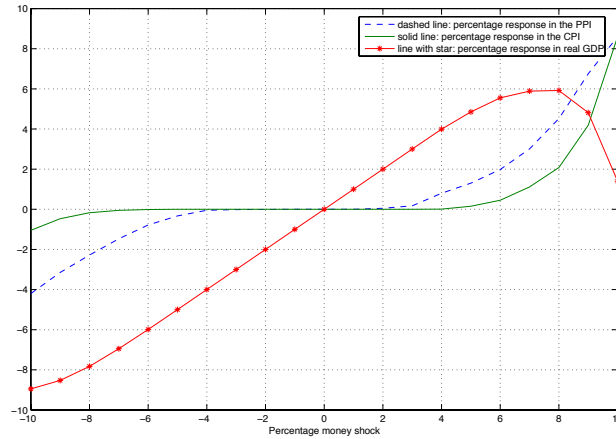

Figure 4 displays the equilibrium responses of the PPI, the CPI, and real GDP to positive and negative money shocks of various sizes. The figure makes clear that the response of the CPI is less volatile than the response of the PPI. Thus, the inflationary or dis-inflationary impacts of the shocks are dampened through the production chain, and this attenuation contributes to generating the real effect of the shocks.

The figure also demonstrates clearly the non-linearities and asymmetries in the responses of equilibrium prices and output to the shocks. The effects of monetary shocks are non-linear as shocks of smaller magnitude lead to less sensitive price responses and greater output multipliers than shocks of greater magnitude, while they are asymmetric as negative shocks have smaller dis-inflationary effects on prices and greater contractionary ef-

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[10]The figure shows that this is the case for a greater than 9.5 percent positive shock. We find (but do not plot in the figure) that this is also true for a larger than 12.5 percent negative shock.

fects on output than the inflations and expansions associated with positive shocks of the same magnitude.

2.4.   Contrast with a One-Sector Model

To further illustrate the significance of the chain of production structure in our baseline model for generating the real effects of monetary shocks, we solve a one-sector version of the baseline model and compare the responses of the CPI and real GDP to money shocks in this degenerate case with those in the baseline model.

FIG. 5. Equilibrium responses of the CPI and real GDP to money shocks of various sizes: contrast between the chain-of-production model and a one-sector model.
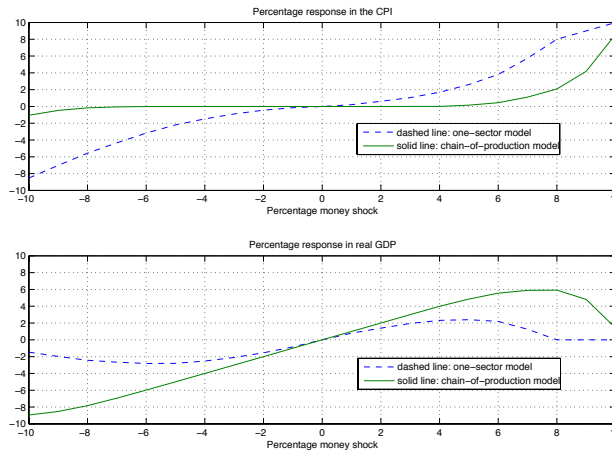
Figure 5 plots these responses. As is clear for the figure, without the production chain, the CPI responses much more and real GDP much less, and the responses are less asymmetric to positive and to negative shocks. The vertical input-output connection in our baseline model effectively attenuates the response of price and magnifies the real effects of money shocks, while the asymmetric strategic interactions between pricing decisions by firms in the producer goods sector work in tandem with the asymmetric strategic interactions between pricing decisions by firms in the consumer goods sector to magnify the degree of asymmetry in the responses of price and output to positive and to negative shocks.

3. A DYNAMIC ECONOMY

The static model presented in the previous section captures some fundamental natures of a chain of production structure and state-dependent

pricing. It is simple and thus useful for analyzing several issues of economic significance in a transparent way. We now extend this static model to a dynamic general equilibrium framework. The extension allows us to study issues such as that concerning persistence in the real effect of monetary shocks, which by its nature can be analyzed only in a dynamic setting.

The extension of the household side is straightforward. We assume that the representative household is infinitely-lived, has a period utility function specified in Section 2, and has a time discount factor $\beta \in (0,1)$. With the representative household, the financial structure of the economy is not important for the subsequent analysis. Also, the fact that the period profit functions are expressed in units of contemporaneous utility makes convenient for the recursive formulation of the dynamic profit-maximization problems for firms.

We describe now the extension of the firm side.

3.1. Serially Correlated Price Adjustment Costs and Nominal Money Growth

In our static model, firms in each sector that choose to change their prices in response to monetary shocks make the same amount of price adjustment. In fact, this is true even in the extended dynamic framework if we were to assume serially uncorrelated price adjustment cost processes, which is a standard assumption in the literature on state-dependent pricing. In these one-sectors models, not only firms that choose to adjust their prices in response to a shock all make the same amount of price adjustment, the size of the adjustment by each firm is typically large compared to the size of the shock. As result, there can be little real effect of monetary shocks in these models.

This homogenous (and large) price adjustment is inconsistent with micro evidence. The typical price adjustment pattern in the data is characterized by many small price changes (which is not predicted by those one-period state-dependent pricing models) coupled with only a few large spikes (e.g., Bils and Klenow, 2003 and Klenow and Krystov, 2003). As an endeavor to match this salient empirical feature, we allow for serial correlations in the price adjustment cost processes, as is motivated by the empirical evidence in Willis (2000a, 2000b). The price adjustment cost may be serially correlated across time and we model it as a stationary AR(1) log-normal process subject to idiosyncratic shocks, which are assumed to be identically and independently distributed among firms in each sector and serially uncorrelated across sectors:

$$\log(\psi_{p,j,t}) = \mu_p + \rho_p \log(\psi_{p,j,t-1}) + \epsilon_{p,j,t}, \tag{45}$$

$$\log(\psi_{o,i,t}) = \mu_o + \rho_o \log(\psi_{o,i,t-1}) + \epsilon_{o,i,t}. \tag{46}$$

The main structural parameters to be estimated are those governing the adjustment cost processes: the mean, persistence, and standard deviation of the innovations to the processes, $\{\mu_p, \rho_p, \sigma_{\epsilon_p}\}$ and $\{\mu_o, \rho_o, \sigma_{\epsilon_o}\}$.

In the dynamic setting, a firm's pricing decision must balance the benefit and the cost of adjusting its price by comparing the discounted expected profit in the case of adjusting and of non-adjusting, while, conditioning on adjusting, the firm needs to compute the optimal amount of price adjustment in order to facilitate the comparison. The decision, on both the extensive margin and the intensive margin, is based upon not only the current realization of the adjustment cost, but also the expected future adjustment costs. The conditional distribution of the adjustment cost process is denoted $\Phi_p(\psi'_{p,j}|\psi_{p,j})$ for a firm in the consumer goods sector and $\Phi_o(\psi'_{o,i}|\psi_{o,i})$ for a firm in the producer goods sector. The conditional distribution has non-trivial implications for pricing behavior if the cost is serially correlated since, with serially correlated adjustment costs, a current realization of a high cost leads to the expectation of a high cost in the future. In particular, along the intensive margin, an adjusting firm currently realizing a high adjustment cost may choose to make a different amount of price adjustment than an adjusting firm currently realizing a low adjustment cost in order to reduce the frequency in which future adjustment cost is paid. Such difference may depend on the current and expected realizations of other shocks that motivate the price adjustments in the first place.

We focus on one type of such shocks, shocks to nominal money growth, which can be viewed more broadly as incorporating the many other variables that shift nominal aggregate demand. We assume that nominal money growth is exogenous and follows a stationary AR(1) process, with mean μ_M, autoregressive coefficient ρ_M, and a white-noise innovation that has a finite standard deviation σ_{ϵ_M}. The nominal money growth process is assumed to be uncorrelated with (45) and (46), and we denote by $\Phi_M(\Delta \log M'|\Delta \log M)$ the conditional distribution of this process.

3.2. Pricing Decisions and Value Functions

At the beginning of any given period, a firm observes the past values of all variables, the current nominal money growth rate, and its current idiosyncratic price adjustment cost that must be paid in the event of adjusting, but not the current values of other variables. To make its pricing decision, the firm needs to forecast other aggregate variables. It is these forecasts along with the observables, as well as the expectations of their future realizations that are relevant for the firm's pricing decision. To derive the firm's minimal state space, denote by $\pi_o(\Omega)$, $\pi_p(\Omega)$, $\pi_q(\Omega)$, the forecasts of the PPI, the CPI, and the auxiliary CPI, respectively, where Ω denotes the information set concerning the aggregate state upon which the forecasts are based. With these forecasts, and in conjunction with the

observables at hand, the forecasts of other relevant stationary variables can be derived immediately:

$$\hat{m} = m_{-1} \frac{1 + \Delta \log M}{1 + \pi_{\mathrm{p}}(\Omega)}, \tag{47}$$

$$\hat{op} = (op)_{-1} \frac{1 + \pi_{\mathrm{o}}(\Omega)}{1 + \pi_{\mathrm{p}}(\Omega)}, \tag{48}$$

$$\hat{qp} = (qp)_{-1} \frac{1 + \pi_{\mathrm{q}}(\Omega)}{1 + \pi_{\mathrm{p}}(\Omega)}. \tag{49}$$

Note that these stationary variables are either real money balances or relative prices, since the levels of nominal variables are not stationary if there is a trend in nominal money growth, or nominal aggregate expenditure.

With this preparation in place, the minimal current state vector can be expressed as

$$S_{\mathrm{p}} = \{p_{-1}, \psi_{p,j}, \Omega\}, \tag{50}$$

for a retailer, and as

$$S_{\mathrm{o}} = \{o_{-1}, \psi_{o,i}, \Omega\}, \tag{51}$$

for a wholesaler. An adjusting firm chooses a percentage change in its nominal price, $\Delta \log(P_i)$ for an adjusting retailer and $\Delta \log(O_i)$ for an adjusting wholesaler. We assume that the firm does not have enough information to perfectly determine the price level in the current period before adjusting. Therefore, the firm's optimization decision is based on inflation forecasts for the consumer and the producer price inflations that are functions of its information set, Ω.

To form the optimization problem in a dynamic programming framework, denote the value of adjusting its price and paying an adjustment cost and the value of non-adjusting by V^{a} and V^{na} for a firm in the CPI sector, and by U^{a} and U^{na} for a firm in the PPI sector, respectively. The value function for a typical firm in the two sectors can then be expressed as

$$V(S_{\mathrm{p}}) = \max\{V^{\mathrm{a}}, V^{\mathrm{na}}\} \tag{52}$$

and

$$U(S_{\mathrm{o}}) = \max\{U^{\mathrm{a}}, U^{\mathrm{na}}\} \tag{53}$$

respectively, where

$$V^{\mathrm{a}}(S_{\mathrm{p}}) = \max_{\Delta \log P_i} \left\{ \mathbf{E}\left[\Pi_{\mathrm{p,j}}(p, op, m)|\Omega\right] - \psi_{p,j} + \beta \mathbf{E}_{S'_{\mathrm{p}}|S_{\mathrm{p}}}[V(S'_{\mathrm{p}})] \right\} \tag{54}$$

$$S'_{\mathrm{p}} = \{p, \psi'_{p,j}, \Omega'\} \tag{55}$$

$$V^{\mathrm{na}}(S_{\mathrm{p}}) = \mathbf{E}\left[\Pi_{\mathrm{p,j}}\left(\frac{p_{-1}}{1 + \pi_{\mathrm{p}}(\Omega)}, op, m\right)|\Omega\right] + \beta\mathbf{E}_{S'_{\mathrm{p}}|S_{\mathrm{p}}}[V(S'_{\mathrm{p}})] \tag{56}$$

$$S'_{\mathrm{p}} = \left\{\frac{p_{-1}}{1 + \pi_{\mathrm{p}}(\Omega)}, \psi'_{p,j}, \Omega'\right\} \tag{57}$$

and

$$U^{\mathrm{a}}(S_{\mathrm{o}}) = \max_{\Delta\log O_i}\left\{\mathbf{E}\left[\Pi_{\mathrm{o,i}}(o, op, qp, m)|\Omega\right] - \psi_{o,i} + \beta\mathbf{E}_{S'_{\mathrm{o}}|S_{\mathrm{o}}}[U(S'_{\mathrm{o}})]\right\} \tag{58}$$

$$S'_{\mathrm{o}} = \{o, \psi'_{o,i}, \Omega'\} \tag{59}$$

$$U^{\mathrm{na}}(S_{\mathrm{o}}) = \mathbf{E}\left[\Pi_{\mathrm{o,i}}\left(\frac{o_{-1}}{1 + \pi_{\mathrm{o}}(\Omega)}, op, qp, m\right)|\Omega\right] + \beta\mathbf{E}_{S'_{\mathrm{o}}|S_{\mathrm{o}}}[U(S'_{\mathrm{o}})] \tag{60}$$

$$S'_{\mathrm{o}} = \left\{\frac{o_{-1}}{1 + \pi_{\mathrm{o}}(\Omega)}, \psi'_{o,i}, \Omega'\right\} \tag{61}$$

where the first expectation is taken over the distribution of the current-period inflation rates in the supports based on the forecasted probabilities (see Section 4.2 below for detail), and the second expectation is taken over the three exogenous variables using the conditional distributions. In the above recursive formulation of the dynamic programming problems, the value functions are expressed in units of utility.

3.3. Equilibrium

A rational expectations equilibrium consists of two pairs of value-policy functions, $\{V, f\}$ and $\{U, g\}$, and a pair of inflation forecast rules, $\{\pi_{\mathrm{p}}(\Omega), \pi_{\mathrm{o}}(\Omega)\}$, such that (i) given the inflation forecast rules, each pair of the value-policy function solves the respective optimization problem, and (ii) the inflation forecasts match the actual inflations in a simulated economy under the corresponding value-policy functions.

For the numerical solution to be conducted below, it is convenient to derive the frictionless steady state values of real money balances and the producer-consumer price ratio. Solving for the first-order conditions of profits in each sector when prices can be costlessly adjusted yields

$$\bar{m} = \left[\frac{A_p^{\eta_o} A_o(\theta_p - 1)(\theta_o - 1)}{\nu^{\eta_p\eta_o}\eta_p\eta_o\Phi\theta_p\theta_o}\right]^{\frac{1}{\eta_p\eta_o}}, \tag{62}$$

$$\bar{op} = \frac{\nu^{\eta_p \eta_o - \eta_p + 1} \eta_o \Phi \theta_o}{A_p^{\eta_o - 1} A_o (\theta_o - 1)} \bar{m}^{\eta_p \eta_o - \eta_p + 1}. \tag{63}$$

We turn now to discussing our data, estimation strategy, and solution method.

4. DATA, ESTIMATION STRATEGY, AND SOLUTION METHOD

We assume that each model period corresponds to a quarter of a year in the data. The first step in our numerical exercise is to assign values to the model's structure parameters. Some of the parameters can be calibrated by matching the model's steady-state values to the historical averages of the actual time series in the quarterly data or by relying on the embodied microeconomic evidence. These include β, which we set to 0.99, so that the steady-state annualized real interest rate is equal to 4 percent, as in the standard business cycle literature. The parameter Φ is chosen so that, in the steady state, a household devotes 1/3 of its time endowment to market activity. We set $\theta_o = \theta_p = 7.5$, as in Section 2, based on the empirical studies by Rotemberg (1996) and Rotemberg and Woodford (1997). Finally, we choose $\mu_M = 0.01$, $\rho_M = 0.6$, and $\sigma_{\epsilon_M} = 0.0092$, based on an autoregression on quarterly M1 data in the postwar U.S. economy.

The difficult job is to estimate the parameters governing the price adjustment cost processes. We will use an indirect inference method proposed by Gourieroux, Monfort, and Renault (1993) to do so. On the other hand, the interlock between idiosyncratic and serially correlated price adjustment costs and sectoral interactions renders our model a high degree of heterogeneity that makes equilibrium price distribution a high dimensional object. Hence we will solve the model using a nonlinear solution method similar to the ones proposed by Haan (1996, 1997) and Krusell and Smith (1997, 1998). Our solution method is embodied in the inference procedure to produce a simulation-based computation algorithm.

We describe now the data and criterion function to be used for the indirect inference.

4.1. Data and Criterion Function

In searching for a set of auxiliary parameters and a criterion function that are closely related to the parameters governing the price adjustment cost processes, it is natural to consider the hazard function which prescribes the conditional probability of a price change.

For this purpose, we appeal to the Federal Reserve Bank of Kansas City's Survey of Tenth District Manufacturers. The survey was conducted quarterly from the fourth quarter of 1995 through the second quarter of 2001 and monthly thereafter. In each quarterly survey, manufacturers were asked to

state whether their prices received for finished product and their prices paid
for raw materials had decreased, increased or remained unchanged in any
one of the previous three months. In each monthly survey, manufacturers
are asked to state whether the prices they received for finished product
and the prices they paid for raw materials have decreased, increased or
remained unchanged relative to prices one month and one year ago. To
be consistent with the quarterly frequency of our theoretical model, we
use these monthly surveys to construct a quarterly series, from 2001:Q3 to
2005:Q1. There are 316 participating firms during these 14 quarters, but
only 45 of them are continuing participants throughout the entire period.
We include these 45 firms into our sample to create a set of panel data.

Based on the optimization model that we have specified for a firm, the
probability of price adjustment is a function of lagged consumer price in-
flation ($\pi_{p,t-1}$), lagged producer price inflation ($\pi_{o,t-1}$), the lagged level of
real money balances (mp_{t-1}), the inverse of the lagged level of the markup
of retail over wholesale prices (op_{t-1}), the current growth rate of the money
supply ($\Delta \log M_t$), and the firm's lagged relative price. Since we as econo-
metricians do not observe the relative prices charged by the firms, we proxy
for this state variable using a combination of four variables that are func-
tions of their pricing decisions in the past. These four variables are the
number of quarters since firm i last changed its price, which occurred in
period $\tilde{t}$, ($T_{i,t,\tilde{t}}$); the cumulative consumer price inflation since the previous
change ($\pi_{p,i,t,\tilde{t}}$), the cumulative producer price inflation since the previous
change ($\pi_{o,i,t,\tilde{t}}$), and the cumulative percentage change in demand for the
sector's goods since the previous price change ($X_{i,t,\tilde{t}}$).

Thus our model implied hazard function can be approximated by a linear
probability model that involves only observables from the above data set:

$$\Pr(y_{i,t} = 1) = \alpha_0 + \alpha_1 \pi_{p,t-1} + \alpha_2 \pi_{o,t-1} + \alpha_3 mp_{t-1} + \alpha_4 op_{t-1} + \alpha_5 \Delta \log M_t$$
$$+ \alpha_6 T_{i,t,\tilde{t}} + \alpha_7 \pi_{p,i,t,\tilde{t}} + \alpha_8 \pi_{o,i,t,\tilde{t}} + \alpha_9 X_{i,t,\tilde{t}} + \epsilon_{i,t}, \tag{64}$$

where $y_{i,t}$ equals 1 if firm i adjusted its price in quarter t, and 0 otherwise.
It is worth noting that, if the adjustment cost process is serially correlated
across time, then the unobserved adjustment cost will be correlated with
the cumulative regressors, which are functions of the firm's pricing decisions
in the past.

Denote by α the auxiliary parameters, Y_T the dependent variables, and
X_T the regressors in (64). We specify a criterion function $Q_T(Y_T, X_T, \alpha)$
as the negative of the sum of squared errors of the linear probability model.
Let $\hat{\alpha}_T$ be the solution to the maximization of the criterion function

$$\hat{\alpha}_T = \text{argmax}_\alpha \, Q_T(Y_T, X_T, \alpha). \tag{65}$$

This solution provides information on the specification of the price adjustment cost process. The estimated coefficients on the cumulative variables are particularly useful for identifying the persistence parameter. Our survey data do not contain information on the magnitude of price changes, which otherwise would also provide useful information on the persistence of the process. Thus in evaluating our model's empirical performance, we also draw on Bils and Klenow (2003) and Klenow and Krystov (2003), who provide information on the frequency and size of price changes in the retail sector of the United States. We will also look at the evidence on the size of the costs reported by Levy, et al. (1997) and Zbaracki, et al. (2004).

4.2. Fixed-Point Inflation Forecast and Value Function Iteration

Our computational procedure starts with a given set of parameters governing the price adjustment cost processes, δ. Given the complexity of equilibrium distributions of firms across prices, the aggregate state vector in our model is a high dimension object, which renders standard nonlinear solution methods impractical. We approximate the aggregate state vector by a lower dimension object and solve the model using a modified nonlinear solution method. A key component of this method involves stochastic forecast rules for sector inflation rates and sector relative prices that capture the perceived laws of motion for some low-order moments of the price distributions that firms will use in solving their dynamic optimization problems. In a rational expectations equilibrium, the forecast rules must be self-validating, in that their predictions about the moments of the aggregate state must be consistent with the aggregations of individual firms' solutions to their optimization problems obtained under the perceived laws of motion for the moments.[11]

We assume that firms approximate the aggregate state vector by the first moments of the distributions of prices and use some linear rules to forecast these moments. Essentially, this boils down to making rational forecasts on the consumer and producer price inflations based on a limited information set.[12] Firms are assumed to base their forecasts on a limited set

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[11] An alternative approach is to take the forecast equations as the criterion functions, given that all variables involved are observable. The auxiliary parameters can then be estimated by running the forecast regressions on actual data. Equilibrium will then be solved by choosing structural parameters such that the estimated forecast equations match closely the same regressions applied to simulated data. One drawback of this approach is that these criterion functions may provide weak identification for the price adjustment cost process.

[12] A wholesaler needs to forecast the auxiliary CPI inflation as well in order to solve its optimization problem. But, as we show in Section 2, up to a first order, the auxiliary CPI and the CPI are identical, and the differences in their second- or higher-order terms only make a fourth or higher-order difference for equilibrium dynamics. This is why the two price indexes track each other closely, and (as a result) so do real GDP C and the linear

of aggregate variables: $\Omega = \{\pi_{p,-1}, \pi_{o,-1}, m_{-1}, op_{-1}, \Delta \log M\}$. Additional variables, including higher moments of the underlying distribution of prices, are added subsequently to see if they help improve the forecasts. Expanding the information set comes with a cost as each variable added becomes a part of the overall state vector. Adding too many variables proves to bring on the curse of dimensionality quickly.

The forecast equations for the consumer and producer price inflations are specified as

$$
\begin{aligned}
\pi_{p,t} &= a_0 + a_1 \pi_{p,t-1} + a_2 \pi_{o,t-1} + a_3 \log m_{t-1} \\
&+ a_4 \log op_{t-1} + a_5 \Delta \log M_t + \epsilon_{p,t}, \tag{66} \\
\pi_{o,t} &= b_0 + b_1 \pi_{p,t-1} + b_2 \pi_{o,t-1} + b_3 \log m_{t-1} \\
&+ b_4 \log op_{t-1} + b_5 \Delta \log M_t + \epsilon_{o,t}. \tag{67}
\end{aligned}
$$

Given these forecast equations along with equations (47) and (48), we can express real money balances and the ratio of producer to consumer prices as a function of the state variables,

$$
\begin{aligned}
\log m_t &= -a_0 - a_1 \pi_{p,t-1} - a_2 \pi_{o,t-1} + (1 - a_3) \log m_{t-1} \\
&- a_4 \log op_{t-1} + (1 - a_5) \Delta \log M_t, \tag{68} \\
\log op_t &= (b_0 - a_0) + (b_1 - a_1) \pi_{p,t-1} + (b_2 - a_2) \pi_{o,t-1} + (b_3 - a_3) \log m_{t-1} \\
&+ (1 + b_4 - a_4) \log op_{t-1} + (b_5 - a_5) \Delta \log M_t. \tag{69}
\end{aligned}
$$

These four equations along with the exogenous money growth equation allow firms to compute the expectations of the future period's state variables conditional on today's realizations of state variables. The firms can use the frictionless steady state values (62) and (63) to solve for the intercept terms in the forecast equations and to guide the specification of the state space for these variables. Using these values and the fact that in the steady state consumer and producer price inflation rates equal the growth rate of nominal money supply, we can solve for the intercept parameters as

$$
\begin{aligned}
a_o &= (1 - a_1 - a_2 - a_5)\mu_M - a_3 \log \bar{m} - a_4 \log \bar{o}p, \tag{70} \\
b_o &= (1 - b_1 - b_2 - b_5)\mu_M - b_3 \log \bar{m} - b_4 \log \bar{o}p. \tag{71}
\end{aligned}
$$

---

aggregate $(1/n) \sum_{j=1}^{n} Y_j$ (see, also, Dotsey, et al., 1999). Using this approximation, we can set Q equal to P in the demand function (6) and in the contemporaneous profit function (15). In consequence, we can drop (49) in its entirety, the terms $(qp)_{-1}$ and $\pi_q(\Omega)$ out of (51), and the terms $\hat{qp}$ and $\pi_q(\Omega')$ out of (58)-(61). After this simplification, the only additional forecast compared to the case with a single-sector model is the forecast of the PPI inflation.

While the intercept terms for the two forecast equations can be derived analytically based on the frictionless steady-state values of the state variables, we need to assign an initial value to each of the other ten forecast parameters to get the algorithm started, and we need to search for a fixed point for these parameters.

Given the initial value of the forecast parameters, each firm's optimization problem can be solved using a value function iteration algorithm built on nonlinear approximations of the value functions, where each of the state variables is discretized onto a finite grid and the exogenous processes is converted into discrete Markov processes following Tauchen (1986). The solution is a fixed point of a contraction mapping. We use the resultant decision rules to simulate the behaviors of a large number of firms over a large number of periods, and run the same forecast regressions on the stationary portion of the simulated data. If the goodness of fit of these regressions is high and the updated forecast parameter values match the initial ones, we stop. Otherwise, we go back to the beginning of the loop and repeat the procedure, this time with the previous forecast parameter values replaced with the updated ones. Repeat this loop until we have a fixed point for the forecast parameters and the goodness of fit of the last regressions is satisfactory.

4.3. Indirect Inference

From each of the simulated datasets generated in the last round of the contraction mapping in 4.2 in which the fixed point for the forecast parameters is obtained, extract a stationary portion of the counterparts of those variables involved in the linear probability model with an equal size, denoted as $(Y_T^s(\delta), X_T^s(\delta)), s = 1, 2, \ldots, S$. For each s, we choose α to maximize the criterion function, with the observed data replaced with the simulated data,

$$\hat{\alpha}_T^s(\delta) = \text{argmax}_\alpha \ Q_T(Y_T^s(\delta), X_T^s(\delta), \alpha), s = 1, 2, \ldots, S. \qquad (72)$$

Our simulation-based estimator of δ is the solution to the following minimization problem,

$$\hat{\delta} = \text{argmin}_\delta \left[\hat{\alpha}_T - \frac{1}{S} \sum_{s=1}^{S} \hat{\alpha}_T^s(\delta) \right]' \hat{\Omega}_T \left[\hat{\alpha}_T - \frac{1}{S} \sum_{s=1}^{S} \hat{\alpha}_T^s(\delta) \right], \qquad (73)$$

where $\hat{\Omega}_T = [T * \text{VAR}(\hat{\alpha}_T)]^{-1}$ is a positive definite matrix that converges asymptotically to a deterministic positive definite matrix Ω.

For a fixed S, the indirect estimator is asymptotically normal where

$$\sqrt{T(\hat{\delta} - \delta_0)} \to \mathrm{N}(0, W(S, \Omega)) \quad \text{in distribution} \tag{74}$$

as $T \to \infty$, where

$$W(S, \Omega) = \left(1 + \frac{1}{S}\right) \left\{ \frac{\partial^2 Q_\infty}{\partial \alpha \partial \delta'} \left[\left(\frac{\partial Q_\infty}{\partial \alpha}\right) \left(\frac{\partial Q_\infty}{\partial \alpha}\right)'\right]^{-1} \frac{\partial^2 Q_\infty}{\partial \alpha \partial \delta'} \right\}^{-1} . \tag{75}$$

The power of this inference depends on the specification of the criterion function and the auxiliary parameters to be estimated, similar to the significance of selecting moments in GMM. The precision of the estimates, measured by the asymptotic variance above, depends on the sensitivity of the auxiliary parameters to movements in the structural parameters through $\frac{\partial^2 Q_\infty}{\partial \alpha \partial \delta'}$. If the sensitivity is low, the derivative will be near zero, indicating a high variance for the structure parameters.

The results of this section is collected in Table 1.

TABLE 1.

Probability Regression, Fixed-Point Forecast, and Price-Adjustment Cost

α_0	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	R^2
-10	0.7	18	0.98	10.8	11	0.006	7.2	-0.6	1.9	**0.55**
(1.35)	(1.25)	(1.27)	(0.68)	(1.45)	(0.58)	(0.0006)	(0.85)	(0.85)	(0.13)	
a_1	a_2	a_3	a_4	a_5	b_1	b_2	b_3	b_4	b_5	R^2
0.362	-0.032	0.212	0.426	0.738	-0.007	0.257	0.394	-0.455	0.759	**0.67**
(0.078)	(0.347)	(0.164)	(0.138)	(0.265)	(0.146)	(0.074)	(0.262)	(0.155)	(0.168)	
μ_p	ρ_p	σ_p	μ_o	ρ_o	σ_o	χ^2				
0.0005	0.5	0.008	0.003	0.7	0.008	**6.6**				
(0.002)	(0.22)	(0.0031)	(0.011)	(0.34)	(0.0022)					

5. SIMULATION RESULTS FOR THE DYNAMIC ECONOMY

We evaluate the performance of our dynamic model based on two sets of empirical evidence. The microeconomic features of the data that we try to match concern the the frequencies of price changes in U.S. retail and manufacturing sectors. Bils and Klenow (2003) and Klenow and Krystov (2003) find that the life of the retail price of a typical good lasts about 4.3 to 5.5 months, depending on whether to count temporary sales, and the monthly frequency of retail price changes is about 0.2, implying a quarterly

adjustment frequency of about 0.49.[13] Our analysis of the Federal Reserve Bank of Kansas City's Survey of Tenth District Manufacturers indicates that the quarterly frequency of price changes for these manufacturers is about 0.37.[14]

Next, we examine the ability of our dynamic model in accounting for the aggregate features of the data. In particular, we will look at two sets of moments for variables of interest: moments that measure volatility (standard deviation) and moments that measure persistence (autocorrelation). The macro data are for the period of Q1:75 to Q3:05 for the United States, which are downloaded from Haver DLXVG3. All data are seasonally adjusted. While the price and inflation data are raw, the output data are HP-filtered. Finally, we generate stochastic impulse response functions to further examine the nonlinearity and asymmetry in the responses of prices and output to monetary policy shocks. In what follows, we assess our model's empirical performance from these three perspectives in sequel.

5.1. Frequency of Price Changes

The upper panel of Table 2 reveals that our dynamic model can almost replicate the observed frequencies of price changes in the retail and manufacturing sectors of the United States. The model implies an average frequency of price adjustment of 0.43 for retailers and 0.35 for manufacturers. These numbers match 88% and 95% of the observed average frequencies of price changes in the two sectors of the U.S. economy.

5.2. Moments

The middle panel of Table 2 illustrates that our dynamic model can come really close to generating the observed volatilities in sector inflation rates. The standard deviation of the CPI inflation rate is 0.81% in the model, which is only 6.6 percentage points above the 0.76% standard deviation of the CPI inflation rate in the data. The model produces a standard deviation of the PPI inflation rate of 0.96%, which is only 6.25 percentage points below the 1.02% standard deviation of the PPI inflation rate in the data. Clearly, the PPI inflation rate is more volatile than the CPI inflation rate in the model, as in the data. The model can also account for almost 20% of the observed volatility in real output.

The lower panel of Table 2 shows that our dynamic model can also account for much of the observed persistence in sector inflation rates and real output. The model generates a first-order autocorrelation of 0.71 for

---

[13]To convert the monthly frequency into a quarterly one, we assume implicitly a flat and independent monthly hazard of not adjusting.

[14]The way we construct the quarterly series does not distinguish cases where a firm reports a price change in only one of the three months within that quarter, in two of the three months, or in all of the three months — they are all counted as one price change in that quarter. Thus the hazard we constructed is likely to be downward biased.

TABLE 2.

Business Cycle Statistics: Model Versus Data

		Two-Sector Model		One-Sector Model	
Statistics	Data	Persistent Cost	IID Cost	Persistent Cost	IID Cost
Frequencies of					
Price Changes					
Retail	0.49	0.43	0.59	0.67	0.76
Manufacturing	0.37	0.35	0.47		
Standard Deviations (%)					
CPI	0.76	0.81	1.17	1.42	1.57
PPI	1.02	0.96	1.23		
GDP	3.03	0.59	0.35	0.28	0.15
Autocorrelations					
CPI	0.80	0.71	0.47	0.38	0.24
PPI	0.64	0.62	0.41		
GDP	0.88	0.82	0.51	0.47	0.31

Notes: Actual data are from Q1:75 to Q3:05 for the United States. Both actual and artificial output data are HP-filtered. Statistics are computed based on simulated series and are averaged over 300 simulations of 100 periods each. The first 30 observations are cut off to remove the impact of initial conditions.

the CPI inflation rate, 0.62 for the PPI inflation rate, and 0.82 for real GDP. The corresponding numbers in the data are 0.80, 0.64, and 0.88, respectively. Hence the model is able to match 89%, 97%, and 93% of the observed persistence in the two sector inflation rates and real output. Clearly, the PPI inflation rate is less persistent than the CPI inflation rate in the model, as in the data.

5.3.   Impulse Response Functions

To further examine the dynamic property of our model's equilibrium, we generate the impulse responses of variables of interest following positive and negative money growth shocks. With the high degree of heterogeneity in the model, the price distribution at the point in time when a shock hits the economy matters a lot for the effect of the shock. To capture such effect, we generate the impulse responses in the following way.

We first simulate the model for a long period of time and generate a time series of each aggregate variable of interest. Keeping this stochastic dynamic equilibrium as a benchmark, we next shock this baseline equilibrium at different points in time, but only one at a time, with the same idiosyncratic price adjustment costs as in the baseline equilibrium, and follow the equilibrium dynamics for a long period to generate a time series

of each aggregate variable (starting from the point when the shock is triggered) for each one of such modified simulations. The difference between the time series in a modified simulation and the baseline time series is defined as an impulse response for this simulation. We then construct the mean impulse response by taking the average of these impulse responses across all modified simulations. We can also compute the standard deviation of these impulse responses and use it to construct a confidence band for a given set of modified simulations.

We find that for positive and negative shocks of a size smaller than one standard deviation in the innovation of the money growth rate, the responses of the economy are almost symmetric. But, for shocks with a larger size, we observe considerably asymmetric responses.

FIG. 6. Mean impulse responses of key variables to 1.25 percent positive and negative shocks to the growth rate of money supply.
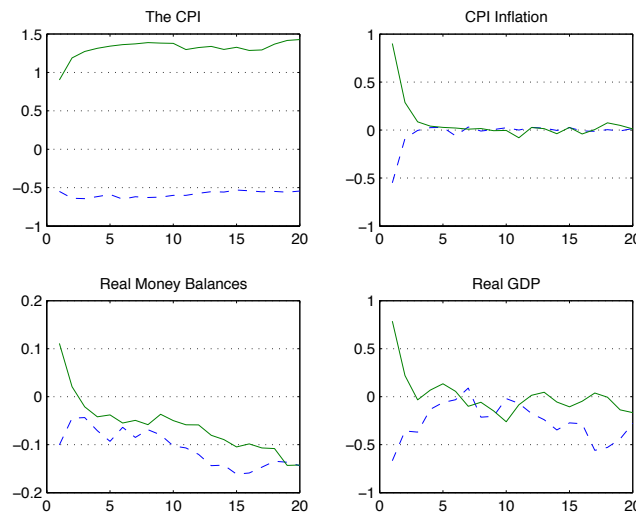
Dashed line: negative shocks; solid line: positive shocks.

Figure 6 displays the mean impulse responses of several key variables, the CPI, CPI inflation rate, real money balances, and real GDP, to $+1.25\%$ and -1.25% shocks to the growth rate of money supply. The dashed line represents responses to negative shocks and the solid line to positive shocks. As can be seen from the figure, there are significant asymmetries in these responses. Price and inflation respond more aggressively to positive shocks than to negative shocks. While the asymmetry in the response of inflation occurs mostly within the the first year after the first shock is triggered, the asymmetry in the response of price can last for a long period of time.

These coupled with the asymmetric response of real money balances suggest a front-loading behavior of firms, which leads to an overshooting in the CPI following positive shocks. The response of real GDP is also asymmetric. Although the initial response is of about the same size following a positive and a negative shock, this impact response dies more quickly following a positive shock than following a negative shock. While the positive impact almost vanishes entirely after three quarters, the negative impact can remain for a long period of time.

6. SENSITIVITY ANALYSIS

In this section, we conduct sensitivity analysis to get a sense about the relative importance of the chain-of-production structure and the persistence in price adjustment costs, and of their interaction, for generating the results in our baseline model.

The upper panel of Table 2 illustrates that there are much more frequent price adjustments in three alternative models, where either the two-stage feature of the production chain, or the persistence feature of the price adjustment cost process, or both, is abstracted from. We note that the average frequency of price changes in our model is an equilibrium property and must be determined endogenously along with other endogenous variables. This allows us to use this dimension to assess the fit of our model to this microeconomic feature of the data.

There is also a stark contrast between the simulated volatilities in our baseline model and in the three alternative models, as is illustrated by the middle panel of Table 2. The baseline model can account for almost the same volatility of CPI inflation, 94% of the volatility of PPI inflation, and almost 20% of the volatility of real GDP. In contrast, the best of the three alternative models can explain only 11% of the output volatility. In particular, the one-sector model with identically and independently distributed price adjustment cost can hardly explain 5% of the observed output volatility. This is not surprising, given that prices respond much more dramatically to monetary shocks and volatilities of inflation rates are much higher in these alternative models than in the baseline model and the data.

The three alternative models also fair much worse than the baseline model in explaining the observed persistence in inflations rates and in real output, as illustrated by the lower panel of Table 2. The baseline model can explain 89% of the persistence in the CPI inflation, 97% of the persistence in the PPI inflation, and 93% of the persistence in real GDP. In contrast, the three alternative models, especially the one-sector model with identically and independently distributed price adjustment cost process, fall much short in explaining the observed persistence in the data.

In sum, our sensitivity analysis shows that both the chain-of-production structure and the persistence in price adjustment cost process play crucial roles in generating the results in our baseline model. A more careful examination of Table 2 suggests that the production chain plays a slightly more significant role than the persistence in adjustment costs, and that the interaction of these two features also make a contribution.

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