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Corporate Investment, Tobin's Q and Liquidity Management under Time-Inconsistent Preferences

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We incorporate time-inconsistent preferences into a dynamic model of corporate investment and liquidity management. For the dividend strategy, we find time-inconsistent preferences always accelerate firm's cash payout. By contrast, the influence of time-inconsistent preferences on the investment policy is ambiguous, which depends on firm's liquidity measured by the cash-capital ratio $w = W/K$. It shows that time-inconsistent preferences induce the shareholder to over-invest when firm's liquidity is essentially low. However, the shareholder prefers under-investing as the firm has sufficient cash. Furthermore, we find time-inconsistent preferences significantly lower a firm's average q and marginal q as well as marginal value of liquidity. Finally, the impact on the investment policy and liquidity management also depends on such factors as whether the shareholder is sophisticated or naive in the expectation regarding his future time-inconsistent behavior.

Key Words: Time-inconsistent preferences; Investment; Liquidity management; Q theory.

JEL Classification Numbers: D91, E22, G30.

1. INTRODUCTION

In most existing models about corporate investment and liquidity management, it is assumed that the entrepreneur has a constant rate of time preference. However, there are two reasons for us to think about departures from this assumption. First, many experimental studies on time preferences suggest that the assumption of time-consistency is unrealistic (see, e.g., Thaler, 1981; Ainslie, 1992; Loewenstein and Prelec, 1992). Conceptually speaking, time-inconsistent preferences refer to that the agent acts more impatiently with short-term decisions. Laibson (1997) initially models such time-varying impatience by using a quasi-hyperbolic discount function, in

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which the discount rate declines as the horizon increases. Second, as documented by Bertrand and Schoar (2003), the entrepreneur fixed effects can explain a significant portion of the heterogeneity in firms' financial and investment practices. In the viewpoint of psychology, personality traits play a vital role in corporate decision-making.

The goal of this paper is to investigate how time-inconsistent preferences distort corporate investment and liquidity management. To characterize this time-varying impatience, we adopt a continuous-time version of quasi-hyperbolic discount functions (see, e.g., Grenadier and Wang, 2007; Harris and Laibson, 2013; Tian, 2016). Following the existing literature, we consider both the naive case and the sophisticated case. For the naive shareholder, he mistakenly believes that the current self can commit all future selves to act as if their discount function remains unchanged. In contrast, the sophisticated shareholder correctly anticipates that their future selves act according to their own preferences. That is to say, naive shareholder regards the future selves as time-consistent agents, while sophisticated shareholder perceives the future selves' time inconsistency. This difference is called "sophisticated effect" in the literature. Therefore, our paper distinguishes the two cases in the following discussion.

As is standard in models of time-inconsistent decision, our problem can be treated as the outcome of an intra-personal game in which the agent is represented by different selves at future periods. In other words, a person cares about not only current self's decision but also future selves' decisions. Our model has two essential building blocks, which are: (1) the workhorse neoclassical q theory model of investment with liquidity constraints; (2) time-inconsistent preferences with quasi-hyperbolic discount functions. For the first block, our model is based on the liquidation case in Bolton et al. (2011) (henceforth BCW). For the second block, we characterize time-inconsistent preferences by two parameters: the additional discount factor η and the future self arrival intensity ξ. This approach is analytically tractable and especially suitable for our continuous time framework.

By comparing our model with BCW, we portray the influence of time-inconsistent preferences on the firm value, investment policy, average q and marginal q. Intuitively, managerial traits generate heterogeneity among otherwise identical firms. Therefore, our results support the empirical evidence that firms with similar fundamentals may choose very different investment and liquidity management decisions.

We find the following main novel results. First, time-inconsistency significantly lowers the firm value and accelerates firm's cash payout. Beyond the standard exponential discounting, the current self values the cash payout obtained from decisions by future selves less than had he distributed himself. Therefore, it provides an extra incentive for the current self to pay out cash before the future selves arrive. Moreover, the firm value in our model is substantially lower than that in BCW due to the additional discount for time-inconsistent preferences. In addition, we find that the endogenous payout boundary in the sophisticated case is smaller than that in the naive case due to the sophisticated effect.

Second, time-inconsistency has ambiguous effects on the investment behavior, which depends on the firm's cash inventory. When the cash-capital ratio $w = W/K$ is close to liquidation boundary, time-inconsistency causes over-investment. This is because the time-inconsistent preferences erode the firm value and create less incentive for the shareholder to sustain the project. Hence time-inconsistency results in over-investment in this region. However, the firm tends to under-invest as the cash inventory is sufficiently high since the time-inconsistent shareholder in this region is more eager to cash payment rather than investment.

Finally, both average q and marginal q in our model are lower than their counterparts in BCW. This is because time-inconsistent preferences lower the total firm value. Furthermore, we find average q in our model is still higher than marginal q, which is consistent with existing literature, such as Demarzo et al. (2012).

Our paper is related to a fast growing literature on dynamic corporate finance with financial friction. BCW propose a tractable dynamic model of corporate investment and risk management for a financially constrained firm. Demarzo et al. (2012) develop a framework integrating dynamic investment theory with dynamic optimal contract theory. Bolton et al. (2013) extend BCW to allow for time-varying investment and financing opportunities. In contrast, this paper extends BCW by considering time-inconsistent preferences.

Our model also relates to recent work on corporate finance under time-inconsistent preferences. Grenadier and Wang (2007) extend the standard real options framework by incorporating time-inconsistent preferences. Chen et al. (2014) study the optimal dividend strategies under time-inconsistent preferences. Tian (2016) provides a tractable framework of entrepreneurial firm's capital structure and investment decisions under time-inconsistent preferences. Liu et al. (2017) study dynamic agency and investment theory under time-inconsistent preferences. In contrast with these papers, our model investigates how time-inconsistent preferences distort firm's investment and liquidity management.

The remainder of the paper is organized as follows: Section 2 describes the model setup, including the time-inconsistent preferences, production technology and liquidity management. Section 3 derives the solution to our model. Quantitative results are presented in Section 4. Section 5 concludes the paper.

2. MODEL SETUP

We incorporate time-inconsistent preferences into BCW. First, we introduce time-inconsistent preferences. Then we describe firm's production technology and liquidity management.

2.1. Time-inconsistent preferences

We assume that the shareholder values cash flow payoffs by using a quasi-hyperbolic discounting function that consists of two time intervals: the present and the future. The shareholder exponentially discounts the flow payoffs in the present period with a discount rate of $r > 0$ and further discounts future cash flow payoffs with an additional discount factor of $\eta \in (0, 1)$, which captures the propensity toward instantaneous satisfaction in human nature. The present period has a stochastic length, and for simplicity, we assume that the length follows an exponential distribution with a hazard rate of ξ. For example, an economic self, named "self 0", is born at t_0. We know that he regards the time-line as the present period $[t_0, t_1)$ and the future period $[t_1, \infty)$, and in the present period, he controls the investment and liquidity management decisions. After t_1, self 0 decays and "self 1" comes into the world. Interestingly, self 1 also regards the time-line as the present period $[t_1, t_2)$ and the future period $[t_2, \infty)$, and he takes over control of the corporate decisions from self 0. Repeating the process, we can generate a sequence of selves $\{0, 1, 2, \ldots\}$ who live in a sequence of periods $\{[t_0, t_1), [t_1, t_2), \ldots\}$. It is obvious that the periods are i.i.d.

Denote self m's quasi-hyperbolic discounting function as $D_m(t, s)$; thus, we have

$$D_m(t, s) = \begin{cases} e^{-r(s-t)} & if \quad s \in [t_m, t_{m+1}) \\ \eta e^{-r(s-t)} & if \quad s \in [t_{m+1}, \infty), \end{cases} \tag{1}$$

where $s > t$ and $t_m < t_{m+1}$. The hazard rate ξ and the additional discount rate η jointly capture the degree of the shareholder's time inconsistency.

In the paper, we discuss two types of beliefs about future selves: *naive* and *sophisticated*. The shareholder is defined as naive when the current self falsely believes that future selves will act in a time-consistent manner despite the absence of a commitment mechanism in our model. However, a sophisticated shareholder correctly foresees that his future selves have different discount rates. We will provide more details regarding naive and sophisticated shareholders in Section 3. Next, we turn to the description of firm's production technology and liquidity management.

2.2. Production technology and liquidity management

The firm employs physical capital for production. We denote the capital stock and investment level as K and I, respectively. Then the firm's capital stock evolves as

$$dK_t = (I_t - \delta K_t)\, dt, \tag{2}$$

where $\delta \geq 0$ is the depreciation rate.

As in the neoclassical investment model, investment entails adjustment cost. We take the conventional assumption that the adjustment cost $G(I, K)$ is convex in investment I and homogeneous of degree one in investment level I and capital stock K. Hence we can write $I + G(I, K) = c(i) K$, where $c(i)$ denotes the total investment cost (including the adjustment cost) per unit of capital and $i = I/K$ is investment-capital ratio. Specifically, we assume $c(i)$ takes the standard quadratic form as

$$c(i) = i + \frac{1}{2}\theta i^2\,, \tag{3}$$

where θ measures the adjustment cost for investment.

We assume the firm's cumulative productivity evolves according to

$$dA_t = \mu dt + \sigma dB_t, \tag{4}$$

where B is a standard Brownian motion under the probability measure $\mathbb{P}$. Over time increment dt, the firm's operating revenue is given by $K_t dA_t$, which is often referred to as the "AK" technology in the macroeconomics literature.

Then the firm's cumulative operating profit evolves as

$$dY_t = K_t\,(dA_t - c(i_t)\, dt)\,, \tag{5}$$

where the first term is the incremental gross output and the second term is total cost of investment. Finally, we assume that the firm can liquidate its assets at any time. To preserve the linear homogeneity of our model, we assume the liquidation value L_t is proportional to the firm's capital with $L_t = lK_t$.

Now we turn to discuss the firm's cash inventory W_t. Following BCW, we model the cash holding cost generated by the agency problem in reduced-form. We assume the rate of return that the firm earns on W_t is the risk-free rate r minus a carry cost $\lambda > 0$. Hence the parameter λ measures the cost of cash holding. Besides cash accumulation, the firm can distribute cash to the shareholders as well. We denote U_t as the firm's cumulative payout to shareholders. Therefore, the firm's cash inventory evolves according to

$$dW_t = dY_t + (r - \lambda)\, W_t dt - dU_t, \tag{6}$$

which is a general accounting identity. In this paper, we only consider the liquidation case in BCW and leave the external financing case for the future research.

2.3. Firm optimality

The firm maximizes shareholder value by choosing its investment I, payout policy U and liquidation time τ:

$$\mathbb{E}\left[\int_0^\tau D_0(0,t)dU_t + D_0(0,\tau)\left(lK_\tau + W_\tau\right)\right]. \tag{7}$$

The expectation is taken under the risk-adjusted probability. In fact, the firm's objective includes the discounted value of net payout to shareholders and the discounted value from liquidation. Moreover, we adopt the quasi-hyperbolic discounting function D_0 due to the existence of time-inconsistent preferences. Let T denote the arrival time of the future self, then we can rewrite the firm's objective as

$$\begin{aligned}&\mathbb{E}\left[1_{\{T\geq\tau\}}\left[\int_0^\tau e^{-rt}dU_t + e^{-r\tau}\left(lK_\tau + W_\tau\right)\right]\right]\\ &+\mathbb{E}\left[1_{\{T<\tau\}}\left[\int_0^T e^{-rt}dU_t + \int_T^\tau \eta e^{-rt}dU_t + \eta e^{-r\tau}\left(lK_\tau + W_\tau\right)\right]\right],\end{aligned} \tag{8}$$

which consists of two terms: (i) when the future self arrives after the liquidation time, the firm's cash flow is discounted exponentially until liquidation; (ii) when the future self arrives before the liquidity time, the firm's cash flow is discounted exponentially until the future self arrives, and further discounted by the factor η from the arrival of the future self until liquidation.

3. MODEL SOLUTIONS

In this section, we first briefly present the main result in BCW without a step-by-step proof. The detailed proof is shown in Subsection 3.2 of the naive case. Since the proof of the sophisticated case is very similar to that of the naive case, we summarize the main result in a proposition in Subsection 3.3 out of space concerns.

3.1. Benchmark: time-consistent preferences

We debote the shareholder value as $P(K,W)$, which is homogenous in capital stock K and cash inventory W. Thus, we have $P(K,W) = p(w)K$, where $w = W/K$ is the firm's cash-capital ratio. As the benchmark, we summarize the optimal investment policy and the solution to the shareholder value function in the following proposition.

Proposition 1. *If the shareholder has time-consistent preferences, $p(w)$ is solved by the following ordinary differential equation (ODE):*

$$rp(w) = (i(w)-\delta)(p(w)-wp'(w))+((r-\lambda)w+\mu-c(i(w)))p'(w)+\frac{\sigma^2}{2}p''(w), \tag{9}$$

subject to the boundary conditions:

$$p(0) = l, \quad p'(\overline{w}) = 1, \quad p''(\overline{w}) = 0. \tag{10}$$

where $\overline{w}$ is the endogenous payout boundary. And the optimal investment-capital ratio $i(w)$ is given by

$$i(w) = \frac{1}{\theta}\left(\frac{p(w)}{p'(w)} - w - 1\right). \tag{11}$$

The details are available in BCW. In the next subsection, we provide the proof for the case of naive shareholder .

3.2. The naive shareholder

The shareholder is naive in that he holds the incorrect belief that his future self will act like a time-consistent agent (Akerlof, 1991). Instead of assuming the shareholder has a different patience level in different periods, it is much more convenient to consider an intra-personal game played by two selves. When the first self decays, the second self appears, and then, the third self will follow. The rest of the analysis can be conducted in the same manner. The exogenous arrival of each self follows a Poisson process with a hazard rate ξ. The present self 0's preference is D_0, and he discounts the cash flow by e^{-rt} in $[t_0, t_1)$ and by ηe^{-rt} in $[t_1, \infty)$. The shareholder is naive since he knows his own inconsistency but does not realize that future selves also have inconsistent preferences. When the future self arrives, the present self achieves a continuation value, which is the present value that depends on the future self's action. We denote the continuation value as $P_b\,(K, W)$. The present self believes that his future selves are consistent and discount the cash flow payoff by ηe^{-rt} in $[t_1, \infty)$, thus we obtain

$$P_b\,(K, W) = \eta P\,(K, W)\,. \tag{12}$$

We solve the shareholder's optimization problem by using dynamic programming. In the interior region, the following Hamilton-Jacobi-Bellman

(HJB) equation holds for the naive shareholder:

$$\begin{aligned} rP_n(K,W) &= \max_{I_n}(I_n - \delta K)P_{n,K} + [(r-\lambda)W + \mu K - c(i_n)K]\, P_{n,W} \\ &\quad + \frac{\sigma^2 K^2}{2} P_{n,WW} + \xi(P_b(K,W) - P_n(K,W)). \end{aligned} \tag{13}$$

The first term on the right side is the marginal effect of net investment on firm value. The second term represents the effect of the firm's expected savings on firm value, and the third term captures the effect of the volatility of cash holdings on firm value. Importantly, the last term emphasizes the difference between the naive case and the time-consistent benchmark. Note that the marginal value of capital is $P_{n,K} = p_n(w) - wp_n'(w)$, the marginal value of cash is $P_{n,W} = p_n'(w)$ and $P_{n,WW} = p_n''(w)/K$. Substituting them into (13), we obtain the following ODE holding for the naive shareholder:

$$\begin{aligned} rp_n(w) &= (i_n(w) - \delta)(p_n(w) - wp_n'(w)) + ((r-\lambda)w + \mu - c(i_n(w)))p_n'(w) \\ &\quad + \frac{\sigma^2}{2} p_n''(w) + \xi(\eta p(w) - p_n(w)), \end{aligned} \tag{14}$$

where the optimal investment-capital ratio $i_n(w)$ is given by

$$i_n(w) = \frac{1}{\theta}\left(\frac{p_n(w)}{p_n'(w)} - w - 1\right). \tag{15}$$

In order to solve this ODE for $p_n(w)$, we need the boundary conditions. Note that for the region $w > \overline{w}_n$, it is optimal for the firm to distribute the excess cash as a lump-sum payoff and bring the cash-capital ratio back to $\overline{w}_n$, which implies

$$p_n(w) = p(\overline{w}_n) + (w - \overline{w}_n). \tag{16}$$

Since (16) holds for all $w > \overline{w}_n$, we take the limit and get the smooth-pasting condition $p_n'(\overline{w}_n) = 1$. In order to determine the optimal payout boundary, we need the super-contact condition $p_n''(\overline{w}_n) = 0$ as well. For the lower boundary, it is optimal for the firm to wait until it runs out of cash and then liquidate its assets, thus we have $p_n(0) = l$.

3.3. The sophisticated shareholder

Different from the naive shareholder, the sophisticated shareholder correctly anticipates that his future self will choose the investment policy and payout boundary that are optimal for the future self but not for the current self (Laibson, 1997). We consider a sophisticated shareholder with an infinite number of selves in this subsection. As in Grenadier and Wang (2007), the investment policy and payout boundary are independent of the number of selves. This means that the stationary solution is a so-called fixed point to the shareholder. Furthermore, let $p_s(w)$ and $p_b(w)$ denote the scaled firm value and the continuation value function, respectively. Using the standard argument, we derive the solutions and summarize them in the following proposition.

Proposition 2. *The sophisticated shareholder's scaled value $p_s(w)$ solves the ODE:*

$$\begin{aligned} rp_s(w) &= (i_s(w)-\delta)(p_s(w)-wp_s'(w)) + ((r-\lambda)w+\mu-c(i_s(w)))p_s'(w) \\ &\quad +\frac{\sigma^2}{2}p_s''(w)+\xi(p_b(w)-p_s(w)), \end{aligned} \tag{17}$$

subject to the boundary conditions:

$$p_s(0)=l, \quad p_s'(\overline{w}_s)=1, \quad p_s''(\overline{w}_s)=0, \tag{18}$$

where $\overline{w}_s$ is the endogenous payout boundary in the sophisticated case. And the optimal investment-capital ratio $i_s(w)$ is given by

$$i_s(w)=\frac{1}{\theta}\left(\frac{p_s(w)}{p_s'(w)}-w-1\right). \tag{19}$$

Additionally, the continuation value function is $p_b(w)=\eta\widetilde{p}(w)$, where $\widetilde{p}(w)$ is solved by

$$r\widetilde{p}(w)=(i_s(w)-\delta)(\widetilde{p}(w)-w\widetilde{p}'(w))+((r-\lambda)w+\mu-c(i_s(w)))\widetilde{p}'(w)+\frac{\sigma^2}{2}\widetilde{p}''(w). \tag{20}$$

where $i_s(w)$ is given by (19) *and the corresponding boundary conditions are*

$$\widetilde{p}(0)=l, \quad \widetilde{p}'(\overline{w}_s)=1, \quad \widetilde{p}''(\overline{w}_s)=0. \tag{21}$$

We highlight the continuation function $p_b(w)=\eta\widetilde{p}(w)$ to illustrate the difference between the naive case and the sophisticated case. From the perspective of sophisticated shareholder, future self's investment policy and payout boundary in ODE (20) is identical to those in (17). This phenomena is called *sophisticated effect*, which states that a sophisticated shareholder knows about his future inconsistency and correctly perceives that future self will choose the same investment policy and payout boundary as current self. This leads to a different result compared with the naive case.

4. QUANTITATIVE RESULTS

In this section, we turn to analysis the quantitative results. Most of the parameter values are borrowed from BCW: the mean and volatility of the risk-adjusted productivity shock are $\mu = 18\%$ and $\sigma = 9\%$; the risk free rate is $r = 6\%$; the rate of depreciation is $\delta = 10.07\%$; the adjustment cost parameter is $\theta = 1.5$; the cash-carrying cost parameter is $\lambda = 1\%$ and the liquidation value is $l = 0.9$.

With respect to time-inconsistent preferences, we set the additional discount factor after the realization of future selves as $\eta = 0.9$. Finally, we set the future self arrival intensity $\xi = 0.2$. Both of the parameter values are consistent with the previous literature (Harris and Laibson, 2013). Moreover, we show the robustness of our results in the comparative statics.

4.1. Scaled firm value

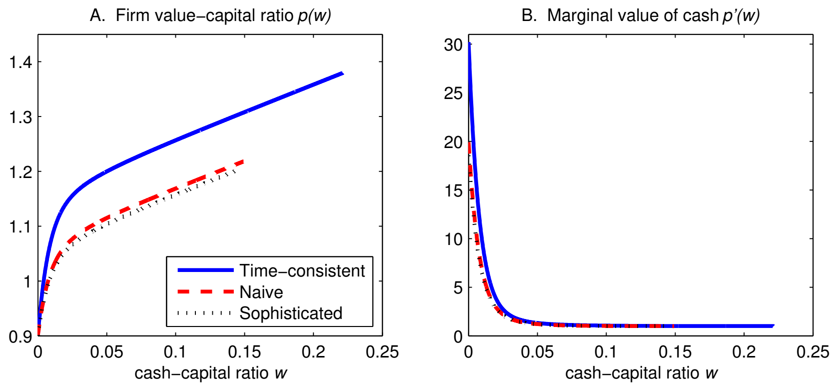

FIG. 1. Firm value-capital ratio $p(w)$ and marginal value of cash $p'(w)$.

Panel A of Figure 1 shows the scaled firm value $p(w)$ with respect to the cash-capital ratio w. In time-consistent benchmark, the endogenous payout boundary is 0.221. In our model, the payout boundary drops to 0.149 for the naive case and 0.144 for the sophisticated case. The intuition is as follows. Beyond the standard exponential discounting, the current self values the cash payout obtained from decisions by future selves less than had he distributed himself. Therefore, it provides an extra incentive for the current self to pay out cash before the future selves arrive. This effect accelerates firm's cash payout, which lowers the endogenous payout boundary. In addition, the firm value in our model is substantially lower than that in BCW due to the additional discount for time-inconsistent preferences.

Furthermore, we find that the payout boundary and the firm value are even lower in the sophisticated case. Note that the sophisticated shareholder correctly forecasts how his future selves behave, i.e., he is not only aware of time inconsistency of current self but also knows the time-inconsistent preferences of future selves. On the other hand, the naive shareholder incorrectly foresees that his future self will act like a time-consistent agent. Hence the sophisticated shareholder is more afraid of his future selves than the naive shareholder. As a result, the sophisticated shareholder hastens the payout decision, which further lowers the firm value.

Panel B of Figure 1 plots the marginal value of cash $p'(w)$ as a function of cash-capital ratio w. We find time inconsistency substantially lower the marginal value of cash. Quantitatively, when firm exhausts the liquidity, $p'(0)$ drops from 30.2 in BCW to 19.9 in the naive case and 18.6 in the sophisticated case. Intuitively, since time inconsistent preferences lowers firm value, it weakens the incentive to avoid liquidation. Thus the marginal value of cash is much lower in our model. Specifically, the marginal value in the sophisticated case is lower than that in the naive case, which is due to the sophisticated effect as well. As the firm has abundant cash, the marginal value approaches one both in BCW and in our model.

4.2. Investment policy

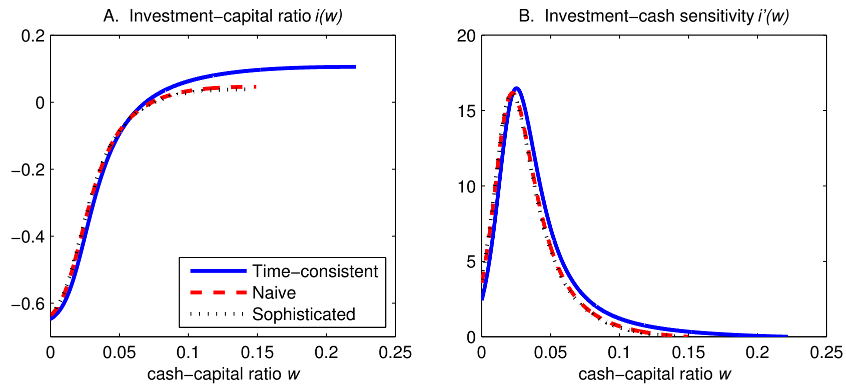

FIG. 2. Investment-capital ratio $i(w)$ and investment-capital sensitivity $i'(w)$.

Panel A of Figure 2 shows the investment-capital ratio $i(w)$ with respect to cash-capital ratio w. We find that as w is close to the payout boundary, time inconsistency leads to under-investment. However, as w approaches zero, time inconsistency causes over-investment.

When the cash inventory is abundant, the shareholder with time-inconsistent preferences is more eager to cash payment. Thus the investment-capital ratio becomes lower. On the other hand, as w approaches the liquidation boundary, it will do disinvestment to acquire cash. Since time-inconsistent preferences lower the firm's continuation value and weaken the incentives to sustain the project, time inconsistency leads to over-investment in this region.

In addition, we compare the difference between the naive case and sophisticated case. We see that the investment-capital ratio in the sophisticated case is higher than that in the naive case when w is low, however, when w is high, it is even lower in the sophisticated case. Recall that a sophisticated shareholder has the incentive to prevent his future selves from making the suboptimal decisions. As the firm becomes close to liquidation, it should invest more in the sophisticated case due to severer erosion in the firm's continuation value. On the other way, the sophisticated shareholder recognizes that future selves are time inconsistent. Therefore, when the cash inventory is high, the sophisticated shareholder further reduces firm's investment.

Now we turn to analyze the investment-cash sensitivity. From equations (11), (15) and (19), taking the first-order derivative with respect to w yields

$$i'_x(w) = -\frac{p_x(w)p''_x(w)}{\theta p'^2_x} > 0, \quad x = \{benchmark, naive, sophisticated\}. \tag{22}$$

Panel B of Figure 2 plots the investment-capital sensitivity $i'(w)$ as a function of the cash-capital ratio w. We find the investment-capital sensitivity is always positive due to the concavity of $p_x(w)$. Interestingly, time-inconsistent preferences do not alter this pattern.

4.3. Average q and marginal q

In this subsection, we turn to exploit the impact of time-inconsistent preferences on the average q and marginal q. Note that the value of capital is the total firm value net of cash, $P(K, W) - W$. Average q, defined as the ratio of the firm value net of cash to its capital stock, is given by

$$q_a(w) = \frac{P(K, W) - W}{K} = p(w) - w. \tag{23}$$

Similarly, marginal q is defined as

$$q_m(w) = \frac{\partial\left(P(K, W) - W\right)}{\partial K} = p(w) - wp'(w). \tag{24}$$

Since the marginal value of cash is always larger than one, we have $q_m(w) < q_a(w)$, which implies that there exists a gap between the average

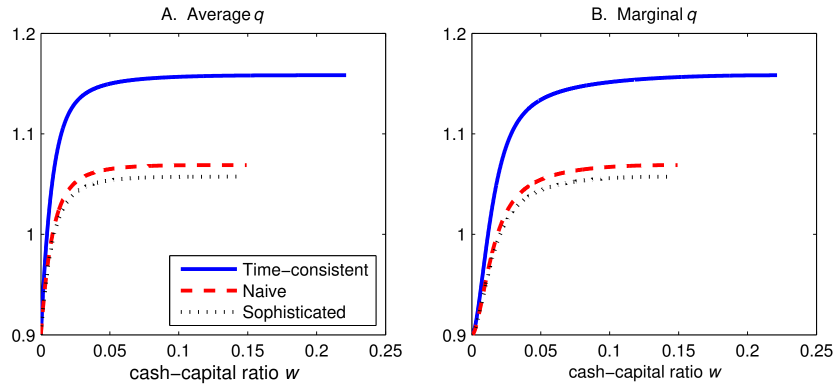

FIG. 3. Average q and marginal q.

q and marginal q. We find time-inconsistent preferences do not alter this pattern.

Figure 3 displays the average q and marginal q as functions of cash-capital ratio w for three cases. By taking time-inconsistency into account, we find both average q and marginal q are lower than their counterparts in BCW. Moreover, due to the sophisticated effect, the average q and marginal q in the sophisticated case are lower than those in the naive case.

4.4. Comparative statics

Since our contribution is embedding time-inconsistent preference into the work of BCW, and all the parameters, apart from η and ξ, have been extensively discussed in their paper, here we only focus on the comparative static effects of the additional discount factor η and the future selves arrival intensity ξ.

Panel A and B of Figure 4 show the investment policies for various values of the additional discount factor η, from 0.7 to 1, where $\eta = 1$ refers to the time-consistent benchmark. First, we find time-inconsistent preferences causes over-investment at the liquidation boundary from Panel A. For instance, when $\eta = 0.7$, the investment-capital ratio for the time-inconsistent manager is -0.52, which is substantially higher than the time-consistent benchmark -0.65. Second, Panel B shows that time inconsistency leads to under-investment at the payout boundary. Specifically, the firm decides to disinvest as long as the additional discount factor η is small enough. Finally, we find that sophisticated shareholder does more investment at the liquidation boundary and less investment around the payout boundary than naive shareholder.

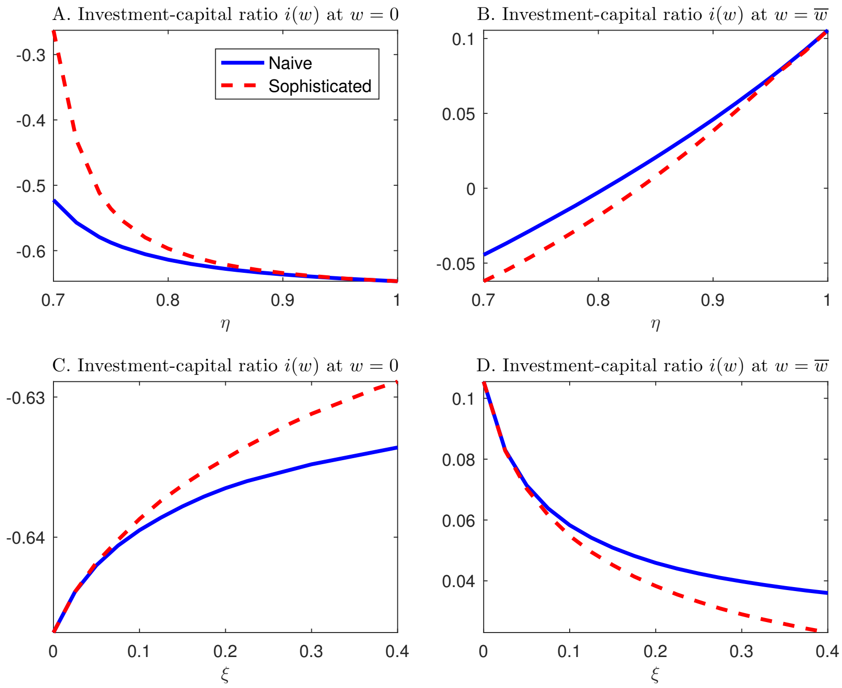

FIG. 4. Investment-capital ratio $i(w)$ at liquidation boundary and payout boundary.

Now we turn to the comparative static effects of the future selves arrival intensity ξ, from 0 to 0.4, where $\xi = 0$ refers to the time-consistent benchmark. As we increase ξ from 0 to 0.4, the firm chooses a higher investment ratio at the liquidation boundary and a lower investment ratio at the payout boundary. Higher ξ will generate more frequent arrival of future selves, which means the agents are more time-inconsistent. The greater time-inconsistency will deviate the shareholder more from the benchmark choice. For the sophisticated shareholder, he deviates from benchmark further than naive one since the sophisticated shareholder does not only know his own time-inconsistency, but also knows the after the realization of future selves he is still time-inconsistent. Accordingly, the investment decision should be even higher around liquidation boundary and lower at the payout boundary.

5. CONCLUSION

We extend a dynamic model of corporate investment and liquidity management by incorporating time-inconsistent preferences. We find time-inconsistent preferences substantially distort the firm's behavior. Our model predicts that time-inconsistent preferences have ambiguous effects on the investment behavior, which depends on the firm's cash inventory. When the cash-capital ratio $w = W/K$ approaches the payout boundary, time-inconsistent preferences leads to under-investment. In this region, the shareholder with time-inconsistent preferences is more eager to cash payment comparing to investment. However, the shareholder prefers over-investment as the cash-capital ratio is close to zero since the additional discount factor lowers firm's continuation value and weakens the shareholder's incentive to sustain the project. Moreover, it shows that time-inconsistent preferences significantly lower a firm's average q and marginal q. Finally, when the shareholder is sophisticated, the resulting distortions in corporate investment and liquidity management are severer than those in the naive case.

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