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Does the Fed Control Trend Inflation?

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The inability of central banks around the globe to increase the rate of inflation within their economies is becoming a serious concern for policy makers. In response to the financial crisis central banks expanded their balance sheets to unprecedented levels and the impact on inflation has been negligible. The failure of standard policy responses to ignite inflation has led several to call for alternative strategies like the Neo-Fisher hypothesis. We test for the necessary causal linkages using a cointegrated structural vector error correction model to determine how much control the Federal Reserve has over the trend rate of inflation in the U.S.

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JEL Classification Numbers: C32, E31, E42.

1. INTRODUCTION

Low and relatively stable inflation in the U.S. has been a hallmark of the economy for over three decades. But during the financial crisis of 2007-09 many economists were concerned about the possibility of a deflationary spiral and the Federal Reserve policy can be interpreted as a response, at least in part, to such concerns. Although economists view inflation as costly to the economy[1], it is fair to say that the fear of a deflation and its effects weighs heavily on the minds of modern central bankers.

Macroeconomics in the 20^{th} century seemed to have a handle on how to create inflation and inflation was the norm throughout the developed world. The standard Monetarist/Keynesian models made it clear that large government deficits and excessive money growth will eventually lead to inflation. But as the century drew to a close Japan entered a period in which the standard economic stories didn't hold true. Even as the Japanese government took on increasing amounts of debt to stimulate domestic spending

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[1]Temple (2002) surveys the literature on inflation and economic growth.

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and the Bank of Japan lowered its policy rate to zero (and lower) the inflation rate has been stubbornly unresponsive.[2]

The situation in Japan went from a curiosity to a global concern with the onset of the financial crisis in 2007. The Federal Reserve and the ECB in particular had adopted low inflation targets, making the prospect of a severe economic downturn leading to a significant deflation a very real possibility. While there is no theoretical reason[3] a deflation should be more costly than inflation, both create adjustment and shoe-leather costs. And while no longer the consensus it once was, the Friedman rule, can still be shown to be optimal monetary policy for a broad class of macro models.[4] But the experience with deflation, especially in the U.S., has been very negative. The Volker deflation of 1981-83 arguably created the worst U.S. recession, up to that time, since the Great Depression. And the Great Depression experience itself has left an indelible mark on the U.S. psyche, especially that of the Federal Reserve, given its role in the disaster. The Fed under Ben Bernanke was not going to allow another bank panic to cripple the economy by creating a deflationary spiral. The Fed expanded its balance sheet by 10 times from 2008 to 2013. And the Federal government increased its indebtedness by over 60% from 2008 to 2018.[5] Even so, measured inflation has averaged 1.56% since 2010 and has not exceeded 3.1%.[6] It seems we have forgotten how to create inflation.

Jim Bullard (2010) raised the idea, based on the simple Fisher equation, that perhaps the inflation rate was being constrained by the low policy rate established by the central bank and that if higher inflation was to be created it would only come after the policy rate was raised, effectively relieving the constraint. Because the idea that the central bank should raise the policy rate in order to create inflation seems counter-intuitive John Cochrane coined the term "Neo-Fisher" to describe the new approach. Cochrane (2014) and Stephen Williamson (2016) provided theoretical support to the idea. Cochrane (2016) shows that virtually all dynamic macro models demonstrate Neo-Fisherian behavior as a consequence of the built-in long-run super-neutrality. But the mechanism by which raising interest rates

[2]See https://fred.stlouisfed.org/graph/fredgraph.png?g=oIwQ and https://fred.stlouisfed.org/graph/fredgraph.png?g=lPlm and https://fred.stlouisfed.org/graph/fredgraph.png?g=oIwY.

[3]We are not saying that no theory exists to explain why deflations are more costly than inflations, or vice versa. We simply suggest that at a fundamental (purely theoretical in nature) level, the costs of either should be mitigatable by creating suitable contingent claims or indexing of prices, wages, etc. Of course, reality may indeed be asymmetric.

[4]See Gahvari (2007) for an analysis of the Friedman rule.

[5]Debt-to-GDP in Q1 2008 was 64.42% and in Q1 2019 it stood at 104.40%.

[6]Values are calculated over the sample January 2010 to December 2018 using the personal consumption expenditure price index PCEPI and inflation is the year-over-year percentage change in the index.

leads to higher inflation is not well developed. In the standard New Keynesian model the channel through which the Neo-Fisher effect operates is via the expectations of future inflation. Williamson (2016) is the only model we can find that explicitly attempts to specify the mechanism by which raising interest rates leads to higher inflation.

The fundamental implication of the Neo-Fisher hypothesis is that changing the policy interest rate (permanently) should lead to a change in inflation in the same direction, in the long run. Crowder (2018) tested this causality implication of the theory using bivariate VAR models and found that inflation causes nominal interest rates in the long run, not the way the Neo-Fisher hypothesis predicts. But that evidence may be inadequate at testing the broader causal pathways since the models are limited to two variables. In this study we allow for multiple channels through which a permanent change in the policy rate may eventually lead to permanent change in inflation and in the same direction. We use a five variable VAR characterized by four long-run equilibrium relationships and test for evidence that the policy rate causes the inflation, at least in the long run.

The next section lays out the specification of the empirical model while providing some theory to motivate the choice of variables. Section 3 discusses the data and the estimation results and section 4 concludes with some discussion.

2. THE EMPIRICAL MODEL

The Neo-Fisher hypothesis implies that raising the policy rate today will eventually lead to an increase in the rate of inflation. This implication of long-run causality can be tested within the time series framework. Equation (1) displays what is commonly referred to as the Fisher equation,

$$i_t = r_t + \pi^e_{t+k} \tag{1}$$

where the nominal interest rate on a k-period bond is i_t, the expected inflation rate is, π^e_{t+k} and the real interest rate is r_t. In a world characterized by long-run super-neutrality of money, the steady state equilibrium implies $r_t = r^*$, the long-run equilibrium or natural rate of interest. In the long run, the equilibrium real interest rate, r^*, should be independent of the inflation rate and outside of the policymakers' control. This super-neutrality is a common feature of modern dynamic macro models. Long-run equilibrium is characterized by a constant real rate[7], so mathematically an increase in i_t must lead to an increase in π^e_{t+k}, eventually. This is the fundamental insight of the Neo-Fisherian hypothesis. In order to raise inflation we must raise the nominal interest rate. And vice versa, to support a higher nom-

[7]In a stochastic environment this translates to a stationary time series process.

inal interest rate in equilibrium there must be higher trend inflation. All of these implications flow from the assumption of super-neutrality and a constant equilibrium real interest rate. The implication is that permanent changes in nominal interest rates can cause changes in trend inflation.[8]

Crowder (2018) used (1) as the foundation for a bivariate VAR to test the long-run causality implications. But there are several other channels by which changes in the policy rate can potentially impact inflation. The simple New Keynesian style Phillips relationship in equation (2) is one such route where the inflation rate is linked to the output gap.

$$\pi_t = \gamma E_t \pi_{t+1} + \kappa(Y_t - Y^*) \tag{2}$$

Equation (2) suggests we should anticipate that the output gap and inflation are positively related, at least in the short run. In addition the New Keynesian IS relationship, which links changes in the real interest rate to the output gap, may depend on more than just the short-term policy real rate, but may in fact rely on the impact on real rates at longer maturities. Furthermore, as will be discussed in more detail, causality in the time series context is about improvements in prediction. There is a long and large literature using the term structure of interest rates to improve the prediction of inflation, e.g. Mishkin (1990) and Gomez, Maheu and Maynard (2008). Including the output gap and longer-maturity interest rates will allow for a more robust test of the Neo-Fisher hypothesis.

We model the relationships in the data using a finite-order vector autoregressive (VAR) process,

$$X_t = \Phi_1 X_{t-1} + \cdots + \Phi_k X_{t-k} + \mu + \varepsilon_t \tag{3}$$

where X_t is a vector of variables integrated of order one or less, Φ_j are square coefficient matrices, μ is a vector of constants and ε_t is a white noise error vector with non-diagonal covariance matrix Ω. The VAR is a powerful and flexible way to model reduced form dynamic time series relationships and it is ubiquitous in applied macroeconomics. In our application $X_t = [gap_t \;\; \pi_t \;\; i_t^l \;\; i_t^s \;\; ff_t]'$ where i_t^l is the long-term nominal interest rate, i_t^s is the short-term nominal interest rate and ff_t is the policy rate. gap_t and π_t are output gap and inflation rate, respectively.

Economic theory and experience provide an initial expectation regarding cointegration such that we expect there to be four stationary or cointegrating relationships among the five variables in X_t.[9] First, we expect gap_t to

[8]The Neo-Fisher hypothesis does not preclude bi-directional long-run causality only that at least one direction of causality should flow from nominal interest rates to inflation.

[9]We conducted a full univariate unit root analysis, but the results are consistent with a priori expectation, namely that the output gap is $I(0)$ and the other four variables are all $I(1)$.

be $I(0)$ or stationary. We also expect to find a Fisher relationship between π_t and ff_t that yields a stationary real interest rate. Finally we expect to find two cointegrating relationships within the term structure group of i_t^l, i_t^s and ff_t such that the respective term premia are also $I(0)$. The set of equations in (4)-(7) shows the long-run relationships implied by economic theory.

$$\begin{aligned} gap_t - \beta_1 ff &= u_{1t} && (4)\\ \pi_t - \beta_2 ff &= u_{2t} && (5)\\ i_t^l - \beta_3 ff_t &= u_{3t} && (6)\\ i_t^s - \beta_4 ff_t &= u_{4t} && (7) \end{aligned}$$

In (4) we expect the output gap to be $I(0)$ implying that $\beta_1 = 0$. Equation (5) is the Fisher relation and equations (6) and (7) capture the term structure premia. Economic theory implies $\beta_2 = \beta_3 = \beta_4 = 1$. These are all testable hypotheses.

If X_t is cointegrated, the VAR in (3) has an equivalent vector error-correction model (VECM) representation that allows one to test for both short-run and long-run Granger causality. Cointegration occurs when two or more $I(1)$ variable share a long-run equilibrium such that deviations from the equilibrium are $I(0)$.

To estimate the potential cointegrating relationships and test hypotheses on the cointegration rank as well as restrictions on the parameters we employ Johansen's (1991) maximum likelihood estimator.[10] This estimator is based on the transformed version of (3) into its VECM form.

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \cdots + \Gamma_{k-1}\Delta X_{t-k+1} + \Pi X_{t-1} + \mu + \varepsilon_t \qquad (8)$$

Assuming there are four stationary relationships or cointegrating vectors among X_t, the long-run multiplier matrix $\Pi = \Phi(1) - I$ can be decomposed into two matrices such that $\alpha\beta' = \Pi$. The $(p \times r)$ matrix β represents the cointegrating vectors or the long-run equilibria of the system of equations. The $(p \times r)$ matrix α is the matrix of error-correction coefficients which measure the rate each variable adjusts towards the long-run equilibrium each period. Within this framework one can test for the number of cointegrating relationships by testing the column rank of Π. The likelihood ratio statistic for the rank of Π is called the trace statistic and is calculated $-T\sum_{i=r+1}^{p}\ln(1-\hat{\lambda}_i)$ where r is the hypothesized rank and $\hat{\lambda}_i$ are the estimated eigenvalues from the Johansen (1991) estimator. The distribution

[10]Besides making inference asymptotically standard in the VECM-MLE, several studies, e.g. Gonzalo(1994) and Haug (1996), using Monte Carlo analysis reveal the procedure to perform well under most data generating processes.

of this statistic is non-standard but simulated critical values are widely available.

From the earlier discussion we hypothesize the form of the cointegrating relationships:

$$\beta' X_t = \begin{bmatrix} 1 & 0 & 0 & 0 & -\beta_1 \\ 0 & 1 & 0 & 0 & -\beta_2 \\ 0 & 0 & 1 & 0 & -\beta_3 \\ 0 & 0 & 0 & 1 & -\beta_4 \end{bmatrix} \begin{bmatrix} gap_t \\ \pi_t \\ i_t^l \\ i_t^s \\ ff_t \end{bmatrix} \sim I(0)$$

where β_j are those from (4)-(7). As discussed earlier, theory further restricts the cointegration space by suggesting the following restrictions; 1) $\beta_1 = 0$, which implies that the output gap is $I(0)$, 2) $\beta_2 = 1$, the long-run (inverse) Fisher effect is one in equation (1), 3) $\beta_3 = \beta_4 = 1$, implied by expectations based theories of the term structure and assuming term premia are stationary. These restrictions can be tested using standard χ^2 distribution theory from the MLE framework.

3. LONG-RUN CAUSALITY IN COINTEGRATED SYSTEMS

The concept of causality in the time series context is tied to the idea of predictability. There is no statistical procedure that can determine actual temporal causality and the best that we can do is to define statistical causality in terms of prediction improvement. Dufour and Renault (1999) review the key concepts surrounding this definition of causality.

Bruneau and Jondeau (1999) also define causality in terms of forecast improvement and provide explicit conditions for long-run causality in the time series context. Consider the vector time series $X = (X_1, \ldots, X_N)'$ potentially integrated of order one, X_j is said to be not unidirectional prior causal for X_i in the long run if and only if, at any date t, the knowledge of lagged $X_{j,t-h}$, $h \geq 0$, does not improve the best linear prediction of X_i. Long-run causality is characterized by the following proposition, the proof of which is given in Bruneau and Jondeau (1999):

Proposition 1. *Consider the VAR process in (3). Then X_j is not unidirectional prior causal to X_i in the long run, if and only if*

$$\left\{ \sum_i C_{ji}(1)\Phi_{ij}(L) = 0 \right\}$$

or equivalently

$$\left\{ C_{ij} = 0 \ \mathit{and} \ \sum_{i \neq j} C_{ji}(1)\Phi_{ij}(L) = 0 \right\}$$

is satisfied from equation (9). Johansen (1991) shows that $C(1) = \beta_{\perp}^{0}\alpha_{\perp}'$, *where* $\beta_{\perp}^{0}$ *is the* (5×1) *orthogonal complement to the matrix of cointegrating relationships* β *and* $\alpha_{\perp}$ *is the* (5×1) *orthogonal complement to the matrix of error-correction parameters* α.[11]

$$X_t = C(1) \sum_{i=1}^{t} \varepsilon_i + C(1)\mu t + C^*(L)(\varepsilon_t + \mu) \tag{9}$$

Restrictions on $C(1)$ can be represented as restrictions on $\alpha_{\perp}'$ and restrictions on $\alpha_{\perp}'$ imply restrictions on α since $\alpha_{\perp}'\alpha = 0$. Proposition 1 relates the long-run causal structure in the VAR to restrictions on the $C(1)$ matrix. Such restrictions can be restated as restrictions on the error-correction space α where hypothesis tests can be carried out in the VECM-MLE using the likelihood ratio procedure yielding tests with standard asymptotic distributions.

This long-run causality result is closely related to the concept of weak exogeneity of variables in the cointegrated system. Engle, Hendry and Richard (1983) introduced a formal definition of weak exogeneity in which statistically efficient estimation and inference can be achieved from considering the conditional model only and ignoring the marginal distribution of the weakly exogenous variables.[12] In the present analysis, the test for weak exogeneity is exactly the same as the test for long-run causality. From (9) the cointegrated system of variables can be decomposed into permanent and transitory components. Those variables that are weakly exogenous (and therefore long-run causally prior to the other variables) have the natural interpretation as the source of the common stochastic trends in the cointegrated system. These common trends represent the long-run equilibrium trend in the system, literally the attractor that pulls the other variables in the system to it over time. Estimates of these common trends will provide a time series of the equilibrium path for all variables in the system.

[11]From the moving average representation of the VECM it can be shown that $C(1) = \beta_{\perp}(\alpha_{\perp}'(I - \sum_{i=1}^{k-1}\Gamma_i)\beta_{\perp})^{-1}\alpha_{\perp}'$. Then setting $\beta_{\perp}^{0} = \beta_{\perp}(\alpha_{\perp}'(I - \sum_{i=1}^{k-1}\Gamma_i)\beta_{\perp})^{-1}$, one can interpret $\alpha_{\perp}'$ as the source of the common trend and $\beta_{\perp}^{0}$ as the loadings of those trends onto each variable in the VAR.

[12]Dolado (1992) and Johansen (1992) provide thorough discussions of weak exogeneity in cointegrated systems and the reader is refereed to these sources for details.

4. THE EMPIRICAL RESULTS

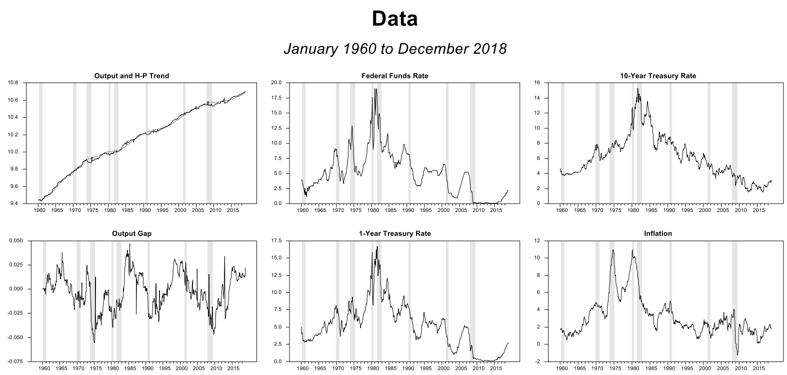
Crowder (2018) tested the implications of the Neo-Fisher hypothesis, i.e., long-run causality of the federal funds to the inflation rate, using a bivariate vector autoregression (VAR) and found strong evidence that inflation is the source of the common trend and thus the long-run causal forcing variable. The criticism is that alternative channels may exist that link the nominal policy interest rate to inflation via an indirect route, yet still ultimately causing inflation in the long run. To address this potential shortcoming, we use an expanded 5-variable VAR that includes the output gap, to capture the Phillips curve relationship such that changes in the policy rate impact the output gap and that in turn changes the inflation rate, and two longer-term nominal Treasury rates to capture any term structure effects that may be present.

The data consist of monthly observations on the inflation rate, the output gap, the federal funds rate, the one-year Treasury rate and the ten-year Treasury rate over the period from January 1960 to December 2018. The inflation rate is calculated as in equation (10),

$$\ln\left(\frac{P_{t+12}}{P_t}\right) \times 100 \tag{10}$$

where P_i is the personal consumption deflator.[13] We measure the output gap as the deviations of the natural log of real personal disposable income per capita from its Hodrick-Prescott filtered trend. Figure 1 displays the data.

FIG. 1.



[13]The data were retrieved from the FRED database at the St. Louis Federal Reserve Bank. The series mnemonics are a229rx0, PCEPI, FEDFUNDS, GS1 and GS10.

We estimate the VECM in (8) where $X_t = [gap_t \;\; \pi_t \;\; i_t^l \;\; i_t^s \;\; ff_t]'$ and we expect to find evidence supporting four cointegrating relationships among the five variables, implying one common trend. To determine the appropriate VAR lag k we used a general-to-specific procedure starting at $k = 24$ to $k = 0$ calculating the AIC statistic at each lag all while maintaining a constant estimation sample size, i.e. all criteria are calculated over the same sample. The resulting choice is $k = 14$.

The results from estimating the VECM using 14 lags in the levels VAR are displayed in table 1. The calculated trace statistics are known to suffer size distortion especially as the dimension of the VAR increases, e.g. Cheung and Lai (1993). Johansen (2000) suggests a Bartlett correction to the raw trace statistic to improve finite sample performance of the test. Consider a test statistic ξ_T that converges in the limit to ξ_∞ which has a known distribution. The statistic ξ_T is consistent of order T^{-1} while a transformed statistic ξ_T^* is of order T^{-2}. This transformed statistic can be based on the expectation of ξ_T since $\frac{\xi_T}{E(\xi_T)} = \frac{\xi_\infty}{E(\xi_\infty)}$ as $T \to \infty$, so that $\xi_T \approx E(\xi_T)\frac{\xi_\infty}{E(\xi_\infty)}$. Under the null hypothesis

$$E(\xi_T) = E(\xi_\infty) + \frac{g(\theta)}{T} + O(T^{-2}) \tag{11}$$

where $g(\theta)$ is known or can be consistently estimated. Dropping the $O(T^{-2})$ term, a little algebra yields the Bartlett-corrected statistic

$$\xi_T^* = \xi_T \left(1 + \frac{g(\theta_0)}{T}\right)^{-1} \tag{12}$$

where $g(\theta_0) = \frac{g(\theta)}{E(\xi_\infty)}$. Johansen (2000) demonstrates that $g(\theta)$ is a function of the parameters in the VECM and will depend on the sample size, the number of common trends and the specification of the deterministic components. The expected value of ξ_∞ is not known in most cases for testing cointegration rank since the distribution of such tests is usually non-standard so we approximate these expectations using simulations. The Bartlett-corrected trace statistics are displayed in Table 1. The evidence is consistent with the maintained assumption there are four cointegrating relationships among the five variables.[14]

[14]The Bartlett-corrected trace statistics were calculated using SVAR, a free stand-alone software package developed by Anders Warne. We thank Professor Warne for making this program available.

TABLE 1.

Unrestricted VECM Estimates

Cointegration Tests		$\hat{\lambda}_i$	Trace Stat[a]	P-value[b]	Bartlett Factor
	$r = 0$	0.1024	144.79	0.00	1.087
	$r \leq 1$	0.0561	76.60	0.00	1.075
	$r \leq 2$	0.0383	39.78	0.00	1.063
	$r \leq 3$	0.0157	14.29	0.08	1.061
	$r \leq 4$	0.0060	3.19	0.17	1.320
Cointegration Parameter Estimates	VECM dependent variable	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$
	gap_t	1.0	0.0	0.0	0.0
	π_t	0.0	1.0	0.0	0.0
	i_t^l	0.0	0.0	1.0	0.0
	i_t^s	0.0	0.0	0.0	1.0
	ff_t	$-7.35E-5$ (0.17)	-0.51 (4.09)	-0.90 (17.92)	-0.96 (48.87)
Weak exogeneity		$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$\hat{\alpha}_4$
63.08 [0.00]	gap_t	-0.15 (6.72)	-0.00 (1.14)	-0.00 (0.37)	0.00 (1.66)
2.81 [0.59]	π_t	0.89 (1.26)	-0.01 (0.95)	0.01 (0.59)	-0.00 (0.21)
14.13 [0.01]	i_t^l	-2.42 (2.83)	0.01 (1.75)	-0.15 (1.22)	-0.03 (0.89)
10.49 [0.03]	i_t^s	-1.04 (0.87)	0.02 (2.25)	0.02 (1.13)	0.02 (0.45)
23.84 [0.00]	ff_t	2.24 (1.62)	0.03 (2.43)	-0.03 (1.54)	0.21 (4.58)

Estimated parameters from the VECM in equation (8). a – Bartlett-corrected trace statistics. b – The p-values are from MacKinnon, et. al. (1999).

The (normalized) estimate of the cointegrating parameters is shown in table 1 and reproduced below,

$$\hat{\beta}' X_t = \begin{bmatrix} 1 & 0 & 0 & 0 & -0.00\ (0.17) \\ 0 & 1 & 0 & 0 & -0.51\ (4.09) \\ 0 & 0 & 1 & 0 & -0.90\ (17.92) \\ 0 & 0 & 0 & 1 & -9.96\ (48.87) \end{bmatrix} \begin{bmatrix} gap_t \\ \pi_t \\ i_t^l \\ i_t^s \\ ff_t \end{bmatrix} \sim I(0)$$

where numbers in parentheses are t-statistics. The first hypothesis we test is whether the output gap is stationary. This is done by testing if all other variables can be excluded from at least one of the cointegrating vectors. Given our normalization of the cointegration space, we can test this hypothesis by testing the null that $\beta_1 = 0$. The t-statistic is very small (-0.17) and the LR test of the same hypothesis is 0.02. The same hypothesis test conducted on each of the other variables yields rejections at high marginal significance levels, revealed by the large t-statistics.[15]

The second hypothesis of interest is the term structure restrictions $\beta_3 = \beta_4 = 1$. The LR test of these two restrictions produces a statistic equal to 2.80 which has a p-value of 25%. Testing the joint null of $\beta_1 = 0$ and $\beta_3 = \beta_4 = 1$ yields a LR statistic of 2.92. Tests of the Fisher restriction $\beta_2 = 1$ yield LR test stats of 3.38, p-value of 0.07, when it's the only restriction, and when we test the joint null $\beta_2 = 1$, $\beta_1 = 0$ and $\beta_3 = \beta_4 = 1$, the statistic is 6.43 with a p-value of 0.16. There is some evidence here that the Fisher effect from equation (1) is greater than one implying $\beta_2 < 1$. This would be consistent with the Darby (1975) effect in which taxes on interest income lead to a Fisher effect equal to $\frac{1}{1-\tau}$ where τ is the average marginal tax rate. Crowder and Wohar (1999) find strong evidence in favor of this effect and Crowder and Hoffman (1996) find that given post-war average marginal tax rates in the United States, the implied Fisher effect is approximately 1.4. Imposing the restriction that $\beta_2 = 0.7 \approx \frac{1}{1.4}$, leaving all other parameters unrestricted, yields a LR test of 1.01 with a p-value of 31%. Including the other restrictions on the cointegrating parameters, i.e. $\beta_1 = 0$ and $\beta_3 = \beta_4 = 1$, yields a test statistic of 4.10 and p-value of 0.39. Since the data slightly prefer the Fisher effect of 1.4 over 1.0, we present the results for that specification.[16]

As a last specification test we conduct the Hansen and Johansen (1999) stability test on the cointegrating vectors. This stability analysis is simply a series of recursive LR tests of the hypothesis that the estimated cointegrating vectors over each sub-sample are equal to the restricted values for the full sample, while treating all of the short-run dynamics as fixed and equal to their full-sample estimates. The results of this analysis and the estimated cointegrating relationships are displayed in Figure 2.[17] There is no evidence that the cointegrating relationships have been unstable over the last 30 years.

From equation (9) it's possible to decompose each series into its permanent and transitory constituent components and the results are shown in

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[15]The calculated test statistics testing the null of stationarity are gap_t 0.02, π_t 4.11, i_t^l 6.72, i_t^s 6.67, and ff_t 6.68. All are distributed as which has 5% critical value of 3.84.

[16]We conducted the complete analysis on the model where the Fisher effect is set at 1.0 and none of the qualitative results depend on this specification choice.

[17]We do not reproduce the first relationship which is just the output gap.

FIG. 2.

Cointegrating Relationships

Restricted

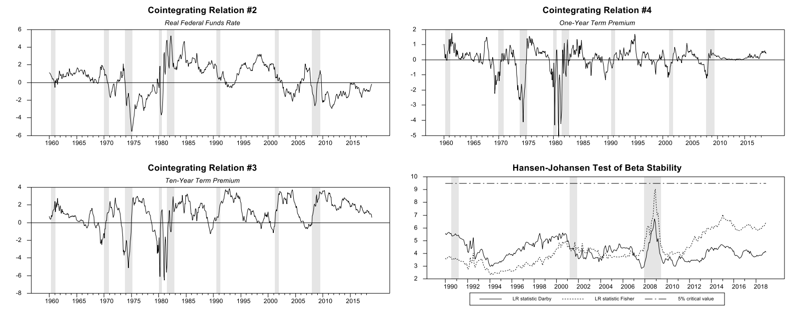


FIG. 3.

Estimated Permannet and Transitory Components

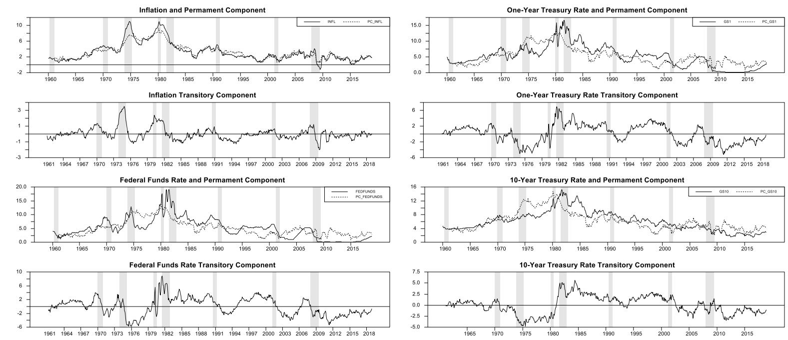


Figure 3. The estimated common trend most closely follows the behavior of the inflation rate. This is not surprising given the results of the weak exogeneity tests. These test values are displayed in Table 1, each testing the null hypothesis that the VAR dependent variable is weakly exogenous. As discussed earlier, weak exogeneity is closely related to long-run causality. A variable that is weakly exogenous cannot be caused, in the long run, by any other variable. Furthermore, weakly exogenous variables can be interpreted as the source of the common trend in the cointegrated system. In the present model there are five variables and four equilibria or cointegrating relationships implying one common trend. From (9) the permanent components of the data are captured by $C(1)\sum_{i=1}^{t}\varepsilon_i$ where the common stochastic trends are given by $\sum_{i=1}^{t}\varepsilon_i$, i.e. the accumulated innovations. Which elements of ε_i are included in the common trends is determined

by $\alpha'_\perp \sum_{i=1}^{t} \varepsilon_i$ and how these common trends are passed to each series is determined by $\beta^0_\perp$ where $C(1) = \beta^0_\perp \alpha'_\perp$. A test for weak exogeneity of the i^{th} variable in X_t is equivalent to a test that the i^{th} column of $\alpha'_\perp$ is not zero or that the i^{th} row of α is equal to zero.

The estimated error-correction parameters in the unrestricted VECM are also shown in Table 1. To the left is the calculated value of the LR test of the joint weak exogeneity restriction $\alpha_{i1} = \alpha_{i2} = \alpha_{i3} = \alpha_{i4} = 0$ for dependent variable $i = gap_t, \pi_t, i^l_t, i^s_t$, and ff_t. Examining the error-correction parameter estimates and their associated t-statistics reveals that each of the variables in the system has at least one, and most have two, significant error-correction coefficients, except for the inflation rate equation. This result is confirmed by examining the joint weak exogeneity tests where only the inflation rate is unresponsive to past equilibrium errors.

Table 2 displays the results from both weak exogeneity restrictions and Granger-causality tests, under both the unrestricted VECM and the VECM where β is restricted based on the results of earlier parameter analyses. Again, all of the evidence implies that inflation is weakly exogenous and is not caused, in the long run, by innovations in any interest rate, including the policy rate. The Granger-causality tests are of the joint significance of the lagged variable i and values in Table 2 are the F-statistics of the null hypothesis. For example, in the first equation of the VECM, where Δgap_t is the dependent variable, testing the restriction that the coefficients on lagged Δgap_{t-j} in that equation are all equal to zero yields an F-statistic of 3.70 which has a p-value of 0. But a similar test on lagged $\Delta \pi_{t-j}$ yields a test stat of 1.10 which is not significant at conventional levels. The estimated parameters and test statistics are similar across the unrestricted and restricted VECM specifications.

All of the results confirm that the common trends in (9) can be written as,

$$C(1)\sum_{i=1}^{t}\varepsilon_i = \beta^0_\perp \alpha'_\perp \sum_{i=1}^{t}\varepsilon_i = \begin{bmatrix} 0.0 \\ 0.7 \\ 1.0 \\ 1.0 \\ 1.0 \end{bmatrix} \begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{t}\varepsilon_{gap} \\ \sum_{i=1}^{t}\varepsilon_{\pi} \\ \sum_{i=1}^{t}\varepsilon_{i^l} \\ \sum_{i=1}^{t}\varepsilon_{i^s} \\ \sum_{i=1}^{t}\varepsilon_{ff} \end{bmatrix} \tag{13}$$

so that only innovations in the inflation equation accumulate to form the common trend. But the Granger-causality results suggest that inflation may be strongly exogenous, i.e., it cannot be forecast in the short or long run. The weak exogeneity of inflation already implies that inflation is long-run causally prior to any of the other VECM components, but the Granger-causality results suggest that inflation may also be short-run causally prior

TABLE 2.

Causality Tests

Weak exogeneity	VAR equation	$\sum gap_j$	$\sum \pi_j$	$\sum i^l_j$	$\sum i^s_j$	$\sum ff_j$
Unrestricted VECM						
63.09 [0.00]	gap_t	3.70 [0.00]	1.10 [0.36]	2.00 [0.02]	1.32 [0.20]	0.93 [0.53]
2.81 [0.59]	π_t	1.32 [0.20]	23.13 [0.00]	1.31 [0.20]	0.95 [0.50]	1.40 [0.15]
14.13 [0.01]	i^l_t	0.97 [0.48]	1.96 [0.02]	5.24 [0.00]	4.97 [0.00]	2.81 [0.00]
10.49 [0.03]	i^s_t	1.09 [0.37]	1.02 [0.43]	1.72 [0.05]	6.79 [0.00]	5.92 [0.00]
23.84 [0.00]	ff_t	1.17 [0.29]	0.98 [0.47]	0.78 [0.68]	5.80 [0.00]	4.35 [0.00]
Restricted VECM						
Weak exogeneity	VAR equation	$\sum gap_j$	$\sum \pi_j$	$\sum i^l_j$	$\sum i^s_j$	$\sum ff_j$
67.24 [0.00]	gap_t	3.85 [0.00]	1.11 [0.35]	2.00 [0.02]	1.32 [0.20]	0.95 [0.50]
3.29 [0.51]	π_t	1.34 [0.19]	22.98 [0.00]	1.32 [0.20]	1.00 [0.45]	1.54 [0.10]
17.44 [0.00]	i^l_t	0.96 [0.49]	1.99 [0.02]	5.27 [0.00]	4.99 [0.00]	2.86 [0.00]
14.37 [0.01]	i^s_t	1.26 [0.23]	0.99 [0.46]	1.75 [0.05]	6.77 [0.00]	5.81 [0.00]
26.30 [0.00]	ff_t	1.24 [0.25]	1.06 [0.39]	0.79 [0.67]	5.82 [0.00]	4.26 [0.00]

Causality tests from VECM in equation (8). Numbers in square brackets are p-values.

to the variables in the model. This result would imply a shocking lack of influence over the path of inflation by policy makers.

But the conclusion cannot be supported since the Granger-causality tests only examine the bivariate causality (predictability) between the dependent variable and the lagged independent variables. It may be that inflation is still caused, in the short run, by the other variables in the VECM through a complex dynamic relationship not well captured by simple bivariate causality tests. The solution is a block exogeneity test in which the VECM is estimated leaving all short-run parameters unrestricted and then re-estimated under the restriction that all other lagged variables can be excluded from the inflation equation. The resulting LR test statistic is 89.56 which is distributed $\chi^2(56)$ and can be rejected at high levels of significance.[18] This result highlights why one cannot rely on the weak exogeneity and bidirectional causality results alone to determine if any causality exists from the federal funds rate to inflation. The block exogeneity test result suggests a more subtle and complicated dynamic relationship exists between the policy variable and inflation, at least in the short run.

To gain some insight regarding these complicated dynamics, we calculate the impulse response functions for a structural version of the VECM in (8). The identification of a structural VAR characterized by cointegration is not much different from the standard structural VAR identification. The VAR in equation (3) can be rewritten as $\Phi(L)X_t = \varepsilon_t$, with the constant terms suppressed for ease of exposition and noting that $E(\varepsilon_t'\varepsilon_t) = \Omega$. We assume that this VAR is just the reduced form of a structural dynamic model with the form $A(L)X_t = \xi_t$ where $E(\xi_t'\xi_t) = I$. The following relationships exist between the structural and reduced form; $\Phi(L) = A(L)A_0^{-1}$ and $\varepsilon_t = \xi_t A_0^{-1}$ implying that $(\xi_t A_0^{-1})'\xi_t A_0^{-1} = (A_0^{-1}\xi_t)'\xi_t A_0^{-1} = (A_0^{-1})'A_0^{-1} = \Omega$, where A_0 is the contemporaneous coefficient matrix of the structural model. In general, the assumption that structural innovations are independent of each other, i.e. $E(\xi_t'\xi_t) = I$, delivers half of the needed restrictions to just identify a structural model from the reduced form. The remaining restrictions needed to identify the structure are usually imposed on A_0. The existence of cointegration, and the restrictions implied by cointegration, effectively reduce the number of a priori restrictions one must impose to achieve a structural interpretation. Since the structural errors are independent of one another, the permanent and transitory innovations in the system are also orthogonal by construction.

The five variable system we are analyzing has four cointegrating relationships and one common trend. Since the common trend is identified by the structure of $\alpha'_\perp$, as described earlier, the assumption of error independence, combined with the restrictions implied by cointegration is sufficient to identify the permanent structural innovation. Specifically, we exploit the result that $\beta_\perp\xi_t = C(1)\varepsilon_t$ where $C(1) = A(1)A_0^{-1} = \beta_\perp\alpha'_\perp$ and is the reduced form long-run impact matrix defined earlier. Since the rank of $C(1)$ is one, only one common stochastic trend or permanent component exists between the five variables in the VAR, knowledge of $\beta_\perp$ and the assumption that the permanent structural errors are independent of the transitory errors is sufficient to identify the permanent innovations.

It is convenient to partition the A_0 matrix into the parameters on the k permanent components versus those on the q transitory components, where

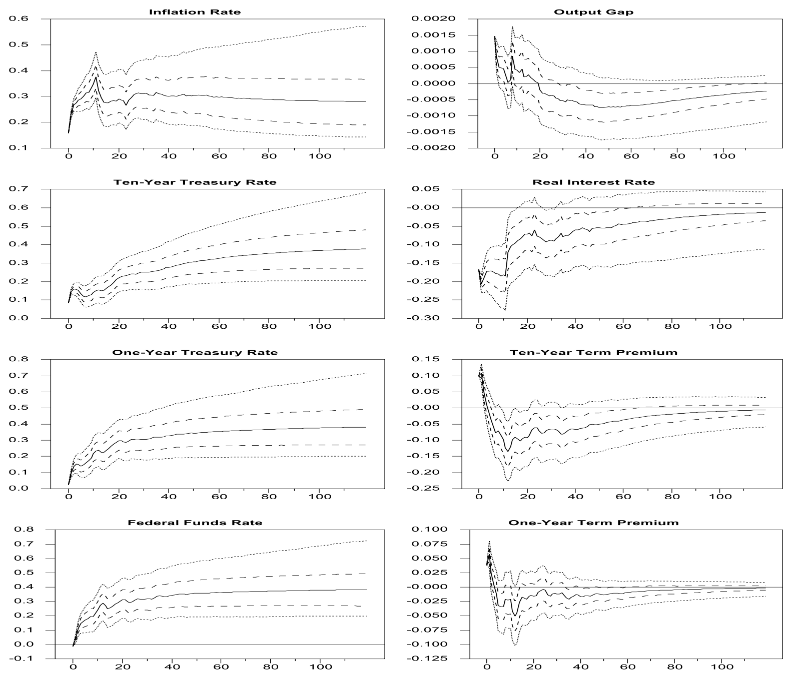
[18] The degrees of freedom for the test are the number of restrictions, in the π_t equation of the VECM all lags (13) of other four variables and all error-correction terms (4) set to zero in the restricted model

$k + q = N$, the dimension of the VAR system. $A_0 = \begin{bmatrix} A_0^k \\ A_0^q \end{bmatrix}$ where A_0 is $(N \times N)$ and $A_0^k = (\beta_\perp' \beta_\perp)^{-1} \beta_\perp' C(1)$. The covariance between the permanent and transitory innovations is $E[A_0^k \varepsilon_t, A_0^q \varepsilon_t] = (\beta_\perp' \beta_\perp)^{-1} \beta_\perp' C(1) \Omega A_0^q$ and we want this to be zero. It makes sense to include Ω^{-1} in A_0^q which allows us to focus on $C(1)$, $A_0^q = H_q \Omega^{-1}$ where $C(1)H_q' = 0$. Any space orthogonal to $C(1)$ will satisfy the requirement of transitory error independence from the permanent structural innovation and a natural choice is to choose the space spanned by the error-correction parameters since cointegration requires that $C(1)\alpha = 0$. All that is left is to ensure the transitory innovations are themselves mutually independent. When the transitory restriction matrix takes the form $H_q = Q^{-1}\alpha$, then the covariance of the transitory errors is $Q^{-1}\alpha'\Omega^{-1}\alpha(Q^{-1})'$ and $Q{-}1$ must be chosen from a suitable decomposition method such that $\alpha'\Omega^{-1}\alpha$ is diagonal.

FIG. 4.

Dynamic Response Functions:

Innovation to Inflation



Figures 4 through 8 display the estimated impulse response functions from the just-identified structural model along with the 68% (one s.d.) and 95% (two s.d.) simulated confidence intervals. Figure 4 shows the responses to a positive innovation in the permanent component, in this case the structural error in the inflation process. The inflation rate rises permanently as a result, as do the interest rates since they are all linked to each other via the Fisher relationship and the term structure. Interestingly the interest rates react to the inflation shock relatively slowly so that the real interest rate declines significantly. As expected, the term premia also rise in response to a positive inflation shock, with the ten-year premium rising more than the one-year term premium. This result is what we would expect as these term premia largely reflect inflation expectations. Expectations appear to adjust quickly since these term premia increases are modest and short lived. Finally, the response of the output gap is also consistent with the New Keynesian IS relationship, inflation shock leads to a temporary decline in the real interest rate which causes a temporary rise in output or a positive output gap. Although the innovation has permanent effects on inflation and the nominal interest rates, the real rate, output gap and term premia are stationary by definition so that even the permanent shock will only have temporary effects on these variables.

FIG. 5.

Dynamic Response Functions:

Innovation to Output Gap

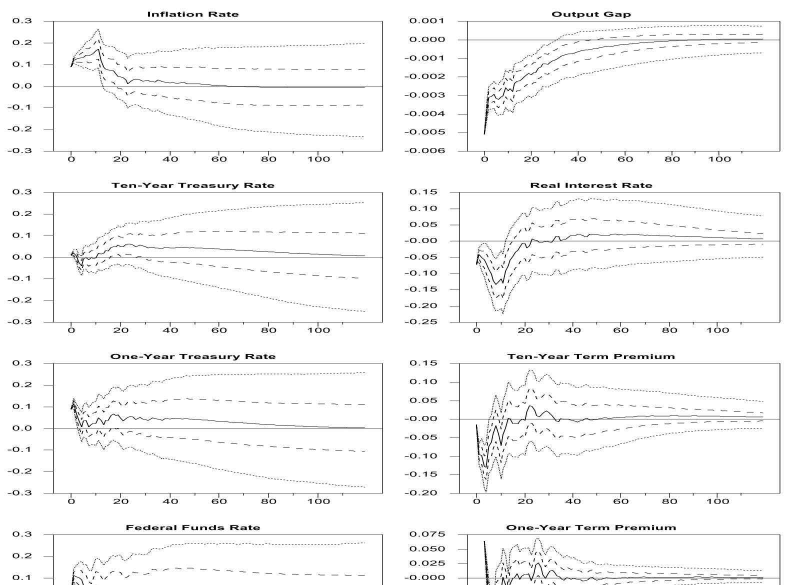
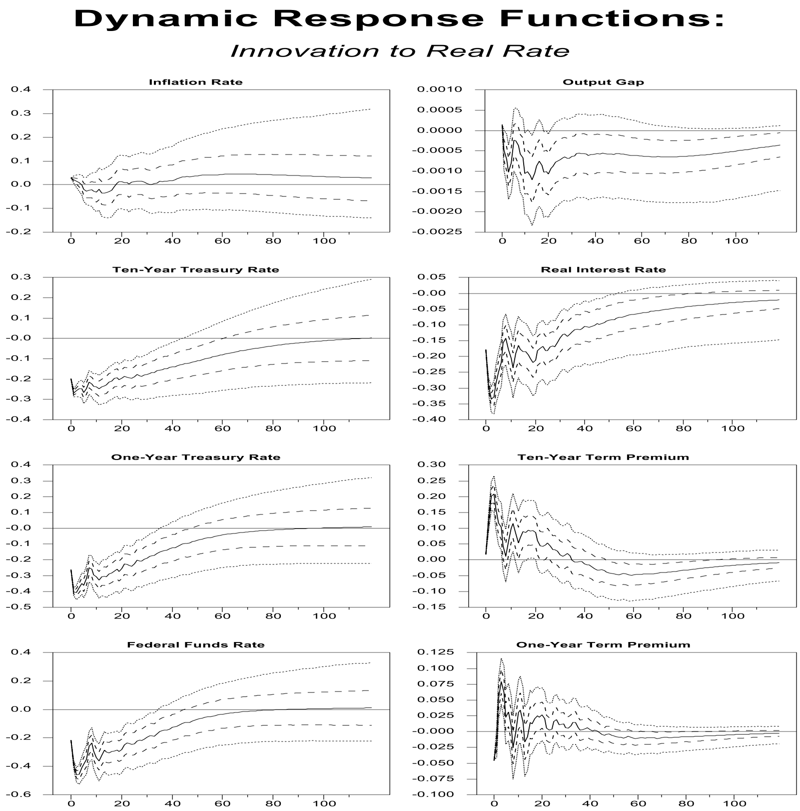
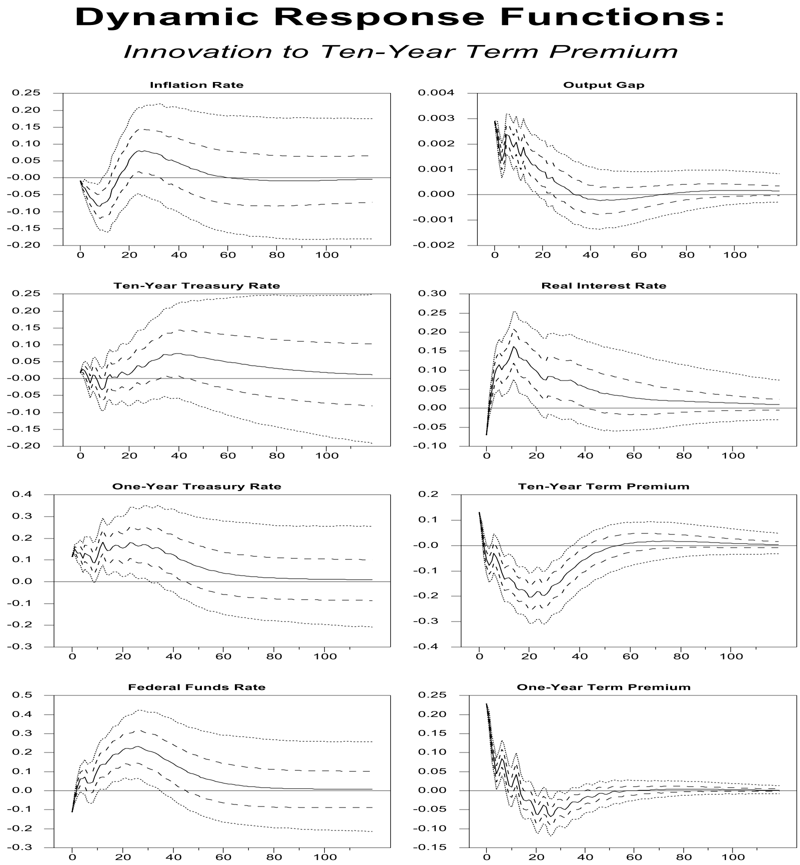


FIG. 6.



The second structural innovation is the first transitory shock and we interpret it as an output gap innovation. Figure 5 displays the dynamic responses of the variables to a negative output gap shock. By construction the responses to this innovation are temporary. The output gap can decline from either a decrease in aggregate demand or aggregate supply. While it was not our explicit intent on separately identifying AD versus AS shocks, it appears that the permanent shock discussed earlier, and the dynamic responses to it, are consistent with an AD shock. This would naturally lead to the interpretation of the second innovation as a (negative) AS shock. The primary evidence to support this interpretation is the response of inflation to the output shock. A negative transitory AS shock leads to a temporary increase in the inflation rate. The modest decline in the real interest rate is also consistent with a supply shock interpretation. The nominal interest rates respond very little to the shock, but the term premia

FIG. 7.

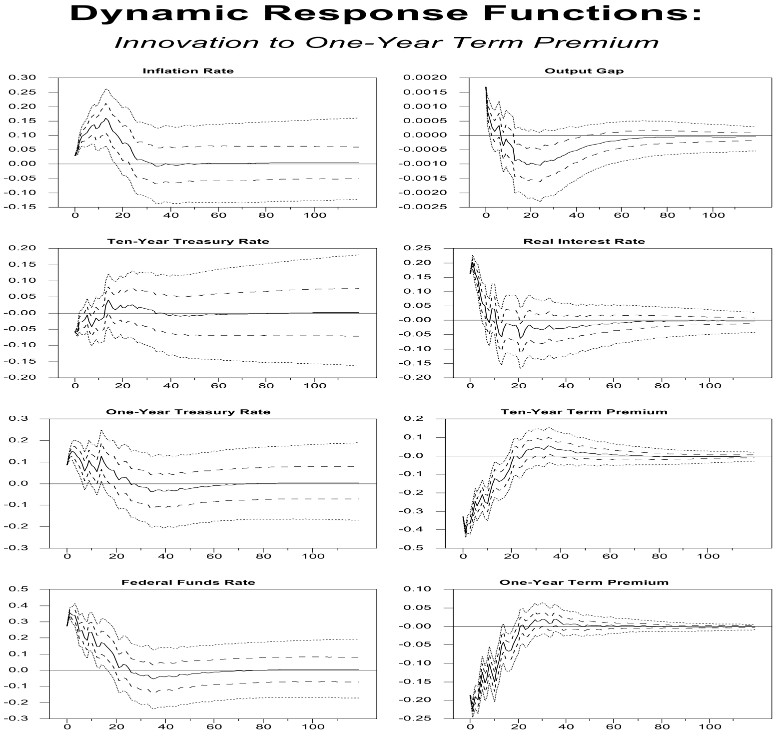


responses are very interesting since the one-year premium rises while the ten-year premium falls.

Figure 8 is also worth examining in some detail. The innovation is to the structural error in the federal funds rate equation of the VECM and we give it the interpretation as a term premium shock, but it is clear it is initiated by changing the federal funds rate. We see that an increase in the policy rate does cause a temporary rise in the inflation rate, consistent with the Neo-Fisher hypothesis.

The dynamic response analysis reveals that innovations in the federal funds rate can lead to a temporary movement of inflation in the same direction, but all permanent changes originate with the inflation process directly. If policy makers at the Fed wish to raise trend inflation, the

FIG. 8.



evidence in this study says it cannot be done solely by changing the policy rate.

5. CONCLUSIONS

We use an expanded VAR specification to test the Neo-Fisher hypothesis implication that permanent changes in the policy rate, i.e. the federal funds rate, cause or predict permanent changes in inflation. The results we find support the conclusions reached by Crowder (2018). The evidence we presented implies the direction of long-run causality is from the inflation rate to the nominal interest rates, not the other way. The evidence we have presented shows that the nominal policy rate can only marginally impact inflation and the effects are short lived. It is difficult to believe what these results mean for monetary policy. The implication that the Fed has almost no control over inflation via changes in the federal funds rate seems incredible, but that is what this evidence suggests.

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