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Liquidity, Efficiency and the 2007-2008 Global Financial Crisis.

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We focus on the relationship between liquidity and market efficiency, and investigate the behavior of three stock market indexes (S&P500, Nasdaq and DAX) before, during and after the global financial crisis. We find that, once accounted for the scale, the two attributes are strongly related and empirical evidence is provided that, when market efficiency is measured through the pointwise regularity of the price, it is a better forecaster of illiquidity than vice versa. We also find that the variation of the illiquidity premium declined to zero during the unconventional interventions that the Federal Reserve launched to face the credit crunch.

Key Words: :Liquidity; Efficiency; Pointwise Regularity; Global Financial Crisis.

JEL Classification Numbers: C22, C58, F37, G01.

1. INTRODUCTION.

Liquidity and efficiency are two of the cornerstones of modern financial theory. Their conceptual depth and their intrinsic difficulty to be measured represent one of the challenges, which has produced even controversial results. Separately, the two topics are extensively studied in literature, in terms of both market microstructure and econometric modeling and estimation.

As for the first strand of research, the contributions have followed three main approaches: a) the analysis of the relation between tick size and the decrease in bid-ask spreads (Bessembinder (2003), Chordia et al. (2005)); b) the return/order flow relation, in connection of specific events or for short spans of time (Lee (1992), Chan and Fong (2000), Hasbrouck and Seppi (2001)); c) the link between liquidity and stock returns in terms of premium demanded for illiquid stocks (Amihud and Mendelson (1986),

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Jones (2002), Amihud (2002), Pastor and Stambaugh (2003), Acharya and Pedersen (2005)).

As for the second line of research, since the seminal work of Fama (1970) a huge number of empirical studies have tried to address the question whether the (discounted) stock prices follow a martingale. It is nearly impossible to summarize in few lines the extent of the literature on efficient markets; globally speaking, the contributions can be grouped into two broad sets: those that test for the strong and semi-strong efficiency, by analyzing how the information flow impacts on the formation of stock prices (e.g. Lo (2004) and references therein); and those that focus on the weak efficiency, by estimating linear and non linear dependence in stock returns (Willinger et al. (1999); Mitra et al. (2017)).

Although the literature on efficient markets is almost overwhelming, the link between liquidity and efficiency is examined in a relatively few number of works, and the question whether changes in liquidity are related to variations in the degree of efficiency is still far from being completely addressed. Chordia et al. (2008) and Chung and Hrazdil (2010) point out that, in Chordia's words, "*In an efficient market, return predictability from past information should be short-lived and minimal. Given th evidence that such predictability does exist in the short run, understanding its time variation and its relation to other financial market attributes, such as liquidity, are of fundamental importance.*". Nonetheless, more than on efficiency, many contributions have focused on the binomial illiquidity-volatility and, even if the debate is still open, some evidence indicates the former as a predictor of the latter (see, e.g., Stoll (1978a), Stoll (1978b), Stoll (2000), Amihud and Mendelson (1989b), Bao and Pan (2013)). The issue here is to clarify whether and to which extent a link exists between volatility and efficiency. This strong relationship will be established in a natural way within the fractional model that will be discussed in Section 3.

Since prices are strongly related to trading volumes and liquidity (Shleifer and Vishny (1992), Stein (1995), and Pulvino (1998)), the relation between liquidity and efficiency is being increasingly studied in recent years. Indeed, they both capture the market sentiment: Baker and Stein (2004) claims that in an unusually liquid market pricing is dominated by irrational investors, who tend to underreact to the information embodied in either order flow or equity issues. Liquidity is therefore a clue that irrational investors have a positive sentiment, which turns into expected returns abnormally low. Furthermore, the 2007-2008 global financial crisis has weakened the idea of a ubiquitous market efficiency, due to the occurrence of extreme events which take place with frequencies and orders of magnitude several times higher than those foreseen by the current paradigm and not explicable in terms of regime-switching. Markets imperfections of recent years have been acknowledged by financial institutions and have motivated large interventions for their correction (e.g. QE or LTRO programs).

Starting from these motivations, we connect the two notions by using a widely employed measure of illiquidity (ILLIQ, introduced by Amihud (2002)) and a topology-based measure of efficiency.

We find that: (a) the two measures are strongly related, and the measure of efficiency is a better forecaster of the measure of illiquidity than vice versa; (b) their correlation increases when markets experience inefficient periods; (c) the two measure reveal that during the Quantitative Easing Programs, the variation of the premium for the illiquidity risk decreases to zero.

These findings corroborate the results in Rogers et al. (2014), who — analyzing bond yields, exchange rates and stock prices from the Euro area, Japan, United Kingdom and the United States — conclude that unconventional monetary policy has proven effective in reducing the term premia and has had some important cross-country spillovers.

The paper is organized as follows: in section 2 the measure of illiquidity is described, along with a concise overview of the main approaches followed to quantify the notion; in section 3 the measure of market efficiency is introduced and its main properties are discussed; section 4 is devoted to the empirical analysis on three main stock indices (S&P500, Nasdaq and DAX) and section 5 concludes.

2. MEASURING ILLIQUIDITY

Despite liquidity represents one of the most debated topic in finance, neither a unique definition nor a single measure have been unanimously accepted (Baker (1996)). In general terms, liquidity is the capability to trade a security easily or, in other words, the ability to buy or sell large quantities of an asset quickly without impacting on the price in a substantial way. According to Sarr and Libek (2002)) and without any claim for the following classification to be thorough, liquidity measures can be broadly grouped into four categories:

1. transaction cost measures, whose aim is to capture the explicit (order processing and taxes) or the implicit (execution) cost of trading financial assets and trading frictions in secondary markets. The measure traditionally used is the *bid-ask spread* and its extensions (Amihud and Mendelson (1986), Amihud (2002), Demesetz (1968)) or the *relative (or inside) spread* (Acker and Tonks (2002), Chordia et al. (2001b), Clark and Kassimatis (2014));

2. volume-based measures, that focus on depth and breadth[1]. Besides the simple *transaction volume* (measured by the total value of trades over a given time interval) and the *turnover* (scaling transaction volumes to the size of the assets (Dennis and Strickland (2003))), the *Hui-Heubel Liquidity Ratio* is one of the most common volume-based measures (Hui and Heubel (1984));

3. price-based measures, that capture the resilience dimension, that is the capability of a market to quickly correct order imbalances, which tend to move prices away from what is predicted by fundamentals. The well-known illiquidity measure introduced by Amihud (2002) ($ILLIQ$, from now on) and the *Market Efficient Coefficient* (MEC), also named *Variance ratio*, proposed by Hasbrouck and Schwartz (1988), are two widely used indicators;

4. market-impact measures, that try to isolate the price movements strictly caused by the degree of liquidity as opposite to those resulting from other factors, such as general market conditions or arrival of new information. An example is given by the *Market-Adjusted liquidity model*, proposed by Hui and Heubel (1984), which attempt to capture the intrinsic illiquidity of a stock using a CAPM-based approach.

As a proxy of illiquidity, in this analysis we will use the measure proposed by Amihud. Even though other finer microstructure measures (requiring a lot of data sometimes not available) are possible, $ILLIQ$ remains a benchmark in the field of liquidity, due to its computational immediacy and parsimony.

The measure is the daily ratio between the absolute stock return and its dollar volume, averaged over some period. Analytically, denoted by R_{itd} the return for stock i, on day d of period t (typically, t is one trading month or one year), by $DVOL_{itd}$ the corresponding dollar trading volume and by D_{it} the number of trading days for stock i in period t, the measure is defined as:

$$ILLIQ_{it} = \frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \frac{|R_{itd}|}{DVOL_{itd}}. \tag{1}$$

Intuitively, $ILLIQ_{it}$ can be interpreted as the daily price response associated with one dollar of trading volume. In our analysis, we will calculate (1) with respect to the whole market, which means assuming the stock i as the market index.

By relating the excess return and $ILLIQ_{it}$, Amihud investigates how the illiquidity represents a source of risk and, as such, investors require

[1] "Depth" refers to a market characterized by a large amount of orders (actually or easily uncovered by potential sellers or buyers); "breadth" indicates that the orders are both numerous and large in volume with minimal impact on prices.

to be compensated for holding illiquid assets. He finds that the expected stock return for a period is an increasing function of the illiquidity forecast in previous period, and that an unexpected rise in illiquidity in a certain period lowers the stock prices in the same period, producing a negative liquidity relationship. These findings confirm previous results due to Amihud and Mendelson (1986) and Anihud and Mendelson (1989a): using data on stocks traded in the NYSE between 1960 and 1980, they provided evidence that across stock portfolios sorted on illiquidity and risk, average stock return is an increasing function of the stock bid-ask spread (after controlling for systematic and unsystematic risk)[2]. Other contributions support the hypothesis of a relationship between expected return and illiquidity. In an intraday analysis, Brennan and Subrahmanyam (1996) use the Kyle's measure of illiquidity[3] and find that both the price impact and the fixed cost are priced. Pastor and Stambaugh (2003) propose that asset prices should reflect a premium for the sensitivity of stock returns to market-wide liquidity; they show that stocks with greater exposure to market liquidity shocks (i.e. with greater systematic liquidity risk) should earn higher returns. Acharya and Pedersen (2005) show that the persistence of liquidity implies that liquidity predicts future returns and co-moves with contemporaneous returns: a high illiquidity today predicts a high expected illiquidity next period, implying a high required return. These findings will be confirmed by our analysis.

International evidence about the association between expected returns and illiquidity are also traced in Amihud et al. (2015) using data from 45 countries divided into 26 developed and 19 emerging markets. They show both that illiquid stocks outperform liquid stocks in nearly all countries and that there is a cross-country commonality in the illiquidity return premium (after controlling for the six global and regional Fama and French (1993) return factors). This approach differs from the commonality in the level of shocks to illiquidity found for the U.S. market by Chordia et al. (2001a), Hasbrouck and Seppi (2001), Huberman and Halka (2001) and for other countries by Brockman et al. (2009) and Karolyi et al. (2012).

[2]Furthermore, the authors also show the existence of liquidity clientele, by which the high-frequency traders choose more liquid assets whereas the low-frequency investors prefer less liquid assets because they can depreciate the transaction costs over longer periods.

[3]In the model of Kyle (1985), λ is a measure of adverse selection, essentially defined as the slope coefficient in a regression relating the price change to trade-by-trade signed order flow. Brennan and Subrahmanyam (1996) use Kyle's λ as a proxy for the trading cost, since they argue that adverse selection is "a primary cause of illiquidity".

3. MEASURING MARKET EFFICIENCY

The idea relies on the assumption that the martingales have a well defined signature in terms of pointwise irregularity of their sample paths, which can be quantified by means of their Hölder exponent.

The transposition of the Efficient Market Hypothesis (EMH) in terms of expected values of discounted payoffs dates back to Fama (1970) and it is "*(...) the unavoidable price one must pay to give the theory of efficient markets empirical content*". In this framework, given the filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F})_t, \mathbb{P})$, efficiency requires that an asset whose price at time t is S_t and whose payoff at time $T > t$ is X_T, fulfills the following relation

$$S_t = \mathbb{E}_t\left(Y_T X_T\right), \tag{2}$$

where Y_T is the stochastic discount factor[4] between t and T. In terms of the asset's gross return $R_T = \frac{X_T}{S_t} = 1 + r_T$, equality (2) becomes

$$1 = \mathbb{E}_t\left(Y_T \frac{X_T}{S_t}\right) = \mathbb{E}_t\left(Y_T R_T\right). \tag{3}$$

A habit in financial literature is to test the EMH by checking the validity of the random walk model through relation (3) or its variants. Even ignoring the so called joint hypothesis problem[5], since $\mathbb{E}_t\left(Y_T R_T\right) = \mathbb{E}_t\left(Y_T\right)\mathbb{E}_t\left(R_T\right) + Cov_t\left(Y_T R_T\right)$, one has

$$\mathbb{E}_t\left(R_T\right) = \frac{1 - Cov_t\left(Y_T R_T\right)}{\mathbb{E}_t\left(Y_T\right)}, \tag{4}$$

stating that the sole predictability of returns, that is the controversial failure of the random walk model, does not prove market inefficiency. Indeed, it suffices the expected conditional return to comply with (4) to save both efficiency and predictability. Likewise, it could also be not so easy to be aware and exploit the predictability following a lack of efficiency; for example, the deviates from the martingale model should be promptly understood by those market participants able to arbitrage the inefficiency away, or the transaction costs could make the trading strategies not profitable enough (see e.g. Guasoni (2006) and Guasoni et al. (2008)).

This suggests to characterize the martingale behavior by means of its path regularity. Indeed, the signature of a Brownian martingale is that

[4]Remind that $Y = (Y_t)$ is a stochastic discount factor (or pricing kernel) if: (a) $\mathbb{P}(Y_t = 1, Y_T > 0) = 1$, and (b) $S_t = \mathbb{E}[Y_T X_T]$.

[5]Any test of market efficiency involves testing at the same time an equilibrium asset pricing model, with the consequence that any anomalous evidence of abnormal market returns can be ascribed to market inefficiency, improper asset pricing model or both.

the *Hölder pointwise regularity* of its trajectories is $1/2$ almost surely[6]. Following Ayache (2013), given the stochastic process $X(t,\omega)$ with a.s. continuous and not differentiable trajectories over the real line $\mathbb{R}$, the local Hölder regularity of the trajectory $t \mapsto X(t,\omega)$ with respect to some fixed point t can be measured through the *pointwise* Hölder exponent, defined as

$$\alpha_X(t,\omega) = \sup\left\{\alpha \geq 0 : \limsup_{h\to 0} \frac{|X(t+h,\omega) - X(t,\omega)|}{|h|^\alpha} = 0\right\}. \quad (5)$$

For certain classes of stochastic processes, remarkably for Gaussian processes, by virtue of zero-one law, there exists a non random quantity $a_X(t)$ such that $\mathbb{P}(a_X(t) = \alpha_X(t,\omega)) = 1$ (see e.g. Ayache (2013)). In addition, when $X(t,\omega)$ is a semimartingale (e.g. Brownian motion), $\alpha_X = \frac{1}{2}$; values different from $\frac{1}{2}$ describe non-Markovian processes, whose smoothness is too high, when $\alpha_X \in \left(\frac{1}{2}, 1\right)$, or too low, when $\alpha_X \in \left(0, \frac{1}{2}\right)$, to satisfy the martingale property. In particular, the quadratic variation of the process can be proven to be zero, if $\alpha_X > \frac{1}{2}$ and infinite, if $\alpha_X < \frac{1}{2}$. Two aspects deserve to be discussed more carefully:

a) Intuitively, the larger the pointwise regularity $\alpha_X(t)$ the lower the volatility process $\sigma_X(t)$. Being an efficient market a semimartingale characterized by $\alpha_X(t) = \frac{1}{2}$, the relation with the volatility suggests the existence of a "physiological" level of volatility for efficiency to hold. When $X(t)$ is a fractional Brownian motion of parameter H (whose pointwise regularity is $\alpha_X(t) = H$ almost everywhere), this relation is known in closed form (Decreusefond and Üstünel (1999)). For this reason, in the following we will refer to pointwise regularity and volatility as the *two sides of the same coin*;

b) the larger the difference $|\alpha_X(t+1) - \alpha_X(t)|$ the greater the impact of the new information on the price process. Obviously, if a great impact event occurs when $\alpha_X(t)$ lies in a neighborhood of $\frac{1}{2}$, the resulting new value of regularity is likely to be inconsistent with the martingale case. This justifies the attention that will be devoted to the study of the pointwise regularity exponents significantly different from $\frac{1}{2}$.

[6]For non Brownian martingales, an analogous result follows from observing that if Z_t is a martingale difference with respect to the filtration $\mathcal{F}_t$ such that $n^{-1}\sum_{t=1}^n \mathbb{E}(Z_t^2|\mathcal{F}_{t-1}) \overset{P}{\to} v$ for a positive constant v, and such that $n^{-1}\sum_{t=1}^n \mathbb{E}(Z_t^2 \mathbb{1}_{|Z_t|>\epsilon\sqrt{n}}|\mathcal{F}_{t-1}) \overset{P}{\to} 0$ for every $\epsilon > 0$, then $\sqrt{n}\bar{Z}_n \overset{d}{\to} \mathcal{N}(0,v)$ (here, $\mathbb{1}$ denotes the indicator function). Given the convergence to the normal law, the proof which covers the Brownian case applies to non Brownian well-behaved martingales (in the sense stated above), see e.g. Revuz and Yor (1999)

3.1. Estimation of the Pointwise Regularity

Many estimators of $\alpha_X(t)$ have been proposed in literature (see, e.g. Péltier and Lévy Véhel (1994), Péltier and Lévy Véhel (1995), Istas and Lang (1997), Benassi et al. (2000), Ayache (2000), Coeurjolly (2008)). Among these, extending the work of Péltier and Lévy Véhel, Bianchi (2005) and Bianchi et al. (2013) have defined the *Absolute Moment Based Estimator, AMBE*.

The explanation of the estimator is beyond the purposes of this article and an extensive discussion can be found in Bianchi et al. (2015). Here, we will be content to recall that, given the time series $X_n, n = 1, \ldots, N$, sampled in discrete time and with unit time variance equal to K, the AMBE of order k and lag q is defined in terms of moving average of size δ as

$$\begin{aligned} h^k_{\delta,q,N,K}(t) &= \frac{\log\left(\frac{\sqrt{\pi}}{\delta-q+1}\sum_{j=t-\delta}^{t-1}|X_{j+q}-X_j|^k / \left(2^{k/2}\Gamma\left(\frac{k+1}{2}\right)K^k\right)\right)}{k\log\left(\frac{q}{N-1}\right)}, \quad (6) \\ t &= \delta+1,\cdots,N+1-q. \end{aligned}$$

It can be proved that the estimator is normally distributed as

$$h^k_{\delta,q,N,K}(t) \sim \mathcal{N}\left(\alpha_X(t), \frac{\pi s^2}{\delta k^2 \log^2(N-1)\, 2^k \left(\Gamma\left(\frac{k+1}{2}\right)\right)^2}\right),$$

where s^2 is the limit variance (increasing with q) of a series of normalized nonlinear functions of a stationary Gaussian sequence with slowly decaying autocorrelation function. Toilsome computations show that, in the martingale case, the variance of the estimator – for $q = 1$ and $K = 1$ – can be explicitly calculated and is given by

$$\sigma^2 := Var\left(h^k_{\delta,1,N,1}(t)\right) = \frac{\sqrt{\pi}\,\Gamma\left(\frac{2k+1}{2}\right)-\Gamma^2\left(\frac{k+1}{2}\right)}{\delta k^2\log^2(N-1)\Gamma^2\left(\frac{k+1}{2}\right)} \quad (7)$$

Relation (7) deserves a few remarks:

- The optimal values minimizing (7) can be proved to be $q = 1$ and $k = 2$;
- the distribution

$$\Phi(z) := \Phi_{(h^k_{\delta,q,N,K}|\alpha_X(t)=\frac{1}{2})}(z) = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{z} e^{-\frac{(x-1/2)^2}{2\sigma^2}}dx, \quad (8)$$

with variance provided by (7), remains the same through time, provided that $\alpha_X(t) = \frac{1}{2}$;

• Even if the variance is calculated with respect to the case $K = 1$, relation (7) holds in general, since different values of K only translate the estimates, as pointed out in Bianchi and Pianese (2014);

• for $\alpha_X(t) \in \left(0, \frac{3}{4}\right)$, $h^k_{\delta,q,N,K}$ has a rate of convergence $O\left(\delta^{-\frac{1}{2}} (\log N)^{-1}\right)$. This ensures reliable estimates even for small values of δ (typical values are $\delta = 20$ or $\delta = 30$, see e.g., Frezza (2012)).

A natural choice to highlight the significant departures from the equilibrium given by the martingale case is to compare $h^k_{\delta,q,N,K}(t)$ with the reference threshold $\alpha_X(t) = \frac{1}{2}$. This is also a way to test the positive association between liquidity and market efficiency, claimed to be amplified during periods that contain new information (see e.g. Chung and Hrazdil (2010)), that is periods in which the pointwise regularity deviates from $\frac{1}{2}$. Once the significance level α has been fixed, denoting by $U_\alpha = (\Phi^{-1}(\alpha/2), \Phi^{-1}(1-\alpha/2))$ the confidence interval, the two functions

$$\gamma_h(t) = h^k_{\delta,q,N,K}(t)\mathbb{1}_{(0,1)\setminus U_\alpha}(h^k_{\delta,q,N,K}(t)) \tag{9}$$

$$\gamma_I(t) = \log ILLIQ(t)\mathbb{1}_{(0,1)\setminus U_\alpha}(h^k_{\delta,q,N,K}(t)) \tag{10}$$

filter out the values of AMBE lying outside the confidence interval (γ_h) and associate the corresponding values of $\log ILLIQ$ (γ_I).

4. EMPIRICAL ANALYSIS

4.1. Data

The analysis concerned the daily closing price and volumes of Standard & Poor's (GSPC), Nasdaq 100 (NDX) and Dax (GDAXI) indexes from September, 2006 to April 2016, for a total of 2,432 observations. Data, whose main descriptive statistics are displayed in Table 1, were obtained electronically from Bloomberg. No pre-filtering were considered necessary, for two reasons: a) the data are stock indexes, therefore they are assumed as portfolios representative of the whole market; b) for our purposes, outliers can be as informative as small sigma events.

4.2. Methodology and analysis

In the first part of the analysis, we investigate the relationship between the illiquidity ($ILLIQ_t$) and the pointwise Hölder regularity, measured by the AMBE (h_t)[7]. In the second part, we show that both the measures provide significant insights about the changes of the illiquidity premium. In particular, we examine the behavior of the binomial liquidity-efficiency

[7] From here on, to keep notation simple, we set $h_t := h^k_{\delta,q,N,K}(t)$

TABLE 1.

Main descriptive statistics of log-return.

	GSPC	NDX	GDAXI
average*	1.8921	4.1585	−2.1845
stdev	0.0133	0.0142	0.0147
maximum	0.1096	0.1185	0.0743
minimum	−0.0947	−0.1111	−0.1080
skewness	−0.3221	−0.1758	−0.0295
kurtosis	12.7994	10.7313	8.5506

$(\times 10^{-4})$

TABLE 2.

Confidence intervals U_α for $N = 2,432$.

	$\delta = 16$	$\delta = 21$	$\delta = 32$	$\delta = 64$
$\alpha = 0.01$	(0.4416, 0.5584)	(0.4490, 0.5510)	(0.4587, 0.5413)	(0.4708, 0.5292)
$\alpha = 0.05$	(0.4556, 0.5444)	(0.4612, 0.5388)	(0.4686, 0.5314)	(0.4778, 0.5222)
$\alpha = 0.10$	(0.4627, 0.5373)	(0.4674, 0.5326)	(0.4736, 0.5264)	(0.4814, 0.5186)

The intervals are calculated as $\frac{1}{2} \pm \Phi^{-1}(\alpha/2)\sigma$, where σ is the square root of relation (7).

TABLE 3.

FED monetary interventions (source Federal Open Market Committee)

Program	**Announcement date**	**Targeted End Date**	**Targeted Total Purchase**
QE1	November 25, 2008	Over Several Quarters	Agency Debt: Up to \$100 bil Agency MBS^a : Up to \$500 bil
QE1	March 18, 2009	March, 2010	Agency Debt: Additional \$100 bil Agency MBS: Additional \$750 bil Longer-Term Treasuries: \$300 bil
QE2	November 3, 2010	June 30, 2011	\$600 bil
MEP^b	September 21, 2011	June 30, 2012	\$400 bil
MEP	June 20, 2012	December 31, 2012	Amount limited by remaining Shorter-Term Treasury Securities^c
QE3	September 13, 2012	October 29, 2014	MBS: \$40-\$45 bil/month

(a): Mortgage Backed Securities
(b): Maturity Extension Program, also named Operation Twist
(c): Shorter-Term Treasury securities are sold or redeemed while an equal amount of longer-Term Treasury securities are purchased resulting in no net increase in balance-sheet size

during the interventions of the Federal Reserve to face the global financial crisis of 2007-2008.

As to the first part of the analysis, we set the estimation windows $\delta = 16, 21, 32, 64$, sizes which are intended to cover up to three trading months;

in particular, the choice of $\delta = 21$ (about one trading month) and $\delta = 64$ (about one trading quarter) is motivated by the analysis of the changes of the illiquidity premium over the whole period of interest. Furthermore, allowing the window to change permits to appreciate the stability of the results. For each fixed window, we regress $ILLIQ_t$ versus h_t, estimated with parameters δ as above, $q = 1$, $k = 2$ and K (varying for each series and δ) valued through the procedure described in Bianchi et al. (2013):

$$\log ILLIQ_t = \alpha_0 + \alpha_1 h_t + \epsilon_t^{(\alpha)}. \tag{11}$$

TABLE 4.

$\log ILLIQ_t = \alpha_0 + \alpha_1 h_t + \epsilon_t^{\alpha}$

		α_0	α_1	Fst	R^2	sse	$rmse$
GSPC	$\delta = 16$	-30.870^{***}	-7.552^{***}	717.53	0.827	8.431	0.237
		(-236.17)	(-26.78)				
	$\delta = 21$	-30.815^{***}	-7.358^{***}	508.59	0.818	6.154	0.233
		(-197.17)	(-22.52)				
	$\delta = 32$	-30.923^{***}	-7.540^{***}	347.17	0.824	3.647	0.222
		(-169.60)	(-18.61)				
	$\delta = 64$	-31.207^{***}	-7.508^{***}	170.76	0.826	1.575	0.209
		(-131.51)	(-13.07)				
NDX	$\delta = 16$	-28.966^{***}	-9.915^{***}	459.61	0.754	17.360	0.340
		(-121.37)	(21.44)				
	$\delta = 21$	-29.258^{***}	-9.621^{***}	319.90	0.739	12.86	0.337
		(-109.32)	$(-17.88\ )$				
	$\delta = 32$	-29.370^{***}	-10.33^{***}	202.98	0.733	8.291	0.335
		(-89.86)	(-14.25)				
	$\delta = 64$	-29.006^{***}	-10.52^{***}	93.67	0.722	3.955	0.332
		(-56.32)	(-9.67)				
GDAXI	$\delta = 16$	-29.310^{***}	-6.567^{***}	265.09	0.639	12.04	0.288
		(-159.57)	(-16.28)				
	$\delta = 21$	-29.246^{***}	-6.427^{***}	189.80	0.627	8.745	0.279
		(-133.20)	(-13.77)				
	$\delta = 32$	-29.114^{***}	-6.441^{***}	127.25	0.668	5.047	0.261
		(-105.15)	(-11.28)				
	$\delta = 64$	-28.860^{***}	-6.381^{***}	62.99	0.637	2.117	0.244
		(-70.82)	(-7.94)				

t-statistics in parentheses. (***), (**) and (*) indicate significance at the 1%, 5%, and 10% levels.

Not surprisingly, the results in Table 4 indicate a strong relationship. The goodness of relation (11) becomes even stronger when regularity exponents significantly different from $\frac{1}{2}$ are considered, as suggested in point b) of Section 3. The evidence is obtained by the following regression[8]

$$\gamma_I(t) = \theta_0 + \theta_1 \gamma_h(t) + \epsilon_t^{(\theta)}, \tag{12}$$

once all the zeros from γ_I and γ_h are removed. Clearly, functions γ require U_α to be determined in advance; for this, Table 2 summarizes the confidence intervals for different values of δ and α, given the sample size N.

TABLE 5.

Out of U_α regression

			θ_0	θ_1	Fst	R^2	sse	$rmse$
GSPC	$\delta = 16$	($\alpha = 0.01$)	-30.915^{***}	-7.392^{***}	381.43	0.862	3.07	0.224
			(-196.7461)	(-19.5303)				
		($\alpha = 0.05$)	-30.921^{***}	-7.382^{***}	515.05	0.869	3.60	0.215
			(-221.82)	(-22.69)				
		($\alpha = 0.10$)	-31.004^{***}	-7.172^{***}	387.32	0.813	5.598	0.250
			(-195.81)	(-19.68)				
	$\delta = 21$	($\alpha = 0.01$)	-30.940^{***}	-6.960^{***}	297.12	0.871	2.676	0.246
			(-169.75)	(-17.24)				
		($\alpha = 0.05$)	-30.951^{***}	-6.980^{***}	389.84	0.866	3.268	0.233
			(-189.07)	(-19.74)				
		($\alpha = 0.10$)	-30.879^{***}	-7.184^{***}	429.42	0.856	3.992	0.235
			(-190.48)	(-20.72)				
	$\delta = 32$	($\alpha = 0.01$)	-31.078^{***}	-7.091^{***}	118.44	0.752	2.107	0.232
			(-117.07)	(-10.88)				
		($\alpha = 0.05$)	-31.040^{***}	-7.214^{***}	176.00	0.792	2.509	0.233
			(-135.51)	(-13.27)				
		($\alpha = 0.10$)	-31.105^{***}	-7.056^{***}	187.42	0.783	2.891	0.236
			(-140.90)	(-13.69)				
	$\delta = 64$	($\alpha = 0.01$)	-31.065^{***}	-7.895^{***}	184.40	0.844	1.318	0.197
			(-132.10)	(-13.58)				
		($\alpha = 0.05$)	-31.065^{***}	-7.895^{***}	184.40	0.844	1.318	0.197
			(-132.10)	(-13.58)				
		($\alpha = 0.10$)	-31.065^{***}	-7.895^{***}	184.40	0.844	1.318	0.197
			(-132.10)	(-13.58)				

t-statistics in parentheses. (***), (**) and (*) indicate significance at the 1%, 5%, and 10% levels.

Using the two measures as delayed regressors and regressands, we analyze also the predictive power of one variable with respect to the other. The

[8]To make the notation clear, in place of the usual subscript, here we use the parenthesis to indicate the dependence from time t.

TABLE 5—*Continued*

			θ_0	θ_1	Fst	R^2	sse	$rmse$
NDX	$\delta = 16$	($\alpha = 0.01$)	-28.993^{***}	-9.857^{***}	319.29	0.872	6.22	0.363
			(-99.54)	(-17.86)				
		($\alpha = 0.05$)	-29.02^{***}	-9.786^{***}	404.64	0.851	8.047	0.337
			(-112.70)	(-20.11)				
		($\alpha = 0.10$)	-29.053^{***}	-9.734^{***}	395.68	0.830	9.572	0.343
			(-112.44)	(-19.89)				
	$\delta = 21$	($\alpha = 0.01$)	-29.314^{***}	-9.413^{***}	265.96	0.875	4.245	0.334
			(-103.94)	(-16.30)				
		($\alpha = 0.05$)	-29.314^{***}	-9.462^{***}	250.62	0.845	5.878	0.358
			(-99.53)	(-15.83)				
		($\alpha = 0.10$)	-29.298^{***}	-9.527^{***}	261.34	0.826	7.103	0.359
			(-100.49)	(-16.17)				
	$\delta = 32$	($\alpha = 0.01$)	-29.128^{***}	-10.954^{***}	71.35	0.652	4.757	0.354
			(-54.65)	(-8.45)				
		($\alpha = 0.05$)	-29.284^{***}	-10.539^{***}	71.02	0.640	5.028	0.354
			(-56.607)	(-8.428)				
		($\alpha = 0.10$)	-29.073^{***}	-11.085^{***}	102.24	0.690	5.963	0.360
			(-62.755)	(-10.11)				
	$\delta = 64$	($\alpha = 0.01$)	-28.922^{***}	-10.688^{***}	68.92	0.792	1.914	0.326
			(-49.90)	(-8.30)				
		($\alpha = 0.05$)	-28.893^{***}	-10.808^{***}	71.32	0.781	2.320	0.341
			(-49.61)	(-8.45)				
		($\alpha = 0.1$)	-28.893^{***}	-10.808^{***}	71.32	0.781	2.320	0.341
			(-49.61)	(-8.45)				

results of the regressions

$$\log ILLIQ_t = \theta_0 + \theta_1 h_{t-j} + v_t, \tag{13}$$

and

$$h_t = \theta_0' + \theta_1' \log ILLIQ_{t-j} + v_t, \tag{14}$$

show that h_t is a better forecaster of $ILLIQ_t$ than vice versa. Interestingly, the goodness of fit changes remarkably with the considered time series (see Table 6).

Since it is well known that $ILLIQ_t$ can be characterized as an AR(1) (see e.g. Amihud (2002), Amihud et al. (2005)), we estimate the autoregressive models for both $\log ILLIQ_t$ and h_t (see (15) and (16), respectively) because: (a) they constitute an additional way to assess the proximity of the two measures; (b) they allow to analyze the relationship between excess returns and liquidity (efficiency). The sample partial autocorrelation

TABLE 5—*Continued*

			θ_0	θ_1	Fst	R^2	sse	$rmse$
GDAXI	$\delta = 16$	($\alpha = 0.01$)	-29.39^{***}	-6.288^{***}	64.87	0.515	7.377	0.348
			(-92.27)	(-8.054)				
		($\alpha = 0.05$)	-29.25^{***}	-6.70^{***}	118.66	0.594	8.524	0.324
			(-112.50)	(-10.89)				
		($\alpha = 0.10$)	-29.280^{***}	-6.620^{***}	152.80	0.619	8.983	0.309
			(-126.97)	(-12.36)				
	$\delta = 21$	($\alpha = 0.01$)	-29.276^{***}	-6.278^{***}	74.74	0.619	5.177	0.335
			(-92.33)	(-8.64)				
		($\alpha = 0.05$)	-29.228^{***}	-6.416^{***}	97.44	0.652	5.254	0.318
			(-101.61)	(-9.87)				
		($\alpha = 0.10$)	-29.19^{***}	-6.530^{***}	127.01	0.672	5.756	0.305
			(-111.72)	(-11.26)				
	$\delta = 32$	($\alpha = 0.01$)	-29.13^{***}	-6.35^{***}	65.64	0.709	2.635	0.312
			(-79.96)	(-8.10)				
		($\alpha = 0.05$)	-29.150^{***}	-6.313^{***}	77.73	0.677	3.422	0.304
			(-86.21)	(-8.82)				
		($\alpha = 0.10$)	-29.144^{***}	-6.347^{***}	89.80	0.681	3.476	0.287
			(-91.88)	(-9.47)				
	$\delta = 64$	($\alpha = 0.01$)	-29.02^{***}	-6.295^{***}	56.90	0.760	1.084	0.245
			(-67.86)	(-7.54)				
		($\alpha = 0.05$)	-28.887^{***}	-6.473^{***}	49.34	0.692	1.656	0.274
			(-61.11)	(-7.024)				
		($\alpha = 0.10$)	-28.893	-6.438	48.14	0.676	1.758	0.277
			(-60.67)	(-6.93)				

t-statistics in parentheses. (***), (**) and (*) indicate significance at the 1%, 5%, and 10% levels.

functions calculated for each δ and stock index indicate that just the one lagged terms are useful predictors of both $\log ILLIQ_t$ and h_t; therefore, the first-order model suffices to account for the dynamics.

$$\log ILLIQ_t = \lambda_0 + \lambda_1 \log ILLIQ_{t-1} + \epsilon_t^{(\lambda)} \tag{15}$$

$$h_t = \rho_0 + \rho_1 h_{t-1} + \epsilon_t^{(\rho)}. \tag{16}$$

Coherently with previous works, we expect a (significant) positive λ_1, as well as a (significant) positive ρ_1. These findings would mean that investors are assumed to predict both illiquidity and pointwise regularity (or volatility) for current window based on information available in previous one.

The second part of the analysis investigates the behavior of the changes in the illiquidity premium before, during and after the global financial

TABLE 6.

$\log ILLIQ_t = \theta_0 + \theta_1 h_{t-j} + v_t$. $h_t = \theta'_0 + \theta'_1 \log ILLIQ_{t-j} + v_t$.

	j	θ_0	θ_1	R^2	Fst	sse	θ'_0	θ'_1	R^2	F	sse
GSPC											
$\delta = 16$	1	-31.768^{***}	-5.609^{***}	0.453	123.13	26.69	-2.193^{***}	-0.077^{***}	0.406	101.98	0.42
		(-135.668)	(-11.096)				(-8.352)	(-10.098)			
	2	-32.189^{***}	-4.689^{***}	0.315	68.21	33.33	-1.791^{***}	-0.065^{***}	0.291	60.97	0.49
		(-122.473)	(-8.269)				(-6.219)	(-7.818)			
	3	-32.467^{***}	-4.080^{***}	0.240	46.23	36.97	-1.586^{***}	-0.060^{***}	0.242	46.87	0.52
		(-116.902)	(-6.799)				(-5.314)	(-6.846)			
	4	-32.502^{***}	-3.996^{***}	0.229	43.36	37.45	-1.348^{***}	-0.053^{***}	0.190	34.31	0.55
		(-115.62)	(-6.585)				(-4.375)	(-5.857)			
	5	-32.618^{***}	-3.736^{***}	0.201	36.18	38.79	-1.202^{***}	-0.048^{***}	0.163	28.33	0.56
		(-113.239)	(-6.015)				(-3.860)	(-5.324)			
$\delta = 21$	1	-31.764^{***}	-5.360^{***}	0.432	85.33	19.21	-2.087^{***}	-0.075^{***}	0.366	64.84	0.32
		(-114.378)	(-9.238)				(-6.562)	(-8.052)			
	2	-32.301^{***}	-4.218^{***}	0.268	40.68	24.70	-1.670^{***}	-0.062^{***}	0.260	39.09	0.37
		(-102.00)	(-6.378)				(-4.872)	(-6.252)			
	3	-32.303^{***}	-4.209^{***}	0.265	39.75	24.78	-1.413^{***}	-0.055^{***}	0.204	28.17	0.39
		(-100.948)	(-6.305)				(-3.978)	(-5.308)			
	4	-32.687^{***}	-3.394^{***}	0.173	22.75	27.89	-0.987^{***}	-0.043^{***}	0.125	15.57	0.42
		(-95.748)	(-4.770)				(-2.671)	(-3.946)			
	5	-32.963^{***}	-2.809^{***}	0.118	14.51	29.64	-0.843^{**}	-0.038^{***}	0.102	12.28	0.42
		(-93.236)	(-3.810)				(-2.249)	(-3.504)			
$\delta = 32$	1	-31.833^{***}	-5.506^{***}	0.433	55.71	11.74	-2.182^{***}	-0.076^{***}	0.399	48.57	0.18
		(-96.107)	(-7.464)				(-5.780)	(-6.969)			
	2	-32.366^{***}	-4.297^{***}	0.264	25.84	15.17	-1.502^{***}	-0.057^{***}	0.224	20.76	0.22
		(-85.21)	(-5.084)				(-3.518)	(-4.556)			
	3	-32.649^{***}	-3.646^{***}	0.190	16.67	16.62	-1.209^{***}	-0.048^{***}	0.169	14.51	0.22
		(-81.270)	(-4.083)				(-2.790)	(-3.808)			
	4	-32.949^{***}	-2.970^{***}	0.126	10.10	17.92	-0.916^{**}	-0.039^{***}	0.113	8.92	0.24
		(-78.407)	(-3.179)				(-2.015)	(-2.987)			
	5	-33.148^{***}	-2.518^{**}	0.091	6.88	18.63	-0.593	-0.030^{**}	0.066	4.85	0.25
		(-76.789)	(-2.624)				(-1.263)	(-2.203)			
$\delta = 64$	1	32.361^{***}	-4.652^{***}	0.318	16.32	6.14	-1.705^{***}	-0.061^{***}	0.264	12.58	0.10
		(-68.601)	(-4.040)				(-2.867)	(-3.546)			
	2	-32.906^{***}	-3.290^{**}	0.159	6.46	7.53	-1.113^{*}	-0.044^{**}	0.139	5.48	0.10
		(-61.974)	(-2.542)				(-1.720)	(-2.341)			
	3	-33.035^{***}	-2.950^{**}	0.128	4.82	7.777	-0.584	-0.028	0.06	2.10	0.11
		(-59.783)	(-2.196)				(-0.861)	(-1.451)			

TABLE 6—*Continued*

	j	θ_0	θ_1	R^2	Fst	sse	θ_0'	θ_1'	R^2	F	sse
	4	-33.108^{***}	-2.795^{**}	0.113	4.09	7.853	-0.465	-0.025	0.044	1.48	0.11
		(-58.405)	(-2.022)				(-0.654)	(-1.219)			
	5	-33.105^{***}	-2.827^{**}	0.115	4.042	7.794	-0.488	-0.026	0.046	1.48	0.11
		(-57.539)	(-2.011)				(-0.667)	(-1.217)			
NDX											
$\delta = 16$	1	-30.218^{***}	-7.481^{***}	0.428	111.52	40.24	-1.283^{***}	-0.052^{***}	0.358	83.20	0.35
		(-82.721)	(-10.560)				(-6.517)	(-9.122)			
	2	-30.610^{***}	-6.720^{***}	0.345	77.90	46.09	-1.017^{***}	-0.045^{***}	0.258	51.48	0.40
		(-77.989)	(-8.826)				(-4.771)	(-7.175)			
	3	-30.941^{***}	-6.076^{***}	0.282	57.75	50.48	-0.878^{***}	-0.041^{***}	0.212	39.45	0.43
		(-75.067)	(-7.599)				(-3.965)	(-6.281)			
	4	-31.022^{***}	-5.914^{***}	0.266	52.92	51.61	-0.737^{***}	-0.036^{***}	0.171	30.17	0.45
		(-73.97)	(-7.27)				(-3.24)	(-5.49)			
	5	$-31.031^{**}*$	-5.892^{***}	0.261	51.32	51.93	-0.650^{***}	-0.034^{***}	0.149	25.36	0.45
		(-73.06)	(-7.16)				(-2.819)	(-5.036)			
$\delta = 21$	1	-30.344^{***}	-7.432^{***}	0.440	88.13	27.50	-1.271^{***}	-0.052^{***}	0.332	55.86	0.26
		(-77.081)	(-9.387)				(-5.382)	(-7.474)			
	2	-30.751^{***}	-6.606^{***}	0.346	58.89	32.10	-1.037^{***}	-0.045^{***}	0.250	36.98	0.29
		(-71.801)	(-7.674)				(-4.120)	(-6.081)			
	3	-30.873^{***}	-6.354^{***}	0.317	51.16	33.51	-0.818^{***}	-0.038^{***}	0.183	24.74	0.32
		(-69.762)	(-7.152)				(-3.101)	(-4.973)			
	4	-31.144^{***}	-5.806^{***}	0.264	39.26	36.10	-0.632^{**}	-0.033^{***}	0.136	17.15	0.33
		(-67.445)	(-6.265)				(-2.326)	(-4.141)			
	5	-31.423^{***}	-5.244^{***}	0.215	29.70	38.51	-0.658^{**}	-0.034^{***}	0.140	17.60	0.33
		(-65.56)	(-5.45)				(-2.40)	(-4.20)			
$\delta = 32$	1	-30.300^{***}	-8.268^{***}	0.466	63.80	16.51	-1.229^{***}	-0.049^{***}	0.345	38.55	0.13
		(-65.01)	(-7.99)				(-4.55)	(-6.20)			
	2	-30.850^{***}	-7.030^{***}	0.336	36.47	20.53	-0.835^{***}	-0.037^{***}	0.202	18.23	0.17
		(-58.815)	(-6.039)				(-2.780)	(-4.269)			
	3	-31.110^{***}	-6.445^{***}	0.280	27.59	22.28	-0.717^{**}	-0.034^{***}	0.168	14.30	0.17
		(-56.171)	(-5.252)				(-2.331)	(-3.782)			
	4	-31.408^{***}	-5.784^{***}	0.224	20.29	23.98	-0.582^{*}	-0.030^{***}	0.128	10.24	0.18
		(-54.231)	(-4.504)				(-1.811)	(-3.200)			
	5	-31.739^{***}	-5.044^{***}	0.171	14.23	25.65	-0.373	-0.024^{**}	0.081	6.05	0.19
		(-52.60)	(-3.77)				(-1.12)	(-2.46)			
$\delta = 64$	1	-30.276^{***}	-7.835^{***}	0.402	23.51	8.50	-0.898^{**}	-0.040^{***}	0.243	11.27	0.07
		(-39.539)	(-4.849)				(-2.204)	(-3.358)			

TABLE 6—*Continued*

	j	θ_0	θ_1	R^2	Fst	sse	θ_0'	θ_1'	R^2	F	sse
	2	-30.955^{***}	-6.388^{***}	0.266	12.33	10.43	-0.583	-0.031^{**}	0.142	5.63	0.08
		(-35.872)	(-3.512)				(-1.313)	(-2.373)			
	3	-31.087^{***}	-6.089^{***}	0.237	10.29	10.82	-0.225	-0.021	0.062	2.20	0.08
		(-34.45)	(-3.21)				(-0.482)	(-1.485)			
	4	-31.518^{***}	-5.206^{**}	0.172	6.67	11.70	-0.166	-0.019	0.049	1.67	0.08
		(-32.943)	(-2.582)				(-0.337)	(-1.291)			
	5	-31.707^{***}	-4.838^{**}	0.147	5.375	11.93	-0.139	-0.018	0.044	1.41	0.08
		(-32.087)	(-2.318)				(-0.272)	(-1.189)			
GDAXI											
$\delta = 16$	1	-29.852^{***}	-5.359^{***}	0.425	110.20	19.72	-1.528^{***}	-0.061^{***}	0.256	51.37	0.37
		(-128.70)	(-10.497)				(-5.537)	(-7.167)			
	2	-30.258^{***}	-4.444^{***}	0.296	62.26	23.76	-0.891^{***}	-0.042^{***}	0.120	20.10	0.44
		(-118.162)	(-7.890)				(-2.981)	(-4.484)			
	3	-30.548^{***}	-3.792^{***}	0.215	40.29	26.40	-0.729^{**}	-0.036^{***}	0.093	15.12	0.44
		(-112.378)	(-6.347)				(-2.406)	(-3.888)			
	4	-30.582^{***}	-3.705^{***}	0.207	37.99	26.34	-0.474	-0.028^{***}	0.058	8.96	0.45
		(-111.701)	(-6.164)				(-1.537)	(-2.992)			
	5	-30.540^{***}	-3.789^{***}	0.215	39.73	25.98	-0.432	-0.027^{***}	0.053	8.11	0.45
		(-111.457)	(-6.303)				(-1.398)	(-2.8477)			
$\delta = 21$	1	-29.777^{***}	-5.281^{***}	0.420	80.99	13.65	-1.410^{***}	-0.058^{***}	0.225	32.42	0.27
		(-107.68)	(-8.910)				(-4.277)	(-5.694)			
	2	-30.166^{***}	-4.435^{***}	0.299	47.39	16.24	-0.921^{***}	-0.043^{***}	0.127	16.14	0.30
		(-99.30)	(-6.88)				(-2.668)	(-4.018)			
	3	-30.341^{***}	-4.048^{***}	0.250	36.45	17.22	-0.470	-0.029^{***}	0.058	6.80	0.31
		(-95.872)	(-6.038)				(-1.312)	(-2.608)			
	4	-30.367^{***}	-3.995^{***}	0.242	34.885	17.38	-0.436	-0.028^{**}	0.054	6.27	0.31
		(-95.101)	(-5.906)				(-1.211)	(-2.505)			
	5	-30.712^{***}	-3.249^{***}	0.162	20.84	19.04	0.043	-0.013	0.012	1.32	0.32
		(-91.443)	(-4.565)				(0.116)	(-1.149)			
$\delta = 32$	1	-29.640^{***}	-5.335^{***}	0.436	56.47	7.67	-1.253^{***}	-0.054^{***}	0.197	17.88	0.16
		(-86.02)	(-7.51)				(-3.05)	(-4.228)			
	2	-30.136^{***}	-4.289^{***}	0.283	28.43	9.57	-0.679	-0.036^{***}	0.091	7.17	0.18
		(-77.100)	(-5.332)				(-1.569)	(-2.678)			
	3	-30.281^{***}	-3.979^{***}	0.243	22.79	10.09	-0.051	-0.016	0.019	1.40	0.19
		(-74.689)	(-4.774)				(-0.114)	(-1.182)			
	4	-30.556^{***}	-3.392^{***}	0.182	15.60	10.56	0.100	-0.012	0.010	0.70	0.19
		(-73.118)	(-3.950)				(0.222)	(-0.837)			

TABLE 6—*Continued*

	j	θ_0	θ_1	R^2	Fst	sse	θ'_0	θ'_1	R^2	F	sse
	5	-30.430^{***}	-3.637^{***}	0.205	17.82	10.214	-0.004	-0.015	0.016	1.13	0.19
		(-72.445)	(-4.222)				(-0.008)	(-1.064)			
$\delta = 64$	1	-29.468^{***}	-5.342^{***}	0.454	29.08	3.07	-0.919	-0.044^{**}	0.135	5.49	0.07
		(-58.402)	(-5.393)				(-1.514)	(-2.344)			
	2	-30.065^{***}	-4.141^{***}	0.279	13.16	3.95	-0.028	-0.017	0.020	0.68	0.08
		(-51.658)	(-3.627)				(-0.044)	(-0.826)			
	3	-30.369^{***}	-3.506^{***}	0.202	8.37	4.20	0.298	-0.006	0.003	0.10	0.08
		(-49.015)	(-2.893)				(0.458)	(-0.312)			
	4	-31.219^{***}	-1.814	0.055	1.87	4.87	1.160	0.021	0.031	1.03	0.08
		(-45.933)	(-1.366)				(1.783)	(1.014)			
	5	-31.962^{***}	-0.319	0.019	0.057	4.72	1.503	0.031	0.073	2.47	0.07
		(-46.939)	(-0.239)				(2.354)	(1.573)			

t-statistics in parentheses. (***), (**) and (*) indicate significance at the 1%, 5%, and 10% levels.

crisis of 2007-2008. The watershed represented by the financial turmoil is of special interest to analyze whether a change occurred in the illiquidity premium: indeed, reasonably the Quantitative Easing Program (QE) run by the Federal Reserve (see Table 3) as a response to the liquidity crunch could have produced a regime-switch revealed by the variations of premium itself. This should not surprise, since it has been argued that the monetary policy is transmitted through the stock market via the "wealth effect" of private portfolios (Bernanke and Kuttner (2004)). In order to test the regime-switching, we split the whole sample into two subsamples: from September 2006 to October 2008 (for a total of $N_1 = 525$ observations) and from November 2008 (beginning of the QE1) to October 2014 (end of the QE3), for a total of $N_2 = 1,512$ observations. During this span of time the Federal Reserve implemented the QE1, QE2, QE3 and the Maturity Extension Program, also named Twist Operation (TO).

To this aim, following a consolidated literature (see e.g., Amihud (2002), Pastor and Stambaugh (2003)), we regress the excess returns (with respect to the Treasury Bill) over both $\log ILLIQ$ and $1-h$ (we recall that $h \in (0,1)$) as

$$\left(\bar{R} - R_f\right)_t = \eta_0 + \eta_1 \log ILLIQ_{t-1} + \epsilon_t^{(\eta)} \tag{17}$$

$$\left(\bar{R} - R_f\right)_t = \kappa_0 + \kappa_1 (1 - h_{t-1}) + \epsilon_t^{(\kappa)} \tag{18}$$

where:

• $t = 1, \delta + 1, \cdots, N_1/\delta$, for the first subsample and $t = (N_1/\delta) + 1, (N_1/\delta) + \delta + 1, \cdots, \frac{N_1+N_2}{\delta})$, for the second subsample, with $\delta = 21$ and $\delta = 64$;

• $\bar{R}$ is the average percentage variation of the stock index in the given window;

• R_f is the one/three-month Treasury Bill at the beginning of the window (data are downloaded from the U.S. Department of Treasury, through http://www.treasury.gov).

TABLE 7.

$\log ILLIQ_t = \lambda_0 + \lambda_1 \log ILLIQ_{t-1} + \epsilon_t^{(\lambda)}$. $h_t = \rho_0 + \rho_1 h_{t-1} + \epsilon_t^{(\rho)}$

	λ_0	λ_1	R^2	Fst	sse	ρ_0	ρ_1	R^2	F	sse
GSPC										
$\delta = 16$	-10.008^{***}	0.709^{***}	0.491	143.99	24.80	0.136^{***}	0.703^{***}	0.495	145.95	0.3537
	(-4.93)	(12)				(5.047)	(12.08)			
$\delta = 21$	-10.336^{***}	0.699^{***}	0.479	102.96	17.64	0.157^{***}	0.669^{***}	0.451	92.09	0.2769
	(-4.37)	(10.15)				(4.70)	(9.60)			
$\delta = 32$	-9.455^{***}	0.7244^{***}	0.505	74.48	5.44	0.140^{***}	0.685^{***}	0.474	65.64	0.1542
	(-3.29)	(8.63)				(3.6844)	(8.1021)			
$\delta = 64$	-12.270^{***}	0.642^{***}	0.396	22.98	21.69	0.188^{***}	0.533^{***}	0.302	15.17	0.08666
	(-2.68)	(4.79)				(3.3427)	(3.8949)			
NDX										
$\delta = 16$	-6.805^{***}	0.800^{***}	0.634	257.94	25.76	0.188^{***}	0.634^{***}	0.401	99.58	0.3241
	(-4.01)	(16.06)				(5.72)	(9.98)			
$\delta = 21$	-5.878^{***}	0.827^{***}	0.676	233.64	15.92	0.193^{***}	0.610^{***}	0.371	66.01	0.2472
	(-3.19)	(15.28)				(5.17)	(8.12)			
$\delta = 32$	-5.461^{***}	0.840^{***}	0.688	161.21	9.65	0.163^{***}	0.637^{***}	0.403	49.18	0.1271
	(-2.43)	(12.70)				(3.98)	(7.01)			
$\delta = 64$	-7.144^{***}	0.790^{***}	0.605	53.66	5.61	0.222^{***}	0.527^{***}	0.281	13.67	0.06609
	(-1.95)	(7.33)				(3.29)	(3.70)			
GDAXI										
$\delta = 16$	-9.049^{***}	0.720^{***}	0.520	160.13	16.54	0.142^{***}	0.684^{***}	0.472	132.95	0.2661
	(-4.93)	(12.64)				(5.27)	(11.52)			
$\delta = 21$	-9.885^{***}	0.693^{***}	0.4721	102.62	12.27	0.159^{***}	0.657^{***}	0.433	85.45	0.2001
	(-4.47)	(10.13)				(4.75)	(9.25)			
$\delta = 32$	-8.919^{***}	0.723^{***}	0.528	81.25	6.439	0.176^{***}	0.631^{***}	0.412	51.04	0.1189
	(-3.45)	(9.01)				(4.11)	(7.14)			
$\delta = 64$	-8.701^{***}	0.729^{***}	0.547	42.32	2.54	0.244^{***}	0.512^{***}	0.281	13.70	0.0599
	(-2.411)	(6.504)				(3.46)	(3.70)			

t-statistics in parentheses. (***), (**) and (*) indicate significance at the 1%, 5%, and 10% levels.

Since models (17) and (18) estimate the market's sensitivity to the illiquidity factor, we expect to find significantly positive η_1 (κ_1) in the former subsample and close to zero η_1 (κ_1) in the latter subsample; indeed, the assumption we want to check is whether the heavy injection of liquidity provided by the monetary authorities has reduced or eliminated at all the illiquidity premium. Tables 8 and 9 exhibit the results. To reinforce the analysis we also investigate more in detail the dynamics through time of the coefficients η_1 and κ_1; since even before the start of QE several actions were taken to face the liquidity crunch[9], we run the regressions (17) and (18) assuming a moving window of length N_1, with increment of one trading month. In this way, we expect to emphasize the behavior of the changes of the illiquidity premium over the whole period; more specifically, we expect to observe the changes of the premium to decay as the interventions get stronger. This is reasonable under the assumption that markets start perceiving the interventions of the central banks as a sort of warranty of liquidity. In other words, no additional reward is required for holding illiquid assets, since the Federal Reserve ensures an "acceptable" level of liquidity in the market. Figure 2 displays the dynamics of the coefficients η_1 (κ_1).

4.3. Discussion of results.

The first point we address is the relationship between the illiquidity ($ILLIQ_t$) and the pointwise Hölder exponent (h_t). For each index, Figure 1 displays the estimates of the pointwise regularity (top-left panels), the illiquidity (bottom-left panels) and the comparison of the three measures

[9]See in this regard the punctual review provided by Brunnermeier (2009), "The first illiquidity wave" on the interbank market started on August 9. At that time, the perceived default and liquidity risks of banks rose significantly, driving up the LIBOR. In response to the freezing up of the interbank market on August 9, the European Central Bank injected 95 billion in overnight credit into the interbank market. The U.S. Federal Reserve followed suit, injecting 24 billion. To alleviate the liquidity crunch, the Federal Reserve reduced the discount rate by half a percentage point to 5.75 percent on August 17, 2007, broadened the type of collateral that banks could post, and lengthened the lending horizon to 30 days. $[\cdots]$ On September 18, the Fed lowered the federal funds rate by half a percentage point (50 basis points) to 4.75 percent and the discount rate to 5.25 percent. $[\cdots]$ Also, various sovereign wealth funds invested a total of more than 38 billion in equity from November until mid-January 2008 in major U.S. banks (IMF, 2008). But matters worsened again starting in that an earlier estimate of the total loss billion, had to be revised upward. $[\cdots]$ The TED spread widened again as the LIBOR peaked in mid December of 2007 $[\cdots]$. This change convinced the Fed to cut the federal funds rate by 0.25 percentage point on December 11, 2007. $[\cdots]$ On December 12, 2007, the Fed announced the creation of the Term Auction Facility (TAF), through which commercial banks could bid anonymously for 28-day loans against a broad set of collateral, including various mortgage-backed securities.$[\cdots]$ At its regular meeting on January 30, the Federal Open Market Committee cut the federal funds rate another 0.5 percentage point. $[\cdots]$ A second event was that of March 11, 2008, when the Federal Reserve announced its 200 billion Term Securities Lending Facility.

TABLE 8.

Excess return with respect to illiquidity and pointwise regularity (monthly data).

	η_0	η_1	R^2	Fst	sse	κ_0	κ_1	R^2	Fst	sse
GSPC										
$Sep06 - Oct08$	0.998^{***}	0.030^{***}	0.494	22.48	0.0026	-0.122^{***}	0.168^{***}	0.584	32.24	0.0022
	(4.58)	(4.74)				(-7.818)	(5.677)			
$Nov08 - Oct14$	0.007	0.0002	0.001	0.12	0.0004	-0.002	0.0031	0.005	0.2036	0.0004
	(0.35)	(0.35)				(-0.48)	(0.45)			
NDX										
$Sep06 - Oct08$	1.149^{***}	0.035^{***}	0.682	49.36	0.0017	-0.153^{***}	0.230^{***}	0.645	41.83	0.0019
	(6.82)	(7.03)				(-8.27)	(6.47)			
$Nov08 - Oct14$	0.01	0.001	0.0052	0.36	0.0004	-0.002	0.004	0.007	0.33	0.0004
	(0.62)	(0.60)				(-0.53)	(0.58)			
GDAXI										
$Sep06 - Oct08$	0.927^{***}	0.030^{***}	0.332	11.43	0.0032	-0.131^{***}	0.189^{***}	0.423	16.85	0.002
	(3.25)	(3.38)				(-5.54)	(4.10)			
$Nov08 - Oct14$	0.023	0.001	0.015	1.05	0.0006	-0.005	0.009	0.021	0.98	0.0004
	(1.02)	(1.02)				(-1.05)	(0.99)			

t-statistics in parentheses. (***), (**) and (*) indicate significance at the 1%, 5%, and 10% levels.

TABLE 9.

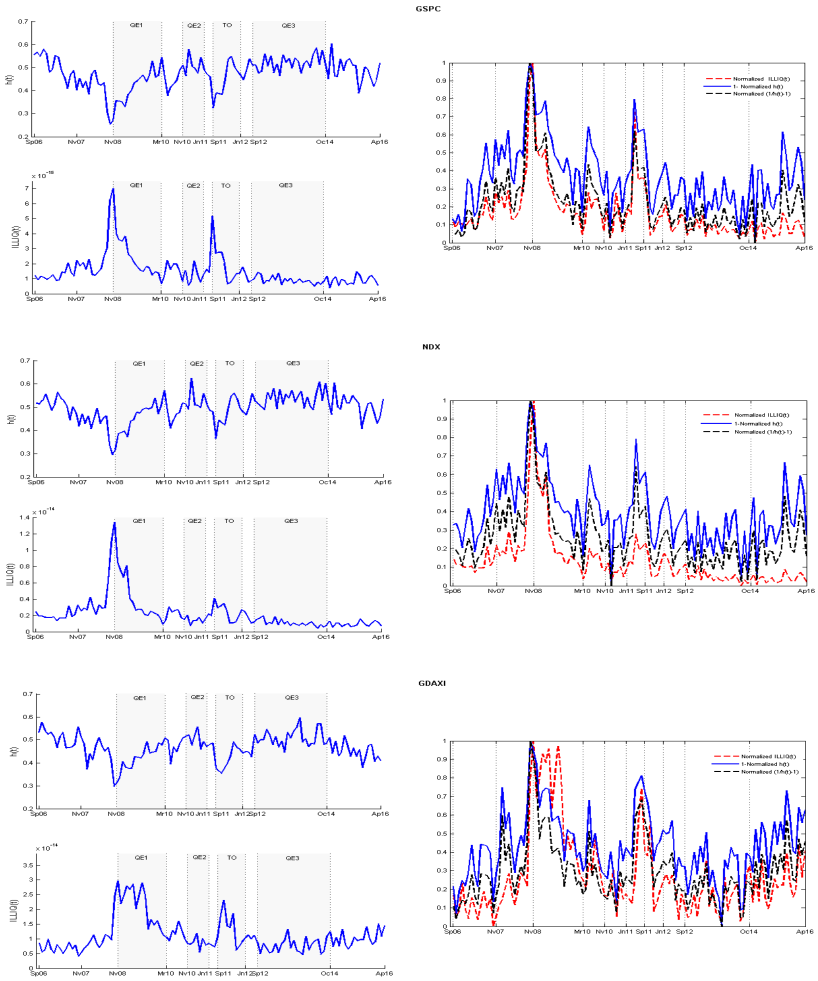
Excess return with respect to illiquidity and pointwise regularity (quarterly data).

	η_0	η_1	R^2	Fst	sse	κ_0	κ_1	R^2	Fst	sse
GSPC										
$Sep06 - Oct08$	1.844^{***}	0.055^{***}	0.768	19.87	0.0004	-0.189^{***}	0.265^{***}	0.758	18.83	0.0004
	(4.38)	(4.46)				(-5.22)	(4.33)			
$Nov08 - Oct14$	0.029^{*}	0.0009^{*}	0.144	3.36	2.06e-05	-0.005^{**}	0.008^{*}	0.173	4.21	1.99e-05
	(1.83)	(1.84)				(-2.11)	(2.05)			
NDX										
$Sep06 - Oct08$	1.98^{***}	0.060^{***}	0.807	25.09	0.0003	-0.238^{***}	0.384^{***}	0.825	28.33	0.0003
	(4.93)	(5.01)				(-6.14)	(5.32)			
$Nov08 - Oct14$	0.022	0.0006	0.102	2.29	2.41e-005	-0.006^{**}	0.011^{**}	0.198	4.95	2.15e-05
	(1.52)	(1.51)				(-2.20)	(2.22)			
GDAXI										
$Sep06 - Oct08$	0.969^{***}	0.031^{***}	0.773	20.40	0.0004	-0.164^{***}	0.275^{***}	0.720	15.19	0.0005
	(4.37)	(4.52)				(-4.83)	(3.90)			
$Nov08 - Oct14$	0.013	0.0004	0.064	1.38	2.69e-05	-0.003	0.006	0.0523	1.11	2.73e-05
	(1.14)	(1.18)				(-1.18)	(1.05)			

t-statistics in parentheses. (***), (**) and (*) indicate significance at the 1%, 5%, and 10% levels.

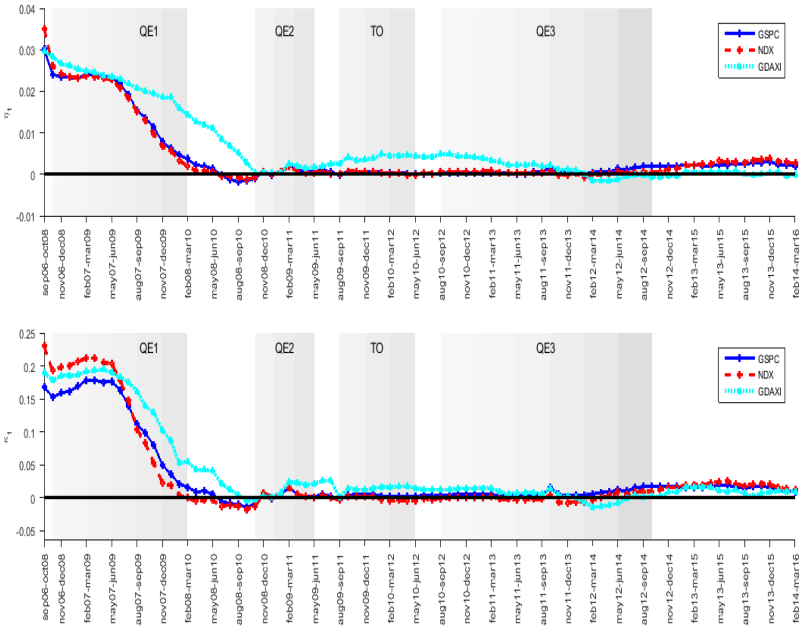
(right panels), properly normalized to ease the visual inspection. It is quite evident that, once the scale is accounted for, the pointwise regularity and

FIG. 1. Estimation of $ILLIQ(t)$ and $h(t)$. Top-left panel: Estimation of the pointwise Hölder regularity $h(t)$; Bottom-left panel: Estimation of the illiquidity measure, $ILLIQ(t)$; right-panel: normalized measures: $ILLIQ(t)$(red-dotted line) and $1 - h(t)$ (blue line). From the left panels it is remarkable the burst of the increasing illiquidity with the simultaneous decreasing pointwise regularity just before November 2008, probably ascribable to the "panic selling" due to Lehman Brother collapse.)



the illiquidity change accordingly, indicating that — even if with different sensitivities — both capture the same market behavior, bringing out the connection between efficiency and liquidity. In Figure 1 the interventions of the Federal Reserve aiming at increasing the liquidity in the markets are identified by the shaded areas. Notice that:

FIG. 2. Changes in the slopes of regressions (17) and (18) when data belong to a monthly sliding window of size 25 trading months, starting from September 2006 and terminating on February 2014. The shaded areas correspond to the interventions of the Federal Reserve and the intensity of grey scale is increasing with the time elapsed from the beginning of each operation.



• soon before the start of the QE1 a sudden burst of illiquidity corresponds to a large downward movement of h_t, which is pushed far away from its physiological value $\frac{1}{2}$. Both the estimates capture the effect of the panic selling triggered by the collapses of Lehman Brothers (September 15, 2008) and the insurance firm AIG (September 16, 2008), two of the catalysts of the volatility explosion;

• at the end of each intervention (QE1, QE2, TO, QE3), not only the liquidity appears to be recovered, but even more informative to us is the fact that h_t is again in a neighborhood of $\frac{1}{2}$, regardless its pre intervention level. This happens systematically at the end of each injection of liquidity for the three indices;

• a difference can be observed between the QE1/TO, on the one side, and QE2/QE3, on the other side. The former two interventions effectively seem to have contributed to increase the liquidity and to push markets back towards efficiency. This effect is particularly pronounced for the QE1, for which the announcement of the Federal Reserve to initiate a program to purchase substantial quantities of mortgage-backed securities surprised the markets Hancock and Passmore (2012), to the extent that the S&P500 grew of 4.5% in two days, before the profit taking of the weekend. On the contrary, the latter two interventions have had a minor impact on the U.S. stock market, which was already efficient when they were launched (h_t fluctuates around $\frac{1}{2}$ at the beginning of QE2 and QE3) and liquidity was not that low. As it will be observed later, these two programs could have provided an excess of liquidity, much to the advantage of abnormal extra returns, as stressed by the values of h_t lying persistently above $\frac{1}{2}$ during the programs themselves. This is also in accordance with Bayoumi and Bui (2011), who analyze the impact of the 2008/2009 and 2010 monetary stimulus packages across a selection of G20 countries and conclude that the QE1 announcements had a strong initial impact on financial conditions, including commodity prices and U.S. and foreign equities. They also find that the effects of QE2 announcements were generally not statistically significant;

- finally, we observe that soon after the end of QE3, h_t restarts slightly decreasing for the three indices, which indicates that markets mean-revert and overreact more than what one would expect if they were efficient.

Table 4 displays the results of the regression (11): a very strong relationship exists between the two measures on all the δ's that have been taken into consideration. This is particularly true for the U.S. market, for which the R^2 ranges from 0.827 to 0.722, with coefficients always significant and stable for the different windows. The relation continues to be significant even when lagged observations are considered (see Table 6 summarizing regressions (13) and (14). In this case, although the the two measures are mutually forecasting to some extent, h_t displays a better goodness of fit in forecasting $\log ILLIQ_t$ than the opposite, as shown by the values of R^2. This is particularly true for the German DAX, index for which the value of $R^{"}$ of h_t as regressor almost double the corresponding value of $\log ILLIQ_t$.

As shown in Table 5, the relationship between $\log ILLIQ_t$ and h_t appears even stronger when one removes the data belonging to the region U_α, which includes the efficient phases of the stock market. Focusing on the inefficient periods, the t-statistics, the standard errors and the p-values all improve; the R^2's of the three indices increase on the average of about 0.024. We ascribe this improvement in the quality of fits to the removal of the noisy information which induces non substantial variations in the stock prices when markets change in a purely random way.

Table 7 summarizes the results of regressions (15) and (16), which test the autoregressive model AR(1) for both the measures. As expected, for all the windows δ, the parameters λ_1 are positive, significant and very close to those estimated in Amihud (2002). Consistently, we also observe a positive and significant ρ_1 for all δ's. These findings confirm that investors predict both illiquidity and pointwise regularity (or volatility, if one changes perspective) for future window, based on information available in the current one.

Tables 8 and 9 summarize the behavior of the variations of the excess returns (with respect to the Treasury Bill) on a monthly and quarterly basis, respectively. The effects generated by the interventions of the Federal Reserve are quite evident. Before the large-scale asset purchases, namely in the period September 2006-October 2008, all the values of η_1 and κ_1 are positive and statistically significant, whereas they change dramatically and fall to zero in the whole period November 2008-October 2014. Notice that in this period the relationship between the excess returns and the illiquidity does not hold any longer, as revealed by all the statistics reported in the tables. This indicates that the premium for illiquidity is completely inelastic during the quantitative easing programs and this result sounds quite reasonable, if one considers that markets perceived that Federal Reserve was acting as a lender of last resort able to provide as much liquidity as needed. This behavior can be appreciated more in detail looking at Figure 2, which displays the evolution of η_1 and κ_1 into a window of 25 points, about two trading years, starting from September 2006 (the shaded areas indicate the interventions). Several remarks can be formulated:

• from September 2006 to October 2008, the increment of illiquidity causes an increment in the excess return, that is lower stock prices[10].

• the impact of QE1 is strong and more pronounced for the two U.S. indexes than for the DAX. This is reasonable in view of the fact that the interventions of the Fed obviously have had a deeper spillover on the U.S. economy and, only by contagion, have caused variations in the European economy. Interestingly, at the end of the QE1, η_1 is zero for both the S&P500 and the NASDAQ, indicating that the intervention indeed worked to the extent that the variation of the excess return with respect to the illiquidity was reduced to zero;

• as of the beginning of the Maturity Extension Program (September 2011) until the end of QE3, the slope is statistically equal to zero for all the indexes (the premium for illiquidity is therefore constant), even if for the German DAX the values are reasonably slightly larger;

[10]The mechanism is well described by Amihud and Mendelson (2015): "[...] The general decline in illiquidity over the recent decades may explain in part the rise in the stock market prices. By the Amihud and Mendelson (1986) theory, investors require lower expected returns on equity claims (relative to Treasury bills, which have the greatest liquidity) when the illiquidity of these claims is lower. This means that for any given cash flows that the stocks generate — in terms of level, risk and growth expectations — the stock value should be higher when illiquidity falls. This is what we have generally witnessed since the end of the early 1980s."

• as of the end of QE3 the slopes increase for the S&P500 and the NASDAQ and become significantly larger than zero at 5% starting from December 2014. It seems very consistent the different behavior of the DAX, whose η_1 remains at zero long after the end of QE3, as a presumable consequence of the two Longer-Term Refinancing Operations (LTRO), which meanwhile the European Central Bank (ECB) had launched[11];

• κ_1 evolves in perfect accordance with η_1 to the point that the Pearson's correlation coefficient estimated between η_1 and κ_1 is equal to 0.9930 for the S&P500, 0.9870 for the NASDAQ and 0.9608 for the DAX, denoting that the two measures indeed characterize the premium for illiquidity almost identically. Furthermore, the positive relationship between the excess return and $(1-h_t)$ means that we can expect higher returns as h_t decreases. This result confirms previous empirical evidence on the U.S. stock market (see e.g. Bianchi et al. (2015)).

The overall behavior of the regressions parameters looks strongly consistent; indeed, starting from a situation where the illiquidity was perceived by financial markets as the main obstacle to their proper functioning (high slope of the excess returns with respect to the illiquidity), the exogenous intervention of the Federal Reserve has had the (hoped) effect to lubricate markets to such an extent to reduce to zero the perceived risk of illiquidity (zero slope). Once the interventions expired, the premium for illiquidity has started growing again, even if at an acceptable level (low slope).

5. CONCLUSIONS

In this paper we have examined the relationship between liquidity and efficiency. When the latter is calculated following a new, topology-based approach, the two notions appear to be intimately related and their relation becomes even stronger when markets experience inefficient periods. We also provide evidence that the efficiency measure is a better forecaster, in terms of its goodness of fit, of the illiquidity measure even when observations lagged up to five days are considered. We have used both the measures to analyze how three main stock indices (S&P500, Nasdaq and DAX) have responded to the unconventional stimuli launched by the Federal Reserve to control the liquidity crunch triggered by the 2007-2008 global financial crisis. Our findings reveal that the variation of the illiquidity premium demanded by investors has become statistically not significant with the in-

[11]The first LTRO program was announced on December 20, 2011, with allotment date on December 21, settlement date on December 22, early repayment date on January 30, 2013 and maturity date set on January 29, 2015; the second LTRO program was announced on February 28, 2012, with allotment date on February 29, settlement date on March 1, early repayment date on February 27, 2013 and maturity date set on February 26, 2015.

crease of the amount of monetary interventions. Consistently with previous studies, we also find that this effect, as it is reasonable, is lagged for the German DAX with respect to the U.S. indices; it is also lightly less pronounced during the Twist Operation and the QE3 and, differently from the behavior of the two U.S. indices (whose variations start increasing a little bit since the end of QE3), becomes statistically zero in the same period, what is ascribable to the LTRO run by the European Central Bank. The relation between the liquidity and the efficiency measures that we use in this work is confirmed by the behavior that we observe for the changes of the premium with respect to the pointwise volatility.

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