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Consumer Loans in a Process of Diffusion of Innovations — an axiomatic analysis

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This paper aims in developing and extending a framework for consumer engagement in the innovative evolution. The starting point is a formal model of the Debreu monetary economy for which the process of diffusion of innovations is defined by using a qualitative theory of dynamical systems. In this process consumers, as well as producers, play an active role. Moreover, an essential part of innovative changes are consumer loans which enhance the purchasing power of households and can affect the process of diffusion of innovations and intensify innovative changes in a whole economy.

Key Words: Diffusion of innovations; Arrow-Debreu model; Consumer loan; Neo-Schumpeterian approach.
JEL Classification Numbers: D11, D50, C6, O30, O31.

1. INTRODUCTION

One of the central subjects of evolutionary economics, especially in the line of thought of Schumpeter's theory of economic development [1912, 1934, 1961], is the analysis of entrepreneurial innovations and evolutionary changes in a production sphere. According to this theory one of the main elements of innovative evolution is a process of diffusion of innovations. In this process not only producers-innovators play an active role by introducing new commodities or new technologies, but also consumers play an important role because their approval of the given innovation is a necessary condition of its market success. This fact is based on two premises: first, nowadays a big part of the innovative effort is devoted to product innovation in capital goods. Second, many firms use innovations to bring to the market ever new varieties of products and create new market niches

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and attracting potential buyers. Moreover, product differentiation on the supply side is the counterpart of the differentiation of demand. Thus, keeping a critical eye open on the mainstream side of economic theory, a neo-Schumpeterian economics with an important role of consumerism, as a valid alternative for a comprehensive study of economic phenomena will be presented.

The phenomenon of consumerism is an important part of the process of economic development, as it leads to increased production, thereby creating more jobs and new markets but it is also a highly complex peculiar process. There is not only an enormous variety of goods and services to choose from, but usually also bounded purchasing power of households. Thus an essential part of innovative evolution is consumer loans as a new means of payments created by banks. Moreover, the purchasing power of households strengthened by consumer loans can affect innovative processes.

The possibility of taking a consumer loan means that for given prices and interest rates, each consumer decides if he should allocate a part of his endowments for savings or to take a consumer loan to enhance his purchasing power and perform an optimal consumption plan better than the one without such a possibility in the real sphere (according to his preference relation). This implies, in fact, the behavior of the bank, because according to the rule of money creation with the given multiplier, called the rate of credit cover, in the first case they have an opportunity to extend the total number of credits. It is worthwhile to notice that bank credits are the only source of money to the economy and both the firms' and the banks' profits are distributed to households (the shareholders).

The idea of an active role of consumers in economic evolution has been reflected in the formal depiction of the neo-Schumpeterian theory initiated in Nelson and Winter [1982] and still continued [i.e. Andersen 2007, Clark and Goldsmith 2006, Green at all 2002, Nelson 2013]. In this trend the research program of studying Schumpeterian evolution in the Arrow-Debreu set up is situated. It was initiated in the 90-s XX century and developed later [Malawski 2004, 2005, Cialowicz and Malawski 2010, 2011, 2012, 2013, 2014, Cialowicz 2015]. This approach is essentially different from the mainstream of modern modelling of Schumpeterian evolution. The difference can be seen in a mathematical setting based on the set-theoretical a topological apparatus, which has been borrowed from the modern general equilibrium theory.

In this context, this paper is an extension of the previous results in a new direction that was earlier neglected and its aim is to develop a framework for consumer engagement in innovation and give an axiomatic analysis of the role of consumer loans in a process of diffusion of innovations. In particular it will be proved that consumers equipped with a loan might enhance their purchasing power and realize an innovative consumption plan, which has an impact on a process of diffusion of innovations initiated by producers-innovators and may intensify innovative changes in a whole economic system. Moreover it will be proved that consumer loans allow obtaining a better state of equilibrium (in a sense of optimality of market participant activities) at the end of the process of diffusion of innovation.

For this purpose the static model of the Debreu monetary economy and definitions of innovative and imitative extensions will be employed. In the next section a process of diffusion of innovations based on imitative changes in the production sector and innovative changes in a consumption sphere is going to be briefly elaborated on. Finally, the role of consumer loans in a process of diffusion of innovations will be analyzed and results will be given in a form of formal theorems.

It is worth emphasizing that there are some advantages referred to formalization in science in general and in economic science, specifically. This kind of formalism increases research efficiency in many ways. At the same time, the results obtained in this research may serve a heuristic base for empirical research, by pointing out the elements of the characteristics of the given model, the most important for the given economic problem, which is designated for observation and testing. It gives the possibility of confronting theories with empirical data.

2. FORMAL MODEL OF A DEBREU MONETARY ECONOMY AND ITS EXTENSIONS

The analysis of the process of diffusion of innovations cannot be described without both real and monetary spheres. Thus the general system approach to this theory in the Arrow-Debreu set up enables us to study the model of a Debreu monetary economy [c.f. Debreu 1959, Cialowicz and Malawski 2011] in the form of a multi-range relational system $E_m = (\mathbb{R}^{l+2}, P_m, C_m, F, \theta, \varpi_m, \mu)$. This model is a combination of production, consumption and financial sectors in which consumers share in producers' and banks' profits and initial total resource is consumer property.

A production system with money has a form of two-range relational system: $P_m = (B, \mathbb{R}^{l+2}, Ch_{P_m})$, where $Ch_{P_m} = (y_m, p_m, \eta_m, \pi_m)$ is a characteristic of the system P_m. In this system, each producer $b \in B$ is represented by feasible technologies he is equipped with, in the form of nonempty production set $y_m(b) := Y_b \subset \mathbb{R}^{l+2}$ in $l + 2$-dimensional commodity space $\mathbb{R}^{l+2} = \mathbb{R}^{l_R} \times \mathbb{R}^{l_F}$, where $\mathbb{R}^{l_R}$ is l-dimensional subspace of "real" commodities and $\mathbb{R}^{l_F}$ is 2-dimensional financial space. It means that in each production plan last two coordinates are assigned to deposits and credits respectively. Moreover, we assume that each producer does not have any deposits and he chooses from the set Y_b those production plans

$y_b = (y_1, \ldots, y_l, 0, -c_b)$, ($c_b$ means producer's b credit) that maximize his profit in a given price system $p_m = (p_1, \ldots, i_s, i_c) \in \mathbb{R}^{l+2}$, where i_s, i_c denote interest rates of savings and credits, respectively. This fact is described by a correspondence of supply η_m such that for each $b \in B, \eta_m(b) := \eta_b(p_m) := \{y'_b \in Y_b : p_m y'_b = \max_{y_b \in Y_b}(p_m y_b)\}$ and maximum profit function π_m such that for each $b \in B, \pi_m(b) := \pi_m(p_m) := \max_{y_b \in Y_b}(p_m y_b)$.

It is worth to mention that banks consider interest rates of credits as being the refund for a risk of a possible default of each applicant to whom they grant loans.

Similarly, a formal model of a consumption sector with money has a form of three-range relational system: $C_m = (A, \mathbb{R}^{l+2}, \mathcal{P}, Ch_{C_m})$, where $Ch_{C_m} = (\chi_m, e_m, \varepsilon_m, p_m, \beta_m, \varphi_m)$ is a characteristic of the system C_m. In this system, each consumer $a \in A$ is characterized by a nonempty consumption set $\chi_m(a) := X_a \subset \mathbb{R}^{l+2}$, such that $x_a = (x_1, \ldots, x_l, s_a, c_a) \in X_a$, where s_a denotes the savings of the consumer a, c_a denotes his loan. Let us note that the savings can be interpreted as outputs for consumers, implying that $s_a \leq 0$, and the consumer's loans as inputs, so that $c_a \geq 0$. Moreover consumer is an owner of an initial endowment $e_m(a) := e_a = (e_1, \ldots, e_l, s_a, c_a) \in X_a$ and is characterized by a preference relation $\varepsilon_m(a) := \preccurlyeq_a \in \mathcal{P}$, where $\mathcal{P}$ is a set of all preference relations in commodity space. Consumer tries to satisfy his preferences in the set X_a subject to his wealth constraint. Each consumer tries to choose and realize a consumption plan x_a^* from his budget set $\beta_m(a)$ the best for him with respect to his preference relation. This fact is described by a correspondence of supply φ_m such that $\varphi_m(a) := \{x_a^* \in \beta_m(a) : \forall x_a \in \beta_m(a) \;\; x_a \preccurlyeq_a x_a^*\}$.

A financial sector F has a form of two-range relational system: $F = (M, \mathbb{R}^{l+2}, Ch_F)$, where $Ch_F = (f, p_m, \gamma, \zeta)$ is a characteristic of the system F. In this system each bank $r \in M$ is characterized by a set of feasible financial plans: $f(r) := F_r \subset \mathbb{R}^{l+2}$. We assume that operations of banks are neutral to real goods, meaning that the financial plan of a bank r has a form $f_r = (0, \ldots, 0, s_r, c_r) \in \mathbb{R}^{l+2}$, where $s_r = \sum_{a \in A} s_{ar}$, $s_{ar} \leq 0$ denotes savings of consumer a in the bank r, $c_r = \sum_{b \in B} c_{br} + \sum_{a \in A} c_{ar}$, $c_{br} \geq 0$ denotes the credit extended to the producer b by the bank r, $c_{ar} \geq 0$ denotes loan of consumer a given by the bank r.

In the given financial plan consumers savings can be interpreted as inputs for a bank (negative coordinate) and banks transform them into outputs (positive coordinate), i.e. credits or loans extended to producers and consumers according to the rule of money creation with the multiplier $\lambda < 0$, called the rate of credit cover. It means that a financial plan f_r is feasible for a bank r, $f_r \in F_r$ if $c_r \leq \lambda s_r$. Notice, that according to the rate of credit cover in case of zero savings $s_r = 0$ cannot extend credits or loans ($c_r = 0$), so in the given model we assume that $s = \sum_{r \in M} s_r \neq 0$. According to previous notation: $c_b = \sum_{r \in M} c_{br}$, $c_a = \sum_{r \in M} c_{ar}$, $s_a = \sum_{r \in M} s_{ar}$.

In the Debreu monetary economy each bank can be interpreted as producer operating on the money markets, so its role is to maximize the profit on financial sets concerning the given prices. Moreover vector of prices has a form $p_m = (p_1, \dots, i_s, i_c)$ and we assume that interest rates of consumer's loans and producer's credits are the same so if $i_s < i_c$, the difference $i_c - i_s$ is the source of bank profits.

Based on the above premises, banks activity is described by a correspondence of optimal financial plans (a money supply correspondence) γ and a maximum profit function ζ, which means that profit of a bank r from realization of a financial plan f_r at the given vector of prices p_m equals: $z_r(p_m, f_r) := p_m f_r = i_c(\sum_{b\in B} c_{br} + \sum_{a\in A} c_{ar}) + i_s(\sum_{a\in A} s_{ar})$. Thus $\gamma(r) = \gamma_r(p_m) := \{f'_r \in F_r : z_r(p_m, f'_r) = \max_{f_r\in F_r} z_r(p_m, f_r)\}$ and $\zeta(r) = \zeta_r(p_m) := \max_{f_r\in F_r} z_r(p_m, f_r)$ for each $r \in M$.

The Debreu monetary economy E_m is a combination of the production system with money P_m, the consumption system with money C_m and the financial system F such that consumers share in producers' and banks' profits. These shares are measured by functions θ and μ respectively, such that for an ordered pair $(a, b) \in A \times B$ a number $\theta_{ab} := \theta(a, b) \in [0, 1]$ describes consumer a share in producer b profit and for each $b \in B, \sum_{a\in A} \theta_{ab} = 1$. Similarly for each pair $(a, r) \in A \times M$ a number $\mu_{ar} \in [0, 1]$ describes consumer a share in bank r profit and for each $r \in M, \sum_{a\in A} \mu_{ar} = 1$. Moreover, an initial total resources $\varpi \in \mathbb{R}^{l+2}$ of the economy E_m are consumers property, which means that $\varpi = (\varpi_1, \varpi_2, \dots, \varpi_l, \varpi_s, \varpi_c) := \sum_{a\in A} e_a$. According to the given assumptions wealth of each consumer in this model is given by the formula:

$$w_a = proj_{\mathbb{R}^l}(p_m) \circ proj_{\mathbb{R}^l}(e_a) + \sum_{b\in B} \theta_{ab}\pi_b(p_m) + \sum_{r\in M} \mu_{ar}\zeta_r(p_m) + s_a + c_a$$

and a budget set has a form $\beta_m(a) := \{x_a \in X_a : p_m x_a \le w_a\}$. Notice, that consumer savings come from unused parts of their endowments: $s_a = p_m x_a - w_a$.

Concluding, in the economic system E_m the role of each market participant is to select and implement the optimal plan in a given price system and individual constraints. Specifically, for a given vector of prices and interest rates, each consumer decides, if he should allocate a part of his endowments for savings or to take consumer loan to enhance his purchasing power and perform an optimal consumption plan better than one without such possibility in the real sphere (according to his preference relation). At the same time producers, whose aim is to choose the production plans maximizing their profits, can get a credit from a bank, to realize an innovative production plan. It is worthwhile to notice that, in the given model sets of agents (set of producers, set of consumers and set of banks) are not disjoint, because both producers and banks are also consumers.

The model described above is a modification of a standard Debreu economy with private ownership $E = (\mathbb{R}^l, P, C, \theta, \varpi)$ [Debreu 1959], because $proj_{\mathbb{R}^l}(E_m) = E$, where $P = proj_{\mathbb{R}^l}(P_m)$, $C = proj_{\mathbb{R}^l}(C_m)$.

Although economic evolution is in principle an easily understandable process, in practice it is surprisingly difficult to analyse. For practical reasons, it remains necessary to develop the theory of economic evolution that might illuminate the problems faced by adopters of innovations, it means consumers equipped with preference relations and loans.

Formal modeling of innovative evolution in Arrow-Debreu set-up is based on the definition of innovative extension of the Debreu monetary economy. Moreover it has to be remembered, that in the Schumpeterian economic thinking, economic development is initiated by producers who become innovators and entrepreneurs [Schumpeter 1961]. Based on the above premises, first the definition of innovative extension of a production sector and production system with money will be introduced and then an innovative extension of a Debreu monetary economy will be defined.

Let $proj_{\mathbb{R}^l}(p)$ stands for the orthogonal projection of a vector p onto the space of commodities $\mathbb{R}^l$ (in the standard basis).

Definition 2.1. [Cialowicz 2015, cf. Malawski 2004] A production system $P' = (B', \mathbb{R}^{l'}, Ch'_{P'})$ is called an innovative extension of a system $P = (B, \mathbb{R}^l, Ch_P)$, in short, $P \subset_i P'$, if:

1) $l \leq l'$
2) $\exists b' \in B' \quad \forall b \in B$

   (2.1) $proj_{\mathbb{R}^l}(Y'_{b'}) \not\subset Y_b$,

   (2.2) $proj_{\mathbb{R}^l}(\eta'_{b'}(p')) \not\subset \eta_b(p)$,

   (2.3) $\pi_b(p) < \pi'_{b'}(p')$.

According to the definition, there may appear in a production system P' at least one new product or commodity (condition 1), which can be interpreted as a better way of meeting the needs present earlier in a system P. At the same time, the definition does not say whether new products are put out by brand new firms or by the ones already existing but modernized. Moreover in a production system P', there is at least one producer-innovator b' whose technological abilities go beyond the abilities of all producers acting within a production system P. Hence, the optimal production plans of the producer b' cannot be reduced to the analogous plans being realized by the producers in a production system P and the fixed producer's maximum profit is greater than the one any of the producers in a system P can make.

In particular, when $l < l'$, Definition 2.1 covers at least four cases of five structural changes characterized by Schumpeter as development [Schumpeter 1961], i.e.

(1) the introduction of a new good — condition 1,
(2) the introduction of a new method of production — condition 2.1,
(3) the opening of a new market — condition 1,
(4) the carrying out of the new organization of any industry — condition 2 as a whole.

Definition 2.2. A production system with money P'_m is called an innovative extension of a system P_m, in short $P_m \subset_i P'_m$, if:

1) $proj_{\mathbb{R}^l}(P) \subset_i proj_{\mathbb{R}^{l'}}(P')$,
2) $\sum_{b \in B} c_b < \sum_{b' \in B'} c'_{b'}$.

According to the definition, innovative changes in a production system with money are determined by adequate changes in its real sphere. The second condition is based on Schumpeter's theory in which bank credit acts as money-capital and, therefore, constitutes the necessary premise for the realization of the innovative processes planned by producer-innovator.

It seems to be clear that changes in the production sphere defined above determine changes in the Debreu economy with private ownership.

Definition 2.3. [Malawski 2004] A Debreu economy with private ownership $E' = (\mathbb{R}^{l'}, P', C', \theta', \varpi')$ is called an innovative extension of an economy $E = (\mathbb{R}^l, P, C, \theta, \varpi)$, in short, $E \subset_i E'$, if $P \subset_i P'$.

Using the above definitions we can formally define the concept of an innovative extension of the Debreu monetary economy, assuming that banks are not innovators and innovative changes are possible only in the real sphere. The innovative changes in the financial sphere are not studied.

Let two Debreu monetary economies: $E_m = (\mathbb{R}^l, P_m, C_m, F, \theta, \varpi_m, \mu)$, $E'_m = (\mathbb{R}^{l'}, P'_m, C'_m, F', \theta', \varpi'_m, \mu')$ be given.

Definition 2.4. [cf. Cialowicz and Malawski, 2011] A Debreu monetary economy E'_m is an innovative extension of an economy E_m (in short: $E_m \subset_i E'_m$), if:

1) $E \subset_i E'$, where $E = proj_{\mathbb{R}^{l_R}}(E_m)$, $E' = proj_{\mathbb{R}^{l_R}}(E'_m)$ (Definition 2.3),
2) $|\sum_{r \in M} s_r| < |\sum_{r' \in M'} s'_{r'}|$.

According to the definition, innovative changes in a Debreu monetary economy are determined by adequate changes in its real sphere E, as well as the total level of savings, and also credits by the rate of credit cover in the absolute value, increase. It is worth noticing that if the level of consumer loans is constant then an innovative extension of the Debreu monetary economy means innovative changes in a production system with money (Definition 2.2).

Remark 2.1. If $E_m \subset_i E'_m$ and $\sum_{a\in A} c_a = \sum_{a'\in A'} c'_{a'}$ then $P_m \subset_i P'_m$.

Let us now consider a different form of a production system extension called an imitative extension which is an important part of a concept of a process of diffusion of innovations.

Let three production systems with money be given: $P_m = (B, \mathbb{R}^{l+2}y, p, \eta, \pi)$, $P'_m = (B', \mathbb{R}^{l'+2}y', p', \eta', \pi')$, $P''_m = (B'', \mathbb{R}^{l''+2}.y'', p'', \eta'', \pi'')$ such that $P_m \subset_i P'_m$, $l \le l' \le l''$, $B = B' = B''$ and B'_i is a set of producers-innovators in the system P'_m.

Definition 2.5. [cf. Cialowicz and Malawski 2014] A production system with money P''_m is called an imitative extension of a system P'_m (in short: $P'_m \subset_{im} P''_m$), if there exists at least one producer-innovator $b' \in B'_i$ and a producer $b'' \in B''$ such that $\exists y''_{b''} \in Y''_{b''} \exists y_b \in Y_b : proj_{\mathbb{R}^{l+2}}(y''_{b''}) = y_b$.

The above definition is consistent with the concept of imitative innovation by Niosi [2012]. It says that in the imitative extension of a production system in P''_m there exists at least one producer b'', who is an imitator of producer-innovator b'_i. This imitator realizes production plan $y''_{b''}$, which is an imitation of the innovative production plan y_b.

Notice, that an imitative extension does not rule out an innovative extension, which means that for production systems from Definition 2.5 it is possible: $P_m \subset_i P''_m$ or $P'_m \subset_i P''_m$.

Definition 2.6. A Debreu monetary economy E'_m is an extension of a system E_m with imitative changes in the production sector (in short: $E_m \subset_{P_{im}} E'_m$) if $P_m \subset_{im} P'_m$.

3. AXIOMATIC ANALYSIS OF INNOVATIVE PRODUCTION AND CONSUMPTION PLANS

Definition 2.2 of innovative extension of a production system with money enables us to define an innovative production plan and innovative products in a space of commodities. In the set of feasible production plans of the producer-innovator $b' \in B'$ we may distinguish an innovative production plan $y'_{b'}$ such that $y'_{b'} \in proj_{\mathbb{R}^{l+2}}(Y'_{b'}) \backslash proj_{\mathbb{R}^{l+2}}(Y_b)$ for each $b \in B$. Moreover innovative changes are observed for a specific (real) commodity called innovative.

Definition 3.1. [cf. Cialowicz 2015] Let two production systems P'_m, P_m be given and $P_m \subset_i P'_m$. A commodity $k \in \{1, 2, \ldots, l\}$ is called innovative, if there exists producer-innovator $b' \in B'$ and an innovative production plan $y'_{b'} = (y'_1, y'_2, \ldots, y'_{l'}, 0, -c'_{b'}) \in Y'_{b'}$ such that for each producer $b \in B$ and production plan $y_b = (y_1, y_2, \ldots, y_l, 0, -c_b) \in Y_b$ there is $y'_k \neq y_k$.

The above definition says that an innovative commodity is a new product introduced in the economy ($l < k \leq l'$) or commodity for which a new methods of production are introduced. It means, that in the innovative extension of a production system with money new products or commodities may appear or innovative changes are observed in the production process of a distinguished commodity k. Moreover for non-innovative products, there is no change in the level of production, it means if $k' \neq k$ then $y'_{k'} = y_{k'}$.

Thus in the space of commodity $\mathbb{R}^{l'+2}$ we may distinguish the subspace of innovative products. Thus, let the space of commodities has a form: $\mathbb{R}^{l'+2} = \mathbb{R}^{l'_I + l'_n + 2} = \mathbb{R}^{l'_I} \times \mathbb{R}^{l'_n} \times \mathbb{R}^2$, where:

- $\mathbb{R}^{l'_I}$ is a space of (real) innovative commodities,
- $\mathbb{R}^{l'_n}$ is a space of (real) non-innovative commodities.

It means that if $l = l'$ then for each innovative production plan $y'_{b'} \in Y'_{b'}$ and each production plan $y_b \in Y_b$ for $k = 1, 2, \ldots, l'$ such that $y'_k \neq y_k$ we have $k \in \{1, 2, \ldots, l_I\}$ and for $\bar{k} \neq k$ such that $\bar{y}_{\bar{k}} = y_{k'}$ we have $\bar{k} \in \{1, 2, \ldots, l_n\}$.

For the purpose of this research we assume, that innovative commodity is input for a consumer or output for a producer (positive coordinate in a consumption or production plan).

Similarly we may introduce subspace of innovative commodities in the whole Debreu monetary economy according to the definition of its innovative extension. Thus it gives the possibility to define innovative consumption plans and innovative extension of a consumption system with money.

In the next definitions the standard inequalities in l-dimensional space $\mathbb{R}^l$ are used:

a) $x \leq y \iff x_k \leq y_k$ for each $k = 1, 2, \ldots, l$,
b) $x < y \iff x \leq y$ and $x \neq y$,
c) $x \ll y \iff x_k < y_k$ for each $k = 1, 2, \ldots, l$,

for $x, y \in \mathbb{R}^l$, where $x = (x_1, x_2, \ldots, x_l)$, $y = (y_1, y_2, \ldots, y_l)$:

Let a Debreu monetary economy E_m and its innovative extension $E'_m (E_m \subset_i E'_m)$ be given.

Definition 3.2. [cf. Cialowicz 2015] A consumption plan $x' \in \mathbb{R}^{l'+2}$ is called innovative if $proj_{\mathbb{R}^{l_I}}(x') > 0$, where $\mathbf{0} = (0, 0, \ldots, 0)$.

According to the given definition, an innovative consumption plan is a plan, in which at least one inputs is an innovative commodity.

Definition 3.3. [cf. Cialowicz 2015] A consumption plan $x'_a = (x'_1, \ldots, x'_{l'}, s'_{a'}, c'_{a'}) \in \mathbb{R}^{l'+2}$ of a consumer $a' \in A'$ is called:

a) at least as innovative as plan $x_a = (x_1, \ldots, x_l, s_a, c_a) \in \mathbb{R}^{l+2}$ of a consumer $a \in A$ (in short: $x_a \leq_I x'_a$) if $proj_{\mathbb{R}^{l_I}}(x_a) \leq proj_{\mathbb{R}^{l_I}}(x'_a)$ and $c_a \leq c'_{a'}$,

b) more innovative than a plan $x_a = (x_1, \ldots, x_l, s_a, c_a) \in \mathbb{R}^{l+2}$ of a consumer $a \in A$ (in short: $x_a <_I x'_a$) if $proj_{\mathbb{R}^{l_I}}(x_a) < proj_{\mathbb{R}^{l_I}}(x'_a)$ and $c_a \leq c'_{a'}$.

This definition says that a consumption plan is more innovative then other plans if all its coordinates represented innovative commodities are not less than corresponding coordinates, at least one of them is greater and the level of consumer loan is not less. It worth to notice that for a non-innovative consumption plan x_a and such that $proj_{\mathbb{R}^{l_I}}(x_a) = 0$ this plan is worse than any innovative plan with the same level of consumption loan.

Definition 3.4. [cf. Cialowicz 2015] A preference relation $\preccurlyeq_a \subset \mathbb{R}^{l+2} \times \mathbb{R}^{l+2}$ of a consumer $a \in A$ is pro-innovative if for each two consumption plans $x_a, x'_a \in X_a$ there is $x_a <_I x'_a \Longrightarrow x_a \prec_a x'_a$.

Notice that if the consumer a is characterized by pro-innovative preference relation then each innovative consumption plan is better than each non-innovative plan and for any two innovative plans better is the one in which there are more innovative commodities. At the basic level, pro-innovative preference relation motivates consumers to look for new, intellectually or emotionally challenges.

Theorem 3.1. *Let a consumer $a \in A$ be given characterized by a consumption set $X_a \neq \emptyset$ and a preference relation $\preccurlyeq_a$. If:*

1)$\exists x_a \in \beta_a \neq \emptyset : x_a = (x_1, \ldots, x_l, s_a, c_a)$ is an innovative plan,
2)$\preccurlyeq_a$ is pro-innovative,

then $\forall x_a^* \in \varphi(a) := \{x_a' \in \beta_a : \forall x_a \in \beta_a \ \ x_a \preccurlyeq_a x_a'\}$, $x_a^* = (x_1^*, \ldots, x_l^*, s_a^*, c_a^*)$, *where* $c_a^* = c_a$ *is an innovative consumption plan.*

Proof. (indirect): Assume, that there exists an optimal consumption plan $x_a^* \in \varphi(a)$, which is non-innovative, it means $proj_{\mathbb{R}^{l_I}}(x_a^*) = 0$ and exists $x_a \in \beta_a \neq \emptyset$ such that x_a is innovative, it means $proj_{\mathbb{R}^{l_I}}(x_a) > 0$. If $proj_{\mathbb{R}^{l_I}}(x_a^*) < proj_{\mathbb{R}^{l_I}}(x_a)$ and $c_a^* = c_a$ then according to the Assumption 2, there is $x_a^* \prec_a x_a$. At the same time, according to the definition of a correspondence of supply $\varphi_m(a)$ there is $x_a \preccurlyeq_a x_a^*$ what gives contradiction to the fact that $x_a^* \prec_a x_a$. ▮

The above theorem says that for a consumer a characterized by pro-innovative preference relation if there are feasible innovative consumption plan x_a in his budget set, each consumption plan x_a^* maximizing his preference relation in the budget set with the same level of consumption loan $c_a^* = c_a$ is innovative.

Definition 3.5. A production plan $y' \in \mathbb{R}^{l+2}$ is called:

a) at least as innovative as a production plan $y \in \mathbb{R}^{l+2}$ (in short: $y \leq_I y'$) if:

$$proj_{\mathbb{R}^{l_I}}(y) \leq proj_{\mathbb{R}^{l_I}}(y') \text{ and } p_m \circ y \leq p_m \circ y',$$

b) more innovative than a production plan $y \in \mathbb{R}^{l+2}$ (in short: $y <_I y'$) if:

$$proj_{\mathbb{R}^{l_I}}(y) < proj_{\mathbb{R}^{l_I}}(y') \text{ and } p_m \circ y < p_m \circ y'.$$

The above definition says that a production plan is more innovative then other plans if all its coordinates represented innovative commodities (outputs) are not less than corresponding coordinates, at least one of them is greater and better production plan gives greater profit then worse production plan.

Remark 3.1. If a production plan $y = (y_1, y_2, \ldots, y_l, 0, -c)$ is not innovative, it means $proj_{\mathbb{R}^{l_I}}(y) = 0$, then for each innovative production plan $y' = (y_1', y_2', \ldots, y_l', 0, -c')$ such that $c' = c$ there is $y <_I y'$, if $proj_{\mathbb{R}^{l_I}}(p_m) \geq 0$ (there are not harmful commodities among innovative commodities).

In the Schumpeterian vision of economic development changes in consumer activities are secondary to respective changes in the production system. Moreover the possibility of an active role of consumers in innovative processes is neglected. This idea was reflected in a large part of mainstream formalizations of the Schumpeterian theory of economic development [cf. Nelson and Winter 1982, 2002]. However, despite originality and theoretical significance of the Schumpeter's approach, the idea that economic evolution is an immensely complex process, demand-side aspects play an important role in this process and innovative changes are possible in a demand sector, has received increased attention in recent years, especially in neo-Schumpeterian modern setting [Andersen 2007, Clark and Goldsmith 2006, Green, Walsh, Tomlinson and McMeekin 2002, Hanush and Pyka 2006, 2007, Saam 2005, Nelson 2013]. In this context, the definition of an innovative extension of a consumption system with money will be formulated as an important element of a process of diffusion of innovations.

Let two consumption systems with money C_m and C'_m be given.

Definition 3.6. A consumption system with money C'_m is an innovative extension of a system C_m, in short $C_m \subset_i C'_m$, if:

1) $l \leq l'$
2) $\exists a' \in A'$

(2.1) there exists at least one $x'_{a'} \in \beta'_{a'} \neq \emptyset$ such that $x'_{a'}$ is an innovative consumption plan,

(2.2) a preference relation $\preccurlyeq_{a'}$ is pro-innovative,

(2.3) $\exists x'_{a'} \in \varphi'_{a'} \quad \forall a \in A \ \forall x_a \in \varphi_a \ \ x_a <_I x'_{a'}$,

(2.4) $\sum_{a \in A} s_a \geq \sum_{a' \in A'} s'_a$, $\sum_{a \in A} c_a \leq \sum_{a' \in A'} c'_{a'}$.

A consumption system $\mathcal{C}'$ is an innovative extension of a system $\mathcal{C}$, if there exists at least one consumer a' with pro-innovative preference relation (Condition 2.2) and feasible innovative consumption plans (Condition 2.1). Moreover this consumer has at least one optimal consumption plan more innovative than the analogous plans being realized by the consumers in a previous consumption system $\mathcal{C}$.

It is easy to see, that in particular, when $l < l'$, Definition 3.5 covers four cases of structural changes in a consumer sphere, i.e.

1) the introduction of a new commodity — condition 1,

2) the introduction of an innovative consumption plans — condition 2.1,

3) the changing of preference relations concerning innovative consumption plans — condition 2.2,

4) the carrying out of the consumption plans more innovative than before — condition 2.3.

Definition 3.7. A Debreu monetary economy E'_m is an extension of a system E_m with innovative changes in the consumption sector (in short: $E_m \subset_{C_i} E'_m$), if $C_m \subset_i C'_m$.

4. CLASSIFICATION OF EQUILIBRIUM STATES IN THE DEBREU MONETARY ECONOMY

In the analysis of the role of consumer's loan in a process of diffusion of innovations, one of the most important, and at the same time, the most difficult problem is to evaluate the results of different processes, it means to compare innovativeness of two economies. To this purpose I suggest to apply a comparison of two different processes with the same starting model taking into account their final equilibrium states.

Definition 4.1.

1) $(m + n + k)$ — elementary sequence of points from a space $\mathbb{R}^{l+2}$ in a form $((x_a), (y_b), (f_r))$ is called an allocation of the Debreu monetary economy E_m.

2) An allocation $((x_a), (y_b), (f_r))$ fulfilled a market equilibrium equality, if:

a) $proj_{\mathbb{R}^l R}(x-y+f) = proj_{\mathbb{R}^l R}(\varpi)$ where $x = \sum_{a\in A} x_a$, $y = \sum_{b\in B} y_b$, $f = \sum_{r\in M} f_r$,

b) $\sum_{a\in A} s_a + \sum_{r\in M} s_r = 2\varpi_s$,

c) $\sum_{r\in M} c_r + \sum_{a\in A} c_a + \sum_{b\in B} c_b = 3\varpi_c$.

3) An allocation $((x_a), (y_b), (f_r))$ is feasible for the economy E_m, if:

a) for each consumer $a \in A$, $x_a \in X_a$,

b) for each producer $b \in B$, $y_b \in Y_b$,

c) for each bank $r \in M$, $f_r \in F_r$,

d) An allocation $((x_a), (y_b), (f_r))$ fulfilled a market equilibrium equality.

According to the above definition an allocation is a sequence of consumption and production plans in a commodity space. If all those plans are feasible concerning technological constraints (for producers and banks) or psycho-physical constraints (for consumers) and a market equilibrium equality is fulfilled, the given allocation is feasible. Among all feasible allocations the most important are allocations formed from optimal consumption, production and financial plans for the fixed vector of prices. If this kind of allocation exists, we said the economy is in equilibrium, i.e., every agent, given the price system and the actions of the other agents, has no incentive to choose a different action, and this allocation together with the vector of prices is called a general (competitive) equilibrium state.

Definition 4.2. $(m + n + k + 1)$ — elementary sequence of points $s = ((x_a^*), (y_b^*), (f_r^*), p^*)$ in the space $\mathbb{R}^{l+2}$ is called a general (competitive) equilibrium state of the Debreu monetary economy E_m, if the following conditions are satisfied:

1) Producer's profit maximization: for each producer $b \in B$ a production plan y_b^* maximizes his profit on the set Y_b at the vector of prices p^*,

2) Bank's profit maximization: for each bank $r \in M$ a financial plan f_r^* maximizes his profit on the set F_r at the vector of prices p^*,

3) Utility maximization: for each consumer $a \in A$ a consumption plan x_a^* is the best (maximal) element for his preference relation $\preccurlyeq_a$ in his budget set $\beta_a := \{x_a \in X_a : p_m x_a \leq pe(a) + \sum_{b \in B} \theta_{ab}\pi_b(p_m) + \sum_{r \in M} \mu_{ar}\zeta_r(p_m) + s_a + c_a\}$,

4) Market clearing: the market equilibrium equality is satisfied.

For this research and particular to study the impact of consumer loans on innovative evolution one of the most important and difficult part is to compare different states of equilibrium according to optimal plans of market participants.

Definition 4.3. An equilibrium state $\bar{s} = ((\bar{x}_a^*), (\bar{y}_b^*), (\bar{f}_r^*), \bar{p}^*)$ of the economy E_m is better than a state $s = ((x_a^*), (y_b^*), (f_r^*), p^*)$, in short, $s \lhd \bar{s}$, if:

1) for each $a \in A$ $x_a^* \preccurlyeq_a \bar{x}_a^*$ and there exists $a' \in A$ $x_{a'}^* \prec_{a'} \bar{x}_a'^*$
2) $\sum_{b \in B} \pi_b(p^*) < \sum_{b \in B} \pi_b(\bar{p}^*)$
3) $\sum_{r \in M} \zeta_r(p^*) \leq \sum_{r \in M} \zeta_r(\bar{p}^*)$.

This definition says that a better state of equilibrium $\bar{s}$ is this one, in which all consumers realize not worse optimal consumption plans than in the state s and at least one of them realizes optimal consumption plan better one, a maximum total profit of the production sector is greater, and a maximum total profit of the financial sector is not less than the analogous profits in the worse equilibrium state.

5. AN AXIOMATIC ANALYSIS OF A PROCESS OF DIFFUSION OF INNOVATIONS

The concept of the process of diffusion of innovations is one of the oldest social science theories. It was first studied in the late 19th century in the book of the French sociologist Gabriel Tarde [1890] and developed later in the 60-s XX century by E.M. Rogers [1962]. It originated in communication to explain how, over time, an idea or product gains momentum and diffuses (or spreads) through a specific population or social system. The result of this diffusion is that people, as part of a social system, adopt a new idea, behavior, or product. Diffusion is the "process by which an innovation is communicated through certain channels over a period of time among the members of a social system". An innovation is "an idea, practice, or object that is perceived to be new by an individual or other unit of adoption". "Communication is a process in which participants create and share information with one another to reach a mutual understanding" [Rogers, 1962].

Many diffusion models have been developed in the last decades, based on various assumptions. One of the most popular is the epidemic model pioneered by Bass [1969]. In this growth model, the initial purchase of new products is based upon the assumption that the probability of purchase at any time is related linearly to the number of previous buyers. Epidemic models have been widely applied in curve-fitting exercises [Bass 1986, Mahajan at al. 1990]. The second type of diffusion model is based on the assumption of rational behavior [David 1969, Leoncini 2001]. The third type of diffusion model is evolutionary or "non-equilibrium" model in which technological change is endogenized [Nelson and Winter 1982, Silverberg 1991, Windrum and Birchenhall 2005].

According to all economic models a process of diffusion of innovation is characterized by multidimensionality and multiphasality. It is initiated by producers-innovators by introducing of new commodities or technologies, which means disturbing of an equilibrium state of the economy. But this process is considerably accelerated in the next phase based on the imitative activity of producers who imitate innovators' strategies by copying their innovative production plans. This leads to an increase in the level of output of the commodity under consideration and an intensification of competition among the firms producing it. Moreover an innovation is profitable if this change is accepted by the demand sector. Hence, the process of diffusion of innovations takes place not only in a production but also in a consumption sphere. The system moves in the direction of a new circular flow and finally, this process brings an economy to a new state of equilibrium.

In this context, the diffusion of innovation is a three-step decision-making process:

1. The analysis starts from a static model of an economy in a state of equilibrium.

2. The evolution of a system is restarted because of innovative disturbance caused by producer-innovator. It starts movements away from equilibrium.

3. The process is continued by imitators of innovator among producers and innovative changes in a consumption sphere. This process brings an economy to a new state of equilibrium.

To grasp the core of Schumpeterian economic evolution and preserve the principles of scientific rationality dominating today's economic theory, such as rigor, generality and analytical simplicity, the standard general equilibrium theory in its Arrow-Debreu set-up seems to be not sufficient tool, because of its static character. According to Schumpeter, standard economics describes how economic agents react to given constraints, and he shows that these reactions are predetermined. In a static economy, which is based on what we today call rational choice theory, nothing new can happen, since agents can only passively adapt to the data. However, the static model of a Debreu economy seems to be a promising point of departure. To dynamize the given model, a qualitative theory of dynamical systems can be employed as a mathematical tool [Sybirskij and Szube 1987], where a (quasi)-semidynamical system is understood as a semigroup of multivalued transformations of a metric space. "...Economic dynamics is an economy considered under a change in time. At a fixed time the people are buyers and sellers. As time moves forward they become consumers and producers as well, consuming the products bought and producing new products to replace product sold..." [Kitchel 2016].

Let me shortly denote a space of all Debreu monetary economies: $\mathbf{E_m} := \{E_m : E_m \text{ is a Debreu monetary economy}\}$.

Definition 5.1. A mapping $f_{E_m} : E_m \times \mathbb{R}_+ \to P(E_m)$ is a (quasi)-semidynamical Debreu monetary economy system, if:

1) $f_{E_m}(E_m, 0) = \{E_m\}$,
2) $f_{E_m}(f_{E_m}(E_m, t_1), t_2) = f_{E_m}(E_m, t_1 + t_2)\ \forall t_1, t_2 \in \mathbb{R}_+$.

In particular a quasi-semidynamical Debreu monetary economy system is called single-valued if $f_{E_m} : \mathbf{E_m} \times \mathbb{R}_+ \to P(\mathbf{E_m})$ is a mapping.

Let a (quasi)- semidynamical system $f_{E_m} : \mathbf{E_m} \times \mathbb{R}_+ \to P(\mathbf{E_m})$ be given.

Definition 5.2. A set $\tau_+(E_m) := \{f_{E_m}(E_m, t) : t \in R_+ \text{ and } f_{E_m}(E_m, 0) = E_m\}$ is called a positive semitrajectory of the (quasi)-semidynamical Debreu monetary economy system.

For the given quasi-semidynamical system I will write in short E_m^t instead of $f_{E_m}(E_m, t)$ for every $E_m \in E_m, t \in R_+$.

Definition 5.3. A positive semitrajectory $\tau_+(E_m)$ is called a process of diffusion of innovations for the Debreu monetary economy E_m, if:

1) $E_m = E_m^0$ and
2) there exists two periods of times $t_1, t_2 \in R_+$ such that $0 < t_1 < t_2$ and $E_m^0 \subset_i E_m^{t_1} \subset_{C_i}^{P_{im}} E_m^{t_2}$, where $E_m^0 \subset_i E_m^{t_1}$ denotes an innovative extension of the economy E_m, $E_m^{t_1} \subset_{C_i}^{P_{im}} E_m^{t_2} \iff (E_m^{t_1} \subset_{P_{im}} E_m^{t_2}$ — Definition 2.6 and $E_m^{t_1} \subset_{C_i} E_m^{t_2}$ — Definition 3.6),
3) there exists a general equilibrium state for the economy $E_m^{t_2}$.

According to the above definition a process of diffusion of the given innovation sets out a technological trajectory or path of innovation. But it worth noticing that for the given starting point in a form of a static model of a monetary economy there are a lot of different possible scenarios of this process. At the same time the different trajectories usually involve different actors, different technologies and models.

It should be remembered that the process of diffusion of innovations in the real economy is inherently uncertain and the given theoretical model contrasted with i.e. Leyden and Link dynamic theory [2014] in which the entrepreneur's decision-making process is based on the maximization of the subjective probability of success.

6. THE ROLE OF CONSUMER LOANS IN A PROCESS OF DIFFUSION OF INNOVATIONS

Consumers loans play an important role in consumers activity, because it gives the possibility to enhance his purchasing power. As a result consumer with the nonzero loan can perform an optimal consumption plan better than one without such possibility according to his preference relation.

The main advantage of consumer credit is that consumers can purchase goods and services and pay for them later. Consumers can purchase items they need when their funds are low. Moreover consumer loan benefits the economy. It allows households to furnish a home, pay for education, and get a car without having to save for them. As long as the economy grows, consumers can pay off this debt more quickly in the future. That's because education allows them a better-paying job, and the cars and all commodities they buy create jobs and new markets. That process creates an upward cycle, boosting the economy even more. Hence cyclical changes in the economy are based on the introduction into the economic system of the new means of payments created by banks in the form of credits/loans. Thus the Federal Reserve reports on consumer debt each month. It's important because consumer loan contributes to economic growth. The latest statistics from the Federal Reserve indicate consumer debt in the United States continues to increase, reaching nearly \$3.6 trillion in April 2016. According to statistics published by the Census Bureau, that works out to over \$11,140 in debt for every man, woman and child that lives in the United States.

Let a Debreu monetary economy $E_m = (\mathbb{R}^{l+2}, P_m, C_m, F, \theta, \varpi_m, \mu)$ be given.

THEOREM 6.2. *Let two consumers $a, a' \in A$ be given such that:*

1)$X_a = X_{a'}$,
2)$\preccurlyeq_a = \preccurlyeq_{a'}$,
3)$proj_{\mathbb{R}^l}(e_a) = proj_{\mathbb{R}^l}(e_{a'})$,
4)$s_a = s_{a'}$,
5)$c_a \neq 0, c_{a'} = 0$,
6)$\theta_{ab} = \theta_{a'b}$ for each $b \in B$
7)$\mu_{ar} = \mu_{a'r}$ for each $r \in M$,
8)$p_m > 0$.

Then:

a)$\beta_m(a') \subsetneq \beta_m(a)$ (a budget set of a consumer with non-zero loan includes a set budget of a consumer without a loan),

b)if a preference relation $\preccurlyeq_a$ is monotone then $x^*_{a'} \prec_a x^*_a$ (optimal consumption plan of a consumer with a non-zero loan is better than optimal consumption plan of a consumer without a loan).

Proof. a) According to Assumptions 3)-7) wealth of given consumers are equal:

$$w_a = proj_{\mathbb{R}^l}(p_m) \circ proj_{\mathbb{R}^l}(e_a) + \sum_{b \in B} \theta_{ab}\pi_b(p_m) + \sum_{r \in M} \mu_{ar}\zeta_r(p_m) + s_a + c_a, \quad (1)$$

$$w_{a'} = proj_{\mathbb{R}^l}(p_m) \circ proj_{\mathbb{R}^l}(e_a) + \sum_{b \in B} \theta_{ab}\pi_b(p_m) + \sum_{r \in M} \mu_{ar}\zeta_r(p_m) + s_a \quad (2)$$

so $w_{a'} < w_a$.

Thus for each $x_{a'} \in \beta_m(a')$ there is $p_m x_{a'} \leq w_{a'} < w_a$ so $x_{a'} \in \beta_m(a)$

$$\beta_m(a) := \{x_a \in X_a : p_m x_a \leq w_a\}$$

b) According to the definition of monotone preference relation for each two consumption plans $x_a, \tilde{x}_a \in X_a$ if $x_a < \tilde{x}_a$ (in commodity bundle x_a

each coordinate is less than or equal corresponding coordinate of the vector $\tilde{x}_a$ and at least one coordinate of the vector x_a is less than the corresponding coordinate of the vector $\tilde{x}_a$) then $x_a \prec_a \tilde{x}_a$ (x_a is worse than $\tilde{x}_a$). Moreover for monotone preference relations an optimal consumption plan x_a^* of the consumer a fulfilled condition $p_m x_a^* = w_a$. Similarly: $p_m x_{a'}^* = w_{a'}$

If $\beta_m(a') \subsetneq \beta_m(a)$ then exists an optimal consumption plan of the consumer a such that $x_a^* \in \beta_m(a)$ and $x_a^* \notin \beta_m(a')$ and for each optimal consumption plan of the consumer a' there is $p_m x_{a'}^* = w_{a'} < w_a = p_m x_a^*$. Thus from the Assumption 8) $p_m > 0$ it follows, that $x_{a'}^* < x_a^*$ and from monotonicity of a preference relations: $x_{a'}^* \prec_a x_a^*$. ▮

Remark 6.1. A preference relation $\preccurlyeq_a \subset \mathbb{R}^{l+2} \times \mathbb{R}^{l+2}$ of consumer a is called monotonic in a set $X_a \subset \mathbb{R}^{l+2}$ if and only if, for any two consumption plans x, $\tilde{x}$ feasible for consumer a for which x is less than $\tilde{x}$ ($x < \tilde{x}$), plan x is worse than plan $\tilde{x}$ ($\forall x, \tilde{x} \in X_a : x < \tilde{x} \Longrightarrow x \prec_a \tilde{x}$). Monotonicity of preference relation reflects principle: "more means better", consumer prefers more of each good to less.

Now it is possible to prove that nonzero consumer loans may improve the final results of the process of diffusion of innovation because it allows obtaining a better state of equilibrium (in a sense of optimality of market participant activities) at the end of this process.

THEOREM 6.3. *Let a Debreu monetary economy E_m and two processes of diffusion of innovations in forms of a positive semitrajectories $\tau_+(E_m)$ and $\bar{\tau}_+(E_m)$ be given. If:*

1)$E_m^{t_1} = \bar{E}_m^{t_1}, F_m^{t_2} = \bar{F}_m^{t_2}$,

2)$l^{t_2} = \bar{l}^{t_2}$, $A = A^{t_2} = \bar{A}^{t_2}$, $B = B^{t_2} = \bar{B}^{t_2}$, $M = M^{t_2} = \bar{M}^{t_2}$, $p_m^{t_0} = \bar{p}_m^{t_0} = p_m^{t_1} = \bar{p}_m^{t_1} = p_m^{t_2} = \bar{p}_m^{t_2} > 0$, $\varpi^{t_2} = \bar{\varpi}^{t_2}$,

3)$\forall a \in A\ \forall b \in B : \theta_{ab}^{t_0} = \theta_{ab}^{t_1} = \theta_{ab}^{t_2}$, $\forall a \in A\ \forall r \in M : \mu_{ar}^{t_0} = \mu_{ar}^{t_1} = \mu_{ar}^{t_2}$,

4)$\sum_{\bar{a} \in \bar{A}^{t_2}} \bar{c}_a^{t_2} > 0$ *and* $\forall \bar{a} \in \bar{A}^{t_2} : \bar{c}_a^{t_2} \geq 0$, $\forall a \in A^{t_2} : c_a^{t_2} = 0$

then $s \lhd \bar{s}$, *where* $\bar{s} = ((\bar{x}_a^*), (\bar{y}_b^*), (\bar{f}_r^*), \bar{p}^*)$ *is the final equilibrium state of the process* $\bar{\tau}_+(E_m)$, $s = ((x_a^*), (y_b^*), (f_r^*), p^*)$ *is the final equilibrium state of the process* $\tau_+(E_m)$.

Proof. **Part I.** According to the Assumption 4) in the process $\bar{\tau}_+(E_m)$ there exists a consumer $\bar{a} \in \bar{A}_i^{t_2}$, for whom $\bar{c}_{\bar{a}}^{t_2} > 0$ (the consumer $\bar{a}$ has nonzero loan), and in the process $\tau_+(E_m)$ each consumer has zero loans. According to the Assumptions 1)-4) and from the Theorem 6.1 it follows $x_{\bar{a}}^* < x_{\bar{a}}^*$ and $x_{\bar{a}}^* \prec_{\bar{a}} \bar{x}_{\bar{a}}^*$ and for each $a \in A$ $x_a^* \preccurlyeq_a \bar{x}_a^*$. It means that the Condition 1) from the Definition 5.3 is fulfilled.

Part II. If $\bar{s}$ is an equilibrium state then it satisfies market equilibrium equality. In particular it means that $proj_{\mathbb{R}^{l_R}}(\bar{x}^{t_2} - \bar{y}^{t_2} + \bar{f}^{t_2}) = proj_{\mathbb{R}^{l_R}}(\bar{\varpi}^{t_2})$, $\sum_{a\in A}\bar{s}_a^{t_2} + \sum_{r\in M}\bar{s}_r^{t_2} = 2\bar{\varpi}_s^{t_2}$, $\sum_{r\in M}\bar{c}_r^{t_2} + \sum_{a\in A}\bar{c}_a^{t_2} + \sum_{b\in B}\bar{c}_b^{t_2} = 3\bar{\varpi}_c^{t_2}$.

Analogously, if s is an equilibrium state (after a period of time t_2) then $proj_{R^{l_R}}(x^{t_2} - y^{t_2} + f^{t_2}) = proj_{R^{l_R}}(\varpi^{t_2})$, $\sum_{r\in M} c_r^{t_2} + \sum_{a\in A} c_a^{t_2} + \sum_{b\in B} c_b^{t_2} = 3\varpi_c^{t_2}$.

By the Assumption 2) we have: $proj_{R^{l_R}}(\bar{x}^{t_2} - \bar{y}^{t_2} + \bar{f}^{t_2}) = proj_{R^{l_R}}(x^{t_2} - y^{t_2} + f^{t_2})$, and by the Assumption 1): $proj_{R^{l_R}}(\bar{x}^{t_2} - \bar{y}^{t_2}) = proj_{R^{l_R}}(x^{t_2} - y^{t_2})$. Therefore, by the Part I of the proof ($x_{\bar{a}}^{*} < x_{\bar{a}}^{*}$) there is: $proj_{R^{l_R}}(-\bar{y}^{t_2}) \leq proj_{R^{l_R}}(-y^{t_2})$ so $proj_{R^{l_R}}(\bar{y}^{t_2}) \geq proj_{R^{l_R}}(y^{t_2})$.

Similarly: $\sum_{r\in M}\bar{c}_r^{t_2} + \sum_{a\in A}\bar{c}_a^{t_2} + \sum_{b\in B}\bar{c}_b^{t_2} = \sum_{r\in M} c_r^{t_2} + \sum_{b\in B} c_b^{t_2}$, according to the Assumption 1) we have: $\sum_{a\in A}\bar{c}_a^{t_2} + \sum_{b\in B}\bar{c}_b^{t_2} = \sum_{b\in B} c_b^{t_2}$. Hence $\sum_{b\in B} c_b^{t_2} - \sum_{b\in B}\bar{c}_b^{t_2} = \sum_{a\in A}\bar{c}_a^{t_2} > 0$ so $\sum_{b\in B} c_b^{t_2} > \sum_{b\in B}\bar{c}_b^{t_2}$.

As a result $\bar{y}^{t_2} > y^{t_2}$ and by taking into consideration the Assumption 2) there is: $\sum_{b\in B}\pi_b^{t_2}(p_m^{t_2}) = y^{t_2}p_m^{t_2} < \sum_{b\in B}\bar{\pi}_b^{t_2}(\bar{p}_m^{t_2}) = \bar{y}^{t_2}\bar{p}_m^{t_2}$ therefore Condition 2) of the Definition 5.3 is fulfilled.

Part III. By the Assumption 1) we have $\sum_{r\in M}\zeta_r^{t_2}(p_m^{t_2}) = \sum_{r\in M}\bar{\zeta}_r^{t_2}(\bar{p}_m^{t_2})$ so Condition 3) of the Definition 5.3 is fulfilled. ∎

According to Theorem 6.2 if during the process of diffusion of innovations in the given model of a Debreu monetary economy there is at least one consumer with a nonzero loan then the final equilibrium state is better than in case when each consumer has zero loans. In general, this theorem shows that the results of the process of diffusion depend on the preferences and behavior of the demand side of the economy.

7. CONCLUSIONS AND CHALLENGES

This paper is devoted to basic blocs of the theory of the process of diffusion of innovations concerning consumer loans and it analyzes human motives as important drivers of development. It draws on a methodological frame of innovative evolution of the Debreu monetary economy based on innovative and imitative activities of all agents and with consumers' behavior as the co-engine in this process.

Additional research is needed to determine how to illustrate a context of applicability of the constructed model. In particular it is worth while to analyze the agent-based simulation of the model to illustrate the theoretical results obtained in this paper and to study the empirical implication of the model.

However, it should be remembered that there are many disadvantages of using consumer credit and the main disadvantage is the cost. If a consumer fails to repay a loan or a credit card balance, this impacts his credit scores, affects terms and conditions, and results in late fees and penalties. Thus the analysis in this paper can be extended in the new direction in which the cost of credits is taken under consideration and consumer's behavior is based on expected utility.

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