ANNALS OF ECONOMICS AND FINANCE **20-2**, 817–845 (2019)

Portfolios Optimizations of Behavioral Stocks with Perception Probability Weightings

Kuo-Hwa Chang and Michael Nayat Young*

Having traceable stock price movements and predictable market trends can always benefit investors. Considering specific irrational investor behaviors that may collectively affect a stock's price movement, we identified the cause-and-effect return patterns of affected stocks (behavioral-stocks) with their corresponding time-to-effects and likelihood-of-effects. Considering different individual investor perceptions on future market performances, we transformed investor perceptions into probability weights on future market performances, wherein the weights are also consistent with the estimated likelihood-of-effects of behavioral-stocks. Utilizing a scenario-based mixed-integer behavioral-stocks portfolio optimization program embedded with a new two-dimensional weightings on scenarios and behavioral-stocks, we obtained portfolios that statistically outperform the market.

Key Words: Portfolio management; Behavioral portfolio optimization; Irrational behaviors; Probability weightings; Mixed integer program.
JEL Classification Numbers: C61, G02, G11, G17.

1. INTRODUCTION

In any investment decision, having good and reliable extra information is always an advantage. Investors all want to know, when is the best time to buy/sell any financial assets such as stocks, bonds, options, futures, etc. The usual go to strategy by individual and professional investors alike are market indicators which may or may not accurately predict the direction of the market. On instances that these indicators accurately predict market direction, the underlying stocks will not necessarily follow the same direction of the market. Therefore, having some mechanism to trace the

* Chang: Corresponding author. Department of Industrial and Systems Engineering, Chung Yuan Christian University, Chung Li District, Taoyuan City, 32023, Taiwan. Email: kuohwa@cycu.edu.tw; Young: Department of Industrial and Systems Engineering, Chung Yuan Christian University, Chung Li District, Taoyuan City, 32023, Taiwan and Department of Industrial Engineering and Engineering Management, Mapua University, Intramuros, 1002, Philippines. Email: michaelnayatyoung@gmail.com.

1529-7373/2019

individual movements or future performances of underlying stocks to better time investment decisions would be of great help to all investors. From this mechanism, we can have clairvoyance on the future performances of the market and underlying stocks, and then we can perfectly time when to buy/sell stocks to have substantial profit. However, we all know that clairvoyance in the financial markets is close to impossible, but still investors and analysts alike would love to have the capability of being able to somewhat predict the trend of the market and simultaneously track the price movements of its underlying stocks. Opportunely, investors have their individual perceptions and biases on future performances of the markets and underlying stocks and may behave irrationally in making investment decisions. From these irrationalities and the collective behavior of investors on specific stocks, they can probably have an effect on the stocks' price movement, which is why in this study following behavioral portfolio theory we study some of the most prevalent irrational behaviors of investors, how they are identified, how they affect, and how long before they affect the stock's price movement to have the extra information on when to buy/sell stocks and have better investment decisions.

A lot of investors have a tendency of over-relying on stereotypes or past events of similar environments which was mentioned by Kahneman and Tversky (1972) as representative bias. A lot of investors also have the preference to readily realize (winners) stocks that made gains than those (losers) stocks that made losses, thus, having the disposition to sell winners and hold losers. A lot of investors also tend to over react to good or bad information and cause temporary large price movements resulting to follow-up reversals and that during the reversals losers outperform winners called as over-reaction by De Bondt and Thaler (1985). A lot of investors are also biased towards their individual confidence which Odean (1998b) calls over-confidence. A lot of investors also have a popular preference called herding bias which is the tendency of individual investors to change their investment views to follow the trading action of the majority. And a lot of them have their own preferences and biases leading to other irrational behaviors when investing on financial assets. These irrational behaviors are well documented over the years and have a lot of available literatures on their existence, like the supporting studies of Barnes (1984), Harless and Peterson (1998), Shefrin (2001), Chen et al. (2007), and Chang et al. (2009) on the representative bias; studies of Odean (1998b), Odean (1999), Barber and Odean (2000), Barber and Odean (2001), and Graham (1999) on over-confidence; studies of Scharfstein and Stein (1990), Graham (1999), Ashiya and Doi (2001), Wylie (2005), Tan et al. (2008), Demirer et al. (2010) on the herding bias, and so on, but for this study, we discuss and focus on the over-reaction and disposition effect bias, and how can these biases be exploited to have extra information on stocks to have better investment decisions.

One of the early works on over-reaction was done by De Bondt and Thaler (1985), who analyzed the abnormal returns of respective portfolios of winners and losers and observed that both portfolios have performance reversal. De Bondt and Thaler (1987) further provided support for the existence of over-reaction as they studied the seasonal patterns of the returns which yielded findings that past losers significantly outperform past winners. Gunaratne and Yonesawa (1997) constructed portfolios consisting respectively of the winners and losers of Tokyo stock exchange market and observed that the losers outperformed the winners after a 4 year period. Bowman and Iverson (1998) spotted return reversals after 1 week of large price changes. Otchere and Chan (2003) checked the winners and losers in the Hong Kong market during the pre- and post-Asian financial crisis periods and found out statistically significant return reversals took place 2 days after the substantial price change. Ma et al. (2005) empirically tested over-reaction on the Nasdaq and NYSE by analyzing the abnormal return of winners and losers and observed significant reversals for both winners and losers of Nasdaq stocks 1-2 day(s) after the large price change while no significant reversals for NYSE stocks. Madura and Richie (2010) studied over-reaction on exchange traded funds(ETF) and observed more pronounce price corrections or reversals after the extreme price movements. Hu et al. (2013), following the method of De Bondt and Thaler (1985) for testing over-reaction, also observed over-reaction in the Shanghai stock market and that the losers outperformed the winners. Accordingly, we try to exploit the information that the winners and losers will have performance reversals.

On another hand, Shefrin and Statman (1985) first coined the tendency of holding on to losers too long and selling winners too soon as the disposition effect. Ferris et al. (1988) tested and determined the existence of the disposition effect by observing abnormal volumes of stocks. From statistical tests, they showed that there are significant evidence to support that the trading volumes of stocks with capital gains (winners) exceed the trading volumes of those stocks with capital losses (losers). Odean (1998a) tested the disposition effect by analyzing the trading records of 10,000 accounts by looking at the frequency and the volume of the winners and losers sold. By calculating the proportions of gains and losses realized (PGR and PLR, respectively) they showed that $PGR > PLR$ throughout the year which demonstrates that the investors tend to sell winners and hold losers. Shapira and Venezia (2001) studied the buy-sell round trips and they found out that the round trips for the winners are shorter than those of losers. Barber and Odean (2011) provided an overview on the researches done on disposition effect and showed that this behavior is prevalent and varies amongst all classes such as gender, education and nationality. Da Costa et al. (2008) observed the relationship of gender and disposition effect. Amongst males and females their statistical tests showed that while males behave as expected of the disposition effect, females on the other hand do not keep losers but sell winners. Goo et al. (2010) analyzed the disposition effect attributes of Taiwanese investors and observed through statistical testing that investors with at least a college degree showed weaker level of disposition effect compared to those investors who don't have any college degrees. They also found out that the disposition effect of a Taiwanese investor towards losers is stronger compared to the winners. Da Costa et al. (2013) observed that experienced investors tend to show lesser levels of disposition effect, moreover Talpsepp (2013) observed that investors of older age groups and females experience lesser levels of disposition effect. Frino et al. (2015) studied the prevalence of disposition effect at the Australian market. Similarly they used PGR/PLR, frequent trading, round size trading heuristics, and the investors' level of diversification to predict the disposition effect considering gender, age and ethnicity which provided similar findings. Similarly, we also try to exploit the information that disposition winners and losers will also have performance reversals.

As mentioned, the over-reaction, disposition effect, and other irrational behaviors caused by the investors' preferences and biases may consequently have a collective effect on the prices of the stocks and knowing this effect can help us in building superior portfolios. This led us to study the cause and effect patterns of the collective irrational behaviors together with the corresponding likelihoods of the effects to occur and respective time durations for the effects to occur after the causes have been identified, and then take advantage of these patterns and extra information to obtain one's optimal portfolio. Most of the studies on irrational behaviors only focused on finding the supporting evidence that these irrational behaviors exist, but only a handful of them tackled the direct impact of these irrational behaviors to the stock prices/returns. Appropriately, we determined and showed how these behaviors (over-reaction and disposition effect) affect individual stock returns, and then used this extra information to time when to buy and sell stocks to improve the portfolio performance.

In this study, first, we practiced the operational definitions(ODs) on observing the cause-and-effect patterns on the price or return to identify the stocks that are impacted by irrational behaviors. Through statistical tests, we classified the stocks whose prices or returns are significantly having the positive effect based on the respective OD and named them behavioral stocks or B-stocks. As mentioned, the focal behaviors of this study are over-reaction and the disposition effect. The tests involve finding the time needed for the effect to occur or the time-to-effect T and the likelihood of the effect to occur after T or likelihood-to-effect P^B. Time-to-effect

T would help in the proper timing of investment transactions, while the likelihood-to-effect P^B would help in the decision making by providing the probability that desired effect or direction of the price movement will occur. If stocks are classified as B-stocks, we can somewhat study and track their price movements. When we are able to find the cause of a B-stock, we are confident with probability P^B that the desired movement (effect) will occur after T days which will be very useful in obtaining optimal portfolios. For obtaining our portfolios, we utilized two kinds of investment pools of B-stocks. The big pool which is comprised of all the B-stocks and the small pool which is the filtered out B-stocks from the big pool whose effects will most likely take place on the next period. The small pool is obtained from the big pool every portfolio formation day during the test period which then serves as our investment pool for that portfolio formation day. Since seeking the optimal portfolio is our second goal in this study, with the available information on B-stocks treated as professional facts, it is only natural to further embed them in finding the optimal portfolio. Furthermore, given that we want to exploit the information on these B-stocks, if the resulting optimal portfolios are outperforming the generic safety-first portfolio and the market, then we have succeeded in extracting useful information from these irrationally affected stocks. If we are outperforming the benchmark and reference portfolios, then we can also say that we are able to somewhat predict the movement of the irrationally affected stocks and we are able to perfectly time our investment transactions to have the optimal returns.

Another factor to consider in investing is the future trend of the market because it greatly influences and affects the price movements of its underlying stocks. Most of the time, individual investors have different perceptions and predictions on the future performances of the market and its underlying stocks and may categorize future returns differently from one another. To our knowledge, there are two categories for describing the nominal occurrences of future returns, one considers continuous states on describing the future returns and the other one considers discrete scenarios. The individual perception of these investors can be modeled by assigning the weighted probabilities or selecting densities of the future returns from their respective viewpoints. These probabilities or densities are obtained through a weight function or utility function in which the investors' characteristics and behaviors will be reflected by the parameters of the functions. Applying on discrete scenarios, Lopes (1987) introduced a psychological theory of choice under uncertainty called security, potential and aspiration theory (SP/A theory). In the framework of SP/A theory, investors tend to make their decisions on taking risks based on both their hope and fear levels. On the other hand, Kahneman and Tversky (1979) introduced prospect theory(PT) mentioning that investors evaluate their prospects from a reference point such that values above (below) the reference point are considered gains (losses) and that the investors' feeling from losses is stronger than the feeling from gains. This means that investors are emotionally affected more by their losses than their gains which results to the behavior that the investors are risk-averse for gains and risk-seeking for losses. Appropriately, they concluded that the corresponding value function is concave for gains and convex for losses. Tversky and Kahneman (1992) argued that the weighting function of individuals are usually in cumulative decision weights instead of separable ones and called it cumulative prospect theory(CPT). Lopes and Oden (1999) compared the performance of SP/A theory against CPT, the results of their experiments showed that SP/A theory outperforms CPT. They claimed that SP/A theory is more useful in modeling investment decision making in viewing the relation between descriptive and normative theories of risky choice. Rieger (2010) validated the credibility of both the SP/A theory and CPT and claimed that although the two came from different psychological ideas they are similar in a certain mathematical framework. In line with these works, we utilized SP/A theory in assigning weights to the scenarios with respect to each individual investor.

Investors view their portfolio not as a whole but as a collection of several mental accounts(MAs) that are associated with specific goals and varying risk attitudes. Friedman and Savage (1948) discussed the insurance and lottery puzzle that an investor may purchase insurance and at the same time he may also buy the high-risk lottery. Thaler (1985) presented the concept of mental accounting and illustrated that decision makers tend to segregate the different types of gambles faced into separate mental accounts(MA) with respective risk attitudes, under which he/she may have different criterion in finding their respective optimal portfolio. For the portfolio optimization problems, Shefrin and Statman (2000) considered SP/A theory and mental accounting to find the optimal portfolio in the framework of behavior portfolio theory (BPT), each account corresponds to a specific goal of the investor such as retirement security, college education plans, or savings for traveling. They favored the safety-first model of Telser (1955) in modeling the optimization problem for each MA, each of which comes with associated tolerance level on taking the risk. The optimization is scenario-based and SP/A weightings are applied on the scenarios. Das et al. (2010) integrated the mean-variance portfolio theory of of Markowitz (1952) and BPT of Shefrin and Statman (2000) into a new mental accounting framework. They showed that solving the mental accounting problem is similar to solving a mean variance problem with a specific risk-aversion coefficient. Baptista (2012) expanded the study of Das et al. (2010) by considering the case wherein the investor faces background risk. Das and Statman (2013) further stretched the scope of BPT and observed that options and structured products have no roles in mean variance portfolios, but they have roles in behavioral portfolios. Fernandes et al. (2010) and Pfiffelmann et al. (2016) applied CPT on the continuous states with a known probability and maximized the expected utility function in the optimization of behavior portfolio. Singer (2009) on the other hand used a rank-dependent utility (RDU) and then applied SP/A theory to assign the probabilities.

The original SP/A weightings on the scenarios is based on subjective investor individuality such as the perspective towards the gain, loss and risk. However, the objective information regarding the B-stocks such as their positive effects on return and corresponding likelihood should also be incorporated in assigning weights to the scenarios. This led us to the idea of a two-dimensional weightings of the probability assignment mechanism in addition to the usual one-dimensional weights on scenarios. With these estimated likelihoods of effects (P^Bs) of B-stock, the new SP/A weightings not only put weights on scenarios but also on the B-stocks to satisfy the estimated likelihoods of effects. We called it the two-dimensional SP/A weightings, where the 1^{st} dimension is the weights on scenarios and the 2^{nd} dimension is the weights on the likelihood-of-effect (P^B) of each B-stock. For the optimization problem, we utilized the model of Shefrin and Statman (2000) considering the safety-first probabilistic constraints and embed it with the two-dimensional weightings into a scenario-based mixed integer program called behavioral stocks portfolio optimization(BSPO) model. A preliminary study on behavioral stocks has been discussed by Chang et al. (2015). They first mentioned the idea of B-stocks and two-dimensional weights. They only considered over-reaction B-stocks and assigned the two-dimensional weights to only one B-stock with the highest likelihood. The mechanism for the assignment is straight forward and can be easily done. We extended their study by considering more types of B-stocks with a more sophisticated and general way to determine the two-dimensional weights for multiple B-stocks.

The main focus of this paper is to introduce the advantages of behavioral stocks (B-stocks) for public use. By presenting the characteristics of B-stocks, investors can take advantage of the information that when they can spot the cause of irrational behavior of a B-stock, they are aware that there is more than a threshold probability chance, say 50% chance, or likelihood-to-effect P^B that this stock will have the desired effect, say a high cumulative abnormal return CAR. Moreover, they will be aware of the time when this high CAR can be realized or the time-to-effect T. This study provides an insight on how to take advantage of the information on T and P^B by presenting a portfolio selection model which is set 1 day before T is reached with the consideration of all the P^Bs of the B-stocks in the investment pool.

The succeeding sections are organized as follows. In Section 2, we determined the ODs of the over-reaction and disposition effect and the corresponding ways to identify B-stocks. We also presented the BSPO model incorporated with the two-dimensional weightings. In Section 3, we defined the empirical tests of our portfolios and made the necessary conclusion for our study in Section 4.

2. METHODOLOGY

In this section, we discussed how we defined our B-stocks and the selection process of B-stocks from the big pool into the small pool. We also explained the idea of the two-dimensional weights and then presented the corresponding BSPO model incorporated with the proposed two-dimensional weightings.

2.1. B-stocks

The B-stocks are identified by verifying the cause-and-effect relation referring to the OD. For over-reaction, we followed the OD of Madura and Richie (2010) such that the cause which is a large positive (negative) price movement is followed by the effect which is a high negative (positive) cumulative abnormal return (CAR). Let AR_t be the abnormal return at time t, which is calculated as $AR_t = R_t - \beta R_{M,t}$, where R_t is the return at time t, $R_{M,t}$ is the market return at time t and β is the Beta value of the stock. CAR is computed as the summation of the abnormal returns over a desired length of time. For the test, we defined large positive (negative) price movement in terms of return rate and calculated CAR starting from the time after a large positive or negative price movement is spotted.

Under the disposition effect, investors sell the winners too soon and hold the losers too long. Selling the winners too soon results to abnormal increase on trading volume and holding the loser too long results to abnormal decrease on trading volume. We followed the definition of Ferris et al. (1988) which used volume along with the price change to identify the disposition effect and we observed the synchronization of the movements of price and volume. For a stock, at each time t we checked the change in price and change in volume in the past T_{RG} period(s). We used the geometric return $R_G = \Pi_{t=1}^{T_{RG}}(R_t + 1) - 1$ to check the price movement and used the abnormal volume, denoted by AV_t at time t, to check the volume movement. Let V_t be the observed relative volume at time t and

$$V_t = \frac{\text{Volume at time t}}{\text{Moving average of the volume from the previous } T_V \text{ periods to time t}}.$$

Let $V_{M,t}$ be the observed relative volume of the market at time t and

$$V_{m,t} = \frac{\text{Market volume at time t}}{\text{Moving average of market volume over previous } T_V \text{ periods to time t}}.$$

Let $\hat{V}_t = a + bV_{M,t}$, where a and b are estimated through the regression

$$V_t = a + bV_{M,t} + \varepsilon, \tag{1}$$

We used $\hat{V}_t$ as the estimated relative volume at time t. The abnormal volume is defined as by Dyl (1977) as $AV_t = V_t - \hat{V}_t$. Let $\overline{AV}$ denote the average of abnormal volume of some T_{AV} periods. At each epoch and for each stock we checked the R_G and $\overline{AV}$. High R_G and high $\overline{AV}$ indicate that this stock experiences high return as a winner but with abnormal frequent transactions underway. We then identified it as a winner being affected by disposition effect. Similarly, the stock with low R_G and low $\overline{AV}$ is identified as a loser being affected by disposition effect. The identification of winners or losers affected by disposition effect serves as the cause when we seek for the B-stock of disposition effect. For the winners, sometimes the investor of disposition effect will hurry to sell them even at a little bit lower price which will make the price to gradually decrease resulting to a negative CAR. Similarly, investor will sell losers if someone can offer a substantially higher price. It will gradually cause price increase resulting to a positive CAR. Frazzini (2006) pointed out that when a winner is identified, the disposition effect investors will want to sell it to lock in the paper gain, which depresses its price temporarily, and from this lower base of price the subsequent returns will be higher; when a loser is identified, the disposition effect investors will be reluctant to sell them, and any trading that will occur will be at a temporarily inflated price, and from that higher base of price the subsequent returns will be lower. The study of Frazzini (2006) was set up to analyze the monthly and yearly return rates while our study focused on a daily trading and we are looking to exploit the respective higher and lower base of the price for the loser and winner mentioned by Frazzini (2006) within the 1^{st} to the 20^{th} day after spotting the loser or winner.

In identifying a B-stock, the significant higher positive CAR or negative CAR serves as the effect accordingly. For our BSPO problem, we only considered the stocks having the effects of positive CAR, that is, for over-reaction we considered stocks with significant negative change in price as the cause and for disposition effect we considered the identification of losers affected by disposition effect, called disposition losers, as the cause. Accordingly, we performed statistical tests (mentioned below) to support our assumption on the price/return movements of the disposition effect B-stocks and also the over-reaction B-stocks. Based on the historical data, for each stock we looked at the cause and effect pattern with time-to-effect T between them. Time-to-effect T is the duration of time needed for the effect of the respective B-stock to occur after its cause has been spotted. Accordingly, at each epoch after spotting the cause, we tested whether the effect will take place and let T be the first or earliest T that $CAR = \sum_{t=1}^{T} AR_t$ is larger than a positive bond, for example, 1% so that the portfolios can be profitable. If T can be found and if the likelihood of effect after T period(s) of time is significantly larger than a threshold percentage, $p_{threshold}$, this stock will then be classified as a B-stock with a corresponding T. Let P_i^B denote the observed likelihood of effect for stock i as the percentage of the cause-effect pattern considering a specific T over all the cause-effect patterns. We applied one-proportion test to test the significance of P_i^B such that

$$\begin{aligned} H_0 &: P_i^B \leq p_{threshold} \\ H_1 &: P_i^B > p_{threshold} \end{aligned} \tag{2}$$

If there were multiple Ts found, we selected the smallest one. For each B-stock, it comes with the respective cause-effect-P^B-T pattern. For obtaining the portfolio at each time period, we further selected those B-stocks from the big pool that will have their respective effect with likelihood P^B to occur on the next period. For a B-stock in the big pool with time-to-effect T, we looked at the $T-1$ period(s) backward and checked whether or not the cause can be identified at the end of that period. If the cause can be spotted, then it means that we have likelihood P_i^B of the effect of a positive CAR on the next period. We then include this B-stock in the small pool, on which the following BSPO model is applied.

In summary, B-stocks are determined by the identification of desired cause-and-effect relations through the respective operational definition (OD) of irrational behaviors. With respect to the irrational behavior considered, we spot the respective cause (high negative change in price for overreaction; disposition losers with low R_G and low $\overline{AV}$ for disposition effect) and see the resulting desired effect ($CAR \geq 1\%$) after a period of time or the time-to-effect T. We then determined whether the respective desired cause-and-effect relation for a specific irrational behavior holds by studying the proportion of the occurrences of the desired effect ($CAR \geq 1\%$) T day(s) after spotting a cause to the total occurrences of the respective cause throughout the historical data. Moreover, we also checked the probability for the effect to occur (P^B) after the cause have been spotted and the time-to-effect T. Time-to-effect (T day(s)) is in fact the actual time needed or the holding period of a B-stock after spotting the cause of the underlying irrational behavior before it realizes the desired effect. A B-stock's T day(s) is the shortest time duration where the proportion of the desired cause-and-effect relation of the B-stock holds and the likelihood-to-effect P^B of the B-stock is significantly larger than a threshold probability through the statistical test (2). For each stock, this statistical test (2) is done considering $T = 1$ to $T = 20$, where the smallest significant T is used for the considered B-stock. Likelihood-to-effect (P^B) is the likelihood or probability of the desired effect ($CAR \geq 1\%$) to occur T day(s) after the cause of the irrational behavior is spotted. P^B is actually the proportion of the desired cause-and-effect relation of a B-stock which is significantly larger than the threshold probability through the statistical test (2).

2.2. Two-dimensional probability weights

Assume that we have m scenarios to represent the m possible collections of returns of all the stocks for the next period (day, week, month, or year). We can think of a scenario as a row vector of returns of all the stocks. According to SP/A theory of Lopes (1987), investors assess scenarios from the most unfavorable ones to most favorable ones. They then weight the scenarios based on their fear(security) and hope(potential) levels. According also to SP/A theory, q_s and q_p are the parameters of the investors reflecting their individual fear and hope levels, respectively, such that the higher the q_s, the more weights are given on the unfavorable outcomes and the higher the q_p, the more weights are given on the favorable outcomes. Higher q_s indicates that the investor has stronger trend to be pessimistic and higher q_p indicates that the investor has stronger trend to be optimistic. On the other hand, given that $q_s > q_p$, the investor is security minded and values loss more than gain, and given that $q_s < q_p$, the investor is potential-minded and values gain more than loss. However, investors with (q_s, q_p) where their $q_s = q_p$ are cautiously hopeful and value the loss and gain equally. Let D be the nominal decumulative probability on a scenario. SP/A theory defines $H_s(D) = D^{1+q_s}$ as the assigned decumulative probability reflecting the fear level of the investor and $H_p(D) = 1 - (1 - D)^{1+q_p}$ as the assigned decumulative probability reflecting his/her hope level. An investor then balance his/her fear and hope levels by selecting another parameter $\theta, 0 \leq \theta \leq 1$. θ will be selected to reflect the dynamics of the investor's outlook on the market and can be changed from time to time. When the investor becomes more potential-minded, he/she will set his/her θ approaching to 1, and when the investor becomes more security-minded, he/she will set his/her θ approaching to 0. The resulting balanced decumulative weight is

$$H(D) = (1 - \theta)H_s(D) + (\theta)H_p(D). \tag{3}$$

Let p_j be the nominal weight on scenario j and let D_j be the nominal decumulative probability on scenario j that $D_j = \sum_{i=j}^{m} p_i$. By (3), the balanced decumulative probability on scenario j is then $H(D_j)$. Let $p_j^{sp/a}$

be the SP/A weight on scenario j and

$$\begin{aligned} p_j^{sp/a} &= H(D_{j-1}) - H(D_j) \\ &= ((1-\theta)H_s(D_{j-1}) + \theta H_p(D_{j-1})) \\ &- ((1-\theta)H_s(D_j) + \theta H_p(D_j)) \end{aligned} \tag{4}$$

Since the framework of the behavioral portfolio selection problem implies that when an investor evaluates the probability weights on the scenarios and expected values of future returns he/she will apply his/her own SP/A weights, therefore for the following proposed optimization model for the individual investors we adopted $p_j^{SP/A}$ as the probability measure. For B-stock i in the small pool, whose effect will take place on the next period with likelihood P_i^B and T day(s) after spotting the cause, the above SP/A weights on the scenarios also need to ensure that the collective weights on this B-stock i in the small pool should be equal to its P_i^B. For B-stock i in the small pool, let z_{ij} be the indicator such that $z_{ij} = 1$ if the effect of B-stock i appears in scenario j, otherwise $z_{ij} = 0$, then $\sum_{j=1}^m z_{ij} p_j^{sp/a}$ can be regarded as the probability based on SP/A weights on the scenarios that B-stock i has the desired effect. Therefore, the SP/A weights should further satisfy

$$\sum_{j=1}^{m} z_{ij} p_j^{sp/a} = P_i^B, \text{ for all the B-stock } i \text{ in small pool }. \tag{5}$$

Furthermore, since the P^Bs of each B-stock are statistically significant through the one proportion test (2), these P^Bs can be considered as professional facts, so these P^Bs should also be reflected in the investor's optimal portfolio. Moreover, since these P^Bs are statistically significant, then it is more likely, that similar results will occur T day(s) after when the causes of these B-stocks are spotted with respective probabilities P^Bs. In an economic intuition standpoint, it is just appropriate that we take advantage of the fact that after spotting the respective cause of these B-stocks there are higher probabilities for these B-stocks to have higher cumulative abnormal returns than those stocks that are not considered as B-stocks, so intuitively, we should consider these P^Bs in the optimization model, and which is why the adjusted SP/A weights should also satisfy and sum up to the individual P^Bs of all B-stocks in the investment pool.

2.3. B-stock portfolio optimization(BSPO) model

The safety-first(SF) MA (e.g. the pension account and education fund account) and the risk-seeking(RS) MA (e.g. one-shot-for-wealth account) are the extremes of MAs. Shefrin and Statman (2000) claimed that the probability constraint of the SF framework is more suitable to represent the behavior of the investors for portfolio optimization. Referring to the model of Shefrin and Statman (2000), their proposed model maximizes the total expected portfolio weighted return and subjected to the constraint satisfying the requirement on the probability of the return over a level, called the aspiration level. The aspiration level in the SF MA will be the lowest return we can tolerate.

Suppose that we have n stocks and m scenarios. Let $X = (x_1, x_2, ..., x_n)$, where $\sum_{i=1}^{n} x_i = 1$, be the portfolio and R_X be the return of portfolio X. Accordingly, the generic SF portfolio selection model is written as

$$\max \quad E[R_X] \tag{6}$$

$$s.t. \quad P(R_X \leq R_L) \leq \alpha, \tag{7}$$

where R_L is the lowest loss level that can be tolerated and α is a probability given by the investor as his/her threshold probability of having returns less than the lowest loss level that can be tolerated. $P(R_X \leq R_L)$ can be regarded as the downside risk of the portfolio and we should keep the downside risk not larger than α.

Scenario j is represented by a row vector of returns $(r_{1j}, r_{2j}, ..., r_{nj})$ where r_{ij} is the return of stock i on scenario j. Let p_j be nominal probability weight on scenario j. For scenario j, let R_{X_j} denote the return of portfolio X on scenario j and $R_{X_j} = \sum_{i=1}^{n} x_i r_{ij}$. The scenario-based SF portfolio selection model is written as

$$\max \quad E[R_X] = \sum_{j=1}^{m} R_{Xj} p_j \tag{8}$$

$$s.t. \quad R_{X_j} = \sum_{i=1}^{n} x_i r_{ij}, j = 1, 2, ..., m$$

$$R_L - R_{X_j} \leq M\omega_j, j = 1, 2, ..., m \tag{9}$$

$$\sum_{j=1}^{m} p_j \omega_j \leq \alpha \tag{10}$$

$$0 \leq x_i \leq 1, \omega_j \text{ is binary}, i = 1, 2, ..., n, j = 1, 2, ..., m,$$

where, for return scenario j, ω_j is an indicator variable such that $\omega_j = 1$ if the return of the portfolio is less than or equal to R_L and $\omega_j = 0$, otherwise. (10) together with (9) are equivalent to (7) in the SF model. The probability of the return to be less than or equal to R_L can be estimated by summing up the weights on the corresponding scenarios that have returns less than or equal to R_L and (10) is the corresponding constraint. In (9)

M is a sufficiently large positive number that makes $\omega_j = 1$(the real 1) in (9) if $R_L > R_{X_j}$. On the other hand, if $R_L \leq R_{X_j}$, ω_j can be 0 or 1(the unwanted 1). However, in the case that the probability of return less than or equal to R_L is truly less than or equal to α, (10) will only be satisfied if the summation on the left in (10) is on the real 1(s). Consequently, in order to satisfy (10), those unwanted 1(s) will eventually be forced to be 0 (s) as many unwanted 1(s) as possible by the objective function until (10) is satisfied. For the other case, (10) will definitely not be satisfied.

The proposed BSPO model includes B-stocks in the investment pool and utilizes SP/A weights (first dimension of weights) on scenarios. It is based on model (8) and by replacing p_j with $p_j^{sp/a}$, the expected return of portfolio X under SP/A weights is

$$E_{sp/a}[R_X] = \sum_{j=1}^{m} R_{Xj} p_j^{sp/a}, \tag{11}$$

where $p_j^{sp/a} = (1-\theta)H_s(D_{j-1})+\theta H_p(D_{j-1})-((1-\theta)H_s(D_j)+\theta H_p(D_j)), j = 1, 2, .., m$ as in (4). With the consideration of P_i^Bs, the weights also have to satisfy equation (5) or the second dimension (2-D) weights. The satisfaction of the 2-D weights will allow for an adjusted probability weights that not only consider the individual perspective (fear and hope levels) of the investor but also consider the actual likelihood (P^Bs) of the B-stocks to have high cumulative returns. In doing so, with the consideration of 2-D weights, we further expect better values for performance measures such as mean expected returns, alphas, betas, etc. compared to just considering the fear and hope levels of the investor. Suppose that there are K B-stocks among the total of n stocks. We let these B-stocks be indexed in the first K stocks. In order to obtain the feasible weights, we further relaxed the required probabilities p_i^B in (5) such that we set a relaxing range for P_i^B where the range starts from $1 - \delta$ of the P_i^B as in (13). We also let the θ in (3) be adjustable and be chosen in a range of the investor's θ in the model. The investor chooses an interval of θ covering his/her θ instead of the exact value of θ. Let these intervals be denoted by $I_i, i = 1, 2, ..., Q$. Given a chosen interval, the resulting θ will be determined in the following BSPO model to make the two-dimensional weights feasible. The value of θ in a range can be regarded as a compromising parameter between the investor's subjective characteristics and the market's objective information. Accordingly, the model called the two-dimensional BSPO model is written as follows:

$$\max \quad E_{sp/a}[R_X] \tag{12}$$

$$s.t. \quad \sum_{j=1}^{m} z_{i,j} p_j^{sp/a} \geq (1-\delta) P_i^B, i = 1, 2, ..., K \tag{13}$$

$$R_{X_j} = \sum_{i=1}^{n} x_i r_{ij}, j = 1, 2, ..., m$$

$$R_L - R_{X_j} \leq M\omega_j, j = 1, 2, ..., m \tag{14}$$

$$\sum_{j=1}^{m} p_j^{sp/a} \omega_j \leq \alpha \tag{15}$$

$$\theta \in I \tag{16}$$

$$0 \leq x_i \leq 1, i = 1, 2, ..., n,$$

$$\omega_j \text{ is binary}, j = 1, 2, ..., m,$$

$$M \text{ is a large positive number.}$$

In (16), interval I is chosen from $\{I_i, i = 1, 2, ..., Q\}$ by the investor such that it covers the investor's θ. $z_{i,j}, i = 1, 2, ..., K$ and $j = 1, 2, ..., m$, is the indicator such that $z_{ij} = 1$ if the desired effect of B-stock i appears on scenario j, otherwise $z_{ij} = 0$. Note that if there will be no B-stock considered on a given day of portfolio formation using the two-dimensional BSPO model, then equation (13) will not be utilized because there will be no P^B to satisfy. Consequently, the resulting portfolio will only have weights on the first dimension.

As mentioned the optimization models presented above all considered a scenario-based approach in selecting the optimal portfolio. The assignment of weights to the scenarios used follows the SP/A theory approach. Accordingly, since the returns of individual stocks are usually affected by the movement of the market, where the underlying assets normally go bull (bear) when we have a bullish (bearish) market, it is just appropriate that the ranking of the scenarios will be dependent on the market returns, such that given a set of historical returns the day with the highest (lowest) market return is considered as the best (worst) scenario. Moreover, in SP/A theory, the scenarios are supposed to be ranked from worst to best before the assignment of weights, therefore the market returns are ranked accordingly and then assigned with the appropriate SP/A weights. Consequently, the portfolio weights will now depend on the satisfaction of the safety-first criterion (1^{st} dimension) on scenarios and/or the P^Bs (2^{nd} dimension) of the B-stocks in the investment pool using the assigned SP/A weights to the ranked scenarios. Note that the portfolio weights are based on the ranked scenarios not the other way around such that the ranking of the scenarios is done first, followed by the assignment of weights, before determining the portfolio weights of the stocks in the portfolio to have the optimum return.

3. EMPIRICAL RESULTS

The B-stocks are chosen from the 888 common stocks in Taiwan stock exchange (TWSE). We considered daily portfolios in testing our proposed portfolio selection framework. Data starting from January 1991 to July 2014 were collected to determine the initial big pool and thereafter the big pool was updated every month. The in-the-sample test or back-test period is from August 1, 2014 to December 31, 2015 (352 trading days). For every daily portfolio, we used the past 500 historical daily data as our return scenarios.

3.1. B-stocks

As mentioned, the B-stocks are determined through statistical tests by studying the cause-and-effect relation patterns for specific irrational behaviors (over-reaction and disposition effect) based from their respective ODs. We count the number of times where the respective causes occur and then check the subsequent respective returns after T day(s) (time-to-effect) if they satisfy the desired resulting effect of a positive cumulative abnormal return ($CAR \geq 1\%$). We then determine and test the proportion, through 1 proportion test (2), if it satisfies a certain threshold probability. During these tests, it is assumed that all stocks tested for consideration to be B-stocks are exposed to similar limits to arbitrage, such that we accept that stocks which are considered as B-stocks will really have $CAR \geq 1\%$ T day(s) after their causes occurred with respective P^Bs (likelihood-of-effects).

As stated, we used a large negative price change and the identification of disposition losers(the stock with low R_G and low $\overline{AV}$) as the cause for B-stocks of over-reaction and disposition effect respectively. For over-reaction, we set the price change of -5% on return as the cause. The reason for this is that there is a mandate in the Taiwan Stock Exchange (TWSE) that stocks should not drop more than 7% in any trading day or else the stock will be delisted, so we believe that a drop of 5% can be considered as an already large enough large negative price change for a given day. Note that when the authors were identifying the over-reaction B-stocks the maximum drop allowed was 7% now it is further relaxed to 10%. For the disposition effect, since Dyl (1977) tested for disposition effect with a $R_G \geq 20\%$ during an 11 month period, we believe that $R_G \geq 10\%$ in a given month can be an appropriate value for testing the disposition effect in a daily setup. Thus, we set $R_G \leq -10\%$ with the negative $\overline{AV}$ as the cause which was verified through the test

$$H_0 \ : \overline{AV} \geq 0 \qquad (17)$$
$$H_1 \ : \overline{AV} < 0.$$

For $\overline{AV}$, V_t and $V_{M,t}$, we used $T_V = 30$ days for calculating the moving averages. Consequently, a and b in (1) were updated every month based on the past 3 years' daily data. For obtaining the R_G we used $T_{RG} = 30$ days. We considered the effects for both B-stocks as $CAR \geq 1\%$. To determine the time-to-effect T, the CAR for each stock was tested from 1 day up to 20 days after the cause has been spotted. The $p_{threshold}$ in the one proportion test (2) was set to be 0.5. The reason for this is that we want a stock that can have a high return with a probability more than tossing a fair coin. Accordingly, the stocks with significant cause-and-effect patterns are classified as the B-stocks and are then included in the big pool. The partial big pool of the first month(August 2014) with the corresponding P_i^Bs, P-values from test (2), and Ts are shown in Table 1.

TABLE 1.

Partial Big Pool of August 2014

Stock ID No.	B-stock	P_i^B	P-Value of test (2)	T
2406	Over-Reaction	0.5758	0.0258	3
2408	Over-Reaction	0.5912	0.0163	4
2471	Over-Reaction	0.5902	0.0795	14
2540	Over-Reaction	0.5884	0.0003	2
3673	Over-Reaction	0.6875	0.0169	5
4938	Disposition Effect	0.6111	0.0912	5
4960	Disposition Effect	0.8182	0.0014	6
5203	Disposition Effect	0.5682	0.0216	10
5264	Disposition Effect	0.7500	0.0786	11
5269	Disposition Effect	0.6579	0.0258	6

From Table 1, we can see 5 of each of the several classified over-reaction B-stocks and disposition effect B-stocks in TWSE with their respective time-to-effect T day(s), likelihood-to-effect P_i^Bs, and their P-values from test (2). Accordingly, this big pool is the initial investment pool where the daily investment pool (small pool) for each trading day is selected. Taking the big pool of the first month(August, 2014) as an example, in total from the 888 stocks tested for consideration of being B-stocks there are 559 identified B-stocks, 172 of which are disposition effect B-stocks, 504 of which are over-reaction B-stocks, and 117 of which are both disposition effect and over-reaction B-stocks. Classifying the 888 stocks by their market capitalization, 48 out of 59 large companies and 511 out of 829 small / medium companies are considered as B-stocks. The detailed classifications are shown in Table 2 in which it shows the number of B-stocks for each industry category and the respective ratio to the total number in that category. The overall percentage of B-stocks is 63%(559 out of 888). We also determined the categories having the percentage more than the overall percentage by performing the two-proportion test with an alternative hypothesis H_1 : category percentage $>$ gross percentage , where the resulting Fisher's P-values for all categories are listed in Table 2. From this same table, it shows that the stocks in Building & Construction, Electronics, Rubber, and Sport & Leisure have ratios more than the overall percentage. Their stocks have more than the average probability to be impacted by irrational behaviors of over-reaction and the disposition effect. Large-size companies also have higher percentage of being identified as B-stocks.

The small pool for each day contains the B-stocks in the big pool that will likely have the effect on the next period/day (tomorrow) T day(s) after the cause has been spotted. For each B-stock in the big pool with time-to-effect T, we looked at the $(T-1)$ day(s) backward to check if a cause can be identified on that day. If a cause can be identified, then we conclude that this B-stock will have the probability P_i^B to have the effect tomorrow and then the stock is included in the small pool. Note that for a given B-stock i, there will be occasions that there are successive days where we can identify a cause, in this case to be consistent with the identification of B-stocks in the big pool, we only consider the cause if it is the cause on the last day of a successive days with identifiable cause, so that the selected B-stock i for the small pool from the big pool will more likely have the desired effect with the corresponding P_i^B. Furthermore, these successive days with identifiable cause can affect the respective P^Bs of their corresponding effect to occur, so only the last cause from the successive days with identifiable cause is considered.

Taking stock 2406 shown in Table 1 as an example, which is an over-reaction B-stock and has a time-to-effect T of 3 days. Considering we are at day 0 we then look at the second day backward. Since that day has a return of -0.0551 as shown in Table 3, which is less than -5%, the cause of the impact of over-reaction has been identified and therefore there is a probability $P_i^B = 0.5758$ which is significantly larger than 0.5 to have the effect $CAR \geq 1\%$ on the next day. Consequently, Stock 2406 is then included in the small pool. Considering stock 2408 as another example, which has a time-to-effect T of 4 days, however, when we look at the third day backwards it has a return of 0.0147, therefore Stock 2408 is not included in the small pool. Moreover, looking at Stock 3673 which has a time-to-effect T of 5 days. Although the fourth day backwards has a return of -0.0691, but looking at the days after it we can see 2 more occurrences of

TABLE 2.

Genealogy Classification of B-Stocks

("DE" denotes disposition effect B-stocks; "OR" denotes over-reaction B-stocks; "Both" denotes OR and DE)

Industry	B-Stocks appearances DE	OR	Both	Total(A)	Total # of stocks (B)	% Percentage A / B	P-Value
Automobile	5	6	3	8	11	73 %	0.372
Building and Construction	5	40	2	43	50	86 %	0.000***
Cement	2	1	1	2	7	29 %	0.987
Chemical Biotech	14	21	6	29	56	52 %	0.964
Electric Machinery	7	23	5	25	47	53 %	0.932
Electrical and Cable	1	8	1	8	16	50 %	0.905
Electronics	89	257	67	279	408	68 %	0.033**
Financial Industry	10	7	3	14	29	48 %	0.962
Foods	1	6	0	7	22	32 %	0.999
Glass and Ceramics	1	3	1	3	5	60 %	0.732
Iron and Steel	0	15	0	15	29	52 %	0.921
Motors and Appliances	0	1	0	1	2	50 %	0.863
Oil, Gas, and Electricity	2	6	2	6	8	75 %	0.382
Paper and Pulp	0	3	0	3	7	43 %	0.929
Plastics	6	16	5	17	23	74 %	0.197
Rubber	3	11	2	12	12	100 %	0.004***
Securities	0	2	0	2	3	67 %	0.690
Shipping and Transport	3	11	3	11	21	52 %	0.887
Spinning	1	4	1	4	6	67 %	0.606
Sport and Leisure	4	4	3	5	5	100 %	0.100*
Steel	0	4	0	4	5	80 %	0.391
Textiles	3	24	2	25	47	53 %	0.932
Tourism	4	5	1	8	14	57 %	0.769
Trading and Consumer	5	9	4	10	18	56 %	0.814
Others	6	17	5	18	37	49 %	0.972
Total	172	504	117	559	888	63 %	
Large Stocks	27	41	20	48	59	81 %	0.002***
Small/Medium Stocks	145	463	97	511	829	62 %	0.729

* indicates significance at 0.1 level, ** indicates significance at 0.05 level, & *** indicates significance at 0.01 level

returns (-0.0698 and -0.0590) that can also be identified as the causes. Therefore, Stock 3673 is not included in the small pool because the effect that will occur on the next day may be affected by these two other causes.

Similarly, considering stock 5264 which is a disposition loser shown in Table 1, having a time-to-effect T of 11 days, we looked at the tenth day backwards. Based on the stock information that R_{GR} is not less than or equal to -10% therefore stock 5264 will not be included in the small pool. However, when checking stock 5269 whose time-to-effect T is 5 days, by looking at the fourth day backwards we can find $R_{GR} = -11.69\%$ and the P-value of testing (17) for a significant average abnormal volume ($\overline{AV}$) is 0.0683 as shown in Table 3, therefore we can identify a cause of a disposition loser and so we must include it in the small pool. The partial first day(August 1, 2014) small pool of the back-test is shown in Table 3.

TABLE 3.

Partial B-stocks Inclusion into small pool

	Over-Reaction			Disposition			
Cause	$R_t \leq -5\%$			$R_{GR} \leq -10\%$, P-value ≤ 0.1			
Stock ID No.	2406	2408	3673	5264		5269	
T	3	4	5	11		6	
Days Backward	R_t	R_t	R_t	R_G	P-Value	R_G	P-Value
0(today)	0.0016	0.0000	-0.0026	0.0410	0.8105	-0.0754	0.5796
1	-0.0065	0.0244	-0.0590	0.0536	0.7639	0.0178	0.5269
2	**-0.0551**	-0.0590	-0.0145	0.0621	0.6416	-0.0643	0.4810
3	-0.0698	**0.0146**	-0.0698	0.1124	0.5653	0.0437	0.3260
4	-0.0250	0.0000	**-0.0691**	0.1383	0.5271	0.0125	0.2087
5	0.0315	-0.0144	-0.0573	0.1343	0.5804	**-0.1169**	**0.0683**
6	0.0235	0.0335	0.0608	0.1744	0.4699	-0.1891	0.0272
7	-0.0087	-0.0037	-0.0124	0.1488	0.3625	-0.1893	0.0006
8	0.0029	-0.0171	0.0211	0.1098	0.2936	-0.2333	0.0004
9	-0.0087	0.0074	-0.0578	0.1458	0.1552	-0.1194	0.0003
10	-0.0086	-0.0158	-0.0492	**0.1243**	**0.0773**	0.0085	0.0000
11	0.0325	0.0000	0.0038	0.0723	0.0907	0.1485	0.0000
12	0.0181	0.0563	-0.0168	0.0506	0.1519	0.0979	0.0000
13	-0.0221	-0.0533	-0.0074	0.1048	0.2890	0.2104	0.0000
14	-0.0116	0.0273	0.0094	0.1308	0.2345	0.2988	0.0000
15	0.0073	-0.0639	-0.0075	0.1393	0.0894	0.3876	0.0000
16	-0.0187	-0.0137	-0.0036	0.0870	0.0964	0.5923	0.0000
17	-0.0347	-0.0034	-0.0452	0.0800	0.1180	0.5910	0.0000
18	-0.0041	0.0103	-0.0237	0.1133	0.1722	0.5873	0.0000
19	0.0197	0.0692	0.0138	0.1111	0.2151	0.6271	0.0001
In Small Pool	Yes	No	No	No		Yes	

3.2. Portfolios

We tested the two-dimensional BSPO model (12) through back-testing. The basic pool for the daily portfolios contain the top 30 stocks from the basket of stocks considered by Taiwan MSCI index but the two-dimensional portfolio selection model further considered the B-stocks in the small pool. Each model utilized the past 500 historical daily data as their return scenarios and the SF parameters are set as $R_L = -2\%$ with $\alpha = 2\%$. These parameters are considered because during the test period the market has returns of -2% on 11 of the 352 (3.13%) trading days.

This study used SP/A theory in the assignment of weights and considered the q_s and q_p parameters similar to those of Lopes and Oden (1999) and Singer (2009). An investor P_1 having a (q_s, q_p) = (3,1) is said to be a security-minded person, an investor P_2 having a (q_s, q_p) = (3,3) is said to be a cautiously-hopeful person, and an investor P_3 having a (q_s, q_p) = (1,3) is said to be a potential-minded person. To cater all types of investors from the most security-minded (risk-averse) investor to the most potential-minded (risk-seeking) investor, we expanded P_1, P_2, and P_3 by varying their θ values. Note that the higher the given θ, the riskier the investor behaves. Accordingly, we further considered seven representative behavioral investors with various θ: P_1 with $\theta = 0.2$ and 0.5, P_2 with $\theta = 0.2, 0.5$ and 0.8 and P_3 with $\theta = 0.5$ and 0.8. For the two-dimensional BSPO model, for testing purposes we considered 3 ranges of θ that can be chosen to satisfy the θ of the investor. They are $I_1 = [0, 0.4], I_2 = [0.4, 0.6]$ and $I_3 = [0.6, 1]$ which cover 0.2, 0.5 and 0.8 of θ, respectively. An investor P_i with θ chooses an I_j covering his/her θ. We used $P_i - I_j$ to denote the portfolio obtained from the two-dimensional BSPO model. Alternatively, we call $P_i - I_j$ as BSPO portfolios. We used the Market as our benchmark and considered the safety-first(SF) model (6) as a reference portfolio. Note that the SF portfolio don't consider SP/A weights on its scenarios meaning the likelihood of all scenarios are equiprobable. All the portfolios utilized the past 500 historical daily data as their return scenarios and the SF parameters were set as $R_L = -2\%$ with $\alpha = 2\%$. The SF portfolio considered the investment pool of only the basic 30 stocks while the $P_i - I_j$ portfolios considered the investment pool of the combined basic 30 stocks and B-stocks. For the two-dimensional BSPO model, $z_{i,j}$ in (13) is the indicator such that $z_{ij} = 1$ if the desired effect($CAR \geq 1\%$) of B-stock i appears in scenario j, otherwise $z_{ij} = 0$. To our knowledge, there is no clear documentation to calculate the CAR on the scenarios, however, it is possible to calculate the abnormal returns. Since T is the first T after the cause has been identified that $CAR = \sum_{t=1}^{T} AR_t$ is larger than or equal to 1%, it means that AR_t must also be positive on the T^{th} day. Accordingly, in this BSPO model, $z_{i,j}$ is relaxed to be the indicator whether the abnormal return of stock i is positive on scenario j on the T^{th} day. The only consequence is that we might have more B-stocks in our portfolios.

Considering the 7 $P_i - I_j$ portfolios ($P_1 - I_1$, $P_1 - I_2$, $P_2 - I_1$, $P_2 - I_2$, $P_2 - I_3$, $P_3 - I_2$, and $P_3 - I_3$), in an economic intuition standpoint, with the identification of significant desired cause-and-effect relations of the B-stocks, investors would now have an extra advantage in the investment world. They can use the information on B-stocks to generate portfolios that could probably outperform the market and SF portfolio. Given a stock and a B-stock with P^B, considering the same starting period, it is safe to assume that after T day(s), the B-stock would probably have a higher chance of having a positive cumulative abnormal return than the stock because of the P^B of the B-stock. Intuitively, by taking advantage of these B-stocks we have more chance of beating the market and generate substantial profits. Accordingly, we expect that the $P_i - I_j$ portfolios can outperform the reference portfolio (SF portfolio) and the Market.

3.3. Back-Test Results

We summarized the descriptive statistics (mean and standard deviation of the returns) of 352 daily returns of the portfolios, market, and SF portfolio. Then we identified the capital asset pricing model (CAPM) regression coefficients and statistics (Alpha, Beta, and R^2) of each portfolio. Moreover, we also studied and showed the distribution of returns, where we counted for each portfolio the number of days that they have positive returns and negative returns. Also, we calculated the cumulative returns and analyzed the cumulative returns observed on every day(daily cumulative return). All these comparison results are shown in Table 4.

TABLE 4.

Return Statistics, Ratio Metrics, and Distributions

Portfolio	Descriptive Statistics Mean daily return	σ	CAPM Coefficients and Statistics Alpha	Beta	R^2	No. of positive daily returns	No. of negative daily returns	No. of days of no portfolios	Cumulative returns	No. of positive daily cumulative return
Market	-3.3153×10^{-4}	0.0096	0.0000	1.0000 ***	1	170	182	0	−0.1243	121
SF	-3.8086×10^{-4}	0.0137	0.0000	1.1471 ***	0.6369	173	179	0	−0.1540	149
P1-I1	–	–	–	–	–	–	–	264	–	–
P1-I2	3.3619×10^{-4}	0.0136	0.0006	0.8467 ***	0.3514	166	150	36	0.0893	324
P2-I1	-1.0149×10^{-4}	0.0118	0.0000	0.5129 ***	0.1714	137	150	65	−0.0585	283
P2-I2	4.6768×10^{-4}	0.0130	0.0008	0.9451 ***	0.4818	167	147	38	0.1444	351
P2-I3	5.7954×10^{-4}	0.0126	0.0009**	1.0278 ***	0.6057	166	146	40	0.1925	352
P3-I2	2.2794×10^{-4}	0.0129	0.0006	0.9829 ***	0.5323	160	129	63	0.0524	351
P3-I3	3.7849×10^{-4}	0.0143	0.0007	1.0487 ***	0.4939	148	139	65	0.1025	352

* indicates significance at 0.1 level, ** indicates significance at 0.05 level, and *** indicates significance at 0.01 level

Since the BSPO model is based on the SF model, the model will choose not to have portfolio if the downside risk estimated by the investor's scenario probability weights exceed the threshold or additionally, there are no feasible two-dimensional weights correctly describing the probabilities (P_i^Bs) of the effects of the B-stocks. This means that there will be instances where specific portfolios will have certain days where there will be no investments or transactions because the safety-first constraints and two-dimensional constraints are not satisfied, thus portfolio returns on those days will be 0s. Among the portfolios, the more conservative the SP/A parameters of the portfolios are, the higher the chance of days of not having a portfolio. From Table 4, portfolio $P_1 - I_1$ have multiple days with no portfolios, so we excluded it in the analysis of the returns.

During the test period, the average daily return of the market is -0.0003 and SF portfolio is -0.0004, while all the BSPO portfolios have positive average returns except the portfolio $P_2 - I_1$. The standard deviations of all the portfolios are similar to the SF portfolio and larger than the Market. From here, it is apparent that the BSPO portfolios can be more profitable than the SF portfolio and the Market.

Further analyzing the returns with respect to CAPM, the Alpha values for the SF portfolio and Market are 0 while the $P_i - I_j$ portfolios except $P_2 - I_1$ all have alpha values greater than 0. Moreover, the Alpha value for $P_2 - I_3$ is also statistically significant. This means that with the right set of SP/A parameters using the BSPO model, it is possible to have statistically significant excess portfolio returns than the market. In addition, the Beta values for $P_i - I_j$ portfolios are very similar to one another and similar to SF's except for $P_2 - I_1$. The Betas of the BSPO portfolios are slightly lower than the Beta of the SF portfolio and are all close to 1 except for $P_2 - I_1$. The Beta values for $P_i - I_j$ portfolios also show that the portfolios are influenced to move on the same direction of the market. With respect to the goodness of fit test with the market returns, the R^2 of the portfolios show that the BSPO portfolios and the SF portfolio do not have a good fit with R^2 values all below 0.7. This is not necessarily a bad thing because we can have excess returns over the market and we want to outperform the Market, therefore the respective R^2 values of the portfolios are acceptable.

By looking at the number of days with positive and negative returns, all the BSPO portfolios have more days having positive returns than having negative returns except for $P_2 - I_1$, while Market and SF portfolio have more days with negative returns. For the cumulative returns, the Market and SF portfolio cumulative returns are all negative, however, among the BSPO portfolios only $P_2 - I_1$ have negative cumulative return. Among the BSPO portfolios, the highest cumulative return can reach 19.25% of $P_3 - I_2$ vs market's -12.43% and SF's -15.40%. When we checked the daily cumulative returns starting from the first day, there are 4 portfolios ($P_2 - I_2, P_2 - I_3, P_3 - I_2$ and $P_3 - I_3$) having 351 or 352 days of positive daily cumulative returns within the 352 test days while the market has 121 days and SF portfolio has 149 days. Again showing the profitability of the BSPO portfolios.

From the observations of the descriptive statistics, CAPM regression coefficients and statistics, and cumulative returns, the BSPO portfolios all showed the superiority of the two-dimensional model to the SF model and Market. Although the comparisons of portfolio performances from Table 4 show that in someway or another $P_i - I_j$ portfolios are superior than the SF portfolio and Market, these comparisons are still not enough to show the dominance of the two-dimensional model, so we further performed statistical tests to conclude more accurate pair comparisons of the daily returns of portfolios. Since the days are heterogeneous and the daily returns of portfolios depend on a particular day, to eliminate the day effects and therefore to reduce the variances of the testing, when comparing the average returns of two portfolios a more rigorous pair-t test should be used. The pair-t test in fact is a t-test on comparing the average of the difference of daily returns of two portfolios with 0. The alternative test hypothesis H_1 is that the average pair difference on return is greater than 0. The results of accepting the alternative hypothesis or rejecting the null hypothesis of the test is justified by sufficient evidence or data. On the other hand, if we fail to accept the alternative hypothesis, it is because the null hypothesis is true or we don't have enough data to conclude the alternative hypothesis. Moreover, when the evidence or data are comparatively not enough this is maybe due to the larger variances from the population. The significant levels considered here are 0.01 and 0.05. We also considered 0.1 as used by Chen et al. (1993) and Spiess and Affleck-Graves (1999).

Appropriately, we performed pair-t tests after deleting the outliers of the difference in returns. Considering a significance level $\alpha = 0.05$, the outliers are determined using Minitab software through the Grubbs' Test for outliers where we have a null hypothesis H_0: all data values come from the same normal population and an alternative hypothesis H_A: smallest or largest data value is an outlier. We compared each BSPO portfolio to the market and SF portfolio. The SF portfolio is also compared to the Market. The comparisons and the resulting P-values are included and showed in Table 5. The pair difference on return is the return of the row portfolio subtracted by the return of the column portfolio.

As shown in Table 5, all $P_i - I_j$ portfolios except $P_2 - I_1$, with strong evidence, outperform the SF portfolio. Moreover, in comparison with the Market the SF portfolio can't outperform the Market while portfolios $P_2 - I_2$, $P_2 - I_3$ and $P_3 - I_2$ outperform the Market. These show that portfolios with B-stocks utilizing two-dimensional weights for some representative portfolios are possible to outperform the Market significantly, while the SF portfolio having no weights on scenarios and P^Bs of B-stocks can't outperform the Market. The reasoning for this is that the $P_i - I_j$ portfolios consider the P^Bs of the B-stocks in the portfolio selection which enable

TABLE 5.

P-values of Pair-t test

	SF	Market
SF		0.544
$P_1 - I_2$	0.067*	0.130
$P_2 - I_1$	0.355	0.357
$P_2 - I_2$	0.033**	0.079*
$P_2 - I_3$	0.016**	0.011**
$P_3 - I_2$	0.061*	0.069*
$P_3 - I_3$	0.064*	0.172

* indicates significance at 0.1 level
** indicates significance at 0.05 level
*** indicates significance at 0.01 level

the $P_i - I_j$ portfolios to fully exploit the information on B-stocks, thus, generating superior portfolios.

4. CONCLUSION

We practiced the portfolio optimization problem by considering the B-stocks, which are impacted by the irrational behaviors of the investors, with the estimated time-to-effect Ts and the likelihood-of-effect P^Bs. Utilizing the cause-effect-P^B-T patterns of these B-stock in obtaining the investors' individual optimal portfolios, the contribution of this paper is three-fold. First is to identify the B-stocks and select those that can help us improve our portfolio. Second is to modify the SP/A weightings to satisfy the P_i^Bs of B-stocks into a two-dimensional SP/A weightings. Third is to have the respective BSPO model that considers not only the subjective characteristics of the investors but also the objective market information. By the empirical tests of the seven representative behavioral investors, the result shows that the two-dimensional BSPO model can outperform the market significantly. It also shows that the inclusion of the B-stocks and the two-dimensional weightings created a good synergy in obtaining the respective optimal portfolios which can be practical and profitable to any types of investor. By considering the investors' parameters as parameters of the model, the framework of our two-dimensional BSPO model can serve as a cornerstone to develop a new generic framework of general portfolio optimization models and these parameters can be updated dynamically.

There are still some suggestions to the future studies, first, although we showed that our two-dimensional BSPO portfolios can outperform the market, however, to be more practical, we need to further consider the impact of transaction costs. In future studies, we should further modify the procedure to have the behavioral portfolios with proper holding periods to compensate for the effect of the transaction costs. Second, there are also opportunities in the timing to update the big pools and the construction of the scenarios. In this study, we updated the big pools at the beginning of every month and we may dynamically update them according to the status of the market or other factors. Third, we used the historical data as our scenarios and we may have a sophisticated model to generate the scenarios. Fourth, later on our model can also be extended to consider multiple B-stocks in the portfolio by exploring more ODs of other irrational behaviors. Fifth, one of the objectives of this study is to generate respective portfolios that reflect the characteristics of the most risk-averse to the most risk-seeking investors by considering 7 sets of SP/A parameters, such that considering a specific set of SP/A parameters the resulting portfolio is the optimal portfolio based from the specific set of fear and hope levels. Although having dynamic SP/A parameters for the optimization could probably generate better performing portfolios than the respective portfolios of individuals with respective SP/A parameters, there are still issues regarding the timing and the appropriateness of the SP/A parameters to be used on a given day of portfolio formation, such that it can also be a future extension of the study wherein the focus would be determining the best combination of SP/A parameters to be used for a given day of portfolio formation considering the market situation and other economic factors. Lastly, for the sixth extension, this study wants to provide individual investors with an alternative investment option, but these individual investors normally don't have the ability to short sale stocks, so we focus on individual investor portfolio management without short selling. Moreover, in the identification of the over-reaction and disposition effect B-stocks, we only considered the significant cause-and-effect relations with a resulting positive cumulative abnormal return. Since we are expecting the B-stocks to have positive cumulative returns after their respective T day(s), then short selling strategies will be irrelevant to be further considered in the portfolio selection. Therefore, short selling can also be considered as a future extension of the study on B-stocks to cater financial institutions that have the capabilities to short sell stocks. This will also allow all investors and everyone to be able to fully exploit the information on B-stocks as they can short-sell (buy) B-stocks that are expected to have negative (positive) cumulative abnormal returns.

ACKNOWLEDGMENTS

This work was partially supported by the Ministry of Science and Technology, Taiwan, under the grant contract MOST 103-2221-E-033 -023.

REFERENCES

Ashiya, M. and T. Doi, 2001. Herd behavior of Japanese economists. *Journal of Economic Behavior and Organization* **46**, 343-346.

Baptista, A. M. 2012. Portfolio selection with mental accounts and background risk. *Journal of Banking and Finance* **36(4)**, 968-980.

Barber, B. M. and T. Odean, 2000. Trading is hazardous to your wealth: the common stock investment performance of individual investors. *The Journal of Finance* **55(2)**, 773-806.

Barber, B. M. and T. Odean, 2001. Boys will be boys: gender, overconfidence, and common stock investment. *The Quarterly Journal of Economics* **116(1)**, 261-292.

Barber, B. M. and T. Odean, 2013. The behavior of individual investors. In Handbook of the Economics of Finance, Vol. 2, 1533-1570. Elsevier.

Barnes, J. H., 1984. Cognitive biases and their impact on strategic planning. *Strategic Management Journal* **5(2)**, 129-137.

Boussaidi, R., 2013. Representativeness heuristic, investor sentiment and overreaction to accounting earnings: the case of the Tunisian stock market. *Procedia - Social and Behavioral Sciences* **81(0)**, 9-21.

Bowman, R. G. and D. Iverson, 1998. Short-run overreaction in the New Zealand stock market. *Pacific-Basin Finance Journal* **6(5)**, 475-491.

Chang, C., Jiang, J., and K. A. Kim, 2009. A test of the representativeness bias effect on stock prices: a study of super bowl commercial likeability. *Economics Letters* **103(1)**, 49-51.

Chang, K.-H., Young, M. N., Hildawa, M. I., Santos, I. J. R., and C.-H. Pan, 2015. Portfolio selection problem considering behavioral stocks. Proceedings of the World Congress on Engineering 2015 Vol II, WCE 2015, July 1 - 3, 2015, London, U.K.

Chen, G. M., Kim, K. A., Nofsinger, J. R., and O. M. Rui, 2007. Trading performance, disposition effect, overconfidence, representativeness bias, and experience of emerging market investors. *Journal of Behavioral Decision Making* **20(4)**, 425-451.

Chen, N. F., Kan, R., and M. H. Miller, 1993. Are the Discounts on Closed End Funds a Sentiment Index?. *The Journal of Finance* **48(2)**, 795-800.

Da Costa, N., Mineto, C., and S. Da Silva, 2008. Disposition effect and gender. *Applied Economics Letters* **15(6)**, 411-416.

Da Costa Jr, N., Goulart, M., Cupertino, C., Macedo Jr, J., and S. Da Silva, 2013. The disposition effect and investor experience. *Journal of Banking and Finance* **37(5)**, 1669-1675.

Das, S., Markowitz, H., Scheid, J., and M. Statman, 2010. Portfolio optimization with mental accounts. *Journal of Financial and Quantitative Analysis* **45(02)**, 311-334.

Das, S. R., and M. Statman, 2013. Options and structured products in behavioral portfolios. *Journal of Economic Dynamics and Control* **37(1)**, 137-153.

De Bondt, W. F. M., and R. Thaler, 1985. Does the stock market overreact? *The Journal of Finance* **40(3)**, 793-805.

De Bondt, W. F. M., and R. Thaler, 1987. Further evidence on investor overreaction and stock market seasonality. *The Journal of Finance* **42(3)**, 557.

Demirer, R., Kutan, A. M., and C.-D. Chen, 2010. Do investors herd in emerging stock markets?: Evidence from the Taiwanese market. *Journal of Economic Behavior and Organization* **76(2)**, 283-295.

Dyl, E. A. 1977. Capital gains taxation and year-end stock market behavior. *The Journal of Finance* **32(1)**, 165-175.

Fernandes, J. L. B., Pena, J. I., and B. M. Tabak, 2010. Behaviour finance and estimation risk in stochastic portfolio optimization. *Applied Financial Economics* **20(9)**, 719-738.

Ferris, S. P., Haugen, R. A., and A. K. Makhija, 1988. Predicting contemporary volume with historic volume at differential price levels: evidence supporting the disposition effect. *The Journal of Finance* **43(3)**, 677-697.

Frazzini, A., 2006. The disposition effect and underreaction to news. *The Journal of Finance* **61(4)**, 2017-2046.

Friedman, M., and L. J. Savage, 1948. The utility analysis of choices involving risk. *Journal of Political Economy* **56(4)**, 279-304.

Frino, A., G. Lepone, and D. Wright, 2015. Investor characteristics and the disposition effect. *Pacific-Basin Finance Journal* **31(0)**, 1-12.

Goo, Y. J., D. H. Chen, S. H. S. Chang, and C. F. Yeh, 2010. A study of the disposition effect for Individual Investors in the Taiwan stock market. *Emerging Markets Finance and Trade* **46(1)**, 108-119.

Gunaratne, P. S. M. and Y. Yonesawa, 1997. Return reversals in the Tokyo stock exchange: a test of stock market overreaction. *Japan and the World Economy* **9(3)**, 363-384.

Graham, J. R., 1999. Herding among investment newsletters: Theory and evidence. *The Journal of Finance* **54(1)**, 237-268.

Graham, J. R., C. R. Harvey, and H. Huang, 2009. Investor competence, trading frequency, and home bias. *Management Science* **55(7)**, 1094-1106.

Harless, D. W. and S. P. Peterson, 1998. Investor behavior and the persistence of poorly-performing mutual funds. *Journal of Economic Behavior and Organization* **37(3)**, 257-276.

Hu, L., Z. J. Sha, X. Y. Liu, and W.-J. Chen, 2013. An empirical study on the overreaction of Shanghai stock market. *Chinese Studies* **2**, 32-35.

Kahneman, D. and A. Tversky, 1972. Subjective probability: A judgment of representativeness. Cognitive Psychology **3(3)**, 430-454.

Kahneman, D. and A. Tversky, 1979. Prospect theory: an analysis of decision under risk. *Econometrica* **47(2)**, 263-291.

Lopes, L. L., 1987. Between hope and fear: the psychology of risk. In B. Leonard (Ed.), Advances in Experimental Social Psychology, Vol. Volume 20, 255-295. Academic Press.

Lopes, L. L. and G. C. Oden, 1999. The role of aspiration level in risky choice: a comparison of cumulative prospect theory and SP/A theory. *Journal of Mathematical Psychology* **43(2)**, 286-313.

Ma, Y., A. P. Tang, and T. Hasan, 2005. The stock price overreaction effect: evidence on Nasdaq stocks. *Quarterly Journal of Business and Economics* **44(3/4)**, 113-127.

Markowitz, H., 1952. Portfolio selection. *The Journal of Finance* **7(1)**, 77-91.

Madura, J. and N. Richie, 2010. Overreaction of exchange-traded funds during the bubble of 1998-2002. Bruce, B.R. Handbook of Behavioral Finance: Edward Elgar Publishing Limited.

Odean, T., 1998a. Are investors reluctant to realize their losses? *The Journal of Finance* **53(5)**, 1775-1798.

Odean, T., 1998b. Volume, volatility, price, and profit when all traders are above average. *The Journal of Finance* **53(6)**, 1887-1934.

Odean, T., 1999. Do investors trade too much? *American Economic Review* **89(5)**, 1279-1298.

Otchere, I. and J. Chan, 2003. Short-term overreaction in the Hong Kong stock market: can a contrarian trading strategy beat the market? *Journal of Behavioral Finance* **4(3)**, 157-171.

Pfiffelmann, M., T. Roger, and O. Bourachnikova, 2016. When behavioral portfolio theory meets Markowitz theory. *Economic Modelling* **53**, 419-435.

Rieger, M. O., 2010. SP/A and CPT: a reconciliation of two behavioral decision theories. *Economics Letters* **108(3)**, 327-329.

Scharfstein, D. S. and J. C. Stein, 1990. Herd behavior and investment. *The American Economic Review* **80(3)**, 465-479.

Shapira, Z. and I. Venezia, 2001. Patterns of behavior of professionally managed and independent investors. *Journal of Banking and Finance* **25(8)**, 1573-1587.

Shefrin, H., 2001. Do investors expect higher returns from safer stocks than from riskier stocks? *Journal of Psychology and Financial Markets* **2(4)**, 176-181.

Shefrin, H. and M. Statman, 1985. The disposition to sell winners too early and ride losers too long: Theory and Evidence. *The Journal of Finance* **40(3)**, 777-790.

Shefrin, H. and M. Statman, 2000. Behavioral portfolio theory. *The Journal of Financial and Quantitative Analysis* **35(2)**, 127-151.

Singer, N., 2009. A behavioral portfolio analysis of retirement portfolios. Thnen-Series of Applied Economic Theory. Working Paper 104.

Spiess, D. K. and J. Affleck-Graves, 1999. The long-run performance of stock returns following debt offerings. *Journal of Financial Economics* **54(1)**, 45-73.

Talpsepp, T., 2013. Does gender and age affect investor performance and the disposition effect?. *Research in Economics and Business: Central and Eastern Europe* **2(1)**.

Tan, L.,T. C. Chiang,J. R. Mason, and E. Nelling, 2008. Herding behavior in Chinese stock markets: An examination of A and B shares. Pacific-Basin Finance Journal **16(1-2)**, 61-77.

Telser, L. G., 1955. Safety-first and hedging. *The Review of Economic Studies* **23(1)**, 1-16.

Thaler, R., 1985. Mental account and consumer choice. *Marketing Science* **4(3)**,199-214.

Tversky, A. and D. Kahneman, 1992. Advances in prospect theory: cumulative representation of uncertainty. *Journal of Risk and Uncertainty* **5(4)**, 297-323.

Wylie, S., 2005. Fund manager herding: A test of the accuracy of empirical results using U.K. data. *The Journal of Business* **78(1)**, 381-403.