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Ambiguity Aversion, Information Acquisition, and Market Opacity

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We investigate the effect of ambiguity on asset pricing and aggregate welfare. There are two types of traders, transparent and opaque, in the economy. The transparent traders face ambiguity about standard deviation (variance) of returns for extra investment opportunities when they trade. This ambiguity restraints their investment decisions, may lead to higher equity premium and loss of social welfare, cause the negative effects on market outcomes. Our analysis demonstrates that how the regulations, increasing information acquisition cost and reducing market opacity, affect traders' behavior, asset pricing and social welfare through transparent traders' ambiguity.

Key Words: Rational expectations equilibrium; Information asymmetry; Switching benefit; Equity premium; Welfare function.
JEL Classification Numbers: D81, G11.

1. INTRODUCTION

Financial markets are entrusted with all the important tasks of intermediating capital to real sector where it is most productive and maintaining a healthy balance between uncertainty (risk and ambiguity, Knight 1921) and reward (Khwaja and Mian, 2011). That the aggregate social welfare gains from efficient capital markets is well known. The more accurately the price reflects, the better it guides the investment allocation in the economy. However, information asymmetry, to some extent, distorts the allocation, and is deemed costly for an economy. Hsieh and Klenow (2009) empirically

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investigate the possible role of such resource misallocation to manufacturing plants leading to huge aggregate total factor productivity difference in China and India, compared with in Unites States.

Due to information asymmetry, investors only have incomplete knowledge of the financial markets. They face uncertainty when they are making investment decisions. In order to mitigate the uncertainty and enhance their own investment performance, investors have incentives to acquire more information to become informational advantage. Generally, agents can acquire private or inside information but they always incur some loss of wealth. Therefore, some are willing to pay for acquiring and others are not. This is a typical strategy to model information heterogeneity of traders in financial markets.

We adopt such a strategy in this study. Differentiating the information set they can observe, the traders are divided into two categories: transparent and opaque. This differentiation is reasonable because there is a remarkable difference between their information search motive and information analysis process, which leads to their different behaviors when information is incorporated into market. Transparent traders are in disadvantageous information position and can observe only the public information dispersed over the financial markets. Opaque traders, besides owning public information after paying for information acquisition, have access to private information not available to transparent traders, are in informational advantage position. Here the terms "opaque" and "transparent" mean that others see them as "opaque" if they have private information to develop complicated trading strategies, thus cannot see through their strategies; and "transparent" if they only have public information to trade risky asset and bond, thus can see through their strategies with ease.

In the model, we assume that transparent traders are ambiguous about the standard deviation or variance of the returns for the extra investment opportunities. Due to incomplete knowledge of the financial markets, they perceive multiple priors over the standard deviation, i.e., the perceived standard deviation belongs to a range including the true one, instead of forming a unique prior. A wider range means a higher dispersion of the beliefs, i.e., higher ambiguity, and vice versa. We assume their investment decisions depend on Maxmin Expected Utility (MEU) Theory (Gilboa and Schmeidler, 1989). That is to say, their portfolios should be selected to maximize the individuals' wealth in the worst pessimistic situation. Thus, they always choose the most conservative positions. Ambiguity has been used extensively to account for position and price behavior that is quantitatively puzzling in light of subjective expected utility theory, and explain many interesting phenomena in financial markets, including incomplete portfolio choice (Condie and Ganguli, 2009; Illeditsch, 2011) and non-participation (Dow and Werlang, 1992; Cao, Wang and Zhang, 2005;

Easley and O'Hara, 2009, 2010; Huang, Zhang and Zhu, 2017). While, opaque traders, with access to private information, are risk averse traders, and make investment decisions depend on Expected Utility Theory.

Opaque and transparent traders in our analysis are somewhat like the institutions and individuals in financial markets, respectively. Generally individuals can only observe public information and rely on it to make investment decisions.[1] They are less-informed and more vulnerable to the influence of psychological biases. Due to ambiguity about the extra investment opportunities, they only trade risky asset and bond in financial markets. While, institutions consist of investment funds, authorized securities firms, hedge funds, and so on. They have more financial strength, stronger information channels and much more professional investment teams. Especially, they are superior in the acquisition of firm-specific information, which is always not available to individuals (Holland and Doran, 1998). From this point, institutions are always in information advantageous position relative to individuals. We assume they need to pay for the private information, or say they pay for collecting information, building professional investment teams to understand and interpret the acquired information and develop into extra investment opportunities. Such opportunities could be direct investment in companies, foreign exchange markets, and assets in which individual traders are not permitted to entry. Thus, with the access to private information, institutions have enlarged investment opportunities.

In this study, we try to investigate the effect of ambiguity about the standard deviation of the returns for extra investment opportunities on asset pricing. Our special focus is on the implication of the policies that increasing information acquisition cost and reducing market opacity for asset pricing and welfare. To clarify this issue, we construct a rational expections equilibrium (REE) model in which opaque traders have private information and face no ambiguity, while transparent traders have no private but only public information and face ambiguity when they trade. The ambiguity captures the market opacity: if the market is with greater opacity, the perceived standard deviation belongs to a wider range, transparent traders face higher ambiguity and they are less confident and reluctant to trade; otherwise if the market is with less opacity, the perceived standard deviation belongs to a narrower range, transparent traders face lower ambiguity and they have more confidence to trade. This uncertainty introduces a variety of complications not captured by traditional classical models of asymmetry information, such as Hellwig (1980), Grossman and Stiglitz (1980) and Diamond and Verrecchia (1981).

[1]Although a vast strand of literature documents that individual investors are uninformed, Kaniel et al (2008, 2012) demonstrates that individual investors in NYSE sometimes are informed and perform well.

With a fixed fraction of opaque traders, we can solve the rational expections equilibrium model. The risky asset's price and equity premium can be determined. Then we use the capital asset pricing model (CAPM) to verify the value of information, i.e., the ex post performance of the both types of traders. It is found that opaque traders can always outperform the market systematically if only they have non-negative initial wealth, while transparent traders earn a market return, no matter what type equilibrium is prevailing in the economy.

However, from a regulatory perspective, it is inappropriate the information advantageous investors can always outperform the market systematically, and to some extent they profit at the expense of the disadvantageous information investors. To mitigate such market outcome due to information asymmetry in financial markets, the government's corresponding regulations includes two aspects: (1) to increase the information acquisition cost, so as to decrease the population of the information advantageous investors, and to keep most of the traders less informed; (2) to reduce the market opacity, so as to reduce ambiguity perceived by less informed investors, and to keep most of the traders informed. How do these policies affect the market outcome? Furthermore, are these policies efficient for the regulations? These questions are important for understanding the economics of market opacity, as well as for guiding reforms that attempt to increase transparency.

Then we investigate the implication of regulations increasing information acquisition cost and reducing market opacity for asset pricing and welfare. In our model, such regulation changes affect the equilibrium fraction of opaque traders, then it needs empirical studies to make clear the final effects on the economy. The analysis results demonstrate that increasing the information acquisition cost will decrease the fraction of opaque traders, increase the equity premium, and uniformly decrease the aggregate social welfare. Thus, the regulatory policies aimed to limit the fraction of opaque traders simply by increasing information acquisition cost is detrimental to the economic entity. Thus, it is inefficient.

Regulatory policies designed to reduce ambiguity perceived by transparent traders, such as enhancing disclosure rules and stringent listing standards to lower market opacity, have complicated effects. Generally, a market with lower opacity lessens the ambiguity perceived by transparent traders. However, such regulations reduce the advantage of opaque traders owning private information and reduce their opportunities to profit, thus decrease the fraction, but the effect on equity premium is complex. On one hand, reducing ambiguity makes transparent traders become confident to trade risky asset, and this tend to reduce the equity premium; on the other hand, reducing ambiguity also reduce the fraction of opaque traders, and this tends to increase the equity premium. Our results demonstrate that regulations for stringent disclosure requirements of the issuing firm and trading information to render a lower market opacity, may induce more opaque traders thus lower equity premium, or less opaque traders thus higher equity premium for different cases. However, there is a consistent conclusion that a lower market opacity will increase the aggregate social welfare.

Our paper is based on the recent papers studying the process of information transmission through prices in the presence of ambiguity (Epstein and Schneider 2008; Caskey 2009; Banerjee and Green 2010; Ozsoylev and Werner 2011; Easley, O'Hara and Yang 2014; Vives 2014; Mele and Ganguli 2015, etc). Market prices aggregate and convey information about the fundamentals of firms although the misalignment of stock prices and fundamentals is possible but typically not long-lived. Epstein and Schneider (2008) propose a model of learning, within the multiple priors framework (Gilboa and Schmeidler, 1989) by assuming investors perceive a range of signal precisions and take a worst case assessment of precision when evaluating actions to investigate the impact of uncertain information quality in financial markets. They find that ambiguity averse investors react asymmetrically to signals: they always react more strongly to bad news than to good news. Caskey (2009) uses smooth ambiguity aversion model (Klibanoff, Marinacci and Mukerji, 2005) to demonstrate that ambiguity averse investors may prefer aggregate information which is nonsufficient statistic for its components even when they have free access to disaggregate information, thus causing prices to underreact to public information. Under this circumstances, equilibrium prices may fail to reflect all the available information so that profit opportunities arise. The model can explain both underreaction, such as the evidence in postearnings announcement, and overrection to accounting accruals. Condie and Ganguli (2009) show that if an ambiguity averse investor has private information, then portfolio inertia can prevent the revelation of information by prices even if there is the same number of uncertain fundamentals and prices. Illeditsch (2011) shows that ambiguous information may lead to risky portfolio that are sensitive to news but insensitive to changes in the stock price, referred as portfolio inertia, and small shocks to cash flow news, asset betas, or market premia, can lead to drastic changes in stock markets and hence to excess volatility. Ju and Miao (2012) use the generalized recursive ambiguity utility model in a general equilibrium setup to study the implications of fragile beliefs for asset pricing. Watanabe (2015) assumes that the investors are ambiguous about asset supply. Then ambiguity averse investors learn less from the market price that equilibrates demand and supply. This hinders coordination of equilibrium beliefs, can make the market unstable and lead to flash crash in the extreme. Mele and Sangiorgi (2015) document that the incentives to reduce risk by acquiring information diminish, just as in Grossman and Stiglitz (1980), but the incentives to reduce ambiguity increase. Thus, due to uncertainty aversion, the incentives to reduce uncertainty by acquiring information increase as more investors acquire information.

This paper is close in methodology to a strand of recent literature, in which rational expectations equilibrium (REE) is adopted. Allen and Jordan (1998) provides an excellent survey of papers on rational expectations equilibrium and its properties in microeconomic models. In rational expectations equilibrium, Ui (2011) considers a stock market with ambiguity averse informed investors under the CARA-normal setting, and studies the relationship between limited market participation and the equity premium which is decomposed into the risk premium and the ambiguity premium. Breon-Drish (2010) studies a standard noisy rational expectations model in the spirit of Grossman and Stiglitz (1980) but relaxes the usual assumption of joint normality of fundamentals and supply. Condie and Ganguli (2011a) show that the existence and robustness of partially revealing rational expectations equilibria in economies with ambiguity-averse preference. This finding illustrates that the presence of ambiguity aversion in markets could have even broader implications. Condie and Ganguli (2011b) establish full revelation for almost all sets of beliefs for Choquet Expected Utility with convex capabilities. Ozsoylev and Werner (2011) propose a novel approach to connect ambiguity to market depth, liquidity risk and trading volume in a typical microstructure model that emphasizes the interaction between uncertainty and private information. Vives (2014) presents a simple large-market REE model which provides conditions to solve the paradoxes associated to fully revealing equilibria in a context where prices aggregate efficiently information. Banerjee and Green (2015) develop a model in which some investors are uncertain whether others are trading on informative signals or noise. Uncertainty about others leads to a nonlinear price that reacts asymmetrically to news. The model connecting rational expectations and differences of opinions to model belief heterogeneity, seems a useful framework for future analysis. Breon-Drish (2015) provides a constructive proof of the existence of equilibria and in the two types and continuum-of-investors settings have given sufficient conditions for uniqueness of this equilibrium within the continuous equilibria. The results open up a broad class of models for applications, including price reaction to information, price drifts and reversals, and the disagreement-return relation. Rahi and Zigrand (2015) provides a framework for studying competitive rational expectations equilibria that encompasses the classical REE models in the CARA (Constant Absolute Risk Aversion)-normal tradition. Condie and Ganguli (2015) show that partially revealing REE arise and affect market variables when private information is received by ambiguity averse investors who exhibit portfolio inertia with respect to information.

This paper is also related to a growing body of literature that studies information acquisition (Barlevy and Veronesi, 2000, 2008; Chamley, 2008; Garcia and Vanden, 2009; Van Nieuwerburgh and Veldkamp, 2010; Colombo et al, 2014; Mele and Sangiorgi, 2015; Huang, 2015; Caplin and Dean, 2015; etc). Barlevy and Veronesi (2000) argue that contrary to the conventional wisdom set forth in Grossman and Stiglitz (1980), it is theoretically possible that as more traders in financial markets acquire information, equilibrium prices would change in such a way that it becomes more difficult for remaining agents to infer the fundamentals from prices. They present an example they thought demonstrates this claim. However, as is subsequently pointed out by Champley (2008), the expression they use for the value of information in that paper is incorrect. As demonstrated by Champley (2007), using the correct expression for the value of learning reveals that learning is in fact a strategic substitute in their example. Garcia and Vanden (2009) study the formation of mutual funds. In their model, informed agents set up mutual funds as a mean of selling their private information to uninformed agents. They investigate the case of imperfect competition among fund managers, where uninformed agents invest simultaneously in multiple mutual funds. The size of assets under management in the mutual fund industry is determined by endogenizing the agents' information acquisition decisions. Mele and Sangiorgi (2015) find that ambiguity aversion could lead to multiple equilibria in the economy, history dependent prices, and large price swings and cause costly information acquisition to be strategic complementary. By identifying the source for strategic complementarities in trading and information acquisition, Goldstein and Yang (2015) show that greater diversity of information in the economy enhances price informativeness and demonstrate that both the size of the informed population and the composition matter in determining traders' behavior and market outcomes. Huang (2015) studies the effect of introducing an options market on investors' incentive to acquire information in a rational expectations equilibrium model.

This paper is most closely related to Easley, O'Hara and Yang (2014), Zhang and Zhu (2016), and Shi and Zhang (2016). Easley, O'Hara and Yang (2014) investigate the implications of regulations affecting the cost of operating a hedge fund and disclosure requirements on asset pricing and aggregate social welfare. It is found that increasing the differential cost of operating a hedge fund decreases the equilibrium fraction of hedge funds, increases the equity premium, and decreases welfare, while, the effects of increasing disclosure are ambiguous. Extending Easley, O'Hara and Yang (2014), Zhang and Zhu (2016) assumes that the transparent traders are ambiguous about the expected return of the extra opportunities to derive the equilibrium. Shi and Zhang (2016) includes a third type of traders, incomplete informed traders, to study the implication of ambiguity on asset pricing.

Our paper differs thus complements the above literature with two points. Firstly, we want to emphasize the source of ambiguity. In the existing studies, investors are ambiguous about asset payoff (Zhang and Zhu, 2016), public fundamental signals, such as earnings report and analyst forecasts (Illeditsch, 2011), or uninformed traders are ambiguous about the trading strategies of informed traders (Easley, O'Hara and Yang, 2014), i.e., the coefficients between the noise and the returns on extra investment opportunities. We assume in this paper that transparent traders are ambiguous about the standard deviation, or variance (risk) of the return for extra investment opportunities. Such ambiguity restraints transparent traders' investment decisions and affects asset pricing and social welfare through a different mechanism. Secondly, the range of the correlation coefficient. Easley, O'Hara and Yang (2014) assume the returns for extra investment opportunities are positively correlated with the noise item in the expression of the asset value. Such assumption limits the correlation to positive and cannot describe full relationship. Thus, in order to explore the correlation for a general case, we extend the coefficient to full range in this paper.

The rest of this paper is organized as follows. Section 2 presents the model, describes the market environment in the economy and the source of ambiguity. Section 3 derives the demand functions of both traders, calculates risky asset prices and equity premiums under rational expectations equilibria, and verifies the ex post performance with Capital Asset Pricing Model for both traders. Section 4 investigates the equilibrium traders distribution. Section 5 studies the implication of changes in information acquisition cost and market opacity to asset pricing and and the social welfare. Section 6 concludes. Appendix presents the proofs.

2. BASIC MODEL

The market environment settings are similar to Easley, O'Hara and Yang (2014). We consider a market for a risky asset, the stock, which has a price of $\tilde{p}$ per unit and an uncertain future value $\tilde{v}$. It is assumed that

$$\tilde{v} = \overline{v} + \tilde{\theta}_O + \tilde{\theta}_T + \tilde{\varepsilon} \tag{1}$$

where $\overline{v} > 0$, $\tilde{\theta}_O \sim N(0, \sigma_{\theta O}^2)$ with $\sigma_{\theta O} > 0$, $\tilde{\theta}_T \sim N(0, \sigma_{\theta T}^2)$ with $\sigma_{\theta T} > 0$, $\tilde{\varepsilon} \sim N(0, \sigma_{\varepsilon}^2)$ with $\sigma_{\varepsilon} > 0$, and that all theses random variables are mutually independent. we can view this risky asset as a proxy for the stock market. A riskless asset, bond, is also available for trading; it has a constant value of 1, with perfectly elastic supply, and yields a rate of return equal to zero.

The market is populated by a continuum of rational traders who are initially identical. Each is endowed with one share of the stock and no bond. They have negative exponential utility u with constant absolute risk aversion 1 over their final wealth, that is, $u(w) = -e^{-w}$. Prior to entering the asset market, all of the traders are transparent. They can choose to become opaque at cost $c > 0$ for private information, or remain transparent. The cost c can be interpreted as information acquisition cost.[2] The difference between these two types of traders are that: transparent traders can only observe public information over asset markets, while opaque traders have access to private information as well as public information. We denote μ as the fraction of opaque traders, then $1 - \mu$ of transparent traders.

Opaque and transparent traders in the model can be viewed as institutions and individuals in asset market, respectively. It is generally known that individuals are often less-informed, make investment decisions only rely on the public information. Institutions, consisting of authorized securities firms, corporate annuity funds, hedge funds, and so on, are always regarded as informed with private, even inside information. Compared with individuals, institutions have more financial strength, stronger information channels and much more professional investment teams. Especially, they are superior in the acquisition of firm-specific information, which is always not available to individuals (Holland and Doran, 1998). Thus, institutions are informed relative to individual traders. They can develop the private information into extra investment opportunities, which could be direct investment in companies, foreign exchange markets, and assets in which individual traders are not permitted to entry. Thus, with the access to private information, institutions have enlarged investment opportunities. Random variables $\tilde{\theta}_O$ and $\tilde{\theta}_T$ in (1) represent information that are observable to opaque traders and transparent traders, respectively, where $\tilde{\theta}_T$ is public information that is observable to all traders, while $\tilde{\theta}_O$ is private information that is observable to only opaque traders.

We assume that all opaque traders have the same investment opportunities, and transparent traders are ambiguous about the standard deviation of the return for extra investment opportunities, due to lack of private information. Let $1 + \tilde{\eta}$ be the (gross) returns of these extra investment opportunities, where $\tilde{\eta} \sim N(m, \sigma_\eta^2)$ with $\sigma_\eta > 0$. In addition, $\tilde{\varepsilon}$ and $\tilde{\eta}$ are correlated with a coefficient of $\rho \in (-1, 1)$, which is restrained as $\rho \in (0, 1)$ in Easley, O'Hara and Yang (2014), thus, opaque traders can use these investment opportunities to hedge stock investments.

[2]Or say the cost is used for collecting information, building professional investment teams to understand and interpret the acquired private information and develop it into extra investment opportunities.

2.1. Ambiguity

Extending the analysis framework in Easley, O'Hara, and Yang (2014), we assume that the transparent traders perceive ambiguity about the standard deviation (or variance) of the returns for the extra investment opportunities, due to lack of private information. That is, although they are unable to assess what σ_η is, they believe the perceived standard deviation σ_η belongs to an interval, which includes the true value $\hat{\sigma}_\eta$. Transparent traders perceive that

$$\sigma_\eta \in [\underline{\sigma}_T, \overline{\sigma}_T] \qquad \text{with} \qquad 0 < \underline{\sigma}_T < \overline{\sigma}_T \qquad \text{and} \qquad \underline{\sigma}_T \leqslant \hat{\sigma}_\eta \leqslant \overline{\sigma}_T$$

and they have multi-prior probabilities on this set and are unable to assign a unique one. We assume that

$$\underline{\sigma}_T \equiv \hat{\sigma}_\eta - \Delta\sigma \qquad \text{and} \qquad \overline{\sigma}_T \equiv \hat{\sigma}_\eta + \Delta\sigma,$$

where $\Delta\sigma$ is an exogenous parameter, related to market opacity or disclosuring rules, which determines the amount of ambiguity perceived by transparent traders. If $\Delta\sigma$ is big (small), the market is with high (low) opacity.

In this research, we assume that traders display preferences in the form of the Maxmin Expected Utility of Gilboa and Schmeidler (1989). This assumption does not allow us to disentangle the notion of ambiguity from that of the attitude towards it as the smooth ambiguity model of Klibanoff, Marinacci and Mukerji (2005), which allows for a separation of tastes and beliefs. Our conclusions about information acquisition and multiple equilibria remain unaffected in this framework.

This research presents the timeline as follows. Firstly, each trader is endowed with one share of the stock and decides whether to pay c for private information to become opaque. Secondly, before the asset opens, transparent traders observe public information $\tilde{\theta}_T$, while opaque traders observe both public and private information $\{\tilde{\theta}_T, \tilde{\theta}_O\}$ and process it into additional investment opportunities. Thirdly, trading starts. All traders trade the stock and bond relying on their strategies. In addition, opaque traders also invest in the additional investment opportunities. Finally, traders receive the payoffs on their portfolio and consume.

To summary, the tuple

$$\mathcal{E} = (\overline{v}, \sigma_{\theta O}, \sigma_{\theta T}, \sigma_\varepsilon, \hat{\sigma}_\eta, \Delta\sigma, \rho, c)$$

defines an economy entity. In this study, we will focus on investigating how transparent traders' ambiguity for the extra investment opportunity affect the equilibrium asset price, trader distribution and the social welfare level. Corresponding to regulatory policies, we are particularly interested in the impact of changing the ambiguity amount $\Delta\sigma$ and the information acquisition cost c.

3. FINANCIAL MARKET EQUILIBRIUM

We adopt Rational Expectations Equilibrium (REE) in the economy, which is generally used to model where investors have private information. The concept of REE considers the role of prices as aggregators of information, and also clearing the markets. In this section we first start by analyzing investors' trading behaviors given some fraction μ of opaque traders, calculate the prices of the risky asset by market clearing condition under equilibrium, then analyze the implication of change in amount of ambiguity to equity premium.

3.1. Demand Function of Opaque Traders

By observing both public and private information $\{\tilde{\theta}_T, \tilde{\theta}_O\}$, opaque traders face no ambiguity. However, they cannot eliminate risk yet. They choose portfolio holdings, D_O of the stock and Z_O of the extra investment opportunities, to maximize the expected utility of their final wealth. Endowed with one share of stocks and paying c for information acquisition, opaque traders' future final wealth after trade is

$$\widetilde{W}_O = (\tilde{p} - c) + (\tilde{v} - \tilde{p})D_O + \tilde{\eta}Z_O \tag{2}$$

and the CARA-normal setup implies that their expected utility is

$$\mathbf{E}\left[-e^{-\widetilde{W}_O}\middle|\tilde{p}, \tilde{\theta}_O, \tilde{\theta}_T\right] = -e^{-\left\{\mathbf{E}[\widetilde{W}_O|\tilde{p},\tilde{\theta}_O,\tilde{\theta}_T] - \frac{1}{2}\mathbf{Var}(\widetilde{W}_O|\tilde{p},\tilde{\theta}_O,\tilde{\theta}_T)\right\}}$$

where $\mathbf{E}[\cdot|\tilde{p}, \tilde{\theta}_O, \tilde{\theta}_T]$ and $\mathbf{Var}(\cdot|\tilde{p}, \tilde{\theta}_O, \tilde{\theta}_T)$ are the conditional mean and variance operators.

By equation (1) and (2), we have

$$\mathbf{E}[\widetilde{W}_O|\tilde{p}, \tilde{\theta}_O, \tilde{\theta}_T] = (\tilde{p} - c) + (\overline{v} + \tilde{\theta}_O + \tilde{\theta}_T - \tilde{p})D_O + mZ_O,$$
$$\mathbf{Var}(\widetilde{W}_O|\tilde{p}, \tilde{\theta}_O, \tilde{\theta}_T) = \sigma_\varepsilon^2 D_O^2 + 2\rho\sigma_\varepsilon\sigma_\eta D_O Z_O + \sigma_\eta^2 Z_O^2.$$

Then, the utility maximization problem is equivalent to the following quadratic optimization

$$\max_{\{D_O, Z_O\}} \left\{\mathbf{E}[\widetilde{W}_O|\tilde{p}, \tilde{\theta}_O, \tilde{\theta}_T] - \frac{1}{2}\mathbf{Var}(\widetilde{W}_O|\tilde{p}, \tilde{\theta}_O, \tilde{\theta}_T)\right\}$$
$$= \max_{\{D_O, Z_O\}} \left\{[(\tilde{p} - c) + (\overline{v} + \tilde{\theta}_O + \tilde{\theta}_T - \tilde{p})D_O + mZ_O] - \frac{1}{2}(\sigma_\varepsilon^2 D_O^2 + 2\rho\sigma_\varepsilon\sigma_\eta D_O Z_O + \sigma_\eta^2 Z_O^2)\right\}.$$

The first order conditions (FOCs) are given by

$$(\overline{v} + \tilde{\theta}_O + \tilde{\theta}_T - \tilde{p}) - \sigma_\varepsilon^2 D_O - \rho\sigma_\varepsilon\sigma_\eta Z_O = 0,$$
$$m - \rho\sigma_\varepsilon\sigma_\eta D_O - \sigma_\eta^2 Z_O = 0.$$

We then have opaque traders' optimal investment strategy as

$$\begin{pmatrix} D_O^* \\ Z_O^* \end{pmatrix} = \frac{1}{1-\rho^2} \begin{pmatrix} \frac{\overline{v}+\tilde{\theta}_O+\tilde{\theta}_T-\tilde{p}}{\sigma_\varepsilon^2} - \frac{\rho m}{\sigma_\varepsilon\sigma_\eta} \\ \frac{m}{\sigma_\eta^2} - \frac{\rho(\overline{v}+\tilde{\theta}_O+\tilde{\theta}_T-\tilde{p})}{\sigma_\varepsilon\sigma_\eta} \end{pmatrix} \tag{3}$$

Note that the demand function of opaque traders for stock in Equation (3), given the first component, the extra investment opportunities always causes opaque traders to trade more or less risky asset, depending on the sign of the product ρm. This is very intuitive, as the opaque traders will use the investment opportunities to hedge the stock investment. If they can benefit more from trading stock, they will trade actively in stock market, and vice versa. In effect, the extra investment opportunities $\tilde{\eta}$ affect opaque traders' stock investments only through its correlation ρ with the stock. Opaque traders' demands function for the extra opportunities are always non-zero because $\frac{m}{\sigma_\eta} - \frac{\rho(\overline{v}+\tilde{\theta}_O+\tilde{\theta}_T-\tilde{p})}{\sigma_\varepsilon} \neq 0$.

We would like to remark that the extra investment opportunity affect opaque traders' demand not only through a multiplier $\frac{1}{1-\rho^2}$, which is interpreted as the *effective* risk tolerance coefficient in Easley, O'Hara and Yang (2014), but also through subtracting a term $\frac{1}{1-\rho^2}\frac{\rho m}{\sigma_\varepsilon\sigma_\eta}$, which will not vanish unless $\rho m = 0$. This subtracting term is not observed in Easley, O'Hara and Yang (2014) since they set the expected payoff m to be zero.

We motivate opaque traders owning extra investment opportunities, it is appropriate because the opaque traders own private information, and they have complete knowledge about the financial market, so that they can develop more investment opportunities to profit. As a consequence, facing no ambiguity, opaque traders will appear to be less uncertainty averse in the financial market than transparent traders.

3.2. Demand Function of Transparent Traders

As the setting in Easley, O'Hara and Yang (2014), for any given prior σ_η, we assume that transparent traders rationally conjecture that the price function of the risky asset is

$$\tilde{p} = \overline{v} + \tilde{\theta}_O + \tilde{\theta}_T - f(\sigma_\eta) \tag{4}$$

where the function $f(\sigma_\eta)$ will be endogenously determined by equilibrium. It is also the perceived equity premium for a given belief prior σ_η of transparent traders: $\mathbf{E}_{\sigma_\eta}[\tilde{v} - \tilde{p}]$, where $\mathbf{E}_{\sigma_\eta}[\cdot]$ denotes the expectation operator under the prior σ_η.

Ambiguity about the standard deviation of the extra investment opportunities, σ_η, leads the transparent traders to choose portfolio holdings by the multiple priors framework by Gilboa and Schmeidler (1989) to optimize

$$\max_{D_T} \min_{\sigma_\eta \in [\underline{\sigma}_T, \overline{\sigma}_T]} \mathbf{E}_{\sigma_\eta}\left[-e^{-\widetilde{W}_T} \mid \tilde{p}, \tilde{\theta}_T\right]$$

subject to standard budget constraint

$$\widetilde{W}_T = \tilde{p} + (\tilde{v} - \tilde{p})D_T \tag{5}$$

where $\mathbf{E}_{\sigma_\eta}[\cdot|\tilde{p}, \theta_T]$ denotes the conditional expectation given the belief prior σ_η, and in Equation (5), the first term, $\tilde{p}$, is the value of the transparent traders' stock endowment, and D_T is their demand for the stock. It follows immediately from our normal distribution structure, that the above decision problem is equivalent to

$$\max_{D_T} \min_{\sigma_\eta \in [\underline{\sigma}_T, \overline{\sigma}_T]} \left\{ \left(\mathbf{E}_{\sigma_\eta}\left[\tilde{v} \mid \tilde{p}, \tilde{\theta}_T\right] - \tilde{p} \right) D_T - \frac{1}{2}\mathbf{Var}_{\sigma_\eta}(\tilde{v} \mid \tilde{p}, \tilde{\theta}_T) D_T^2 \right\}.$$

$\mathbf{E}_{\sigma_\eta}[\tilde{v}|\tilde{p}, \theta_T]$ and $\mathbf{Var}_{\sigma_\eta}(\tilde{v}|\tilde{p}, \theta_T)$, the conditional moments of $\tilde{v}$ taken under a particular belief σ_η, are given by

$$\mathbf{E}_{\sigma_\eta}[\tilde{v}|\tilde{p}, \theta_T] = \tilde{p} + f(\sigma_\eta) \qquad \text{and} \qquad \mathbf{Var}_{\sigma_\eta}(\tilde{v}|\tilde{p}, \theta_T) = \sigma_\varepsilon^2$$

and as a result, for a fixed investment D_T, we have

$$\left(\mathbf{E}_{\sigma_\eta}\left[\tilde{v} \mid \tilde{p}, \tilde{\theta}_T\right] - \tilde{p} \right) D_T - \frac{1}{2}\mathbf{Var}_{\sigma_\eta}(\tilde{v} \mid \tilde{p}, \tilde{\theta}_T) D_T^2 = f(\sigma_\eta) D_T - \frac{1}{2}\sigma_\varepsilon^2 D_T^2.$$

Define the minimum and maximum values that $f(\cdot)$ takes as

$$\underline{f} \equiv \min_{\sigma_\eta \in [\underline{\sigma}_T, \overline{\sigma}_T]} f(\sigma_\eta) \qquad \text{and} \qquad \overline{f} \equiv \max_{\sigma_\eta \in [\underline{\sigma}_T, \overline{\sigma}_T]} f(\sigma_\eta).$$

Then the objective function of a transparent trader can be written as

$$\begin{aligned} &\min_{\sigma_\eta \in [\underline{\sigma}_T, \overline{\sigma}_T]} \left(\mathbf{E}_{\sigma_\eta}\left[\tilde{v}|\tilde{p}, \tilde{\theta}_T\right] - \tilde{p} \right) D_T - \frac{1}{2} Var_{\sigma_\eta}(\tilde{v}|\tilde{p}, \tilde{\theta}_T) D_T^2 \\ &= \begin{cases} \overline{f} D_T - \frac{1}{2}\sigma_\varepsilon^2 D_T^2, & \text{if} \quad D_T < 0 \\ 0, & \text{if} \quad D_T = 0 \\ \underline{f} D_T - \frac{1}{2}\sigma_\varepsilon^2 D_T^2, & \text{if} \quad 0 < D_T. \end{cases} \end{aligned} \tag{6}$$

Solving the whole optimization problem case by case, we have transparent traders' demand function

$$D_T^*(\tilde{p}, \theta_T) = \begin{cases} \frac{\overline{f}}{\sigma_\varepsilon^2}, & \text{if} \quad \overline{f} < 0 \\ 0, & \text{if} \ \underline{f} \leqslant 0 \leqslant \overline{f} \\ \frac{\underline{f}}{\sigma_\varepsilon^2}, & \text{if} \quad 0 < \underline{f}. \end{cases} \tag{7}$$

This demand function is very intuitive, and it is consistent with the counterpart in Easley, O'Hara, and Yang (2014). From previous setting we know the function $f(\sigma_\eta)$ represents the transparent traders' perceived equity premium for a given belief σ_η. Thus, if the minimum perceived equity premium across all possible beliefs is still positive, then the transparent traders know that the stock is undervalued and they are confident to buy in. If the maximum perceived equity premium is negative, the transparent traders know that the stock is overvalued relative to the fundamentals and they are going to short. As for the betweenness (i.e., $\underline{f} \leqslant 0 \leqslant \overline{f}$), the transparent traders are not sure about whether the stock is undervalued or overvalued, and they are reluctant to hold any stock. This is called portfolio inertia, i.e., transparent traders strictly prefers to maintain a zero rather than a non-zero position of the risky asset when the asset price lies with a given (non-trivial) interval (Mukerji and Tallon, 2003). Such situations of portfolio inertia were first linked with Knightian uncertainty by Dow and Werlang (1992). This inertia phenomenon is also observed in the demand function in Easley, O'Hara, and Yang (2014), but they rule out this scenario in equilibrium. However, we will show later that the portfolio inertia of transparent traders is a possible equilibrium outcome.

All these three cases can occur because a transparent trader is heavily influenced by the worst possible distribution of the standard deviation, and what is the worst depends on the investor's perceived asset premium. Due to ambiguity averse, transparent traders want to be compensated for bearing more uncertainty, they will require an additional discount on the price of the asset. This demand function contrasts with the opaque traders who hold a non-zero position in any asset as long as its price is not equal to its fundamentals. As for the transparent traders will short the risky asset only if the maximum equity premium $\overline{f} < 0$ (the asset price is above $\overline{f}$), this case can occur in equilibrium in the economy, because as risk averse investors, opaque traders have the extra investment opportunities to hedge to avoid very low levels of consumptions in the future. By their optimal portfolios, they might suffer some loss in stock market, but earn great from the extra investment opportunities.

Condie and Ganguli (2015) show that partially revealing REE arise and affect market variables when private information is received by ambiguity averse investors who exhibit portfolio inertia with respect to information.

Such investor behavior arises, due to subadditive probabilities utility (Simonsen 1991), including Choquet Expected Utility (CEU) by Schmeidler (1989) and Maxmin Expected Utility (MEU) by Gilboa and Schmeidler (1989). While, smooth preference representations such as Klibanoff, Marinacci, and Mukerji (2005), Maccheroni, Marinacci and Rustichini (2006), and Hansen and Sargent (2007) do not yield portfolio inertia.

3.3. Rational Expectations Equilibrium

By using the demand functions of both traders, Equations (3) and (7), we can determine the functional form of $f(\sigma_\eta)$ from the market-clearing condition

$$\mu D_O(\tilde{p}, \tilde{\theta}_O, \tilde{\theta}_T) + (1-\mu) D_T(\tilde{p}, \tilde{\theta}_T) = 1. \tag{8}$$

The demand function of transparent traders can take the three possible values in Equation (7), so we will discuss market clearing case by case.

Case 1. Suppose transparent traders' demand function is taken as Equation (7) that transparent traders hold short positions of risky asset, $D_T(\tilde{p}, \tilde{\theta}_T) = \frac{\overline{f}}{\sigma_\varepsilon^2}$ for $\overline{f} < 0$, and opaque traders' demand function is as Equation (3), $D_O = \frac{1}{1-\rho^2}\left[\frac{\overline{v}+\tilde{\theta}_O+\tilde{\theta}_T-\tilde{p}}{\sigma_\varepsilon^2} - \frac{\rho m}{\sigma_\varepsilon \sigma_\eta}\right]$. Then, substituting the demand functions into the market clearing condition, Equation (8), we can solve for the equilibrium price $\tilde{p}$.

$$\tilde{p} = \overline{v} + \tilde{\theta}_O + \tilde{\theta}_T - \left[\rho m \frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{(1-\rho^2)\left[\sigma_\varepsilon^2 - (1-\mu)\overline{f}\right]}{\mu}\right].$$

Comparing this price with the conjectured price function in Equation (4), if the transparent traders' conjecture is rational, we can find that

$$f(\sigma_\eta) = \rho m \frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{(1-\rho^2)\left[\sigma_\varepsilon^2 - (1-\mu)\overline{f}\right]}{\mu}. \tag{9}$$

In the current case, $D_T(\tilde{p}, \tilde{\theta}_T) = \frac{\overline{f}}{\sigma_\varepsilon^2}$, we must have $\overline{f} < 0$, therefore, $f(\sigma_\eta) < 0$ for all possible σ_η. We consider the following two settings:

Case 1.1. If $\rho m < 0$, then $f(\sigma_\eta)$ achieves its maximum at $\overline{\sigma}_T$ in Equation (9),

$$\overline{f} = \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} + \frac{(1-\rho^2)\left[\sigma_\varepsilon^2 - (1-\mu)\overline{f}\right]}{\mu}.$$

$\overline{f}$ can be calculated, and the result is

$$\begin{aligned}\overline{f} &= \frac{\rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T}\mu + (1-\rho^2)\sigma_\varepsilon^2}{\mu + (1-\rho^2)(1-\mu)} \\ &= \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu + (1-\rho^2)(1-\mu)} + \frac{\mu}{\mu + (1-\rho^2)(1-\mu)}\rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T}, \end{aligned} \tag{10}$$

where $\overline{f} < 0$ is equivalent to $\frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon \overline{\sigma}_T}{\mu}$.

Substituting Equation (10) into Equation (9), we calculate $f(\sigma_\eta)$ as

$$\begin{aligned} f(\sigma_\eta) &= \rho m\frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{1-\rho^2}{\mu + (1-\rho^2)(1-\mu)}\left[\sigma_\varepsilon^2 - (1-\mu)\rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T}\right], \\ &\text{if } \frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon\overline{\sigma}_T}{\mu}. \end{aligned} \tag{11}$$

Case 1.2. If $\rho m \geqslant 0$, then $f(\sigma_\eta)$ achieves its maximum at $\underline{\sigma}_T$ in Equation (9),

$$\overline{f} = \rho m\frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \frac{(1-\rho^2)\left[\sigma_\varepsilon^2 - (1-\mu)\overline{f}\right]}{\mu}.$$

$\overline{f}$ can be calculated, and the result is

$$\overline{f} = \frac{\rho m\frac{\sigma_\varepsilon}{\underline{\sigma}_T}\mu + (1-\rho^2)\sigma_\varepsilon^2}{\mu + (1-\rho^2)(1-\mu)} = \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu + (1-\rho^2)(1-\mu)} + \frac{\mu}{\mu + (1-\rho^2)(1-\mu)}\rho m\frac{\sigma_\varepsilon}{\underline{\sigma}_T} > 0$$

which contradicts the fact $\overline{f} < 0$.

Case 2. Suppose transparent traders' demand function is taken as Equation (7) that transparent traders do not trade risky asset, $D_T(\tilde{p}, \tilde{\theta}_T) = 0$ for $\underline{f} \leqslant 0 \leqslant \overline{f}$, and opaque traders' demand function is as Equation (3) . Then, substituting the demand functions into the market clearing condition, Equation (8), we can solve for the equilibrium price $\tilde{p}$.

$$\tilde{p} = \overline{v} + \tilde{\theta}_O + \tilde{\theta}_T - \left[\rho m\frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu}\right].$$

Comparing this price with the conjectured price function in Equation (4), if the transparent traders' conjecture is rational, we can find that

$$f(\sigma_\eta) = \rho m\frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu}.$$

Case 2.1. If $\rho m < 0$, then $f(\sigma_\eta)$ achieves its minimum at $\underline{\sigma}_T$ and its maximum at $\overline{\sigma}_T$,

$$\rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu} = \underline{f} \leqslant 0 \leqslant \overline{f} = \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu}. \quad (12)$$

Then, we can get $-\frac{\sigma_\varepsilon \overline{\sigma}_T}{\mu} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu}$ and $f(\sigma_\eta)$ as follows

$$f(\sigma_\eta) = \rho m \frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu}, \quad \text{if} \quad -\frac{\sigma_\varepsilon \overline{\sigma}_T}{\mu} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu}. \quad (13)$$

Case 2.2. If $\rho m \geqslant 0$, then $f(\sigma_\eta) > 0$, which contradicts $\underline{f} \leqslant 0 \leqslant \overline{f}$.

Case 3. Suppose transparent traders' demand function is taken as Equation (7) that transparent traders hold long positions of risky asset, $D_T(\tilde{p}, \tilde{\theta}_T) = \frac{\underline{f}}{\sigma_\varepsilon^2}$ for $\underline{f} > 0$, and opaque traders' demand function is as Equation (3) . Then, substituting the demand functions into the market clearing condition, Equation (8), we can solve for the equilibrium price $\tilde{p}$.

$$\tilde{p} = \overline{v} + \tilde{\theta}_O + \tilde{\theta}_T - \left[\rho m \frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{(1-\rho^2)\left[\sigma_\varepsilon^2 - (1-\mu)\underline{f}\right]}{\mu} \right].$$

Comparing this price with the conjectured price function in Equation (4), if the transparent traders' conjecture is rational, we can find that

$$f(\sigma_\eta) = \rho m \frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{(1-\rho^2)\left[\sigma_\varepsilon^2 - (1-\mu)\underline{f}\right]}{\mu}. \quad (14)$$

In the current case, $D_T(\tilde{p}, \tilde{\theta}_T) = \frac{\underline{f}}{\sigma_\varepsilon^2}$, we must have $\underline{f} > 0$, therefore, $f(\sigma_\eta) > 0$ for all possible σ_η. We consider the following two settings:

Case 3.1. If $\rho m < 0$, then $f(\sigma_\eta)$ achieves its minimum at $\underline{\sigma}_T$ in Equation (14),

$$\underline{f} = \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \frac{(1-\rho^2)\left[\sigma_\varepsilon^2 - (1-\mu)\underline{f}\right]}{\mu}.$$

$\underline{f}$ can be calculated, and the result is

$$\begin{aligned} \underline{f} &= \frac{\rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T}\mu + (1-\rho^2)\sigma_\varepsilon^2}{\mu + (1-\rho^2)(1-\mu)} \\ &= \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu + (1-\rho^2)(1-\mu)} + \frac{\mu}{\mu + (1-\rho^2)(1-\mu)} \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} \end{aligned} \quad (15)$$

where $\underline{f} > 0$ means that $-\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu} < \frac{\rho m}{1-\rho^2} < 0$.

Substituting Equation (15) into Equation (14), we calculate $f(\sigma_\eta)$ as

$$\begin{aligned} f(\sigma_\eta) &= \rho m \frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{1-\rho^2}{\mu + (1-\rho^2)(1-\mu)} \left[\sigma_\varepsilon^2 - (1-\mu)\rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} \right], \quad (16) \\ & \text{if } -\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu} < \frac{\rho m}{1-\rho^2} < 0. \end{aligned}$$

Case 3.2. If $\rho m \geqslant 0$, $f(\sigma_\eta)$ achieves its minimum at $\overline{\sigma}_T$ in Equation (A1),

$$\underline{f} = \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} + \frac{(1-\rho^2)\left[\sigma_\varepsilon^2 - (1-\mu)\underline{f}\right]}{\mu}.$$

$\underline{f}$ can be calculated, and the result is

$$\begin{aligned} \underline{f} &= \frac{\rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T}\mu + (1-\rho^2)\sigma_\varepsilon^2}{\mu + (1-\rho^2)(1-\mu)} \\ &= \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu + (1-\rho^2)(1-\mu)} + \frac{\mu}{\mu + (1-\rho^2)(1-\mu)} \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T}. \quad (17) \end{aligned}$$

It's obvious that $\underline{f} > 0$ when $\rho m \geqslant 0$.

Substituting Equation (17) into Equation (14), we calculate $f(\sigma_\eta)$ as

$$\begin{aligned} f(\sigma_\eta) &= \rho m \frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{1-\rho^2}{\mu + (1-\rho^2)(1-\mu)} \left[\sigma_\varepsilon^2 - (1-\mu)\rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} \right], \quad (18) \\ & \text{if } 0 \leqslant \frac{\rho m}{1-\rho^2}. \end{aligned}$$

The above results can be summarized in Theorem 1 on existence of rational expectations equilibrium. So, Theorem 1 defines three types of equilibria.

THEOREM 1. *Suppose* $0 < \mu < 1$. *There exists an REE in which the price function is*

$$\tilde{p} = \overline{v} + \tilde{\theta}_O + \tilde{\theta}_T - f(\sigma_\eta),$$

where $f(\sigma_\eta)$ *is given as follows*

[1]Transparent traders hold short position of risky asset: $\frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon \overline{\sigma}_T}{\mu}$, *then*

$$f(\sigma_\eta) = \rho m \frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{1-\rho^2}{\mu + (1-\rho^2)(1-\mu)} \left[\sigma_\varepsilon^2 - (1-\mu)\rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} \right]. \quad (19)$$

[2]Transparent traders do not trade risky asset: $-\frac{\sigma_\varepsilon \overline{\sigma}_T}{\mu} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu}$, *then*

$$f(\sigma_\eta) = \rho m \frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu}. \tag{20}$$

[3]Transparent traders hold long position of risky asset: $-\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu} < \frac{\rho m}{1-\rho^2}$, *then*

$$f(\sigma_\eta) = \begin{cases} \rho m \frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{1-\rho^2}{\mu+(1-\rho^2)(1-\mu)} \left[\sigma_\varepsilon^2 - (1-\mu)\rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T}\right], & \text{if } -\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu} < \frac{\rho m}{1-\rho^2} < 0 \\ \rho m \frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{1-\rho^2}{\mu+(1-\rho^2)(1-\mu)} \left[\sigma_\varepsilon^2 - (1-\mu)\rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T}\right], & \text{if } 0 \leqslant \frac{\rho m}{1-\rho^2}. \end{cases} \tag{21}$$

Remark 1. The parameter product ρm as the product of ρ, the correlation coefficient between risky asset and extra investment opportunities, and m, the mean of extra investment opportunities, plays an important role for the pricing. Notice that the pricing function $f(\cdot)$ is the linear function of $\frac{1}{\sigma_\eta}$, and the slope is given by $\rho m \sigma_\epsilon$. Thus given σ_ϵ, the absolute value of ρm acts as a measure of how sensitive the equity market will be to the ambiguity amount, and the sign of ρm determines the direction in which ambiguity will drive the asset price.

Remark 2. As we mention in the above subsection, portfolio inertia (or non-participation) phenomenon is shown to be one of the possible equilibrium outcome, which is different from the result in Easley, O'Hara and Yang (2014). Portfolio inertia has proven to be a major contribution of ambiguity models to the financial markets, and is the common feature for literature investigating ambiguity with the multiple-prior model, such as Dow and Werlang (1992), Mukerji and Tallon (2003), Easley and O'Hara (2009, 2010) and Huang, Zhang and Zhu (2017).

Remark 3. From these three cases in Theorem 1, we analyze transparent and opaque traders' equilibrium positions on risky asset.

[1] In type [1] equilibrium, transparent traders hold short positions of risky asset.

$$D_T^* = \frac{\overline{f}}{\sigma_\varepsilon^2} = \frac{\rho m \frac{1}{\sigma_\varepsilon \overline{\sigma}_T}\mu + (1-\rho^2)}{\mu + (1-\rho^2)(1-\mu)} = \frac{1-\rho^2}{(1-\rho^2)+\rho^2\mu} + \frac{\mu}{(1-\rho^2)+\rho^2\mu}\frac{\rho m}{\sigma_\varepsilon \overline{\sigma}_T} < 0$$

which is equivalent to $\frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon \overline{\sigma}_T}{\mu}$. Opaque traders hold large long positions of risky asset,

$$D_O^* = \frac{1}{\mu}\left[1-(1-\mu)D_T^*\right] = \frac{1}{(1-\rho^2)+\rho^2\mu}\left[1-(1-\mu)\frac{\rho m}{\sigma_\varepsilon \overline{\sigma}_T}\right] > \frac{1}{\mu}.$$

[2] In type [2] equilibrium, transparent traders do not trade risky asset.

$$D_T^* = 0 \qquad \text{and} \qquad \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu} = \underline{f} \leqslant 0 \leqslant \overline{f} = \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu}$$

which is equivalent to $-\frac{\sigma_\varepsilon \overline{\sigma}_T}{\mu} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu}$. Opaque traders hold long positions of risky asset,

$$D_O^* = \frac{1}{\mu}\left[1-(1-\mu)D_T^*\right] = \frac{1}{\mu}.$$

[3] In type [3] equilibrium, transparent traders hold long positions of risky asset.

$$D_T^* = \frac{f}{\sigma_\varepsilon^2} > 0.$$

[3.1] If $\rho m < 0$, then

$$D_T^* = \frac{\underline{f}}{\sigma_\varepsilon^2} = \frac{\rho m \frac{1}{\sigma_\varepsilon \underline{\sigma}_T}\mu + (1-\rho^2)}{\mu + (1-\rho^2)(1-\mu)} = \frac{1-\rho^2}{(1-\rho^2)+\rho^2\mu} + \frac{\mu}{(1-\rho^2)+\rho^2\mu}\frac{\rho m}{\sigma_\varepsilon \underline{\sigma}_T} > 0$$

which is equivalent to $-\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu} < \frac{\rho m}{1-\rho^2}$. Therefore $-\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu} < \frac{\rho m}{1-\rho^2} < 0$, and hence opaque traders long buy risky asset,

$$D_O^* = \frac{1}{\mu}\left[1-(1-\mu)D_T^*\right] = \frac{1}{1-\rho^2+\rho^2\mu}\left[1-(1-\mu)\frac{\rho m}{\sigma_\varepsilon \underline{\sigma}_T}\right]$$

with $\frac{1}{1-\rho^2+\rho^2\mu} < D_O^* < \frac{1}{\mu}$.

[3.2] If $\rho m \geqslant 0$, then

$$D_T^* = \frac{\underline{f}}{\sigma_\varepsilon^2} = \frac{\rho m \frac{1}{\sigma_\varepsilon \overline{\sigma}_T}\mu + (1-\rho^2)}{\mu + (1-\rho^2)(1-\mu)} = \frac{1-\rho^2}{(1-\rho^2)+\rho^2\mu} + \frac{\mu}{(1-\rho^2)+\rho^2\mu}\frac{\rho m}{\sigma_\varepsilon \overline{\sigma}_T} > 0$$

holds obviously. Therefore $0 \leqslant \frac{\rho m}{1-\rho^2}$, and hence opaque traders hold long positions of risky asset,

$$D_O^* = \frac{1}{\mu}\left[1-(1-\mu)D_T^*\right] = \frac{1}{(1-\rho^2)+\rho^2\mu}\left[1-(1-\mu)\frac{\rho m}{\sigma_\varepsilon \overline{\sigma}_T}\right] \leqslant \frac{1}{(1-\rho^2)+\rho^2\mu}.$$

[3.2.1] If $0 = \rho m$, then $D_O^* = \frac{1}{(1-\rho^2)+\rho^2\mu}$;

[3.2.2] If $0 < \rho m < \rho^2 \sigma_\varepsilon \overline{\sigma}_T$, then $1 < D_O^* < \frac{1}{(1-\rho^2)+\rho^2\mu}$;

[3.2.3] If $\rho m = \rho^2\sigma_\varepsilon\overline{\sigma}_T$, then $D_O^* = 1$;
[3.2.4] If $\rho^2\sigma_\varepsilon\overline{\sigma}_T < \rho m$, then $D_O^* < 1$.

3.4. Implication for Equity Premium

An outside econometrician who has access to the rational belief of the equity market will evaluate the equilibrium equity premium of the stock as

$$\mathbf{EP} \equiv \mathbf{E}_{\hat{\sigma}_\eta}[\tilde{v} - \tilde{p}] = f(\hat{\sigma}_\eta)$$

where boldface $\mathbf{E}_{\hat{\sigma}_\eta}[\cdot]$ indicates that the expectation is taken under the true value $\hat{\sigma}_\eta$ of the parameter σ_η. Following Theorem 1, we can get that

$$\mathbf{EP} = \begin{cases} \rho m\frac{\sigma_\varepsilon}{\hat{\sigma}_\eta} + \frac{1-\rho^2}{\mu+(1-\rho^2)(1-\mu)}\left[\sigma_\varepsilon^2 - (1-\mu)\rho m\frac{\sigma_\varepsilon}{\hat{\sigma}_\eta+\Delta\sigma}\right], & \text{if } \frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}{\mu} \\ \rho m\frac{\sigma_\varepsilon}{\hat{\sigma}_\eta} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu}, & \text{if } -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}{\mu} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}{\mu} \\ \rho m\frac{\sigma_\varepsilon}{\hat{\sigma}_\eta} + \frac{1-\rho^2}{\mu+(1-\rho^2)(1-\mu)}\left[\sigma_\varepsilon^2 - (1-\mu)\rho m\frac{\sigma_\varepsilon}{\hat{\sigma}_\eta-\Delta\sigma}\right], & \text{if } -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}{\mu} < \frac{\rho m}{1-\rho^2} < 0 \\ \rho m\frac{\sigma_\varepsilon}{\hat{\sigma}_\eta} + \frac{1-\rho^2}{\mu+(1-\rho^2)(1-\mu)}\left[\sigma_\varepsilon^2 - (1-\mu)\rho m\frac{\sigma_\varepsilon}{\hat{\sigma}_\eta+\Delta\sigma}\right], & \text{if } 0 \leqslant \frac{\rho m}{1-\rho^2}. \end{cases} \tag{22}$$

Let us consider a small change in the amount of ambiguity $\Delta\sigma$ (keeping the equilibrium type unchanged). If the equilibrium is at type [1], then **EP** is negative, indicating that the asset is over-valued. In this scenario transparent traders are shorting the asset while opaque traders are holding long positions. A decrease in $\Delta\sigma$ will make transparent traders short less and drive the market price closer to the fundamental value, thus increasing **EP**. If the equilibrium is at type [2], transparent does not participate in the market, thus changing $\Delta\sigma$ will not affect market prices. If the equilibrium is at type [3], transparent traders hold long positions of the asset. Decreasing $\Delta\sigma$ will reduce market opacity and encouraging the transparent traders to trade more actively, thus decreasing **EP**. Although the above analysis assume that the equilibrium type is unchanged, but we can conclude that reducing the ambiguity amount will in general drive **EP** closer to zero.

Now, for any fixed $\mu \in (0,1)$, we consider how the amount of ambiguity ($\Delta\sigma$) affects the equity premium.

$$\frac{\partial\mathbf{EP}}{\partial\Delta\sigma} = \begin{cases} \frac{(1-\rho^2)(1-\mu)}{\mu+(1-\rho^2)(1-\mu)}\rho m\frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta+\Delta\sigma)^2} < 0, & \text{if } \frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}{\mu} \\ 0, & \text{if } -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}{\mu} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}{\mu} \\ -\frac{(1-\rho^2)(1-\mu)}{\mu+(1-\rho^2)(1-\mu)}\rho m\frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta-\Delta\sigma)^2} > 0, & \text{if } -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}{\mu} < \frac{\rho m}{1-\rho^2} < 0 \\ \frac{(1-\rho^2)(1-\mu)}{\mu+(1-\rho^2)(1-\mu)}\rho m\frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta+\Delta\sigma)^2} > 0, & \text{if } 0 \leqslant \frac{\rho m}{1-\rho^2}. \end{cases} \tag{23}$$

First, for the case where transparent traders hold short positions of risky asset, the equity premium is expressed as Equation (22). Under this case,

$\overline{\sigma}_T = \hat{\sigma}_\eta + \Delta\sigma < \frac{-\rho m}{1-\rho^2}\frac{\mu}{\sigma_\varepsilon}$ for $\frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}{\mu} < 0$, there always exists $\frac{\partial \mathbf{EP}}{\partial \Delta\sigma} < 0$ in Equation (3.22.1), so a decrease in the amount of ambiguity (more specifically the difference of ambiguity between the upper bound and the true value of the standard deviation) will increase the equity premium. The intuition is that, with a decrease of the amount of ambiguity, the equity premium increases. As the premium is negative, the price of the risky asset deviates less from its value. That is, the price reflects the fundamentals better with a lower market opacity. This situation implies that: transparent traders short the risky asset, and opaque traders long it. As opaque traders have complete knowledge of the market, they know exactly the risky asset is overvalued. While, according to their optimal portfolios, opaque traders still invest in risky asset and use the extra investment opportunities to hedge it.

Second, for the case where transparent traders' do not trade risky asset, the equity premium is shown in Equation (22). Under such a case, $\underline{\sigma}_T = \hat{\sigma}_\eta - \Delta\sigma \leqslant \frac{-\rho m}{1-\rho^2}\frac{\mu}{\sigma_\varepsilon} \leqslant \hat{\sigma}_\eta + \Delta\sigma = \overline{\sigma}_T$ for $\rho m < 0$, then $\frac{\partial \mathbf{EP}}{\partial \Delta\sigma} = 0$ in Equation (23), transparent traders will not trade in the stock, but only the opaque traders absorb all the risky asset in the market. So, transparent traders' beliefs will neither influence the asset equilibrium prices nor the equity premium directly. However, changes of amount of ambiguity (if big enough) will update the transparent traders' beliefs and alter the equilibrium type.

Third, for the case where transparent traders hold long positions of risky asset, the equity premium is shown in Equations (22) and (23). For this case, $\frac{-\rho m}{1-\rho^2}\frac{\mu}{\sigma_\varepsilon} < \hat{\sigma}_\eta - \Delta\sigma = \underline{\sigma}_T$ for $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}{\mu} < \frac{\rho m}{1-\rho^2} < 0$, then $\frac{\partial \mathbf{EP}}{\partial \Delta\sigma} > 0$. In addition, for the case $0 \leqslant \frac{\rho m}{1-\rho^2}$, we also have $\frac{\partial \mathbf{EP}}{\partial \Delta\sigma} > 0$ in Equations (22) and (23), so a decrease in the amount of ambiguity will decrease the equity premium. As the premium is positive, the price of the risky asset deviates less from its value. That is, the price reflects the fundamentals better in the market with a lower opacity. This situation implies that: transparent traders buy in the risky asset, and opaque traders sell out it. As opaque traders have complete knowledge of the market, they know exactly the risky asset is undervalued. While, according to their optimal portfolios, opaque traders still sell out the risky asset to raise fund and invest in the extra investment opportunities. With the decrease of the opacity, transparent traders are more confident and optimistic to trade more. So, there are more asset demands, then a lower equity premium.

PROPOSITION 1. *Fix traders distribution* $\mu \in (0, 1)$.

[1]Transparent traders hold short positions of risky asset, $\frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}{\mu}$, *then* $\frac{\partial \mathbf{EP}}{\partial \Delta\sigma} < 0$, *reducing the amount of ambiguity will raise the equity premium.*

[2]Transparent traders do not trade risky asset, $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}{\mu} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}{\mu}$*, then* $\frac{\partial \mathbf{EP}}{\partial \Delta\sigma} = 0$*, reducing ambiguity will not change the transparent traders' beliefs, therefore not affect the equity premium.*

[3]Transparent traders hold long positions of risky asset, $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}{\mu} < \frac{\rho m}{1-\rho^2}$*, then* $\frac{\partial \mathbf{EP}}{\partial \Delta\sigma} > 0$*, reducing the amount of ambiguity will lower the equity premium.*

In Proposition 1, the condition is relevant to the amount of ambiguity, because a large change of the amount of ambiguity will update transparent traders' beliefs, thus change the equilibrium type, therefore alter the expression of equity premium and the sign of $\frac{\partial \mathbf{EP}}{\partial \Delta\sigma}$. However, there is a consistent conclusion: the risky asset's price deviates less from its fundamentals with the reduction of market opacity.

3.5. Ex Post Performance Analysis

For a variety of reasons, individual and institutions are viewed differently in the studies. And institutions are viewed as informed investors, while individuals are believed to have psychological biases and are often thought as noise traders in the sense of Kyle (1985) or Black (1986). In this section, according to both traders' portfolio, we show that the difference in returns occurs in equilibrium in the model. It is not surprising as institutions in our economy benefit from information advantage and are actively developing extra investment opportunities that produce alpha returns.

Acquiring private information allows opaque traders face no ambiguity but only risk when trade. And with the acquired information they can develop extra investment opportunities to hedge their stock investments. While trading strategies of transparent traders are only the stock and bond, due to their incomplete knowledge of the extra investment opportunities and ambiguity aversion. In this section, we compare ex post performance of both types of traders by using Capital Asset Pricing Model (CAPM) by Sharpe (1964) to show the value of information advantage.

Previously we have treated the tradable risky asset as the proxy of the stock market, so the return on market portfolio in excess of the interest rate (which is normalized to 1 in our model) is

$$\tilde{R}_M = \frac{\tilde{v}}{\tilde{p}} - 1 = \frac{\tilde{\varepsilon} + f(\sigma_\eta)}{\tilde{p}}.$$

Substituting the relevant branch of the transparent traders' demand function into their budget constraint, and dividing by their initial wealth $\tilde{p}$ (the initial endowed one share of stock) shows that the excess return of transparent traders is

$$\tilde{R}_T = \frac{\tilde{W}_T}{\tilde{p}} - 1 = \tilde{R}_M D_T = \begin{cases} \tilde{R}_M \frac{\overline{f}}{\sigma_\varepsilon^2}, & \text{if } \quad \overline{f} < 0 \\ 0, & \text{if } \underline{f} \leqslant 0 \leqslant \overline{f} \\ \tilde{R}_M \frac{\underline{f}}{\sigma_\varepsilon^2}, & \text{if } \quad 0 < \underline{f}. \end{cases}$$

By CAPM, the beta of transparent traders' portfolio can be computed as

$$\beta_T \equiv \frac{\mathbf{cov}_{\hat{\sigma}_\eta}(\tilde{R}_T, \tilde{R}_M|\tilde{p})}{\mathbf{Var}_{\hat{\sigma}_\eta}(\tilde{R}_M|\tilde{p})} = D_T = \begin{cases} \frac{\overline{f}}{\sigma_\varepsilon^2}, & \text{if } \quad \overline{f} < 0 \\ 0, & \text{if } \underline{f} \leqslant 0 \leqslant \overline{f} \\ \frac{\underline{f}}{\sigma_\varepsilon^2}, & \text{if } \quad 0 < \underline{f} \end{cases}$$

where the boldface $\mathbf{cov}_{\hat{\sigma}_\eta}(\cdot, \cdot|\tilde{p})$ and $\mathbf{Var}_{\hat{\sigma}_\eta}(\cdot|\tilde{p})$ indicate conditional moments under the true value $\hat{\sigma}_\eta$ of the parameter σ_η.[3] Thus, by CAPM, the market-adjusted returns on transparent traders' equilibrium portfolio is

$$\alpha_T \equiv \mathbf{E}[\tilde{R}_T|\tilde{p}] - \beta_T \mathbf{E}[\tilde{R}_M|\tilde{p}] = 0$$

for all the three cases. This result is not surprising and can be understood that: less informed traders only trade bond with zero return and market portfolio, so their portfolio returns surely do not outperform the market.

And as for opaque traders, including both the optimal investment in the tradable assets and the extra investment opportunities (3) into the opaque traders' budget constraint (2) and dividing by their total invested capital $(\tilde{p} - c)$ shows that the excess return on the equilibrium portfolio of opaque traders is

$$\tilde{R}_O = \frac{\tilde{W}_O}{\tilde{p} - c} - 1 = \frac{1}{\tilde{p} - c} \left\{ \frac{\tilde{\varepsilon} + f(\sigma_\eta)}{1 - \rho^2} \left[\frac{f(\sigma_\eta)}{\sigma_\varepsilon^2} - \frac{\rho m}{\sigma_\varepsilon \sigma_\eta} \right] + \frac{\tilde{\eta}}{1 - \rho^2} \left[\frac{m}{\sigma_\eta^2} - \frac{\rho f(\sigma_\eta)}{\sigma_\varepsilon \sigma_\eta} \right] \right\}.$$

In Appendix A1, the beta and alpha of opaque traders' portfolio can be computed as

$$\beta_O \equiv \frac{\mathbf{cov}_{\hat{\sigma}_\eta}(\tilde{R}_O, \tilde{R}_M|\tilde{p})}{\mathbf{Var}_{\hat{\sigma}_\eta}(\tilde{R}_M|\tilde{p})} = \frac{\tilde{p}}{\tilde{p} - c} \frac{f(\hat{\sigma}_\eta)}{\sigma_\varepsilon^2} = \frac{\tilde{p}}{\tilde{p} - c} \frac{\mathbf{EP}}{\sigma_\varepsilon^2}$$

$$\alpha_O \equiv \mathbf{E}[\tilde{R}_O|\tilde{p}] - \beta_O \mathbf{E}[\tilde{R}_M|\tilde{p}] = \frac{1}{1 - \rho^2} \frac{1}{\tilde{p} - c} \left(\frac{\rho f(\hat{\sigma}_\eta)}{\sigma_\varepsilon} - \frac{m}{\hat{\sigma}_\eta} \right)^2$$

where **EP** is the equity premium of the stock. Thus α_O is strictly positive as long as the net wealth of opaque traders is strictly positive, $\tilde{p} - c > 0$.

[3]We assume that the beta is computed on price $\tilde{p}$ because in practice, price information is indeed available to econometricians.

Proposition 2. *Under Rational Expectations Equilibrium (REE), if the net wealth of opaque traders is strictly positive, $\tilde{p} - c > 0$, then they appear to generate strictly positive alphas, no matter what type of equilibrium is prevailing in the economy.*

From Proposition 2, we find that opaque traders can always beat the market to earn positive, due to their informational advantage, only if they have a positive net wealth, no matter what type of equilibrium in the economy. Knowledge or information about the probability distribution of the extra investment opportunities is valuable, because it provides the opaque traders with a "code" for correctly interpreting the information conveyed through the price.

4. TRADERS DISTRIBUTION EQUILIBRIUM

In this section, we now analyze traders' decisions whether to pay for information acquisition to become opaque or remain transparent prior to trading. They assess such decisions relying on the comparison between the ex ante expected utility of becoming opaque and that of staying transparent.

We focus on the economy with interior $\mu^* \in (0, 1)$. Substituting the optimal investment of an opaque trader, Equation (3), into Equation (2), we can obtain the indirect utility function of an opaque trader in a market with asset price $\tilde{p}$ is

$$\begin{aligned} V_{O1}(\tilde{p}, \tilde{\theta}_O, \tilde{\theta}_T; \sigma_\eta) &= \mathbf{E}\left[-e^{-\widetilde{W}_O} \middle| \tilde{p}, \tilde{\theta}_O, \tilde{\theta}_T\right] \qquad (24) \\ &= -e^{-\left\{\overline{v}+\tilde{\theta}_O+\tilde{\theta}_T-c-f(\sigma_\eta)+\frac{1}{2(1-\rho^2)}\left[\frac{f^2(\sigma_\eta)}{\sigma_\varepsilon^2}-2\rho m\frac{f(\sigma_\eta)}{\sigma_\varepsilon\sigma_\eta}+\frac{m^2}{\sigma_\eta^2}\right]\right\}} \end{aligned}$$

and according to Maxmin Expected Utility Theory by Gilboa and Schmeilder (1989), the ex ante expected utility function of becoming opaque is

$$\begin{aligned} V_{O0} &= \min_{\sigma_\eta \in [\underline{\sigma}_T, \overline{\sigma}_T]} \mathbf{E}_{\sigma_\eta}\left[V_{O1}(\tilde{p}, \tilde{\theta}_O, \tilde{\theta}_T; \sigma_\eta)\right] \qquad (25) \\ &= -e^{-\left\{\overline{v}-c+\min_{\sigma_\eta\in[\underline{\sigma}_T,\overline{\sigma}_T]}\left[-f(\sigma_\eta)+\frac{1}{2(1-\rho^2)}\left(\frac{f^2(\sigma_\eta)}{\sigma_\varepsilon^2}-2\rho m\frac{f(\sigma_\eta)}{\sigma_\varepsilon\sigma_\eta}+\frac{m^2}{\sigma_\eta^2}\right)\right]-\frac{1}{2}\left(\sigma_{\theta O}^2+\sigma_{\theta T}^2\right)\right\}} \end{aligned}$$

from equilibrium price function in Theorem 1.

Similarly, the indirect utility function of being a transparent trader in a market with asset price $\tilde{p}$ is

$$V_{T1}(\tilde{p},\tilde{\theta}_T) = \min_{\sigma_\eta\in[\underline{\sigma}_T,\overline{\sigma}_T]} \mathbf{E}_{\sigma_\eta}\left[-e^{-(\tilde{p}+D_T(\tilde{p},\tilde{\theta}_T)(\tilde{v}-\tilde{p}))}|\tilde{p},\tilde{\theta}_T\right]$$
$$= \begin{cases} -e^{-\left(\tilde{p}+\frac{\overline{f}^2}{2\sigma_\varepsilon^2}\right)}, & \text{if} \quad \overline{f}<0 \\ -e^{-\tilde{p}}, & \text{if} \quad \underline{f}\leqslant 0\leqslant \overline{f} \\ -e^{-\left(\tilde{p}+\frac{\underline{f}^2}{2\sigma_\varepsilon^2}\right)}, & \text{if} \quad 0<\underline{f}. \end{cases} \tag{26}$$

Given the recursive multiple-priors utility representation, the ex ante utility of staying transparent is

$$V_{T0} = \min_{\sigma_\eta\in[\underline{\sigma}_T,\overline{\sigma}_T]} \mathbf{E}_{\sigma_\eta}\left[V_{T1}(\tilde{p},\tilde{\theta}_T)\right] = \begin{cases} -e^{-\left[\overline{v}-\overline{f}+\frac{\overline{f}^2}{2\sigma_\varepsilon^2}-\frac{1}{2}\left(\sigma_{\theta O}^2+\sigma_{\theta T}^2\right)\right]}, & \text{if} \quad \overline{f}<0 \\ -e^{-\left[\overline{v}-\overline{f}-\frac{1}{2}\left(\sigma_{\theta O}^2+\sigma_{\theta T}^2\right)\right]}, & \text{if} \quad \underline{f}\leqslant 0\leqslant \overline{f} \\ -e^{-\left[\overline{v}-\overline{f}+\frac{\underline{f}^2}{2\sigma_\varepsilon^2}-\frac{1}{2}\left(\sigma_{\theta O}^2+\sigma_{\theta T}^2\right)\right]}, & \text{if} \quad 0<\underline{f}. \end{cases}$$

It can be rewritten as

$$V_{T0} = \begin{cases} -e^{-\left[\overline{v}+\min\limits_{\sigma_\eta}[-f(\sigma_\eta)]+\min\limits_{\sigma_\eta}\frac{[f(\sigma_\eta)]^2}{2\sigma_\varepsilon^2}-\frac{1}{2}\left(\sigma_{\theta O}^2+\sigma_{\theta T}^2\right)\right]}, & \text{if} \quad \overline{f}<0 \\ -e^{-\left[\overline{v}+\min\limits_{\sigma_\eta}[-f(\sigma_\eta)]-\frac{1}{2}\left(\sigma_{\theta O}^2+\sigma_{\theta T}^2\right)\right]}, & \text{if} \quad \underline{f}\leqslant 0\leqslant \overline{f} \\ -e^{-\left[\overline{v}+\min\limits_{\sigma_\eta}[-f(\sigma_\eta)]+\min\limits_{\sigma_\eta}\frac{[f(\sigma_\eta)]^2}{2\sigma_\varepsilon^2}-\frac{1}{2}\left(\sigma_{\theta O}^2+\sigma_{\theta T}^2\right)\right]}, & \text{if} \quad 0<\underline{f}. \end{cases} \tag{27}$$

Note that, for type 1 equilibrium, $\overline{f}<0$, the risky asset is strictly overvalued. $f(\sigma_\eta)$ plays the two different roles in affecting the ex ante expected utilities V_{O0} and V_{T0}. First, an increase in $f(\sigma_\eta)$ reduces its absolute value, a high $f(\sigma_\eta)$ represents a less deviation from the fundamentals, thereby reducing ex ante utilities. This role is reflected by the item $-f(\sigma_\eta)$ in Equations (25) and (27). Second, a high $f(\sigma_\eta)$ decreases the value of $\frac{f^2(\sigma_\eta)}{2\sigma_\varepsilon^2}$, the attractive trading opportunities for traders. This second role of the equity premium is captured by the quadratic terms $\frac{f^2(\sigma_\eta)}{2\sigma_\varepsilon^2}$ in Equations (25) and (27).

For type 2 equilibrium, $\underline{f}\leqslant 0\leqslant \overline{f}$, because of the ambiguity faced by transparent traders, they are reluctant to invest in risky asset. Only the perceived equity premium $f(\sigma_\eta)$ affects the ex ante expected utility V_{O0} and V_{T0}, which is only the discount on the expected wealth level associated with the traders' unit stock endowment, thereby reducing ex ante utilities.

For type 3 equilibrium, $\underline{f} > 0$, this case is similar to the one in Easley, O'Hara and Yang (2014), the perceived equity premium $f(\sigma_\eta)$ plays two different roles in affecting the ex ante expected utilities V_{O0} and V_{T0}.

Equation (25) and (27) replicate the results in Easley, O'Hara and Yang (2014). Firstly, there are two ways for the pricing function $f(\cdot)$ to affect the ex ante utility of both types of traders. The linear term $-f(\sigma_\eta)$ in (25) and (27) represents the discount on the expected wealth level associated with traders unit stock endowment, while the quadratic terms $\frac{1}{2(1-\rho^2)}\left(\frac{f^2(\sigma_\eta)}{\sigma_\varepsilon^2} - 2\rho m \frac{f(\sigma_\eta)}{\sigma_\varepsilon \sigma_\eta} + \frac{m^2}{\sigma_\eta^2}\right)$ in (25) and $\frac{f(\sigma_\eta)^2}{2\sigma_\epsilon^2}$ in (27) represents the perceived benefit resulting from future trading in the asset market. Secondly, opaque traders and transparent traders evaluate these two roles of the equity premium $f(\sigma_\eta)$ differently. Transparent traders face ambiguity both at the type-decision stage and at the trading stage, hence employing two separate minimum operators in (27), $\min\limits_{\sigma_\eta \in [\underline{\sigma}_T, \overline{\sigma}_T]} (-f(\sigma_\eta)) + \min\limits_{\sigma_\eta \in [\underline{\sigma}_T, \overline{\sigma}_T]} \frac{f(\sigma_\eta)^2}{2\sigma_\epsilon^2}$, while opaque traders face ambiguity only at the type-decision stage and resolve their ambiguity at the trading stage, hence applying only one minimum operator jointly to the linear and quadratic terms, $\min\limits_{\sigma_\eta \in [\underline{\sigma}_T, \overline{\sigma}_T]} \left[-f(\sigma_\eta) + \frac{1}{2(1-\rho^2)}\left(\frac{f^2(\sigma_\eta)}{\sigma_\varepsilon^2} - 2\rho m \frac{f(\sigma_\eta)}{\sigma_\varepsilon \sigma_\eta} + \frac{m^2}{\sigma_\eta^2}\right)\right]$.

Inspired by Easley, O'Hara and Yang (2014), according to Equations (25) and (27), we define the switching benefit from being a transparent trader to an opaque trader as a fraction μ of opaque traders as

$$B(\mu) = \begin{cases} \min\limits_{\sigma_\eta \in [\underline{\sigma}_T, \overline{\sigma}_T]} \left[-f(\sigma_\eta) + \frac{1}{2(1-\rho^2)}\left(\frac{f^2(\sigma_\eta)}{\sigma_\varepsilon^2} - 2\rho m \frac{f(\sigma_\eta)}{\sigma_\varepsilon \sigma_\eta} + \frac{m^2}{\sigma_\eta^2}\right)\right] - \left[-\overline{f} + \frac{\overline{f}^2}{2\sigma_\varepsilon^2}\right], & \text{if } \overline{f} < 0 \\ \min\limits_{\sigma_\eta \in [\underline{\sigma}_T, \overline{\sigma}_T]} \left[-f(\sigma_\eta) + \frac{1}{2(1-\rho^2)}\left(\frac{f^2(\sigma_\eta)}{\sigma_\varepsilon^2} - 2\rho m \frac{f(\sigma_\eta)}{\sigma_\varepsilon \sigma_\eta} + \frac{m^2}{\sigma_\eta^2}\right)\right] - \left[-\overline{f}\right], & \text{if } \underline{f} \leqslant 0 \leqslant \overline{f} \\ \min\limits_{\sigma_\eta \in [\underline{\sigma}_T, \overline{\sigma}_T]} \left[-f(\sigma_\eta) + \frac{1}{2(1-\rho^2)}\left(\frac{f^2(\sigma_\eta)}{\sigma_\varepsilon^2} - 2\rho m \frac{f(\sigma_\eta)}{\sigma_\varepsilon \sigma_\eta} + \frac{m^2}{\sigma_\eta^2}\right)\right] - \left[-\overline{f} + \frac{\underline{f}^2}{2\sigma_\varepsilon^2}\right], & \text{if } 0 < \underline{f}. \end{cases} \quad (28)$$

Appendix A2 provides the closed-form solution in (A8).

$$B(\mu) = \begin{cases} \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)\right]^2, & \text{if } \frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon \overline{\sigma}_T}{\mu} \\ \frac{1-\rho^2}{2\mu^2}\sigma_\varepsilon^2 + \frac{1}{2}\left(\frac{m}{\overline{\sigma}_T}\right)^2, & \text{if } -\frac{\sigma_\varepsilon \overline{\sigma}_T}{\mu} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu} \\ \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon - \frac{m}{\underline{\sigma}_T}\right)\right]^2 + \frac{1}{2}\left(\frac{m}{\overline{\sigma}_T}\right)^2 - \frac{1}{2}\left(\frac{m}{\underline{\sigma}_T}\right)^2, & \text{if } -\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu} < \frac{\rho m}{1-\rho^2} < 0 \\ \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)\right]^2 + \frac{1}{2}\left(\frac{m}{\underline{\sigma}_T}\right)^2 - \frac{1}{2}\left(\frac{m}{\overline{\sigma}_T}\right)^2, & \text{if } 0 \leqslant \rho m \leqslant \rho^2\sigma_\varepsilon \underline{\sigma}_T \\ \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)\right]^2 - \frac{1}{2}\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)^2 + \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} - \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T}, & \text{if } \rho^2\sigma_\varepsilon \underline{\sigma}_T < \rho m < \rho^2 \sigma_\varepsilon \overline{\sigma}_T \\ \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)\right]^2 + \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} - \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T}, & \text{if } \rho^2 \sigma_\varepsilon \overline{\sigma}_T \leqslant \rho m \end{cases} \quad (29)$$

After case-by-case study, we demonstrate the explicit form of the benefit function $B(\cdot)$ in Proposition 3, which also reveals its basic properties as a function of μ.

PROPOSITION 3. *The benefit of switching from being a transparent trader to an opaque trader* $B : [0,1] \to \mathbb{R}$ *is a non-negative, continuous, and* *strictly decreasing function. The switching benefit function $B(\cdot)$ is given by an explicit form as*

$$B(\mu) = \begin{cases} \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon - \frac{m}{\underline{\sigma}_T}\right)\right]^2 + \frac{1}{2}\left(\frac{m}{\overline{\sigma}_T}\right)^2 - \frac{1}{2}\left(\frac{m}{\underline{\sigma}_T}\right)^2, & \textit{if } \rho m < 0 \textit{ and } \mu < (1-\rho^2)\frac{\sigma_\varepsilon\underline{\sigma}_T}{-\rho m} \\ \frac{1-\rho^2}{2\mu^2}\sigma_\varepsilon^2 + \frac{1}{2}\left(\frac{m}{\overline{\sigma}_T}\right)^2, & \textit{if } \rho m < 0 \textit{ and } (1-\rho^2)\frac{\sigma_\varepsilon\underline{\sigma}_T}{-\rho m} \leqslant \mu \leqslant (1-\rho^2)\frac{\sigma_\varepsilon\overline{\sigma}_T}{-\rho m} \\ \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)\right]^2, & \textit{if } \rho m < 0 \textit{ and } (1-\rho^2)\frac{\sigma_\varepsilon\overline{\sigma}_T}{-\rho m} < \mu \\ \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)\right]^2 + \frac{1}{2}\left(\frac{m}{\underline{\sigma}_T}\right)^2 - \frac{1}{2}\left(\frac{m}{\overline{\sigma}_T}\right)^2, & \textit{if } 0 \leqslant \rho m \leqslant \rho^2\sigma_\varepsilon\underline{\sigma}_T \\ \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)\right]^2 & \\ -\frac{1}{2}\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)^2 + \rho m\frac{\sigma_\varepsilon}{\underline{\sigma}_T} - \rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T}, & \textit{if } \rho^2\sigma_\varepsilon\underline{\sigma}_T < \rho m < \rho^2\sigma_\varepsilon\overline{\sigma}_T \\ \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)\right]^2 + \rho m\frac{\sigma_\varepsilon}{\underline{\sigma}_T} - \rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T}, & \textit{if } \rho^2\sigma_\varepsilon\overline{\sigma}_T \leqslant \rho m \end{cases} \tag{30}$$

In order for an interior equilibrium fraction $\mu^* \in (0,1)$, it must be that $B(\mu^*) = c$, i.e, each trader is indifferent between becoming opaque versus remaining transparent. It is shown that such an equilibrium exists when c takes values in a range whose size is associated with the degree of ambiguity $\Delta\sigma$. Additionally, it is shown that the switching benefit function $B(\cdot)$ is decreasing in μ, and any interior fraction of opaque traders ($0 < \mu^* < 1$) must be unique.

Let $\overline{c}$ and $\underline{c}$ denote the upper and lower bounds of the range of the possible cost of c. We can obtain $\overline{c} = B(0+)$, $\underline{c} = B(1-)$, and how the range of cost, $\overline{c} - \underline{c}$ will be affected by the ambiguity amount $\Delta\sigma$ is shown in Proposition 4 regarding equilibrium fraction μ^* of opaque traders.

Proposition 4. *Denote $B(0+) = \overline{c}$ and $B(1-) = \underline{c}$. When $\underline{c} < c < \overline{c}$, there exists a unique interior trader distribution $0 < \mu^* < 1$.*

[1]If $\rho m < 0$, then $\frac{\partial(\overline{c}-\underline{c})}{\partial\Delta\sigma} > 0$, the size of the range $(\underline{c}, \overline{c})$ increase with degree of ambiguity.

[2]If $0 < \rho m < \rho^2\sigma_\varepsilon\overline{\sigma}_T$, then $\frac{\partial(\overline{c}-\underline{c})}{\partial\Delta\sigma} > 0$, the size of the range $(\underline{c}, \overline{c})$ increase with degree of ambiguity.

[3]If $\rho^2\sigma_\varepsilon\overline{\sigma}_T < \rho m$, then $\frac{\partial(\overline{c}-\underline{c})}{\partial\Delta\sigma} < 0$, the size of the range $(\underline{c}, \overline{c})$ decrease with degree of ambiguity.

Remark 3 of Theorem 1 reports that [1] if $\rho m < 0$, then $\frac{1}{1-\rho^2+\rho^2\mu} < D_O^*$; [2] if $0 < \rho m < \rho^2\sigma_\varepsilon\overline{\sigma}_T$, then $1 < D_O^* < \frac{1}{1-\rho^2+\rho^2\mu}$; and [3] if $\rho^2\sigma_\varepsilon\overline{\sigma}_T < \rho m$, then $D_O^* < 1$. Thus, in order to buy in risky asset, opaque traders will bear market risk. However, when they sell out risky asset, opaque traders are not willing to bear market risk.

5. REGULATION, PRICES, AND WELFARE

In subsection 3.5, it is shown that only if opaque traders have a positive net wealth, no matter what type equilibrium of the economy, they can always outperform the market. From a regulatory perspective, it is inappropriate that the information advantageous investors always beat the market to gain positive abnormal return, while disadvantageous traders are exploited by the information advantageous to some extent. To mitigate such market outcome resulting from information asymmetry in financial market, the government's corresponding regulations includes two aspects: (1) to raise the cost for information acquisition, so as to decrease the fraction of the information advantageous investors, thus to keep most of the traders less informed; (2) to reduce the market opacity, so as to reduce ambiguity perceived by information disadvantageous investors, thus to keep most of the traders informed.

There are hot discussions, both in academic circle and in the financial industry, about whether the policy-makers should put more regulatory restrains on the everyday operations of hedge funds. Inspired by Easley, O'Hara and Yang (2014), we will focus on the changes in c, the cost for information acquisition to become opaque, and the changes in the amount of ambiguity, $\Delta\sigma$, since these two aspects are closely related to regulations. We will analyze how these two regulations affect the equilibrium trader distribution μ^*, the equity premium $\mathbf{EP}^*$, and the welfare $\mathbf{WEL}^*$.

5.1. Implication of Information Acquisition Cost: *c*

In this subsection, we focus on the impact of increasing c while keeping $\Delta\sigma$ fixed, and examine the effect on the equilibrium fraction μ^* of opaque traders and the changes of benefit function, equity premium, and welfare function.

5.1.1. Trader Distribution μ^*

We begin by analyzing the implication of increasing information acquisition cost c on the equilibrium fraction μ^* of opaque traders, which is determined by the condition $B(\mu^*) = c$.

From Equation (29), we have the derivative of switching benefit function $B(\mu^*)$,

$$\frac{\partial B(\mu^*)}{\partial \mu^*} = \begin{cases} -\frac{\rho^2(1-\rho^2)}{[(1-\rho^2)+\rho^2\mu^*]^3}\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)^2, & \text{if } \frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon\overline{\sigma}_T}{\mu^*} \\ -\frac{1-\rho^2}{\mu^{*3}}\sigma_\varepsilon^2, & \text{if } -\frac{\sigma_\varepsilon\overline{\sigma}_T}{\mu^*} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon\underline{\sigma}_T}{\mu^*} \\ -\frac{\rho^2(1-\rho^2)}{[(1-\rho^2)+\rho^2\mu^*]^3}\left(\rho\sigma_\varepsilon - \frac{m}{\underline{\sigma}_T}\right)^2, & \text{if } -\frac{\sigma_\varepsilon\underline{\sigma}_T}{\mu^*} < \frac{\rho m}{1-\rho^2} < 0 \\ -\frac{\rho^2(1-\rho^2)}{[(1-\rho^2)+\rho^2\mu^*]^3}\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)^2, & \text{if } 0 \leqslant \rho m \end{cases}$$

Fix the amount of ambiguity $\Delta\sigma$ perceived by transparent traders, then $\frac{\partial B(\mu^*)}{\partial \mu^*} < 0$, i.e., the switching benefit function is decreasing with equilibrium fraction of opaque traders.

Note that the only way a change in c affect the equilibrium trader distribution is through the equilibrium condition $B(\mu^*) = c$, since the function form of $B(\cdot)$ does not concern the cost c. If c increases, μ^* must decrease to restore the equality $B(\mu^*) = c$, which is consistent with the intuition that an increase in c will reduce the incentive of transparent traders to become opaque, leading to fewer opaque traders, that is, a lower μ^*.

5.1.2. *Equity Premium*

The equilibrium equity premium is given by Equation (22) evaluated at the equilibrium μ^*.

$$\mathbf{EP}^* = \begin{cases} \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta} + \frac{1-\rho^2}{(1-\rho^2)+\rho^2\mu^*}\left[\sigma_\varepsilon^2 - (1-\mu)\rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta+\Delta\sigma}\right], & \text{if } \frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}{\mu^*} \\ \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu^*}, & \text{if } -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}{\mu^*} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}{\mu^*} \\ \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta} + \frac{1-\rho^2}{(1-\rho^2)+\rho^2\mu^*}\left[\sigma_\varepsilon^2 - (1-\mu)\rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta-\Delta\sigma}\right], & \text{if } -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}{\mu^*} < \frac{\rho m}{1-\rho^2} < 0 \\ \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta} + \frac{1-\rho^2}{(1-\rho^2)+\rho^2\mu^*}\left[\sigma_\varepsilon^2 - (1-\mu)\rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta+\Delta\sigma}\right], & \text{if } 0 \leqslant \frac{\rho m}{1-\rho^2}. \end{cases}$$

The only way that a change in the cost c will affect the equilibrium equity premium is through affecting μ^*, and how μ^* affects $\mathbf{EP}^*$ depending upon the equilibrium type. In some equilibrium type, opaque traders may not trade more aggressively than transparent traders, so increasing the proportion of opaque traders may not result in a decrease in the $\mathbf{EP}^*$. Appendix A4 checks the three cases and proves the result as follows.

Proposition 5. *Fix the amount of ambiguity $\Delta\sigma$ perceived by transparent traders.*

[1]If $\rho m < \rho^2\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma)$, then $\frac{\partial \mathbf{EP}^*}{\partial \mu^*} < 0$, increasing the proportion of opaque traders will decrease the equity premium.

[2]If $\rho^2\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma) < \rho m$, then $\frac{\partial \mathbf{EP}^*}{\partial \mu^*} > 0$, increasing the proportion of opaque traders will increase the equity premium.

To understand the result intuitively, we can look into the positions of opaque traders in these two different scenarios. If $\rho m < \rho^2\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma)$, then the opaque traders' demand is $D_O^* > 1$, i.e., compared with the initial unit stock endowment, opaque traders will buy in and transparent traders sell out the stock. Thus, an increase in μ^* due to a decrease in c increases the aggregated demand for the risky asset, which will increase the asset price and decrease the equity premium. On the other hand, if $\rho^2\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma) < \rho m$, $D_O^* < 1$ while $D_T^* > 1$, thus an increase in μ^* due to a decrease in c decreases the aggregated demand for the risky asset, decreases the asset price and increases the equity premium.

5.1.3. Welfare Function

Following Easley, O'Hara and Yang (2014), we measure the welfare level of the entire equity market by the certainty equivalent of transparent traders' ex ante equilibrium utility (adjusted by subtracting a constant $\overline{v} - \frac{1}{2}\left[\sigma_{\theta O}^2 + \sigma_{\theta T}^2\right]$), which is, by Equation (27), as follows

$$\mathbf{WEL}^* = \begin{cases} -\overline{f}^* + \frac{\overline{f}^{*2}}{2\sigma_\varepsilon^2}, & \text{if} \quad \frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon \overline{\sigma}_T}{\mu^*} \\ -\overline{f}^*, & \text{if} \quad -\frac{\sigma_\varepsilon \overline{\sigma}_T}{\mu^*} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu^*} \\ -\overline{f}^* + \frac{\underline{f}^{*2}}{2\sigma_\varepsilon^2}, & \text{if} \quad -\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu^*} < \frac{\rho m}{1-\rho^2} \end{cases} \tag{31}$$

where $\overline{f}^*$ and $\underline{f}^*$ are transparent traders perceived maximum and minimum equilibrium equity premium, defined in Section 3.3 evaluated at an equilibrium fraction μ^*. In Equation (31), the linear term represents the discount effect of equity premium on wealth perceived by transparent traders, and the second term represents the potential benefit from future trading. Appendix A5 calculates welfare function.

$$\mathbf{WEL} = \begin{cases} \frac{1}{2}\left\{\left[\frac{\rho\mu}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)\right]^2 - \sigma_\varepsilon^2\right\}, & \text{if} \quad \frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon \overline{\sigma}_T}{\mu} \\ -\rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} - \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu}, & \text{if} \quad -\frac{\sigma_\varepsilon \overline{\sigma}_T}{\mu} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu} \\ -\rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} + \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \frac{1}{2}\left\{\left[\frac{\rho\mu}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon - \frac{m}{\underline{\sigma}_T}\right)\right]^2 - \sigma_\varepsilon^2\right\}, & \text{if} \quad -\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu} < \frac{\rho m}{1-\rho^2} < 0 \\ -\rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} + \frac{1}{2}\left\{\left[\frac{\rho\mu}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)\right]^2 - \sigma_\varepsilon^2\right\}, & \text{if} \quad 0 \leqslant \frac{\rho m}{1-\rho^2} \end{cases} \tag{32}$$

The net effect of an increase in the cost c on welfare depends on the its relative impact on these two terms.

$$\frac{\partial \mathbf{WEL}^*}{\partial \mu^*} = \begin{cases} \frac{1}{\sigma_\varepsilon^2}\frac{(1-\rho^2)\mu^*}{[(1-\rho^2)+\rho^2\mu^*]^3}\left(\rho^2\sigma_\varepsilon^2 - \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T}\right)^2 > 0, & \text{if} \quad \frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma)}{\mu^*} \\ \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu^{*2}} > 0, & \text{if} \quad -\frac{\sigma_\varepsilon \overline{\sigma}_T}{\mu^*} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu^*} \\ \frac{1}{\sigma_\varepsilon^2}\frac{(1-\rho^2)\mu^*}{[(1-\rho^2)+\rho^2\mu^*]^3}\left(\rho^2\sigma_\varepsilon^2 - \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T}\right)^2 > 0, & \text{if} \quad -\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu^*} < \frac{\rho m}{1-\rho^2} < 0 \\ \frac{1}{\sigma_\varepsilon^2}\frac{(1-\rho^2)\mu^*}{[(1-\rho^2)+\rho^2\mu^*]^3}\left(\rho^2\sigma_\varepsilon^2 - \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T}\right)^2 > 0, & \text{if} \quad 0 \leqslant \frac{\rho m}{1-\rho^2} \end{cases} \tag{33}$$

With Equation (33), the transparent traders' demand function is $\frac{\overline{f}^*}{\sigma_\varepsilon^2}$ with $\overline{f}^* < 0$. An increase in the $\overline{f}^*$ reduces its absolute value, thus decreases $\frac{\overline{f}^{*2}}{2\sigma_\varepsilon^2}$, decrease the expected benefit from the trading. Together with the discount effect, an increase in $\overline{f}^*$ reduces the welfare function. With Equation (33), the transparent traders will not participate with $\underline{f}^* \leqslant 0 \leqslant \overline{f}^*$. Thus, the welfare function has the only component, $-\overline{f}^*$. And with Equation (33), the transparent traders' demand function is $\frac{\underline{f}^*}{\sigma_\varepsilon^2}$ with $0 < \underline{f}^*$. The second part of the welfare function, $\frac{\underline{f}^{*2}}{2\sigma_\varepsilon^2}$, is increasing in $\underline{f}^*$. Meanwhile, an increase in $\overline{f}^*$ reduces welfare, because it reflects transparent traders' perceived discount of their expected wealth.

PROPOSITION 6. *Fix the amount of ambiguity $\Delta\sigma$ perceived by transparent traders. Then the welfare increases in fraction of opaque traders,* $\frac{\partial \mathbf{WEL}^*}{\partial \mu^*} > 0$.

The above results demonstrate that increasing information acquisition cost might decrease the population of opaque traders, have different impacts on the equilibrium equity premium, however, it always decreases the aggregate social welfare. Thus, regulations designed with this effect can be detrimental to the economy. On the contrary, regulation decreasing information acquisition cost might induce more opaque traders, decrease equity premium and increase social welfare, will benefit the economy.

5.2. Implication of disclosure: $\Delta\sigma$

Market opacity is a fundamental issue in the design and regulation of markets. It's of considerable importance to investors, academics and regulators. In this subsection we examine the implication of decreasing the amount of ambiguity, $\Delta\sigma$, while keeping c fixed, for investors behavior, asset pricing and welfare.

5.2.1. Investors Behavior

From the switching benefit function $B(\mu^*, \Delta\sigma) = c$, Equation (29), with $c \in (\underline{c}, \overline{c})$ and c fixed, it is found that a decrease in the amount of ambiguity $\Delta\sigma$ will determine an endogenous equilibrium fraction μ^*. For opaque traders, a decrease in $\Delta\sigma$ generally reduces the benefit of the switch, as the reduced $\Delta\sigma$ weakens their informational advantage. On the other hand, a decrease in $\Delta\sigma$ affects the switching benefit directly through $\overline{\sigma}_T$ and $\underline{\sigma}_T$. The total effect of decreasing $\Delta\sigma$ on μ^* depends on the synthesis of these two effects. In contrast, for the transparent traders, a decrease in $\Delta\sigma$ always reduces their ambiguity, increases their confidence to trade and adjust their trading strategies. In order to investigate the detailed relationship between a decrease in the amount of ambiguity $\Delta\sigma$ and the change of equilibrium fraction μ^*, we can derive the benefit function $B(\mu^*, \Delta\sigma) = c$ in $\Delta\sigma$ to obtain $\frac{\partial\mu^*}{\partial\Delta\sigma}$ for each case.

From Equation (29), we can obtain the switching benefit function for an endogenous equilibrium fraction μ^* as

$$B(\mu^*, \Delta\sigma) = \begin{cases} \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu^*}\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta+\Delta\sigma}\right)\right]^2, & \text{if } \frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}{\mu^*} \\ \frac{1-\rho^2}{2\mu^{*2}}\sigma_\varepsilon^2 + \frac{1}{2}\left(\frac{m}{\hat{\sigma}_\eta+\Delta\sigma}\right)^2, & \text{if } -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}{\mu^*} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}{\mu^*} \\ \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu^*}\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta-\Delta\sigma}\right)\right]^2 & \\ +\frac{1}{2}\left(\frac{m}{\hat{\sigma}_\eta+\Delta\sigma}\right)^2 - \frac{1}{2}\left(\frac{m}{\hat{\sigma}_\eta-\Delta\sigma}\right)^2, & \text{if } -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}{\mu^*} < \frac{\rho m}{1-\rho^2} < 0 \\ \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu^*}\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta+\Delta\sigma}\right)\right]^2 & \\ +\frac{1}{2}\left(\frac{m}{\hat{\sigma}_\eta-\Delta\sigma}\right)^2 - \frac{1}{2}\left(\frac{m}{\hat{\sigma}_\eta+\Delta\sigma}\right)^2, & \text{if } 0 \leqslant \rho m \leqslant \rho^2\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma) \\ \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu^*}\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta+\Delta\sigma}\right)\right]^2 & \\ -\frac{1}{2}\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta+\Delta\sigma}\right)^2 + \rho m\frac{\sigma_\varepsilon}{\hat{\sigma}_\eta-\Delta\sigma} - \rho m\frac{\sigma_\varepsilon}{\hat{\sigma}_\eta+\Delta\sigma}, & \text{if } \rho^2\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma) < \rho m < \rho^2\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma) \\ \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu^*}\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta+\Delta\sigma}\right)\right]^2 & \\ +\rho m\frac{\sigma_\varepsilon}{\hat{\sigma}_\eta-\Delta\sigma} - \rho m\frac{\sigma_\varepsilon}{\hat{\sigma}_\eta+\Delta\sigma}, & \text{if } \rho^2\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma) \leqslant \rho m \end{cases} \tag{34}$$

We then consider transparent traders' investment opportunities on market opacity case by case.

Case 1. Transparent traders hold short positions of risky asset. For $\frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}{\mu^*}$, then

$$B(\mu^*, \Delta\sigma) = \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu^*}\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta+\Delta\sigma}\right)\right]^2.$$

Iso-benefit curve $B(\mu^*, \Delta\sigma) = \text{constant}$ implies

$$0 = \frac{m}{(\hat{\sigma}_\eta+\Delta\sigma)^2}[(1-\rho^2)+\rho^2\mu^*] - \rho^2\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta+\Delta\sigma}\right)\frac{\partial\mu^*}{\partial\Delta\sigma},$$

then

$$\frac{\partial\mu^*}{\partial\Delta\sigma} = \frac{[(1-\rho^2)+\rho^2\mu^*]\frac{m}{(\hat{\sigma}_\eta+\Delta\sigma)^2}}{\rho^2\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta+\Delta\sigma}\right)} = \frac{[(1-\rho^2)+\rho^2\mu^*]\frac{\rho m}{(\hat{\sigma}_\eta+\Delta\sigma)^2}}{\rho^2\left(\rho^2\sigma_\varepsilon - \frac{\rho m}{\hat{\sigma}_\eta+\Delta\sigma}\right)} < 0. \tag{35}$$

That is, decreasing market opacity induces transparent traders greater incentive to pay for information acquisition to enlarge their investment opportunities.

For this case, risky asset is extremely overvalued. From Proposition 1 we know that with the reduction of market opacity, the price of risky asset deviates less from its value. While, according to transparent traders' demand function, $D_T^*(\tilde{p}, \theta_T) = \frac{\overline{f}}{\sigma_\varepsilon^2}$ with $\overline{f} < 0$, they will short less risky asset, thus will receive less payoffs in the end. However, as rational investors, transparent traders have incentive for information acquisition to enlarge their investment opportunities for more final consumption. From another point, transparent traders short the largest amount of risky asset for this case. They don't know the asset is overvalued, just due to a good fundamental or only over heated economy. And they are afraid if their investments are safe. Thus, transparent traders have greater incentive for information acquisitions to develop extra investment opportunities to hedge their risky asset positions.

Case 2. Transparent traders do not trade risky asset. For $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}{\mu^*} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}{\mu^*}$, then

$$B(\mu^*, \Delta\sigma) = \frac{1-\rho^2}{2\mu^{*2}}\sigma_\varepsilon^2 + \frac{1}{2}\left(\frac{m}{\hat{\sigma}_\eta + \Delta\sigma}\right)^2.$$

Iso-benefit curve $B(\mu^*, \Delta\sigma) = \text{constant}$ implies

$$0 = -\frac{1-\rho^2}{\mu^{*3}}\sigma_\varepsilon^2 \frac{\partial\mu^*}{\partial\Delta\sigma} - \frac{m^2}{(\hat{\sigma}_\eta + \Delta\sigma)^3},$$

then

$$\frac{\partial\mu^*}{\partial\Delta\sigma} = -\frac{\frac{m^2}{(\hat{\sigma}_\eta+\Delta\sigma)^3}}{\frac{1-\rho^2}{\mu^{*3}}\sigma_\varepsilon^2} < 0. \tag{36}$$

That is, decreasing market opacity induces transparent traders greater incentive to pay for information acquisition to enlarge their investment opportunities.

For this case, with the reduction of market opacity, the prices of risky asset is closer to its value. Due to ambiguity, transparent traders exhibit portfolio inertia to maintain a zero holding rather than a non-zero position of risky asset. While, "no pays no gains", as rational investors, transparent traders have incentive for information acquisition to enlarge their investment opportunities for more final consumptions.

Case 3. Transparent traders hold long positions of risky asset. For $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}{\mu^*} < \frac{\rho m}{1-\rho^2}$, we consider the following two cases.

Case 3.1. If $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}{\mu^*} < \frac{\rho m}{1-\rho^2} < 0$, then

$$B(\mu^*, \Delta\sigma) = \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu^*}\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta - \Delta\sigma}\right)\right]^2 + \frac{1}{2}\left(\frac{m}{\hat{\sigma}_\eta+\Delta\sigma}\right)^2 - \frac{1}{2}\left(\frac{m}{\hat{\sigma}_\eta-\Delta\sigma}\right)^2.$$

Iso-benefit curve $B(\mu^*, \Delta\sigma) =$ constant implies

$$\begin{aligned} & -\frac{\rho^2}{(1-\rho^2)+\rho^2\mu^*}\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta - \Delta\sigma}\right)^2 \frac{\partial\mu^*}{\partial\Delta\sigma} \\ = & \left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta - \Delta\sigma}\right)\frac{m}{(\hat{\sigma}_\eta - \Delta\sigma)^2} + \frac{[(1-\rho^2)+\rho^2\mu^*]^2}{1-\rho^2}\left(\frac{m^2}{(\hat{\sigma}_\eta + \Delta\sigma)^3} + \frac{m^2}{(\hat{\sigma}_\eta - \Delta\sigma)^3}\right) \\ = & \frac{\rho m}{\rho^2(\hat{\sigma}_\eta - \Delta\sigma)^3}\left[[(1-\rho^2)+\rho^2\mu^*]^2\left(\frac{(\hat{\sigma}_\eta - \Delta\sigma)^3}{(\hat{\sigma}_\eta + \Delta\sigma)^3}+1\right)-(1-\rho^2)\right]\left[\frac{\rho m}{1-\rho^2} + M_1^{BF}(\mu^*, \Delta\sigma)\right] \end{aligned} \tag{37}$$

where

$$M_1^{BF}(\mu^*, \Delta\sigma) = \frac{\rho^2\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)}{[(1-\rho^2)+\rho^2\mu^*]^2\left(\frac{(\hat{\sigma}_\eta - \Delta\sigma)^3}{(\hat{\sigma}_\eta + \Delta\sigma)^3}+1\right)-(1-\rho^2)}.$$

Then

$$\frac{\partial\mu^*}{\partial\Delta\sigma} \lesseqqgtr 0 \qquad \text{if and only if} \qquad \frac{\rho m}{1-\rho^2} \lesseqqgtr -M_1^{BF}(\mu^*, \Delta\sigma).$$

Note that $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)}{\mu^*} \lesseqqgtr -M_1^{BF}(\mu^*, \Delta\sigma)$ if and only if $1-\frac{(\hat{\sigma}_\eta - \Delta\sigma)^3}{\rho^2[(\hat{\sigma}_\eta - \Delta\sigma)^3+(\hat{\sigma}_\eta + \Delta\sigma)^3]} \lesseqqgtr \mu^*$, then we consider the following two settings.

Case 3.1.1. $\mu^* \leqslant 1 - \frac{(\hat{\sigma}_\eta - \Delta\sigma)^3}{\rho^2[(\hat{\sigma}_\eta - \Delta\sigma)^3+(\hat{\sigma}_\eta + \Delta\sigma)^3]}$. Then $-M_1^{BF}(\mu^*, \Delta\sigma) \leqslant -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)}{\mu^*} < \frac{\rho m}{1-\rho^2} < 0$ and hence $\frac{\partial\mu^*}{\partial\Delta\sigma} > 0$, i.e., increasing market opacity induces traders greater incentive to pay for information acquisition.

Case 3.1.2. $1-\frac{(\hat{\sigma}_\eta - \Delta\sigma)^3}{\rho^2[(\hat{\sigma}_\eta - \Delta\sigma)^3+(\hat{\sigma}_\eta + \Delta\sigma)^3]} < \mu^*$, then $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)}{\mu^*} < -M_1^{BF}(\mu^*, \Delta\sigma)$, then

[3.1.2.1] If $\frac{\rho m}{1-\rho^2} < -M_1^{BF}(\mu^*, \Delta\sigma)$, then $\frac{\partial\mu^*}{\partial\Delta\sigma} < 0$, i.e., decreasing market opacity induces traders greater incentive to pay for information acquisition.

[3.1.2.2] If $\frac{\rho m}{1-\rho^2} = -M_1^{BF}(\mu^*, \Delta\sigma)$, then $\frac{\partial\mu^*}{\partial\Delta\sigma} = 0$, i.e., decreasing market opacity does not affect traders equilibrium distribution.

[3.1.2.3] If $\frac{\rho m}{1-\rho^2} > -M_1^{BF}(\mu^*, \Delta\sigma)$, then $\frac{\partial\mu^*}{\partial\Delta\sigma} > 0$, i.e., decreasing market opacity induces traders less incentive to pay for information acquisition.

From the above analysis for this case, there is a threshold value to differentiate the sign of $\frac{\partial\mu^*}{\partial\Delta\sigma}$. On the left side of the threshold value, with the reduction of market opacity, according to transparent traders' demand function, $D_T^*(\tilde{p}, \theta_T) = \frac{\underline{f}}{\sigma_\varepsilon^2}$ with $\underline{f} > 0$, they will hold small long positions of risky asset, thus will receive less payoffs in the end. But, as rational investors, transparent traders have incentive for information acquisition to enlarge their investment opportunities for more final consumptions. It's very similar to Case 1 and Case 2. While, on the right side of the threshold, the effect that reducing market opacity weakens the advantage of opaque traders dominates. Transparent traders can free-ride on learning of other merely by observing the asset price, thus have less incentive to pay for information acquisition. It's similar to the claim in Grossman and Stiglitz (1980).

Case 3.2. If $0 \leqslant \frac{\rho m}{1-\rho^2}$, there are three settings as follows.

Case 3.2.1. If $0 \leqslant \rho m \leqslant \rho^2 \sigma_\varepsilon (\hat{\sigma}_\eta - \Delta\sigma)$, then

$$B(\mu^*, \Delta\sigma) = \frac{1-\rho^2}{2} \left[\frac{1}{(1-\rho^2)+\rho^2\mu^*} \left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta + \Delta\sigma} \right) \right]^2 + \frac{1}{2} \left(\frac{m}{\hat{\sigma}_\eta - \Delta\sigma} \right)^2 - \frac{1}{2} \left(\frac{m}{\hat{\sigma}_\eta + \Delta\sigma} \right)^2 .$$

Iso-benefit curve $B(\mu^*, \Delta\sigma) = $ constant implies

$$\begin{aligned} & \frac{\rho^2}{(1-\rho^2)+\rho^2\mu^*} \left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta + \Delta\sigma} \right)^2 \frac{\partial \mu^*}{\partial \Delta\sigma} \qquad (38) \\ = & \left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta + \Delta\sigma} \right) \frac{m}{(\hat{\sigma}_\eta + \Delta\sigma)^2} + \frac{[(1-\rho^2)+\rho^2\mu^*]^2}{1-\rho^2} \left(\frac{m^2}{(\hat{\sigma}_\eta - \Delta\sigma)^3} + \frac{m^2}{(\hat{\sigma}_\eta + \Delta\sigma)^3} \right). \end{aligned}$$

Thus $\frac{\partial \mu^*}{\partial \Delta\sigma} > 0$, i.e., decreasing market opacity induces transparent traders less incentive to pay for information acquisition.

For this case, transparent traders hold moderate long positions of risky asset. The effect that reducing market opacity weakens opaque traders' advantage not facing ambiguity when trading the stock dominates. The price of risky asset is informative, in the sense that transparent traders can free-ride on learning from others. Therefore, transparent traders have less incentive to acquire information. It is very similar to Easley, O'Hara and Yang (2014).

Case 3.2.2. If $\rho^2 \sigma_\varepsilon (\hat{\sigma}_\eta - \Delta\sigma) < \rho m < \rho^2 \sigma_\varepsilon (\hat{\sigma}_\eta + \Delta\sigma)$, then

$$\begin{aligned} B(\mu^*, \Delta\sigma) \;=\; & \frac{\rho^2[(1-\rho^2)(1-\mu^*)^2 - \mu^{*2}]}{2[(1-\rho^2)+\rho^2\mu^*]^2} \left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta + \Delta\sigma} \right)^2 \\ & + \; \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta - \Delta\sigma} - \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta + \Delta\sigma}. \end{aligned}$$

Iso-benefit curve $B(\mu^*, \Delta\sigma) = $ constant implies

$$\begin{aligned} & \frac{\rho^2}{(1-\rho^2)+\rho^2\mu^*} \left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta + \Delta\sigma} \right)^2 \frac{\partial \mu^*}{\partial \Delta\sigma} \qquad (39) \\ = & \left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta + \Delta\sigma} \right) \frac{m}{(\hat{\sigma}_\eta + \Delta\sigma)^2} + \frac{[(1-\rho^2)+\rho^2\mu^*]^2}{1-\rho^2} \left\{ \rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta - \Delta\sigma)^2} + \frac{m^2}{(\hat{\sigma}_\eta + \Delta\sigma)^3} \right\}. \end{aligned}$$

Thus $\frac{\partial \mu^*}{\partial \Delta\sigma} > 0$, i.e., decreasing market opacity induces transparent traders less incentive to pay for information acquisition. This case is very similar to Case 3.2.1.

Case 3.2.3. If $\rho^2\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma) \leqslant \rho m$, then

$$B(\mu^*, \Delta\sigma) = \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu^*}\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta + \Delta\sigma}\right)\right]^2 + \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta - \Delta\sigma} - \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta + \Delta\sigma}.$$

Iso-benefit curve $B(\mu^*, \Delta\sigma) =$ constant implies

$$\begin{aligned}
&\frac{\rho^2}{(1-\rho^2)+\rho^2\mu^*}\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta + \Delta\sigma}\right)^2 \frac{\partial \mu^*}{\partial \Delta\sigma} \\
= &\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta + \Delta\sigma}\right)\frac{m}{(\hat{\sigma}_\eta + \Delta\sigma)^2} + \frac{[(1-\rho^2)+\rho^2\mu^*]^2}{1-\rho^2}\left(\rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta - \Delta\sigma)^2} + \rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta + \Delta\sigma)^2}\right) \\
= &\frac{m}{\rho}\frac{1}{(\hat{\sigma}_\eta + \Delta\sigma)^3}\left[M_2^{BF}(\mu^*, \Delta\sigma) - \rho m\right] \qquad (40)
\end{aligned}$$

where

$$M_2^{BF}(\mu^*, \Delta\sigma) = \rho^2\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma)\left[1 + \frac{[(1-\rho^2)+\rho^2\mu^*]^2}{1-\rho^2}\left(\frac{(\hat{\sigma}_\eta + \Delta\sigma)^2}{(\hat{\sigma}_\eta - \Delta\sigma)^2} + 1\right)\right].$$

Then

$$\frac{\partial \mu^*}{\partial \Delta\sigma} \lesseqqgtr 0 \qquad \text{if and only if} \qquad M_2^{BF}(\mu^*, \Delta\sigma) \lesseqqgtr \rho m$$

and hence we have a threshold to differentiate the sign of $\frac{\partial \mu^*}{\partial \Delta\sigma}$.

[3.2.3.1] If $\rho m < M_2^{BF}(\mu^*, \Delta\sigma)$, then $\frac{\partial \mu^*}{\partial \Delta\sigma} > 0$, i.e., decreasing market opacity decreases transparent traders' incentive to pay for information acquisition.

[3.2.3.2] If $\rho m = M_2^{BF}(\mu^*, \Delta\sigma)$, then $\frac{\partial \mu^*}{\partial \Delta\sigma} = 0$, i.e., decreasing market opacity does not affect traders equilibrium distribution.

[3.2.3.3] If $\rho m > M_2^{BF}(\mu^*, \Delta\sigma)$, then $\frac{\partial \mu^*}{\partial \Delta\sigma} < 0$, i.e., decreasing market opacity increases transparent traders' greater incentive to pay for information acquisition to enlarge their investment opportunities.

On the left side of the threshold, the effect that reducing market opacity weakens the advantage of opaque traders dominates. Transparent traders can free-ride on learning from other, thus have less incentive to pay for information acquisition. It's very similar to Case 3.2.1 and 3.2.2. While, on the right side of the threshold value, transparent traders hold the largest long positions of risky asset. With the reduction of market opacity, transparent traders know exactly risky asset is extremely undervalued, and they want to invest more in it. However, due to uncertainty aversion, they have greater incentive to pay for information acquisition to use extra opportunities to hedge their investment in risky asset. It's very similar to Case 1 and Case 2.

Summarizing the above analysis, we have the following conclusions:

[1] Case: $\mu^* \leqslant 1 - \frac{(\hat{\sigma}_\eta - \Delta\sigma)^3}{\rho^2[(\hat{\sigma}_\eta - \Delta\sigma)^3 + (\hat{\sigma}_\eta + \Delta\sigma)^3]}$.

[Case 1] Transparent traders hold short positions of risky asset. For $\frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma)}{\mu^*}$, then $\frac{\partial \mu^*}{\partial \Delta\sigma} < 0$.

[Case 2] Transparent traders do not trade risky asset. For $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma)}{\mu^*} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)}{\mu^*}$, then $\frac{\partial \mu^*}{\partial \Delta\sigma} < 0$.

[Case 3] Transparent traders hold long positions of risky asset.

[Case 3.1] If $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)}{\mu^*} < \frac{\rho m}{1-\rho^2} < 0$, then $\frac{\partial \mu^*}{\partial \Delta\sigma} > 0$.

[Case 3.2] If $0 \leqslant \frac{\rho m}{1-\rho^2}$, then we have the following conclusions:

[Case 3.2.1] If $0 \leqslant \rho m \leqslant \rho^2\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)$, then $\frac{\partial \mu^*}{\partial \Delta\sigma} > 0$.

[Case 3.2.2] If $\rho^2\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma) < \rho m < \rho^2\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma)$, then $\frac{\partial \mu^*}{\partial \Delta\sigma} > 0$.

[Case 3.2.3] If $\rho^2\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma) \leqslant \rho m$, then $\frac{\partial \mu^*}{\partial \Delta\sigma} \gtreqqless 0$ if and only if $\rho m \lesseqqgtr M_2^{BF}(\mu^*, \Delta\sigma)$.

[2] Case: $1 - \frac{(\hat{\sigma}_\eta - \Delta\sigma)^3}{\rho^2[(\hat{\sigma}_\eta - \Delta\sigma)^3 + (\hat{\sigma}_\eta + \Delta\sigma)^3]} < \mu^*$.

[Case 1] Transparent traders hold short positions of risky asset. For $\frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma)}{\mu^*}$, then $\frac{\partial \mu^*}{\partial \Delta\sigma} < 0$.

[Case 2] Transparent traders do not trade risky asset. For $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma)}{\mu^*} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)}{\mu^*}$, then $\frac{\partial \mu^*}{\partial \Delta\sigma} < 0$.

[Case 3] Transparent traders hold long positions of risky asset.

[Case 3.1] If $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)}{\mu^*} < \frac{\rho m}{1-\rho^2} < 0$, then $\frac{\partial \mu^*}{\partial \Delta\sigma} \lesseqqgtr 0$ if and only if $\frac{\rho m}{1-\rho^2} \lesseqqgtr -M_1^{BF}(\mu^*, \Delta\sigma)$.

[Case 3.2] If $0 \leqslant \frac{\rho m}{1-\rho^2}$, then we have the following conclusions:

[Case 3.2.1] If $0 \leqslant \rho m \leqslant \rho^2\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)$, then $\frac{\partial \mu^*}{\partial \Delta\sigma} > 0$.

[Case 3.2.2] If $\rho^2\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma) < \rho m < \rho^2\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma)$, then $\frac{\partial \mu^*}{\partial \Delta\sigma} > 0$.

[Case 3.2.3] If $\rho^2\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma) \leqslant \rho m$, then $\frac{\partial \mu^*}{\partial \Delta\sigma} \gtreqqless 0$ if and only if $\rho m \lesseqqgtr M_2^{BF}(\mu^*, \Delta\sigma)$.

Proposition 7. *Fix information acquisition cost c. We have the relation between trader distribution and amount of ambiguity.*

[1]For $\mu^* \leqslant 1 - \frac{(\hat{\sigma}_\eta - \Delta\sigma)^3}{\rho^2((\hat{\sigma}_\eta - \Delta\sigma)^3 + (\hat{\sigma}_\eta + \Delta\sigma)^3)}$*, we then have*

[1.1]if $\rho m \leqslant -(1-\rho^2)\frac{\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)}{\mu^*}$*, then* $\frac{\partial \mu^*}{\partial \Delta\sigma} < 0$*;*

[1.2]if $-(1-\rho^2)\frac{\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)}{\mu^*} < \rho m < M_2^{BF}(\mu^*, \Delta\sigma)$*, then* $\frac{\partial \mu^*}{\partial \Delta\sigma} > 0$*;*

[1.3]if $M_2^{BF}(\mu^*, \Delta\sigma) < \rho m$*, then* $\frac{\partial \mu^*}{\partial \Delta\sigma} < 0$*.*

[2]For $1 - \frac{(\hat{\sigma}_\eta - \Delta\sigma)^3}{\rho^2((\hat{\sigma}_\eta - \Delta\sigma)^3 + (\hat{\sigma}_\eta + \Delta\sigma)^3)} < \mu^*$*, we then have*

[2.1]if $\rho m < -(1-\rho^2)M_1^{BF}(\mu^*, \Delta\sigma)$*, then* $\frac{\partial \mu^*}{\partial \Delta\sigma} < 0$*;*

[2.2]if $-(1-\rho^2)M_1^{BF}(\mu^*, \Delta\sigma) < \rho m < M_2^{BF}(\mu^*, \Delta\sigma)$*, then* $\frac{\partial \mu^*}{\partial \Delta\sigma} > 0$*;*

[2.3]if $M_2^{BF}(\mu^*, \Delta\sigma) < \rho m$*, then* $\frac{\partial \mu^*}{\partial \Delta\sigma} < 0$*.*

According to whether the transparent traders have incentive to pay for information acquisition or not, we find that there are two critical values of ρm, therefore three intervals for the analysis of $\frac{\partial \mu^*}{\partial \Delta\sigma}$:

(1) ρm is small. If $\rho m \leqslant -(1-\rho^2)\frac{\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)}{\mu^*}$, and $\rho m < -(1-\rho^2)M_1^{BF}(\mu^*, \Delta\sigma)$ for a big fraction of opaque traders, $1 - \frac{(\hat{\sigma}_\eta - \Delta\sigma)^3}{\rho^2((\hat{\sigma}_\eta - \Delta\sigma)^3 + (\hat{\sigma}_\eta + \Delta\sigma)^3)} < \mu^*$, which is corresponding to transparent traders' short positions, non-participation, and small long positions of risky asset, then $\frac{\partial \mu^*}{\partial \Delta\sigma} < 0$. Decreasing market opacity induces transparent traders greater incentive to pay for information acquisition to enlarge their investment opportunities. This case is similar to the learning as a strategic complement discussed in Barlevy and Veronesi (2000), Mele and Sangiorgi (2015).

When the economy experience an irrational exuberance, the risky asset is always overvalued. There is an intriguing empirical fact that complex assets have been traded at a premium, rather than a discount before 2007. Coval, Jurek, and Stafford (2009) find that senior collateralized debt obligation tranches were significantly overpriced. Henderson and Pearson (2011) report that a retail structured equity product's price is almost 8% greater than its fair value. Celerier and Vallee (2013) find that structured products are traded at a premium in Europe; the more complex a product, the more pronounced its overpricing.

Although the price of the risky asset is to the extreme, the informed opaque traders buy in the risky asset and use the extra investment opportunities to hedge. With the reduction of market opacity, transparent traders know the asset is overvalued. However, it is difficult for these uninformed traders to identify what the extremely high price means, a good fundamental or only over heated economy. Hence, the fact that more traders acquire information and cause prices to be more extreme means that remaining transparent traders will have more difficulty in identifying what price reflects, and so their incentive to acquire information increases.

(2) ρm is moderate. If $-(1-\rho^2)\frac{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}{\mu^*} < \rho m < M_2^{BF}(\mu^*,\Delta\sigma)$ for a small fraction of opaque traders, $\mu^* \leqslant 1-\frac{(\hat{\sigma}_\eta-\Delta\sigma)^3}{\rho^2((\hat{\sigma}_\eta-\Delta\sigma)^3+(\hat{\sigma}_\eta+\Delta\sigma)^3)}$, and $-(1-\rho^2)M_1^{BF}(\mu^*,\Delta\sigma) < \rho m < M_2^{BF}(\mu^*,\Delta\sigma)$ for a big fraction of opaque traders, $1-\frac{(\hat{\sigma}_\eta-\Delta\sigma)^3}{\rho^2((\hat{\sigma}_\eta-\Delta\sigma)^3+(\hat{\sigma}_\eta+\Delta\sigma)^3)} < \mu^*$, which corresponds to the transparent traders' moderate positions of risky asset. This case is similar to Easley, O'Hara and Yang (2014). As the effect that decreasing market opacity weakens the information advantage of opaque traders dominates, a reduced $\Delta\sigma$ decreases the fraction of opaque traders. When more traders acquire information, price of risky asset becomes more informative, in the sense that it is easier for uninformed traders to free-ride on the learning from others.

(3) ρm is big. If $M_2(\mu^*,\Delta\sigma) < \rho m$, which corresponds to transparent traders' largest positions of risky asset, then $\frac{\partial\mu^*}{\partial\Delta\sigma} < 0$. Decreasing market opacity induces transparent traders greater incentive to pay for information acquisition and use extra investment opportunities to hedge their risky asset positions. This case is similar to the learning as a strategic complement discussed in Mele and Sangiorgi (2015): the larger the fraction of opaque traders, the higher the benefit to become informed.

This case can occur when the economy experiences a hard time, such as a recession, or there is a large (but not fully observed) exogenous supply of stocks. However, the uninformed transparent traders have difficulty in identifying what the extremely low price reflects, liquidity constrained individuals for cash or low fundamental value. Thus, transparent traders have greater incentive to pay for information acquisition to develop extra investment opportunities to hedge their risky asset positions.

5.2.2. Equity Premium

However, the impact of a change in $\Delta\sigma$ on the equity premium $\mathbf{EP}^*$ is also very complicated, because according to the expression of equity premium in Section 5.1.2, $\Delta\sigma$ affects the equity premium in two ways. Firstly, as captured by $\overline{\sigma}_T$ and $\underline{\sigma}_T$, when μ^* is fixed, a change in $\Delta\sigma$ means a directly effect on equity premium $\mathbf{EP}^*$. Secondly, there is an indirect effect, as captured by the endogenous μ^*: a change in $\Delta\sigma$ causes μ^* to change, which, in turn, tends to change the equity premium. The total effect of a change in $\Delta\sigma$ on $\mathbf{EP}^*$ depends on the synthesis of these two effects. We will discuss it for each case.

Case 1. Transparent traders hold short positions of risky asset. For $\frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}{\mu^*}$, then

$$\mathbf{EP}^* = \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta} + \frac{1-\rho^2}{(1-\rho^2)+\rho^2\mu^*}\left[\sigma_\varepsilon^2 - (1-\mu^*)\rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta+\Delta\sigma}\right]$$

and, from (35),

$$\begin{aligned}
&\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} \\
= {} & \frac{1-\rho^2}{[(1-\rho^2)+\rho^2\mu^*]^2}\left\{\left[\rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta+\Delta\sigma} - \rho^2\sigma_\varepsilon^2\right]\frac{\partial \mu^*}{\partial \Delta\sigma} + (1-\mu^*)\rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta+\Delta\sigma)^2}[(1-\rho^2)+\rho^2\mu^*]\right\} \\
= {} & \frac{1-\rho^2}{(1-\rho^2)+\rho^2\mu^*}\left\{-\frac{1}{\rho^2}\rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta+\Delta\sigma)^2} + (1-\mu^*)\rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta+\Delta\sigma)^2}\right\} = -\frac{1-\rho^2}{\rho^2}\rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta+\Delta\sigma)^2} > 0.
\end{aligned}$$

That is, reducing market opacity decreases the equity premium.

For this case, transparent traders hold short positions of risky asset, and opaque traders absorb all the risky asset. From Remark 3 of Theorem 1, we know $D_O^* > \frac{1}{\mu^*} > 1$, opaque traders buy in high positions of risky asset. From section 5.2.1, we know that $\frac{\partial \mu^*}{\partial \Delta\sigma} < 0$. Reducing market opacity induces more opaque traders, therefore lower equity premium (negative), that is, $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} > 0$.

Case 2. Transparent traders do not trade risky asset. For $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}{\mu^*} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}{\mu^*}$, then

$$\mathbf{EP}^* = \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu^*}$$

and, from (36),

$$\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} = -\frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu^{*2}}\frac{\partial \mu^*}{\partial \Delta\sigma} = \frac{m^2}{(\hat{\sigma}_\eta+\Delta\sigma)^3}\mu^* > 0.$$

That is, reducing market opacity decreases the equity premium.

For this case, transparent traders exhibit portfolio inertia and maintain a zero holding of risky asset, and opaque traders absorb all the risky asset. From section 5.2.1, we know that $\frac{\partial \mu^*}{\partial \Delta\sigma} < 0$. Reducing market opacity induces more opaque traders, therefore lower equity premium (negative), that is, $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} > 0$.

Case 3. Transparent traders hold long positions of risky asset. For $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}{\mu^*} < \frac{\rho m}{1-\rho^2}$, we consider the following two cases.

Case 3.1. If $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}{\mu^*} < \frac{\rho m}{1-\rho^2} < 0$, then

$$\mathbf{EP}^* = \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta} + \frac{1-\rho^2}{(1-\rho^2)+\rho^2\mu^*}\left[\sigma_\varepsilon^2 - (1-\mu^*)\rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta-\Delta\sigma}\right]$$

and, from (37),

$$\begin{aligned} & \frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} \\ = & \frac{1-\rho^2}{[(1-\rho^2)+\rho^2\mu^*]^2}\left\{\left[\rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta-\Delta\sigma} - \rho^2\sigma_\varepsilon^2\right]\frac{\partial \mu^*}{\partial \Delta\sigma} - [(1-\rho^2)+\rho^2\mu^*](1-\mu^*)\rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta-\Delta\sigma)^2}\right\} \\ = & \frac{1-\rho^2}{\rho^2}\rho m \sigma_\varepsilon \frac{\frac{(1-\rho^2)+\rho^2\mu^*}{(\hat{\sigma}_\eta+\Delta\sigma)^3} + \frac{\rho^2\mu^*}{(\hat{\sigma}_\eta-\Delta\sigma)^3}}{\rho\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta-\Delta\sigma}\right)}\left[\frac{\rho m}{1-\rho^2} + M_1^{EP}(\mu^*,\Delta\sigma)\right] \end{aligned}$$

where

$$M_1^{EP}(\mu^*,\Delta\sigma) = \frac{\frac{\rho^2\sigma_\varepsilon}{(\hat{\sigma}_\eta-\Delta\sigma)^2}}{\frac{(1-\rho^2)+\rho^2\mu^*}{(\hat{\sigma}_\eta+\Delta\sigma)^3} + \frac{\rho^2\mu^*}{(\hat{\sigma}_\eta-\Delta\sigma)^3}}.$$

Then

$$\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} \lesseqqgtr 0 \qquad \text{if and only if} \qquad \frac{\rho m}{1-\rho^2} \gtreqqless -M_1^{EP}(\mu^*,\Delta\sigma).$$

and hence we have a threshold to differentiate the sign of $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma}$.

[3.1.1] If $\frac{\rho m}{1-\rho^2} < -M_1^{EP}(\mu^*,\Delta\sigma)$, then $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} > 0$, i.e., decreasing market opacity decreases equity premium.

[3.1.2] If $\frac{\rho m}{1-\rho^2} = -M_1^{EP}(\mu^*,\Delta\sigma)$, then $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} = 0$, i.e., decreasing market opacity does not affect equity premium.

[3.1.3] If $\frac{\rho m}{1-\rho^2} > -M_1^{EP}(\mu^*,\Delta\sigma)$, then $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} < 0$, i.e., decreasing market opacity increases equity premium.

On the left side of the threshold, transparent traders hold very small long positions of risky asset, and opaque traders absorb almost all risky asset. Precisely,

$$\begin{aligned} D_O^* &= \frac{1}{1-\rho^2+\rho^2\mu^*}\left[1-(1-\mu^*)\frac{\rho m}{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}\right] \\ &> \frac{1}{1-\rho^2+\rho^2\mu^*}\left[1+(1-\mu^*)\frac{(1-\rho^2)\frac{\rho^2}{(\hat{\sigma}_\eta-\Delta\sigma)^3}}{\frac{(1-\rho^2)+\rho^2\mu^*}{(\hat{\sigma}_\eta+\Delta\sigma)^3}+\frac{\rho^2\mu^*}{(\hat{\sigma}_\eta-\Delta\sigma)^3}}\right] \\ &= \frac{\frac{1}{(\hat{\sigma}_\eta+\Delta\sigma)^3}+\frac{\rho^2}{(\hat{\sigma}_\eta-\Delta\sigma)^3}}{\frac{(1-\rho^2)+\rho^2\mu^*}{(\hat{\sigma}_\eta+\Delta\sigma)^3}+\frac{\rho^2\mu^*}{(\hat{\sigma}_\eta-\Delta\sigma)^3}} > \frac{1}{1-\rho^2+\rho^2\mu^*} > 1. \end{aligned}$$

Reducing market opacity induces more opaque traders, thus less equity premium and higher price level, i.e., $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} > 0$.

While on the right side of the threshold, $1 < D_O^* < \frac{1}{\mu^*}$, from Remark 3 of Theorem 1, and opaque traders buy in small positions of risky asset. In this case, $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} < 0$. Reducing market opacity induces increases of equity premium, and decreases of price of risky asset.

Case 3.2. If $0 \leqslant \frac{\rho m}{1-\rho^2}$, then

$$\mathbf{EP}^* = \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta} + \frac{1-\rho^2}{(1-\rho^2)+\rho^2\mu^*}\left[\sigma_\varepsilon^2 - (1-\mu^*)\rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta + \Delta\sigma}\right]$$

and

$$\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} = \frac{1-\rho^2}{[(1-\rho^2)+\rho^2\mu^*]^2}\left\{\left[\rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta + \Delta\sigma} - \rho^2\sigma_\varepsilon^2\right]\frac{\partial \mu^*}{\partial \Delta\sigma} + [(1-\rho^2)+\rho^2\mu^*](1-\mu^*)\rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta + \Delta\sigma)^2}\right\}.$$

Case 3.2.1. $0 \leqslant \rho m \leqslant \rho^2\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)$, then, from (37),

$$\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} = -\frac{1-\rho^2}{\rho^2}\rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta+\Delta\sigma)^2} - \sigma_\varepsilon \frac{[(1-\rho^2)+\rho^2\mu^*]\left(\frac{m^2}{(\hat{\sigma}_\eta-\Delta\sigma)^3} + \frac{m^2}{(\hat{\sigma}_\eta+\Delta\sigma)^3}\right)}{\rho\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta+\Delta\sigma}\right)} < 0.$$

That is, reducing market opacity increases the equity premium.

For this case, transparent traders hold small long positions of risky asset. From Remark 3 of in Theorem1, we know $1 < D_O^* < \frac{1}{1-\rho^2-\rho^2\mu^*}$, opaque traders buy in small positions of risky asset. From section 3.2.1, we know that $\frac{\partial \mu^*}{\partial \Delta\sigma} > 0$. Reducing market opacity induces less opaque traders, therefore higher equity premium, that is, $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} < 0$.

Case 3.2.2. $\rho^2\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma) < \rho m < \rho^2\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma)$, then, from (39),

$$\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} = -\frac{1-\rho^2}{\rho^2}\rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta+\Delta\sigma)^2} - \sigma_\varepsilon \frac{[(1-\rho^2)+\rho^2\mu^*]\left(\rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta-\Delta\sigma)^2} + \frac{m^2}{(\hat{\sigma}_\eta+\Delta\sigma)^3}\right)}{\rho\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta+\Delta\sigma}\right)} < 0.$$

That is, reducing market opacity increases the equity premium. This case is very similar to case 3.2.1

Case 3.2.3. $\rho^2\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)\leqslant\rho m$, then, from (40),

$$\begin{aligned}\frac{\partial\mathbf{EP}^*}{\partial\Delta\sigma} &= -\frac{\rho m\sigma_\varepsilon}{\rho\left(\rho\sigma_\varepsilon-\frac{m}{\hat{\sigma}_\eta+\Delta\sigma}\right)}\left\{\left(\frac{(1-\rho^2)+\rho^2\mu^*}{(\hat{\sigma}_\eta-\Delta\sigma)^2}+\frac{2(1-\rho^2)+\rho^2\mu^*}{(\hat{\sigma}_\eta+\Delta\sigma)^2}\right)\sigma_\varepsilon-\frac{1-\rho^2}{\rho^2}\frac{\rho m}{(\hat{\sigma}_\eta+\Delta\sigma)^3}\right\} \\ &= \frac{\rho m\sigma_\varepsilon}{\rho\left(\rho\sigma_\varepsilon-\frac{m}{\hat{\sigma}_\eta+\Delta\sigma}\right)}\frac{1-\rho^2}{\rho^2}\frac{1}{(\hat{\sigma}_\eta+\Delta\sigma)^3}\left\{\rho m-M_2^{EP}(\mu^*,\Delta\sigma)\right\}\end{aligned}$$

where

$$M_2^{EP}(\mu^*,\Delta\sigma)=\frac{\rho^2}{1-\rho^2}\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)^3\left(\frac{(1-\rho^2)+\rho^2\mu^*}{(\hat{\sigma}_\eta-\Delta\sigma)^2}+\frac{2(1-\rho^2)+\rho^2\mu^*}{(\hat{\sigma}_\eta+\Delta\sigma)^2}\right).$$

Then

$$\frac{\partial\mathbf{EP}^*}{\partial\Delta\sigma}\lesseqqgtr 0 \qquad \text{if and only if} \qquad \rho m \gtreqqless M_2^{BF}(\mu^*,\Delta\sigma)$$

and hence we have a threshold to differentiate the sign of $\frac{\partial\mathbf{EP}^*}{\partial\Delta\sigma}$.

[3.2.3.1] If $\rho m<M_2^{EP}(\mu^*,\Delta\sigma)$, then $\frac{\partial\mathbf{EP}^*}{\partial\Delta\sigma}>0$, i.e., decreasing market opacity decreases equity premium.

[3.2.3.2] If $\rho m=M_2^{EP}(\mu^*,\Delta\sigma)$, then $\frac{\partial\mathbf{EP}^*}{\partial\Delta\sigma}=0$, i.e., decreasing market opacity does not affect equity premium.

[3.2.3.3] If $\rho m>M_2^{EP}(\mu^*,\Delta\sigma)$, then $\frac{\partial\mathbf{EP}^*}{\partial\Delta\sigma}<0$, i.e., decreasing market opacity increases equity premium.

On the left side of the threshold, transparent traders hold moderate long positions of risky asset, and opaque traders sell out a fraction of risky asset. Precisely,

$$\begin{aligned}D_O^* &= \frac{1}{1-\rho^2+\rho^2\mu^*}\left[1-(1-\mu^*)\frac{\rho m}{\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}\right] \\ &> \frac{1}{1-\rho^2+\rho^2\mu^*}\left[1-(1-\mu^*)\frac{\rho^2}{1-\rho^2}(\hat{\sigma}_\eta+\Delta\sigma)^2\left(\frac{(1-\rho^2)+\rho^2\mu^*}{(\hat{\sigma}_\eta-\Delta\sigma)^2}+\frac{2(1-\rho^2)+\rho^2\mu^*}{(\hat{\sigma}_\eta+\Delta\sigma)^2}\right)\right] \\ &= \left(\frac{1-2\rho^2+\rho^2\mu^*}{(\hat{\sigma}_\eta+\Delta\sigma)^2}-\frac{\rho^2(1-\mu^*)}{(\hat{\sigma}_\eta-\Delta\sigma)^2}\right)\frac{(\hat{\sigma}_\eta+\Delta\sigma)^2}{1-\rho^2}=1-\frac{\rho^2(1-\mu^*)}{1-\rho^2}\left(1+\frac{(\hat{\sigma}_\eta+\Delta\sigma)^2}{(\hat{\sigma}_\eta-\Delta\sigma)^2}\right)\end{aligned}$$

from Theorem1, then $1-\frac{\rho^2(1-\mu^*)}{1-\rho^2}\left(1+\frac{(\hat{\sigma}_\eta+\Delta\sigma)^2}{(\hat{\sigma}_\eta-\Delta\sigma)^2}\right)<D_O^*\leqslant 1$, opaque traders sell small positions of risky asset. In this case, transparent traders have less incentive to acquire information, thus, $\frac{\partial\mathbf{EP}^*}{\partial\Delta\sigma}>0$. Reducing market opacity induces decreases of equity premium, and increases of price of risky asset.

While on the right side of the threshold, transparent traders hold huge long positions of risky asset. From Remark 3 of Theorem 1, opaque traders sell big positions of risky asset, more precisely,

$$D_O^* < 1 - \frac{\rho^2(1-\mu^*)}{1-\rho^2}\left(1 + \frac{(\hat{\sigma}_\eta + \Delta\sigma)^2}{(\hat{\sigma}_\eta - \Delta\sigma)^2}\right)$$

In this case, in order to hedge their investment in risky asset, transparent traders have greater incentive to acquire information to enlarge their investment opportunities, thus $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} < 0$. Reducing market opacity induces increases of equity premium, and decreases of price of risky asset.

Summarizing our analysis, we have the following conclusions:

[Case 1] Transparent traders hold short positions of risky asset. For $\frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}{\mu^*}$, then $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} > 0$.

[Case 2] Transparent traders do not trade risky asset. For $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}{\mu^*} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}{\mu^*}$, then $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} > 0$.

[Case 3] Transparent traders hold long positions of risky asset.

[Case 3.1] If $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}{\mu^*} < \frac{\rho m}{1-\rho^2} < 0$, then $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} \gtreqless 0$ if and only if $\frac{\rho m}{1-\rho^2} \lesseqgtr -M_1^{EP}(\mu^*, \Delta\sigma)$.

[Case 3.2] If $0 \leqslant \frac{\rho m}{1-\rho^2}$, then we have the following conclusions:

[Case 3.2.1] If $0 \leqslant \rho m \leqslant \rho^2\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)$, then $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} < 0$.

[Case 3.2.2] If $\rho^2\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma) < \rho m < \rho^2\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma)$, then $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} < 0$.

[Case 3.2.3] If $\rho^2\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma) \leqslant \rho m$, then $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} \lesseqgtr 0$ if and only if $\rho m \gtreqless M_2^{BF}(\mu^*, \Delta\sigma)$.

Proposition 8. *Fix information acquisition cost c. We have the relation between equity premium and amount of ambiguity.*

[1]If $\frac{\rho m}{1-\rho^2} < -M_1^{EP}(\mu^*, \Delta\sigma)$, *then* $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} > 0$.
[2]If $-M_1^{EP}(\mu^*, \Delta\sigma) < \frac{\rho m}{1-\rho^2} < \frac{\rho^2\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}{1-\rho^2}$, *then* $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} < 0$.
[3]If $\rho^2\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma) \leqslant \rho m < M_2^{EP}(\mu^*, \Delta\sigma)$, *then* $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} > 0$.
[4]If $M_2^{EP}(\mu^*, \Delta\sigma) < \rho m$, *then* $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} < 0$.

From the above analysis, we find there are two thresholds of $\frac{\rho m}{1-\rho^2}$ and four intervals for the analysis of $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma}$:

(1) When $\frac{\rho m}{1-\rho^2} < -M_1^{EP}(\mu^*, \Delta\sigma)$, then transparent traders hold short, no, or very small long positions of risky asset. More precisely,

$$\begin{aligned} D_O^* &= \frac{1}{1-\rho^2+\rho^2\mu^*}\left[1-(1-\mu^*)\frac{\rho m}{\sigma_\varepsilon(\hat\sigma_\eta-\Delta\sigma)}\right] \\ &> \frac{1}{1-\rho^2+\rho^2\mu^*}\left[1+(1-\mu^*)\frac{(1-\rho^2)\frac{\rho^2}{(\hat\sigma_\eta-\Delta\sigma)^3}}{\frac{(1-\rho^2)+\rho^2\mu^*}{(\hat\sigma_\eta+\Delta\sigma)^3}+\frac{\rho^2\mu^*}{(\hat\sigma_\eta-\Delta\sigma)^3}}\right] \\ &= \frac{\frac{1}{(\hat\sigma_\eta+\Delta\sigma)^3}+\frac{\rho^2}{(\hat\sigma_\eta-\Delta\sigma)^3}}{\frac{(1-\rho^2)+\rho^2\mu^*}{(\hat\sigma_\eta+\Delta\sigma)^3}+\frac{\rho^2\mu^*}{(\hat\sigma_\eta-\Delta\sigma)^3}} > \frac{1}{1-\rho^2+\rho^2\mu^*}, \end{aligned}$$

opaque traders buy in big positions of risky asset. In this case, $\frac{\partial \mathbf{EP}^*}{\partial\Delta\sigma} > 0$. Reducing market opacity induces decreases of equity premium, and the price of risky asset deviates less from its value.

(2) When $-M_1^{EP}(\mu^*, \Delta\sigma) < \frac{\rho m}{1-\rho^2} < \frac{\rho^2\sigma_\varepsilon(\hat\sigma_\eta+\Delta\sigma)}{1-\rho^2}$, then transparent traders hold small long positions of risky asset, equivalently,

$$1 < D_O^* < \frac{\frac{1}{(\hat\sigma_\eta+\Delta\sigma)^3}+\frac{\rho^2}{(\hat\sigma_\eta-\Delta\sigma)^3}}{\frac{(1-\rho^2)+\rho^2\mu^*}{(\hat\sigma_\eta+\Delta\sigma)^3}+\frac{\rho^2\mu^*}{(\hat\sigma_\eta-\Delta\sigma)^3}},$$

opaque traders purchase small positions of risky asset. In this case, $\frac{\partial \mathbf{EP}^*}{\partial\Delta\sigma} < 0$. Reducing market opacity induces increases of equity premium, and decreases of price of risky asset.

(3) When $\rho^2\sigma_\varepsilon(\hat\sigma_\eta+\Delta\sigma) \leqslant \rho m < M_2^{EP}(\mu^*, \Delta\sigma)$, then transparent traders hold big long positions of risky asset, equivalently,

$$\begin{aligned} D_O^* &= \frac{1}{1-\rho^2+\rho^2\mu^*}\left[1-(1-\mu^*)\frac{\rho m}{\sigma_\varepsilon(\hat\sigma_\eta+\Delta\sigma)}\right] \\ &> \frac{1}{1-\rho^2+\rho^2\mu^*}\left[1-(1-\mu^*)\frac{\rho^2}{1-\rho^2}(\hat\sigma_\eta+\Delta\sigma)^2\left(\frac{(1-\rho^2)+\rho^2\mu^*}{(\hat\sigma_\eta-\Delta\sigma)^2}+\frac{2(1-\rho^2)+\rho^2\mu^*}{(\hat\sigma_\eta+\Delta\sigma)^2}\right)\right] \\ &= \left(\frac{1-2\rho^2+\rho^2\mu^*}{(\hat\sigma_\eta+\Delta\sigma)^2}-\frac{\rho^2(1-\mu^*)}{(\hat\sigma_\eta-\Delta\sigma)^2}\right)\frac{(\hat\sigma_\eta+\Delta\sigma)^2}{1-\rho^2} = 1-\frac{\rho^2(1-\mu^*)}{1-\rho^2}\left(1+\frac{(\hat\sigma_\eta+\Delta\sigma)^2}{(\hat\sigma_\eta-\Delta\sigma)^2}\right), \end{aligned}$$

then $1-\frac{\rho^2(1-\mu^*)}{1-\rho^2}\left(1+\frac{(\hat\sigma_\eta+\Delta\sigma)^2}{(\hat\sigma_\eta-\Delta\sigma)^2}\right) < D_O^* \leqslant 1$, opaque traders sell small positions of risky asset. In this case, $\frac{\partial \mathbf{EP}^*}{\partial\Delta\sigma} > 0$. Reducing market opacity induces decreases of equity premium, and increases of price of risky asset.

(4) When $M_2^{EP}(\mu^*, \Delta\sigma) < \rho m$, then transparent traders hold huge long positions of risky asset, more precisely,

$$D_O^* < 1-\frac{\rho^2(1-\mu^*)}{1-\rho^2}\left(1+\frac{(\hat\sigma_\eta+\Delta\sigma)^2}{(\hat\sigma_\eta-\Delta\sigma)^2}\right),$$

opaque traders sell big positions of risky asset. In this case, $\frac{\partial \mathbf{EP}^*}{\partial \Delta\sigma} < 0$. Reducing market opacity induces increases of equity premium, and decreases of price of risky asset.

5.2.3. Welfare Function

We now explore how welfare function depends upon market opacity via fraction of opaque traders.

Case 1. Transparent traders hold short positions of risky asset. For $\frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}{\mu^*}$, then

$$f(\sigma_\eta) = \rho m \frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{1-\rho^2}{(1-\rho^2)+\rho^2\mu}\left[\sigma_\varepsilon^2 - (1-\mu)\rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta+\Delta\sigma}\right]$$

and

$$\underline{f}^* = \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \frac{1-\rho^2}{\mu^*+(1-\rho^2)(1-\mu^*)}\left[\sigma_\varepsilon^2 - (1-\mu^*)\rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta+\Delta\sigma}\right]$$
$$\overline{f}^* = \frac{(1-\rho^2)\sigma_\varepsilon^2}{(1-\rho^2)+\rho^2\mu^*} + \frac{\mu^*}{(1-\rho^2)+\rho^2\mu^*}\rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta+\Delta\sigma},$$

then, from (35),

$$\begin{aligned}
&\frac{\partial \overline{f}^*}{\partial \Delta\sigma} \\
&= \frac{1}{[(1-\rho^2)+\rho^2\mu^*]^2}\left\{-(1-\rho^2)\left(\rho^2\sigma_\varepsilon^2 - \rho m\frac{\sigma_\varepsilon}{\hat{\sigma}_\eta+\Delta\sigma}\right)\frac{\partial\mu^*}{\partial\Delta\sigma} - [(1-\rho^2)+\rho^2\mu^*]\mu^*\rho m\frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta+\Delta\sigma)^2}\right\} \\
&= \frac{1}{(1-\rho^2)+\rho^2\mu^*}\left\{-\frac{1-\rho^2}{\rho^2}\frac{\rho m\sigma_\varepsilon}{(\hat{\sigma}_\eta+\Delta\sigma)^2} - \mu^*\rho m\frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta+\Delta\sigma)^2}\right\} = -\frac{1}{\rho^2}\frac{\rho m\sigma_\varepsilon}{(\hat{\sigma}_\eta+\Delta\sigma)^2} > 0
\end{aligned}$$

$$\frac{\partial \mathbf{WEL}^*}{\partial \Delta\sigma} = \left[-1 + \frac{\overline{f}^*}{\sigma_\varepsilon^2}\right]\frac{\partial \overline{f}^*}{\partial \Delta\sigma} = -\frac{\mu^*}{(1-\rho^2)+\rho^2\mu^*}\left[\rho^2 - \frac{\rho m}{\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}\right]\frac{\partial \overline{f}^*}{\partial \Delta\sigma} < 0.$$

Thus, for this case, reducing market opacity, i.e., $\Delta\sigma$ decreases, renders equity premium deviates less from its fundamental value, and improves traders' welfare ex ante. When this positive effect is strong enough, traders are made better off by more stringent disclosure. Totally, disclosure is welfare-improving.

Case 2. Transparent traders do not trade risky asset. For $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta+\Delta\sigma)}{\mu^*} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta-\Delta\sigma)}{\mu^*}$, then

$$f(\sigma_\eta) = \rho m \frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu}$$

and

$$\underline{f}^* = \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta - \Delta\sigma} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu^*} \leqslant 0 \qquad \text{and} \qquad \overline{f}^* = \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta + \Delta\sigma} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu^*} \geqslant 0.$$

Hence, from (36),

$$\begin{aligned}\frac{\partial \overline{f}^*}{\partial \Delta\sigma} &= -\rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta + \Delta\sigma)^2} - \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu^{*2}} \frac{\partial \mu^*}{\partial \Delta\sigma} \\ &= -\rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta + \Delta\sigma)^2} + \mu^* \frac{m^2}{(\hat{\sigma}_\eta + \Delta\sigma)^3} > 0\end{aligned}$$

$$\frac{\partial \mathbf{WEL}^*}{\partial \Delta\sigma} = -\frac{\partial \overline{f}^*}{\partial \Delta\sigma} < 0.$$

For this case, reducing market opacity improves traders' welfare ex ante. Traders are made better off by more stringent disclosure. Therefore, disclosure is welfare-improving.

Case 3. Transparent traders hold long positions of risky asset. For $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)}{\mu^*} < \frac{\rho m}{1-\rho^2}$, we consider the following two cases.

Case 3.1. If $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)}{\mu^*} < \frac{\rho m}{1-\rho^2} < 0$, then

$$f(\sigma_\eta) = \rho m \frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{1-\rho^2}{\mu + (1-\rho^2)(1-\mu)} \left[\sigma_\varepsilon^2 - (1-\mu)\rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta - \Delta\sigma}\right]$$

and

$$\begin{aligned}\underline{f}^* &= \frac{\rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta - \Delta\sigma} \mu^* + (1-\rho^2)\sigma_\varepsilon^2}{(1-\rho^2) + \rho^2\mu^*} = \frac{(1-\rho^2)\sigma_\varepsilon^2}{(1-\rho^2)+\rho^2\mu^*} + \frac{\mu^*}{(1-\rho^2)+\rho^2\mu^*} \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta - \Delta\sigma} \\ \overline{f}^* &= \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta + \Delta\sigma} + \frac{1-\rho^2}{(1-\rho^2)+\rho^2\mu^*} \left[\sigma_\varepsilon^2 - (1-\mu^*)\rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta - \Delta\sigma}\right] \\ &= \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta + \Delta\sigma} - \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta - \Delta\sigma} + \underline{f}.\end{aligned}$$

Hence, from (37),

$$\begin{aligned}&\frac{\partial \underline{f}^*}{\partial \Delta\sigma} \\ &= \frac{1}{[(1-\rho^2)+\rho^2\mu^*]^2} \left\{-(1-\rho^2)\left(\rho^2\sigma_\varepsilon^2 - \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta - \Delta\sigma}\right) \frac{\partial \mu^*}{\partial \Delta\sigma} + [(1-\rho^2)+\rho^2\mu^*]\mu^* \rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta - \Delta\sigma)^2}\right\} \\ &= \frac{\sigma_\varepsilon}{\rho} \frac{\rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta - \Delta\sigma)^2} + m^2 \left(\frac{(1-\rho^2)+\rho^2\mu^*}{(\hat{\sigma}_\eta + \Delta\sigma)^3} - \frac{\rho^2(1-\mu^*)}{(\hat{\sigma}_\eta - \Delta\sigma)^3}\right)}{\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta - \Delta\sigma}}\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathbf{WEL}^*}{\partial \Delta\sigma} &= \rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta + \Delta\sigma)^2} + \rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta - \Delta\sigma)^2} + \left[-1 + \frac{\underline{f}^*}{\sigma_\varepsilon^2}\right] \frac{\partial \underline{f}^*}{\partial \Delta\sigma} \\ &= \frac{\rho m \sigma_\varepsilon}{(\hat{\sigma}_\eta + \Delta\sigma)^2} - \frac{\mu^* m^2}{(\hat{\sigma}_\eta + \Delta\sigma)^3} \\ &+ \rho m \frac{(1-\rho^2)(1-\mu^*)}{(1-\rho^2)+\rho^2\mu^*} \left(\frac{\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)}{\mu^*} + \frac{\rho m}{1-\rho^2}\right) \frac{\mu^*}{(\hat{\sigma}_\eta - \Delta\sigma)^3} < 0.\end{aligned}$$

For this case, reducing market opacity improves traders' welfare ex ante. When this positive effect is strong enough to dominate the potential effect of the increased equity premium, traders are actually made better off by more stringent disclosure. Totally, disclosure is welfare-improving.

Case 3.2. If $0 \leqslant \frac{\rho m}{1-\rho^2}$, then

$$f(\sigma_\eta) = \rho m \frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{1-\rho^2}{(1-\rho^2)+\rho^2\mu} \left[\sigma_\varepsilon^2 - (1-\mu)\rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta + \Delta\sigma}\right]$$

and

$$\begin{aligned}\underline{f}^* &= \frac{\rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta + \Delta\sigma}\mu^* + (1-\rho^2)\sigma_\varepsilon^2}{(1-\rho^2)+\rho^2\mu^*} = \frac{(1-\rho^2)\sigma_\varepsilon^2}{(1-\rho^2)+\rho^2\mu^*} + \frac{\mu^*}{(1-\rho^2)+\rho^2\mu^*}\rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta + \Delta\sigma} \\ \overline{f}^* &= \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta - \Delta\sigma} + \frac{1-\rho^2}{(1-\rho^2)+\rho^2\mu^*} \left[\sigma_\varepsilon^2 - (1-\mu^*)\rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta + \Delta\sigma}\right] \\ &= \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta - \Delta\sigma} - \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta + \Delta\sigma} + \underline{f}^*.\end{aligned}$$

Thus

$$\frac{\partial \underline{f}^*}{\partial \Delta\sigma} = \frac{1}{[(1-\rho^2)+\rho^2\mu^*]^2} \left\{-(1-\rho^2)\left(\rho^2\sigma_\varepsilon^2 - \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta + \Delta\sigma}\right)\frac{\partial \mu^*}{\partial \Delta\sigma} - [(1-\rho^2)+\rho^2\mu^*]\mu^* \rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta + \Delta\sigma)^2}\right\}$$

and

$$\begin{aligned}\frac{\partial \mathbf{WEL}^*}{\partial \Delta\sigma} &= -\rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta - \Delta\sigma)^2} - \rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta + \Delta\sigma)^2} + \left[-1 + \frac{f^*_{min}}{\sigma_\varepsilon^2}\right] \frac{\partial \underline{f}^*}{\partial \Delta\sigma} \\ &= -\rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta - \Delta\sigma)^2} - \rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta + \Delta\sigma)^2} - \frac{\mu^*}{(1-\rho^2)+\rho^2\mu^*}\left[\rho^2 - \frac{\rho m}{\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma)}\right] \frac{\partial \underline{f}^*}{\partial \Delta\sigma}\end{aligned}$$

Case 3.2.1. $0 \leqslant \rho m \leqslant \rho^2 \sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)$, then, from (38),

$$\frac{\partial \underline{f}^*}{\partial \Delta\sigma} = -\frac{\sigma_\varepsilon}{\rho^2}\left\{\frac{\rho m}{(\hat{\sigma}_\eta + \Delta\sigma)^2} + \frac{\rho^2[(1-\rho^2)+\rho^2\mu^*]\left(\frac{m^2}{(\hat{\sigma}_\eta - \Delta\sigma)^3} + \frac{m^2}{(\hat{\sigma}_\eta + \Delta\sigma)^3}\right)}{\rho\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta + \Delta\sigma}\right)}\right\} < 0$$

and

$$\begin{aligned}
&\frac{\partial \mathbf{WEL}^*}{\partial \Delta\sigma} \\
= & -\left(\rho\sigma_\varepsilon - \mu^* \frac{m}{\hat{\sigma}_\eta - \Delta\sigma}\right) \frac{m}{(\hat{\sigma}_\eta - \Delta\sigma)^2} - \left(\rho\sigma_\varepsilon - \mu^* \frac{m}{\hat{\sigma}_\eta + \Delta\sigma}\right) \frac{m}{(\hat{\sigma}_\eta + \Delta\sigma)^2} \\
+ & \frac{\mu^*}{(1-\rho^2) + \rho^2\mu^*} \left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta + \Delta\sigma}\right) \frac{m}{(\hat{\sigma}_\eta + \Delta\sigma)^2} \\
\leqslant & -\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta - \Delta\sigma}\right) \frac{m}{(\hat{\sigma}_\eta - \Delta\sigma)^2} - \left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta + \Delta\sigma}\right) \frac{m}{(\hat{\sigma}_\eta + \Delta\sigma)^2} \\
+ & \frac{\mu^*}{(1-\rho^2) + \rho^2\mu^*} \left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta + \Delta\sigma}\right) \frac{m}{(\hat{\sigma}_\eta + \Delta\sigma)^2} \\
= & -\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta - \Delta\sigma}\right) \frac{m}{(\hat{\sigma}_\eta - \Delta\sigma)^2} - \frac{(1-\rho^2)(1-\mu^*)}{(1-\rho^2) + \rho^2\mu^*} \left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta + \Delta\sigma}\right) \frac{m}{(\hat{\sigma}_\eta + \Delta\sigma)^2} < 0.
\end{aligned}$$

This case is very similar to Case 3.1, disclosure is welfare-improving.

Case 3.2.2. $\rho^2\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma) < \rho m < \rho^2\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma)$, then, from (39),

$$\frac{\partial \underline{f}^*}{\partial \Delta\sigma} = -\frac{\rho m \sigma_\varepsilon}{\rho^2} \left\{ \frac{1}{(\hat{\sigma}_\eta + \Delta\sigma)^2} + \frac{[(1-\rho^2) + \rho^2\mu^*]\left(\frac{\rho^2\sigma_\varepsilon}{(\hat{\sigma}_\eta - \Delta\sigma)^2} + \frac{\rho m}{(\hat{\sigma}_\eta + \Delta\sigma)^3}\right)}{\rho\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta + \Delta\sigma}\right)} \right\} < 0$$

and

$$\begin{aligned}
&\frac{\partial \mathbf{WEL}^*}{\partial \Delta\sigma} \\
= & -(1-\mu^*)\rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta - \Delta\sigma)^2} - \left(\rho\sigma_\varepsilon - \mu^* \frac{m}{\hat{\sigma}_\eta + \Delta\sigma}\right) \frac{m}{(\hat{\sigma}_\eta + \Delta\sigma)^2} \\
+ & \frac{\mu^*}{(1-\rho^2) + \rho^2\mu^*} \left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta + \Delta\sigma}\right) \frac{m}{(\hat{\sigma}_\eta + \Delta\sigma)^2} \\
\leqslant & -(1-\mu^*)\rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta - \Delta\sigma)^2} - \left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta + \Delta\sigma}\right) \frac{m}{(\hat{\sigma}_\eta + \Delta\sigma)^2} \\
+ & \frac{\mu^*}{(1-\rho^2) + \rho^2\mu^*} \left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta + \Delta\sigma}\right) \frac{m}{(\hat{\sigma}_\eta + \Delta\sigma)^2} \\
= & -(1-\mu^*)\rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta - \Delta\sigma)^2} - \frac{(1-\rho^2)(1-\mu^*)}{(1-\rho^2) + \rho^2\mu^*} \left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta + \Delta\sigma}\right) \frac{m}{(\hat{\sigma}_\eta + \Delta\sigma)^2} < 0.
\end{aligned}$$

This case is very similar to Case 3.1, disclosure is welfare-improving.

Case 3.2.3. $\rho^2\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma) \leqslant \rho m$, then, from (40),

$$\frac{\partial \underline{f}^*}{\partial \Delta\sigma} = -\frac{\rho m \sigma_\varepsilon}{\rho^2} \left\{ \frac{1}{(\hat{\sigma}_\eta + \Delta\sigma)^2} + \frac{[(1-\rho^2) + \rho^2\mu^*]\left(\frac{\rho^2\sigma_\varepsilon}{(\hat{\sigma}_\eta - \Delta\sigma)^2} + \frac{\rho^2\sigma_\varepsilon}{(\hat{\sigma}_\eta + \Delta\sigma)^2}\right)}{\rho\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta + \Delta\sigma}\right)} \right\} < 0$$

and

$$\frac{\partial \mathbf{WEL}^*}{\partial \Delta\sigma} = -(1-\mu^*)\rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta - \Delta\sigma)^2} - (1-\mu^*)\rho m \frac{\sigma_\varepsilon}{(\hat{\sigma}_\eta + \Delta\sigma)^2} + \frac{\mu^*}{(1-\rho^2)+\rho^2\mu^*}\left(\rho\sigma_\varepsilon - \frac{m}{\hat{\sigma}_\eta + \Delta\sigma}\right)\frac{m}{(\hat{\sigma}_\eta + \Delta\sigma)^2} < 0.$$

For this case, reducing market opacity improves traders' welfare ex ante. When this positive effect is strong enough to dominate the potential effect of the increased equity premium, traders are actually made better off by more stringent disclosure. Still in total, disclosure is welfare-improving.

Our analysis reports that market opacity does matter, affecting market equilibrium and the aggregate social welfare. In total, there is a consistent conclusion: a lower market opacity may consistently increase the aggregate social welfare, i.e., disclosure is welfare-improving.

PROPOSITION 9. *Fix information acquisition cost c. Suppose* $0 < \mu^* < 1$, *then a decrease in* $\Delta\sigma$ *will consistently increase welfare,* $\frac{\partial \mathbf{WEL}^*}{\partial \Delta\sigma} < 0$.

6. CONCLUSION

This paper investigates the effect of ambiguity about variance of extra investment opportunities on asset pricing. Due to incomplete knowledge about the extra investment opportunities and ambiguity aversion, transparent traders make decisions on MEU by Gilboa and Smeidler (1989). Their demand functions demonstrate piecewise linear (continuous but not smooth) according to different equity premium level, that is, transparent traders may hold short, zero, or long positions of risky asset. While, traders may also choose to pay for information acquisition to become opaque, so as to have enlarged investment opportunities to hedge their risky asset. Opaque traders are risk averse and make decisions by standard expected utility theory. Corresponding to transparent traders' demand functions, there exists one unique of the three different types of rational expectations equilibria. Our analysis shows that if only opaque traders have positive net wealth, no matter at which equilibrium type of the economy, they can always outperform the market. From a regulatory point, it is inappropriate that the informed traders can always beat the market to gain positive abnormal return. We analyze the implication of regulations increasing the information acquisition cost and reducing market opacity to investor behavior, asset pricing and the aggregate social welfare.

Our analysis results demonstrate that the implication of increasing the information acquisition cost will decrease the fraction of opaque traders, increase the equity premium, and decrease the aggregate social welfare uniformly. Thus, regulatory policies designed to limit the number of informed traders by increasing information acquisition costs seem ill advised. On the contrary, decreasing such costs to induce more informed traders seems a good policy. Regulations for stringent disclosure requirements of the issuing firm and trading information to render a lower market opacity, may induce more opaque traders thus lower equity premium, or less opaque traders thus higher equity premium for different cases, however, there is a consistent conclusion that a lower market opacity will increase the aggregate social welfare.

Thus, from the regulatory point, in order to restore financial market efficient allocation function, it is necessary to decrease information acquisition cost and reduce market opacity, so as to mitigate information asymmetry to induce more participation and restore market efficiency.

APPENDIX

Appendix A1: Proposition 2 on Beta and Alpha of Opaque Traders' Portfolio

From definition we have

$$\begin{aligned}
&\mathbf{Cov}_{\hat{\sigma}_\eta}\left(\tilde{R}_O, \tilde{R}_M \middle| \tilde{p}\right) \\
=\ &\mathbf{Cov}_{\hat{\sigma}_\eta}\left(\frac{1}{\tilde{p}-c}\left\{\frac{\tilde{\varepsilon}+f(\hat{\sigma}_\eta)}{1-\rho^2}\left[\frac{f(\hat{\sigma}_\eta)}{\sigma_\varepsilon^2}-\frac{\rho m}{\sigma_\varepsilon\hat{\sigma}_\eta}\right]+\frac{\tilde{\eta}}{1-\rho^2}\left[\frac{m}{\hat{\sigma}_\eta^2}-\frac{\rho f(\hat{\sigma}_\eta)}{\sigma_\varepsilon\hat{\sigma}_\eta}\right]\right\}, \frac{\tilde{\varepsilon}+f(\hat{\sigma}_\eta)}{\tilde{p}}\middle|\tilde{p}\right) \\
=\ &\frac{1}{1-\rho^2}\frac{1}{\tilde{p}(\tilde{p}-c)}\left\{\left[\frac{f(\hat{\sigma}_\eta)}{\sigma_\varepsilon^2}-\frac{\rho m}{\sigma_\varepsilon\hat{\sigma}_\eta}\right]\mathbf{Cov}_{\hat{\sigma}_\eta}\left(\tilde{\varepsilon},\tilde{\varepsilon}\middle|\tilde{p}\right)+\left[\frac{m}{\hat{\sigma}_\eta^2}-\frac{\rho f(\hat{\sigma}_\eta)}{\sigma_\varepsilon\hat{\sigma}_\eta}\right]\mathbf{Cov}_{\hat{\sigma}_\eta}\left(\tilde{\eta},\tilde{\varepsilon}\middle|\tilde{p}\right)\right\}
\end{aligned}$$

Since the price defined in Equation (4) only contains information about $\tilde{\theta}_T$ and $\tilde{\theta}_O$, $\mathbf{Cov}_{\hat{\sigma}_\eta}(\tilde{\varepsilon},\tilde{\varepsilon}) = \sigma_\varepsilon^2$ and $\mathbf{Cov}_{\hat{\sigma}_\eta}(\tilde{\eta},\tilde{\varepsilon}) = \rho\sigma_\varepsilon\hat{\sigma}_\eta$, then

$$\mathbf{Cov}_{\hat{\sigma}_\eta}(\tilde{R}_O, \tilde{R}_M|\tilde{p}) = \frac{f(\hat{\sigma}_\eta)}{\tilde{p}(\tilde{p}-c)}.$$

As we know,

$$\mathbf{Var}_{\hat{\sigma}_\eta}(\tilde{R}_M \middle| \tilde{p}) = \mathbf{Cov}_{\hat{\sigma}_\eta}\left(\frac{\tilde{\varepsilon}+f(\hat{\sigma}_\eta)}{\tilde{p}}\middle|\tilde{p}\right) = \frac{\sigma_\varepsilon^2}{\tilde{p}^2}.$$

Then

$$\beta_O \equiv \frac{\mathbf{Cov}_{\hat{\sigma}_\eta}(\tilde{R}_O, \tilde{R}_M|\tilde{p})}{\mathbf{Var}_{\hat{\sigma}_\eta}(\tilde{R}_M\big|\tilde{p})} = \frac{\frac{f(\hat{\sigma}_\eta)}{\tilde{p}(\tilde{p}-c)}}{\frac{\sigma_\varepsilon^2}{\tilde{p}^2}} = \frac{\tilde{p}}{\tilde{p}-c}\frac{f(\hat{\sigma}_\eta)}{\sigma_\varepsilon^2} = \frac{\tilde{p}}{\tilde{p}-c}\frac{\mathbf{EP}}{\sigma_\varepsilon^2}.$$

On the other hand,

$$\begin{aligned}
&\mathbf{E}_{\hat{\sigma}_\eta}[\tilde{R}_O|\tilde{p}] \\
=\ & \mathbf{E}_{\hat{\sigma}_\eta}\left[\frac{1}{\tilde{p}-c}\left\{\frac{\tilde{\varepsilon}+f(\hat{\sigma}_\eta)}{1-\rho^2}\left[\frac{f(\hat{\sigma}_\eta)}{\sigma_\varepsilon^2}-\frac{\rho m}{\sigma_\varepsilon\hat{\sigma}_\eta}\right]+\frac{\tilde{\eta}}{1-\rho^2}\left[\frac{m}{\hat{\sigma}_\eta^2}-\frac{\rho f(\hat{\sigma}_\eta)}{\sigma_\varepsilon\hat{\sigma}_\eta}\right]\right\}\Bigg|\tilde{p}\right] \\
=\ & \frac{1}{\tilde{p}-c}\left\{\frac{f(\hat{\sigma}_\eta)}{1-\rho^2}\left[\frac{f(\hat{\sigma}_\eta)}{\sigma_\varepsilon^2}-\frac{\rho m}{\sigma_\varepsilon\hat{\sigma}_\eta}\right]+\frac{m}{1-\rho^2}\left[\frac{m}{\hat{\sigma}_\eta^2}-\frac{\rho f(\hat{\sigma}_\eta)}{\sigma_\varepsilon\hat{\sigma}_\eta}\right]\right\} \\
=\ & \frac{1}{1-\rho^2}\frac{1}{\tilde{p}-c}\left(\frac{f^2(\hat{\sigma}_\eta)}{\sigma_\varepsilon^2}-\frac{2\rho m f(\hat{\sigma}_\eta)}{\sigma_\varepsilon\hat{\sigma}_\eta}+\frac{m^2}{\hat{\sigma}_\eta^2}\right)
\end{aligned}$$

and

$$\mathbf{E}_{\hat{\sigma}_\eta}[\tilde{R}_M|\tilde{p}] = \mathbf{E}_{\hat{\sigma}_\eta}\left[\frac{\tilde{\varepsilon}+f(\hat{\sigma}_\eta)}{\tilde{p}}\Bigg|\tilde{p}\right] = \frac{f(\hat{\sigma}_\eta)}{\tilde{p}}.$$

Thus

$$\begin{aligned}
\alpha_O &\equiv \mathbf{E}_{\hat{\sigma}_\eta}[\tilde{R}_O|\tilde{p}] - \beta_O\mathbf{E}_{\hat{\sigma}_\eta}[\tilde{R}_M|\tilde{p}] \\
&= \frac{1}{1-\rho^2}\frac{1}{\tilde{p}-c}\left(\frac{f^2(\hat{\sigma}_\eta)}{\sigma_\varepsilon^2}-\frac{2\rho m f(\hat{\sigma}_\eta)}{\sigma_\varepsilon\hat{\sigma}_\eta}+\frac{m^2}{\hat{\sigma}_\eta^2}\right)-\frac{\tilde{p}}{\tilde{p}-c}\frac{f(\hat{\sigma}_\eta)}{\sigma_\varepsilon^2}\frac{f(\hat{\sigma}_\eta)}{\tilde{p}} \\
&= \frac{1}{1-\rho^2}\frac{1}{\tilde{p}-c}\left(\frac{\rho^2 f^2(\hat{\sigma}_\eta)}{\sigma_\varepsilon^2}-\frac{2\rho m f(\hat{\sigma}_\eta)}{\sigma_\varepsilon\hat{\sigma}_\eta}+\frac{m^2}{\hat{\sigma}_\eta^2}\right) \\
&= \frac{1}{1-\rho^2}\frac{1}{\tilde{p}-c}\left(\frac{\rho f(\hat{\sigma}_\eta)}{\sigma_\varepsilon}-\frac{m}{\hat{\sigma}_\eta}\right)^2
\end{aligned}$$

From Theorem 1, we have $\frac{\rho f(\hat{\sigma}_\eta)}{\sigma_\varepsilon} - \frac{m}{\hat{\sigma}_\eta} \neq 0$. Therefore α_O is strictly positive as long as $\tilde{p} - c > 0$.

Appendix A2: Proposition 3 on Benefit Function

We consider the objective function in the minimization problem in Equation (28),

$$\begin{aligned} & -f(\sigma_\eta)+\frac{1}{2(1-\rho^2)}\left(\frac{f^2(\sigma_\eta)}{\sigma_\varepsilon^2}-2\rho m\frac{f(\sigma_\eta)}{\sigma_\varepsilon\sigma_\eta}+\frac{m^2}{\sigma_\eta^2}\right) \\ = \ & \frac{1}{2(1-\rho^2)}\left\{\frac{1}{\sigma_\varepsilon^2}\left[f(\sigma_\eta)-\rho m\frac{\sigma_\varepsilon}{\sigma_\eta}-(1-\rho^2)\sigma_\varepsilon^2\right]^2+(1-\rho^2)\left[\left(\rho\sigma_\varepsilon-\frac{m}{\sigma_\eta}\right)^2-\sigma_\varepsilon^2\right]\right\}. \end{aligned}$$

From the three cases in Theorem 1, $f(\sigma_\eta)-\rho m\frac{\sigma_\varepsilon}{\sigma_\eta}$ is independent of σ_η, then the benefit function in Equation (28) can be rewritten as

$$B(\mu)=\begin{cases} \frac{1}{2(1-\rho^2)\sigma_\varepsilon^2}\left[f(\sigma_\eta)-\rho m\frac{\sigma_\varepsilon}{\sigma_\eta}-(1-\rho^2)\sigma_\varepsilon^2\right]^2 & \\ +\frac{1}{2}\min\limits_{\sigma_\eta\in[\underline{\sigma}_T,\overline{\sigma}_T]}\left(\rho\sigma_\varepsilon-\frac{m}{\sigma_\eta}\right)^2-\frac{1}{2}\sigma_\varepsilon^2-\left[-\overline{f}+\frac{\overline{f}^2}{2\sigma_\varepsilon^2}\right], & \text{if } \overline{f}<0 \\ \frac{1}{2(1-\rho^2)\sigma_\varepsilon^2}\left[f(\sigma_\eta)-\rho m\frac{\sigma_\varepsilon}{\sigma_\eta}-(1-\rho^2)\sigma_\varepsilon^2\right]^2 & \\ +\frac{1}{2}\min\limits_{\sigma_\eta\in[\underline{\sigma}_T,\overline{\sigma}_T]}\left(\rho\sigma_\varepsilon-\frac{m}{\sigma_\eta}\right)^2-\frac{1}{2}\sigma_\varepsilon^2-\left[-\overline{f}\right], & \text{if } \underline{f}\leqslant 0\leqslant\overline{f} \\ \frac{1}{2(1-\rho^2)\sigma_\varepsilon^2}\left[f(\sigma_\eta)-\rho m\frac{\sigma_\varepsilon}{\sigma_\eta}-(1-\rho^2)\sigma_\varepsilon^2\right]^2 & \\ +\frac{1}{2}\min\limits_{\sigma_\eta\in[\underline{\sigma}_T,\overline{\sigma}_T]}\left(\rho\sigma_\varepsilon-\frac{m}{\sigma_\eta}\right)^2-\frac{1}{2}\sigma_\varepsilon^2-\left[-\overline{f}+\frac{\underline{f}^2}{2\sigma_\varepsilon^2}\right], & \text{if } 0<\underline{f}. \end{cases} \tag{A.1}$$

We now solve the benefit function from the three cases on what position transparent traders hold risky asset in the proof process of Theorem 1.

Case 1. Transparent traders hold short position of risky asset. For $\frac{\rho m}{1-\rho^2}<-\frac{\sigma_\varepsilon\overline{\sigma}_T}{\mu}$,

$$f(\sigma_\eta)=\rho m\frac{\sigma_\varepsilon}{\sigma_\eta}+\frac{1-\rho^2}{\mu+(1-\rho^2)(1-\mu)}\left[\sigma_\varepsilon^2-(1-\mu)\rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T}\right]$$

and

$$\overline{f}=\frac{\rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T}\mu+(1-\rho^2)\sigma_\varepsilon^2}{\mu+(1-\rho^2)(1-\mu)}=\frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu+(1-\rho^2)(1-\mu)}+\frac{\mu}{\mu+(1-\rho^2)(1-\mu)}\rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T},$$

then $\min\limits_{\sigma_\eta\in[\underline{\sigma}_T,\overline{\sigma}_T]}\left(\rho\sigma_\varepsilon-\frac{m}{\sigma_\eta}\right)^2=\left(\rho\sigma_\varepsilon-\frac{m}{\overline{\sigma}_T}\right)^2$ and

$$B(\mu)=\frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon-\frac{m}{\overline{\sigma}_T}\right)\right]^2. \tag{A.2}$$

Case 2. Transparent traders do not trade risky asset. For $-\frac{\sigma_\varepsilon\overline{\sigma}_T}{\mu} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon\underline{\sigma}_T}{\mu}$,

$$f(\sigma_\eta) = \rho m\frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu}$$

and

$$\underline{f} = \rho m\frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu} \leqslant 0 \qquad \text{and} \qquad \overline{f} = \rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu} \geqslant 0,$$

then $\min\limits_{\sigma_\eta\in[\underline{\sigma}_T,\overline{\sigma}_T]}\left(\rho\sigma_\varepsilon - \frac{m}{\sigma_\eta}\right)^2 = \left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)^2$ and

$$B(\mu) = \frac{1-\rho^2}{2\mu^2}\sigma_\varepsilon^2 + \frac{1}{2}\left(\frac{m}{\overline{\sigma}_T}\right)^2. \tag{A.3}$$

Case 3. Transparent traders hold long position of risky asset. For $-\frac{\sigma_\varepsilon\underline{\sigma}_T}{\mu} < \frac{\rho m}{1-\rho^2}$,

$$f(\sigma_\eta) = \begin{cases} \rho m\frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{1-\rho^2}{\mu+(1-\rho^2)(1-\mu)}\left[\sigma_\varepsilon^2 - (1-\mu)\rho m\frac{\sigma_\varepsilon}{\underline{\sigma}_T}\right] & \text{if} \quad -\frac{\sigma_\varepsilon\underline{\sigma}_T}{\mu} < \frac{\rho m}{1-\rho^2} < 0 \\ \rho m\frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{1-\rho^2}{\mu+(1-\rho^2)(1-\mu)}\left[\sigma_\varepsilon^2 - (1-\mu)\rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T}\right] & \text{if} \quad 0 \leqslant \frac{\rho m}{1-\rho^2} \end{cases}$$

and

$$\underline{f} = \begin{cases} \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu+(1-\rho^2)(1-\mu)} + \frac{\mu}{\mu+(1-\rho^2)(1-\mu)}\rho m\frac{\sigma_\varepsilon}{\underline{\sigma}_T} & \text{if} \quad -\frac{\sigma_\varepsilon\underline{\sigma}_T}{\mu} < \frac{\rho m}{1-\rho^2} < 0 \\ \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu+(1-\rho^2)(1-\mu)} + \frac{\mu}{\mu+(1-\rho^2)(1-\mu)}\rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T} & \text{if} \quad 0 \leqslant \frac{\rho m}{1-\rho^2} \end{cases}$$

$$\overline{f} = \begin{cases} \rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T} + \frac{1-\rho^2}{\mu+(1-\rho^2)(1-\mu)}\left[\sigma_\varepsilon^2 - (1-\mu)\rho m\frac{\sigma_\varepsilon}{\underline{\sigma}_T}\right] & \\ = \rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T} - \rho m\frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \underline{f} & \text{if} \quad -\frac{\sigma_\varepsilon\underline{\sigma}_T}{\mu} < \frac{\rho m}{1-\rho^2} < 0 \\ \rho m\frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \frac{1-\rho^2}{\mu+(1-\rho^2)(1-\mu)}\left[\sigma_\varepsilon^2 - (1-\mu)\rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T}\right] & \\ = \rho m\frac{\sigma_\varepsilon}{\underline{\sigma}_T} - \rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T} + \underline{f} & \text{if} \quad 0 \leqslant \frac{\rho m}{1-\rho^2}. \end{cases}$$

Case 3.1. For $-\frac{\sigma_\varepsilon\underline{\sigma}_T}{\mu} < \frac{\rho m}{1-\rho^2} < 0$, then $\min\limits_{\sigma_\eta\in[\underline{\sigma}_T,\overline{\sigma}_T]}\left(\rho\sigma_\varepsilon - \frac{m}{\sigma_\eta}\right)^2 = \left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)^2$ and

$$B(\mu) = \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon - \frac{m}{\underline{\sigma}_T}\right)\right]^2 + \frac{1}{2}\left(\frac{m}{\overline{\sigma}_T}\right)^2 - \frac{1}{2}\left(\frac{m}{\underline{\sigma}_T}\right)^2 \tag{A.4}$$

Case 3.2. For $0 \leqslant \frac{\rho m}{1-\rho^2}$,

$$B(\mu) = \frac{\rho^2[(1-\rho^2)(1-\mu)^2 - \mu^2]}{2[\mu + (1-\rho^2)(1-\mu)]^2}\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)^2 + \frac{1}{2}\min_{\sigma_\eta \in [\underline{\sigma}_T, \overline{\sigma}_T]}\left(\rho\sigma_\varepsilon - \frac{m}{\sigma_\eta}\right)^2 + \rho m\frac{\sigma_\varepsilon}{\underline{\sigma}_T} - \rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T}.$$

Then we consider the following three cases.

Case 3.2.1. If $0 \leqslant \rho\left(\rho\sigma_\varepsilon - \frac{m}{\underline{\sigma}_T}\right)$, then $\min_{\sigma_\eta \in [\underline{\sigma}_T, \overline{\sigma}_T]}\left(\rho\sigma_\varepsilon - \frac{m}{\sigma_\eta}\right)^2 = \left(\rho\sigma_\varepsilon - \frac{m}{\underline{\sigma}_T}\right)^2$,

$$B(\mu) = \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)\right]^2 + \frac{1}{2}\left(\frac{m}{\underline{\sigma}_T}\right)^2 - \frac{1}{2}\left(\frac{m}{\overline{\sigma}_T}\right)^2 \tag{A.5}$$

Case 3.2.2. If $\rho\left(\rho\sigma_\varepsilon - \frac{m}{\underline{\sigma}_T}\right) < 0 < \rho\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)$, then $\min_{\sigma_\eta \in [\underline{\sigma}_T, \overline{\sigma}_T]}\left(\rho\sigma_\varepsilon - \frac{m}{\sigma_\eta}\right)^2 =$ 0,

$$\begin{aligned} B(\mu) &= \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)\right]^2 \\ &- \frac{1}{2}\left(\rho\sigma_\varepsilon - \frac{m}{\underline{\sigma}_T}\right)^2 + \frac{1}{2}\left(\frac{m}{\underline{\sigma}_T}\right)^2 - \frac{1}{2}\left(\frac{m}{\overline{\sigma}_T}\right)^2. \end{aligned} \tag{A.6}$$

Case 3.2.3. If $\rho\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right) \leqslant 0$, then $\min_{\sigma_\eta \in [\underline{\sigma}_T, \overline{\sigma}_T]}\left(\rho\sigma_\varepsilon - \frac{m}{\sigma_\eta}\right)^2 = \left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)^2$,

$$\begin{aligned} B(\mu) &= \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)\right]^2 \\ &+ \rho m\frac{\sigma_\varepsilon}{\underline{\sigma}_T} - \rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T}. \end{aligned} \tag{A.7}$$

We summarize the above analysis and obtain the benefit function as follows

$$B(\mu)=\begin{cases}\frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon-\frac{m}{\overline{\sigma}_T}\right)\right]^2, & \text{if}\quad \frac{\rho m}{1-\rho^2}<-\frac{\sigma_\varepsilon\overline{\sigma}_T}{\mu}\\ \frac{1-\rho^2}{2\mu^2}\sigma_\varepsilon^2+\frac{1}{2}\left(\frac{m}{\overline{\sigma}_T}\right)^2, & \text{if}\quad -\frac{\sigma_\varepsilon\overline{\sigma}_T}{\mu}\leqslant\frac{\rho m}{1-\rho^2}\leqslant-\frac{\sigma_\varepsilon\underline{\sigma}_T}{\mu}\\ \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon-\frac{m}{\underline{\sigma}_T}\right)\right]^2 & \\ +\frac{1}{2}\left(\frac{m}{\overline{\sigma}_T}\right)^2-\frac{1}{2}\left(\frac{m}{\underline{\sigma}_T}\right)^2, & \text{if}\quad -\frac{\sigma_\varepsilon\underline{\sigma}_T}{\mu}<\frac{\rho m}{1-\rho^2}<0\\ \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon-\frac{m}{\overline{\sigma}_T}\right)\right]^2 & \\ +\frac{1}{2}\left(\frac{m}{\underline{\sigma}_T}\right)^2-\frac{1}{2}\left(\frac{m}{\overline{\sigma}_T}\right)^2, & \text{if}\quad 0\leqslant\rho m\leqslant\rho^2\sigma_\varepsilon\underline{\sigma}_T\\ \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon-\frac{m}{\overline{\sigma}_T}\right)\right]^2 & \\ -\frac{1}{2}\left(\rho\sigma_\varepsilon-\frac{m}{\overline{\sigma}_T}\right)^2+\rho m\frac{\sigma_\varepsilon}{\underline{\sigma}_T}-\rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T}, & \text{if}\quad \rho^2\sigma_\varepsilon\underline{\sigma}_T<\rho m<\rho^2\sigma_\varepsilon\overline{\sigma}_T\\ \frac{1-\rho^2}{2}\left[\frac{1}{(1-\rho^2)+\rho^2\mu}\left(\rho\sigma_\varepsilon-\frac{m}{\overline{\sigma}_T}\right)\right]^2+\rho m\frac{\sigma_\varepsilon}{\underline{\sigma}_T}-\rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T}, & \text{if}\quad \rho^2\sigma_\varepsilon\overline{\sigma}_T\leqslant\rho m\end{cases}\tag{A.8}$$

Appendix A3: Proposition 4 on Size of Cost Range

Case 1. $\frac{\rho m}{1-\rho^2}<-\sigma_\varepsilon\overline{\sigma}_T$, that is, $\rho m<0$ and $(1-\rho^2)\frac{\sigma_\varepsilon\overline{\sigma}_T}{-\rho m}<1$,

$$\bar{c}=B(0+)=\frac{1}{2(1-\rho^2)}\left(\rho\sigma_\varepsilon-\frac{m}{\underline{\sigma}_T}\right)^2+\frac{1}{2}\left(\frac{m}{\overline{\sigma}_T}\right)^2-\frac{1}{2}\left(\frac{m}{\underline{\sigma}_T}\right)^2$$
$$\underline{c}=B(1-)=\frac{1-\rho^2}{2}\left(\rho\sigma_\varepsilon-\frac{m}{\overline{\sigma}_T}\right)^2$$

then

$$\bar{c}-\underline{c}=\frac{1}{2(1-\rho^2)}\left(\rho\sigma_\varepsilon-\frac{m}{\underline{\sigma}_T}\right)^2+\frac{1}{2}\left(\frac{m}{\overline{\sigma}_T}\right)^2-\frac{1}{2}\left(\frac{m}{\underline{\sigma}_T}\right)^2-\frac{1-\rho^2}{2}\left(\rho\sigma_\varepsilon-\frac{m}{\overline{\sigma}_T}\right)^2$$

and

$$\frac{\partial(\bar{c}-\underline{c})}{\partial\Delta\sigma}=-\frac{\rho m}{1-\rho^2}\frac{\sigma_\varepsilon}{\underline{\sigma}_T^2}-(1-\rho^2)\rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T^2}+\left[\frac{\rho^2}{1-\rho^2}\frac{m^2}{\underline{\sigma}_T^3}-\rho^2\frac{m^2}{\overline{\sigma}_T^3}\right]>0,$$

the size of the range $(\underline{c},\bar{c})$ increase with degree of ambiguity.

Case 2. $-\sigma_\varepsilon\overline{\sigma}_T\leqslant\frac{\rho m}{1-\rho^2}\leqslant-\sigma_\varepsilon\underline{\sigma}_T$, that is, $\rho m<0$ and $(1-\rho^2)\frac{\sigma_\varepsilon\underline{\sigma}_T}{-\rho m}\leqslant 1\leqslant(1-\rho^2)\frac{\sigma_\varepsilon\overline{\sigma}_T}{-\rho m}$.

$$\bar{c}=B(0+)=\frac{1}{2(1-\rho^2)}\left(\rho\sigma_\varepsilon-\frac{m}{\underline{\sigma}_T}\right)^2+\frac{1}{2}\left(\frac{m}{\overline{\sigma}_T}\right)^2-\frac{1}{2}\left(\frac{m}{\underline{\sigma}_T}\right)^2$$
$$\underline{c}=B(1-)=\frac{1-\rho^2}{2}\sigma_\varepsilon^2+\frac{1}{2}\left(\frac{m}{\overline{\sigma}_T}\right)^2$$

then

$$\overline{c} - \underline{c} = \frac{1}{2(1-\rho^2)}\left(\rho\sigma_\varepsilon - \frac{m}{\underline{\sigma}_T}\right)^2 - \frac{1}{2}\left(\frac{m}{\underline{\sigma}_T}\right)^2 - \frac{1-\rho^2}{2}\sigma_\varepsilon^2$$

and

$$\frac{\partial(\overline{c}-\underline{c})}{\partial\Delta\sigma} = -\frac{\rho m}{1-\rho^2}\frac{\sigma_\varepsilon}{\underline{\sigma}_T^2} + \frac{\rho^2}{1-\rho^2}\frac{m^2}{\underline{\sigma}_T^3} > 0,$$

the size of the range $(\underline{c}, \overline{c})$ increase with degree of ambiguity.

Case 3. $-\sigma_\varepsilon\underline{\sigma}_T < \frac{\rho m}{1-\rho^2} < 0$, that is, $\rho m < 0$ and $1 < (1-\rho^2)\frac{\sigma_\varepsilon\underline{\sigma}_T}{-\rho m}$.

$$\begin{aligned}\overline{c} = B(0+) &= \tfrac{1}{2(1-\rho^2)}\left(\rho\sigma_\varepsilon - \tfrac{m}{\underline{\sigma}_T}\right)^2 + \tfrac{1}{2}\left(\tfrac{m}{\overline{\sigma}_T}\right)^2 - \tfrac{1}{2}\left(\tfrac{m}{\underline{\sigma}_T}\right)^2 \\ \underline{c} = B(1-) &= \tfrac{1-\rho^2}{2}\left(\rho\sigma_\varepsilon - \tfrac{m}{\underline{\sigma}_T}\right)^2 + \tfrac{1}{2}\left(\tfrac{m}{\overline{\sigma}_T}\right)^2 - \tfrac{1}{2}\left(\tfrac{m}{\underline{\sigma}_T}\right)^2\end{aligned}$$

then

$$\overline{c} - \underline{c} = \frac{\rho^2(2-\rho^2)}{2(1-\rho^2)}\left(\rho\sigma_\varepsilon - \frac{m}{\underline{\sigma}_T}\right)^2$$

and

$$\frac{\partial(\overline{c}-\underline{c})}{\partial\Delta\sigma} = -\frac{\rho^2(2-\rho^2)}{1-\rho^2}\left(\rho m\sigma_\varepsilon - \frac{m^2}{\underline{\sigma}_T}\right)\frac{1}{\underline{\sigma}_T^2} > 0,$$

the size of the range $(\underline{c}, \overline{c})$ increase with degree of ambiguity.

Case 4. $0 \leqslant \rho m$ including three settings: $0 \leqslant \rho m \leqslant \rho^2\sigma_\varepsilon\underline{\sigma}_T$, $\rho^2\sigma_\varepsilon\underline{\sigma}_T < \rho m < \rho^2\sigma_\varepsilon\overline{\sigma}_T$, and $\rho^2\sigma_\varepsilon\overline{\sigma}_T \leqslant \rho m$.

$$\overline{c} - \underline{c} = \frac{\rho^2(2-\rho^2)}{2(1-\rho^2)}\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)^2$$

then

$$\frac{\partial(\overline{c}-\underline{c})}{\partial\Delta\sigma} = \frac{(2-\rho^2)}{1-\rho^2}\left(\rho^2\sigma_\varepsilon - \frac{\rho m}{\overline{\sigma}_T}\right)\frac{\rho m}{\overline{\sigma}_T^2}$$

[4.1] If $\rho m = 0$, then $\frac{\partial(\overline{c}-\underline{c})}{\partial\Delta\sigma} = 0$, the size of the range $(\underline{c}, \overline{c})$ is independent of degree of ambiguity.

[4.2] If $0 < \rho m < \rho^2\sigma_\varepsilon\overline{\sigma}_T$, then $\frac{\partial(\overline{c}-\underline{c})}{\partial\Delta\sigma} > 0$, the size of the range $(\underline{c}, \overline{c})$ increase with degree of ambiguity.

[4.3] If $\rho m = \rho^2 \sigma_\varepsilon \overline{\sigma}_T$, then $\frac{\partial(\overline{c}-\underline{c})}{\partial \Delta\sigma} = 0$, the size of the range $(\underline{c}, \overline{c})$ is independent of degree of ambiguity.

[4.4] If $\rho^2 \sigma_\varepsilon \overline{\sigma}_T < \rho m$, then $\frac{\partial(\overline{c}-\underline{c})}{\partial \Delta\sigma} < 0$, the size of the range $(\underline{c}, \overline{c})$ decrease with degree of ambiguity.

Appendix A4: Proposition 5 on Monotonicity of Equilibrium Equity Premium in Fraction of Opaque Traders

Case 1. Transparent traders hold short positions of risky asset. For $\frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma)}{\mu^*}$,

$$\frac{\partial \mathbf{EP}^*}{\partial \mu^*} = -\frac{1-\rho^2}{[(1-\rho^2)+\rho^2\mu^*]^2}\left[\rho^2\sigma_\varepsilon^2 - \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta + \Delta\sigma}\right] < 0$$

which means that increasing the fraction of opaque traders will decrease the equity premium. Within this type equilibrium, transparent traders are confident that the risky asset are overvalued, then they short the stock. And the opaque traders hold long positions by their optimal portfolios. As the fraction of opaque traders increases, so does the demand for risky asset, then the price becomes higher, thus the premium (negative) decreases, i.e., $\frac{\partial \mathbf{EP}^*}{\partial \mu^*} < 0$.

Case 2. Transparent traders do not trade risky asset. For $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma)}{\mu^*} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)}{\mu^*}$,

$$\frac{\partial \mathbf{EP}^*}{\partial \mu^*} = -\frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu^{*2}} < 0$$

which means that increasing the fraction of opaque traders will decrease the equity premium. Within this type equilibrium, transparent traders will not participate. Opaque traders absorb all the supply of the stock in the market, thus increasing the proportion of opaque traders will increase the demand, then increase the asset price even overvalue, therefore decrease the equity premium, i.e., $\frac{\partial \mathbf{EP}^*}{\partial \mu^*} < 0$.

Case 3. Transparent traders hold long positions of risky asset. For $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)}{\mu^*} < \frac{\rho m}{1-\rho^2}$, we consider the following two cases.

Case 3.1 If $-\frac{\sigma_\varepsilon(\hat{\sigma}_\eta - \Delta\sigma)}{\mu^*} < \frac{\rho m}{1-\rho^2} < 0$, then

$$\frac{\partial \mathbf{EP}^*}{\partial \mu^*} = -\frac{1-\rho^2}{[(1-\rho^2)+\rho^2\mu^*]^2}\left[\rho^2\sigma_\varepsilon^2 - \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta - \Delta\sigma}\right] < 0$$

which means that increasing the fraction of opaque traders will decrease the equity premium. For this setting, $1 < D_O^* < \frac{1}{\mu^*}$ from Appendix A3.

That is, compared with the initial endowed one share of stock, opaque traders will buy in and transparent traders sell out a proportion of the risky asset. Therefore, an increase in μ^* due to a decrease in c means the aggregated demand for risky asset by opaque traders becomes more and the aggregated supply by transparent traders becomes less, which will increase asset price, therefore, decrease the equity premium, i.e., $\frac{\partial \mathbf{EP}^*}{\partial \mu^*} < 0$.

Case 3.2 If $0 \leqslant \frac{\rho m}{1-\rho^2}$, then

$$\frac{\partial \mathbf{EP}^*}{\partial \mu^*} = -\frac{1-\rho^2}{[(1-\rho^2)+\rho^2\mu^*]^2}\left[\rho^2\sigma_\varepsilon^2 - \rho m \frac{\sigma_\varepsilon}{\hat{\sigma}_\eta + \Delta\sigma}\right]$$

If $\rho m < \rho^2\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma)$, then $\frac{\partial \mathbf{EP}^*}{\partial \mu^*} < 0$, which means that increasing the fraction of opaque traders will decrease the equity premium. For this setting, then $1 < D_O^* < \frac{1}{1-\rho^2+\rho^2\mu^*}$. That is, compared with the initial endowed one share of stock, opaque traders will buy in and transparent traders sell out a proportion of the risky asset. Therefore, an increase in μ^* due to a decrease in c means the aggregated demand for risky asset by opaque traders becomes more and the aggregated supply by transparent traders becomes less, which will increase asset price, therefore, decrease the equity premium, i.e., $\frac{\partial \mathbf{EP}^*}{\partial \mu^*} < 0$.

If $\rho m > \rho^2\sigma_\varepsilon(\hat{\sigma}_\eta + \Delta\sigma)$, then $\frac{\partial \mathbf{EP}^*}{\partial \mu^*} > 0$, which means that increasing the fraction of opaque traders will increase the equity premium. For this setting, then $D_O^* < 1$. That is, compared with the initial endowed one share of stock, opaque traders will sell out and transparent traders will buy in a proportion of the risky asset. Therefore, an increase in μ^* due to a decrease in c means the aggregated supply of risky asset by opaque traders becomes more and the aggregated supply by transparent traders becomes less, which will lower the asset price, therefore increase equity premium, i.e., $\frac{\partial \mathbf{EP}^*}{\partial \mu^*} > 0$.

Appendix A5: Welfare Function

We now calculate welfare function (31) according to three cases.

Case 1. Transparent traders hold short positions of risky asset. For $\frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon \overline{\sigma}_T}{\mu}$,

$$f(\sigma_\eta) = \rho m \frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{1-\rho^2}{\mu + (1-\rho^2)(1-\mu)}\left[\sigma_\varepsilon^2 - (1-\mu)\rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T}\right]$$

and hence

$$\underline{f} = \rho m\frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \frac{1-\rho^2}{\mu+(1-\rho^2)(1-\mu)}\left[\sigma_\varepsilon^2 - (1-\mu)\rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T}\right]$$

$$\overline{f} = \frac{\rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T}\mu + (1-\rho^2)\sigma_\varepsilon^2}{\mu+(1-\rho^2)(1-\mu)} = \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu+(1-\rho^2)(1-\mu)} + \frac{\mu}{\mu+(1-\rho^2)(1-\mu)}\rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T}$$

then

$$\begin{aligned}\mathbf{WEL} &= -\overline{f} + \frac{\overline{f}^2}{2\sigma_\varepsilon^2} = \frac{1}{2}\left\{\left[\sigma_\varepsilon - \frac{\overline{f}}{\sigma_\varepsilon}\right]^2 - \sigma_\varepsilon^2\right\} \\ &= \frac{1}{2}\left\{\left[\frac{\rho\mu}{\mu+(1-\rho^2)(1-\mu)}\left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T}\right)\right]^2 - \sigma_\varepsilon^2\right\}.\end{aligned}$$

Case 2. Transparent traders do not trade risky asset. For $-\frac{\sigma_\varepsilon\overline{\sigma}_T}{\mu} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon\underline{\sigma}_T}{\mu}$,

$$f(\sigma_\eta) = \rho m\frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu}$$

and hence

$$\underline{f} = \rho m\frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu} \leqslant 0 \qquad \text{and} \qquad \overline{f} = \rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu} \geqslant 0$$

then

$$\mathbf{WEL} = -\overline{f} = -\rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T} - \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu}.$$

Case 3. Transparent traders hold long positions of risky asset. For $-\frac{\sigma_\varepsilon\underline{\sigma}_T}{\mu} < \frac{\rho m}{1-\rho^2}$,

$$f(\sigma_\eta) = \begin{cases} \rho m\frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{1-\rho^2}{\mu+(1-\rho^2)(1-\mu)}\left[\sigma_\varepsilon^2 - (1-\mu)\rho m\frac{\sigma_\varepsilon}{\underline{\sigma}_T}\right], & \text{if} \quad -\frac{\sigma_\varepsilon\underline{\sigma}_T}{\mu} < \frac{\rho m}{1-\rho^2} < 0 \\ \rho m\frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{1-\rho^2}{\mu+(1-\rho^2)(1-\mu)}\left[\sigma_\varepsilon^2 - (1-\mu)\rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T}\right], & \text{if} \quad 0 \leqslant \frac{\rho m}{1-\rho^2} \end{cases}$$

and hence

$$\underline{f} = \begin{cases} \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu+(1-\rho^2)(1-\mu)} + \frac{\mu}{\mu+(1-\rho^2)(1-\mu)}\rho m\frac{\sigma_\varepsilon}{\underline{\sigma}_T}, & \text{if} \quad -\frac{\sigma_\varepsilon\underline{\sigma}_T}{\mu} < \frac{\rho m}{1-\rho^2} < 0 \\ \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu+(1-\rho^2)(1-\mu)} + \frac{\mu}{\mu+(1-\rho^2)(1-\mu)}\rho m\frac{\sigma_\varepsilon}{\overline{\sigma}_T}, & \text{if} \quad 0 \leqslant \frac{\rho m}{1-\rho^2} \end{cases}$$

$$\overline{f} = \begin{cases} \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} + \frac{1-\rho^2}{\mu+(1-\rho^2)(1-\mu)} \left[\sigma_\varepsilon^2 - (1-\mu)\rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} \right] & \\ = \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} - \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \underline{f}, & \text{if} \quad -\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu} < \frac{\rho m}{1-\rho^2} < 0 \\ \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \frac{1-\rho^2}{\mu+(1-\rho^2)(1-\mu)} \left[\sigma_\varepsilon^2 - (1-\mu)\rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} \right] & \\ = \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} - \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} + \underline{f}, & \text{if} \quad 0 \leqslant \frac{\rho m}{1-\rho^2}. \end{cases}$$

We consider the following two cases.

Case 3.1 If $-\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu} < \frac{\rho m}{1-\rho^2} < 0$, then

$$\begin{aligned} \mathbf{WEL} &= -\overline{f} + \frac{\underline{f}^2}{2\sigma_\varepsilon^2} = -\rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} + \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} - \underline{f} + \frac{\underline{f}^2}{2\sigma_\varepsilon^2} \\ &= -\rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} + \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \frac{1}{2} \left\{ \left[\sigma_\varepsilon - \frac{\underline{f}}{\sigma_\varepsilon} \right]^2 - \sigma_\varepsilon^2 \right\} \\ &= -\rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} + \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \frac{1}{2} \left\{ \left[\frac{\rho\mu}{\mu + (1-\rho^2)(1-\mu)} \left(\rho\sigma_\varepsilon - \frac{m}{\underline{\sigma}_T} \right) \right]^2 - \sigma_\varepsilon^2 \right\}. \end{aligned}$$

Case 3.2 If $0 \leqslant \frac{\rho m}{1-\rho^2}$, then

$$\begin{aligned} \mathbf{WEL} &= -\overline{f} + \frac{\underline{f}^2}{2\sigma_\varepsilon^2} = -\rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} - \underline{f} + \frac{\underline{f}^2}{2\sigma_\varepsilon^2} \\ &= -\rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} + \frac{1}{2} \left\{ \left[\sigma_\varepsilon - \frac{\underline{f}}{\sigma_\varepsilon} \right]^2 - \sigma_\varepsilon^2 \right\} \\ &= -\rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} + \frac{1}{2} \left\{ \left[\frac{\rho\mu}{\mu + (1-\rho^2)(1-\mu)} \left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T} \right) \right]^2 - \sigma_\varepsilon^2 \right\}. \end{aligned}$$

Thus we have

$$\mathbf{WEL} = \begin{cases} \frac{1}{2} \left\{ \left[\frac{\rho\mu}{(1-\rho^2)+\rho^2\mu} \left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T} \right) \right]^2 - \sigma_\varepsilon^2 \right\}, & \text{if} \quad \frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon \overline{\sigma}_T}{\mu} \\ -\rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} - \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu}, & \text{if} \quad -\frac{\sigma_\varepsilon \overline{\sigma}_T}{\mu} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu} \\ -\rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} + \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} & \\ +\frac{1}{2} \left\{ \left[\frac{\rho\mu}{(1-\rho^2)+\rho^2\mu} \left(\rho\sigma_\varepsilon - \frac{m}{\underline{\sigma}_T} \right) \right]^2 - \sigma_\varepsilon^2 \right\}, & \text{if} \quad -\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu} < \frac{\rho m}{1-\rho^2} < 0 \\ -\rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} & \\ +\frac{1}{2} \left\{ \left[\frac{\rho\mu}{(1-\rho^2)+\rho^2\mu} \left(\rho\sigma_\varepsilon - \frac{m}{\overline{\sigma}_T} \right) \right]^2 - \sigma_\varepsilon^2 \right\}, & \text{if} \quad 0 \leqslant \frac{\rho m}{1-\rho^2} \end{cases} \quad \text{(A.9)}$$

Appendix A6: Proposition 6 on Monotonicity of Welfare Function in Fraction of Opaque Traders

Case 1. Transparent traders hold short positions of risky asset. For $\frac{\rho m}{1-\rho^2} < -\frac{\sigma_\varepsilon \overline{\sigma}_T}{\mu^*}$,

$$f(\sigma_\eta) = \rho m \frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{1-\rho^2}{\mu^* + (1-\rho^2)(1-\mu^*)} \left[\sigma_\varepsilon^2 - (1-\mu^*)\rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T}\right]$$

and hence

$$\begin{aligned} \underline{f}^* &= \rho m \tfrac{\sigma_\varepsilon}{\underline{\sigma}_T} + \tfrac{1-\rho^2}{\mu^*+(1-\rho^2)(1-\mu^*)} \left[\sigma_\varepsilon^2 - (1-\mu^*)\rho m \tfrac{\sigma_\varepsilon}{\overline{\sigma}_T}\right] \\ \overline{f}^* &= \tfrac{(1-\rho^2)\sigma_\varepsilon^2}{(1-\rho^2)+\rho^2\mu^*} + \tfrac{\mu^*}{(1-\rho^2)+\rho^2\mu^*} \rho m \tfrac{\sigma_\varepsilon}{\overline{\sigma}_T}, \end{aligned}$$

then

$$\frac{\partial \overline{f}^*}{\partial \mu^*} = -\frac{1-\rho^2}{[(1-\rho^2)+\rho^2\mu^*]^2} \left(\rho^2\sigma_\varepsilon^2 - \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T}\right) < 0$$

and

$$\frac{\partial \mathbf{WEL}^*}{\partial \mu^*} = \left[-1 + \frac{\overline{f}^*}{\sigma_\varepsilon^2}\right] \frac{\partial \overline{f}^*}{\partial \mu^*} = \frac{1}{\sigma_\varepsilon^2} \frac{(1-\rho^2)\mu^*}{[(1-\rho^2)+\rho^2\mu^*]^3} \left(\rho^2\sigma_\varepsilon^2 - \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T}\right)^2 > 0.$$

Case 2. Transparent traders do not trade risky asset. For $-\frac{\sigma_\varepsilon \overline{\sigma}_T}{\mu^*} \leqslant \frac{\rho m}{1-\rho^2} \leqslant -\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu^*}$,

$$f(\sigma_\eta) = \rho m \frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu^*}$$

and hence

$$\underline{f}^* = \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu^*} \qquad \text{and} \qquad \overline{f}^* = \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} + \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu^*},$$

then

$$\frac{\partial \overline{f}^*}{\partial \mu^*} = -\frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu^{*2}} \qquad \text{and} \qquad \frac{\partial \mathbf{WEL}^*}{\partial \mu^*} = -\frac{\partial \overline{f}^*}{\partial \mu^*} = \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu^{*2}} > 0.$$

Case 3. Transparent traders hold long positions of risky asset. For $-\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu^*} < \frac{\rho m}{1-\rho^2}$,

$$f(\sigma_\eta) = \begin{cases} \rho m \frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{1-\rho^2}{\mu^*+(1-\rho^2)(1-\mu^*)} \left[\sigma_\varepsilon^2 - (1-\mu^*)\rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T}\right], & \text{if} \quad -\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu^*} < \frac{\rho m}{1-\rho^2} < 0 \\ \rho m \frac{\sigma_\varepsilon}{\sigma_\eta} + \frac{1-\rho^2}{\mu^*+(1-\rho^2)(1-\mu^*)} \left[\sigma_\varepsilon^2 - (1-\mu^*)\rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T}\right], & \text{if} \quad 0 \leqslant \frac{\rho m}{1-\rho^2} \end{cases}$$

and hence

$$\underline{f}^* = \begin{cases} \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu^*+(1-\rho^2)(1-\mu^*)} + \frac{\mu^*}{\mu^*+(1-\rho^2)(1-\mu^*)} \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T}, & \text{if} \quad -\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu^*} < \frac{\rho m}{1-\rho^2} < 0 \\ \frac{(1-\rho^2)\sigma_\varepsilon^2}{\mu^*+(1-\rho^2)(1-\mu^*)} + \frac{\mu^*}{\mu^*+(1-\rho^2)(1-\mu^*)} \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T}, & \text{if} \quad 0 \leqslant \frac{\rho m}{1-\rho^2} \end{cases}$$

$$\overline{f}^* = \begin{cases} \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} + \frac{1-\rho^2}{\mu^*+(1-\rho^2)(1-\mu^*)} \left[\sigma_\varepsilon^2 - (1-\mu^*)\rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T}\right] & \\ = \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} - \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \underline{f}, & \text{if} \quad -\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu^*} < \frac{\rho m}{1-\rho^2} < 0 \\ \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} + \frac{1-\rho^2}{\mu^*+(1-\rho^2)(1-\mu^*)} \left[\sigma_\varepsilon^2 - (1-\mu^*)\rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T}\right] & \\ = \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T} - \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T} + \underline{f}, & \text{if} \quad 0 \leqslant \frac{\rho m}{1-\rho^2}. \end{cases}$$

We consider the following two cases.

Case 3.1 If $-\frac{\sigma_\varepsilon \underline{\sigma}_T}{\mu^*} < \frac{\rho m}{1-\rho^2} < 0$, then

$$\frac{\partial \underline{f}^*}{\partial \mu^*} = -\frac{1-\rho^2}{[(1-\rho^2)+\rho^2\mu^*]^2} \left(\rho^2\sigma_\varepsilon^2 - \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T}\right) < 0$$

and

$$\begin{aligned} \frac{\partial \mathbf{WEL}^*}{\partial \mu^*} &= -\frac{\partial \overline{f}^*}{\partial \mu^*} + \frac{\underline{f}^*}{\sigma_\varepsilon^2} \frac{\partial \underline{f}^*}{\partial \mu^*} = \left[-1 + \frac{\underline{f}^*}{\sigma_\varepsilon^2}\right] \frac{\partial \underline{f}^*}{\partial \mu^*} \\ &= \frac{1}{\sigma_\varepsilon^2} \frac{(1-\rho^2)\mu^*}{[(1-\rho^2)+\rho^2\mu^*]^3} \left(\rho^2\sigma_\varepsilon^2 - \rho m \frac{\sigma_\varepsilon}{\underline{\sigma}_T}\right)^2 > 0. \end{aligned}$$

Case 3.2 If $0 \leqslant \frac{\rho m}{1-\rho^2}$, then

$$\frac{\partial \underline{f}^*}{\partial \mu^*} = -\frac{1-\rho^2}{[(1-\rho^2)+\rho^2\mu^*]^2} \left(\rho^2\sigma_\varepsilon^2 - \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T}\right)$$

and

$$\begin{aligned} \frac{\partial \mathbf{WEL}^*}{\partial \mu^*} &= -\frac{\partial \overline{f}^*}{\partial \mu^*} + \frac{\underline{f}^*}{\sigma_\varepsilon^2} \frac{\partial \underline{f}^*}{\partial \mu^*} = \left[-1 + \frac{\underline{f}^*}{\sigma_\varepsilon^2}\right] \frac{\partial \underline{f}^*}{\partial \mu^*} \\ &= \frac{1}{\sigma_\varepsilon^2} \frac{(1-\rho^2)\mu^*}{[(1-\rho^2)+\rho^2\mu^*]^3} \left(\rho^2\sigma_\varepsilon^2 - \rho m \frac{\sigma_\varepsilon}{\overline{\sigma}_T}\right)^2 > 0. \end{aligned}$$

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