ANNALS OF ECONOMICS AND FINANCE **20-2**, 525–540 (2019)

The Balancing Act: The Optimal Assignment of New Players in Sports Leagues*

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In this paper, we investigate the optimal allocation of new players in professional sports leagues. A new player is to be allocated to the teams in a league. The league maximizes the attractiveness of the contests, which is equivalent to suspense and competitive balance in our model. Meanwhile, the teams maximize their winning probabilities. In the static model, we show that it is always optimal for the league to allocate the new player to the weakest team. However, competition between teams for this new player may or may not lead to this optimal allocation. In the dynamic model, we show that allocating the new player to the weakest team with probability one may lead to shirking in the teams in the initial periods; this probability must be low enough to induce full effort from the teams.

Key Words: Sports leagues; Competitive balance; Optimal allocation.
JEL Classification Numbers: J44, Z22, Z28.

1. INTRODUCTION

Sports leagues are a dominant form in the world of professional sports. These sports leagues are in charge of organizing the contests between teams, as well as selling the rights of broadcasting the contests and various kinds of advertisement. A team in a league certainly cares mostly about win-

* Song would like to acknowledge the financial support from the National Natural Science Foundation of China Project (No. 71232013), Zhongying Young Scholar Program, and Social Science Fund of Zhejiang Province (No. 17NDJC191YB). Wang would like to acknowledge the financial support from Zhejiang University during his stay there and from SSHRC of Canada. Ye would like to acknowledge the financial support from the National Natural Science Foundation of China Project (No. 71773112) and Hengyi Fund of Zhejiang University.

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1529-7373/2019

ning the contests. When it wins more contests, it receives more corporate sponsorships, sells more team related merchandizes, has a larger chance for advancing to higher divisions, and even increases its stock price if the team is publicly listed[1].

The leagues, on the other hand, have different objectives. Their goal is to produce attractive games. Fans have a strong preference for uncertain outcomes. Quirk and Fort (1995) argue that "one of the key ingredients of the demand by fans for team sports is the excitement generated because of the uncertainty of outcome of league games." The main generator of this excitement is called the "competitive balance." When the strengths of the teams are more balanced, the games are more competitive, and the outcomes are more uncertain. In contrast, when the strengths of the teams are more unevenly distributed, the games are more lopsided, and the outcomes are more predictable.

This competitive balance is vital to sports leagues. The elimination of competition in professional sports effectively eliminates the industry, as noted by Neale (1964) and El-Hodiri and Quirk (1971). MLB's Blue Ribbon Panel (2000) argue that the lack of competitive balance during the 1990s severely harmed the game of professional baseball. To help maintain the competitive balance in games, a league may regulate the movement of players already in the league, and put in place a draft system for players new to the league.

The draft rules in the NBA are a typical example. In the early days, top new players were assigned to the lowest ranked teams. This was certainly a consideration to balance the competitive strengths of the teams in NBA. But some authors find the perverse incentive effects associated with such draft system. Taylor and Trogdon (2002) find that the draft for assigning new players appears to have given an incentive to teams eliminated from contention for play-offs to lose matches. Price et al (2010) find that this match losing behavior is more significant near the end of the season.

In this paper, we examine a sports league with teams competing in contests. Each team is endowed with a strength. In a contest, teams compete against each other. The winning probability of a team depends on its own strength and the total strength of the teams. We focus on the allocation of new players. To be precise, we assume that there is only one new player with a certain strength to be allocated. This new player represents the difference in strength between the best new player and the rest of the players. If this new player joins a particular team, the strength of that team is increased by the strength of this new player. The league's objective is to maximize the attractiveness of the contests, which is the same as maximiz-

[1]See Andreff and Saudohar (2000).

ing the competitive balance of the teams. Meanwhile, each team's objective is to maximize its winning probabilities in the contests.

We first examine the optimal allocation of the new player for the league. Not surprisingly, we find that in either the two-team case or the three-team case, it is always optimal to allocate the new player to the weakest team. We then investigate whether a competitive mechanism such as an English auction can achieve the league's objective. We calculate the willingness to pay for the new player by each team, and the team with the highest willingness to pay wins the new player. We find that, in the two-team case, the two teams have the same willingness to pay for the new player. Therefore, the weaker term wins only with 50% probability, and the league's objective is not achieved. However, when there are three teams, we find that a weaker team has a higher willingness to pay for the new player. Moreover, the stronger a team is, the lower its willingness to pay for the new player. Therefore, the weakest team will definitely win the player in a competitive mechanism, and the league's object is achieved.

Since a competitive mechanism to allocate the new player may or may not achieve the league's objective, a league usually has some draft rules to allocate a new player. We investigate the optimal draft rule in the two-team case and determine the optimal probability for the losing team to be allocated the new player. In this case, a team's incentive to play with full strength becomes relevant. We show that in a two-period model, the teams always have incentive to use their full strength in the contest. However, in a multi-period model, the teams may not use their full strength in the contest in order to lose, if the losing team has a high enough probability of being allocated the new player according to the draft rule. To discourage such incentive to lose (i.e., not using the full strength), the league must reduce the probability of the losing team being allocated the new player. This last conclusion has strong support from the evolution of the NBA draft rules.[2]

Our paper is related to Grier and Tollison (1994), who examine the empirical evidence from the winning records in the National Football League and find that rookie draft promotes competitive balance and is indeed a balancing institution. Meanwhile, Fort and Quirk (1995) assess the degree to which different mechanisms create greater balance. They conclude that neither the reserve clause nor the reverse-order amateur draft aid balance.[3]

[2]Taylor and Trogdon (2002) examine three NBA seasons to determine whether team performance responded to changes in the underlying tournament incentives provided by the NBA's introduction and restructuring of the lottery system to determine draft order. They find strong evidence that NBA teams are more likely to lose when incentives to lose are present.

[3]Fort and Quirk (2011) study whether or not an increase in balance will increase social welfare, an issue not examined in our paper.

Later, Borland et al. (2009) provide an important empirical study on the Australian National Football League and show how the effect of a draft mechanism will depend on the environment in which it applied and the details of the mechanism. In particular, they offer explanation for why the draft system is more important for small team game like NBA than for large team game like the football in Australia, as the effect of an extra high-ability player on club performance may be relatively high in the former.

Our paper is also related to the literature on auctions with externality. The allocation of an essential input to firms imposes externality on other firms. Jehiel, Moldovanu, and Stacchetti (1996), for example, consider mechanisms that take into consideration of the externality on valuations that depend on the allocation of the good being auctioned. In our three-team model, the willingness to pay for the new player by a particular team depends on which other team will get the player if it does not get the player.

The incentive analysis in our paper is related to Taylor and Trogdon (2002), who point out that while the allocation of new talent to lower-ranked teams can contribute to the overall league parity, the shirking effect can be detrimental to the health of any sports league. We show that teams will play with full strength only if the probability of allocating the new player to the losing team is low enough.

The rest of the paper is organized as follows. In Section 2, we set up a static model and examine the optimal allocation of a new player to the league. We also examine whether a competitive mechanism for the new player can achieve the league's objective. In Section 3, we set up a dynamic model. In this section, we investigate whether teams have the incentive to play with full strength. We also establish the optimal draft rule for the league. In Section 4, we offer a simple extension regarding a team's objective and conclude.

2. THE STATIC MODEL

In this section, we will investigate the optimal assignment of a new player in a static model. We first suppose that there are two teams in a league. We then generalize the analysis to three teams. Here, our central question is whether competition between the teams for the player can achieve the league's objective.

2.1. The Two-Team Case

Consider a league consisting of two teams, team 1 and team 2. Team i has some existing players, with their total strength denoted by T_i, $i = 1, 2$. Without loss of generality, we assume that team 1 is weakly stronger than team 2, i.e., $T_1 \geq T_2$.

These two teams compete against each other in a sport contest. Following the contest literature, we adopt the following widely used Contest Success Function (CSF) to denote the probability of winning for team i:

$$P_i = \frac{T_i}{T_i + T_j}, \quad i \neq j.$$

Suppose that a new player of strength $a > 0$ becomes available to the league. In reality, there could be many new players available for the teams to choose from. But some players are obviously better than others. If each team can obtain one new player, then this a represents the difference in strength between the best player and the second best player. Then the question becomes which team should obtain the best player.

Suppose that team i obtains the new player. Then team i's total strength is increased to $T_i + a$ and the team's probability of winning in the contest becomes

$$P_i(i) = \frac{T_i + a}{(T_i + a) + T_j}.$$

In this case, team j's probability of winning becomes $P_j(i) = 1 - P_i(i)$. Here, we implicitly assume that the strength of the new player is team-independent.

How should the league assign this new player? The contest between these two teams attracts viewers. The league can sell the rights to broadcast this contest to a TV network. How much the TV network is willing to pay for the rights depends on how much advertising revenue it can generate, which in turn depends on how attractive the contest is. The league's objective is to maximize the attractiveness of the contest.

In the sports psychology literature, when the two sides of a contest are close in strength, suspense is generated, as the contest outcome becomes uncertain. In a contest between two teams, if P denotes the probability of winning for a team, then we define $S = P(1 - P)$ as the amount of suspense in the contest.[4] In this paper, we use this S as the index for the attractiveness of the contest. Note that the closer the probabilities of winning of these two teams, the higher the index of attractiveness. This index is maximized at $P = \frac{1}{2}$, and minimized when $P = 0$, or $P = 1$. The league's objective is to maximize this S by allocating the new player to the appropriate team, which is denoted by i:

$$\max_{i=1,2} S(i) = P_i(i)[1 - P_i(i)]$$

[4]This is called the competitive balance in Palominoa and Sakovics (2004).

Since

$$\begin{aligned} S(2)-S(1) &= P_1(2)[1-P_1(2)]-P_1(1)[1-P_1(1)] \\ &= \frac{a(T_1-T_2)}{(T_1+T_2+a)^2} \geq 0, \end{aligned}$$

where the above inequality holds because $T_1 \geq T_2$, the weak team (team 2) should obtain the new player to maximize the attractiveness. We have the following lemma.

Lemma 1. *Assigning the new player to the weak team maximizes the attractiveness of the contest and thus also maximizes the league's objective.*

This lemma is easy to understand. Increasing the strength of the weaker team would increase the competitive balance of the contest, while increasing the strength of the stronger team would make the contest lopsided and reduce the suspense of it.

The league can assign the new player to the weak team directly, or allow the teams to compete for the new player. The question is, whether competition for the new player will enable the weak team to obtain him.

For simplicity, assume that a team's willingness to pay for the player is equal to the difference in probability between winning him and losing him. For example, for team i, the willingness to pay for the new player is given by

$$V_i \equiv P_i(i)-P_i(j)=\frac{T_i+a}{(T_i+a)+T_j}-\frac{T_i}{T_i+(T_j+a)}=\frac{a}{T_i+T_j+a}.$$

Note that if firm i loses the player, he will be obtained by firm j. Similarly,

$$V_j \equiv P_j(j)-P_j(i)=\frac{T_j+a}{(T_j+a)+T_i}-\frac{T_j}{T_j+(T_i+a)}=\frac{a}{T_i+T_j+a}.$$

Since the two teams have the same willingness to pay for the new player, the competition for the player (say, in an English auction) ends up in a tie. With a 50-50 tie breaking rule, the player goes to each team with probability 0.5. In this case, the attractiveness of the contests becomes $\frac{1}{2}S(1)+\frac{1}{2}S(2)$. Since we learn from Lemma 1 that $S(1) \leq S(2)$, the competition between teams for the new player cannot reach the maximum attractiveness of the league, which is equal to $S(2)$. Consequently, a competitive mechanism fails to maximize the league's objective.

Proposition 1. *Suppose that a team's willingness to pay for the new player is equal to the team's incremental winning probability contributed by* *the new player, then a competitive mechanism such as an English auction cannot achieve the league's objective.*

2.2. The Three-Team Case

In the last subsection, we show that a competitive mechanism does not maximize the league's objective, which is equal to the attractiveness of the contest. In this subsection, we will examine the case of three teams to see if the same conclusion holds.

Suppose that there are three teams: 1, 2 and 3. Without loss of generality, assume that $T_1 \geq T_2 \geq T_3$. Therefore, team 1 is the strongest, and team 3 is the weakest. Teams compete against each other in pair-wise contests; each team will compete against two other teams in two separate contests. Therefore, there are a total of three separate contests.

We first calculate the winning probability for each team in each contest. The winning probability for team i competing against team j when the new player joins team k is denoted by $P_{ij}(k)$, $i \neq j$. Then we have

$$\begin{aligned} P_{ij}(i) &= \frac{T_i + a}{(T_i + a) + T_j}, \\ P_{ij}(j) &= \frac{T_i}{T_i + (T_j + a)}, \\ P_{ij}(k) &= \frac{T_i}{T_i + T_j}, \; k \neq i,\, j. \end{aligned}$$

Suppose that the new player joins team i. In the case of three teams, we define the attractiveness of the league as the total attractiveness of the three separate contests between the teams. That is,

$$\begin{aligned} S(i) &= P_{ij}(i)[1 - P_{ij}(i)] + P_{ik}(i)[1 - P_{ik}(i)] + P_{jk}(i)[1 - P_{jk}(i)] \\ &= P_{ij}(i)P_{ji}(i) + P_{ik}(i)P_{ki}(i) + P_{jk}(i)P_{kj}(i) \\ &= \frac{T_i + a}{T_i + a + T_j}\frac{T_j}{T_i + T_j + a} + \frac{T_i + a}{T_i + T_k + a}\frac{T_k}{T_i + T_k + a} + \frac{T_j}{T_j + T_k}\frac{T_k}{T_j + T_k} \\ &= \frac{(T_i + a)T_j}{(T_i + T_j + a)^2} + \frac{(T_i + a)T_k}{(T_i + T_k + a)^2} + \frac{T_j T_k}{(T_j + T_k)^2}. \end{aligned}$$

We have the following lemma.

Lemma 2. *Assume that $a \leq T_2 - T_3$. Then assigning the new player to the weakest team (team 3) will maximize the league's attractiveness.*

Proof. We first calculate the difference in attractiveness for the new player joining different teams. We have

$$\begin{aligned} S(i) - S(k) &= \left[\frac{(T_i+a)T_j}{(T_i+T_j+a)^2} + \frac{(T_i+a)T_k}{(T_i+T_k+a)^2} + \frac{T_jT_k}{(T_j+T_k)^2}\right] \\ &\quad - \left[\frac{(T_k+a)T_j}{(T_k+T_j+a)^2} + \frac{(T_k+a)T_i}{(T_i+T_k+a)^2} + \frac{T_jT_i}{(T_j+T_i)^2}\right] \\ &= \frac{a(T_k-T_i)}{(T_i+T_k+a)^2} + \left[\frac{(T_i+a)T_j}{(T_i+T_j+a)^2} - \frac{(T_k+a)T_j}{(T_k+T_j+a)^2}\right] \\ &\quad - \left[\frac{T_jT_i}{(T_j+T_i)^2} - \frac{T_jT_k}{(T_j+T_k)^2}\right]. \end{aligned}$$

Suppose that team k is the weakest team; that is, $T_k \le T_i$ and $T_k \le T_j$. Then the first term in the RHS of the above expression is negative. Define

$$f(a) = \frac{1}{T_j}\left[\frac{(T_i+a)T_j}{(T_i+T_j+a)^2} - \frac{(T_k+a)T_j}{(T_k+T_j+a)^2}\right].$$

Then $f(0) = \frac{1}{T_j}\left[\frac{T_jT_i}{(T_j+T_i)^2} - \frac{T_jT_k}{(T_j+T_k)^2}\right]$ and $f\prime(a) = \frac{(T_j-T_i-a)}{(T_i+T_j+a)^3} - \frac{(T_j-T_k-a)}{(T_j+T_k+a)^3}$. Note that $T_j - T_i - a \le T_j - T_k - a$ and $T_j + T_i + a \ge T_j + T_k + a$. If $T_j - T_k - a \ge 0$, then $f\prime(a) \le 0$, and $f(a) \le f(0)$. Therefore, $S(i) \le S(k)$.

To guarantee that $S(1) \le S(3)$ and $S(2) \le S(3)$, we need to have $T_2 - T_3 - a \ge 0$, and $T_1 - T_3 - a \ge 0$. Since $T_1 \ge T_2$, we only need $T_2 - T_3 - a \ge 0$. This inequality guarantees that $S(3)$ is higher than $S(1)$ and $S(2)$; that is, the new player joining team 3 achieves the league's maximum attractiveness. ▮

To investigate how a competitive mechanism will allocate the player, we need to define a team's objective in the three-team case. Similarly to the two-firm case, assume that a team's payoff is equal to the sum of its winning probabilities in its two contests. Then team i's payoff when it obtains the new player is given by

$$P_i(i) \equiv P_{ij}(i) + P_{ik}(i) = \frac{T_i+a}{T_i+a+T_j} + \frac{T_i+a}{T_i+a+T_k}.$$

Similarly, team i's payoff when team $j \ne i$ obtains the new player is

$$P_i(j) = P_{ij}(j) + P_{ik}(j) = \frac{T_i+a}{T_i+T_j+a} + \frac{T_i}{T_i+T_k}.$$

Suppose that $k \ne i$, j. Then team i's difference in payoffs between the situations where team j obtains the new player and where team k obtains the new player is given by the following:

$$\begin{aligned} P_i(j) - P_i(k) &= T_i\left[\frac{1}{T_i+T_j+a} + \frac{1}{T_i+T_k} - \frac{1}{T_i+T_k+a} - \frac{1}{T_i+T_j}\right] \\ &= T_i\left[\left(\frac{1}{T_i+T_j+a} - \frac{1}{T_i+T_k+a}\right) - \left(\frac{1}{T_i+T_j} - \frac{1}{T_i+T_k}\right)\right]. \end{aligned}$$

Define $h(a) = \frac{1}{T_i+T_j+a} - \frac{1}{T_i+T_k+a}$. Then $h(0) = \frac{1}{T_i+T_j} - \frac{1}{T_i+T_k}$, and $h\prime(a) = -\frac{1}{(T_i+T_j+a)^2} + \frac{1}{(T_j+T_k+a)^2} \geq 0$ if $T_j \geq T_k$. Therefore, $h(a) > h(0)$, and thus $P_i(j) - P_i(k) \geq 0$ if $T_j \geq T_k$. We have the following lemma.

LEMMA 3. *If $T_j \geq T_k$, then $P_i(j) - P_i(k) \geq 0$, where $i \neq j$, k. That is, suppose that a team does not obtain the new player, then its payoff is higher if the remaining stronger team obtains the new player.*

We next examine which team has the highest willingness to pay for the new player. Note that in this three-team situation, team i's willingness to pay depends on which other team will obtain the new player if team i does not obtain him. Suppose that if team i does not obtain the new player, then team j will obtain him. (In this case, team i is effectively competing against team j for the new player.) Then team i's willingness to pay V_{ij} is the difference in its payoffs between the situations where team i obtains the player and where team j obtains him. That is,

$$\begin{aligned} V_{ij} &\equiv P_i(i) - P_i(j) \\ &= \left(\frac{T_i+a}{T_i+a+T_j} + \frac{T_i+a}{T_i+a+T_k}\right) - \left(\frac{T_i}{T_i+T_j+a} + \frac{T_i}{T_i+T_k}\right). \end{aligned}$$

Similarly, team j's willingness to pay V_{ji} when it competes against team i for the new player is given by

$$\begin{aligned} V_{ji} &= P_j(j) - P_j(i) \\ &= \left(\frac{T_j+a}{T_i+T_j+a} + \frac{T_j+a}{T_j+a+T_k}\right) - \left(\frac{T_j}{T_i+T_j+a} + \frac{T_j}{T_j+T_k}\right). \end{aligned}$$

To see which willingness to pay is higher, we take the difference of the two:

$$\begin{aligned} & V_{ij} - V_{ji} \\ &= \frac{T_i+a}{T_i+a+T_k} - \frac{T_i}{T_i+T_k} + \frac{T_j}{T_j+T_k} - \frac{T_j+a}{T_j+a+T_k} \\ &= \left(\frac{T_i+a}{T_i+a+T_k} - \frac{T_j+a}{T_j+a+T_k}\right) - \left(\frac{T_i}{T_i+T_k} - \frac{T_j}{T_j+T_k}\right). \end{aligned}$$

Define $g(a) = \frac{T_i+a}{T_i+a+T_k} - \frac{T_j+a}{T_j+a+T_k}$. Then $g(0) = \frac{T_i}{T_i+T_k} - \frac{T_j}{T_j+T_k}$, and $g\prime(a) = \frac{T_k}{(T_i+T_k+a)^2} - \frac{T_k}{(T_j+T_k+a)^2} \leq 0$ if $T_i \geq T_j$. Therefore, $g(a) < g(0)$, and thus $V_{ij} \leq V_{ji}$ if $T_i \geq T_j$. We have the following lemma.

LEMMA 4. *If $T_i \geq T_j$, then $V_{ij} \leq V_{ji}$. That is, when two teams are competing against each other for the new player, the weaker team's willingness to pay is higher.*

From Lemmas 3 and 4, we can conclude the following:

$$V_{12} \leq V_{21} \leq V_{23} \leq V_{32}$$

and

$$V_{12} \leq V_{13} \leq V_{31} \leq V_{32}.$$

We have the following proposition.

PROPOSITION 2. *Suppose that $T_1 \geq T_2 \geq T_3$. Then team 3 has the highest willingness to pay for the new player. If a competitive mechanism such as an English auction is used to allocate the new player, then team 3 wins the new player. The objective of the league is achieved.*

This proposition shows that the weakest team values the new player the most. This result implies that the most talented players in a league would flow to the weakest team if the teams are allowed to compete for the players and if the teams' budget constraints are not binding. Traditional arguments against free agency state that without the binding restrictions of the reserve clause, the most talented players would gravitate toward the large market franchises (strong teams). According to our analysis here, in the contrary, talented players (i.e., playing strengths) are dispersed among teams over time. Our analysis is in line with the empirical study by Vrooman (1996), who finds that competitive balance (measured as season-to-season performance discontinuity) is not achieved during the pre-free-agency period in the sense that it is characterized by significant season-to-season continuity and BIG4 dominance. However, after the reform, a gradual erosion of season-to-season continuity of performance appeared.

3. THE DYNAMIC MODEL

In the previous section, we conclude that assigning the new player to the weakest team would achieve the league's objective, which is to maximize the attractiveness of the contests, and thus the competitive balance of the teams. In this section, we will investigate the effect of this kind of draft rules on the behavior of the teams in a dynamic setting. Teams in a professional sports league compete for many seasons, and a new player assigned to a team will play for that team for many seasons. In this section, we assume that the teams first compete in period 1, and at the beginning of period 2, a new player is assigned to a team according to the result of the contest in period 1. Once assigned, the new player will play for that team for N periods.

3.1. The Strategic Behavior of the Teams

We first suppose that $N = 1$ and teams 1 and 2 compete for a total of $1 + 1$ periods. Again, let T_i be the strength of team i, with $T_1 \geq T_2$. In period 1, team i chooses $\tilde{T}_i \in [\epsilon T_i, T_i]$ in the contest, where $\epsilon \in (0, 1)$. In this section, we allow a team to use only partial strength in the contest. We assume that the minimum strength a team can use is proportional to its real strength. This would make some later analysis much simpler.

There is no cost saved for a team not using its full strength. But as we shall see, a team may want to do this to lower the probability of winning in the first period, so that it has a better chance to get the new player in the next period.[5]

A new player is available in period 2. In period 2, we assume that the team who lost in period 1 will get the new player with probability $q \in [0, 1]$, while the team who won in period 1 will get him with probability $1 - q$. A team's payoff is equal to the sum of its winning probabilities in the two periods, and therefore it would choose its playing strength to maximize this sum.

Suppose that team i chooses $\tilde{T}_i$ in period 1. Then the winning probability is $\frac{\tilde{T}_i}{\tilde{T}_1+\tilde{T}_2}$ for team i, $i = 1, 2$. Consider team 1. If it wins in period 1, with probability $1 - q$ it will get the new player in period 2 and its winning probability in period 2 will become $\frac{T_1+a}{T_1+T_2+a}$. This is because each team will choose its full strength in the contest in period 2, as there is no gain for strength manipulation. Similarly, with probability q, team 1 will not get the new player, and its winning probability will become $\frac{T_1}{T_1+T_2+a}$ in period 2. Meanwhile, if team 1 loses in period 1, it will get the new player with probability q and its winning probability in period 2 will become $\frac{T_1+a}{T_1+T_2+a}$. Similarly, with probability $1 - q$, it will not get the new player and its winning probability in period 2 will become $\frac{T_1}{T_1+T_2+a}$. So the expected sum

[5]To legally avoid using full strength in a competition, as Borland et al (2009) put it, a team "... exerting more 'losing effort' might, for example, involve choosing a team or adopting a player management regime that does not maximize a club's chance of winning a match (such as allowing star players to have season-ending surgery to deal with injuries)."

of winning probabilities in the two periods for team 1 can be written as

$$\Pi_1 = \tilde{P}_1 + \tilde{P}_1[(1-q)P_1(1) + qP_1(2)] + (1-\tilde{P}_1)[qP_1(1) + (1-q)P_1(2)]$$

where

$$\tilde{P}_1 = \frac{\tilde{T}_1}{\tilde{T}_1 + \tilde{T}_2}, \quad P_1(1) = \frac{T_1 + a}{T_1 + T_2 + a}, \quad P_1(2) = \frac{T_1}{T_1 + T_2 + a}.$$

Taking the derivative of Π_1 with respect to team 1's choice $\tilde{T}_1$, we have

$$\frac{\partial \Pi_1}{\partial \tilde{T}_1} = \frac{\tilde{T}_2}{(\tilde{T}_1 + \tilde{T}_2)^2}\left(1 + \frac{a(1-2q)}{T_1 + T_2 + a}\right) > 0.$$

where the last inequality is obtained as $|1-2q| < 1$. Therefore, team 1 (and thus team 2) would use its full strength in the contest in period 1.

Now we consider a $1+N$ period game. Suppose that these two teams play for N periods after period 1. In period 1, team i chooses $\tilde{T}_i \in [\epsilon T_i, T_i]$ in the contest. Again, a new player is available at the beginning of period 2. If a team gets the new player, the new player will play for that team for N periods. A team's payoff is equal to the sum of its winning probabilities in these $N+1$ periods. It is obvious that each team will play with full strength from period 2 to period $N+1$, since there is no gain for strength manipulation. Therefore, team 1's payoff in this game becomes

$$\Pi_1(N) = \tilde{P}_1 + \tilde{P}_1[(1-q)NP_1(1) + qNP_1(2)] + (1-\tilde{P}_1)[qNP_1(1) + (1-q)NP_1(2)],$$

that is, the period 2 winning probabilities in the above Π_1 are all multiplied by N. The corresponding derivative of $\Pi_1(N)$ with respect to $\tilde{T}_1$ becomes

$$\frac{\partial \Pi_1(N)}{\partial \tilde{T}_1} = \frac{\tilde{T}_2}{(\tilde{T}_1 + \tilde{T}_2)^2}\left(1 + \frac{Na(1-2q)}{T_1 + T_2 + a}\right).$$

Similarly, $\frac{\partial \Pi_2(N)}{\partial \tilde{T}_2}$ can be obtained by switching 1 and 2 in the above expression. We have the following lemma.

LEMMA 5. *If $q > \frac{T_1+T_2+(N+1)a}{2Na}$, then $\frac{\partial \Pi_1}{\partial \tilde{T}_1} < 0$ and $\frac{\partial \Pi_2}{\partial \tilde{T}_2} < 0$. In this case, each team plays with its minimum strength in the first period. If $q \leq \frac{T_1+T_2+(N+1)a}{2Na}$, then $\frac{\partial \Pi_1}{\partial \tilde{T}_1} \geq 0$ and $\frac{\partial \Pi_2}{\partial \tilde{T}_2} \geq 0$. In this case, each team plays with its maximum strength in the first period.*

When q is larger, the benefit to losing in period 1 is higher, as the probability becomes larger for the losing team to get the new player who will play for the team for N periods. Therefore, a team will try its best to lose the first period contest by not playing with full strength. When q is smaller, the benefit to losing in period 1 is lower, as the probability of getting the new player is lower. In this case, a team will play with full strength in period 1 to maximize the probability of winning.

3.2. The Optimal Allocation Mechanism

The objective of the league is to maximize the total attractiveness of the contests in the $N+1$ periods by choosing q, taking into consideration of the strategic behavior of the two teams in the first period:

$$\begin{aligned} TA \;=\; & \tilde{P}_1(1-\tilde{P}^1) + \tilde{P}_1[(1-q)NS(1) + qNS(2)] \\ & + (1-\tilde{P}_1)[qNS(1) + (1-q)NS(2)]. \end{aligned}$$

where $S(1) = P_1(1)[1-P_1(1)]$ and $S(2) = P_1(2)[1-P_1(2)]$ are the attractiveness of the contests when the new player joins team 1 and team 2, respectively.

According to Lemma 5, there are two cases. When $q \leq \frac{T_1+T_2+(N+1)a}{2Na}$, we have $\frac{\partial \Pi_1}{\partial \tilde{T}_1} \geq 0$ and $\frac{\partial \Pi_2}{\partial \tilde{T}_2} \geq 0$. In this case, a team's payoff increases with its period 1 playing strength, and therefore it will play with full strength in period 1 in the equilibrium. When $q > \frac{T_1+T_2+(N+1)a}{2Na}$, we have $\frac{\partial \Pi_1}{\partial \tilde{T}_1} < 0$ and $\frac{\partial \Pi_2}{\partial \tilde{T}_2} < 0$. In this case, a team's payoff decreases with its period 1 playing strength, and therefore it will play with minimum strength $\tilde{T}_i = \epsilon T_i$ in period 1 in the equilibrium.

What would be the optimal q for the league to maximize its objective? When $q > \frac{T_1+T_2+(N+1)a}{2Na}$, we have $\tilde{T}_i = \epsilon T_i$. Therefore, $\tilde{P}_1 = \frac{\tilde{T}_1}{\tilde{T}_1+\tilde{T}_2} = \frac{T_1}{T_1+T_2} \geq \frac{1}{2}$. In this case,

$$\frac{dTA}{dq} = N[\tilde{P}_1 - (1-\tilde{P}_1)][S(2) - S(1)] \geq 0,$$

where $S(2) \geq S(1)$ from Lemma 1. Therefore, $q = 1$ maximizes TA in this case.

When $q \leq \frac{T_1+T_2+(N+1)a}{2Na}$, we have $\tilde{T}_i = T_i$. Therefore, $\tilde{P}_1 = \frac{\tilde{T}_1}{\tilde{T}_1+\tilde{T}_2} = \frac{T_1}{T_1+T_2} \geq \frac{1}{2}$. Again, in this case,

$$\frac{dTA}{dq} = N[\tilde{P}_1 - (1-\tilde{P}_1)][S(2) - S(1)] \geq 0,$$

Therefore, $q = \frac{T_1+T_2+(N+1)a}{2Na}$ maximizes TA in this case. Hence, $q = 1$ maximizes TA overall. We have the following proposition.

Proposition 3. *The league's optimal probability of the losing team getting the new player is 1.*

When $q = 1$, the teams will not use their maximum strength in period 1. Because $q = 1$ maximizes the chance that the weaker team will get the new player, it is still optimal for the league to do so. In this way, the attractiveness of the contests from period 2 to period N is maximized.

From Lemma 5, we know that with the optimal q, the above proposition, the two teams will play with their minimum strength in period 1. If the league would like the teams to play with their maximum strength in period 1, the maximum q the league can set is given by the following proposition.

Proposition 4. *The optimal q maximizes the league's objective conditional on the teams are still induced to play with their maximum strength in period 1 is given by* $q = \frac{T_1+T_2+(N+1)a}{2Na}$.

This conclusion can be seem from the fact that the league's objective is increasing in q. From Lemma 5, the q in the above proposition is the maximum q with which the teams will still play with full strength in period 1. Setting $q = 1$ provides too much incentive for the teams not to play with their maximum strength, as the losing team is allocated with the new player with probability 1. This is probably the reason why most sports leagues do not set $q = 1$ in the draft of new players.

4. CONCLUSION

In this paper, we develop a model of allocating new players in a sports league. We analyze both a static model and a dynamic model. In the static model, teams bid for the new player and then compete in contests. In the dynamic model, each team's chance to be allocated with the new player in the subsequent periods depends on the result of the contest in the first period.

In the static model, we investigate the allocation of a new player among teams through two mechanisms: a competitive mechanism and a centralized mechanism. We find that, if the league has the authority to assign the new player, then it will always assign this player to the weakest team. For teams that maximize their own winning probabilities, both teams have the same willingness to pay for the new player in the two-team case, and the weakest team has the highest willingness to pay in the three-team case. Furthermore, the higher the strength a team has, the lower its willingness to pay becomes.

In the dynamic model, we investigate the typical draft system in professional sports leagues. If the new player is always assigned to the weakest team, then teams have incentive not to play with full strength. Given that the league's objective is to maximize the suspense in the contests (i.e. the competitive balance of the contests), it is optimal for the league to assign the new player to the losing team with probability 1. But if the league wants the teams to always play with maximum strength, then that probability has to be less than 1. This result is supported by the evolution of the draft systems in many professional sports leagues.

In the analysis in this paper, we assume that a team's objective is to maximize its winning probability. In reality, teams often get a share of the revenue from the league in additional to the benefit from winning a contest.[6] Suppose that we extend a team's objective function to a linear combination of the league's revenue and the team's winning probability. Consider the two-team case. Suppose that we change team i's objective function to $W_i(i) = S(i) + \beta P_i(i)$, and $W_i(j) = S(j) + \beta P_i(j)$, where $\beta > 0$. Then team i's willingness to pay for the new player becomes

$$\begin{aligned} V_i &\equiv W_i(i) - W_i(j) = S(i) - S(j) + \beta[P_i(i) - P_i(j)] \\ &= \frac{a(T_j - T_i)}{(T_i + T_j + a)^2} + \beta \frac{a}{T_i + T_j + a}. \end{aligned}$$

From $T_1 \geq T_2$, we have $V_1 \leq V_2$. Therefore, a competitive mechanism such as an English auction would allocate the new player to the weaker team, and thus maximize the league's objective. Therefore, adding a very small weight on the league's revenue will make a competitive mechanism optimal in the two-team case. This property remains valid in the three-team case. It is in line with the findings by Demmert (1974), who examines the revenue sharing as a mechanism by which the league internalizes the externalities among teams. He finds that more equal sharing alleviates market differences and equalizes the distribution of playing skills among teams. It is also in accordance with Madden (2011) who finds that the introduction of revenue sharing always causes win percentages and competitive balance to move toward their efficient level.

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