

## Saving-Based Asset Pricing and Leisure

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This paper integrates two strands of the asset-pricing literature. Dreyer et al. (2013) developed and estimated a model of “saving-based” preferences that provides a plausible resolution of the equity premium paradox; Uhlig (2007) has emphasized the importance of incorporating labor supply into models of asset pricing. Here we analyze the implications for asset pricing of incorporating non-separable leisure into a model with saving-based preferences. We derive the Euler equations for this class of preferences and show that our parameter estimates are statistically significant, indicating that investors possess both preferences for savings and for leisure in the American economy.

*Key Words:* Equity premium puzzle; CCAPM; Leisure; Wealth; Saving-based preference; Asset pricing.

*JEL Classification Numbers:* G10, G11, G12.

### 1. INTRODUCTION

To the classical economists, saving was merely consumption postponed: One abstains from consumption now only in order to consume more in the future. Marshall (1920), offered a richer interpretation of why people save, suggesting that there might be pleasure in the very act of saving. To make his point, he told the following parable:

The extra pleasure which a peasant who has built a weatherproof hut derives from its use, while the snow is drifting into those of his neighbors who have spent less on building theirs, is the price earned by his working and waiting. It represents the extra productiveness of efforts wisely spent in providing against distant evils, or the satisfaction of future wants, as compared with what would have been derived from an impulsive grasping at immediate satisfactions.

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As another example, he described a retired physician who lends money to a factory to help it acquire some new machinery. The act of lending itself gives him pleasure. In modern parlance, saving itself might be an argument in the utility function, in addition to consumption. If we denote savings by  $s_t$ , we can express Marshall's idea succinctly by writing the utility function as  $u(c_t, s_t)$ , rather than the canonical  $u(c_t)$ . Dreyer et al. (2013) called these "saving-based preferences"<sup>1</sup> and developed a theory of asset-pricing in the presence of such preferences.

The authors showed that this theory in principle could explain the equity premium puzzle [Mehra and Prescott (1985)]:<sup>2</sup> Saving-based preferences cause the premium to depend upon the covariance of the risky rate of return with not only consumption growth, but also with the current and future growth of wealth. In other words, saving-based preferences add two extra "factors" to the premium, in addition to the familiar one from consumption growth. They estimated the Euler equation using the Generalized Method of Moments (GMM). A version of the model without human capital performed relatively well, and there was evidence that the preference for saving is significant. However, the estimated effective coefficient of relative risk aversion (RRA) was too low (only 10.5) to resolve the puzzle of the premium [in their original paper Mehra and Prescott (1985) required an RRA of 25 to explain it].

In this paper we enrich the model by using a utility function defined over both saving and leisure. The labor/leisure decision is a fundamental element of consumer behavior and plays a crucial role in explaining business cycles [Hansen (1985), Lettau and Uhlig (2000)]. Uhlig (2007) has also emphasized the need to incorporate leisure into asset-pricing models, since it constitutes a form of self-insurance that can alter the demand for risky assets. He finds empirical evidence supporting the idea that the long-run risk price caused by changes in leisure is large. Other authors also echo the need to put leisure in the investor's utility function [Campbell (1994), Chiarolla and Hausmann (2001), Lettau and Uhlig (2000), Dittmar and Palomino (2010), and Prescott (1986), *inter alia*]. Models of leisure with

<sup>1</sup>Gootzeit et al. (2002) originally called them "Marshallian recursive preferences," since their incarnation in continuous time constitutes a form of recursive utility. Other authors such as Cui et al. (2008), Cui and Gong (2008) and Dreyer (2012) develop upon this idea. We refer the reader to Dreyer et al. (2013) for a broader philosophical discussion of how these preferences can be interpreted and related to models of internal habit formation [Abel (1990), Carroll (2000), *inter alia*], the spirit of capitalism [Zou (1994), Smith (2001), *inter alia*], and prospect theory [Barberis et al (2001)].

<sup>2</sup>There have been notable attempts to solve the puzzle, using models of habit persistence [Campbell and Cochrane (1999)] and of uninsured idiosyncratic risks [Constantinides and Duffie (1996)]. However, these models still require implausibly high degrees of risk aversion in order to explain stock returns [Cochrane (2001)]. The puzzle is still a puzzle [Kocherlakota (1996), Mehra and Prescott (2003), Mehra (2008)].

non-separable utility, however generally do badly in explaining the equity premium [Eichenbaum et al. (1988), Lettau (2003)].

One might expect that introducing labor supply would raise the demand for risky assets and so lower the equity premium. In other words, it would make the puzzle even more puzzling. However, merging leisure with saving-based preferences helps better explain the equity premium puzzle in two ways. First, the presence of a risky wage adds yet another “factor” to the premium — the covariance of wage growth with the risky return. Second, it turns out the presence of having consumption, saving, and leisure in the utility function together dramatically increases the factor weight attached to the covariance of consumption growth with the risky return. Leisure enters directly into the stochastic discount factor along with consumption and saving. In this multi-variate utility function, the interaction of leisure with saving-based preferences generates a coefficient of relative risk aversion large enough to explain the puzzle.

We estimate the Euler equations for this model with GMM replicating the study of Dreyer et al. (2013), but with the inclusion of leisure. This yields strong empirical results: First, in line with what is suggested by the literature [Uhlig (2006, 2007), Lettau and Uhlig (2000), Dittmar and Palomino (2010), and Prescott (1986), *inter alia*], we verify that Americans possess a significant taste for leisure, as well as saving. Second, we show that endogenizing leisure in a saving-based model yields an appreciably higher estimate of effective relative risk aversion, large enough to explain the equity premium. Calibrating the model with the estimated preference parameters predicts an equity premium that is quite close to the observed premium.

## 2. THEORY

### 2.1. The Model

In each period  $t$  a consumer works  $n_t$  hours at an hourly wage of  $\omega_t$ . We normalize the endowment of time to 1 hour, so that his leisure is  $l_t = 1 - n_t$ . The wage may evolve stochastically, but the wage at time  $t$  is known at  $t$ . The investor can invest in either a bond or a stock (or a portfolio of stocks). The bond pays a risk-free rate of  $R_{t+1}^f$  in the next period, while the stock pays a risky return of  $R_{t+1}$ . His wealth then accumulates according to:

$$w_{t+1} = (w_t + \omega_t n_t - c_t) R_{t+1}^w, \quad (1)$$

where  $c_t$  is consumption,  $w_t + \omega_t n_t - c_t$  is cash-on-hand, and  $R_{t+1}^w = (1 - \lambda_t) R_{t+1}^f + \lambda_t R_{t+1}$  is the return of his portfolio, a weighted average of the returns of the financial assets where the cash on hand is invested.

The consumer's preferences differ from those of the canonical CCAPM in two ways:

- First, as in Dreyer et al. (2013), the consumer derives utility not only from consumption but also from the act of saving (the accumulation of wealth). Since saving represents the accumulation of wealth, we assume that it is the gross growth rate of wealth,  $w_{t+1}/w_t$ , that enters the utility function. We could also use the level of wealth instead of its growth rate, as in between others, such as Airaudo (2016). However, we decided to keep with the latter case in order to be consistent with Dreyer et al. (2013).
- Second, as in the models of Hansen (1985), Cochrane (1997), Lettau and Uhlig (2000), Lettau (2003), and many others, our consumer also derives utility from leisure. As mentioned above, we permit leisure to enter the utility function non-separably.

In sum, the period utility function is given by  $u(c_t, l_t, w_{t+1}/w_t)$ . From now on, we will use  $u_1$ ,  $u_2$  and  $u_3$  to denote the partial derivatives of the consumer's preferences (marginal utilities) with respect to consumption, leisure, and savings respectively. We assume that the marginal utility is positive ( $u_1, u_2, u_3 > 0$ ) and diminishing ( $u_{11}, u_{22}, u_{33} < 0$ ) for all three arguments; we do not restrict the signs of the cross-partials ( $u_{12}, u_{13}, u_{23}$ ), but do assume the utility function is strictly concave.<sup>3</sup>

In our empirical work we will use the following specification of preferences:

$$u\left(c_t, l_t, \frac{w_{t+1}}{w_t}\right) = \begin{cases} \frac{[c_t l_t^\varphi (\frac{w_{t+1}}{w_t})^\theta]^{1-\eta}}{1-\eta}, & \eta > 0, \eta \neq 1 \\ \ln c_t + \varphi \ln l_t + \theta \ln\left(\frac{w_{t+1}}{w_t}\right), & \eta = 1. \end{cases} \quad (2)$$

The parameters  $\varphi$  and  $\theta$  govern the strength of preferences for leisure and saving. If  $\varphi = \theta = 0$  we revert to the canonical CCAPM model with constant relative risk aversion (CRRA). If  $\varphi > 0$  but  $\theta = 0$  we have endogenous labor supply and no taste for saving, as in Uhlig (2007), among others; conversely, if  $\theta > 0$  but  $\varphi = 0$  there are saving-based preferences with exogenous labor supply, as in Dreyer et al. (2013). The parameter  $\eta$  is the coefficient of relative risk aversion with respect to the aggregator in braces.

There is a tricky question of interpretation here, since the parameters of Equation (2) affect both risk aversion and strength of preference over the

<sup>3</sup>It is common to see leisure modeled using separable preferences [Hansen (1985), Lettau (2003)]. Models of habit formation also normally assume separable preferences [Boldrin (2001)]. However, the use of separable preferences with leisure diminishes the possibilities for investors to smooth consumption. For us, the interaction of consumption, saving, and leisure in the stochastic discount factor is vital.

three variables.<sup>4</sup> To disentangle risk preferences from ordinal preferences over the three “goods”  $c_t$ ,  $l_t$ , and  $w_{t+1}/w_t$  we invoke the classic analysis of multivariate risk aversion developed by Kihlstrom and Mirman (1974, 1981). To this end rewrite Equation (2) as follows:

$$u\left(c_t, l_t, \frac{w_{t+1}}{w_t}\right) = \frac{\left[ c_t^{\frac{1}{1+\varphi+\theta}} l_t^{\frac{\varphi}{1+\varphi+\theta}} \left(\frac{w_{t+1}}{w_t}\right)^{\frac{\theta}{1+\varphi+\theta}} \right]^{(1-\eta)(1+\varphi+\theta)}}{1-\eta} \quad (3)$$

Kihlstrom and Mirman (1974, 1981) show that the exponent attached to the brackets governs risk aversion with respect to the aggregator inside the bracket, while the exponents on the terms in this aggregator then govern the relative “ordinal” preferences over consumption, leisure, and saving. The “effective” coefficient of relative risk aversion in this multivariate setting [see Smith (2001)] is then  $\Gamma = 1 - (1 - \eta)(1 + \varphi + \theta)$ . Notice that increases in the weights attached to either leisure or saving ( $\varphi$  and  $\theta$ ) will increase or decrease risk aversion depending upon whether  $\eta > \leq 1$ .

## 2.2. Consumption, Leisure and Portfolio Decisions

Define the consumer’s information set at time  $t$  as  $\Omega_t$ ; it contains all current and lagged data available in period  $t$ . The consumer chooses profiles of consumption and leisure to maximize expected lifetime utility,

$$\max \sum_{t=0}^{\infty} \beta^t E \left\{ u\left(c_t, l_t, \frac{w_{t+1}}{w_t}\right) \mid \Omega_t \right\}, \quad (4)$$

subject to the constraint given by Equation (1) and given initial financial wealth  $w_0$ . There is a formal solution for this problem in the Appendix; here we provide an intuitive explanation of it.

Consider first the consumption decision. How does consuming more today affect the utility of the investor? According to the structure of the saving-based preference with leisure, consuming one more unit today will increase the total utility of the investor by  $E \left\{ u_1\left(c_t, l_t, \frac{w_{t+1}}{w_t}\right) \mid \Omega_t \right\}$  and decrease it by  $\beta E \left\{ u_1\left(c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}}\right) R_{t+1}^w \mid \Omega_t \right\}$ . The decrease of future consumption or future leisure is explained by the decrease of a unit of current savings (used to increase current consumption) that could have earned  $R_{t+1}^w$ .

<sup>4</sup>Indeed, in this time-separable setting they also confound willingness to substitute over time with risk aversion and ordinal preferences over the different goods. It is possible to unravel these three distinct aspects of preferences by adopting a recursive preference structure [see Smith (2001) and Nocetti and Smith (2011)]. It is well known [Weil (1990)] that preferences for intertemporal substitution have no effect on the equity premium, so we adhere to a time-separable structure here in the interests of simplicity.

On the other hand, because our utility function also depends on savings, one extra unit of consumption today will impact the gross growth rate of wealth by  $\frac{R_{t+1}^w}{w_t}$ . Thus, since savings decrease, the current utility of the consumer also decreases by  $E \left\{ u_3 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \frac{R_{t+1}^w}{w_t} \middle| \Omega_t \right\}$ . However, because future consumption or future leisure decreases, saving in  $t + 1$  increases. The decrease of the gross rate of wealth in  $t + 2$  is given by  $\frac{w_{t+1}}{w_t} \frac{R_{t+1}^w}{w_t}$ . This implies that the consumer experiences a lower reduction in his discounted utility in  $t + 1$  because it depends on  $\frac{w_{t+2}}{w_{t+1}}$ . Thus, the reduction in discounted utility equals  $\beta E \left\{ u_3 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \frac{R_{t+1}^w}{w_{t+1}} \frac{w_{t+2}}{w_{t+1}} \middle| \Omega_t \right\}$ .

The consumer will choose consumption so that the expected marginal benefits and costs of doing so are equal. This yields the following first-order condition:

$$E \left\{ u_1 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) - u_3 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \frac{R_{t+1}^w}{w_t} \middle| \Omega_t \right\} = \beta E \left\{ \begin{array}{l} u_1 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) R_{t+1}^w \\ - u_3 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \frac{R_{t+1}^w}{w_{t+1}} \frac{w_{t+2}}{w_{t+1}} \end{array} \middle| \Omega_t \right\} \quad (5)$$

The same kind of argument applies to the labor supply decision. What if the investor chooses to vary leisure by one unit today, instead of consumption? This would mean that he would work one less hour today, which implies that he would gain current utility of  $E \left\{ E u_2 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \middle| \Omega_t \right\}$ . On the other hand, one hour less of work today would cost him a decrease in his salary of  $\omega_t$ . This would reduce current savings by  $\omega_t$  and future savings by  $\omega_t R_{t+1}^w$ , which would entail lower consumption or leisure level in the future. Thus, this difference in savings would decrease his inter-temporal utility by  $\beta E \left\{ u_2 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \frac{\omega_t R_{t+1}^w}{\omega_{t+1}} \middle| \Omega_t \right\}$ .

Since our utility function also depends on savings, one extra unit of leisure today will impact the gross growth rate of wealth by  $\frac{\omega_t R_{t+1}^w}{w_t}$ . Thus, since savings decrease, the current utility of the consumer also decreases by  $E \left\{ u_3 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \frac{\omega_t R_{t+1}^w}{w_t} \middle| \Omega_t \right\}$ . However, because future consumption or future leisure decrease, saving in  $t + 1$  increases. The decrease of the gross rate of wealth in  $t + 2$  is given by  $\frac{w_{t+1}}{w_t} \frac{\omega_t R_{t+1}^w}{w_t}$ . This implies that the consumer experiences a lower reduction in his discounted utility in  $t + 1$ , because it depends on  $\frac{w_{t+2}}{w_{t+1}}$ . The reduction in discounted utility therefore equals  $\beta E \left\{ \beta E u_3 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \frac{w_{t+2}}{w_{t+1}} \frac{\omega_t R_{t+1}^w}{w_{t+1}} \middle| \Omega_t \right\}$ .

The consumer will choose leisure to equate its expected marginal benefits and costs. This yields the first-order condition:

$$E \left\{ u_1 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) - u_3 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \frac{\omega_t R_{t+1}^w}{w_t} \middle| \Omega_t \right\} = \beta E \left\{ \begin{array}{l} u_1 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \frac{w_t R_{t+1}^w}{w_{t+1}} \\ - u_3 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \frac{w_{t+2}}{w_{t+1}} \frac{\omega_t R_{t+1}^w}{w_{t+1}} \end{array} \middle| \Omega_t \right\} \quad (6)$$

Comparing Equations (5) and (6), it is evident that in each period  $t$

$$\frac{Eu_2 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right)}{Eu_1 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right)} = \omega_t. \quad (7)$$

Intuitively, the consumer will equate the appropriately defined marginal rate of substitution between consumption and leisure to the wage in every period.<sup>5</sup>

Finally consider the portfolio decision. The rational investor calibrates his portfolio between risky stocks and the riskless asset until his expected utility is maximized. Raising the share of the risky asset a little bit increases the overall return premium  $E\{R_{t+1} - R_{t+1}^f | \Omega_t\}$  of the portfolio. This in turn increases wealth accumulation in time  $t$  and lowers it in time  $t + 1$ . Trading off the marginal costs and benefits implies that, for our extended saving-based preferences,

$$E \left\{ (R_{t+1} - R_{t+1}^f) \left[ \begin{array}{l} \beta u_1 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \\ + u_3 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \frac{1}{w_t} \\ - \beta u_3 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \frac{w_{t+2}}{w_{t+1}^2} \end{array} \right] \middle| \Omega_t \right\} = 0. \quad (8)$$

We can rewrite Equation (8) using the preferences described by Equation (2):

$$E \left\{ \left[ \begin{array}{l} (R_{t+1} - R_{t+1}^f) \left( \frac{w_{t+1}}{w_t} \right)^{\theta(1-\eta)} \\ \left[ \theta \frac{c_t}{w_t} \frac{w_t}{w_{t+1}} + \beta \left( \frac{c_{t+1}}{c_t} \right)^{\varphi(1-\eta)-\eta} \left( \frac{w_t}{w_{t+1}} \right)^{\varphi(1-\eta)} \left( \frac{w_{t+2}}{w_{t+1}} \right)^{\theta(1-\eta)} \left( 1 - \theta \frac{c_{t+1}}{w_{t+1}} \right) \right] \end{array} \right] \middle| \Omega_t \right\} = 0. \quad (9)$$

<sup>5</sup>Why “appropriately defined?” Notice that the wage does not equal the actual marginal rate of substitution  $u_2/u_1$  in each period, nor does it equal the expected marginal rate of substitution,  $E(u_2/u_1)$ . Instead it equals the ratio of the expected marginal utilities  $Eu_2/Eu_1$ . This occurs because the two marginal utilities are random at time  $t$ .

If we set  $\varphi = 0$ , this reduces to the Euler equation with saving-based utility but without leisure, Equation (10) of Dreyer et al. (2013); by setting both  $\varphi = \theta = 0$ , it reduces still further to the familiar  $E \left\{ (R_{t+1} - R_{t+1}^f) \left( \frac{c_{t+1}}{c_t} \right)^{-\eta} \middle| \Omega_t \right\} = 0$  of the CCAPM.

Equation (9) shows us that the stochastic discount factor is a function of the gross growth rates of consumption ( $c_{t+1}/c_t$ ), current wealth ( $w_{t+1}/w_t$ ), and future wealth ( $w_{t+2}w_{t+1}$ ) [as in Dreyer et al. (2013)], as well as upon that of wage income ( $\omega_{t+1}/\omega_t$ ). This has important implications for the equity premium. To see this, note that, for small growth rates and a small equity premium, Equation (9) can be expressed with the following approximation:<sup>6</sup>

$$\begin{aligned} & E(R_{t+1} - R_{t+1}^f) \\ &= \Omega_c \left( \frac{c_t}{w_t} \right) \text{cov} \left( \frac{c_{t+1}}{c_t}, R_{t+1} \middle| \Omega_t \right) \\ &+ \Omega_{w1} \left( \frac{c_t}{w_t} \right) \text{cov} \left( \frac{w_{t+1}}{c_t}, R_{t+1} \middle| \Omega_t \right) \\ &+ \Omega_{w1} \left( \frac{c_t}{w_t} \right) \text{cov} \left( \frac{w_{t+2}}{w_{t+1}}, R_{t+1} \middle| \Omega_t \right) \\ &+ \Omega_\omega \left( \frac{c_t}{w_t} \right) \text{cov} \left( \frac{\omega_{t+1}}{\omega_t}, R_{t+1} \middle| \Omega_t \right), \end{aligned} \quad (10)$$

where the omegas are (potentially time-varying) factor weights,<sup>7</sup>

$$\Omega_c \left( \frac{c_t}{w_t} \right) = \frac{\eta + \theta(1 - \eta)(1 + \varphi) \frac{c_t}{w_t} - \varphi(1 - \eta)}{\theta \frac{c_t}{w_t} + \beta \left( 1 - \theta \frac{c_t}{w_t} \right)} \quad (11)$$

$$\Omega_{w1} \left( \frac{c_t}{w_t} \right) = \theta \frac{c_t}{w_t} \frac{1 - \beta - \theta(1 - \eta)}{\theta \frac{c_t}{w_t} + \beta \left( 1 - \theta \frac{c_t}{w_t} \right)} \quad (12)$$

<sup>6</sup>Define the growth rates of consumption, current wealth, future wealth, and wage income by  $g_c = (c_{t+1} - c_t)/c_t$ ,  $g_{w1} = (w_{t+1} - w_t)/w_t$ ,  $g_{w2} = (w_{t+2} - w_{t+1})/w_{t+1}$ , and  $g_\omega = (\omega_{t+1} - \omega_t)/\omega_t$ , respectively. Equation (10) follows by taking a second-order Taylor series of Equation (9) around  $R_{t+1} - R_{t+1}^f = g_c = g_{w1} = g_{w2} = g_\omega = 0$ .

<sup>7</sup>We say ‘‘potentially’’ time-varying because the consumption/wealth ratio may be constant when innovations to consumption, wealth, and wage income are i.i.d.



$$\Omega_{w2} \left( \frac{c_t}{w_t} \right) = \beta \theta \frac{(1 - \eta) \left( 1 - \theta \frac{c_t}{w_t} \right)}{\theta \frac{c_t}{w_t} + \beta \left( 1 - \theta \frac{c_t}{w_t} \right)} \quad (13)$$

$$\Omega_{\omega} \left( \frac{c_t}{w_t} \right) = \beta \varphi \frac{(1 - \eta) \left( 1 - \theta \frac{c_t}{w_t} \right)}{\theta \frac{c_t}{w_t} + \beta \left( 1 - \theta \frac{c_t}{w_t} \right)}. \quad (14)$$

As predicted by the CCAPM, the equity premium should depend upon the covariance between consumption growth and the risky return. However, it should also depend upon the covariances of the rate of return with the growth of both current and future wealth and with the growth of wage income.

Notice that the factor weights attached to wealth growth disappear [Equations (12) and (13)] if there is no utility from saving ( $\theta = 0$ ), while the factor weight attached to wage growth [Equation (14)] disappears if there is no utility from leisure ( $\varphi = 0$ ). Notice also, from Equation (11), that the interaction of preferences for saving and leisure affects the factor weight on consumption growth. When we calibrate the model, we will see that, empirically, the tastes for saving ( $\theta$ ) and leisure ( $\varphi$ ) are large enough to cause a large increase in the factor weight attached to consumption growth, relative to the canonical model.

### 3. ESTIMATION

#### 3.1. Data

We will estimate the model using GMM. To facilitate comparison, we employ the same data for the United States from 1947 to 2009 as in Dreyer et al. (2013). Details about the data are available in that paper. Because of the empirical problems associated with the use of the series of leisure in the estimation of asset pricing models, however, we employ labor supply in our empirical tests — in other words, we simply assume that leisure is time spent not working. Following Mehra (2008) and his critique of the correct proxy for the riskless asset, we use the rates of return of long duration T-Bills as the riskless rate of return and the S&P 500 as a proxy for the risky market return.

#### 3.2. Estimation of the Euler Equations

Assuming the utility function of the investor is given by Equation (3), Equations (5), (7) and (8) imply the following Euler equations<sup>8</sup>:

<sup>8</sup>Notice that these Euler equations for Saving-Based preferences, equations (15) and (16), are similar to the Euler equation using recursive preferences of Epstein and Zin

$$E \left\{ \left( \frac{w_{t+1}}{w_t} \right)^{\theta(1-\eta)} \left[ \begin{array}{c} \theta \frac{c_t}{w_t} \frac{w_t}{w_{t+1}} + \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\eta} \left( \frac{\omega_t}{\omega_{t+1}} \right)^{\varphi(1-\eta)} \\ \left( \frac{c_{t+1}}{c_t} \right)^{\varphi(1-\eta)} \left( \frac{w_{t+2}}{w_{t+1}} \right)^{\theta(1-\eta)} \left( 1 - \theta \frac{c_{t+1}}{w_{t+1}} \right) \end{array} \right] R_{t+1} \middle| \Omega_t \right\} = 1$$

$$E \left\{ \left( \frac{w_{t+1}}{w_t} \right)^{\theta(1-\eta)} \middle| \Omega_t \right\} = 1 \quad (15)$$

$$E \left\{ \left( \frac{w_{t+1}}{w_t} \right)^{\theta(1-\eta)} \left[ \begin{array}{c} \theta \frac{c_t}{w_t} \frac{w_t}{w_{t+1}} + \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\eta} \left( \frac{\omega_t}{\omega_{t+1}} \right)^{\varphi(1-\eta)} \\ \left( \frac{c_{t+1}}{c_t} \right)^{\varphi(1-\eta)} \left( \frac{w_{t+2}}{w_{t+1}} \right)^{\theta(1-\eta)} \left( 1 - \theta \frac{c_{t+1}}{w_{t+1}} \right) \end{array} \right] R_{t+1}^f \middle| \Omega_t \right\} = 1$$

$$E \left\{ \left( \frac{w_{t+1}}{w_t} \right)^{\theta(1-\eta)} \middle| \Omega_t \right\} = 1 \quad (16)$$

We will focus our study on these two equations by estimating their parameters with GMM following the method of Dreyer et al. (2013) in order to have consistency in the comparison of our results. As the those authors do, we restrict our parameters to “economically significant boundaries”:  $\beta = [.9; 1.5]$ ,  $\eta = [.01; 100]$ ,  $\varphi = [.01; 100]$ ,  $\theta = [.01; 100]$  We define the different alternatives for our vector of instruments using the following variables:

$$z = \left[ k, \frac{c_t}{c_{t-1}}, \frac{c_{t-1}}{c_{t-2}}, \frac{w_t}{w_{t-1}}, \frac{w_{t-1}}{w_{t-2}}, \frac{\omega_t}{\omega_{t-1}}, \frac{\omega_{t-1}}{\omega_{t-2}}, R_t^f, R_{t-1}^f, R_t, R_{t-1} \right]$$

We select the vector of instruments of the GMM in a systematic way. We start by selecting alternatives of vectors with few instruments. By doing so, we can identify the instrument which causes the rejection of the GMM estimates in case the model is rejected by the J statistics. This way, we add only relevant instruments, so that the condition of identification of the GMM is met. Therefore, alternatives with significant J statistics are ignored. Table 1 gives us the alternatives of instruments used in our estimations.

Before we run our estimations, it is important to make sure that our variables are stationary. However, when running the Augmented Dickey-Fuller (ADF) test for our variables, we observe that two variables are not stationary in our data period, namely return on government bonds and the relation between consumption and wealth. In order to deal with this

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(1991). Both specifications of preferences generate Euler equations that include the growth rate of wealth.

**TABLE 1.**

Alternatives of vectors of instruments used in the GMM estimations.

Alternatives	Instrument Vector
1	$z = \left[ k, \frac{c_t}{c_{t-1}}, \frac{\omega_t}{\omega_{t-1}}, \frac{\omega_{t-1}}{\omega_{t-2}}, R_t^f, R_{t-1}^f \right]$
2	$z = \left[ k, \frac{c_t}{c_{t-1}}, \frac{\omega_t}{\omega_{t-1}}, R_t^f, R_{t-1}^f, R_t \right]$
3	$z = \left[ k, \frac{c_t}{c_{t-1}}, \frac{\omega_t}{\omega_{t-1}}, \frac{\omega_{t-1}}{\omega_{t-2}}, R_t^f, R_{t-1}^f, R_t, R_{t-1} \right]$
4	$z = \left[ k, \frac{c_t}{c_{t-1}}, \frac{c_{t-1}}{c_{t-2}}, \frac{\omega_t}{\omega_{t-1}}, \frac{\omega_{t-1}}{\omega_{t-2}}, R_t^f, R_t, R_{t-1} \right]$
5	$z = \left[ k, \frac{c_t}{c_{t-1}}, \frac{c_{t-1}}{c_{t-2}}, \frac{\omega_t}{\omega_{t-1}}, \frac{\omega_{t-1}}{\omega_{t-2}}, R_{t-1}^f, R_t, R_{t-1} \right]$
6	$z = \left[ k, \frac{c_t}{c_{t-1}}, \frac{c_{t-1}}{c_{t-2}}, \frac{\omega_t}{\omega_{t-1}}, \frac{\omega_{t-1}}{\omega_{t-2}}, R_t^f, R_{t-1}^f, R_t, R_{t-1} \right]$

problem, we remove some periods in the beginning and in the end of the data sample exactly as in Dreyer et al. (2013). The final ADF test for our variables is provided in Table 2. Notice that, after leaving part of the series out of our sample, we can reject the null hypothesis of non-stationarity for all model variables at the 10% significance level.

**TABLE 2.**

Augmented Dickey-Fuller test of stationarity for the model's variables.

Variables	$\frac{c_{t+1}}{c_t}$	$\frac{\omega_{t+1}}{\omega_t}$	$R_{t+1}^f$	$R_{t+1}$	$\frac{w_{t+1}}{w_t}$	$\frac{w_{t+2}}{w_{t+1}}$	$\frac{c_t}{w_t}$
p-values	0.06	0.02	0.01	0.01	0.01	0.01	0.09

Now that we have verified stationarity of our variables, we can proceed with the GMM estimations. We use the covariance matrix as weighting matrix of the GMM in order to find the most efficient estimates with a Parzen kernel according to Smith (2004). In the first step, for every selection of instruments the residuals of our estimations presented a high level of autocorrelation, so we decided to apply a pre-whitening correction according to Andrews and Monahan (1992). We used their “improved HAC covariance matrix” for our estimations with an autoregressive vector of order one. Because our model is highly nonlinear, we choose to use the cumulative updating method to run the GMM using the R-project according to Chaussé (2010). Results for our GMM estimations are provided in Table 3, where all coefficients are presented in annual terms:

Observing the estimation results in Table 3, none of the alternatives of instruments are rejected by the J-statistic of the GMM, which implies that the identification condition of the GMM method is satisfied for them. Besides, none of the estimated parameters have averages above or below

**TABLE 3.**

Estimation results.

Alternatives	Estimation				p-values				
	Parameters				$H0 = 0$				
	$\beta$	$\eta$	$\theta$	$\varphi$	J-test	$\beta$	$\eta$	$\theta$	$\varphi$
1	0.922	4.89	4.62	6.86	0.12	< 0.01	< 0.01	< 0.01	< 0.01
2	0.988	5.06	3.77	9.96	0.26	< 0.01	< 0.01	< 0.01	0.055
3	0.975	5.45	3.91	3.23	0.25	< 0.01	< 0.01	< 0.01	0.02
4	0.981	5.53	4.00	2.39	0.13	< 0.01	< 0.01	< 0.01	0.05
5	0.962	5.18	4.29	2.67	0.14	< 0.01	< 0.01	< 0.01	0.04
6	0.979	5.40	4.22	2.25	0.17	< 0.01	< 0.01	< 0.01	0.05

the boundary restriction (corner results), what implies that the estimation errors are minimized for our different alternatives<sup>9</sup>.

All our coefficients are statistically significant at the 10% level. The estimated Betas have a reasonable average of around 0.968, which is the same value found by Dreyer et al. (2013).  $\eta$ ,  $\theta$ , and  $\varphi$  have averages of 5.25, 4.13 and 4.56 respectively. Besides, these coefficients imply an effective coefficient of relative risk aversion [Kihlstrom and Mirman (1974, 1981) Smith (2001)], of  $\Gamma = 1 - (1 - \eta)(1 + \varphi + \theta)$ , which equals 42. This number is higher than that in Dreyer et al. (2013), which was only 10.5. Significant estimates for  $\varphi$  and  $\theta$  give evidence that the tastes for savings and leisure exist and are positive in the American economy, and that these variables should not be ignored in asset pricing.

The canonical equity premium puzzle arises from the fact that the observed premium is far too large to be explained by conventional estimates of relative risk aversion. Our estimate of an effective coefficient of relative risk aversion of 42 is sufficiently large to explain the paradox. What the saving-based model with leisure does is, by adding the richer preference structure, to endogenously generate an effective CRRA that actually matches what it would take to explain the equity premium.

<sup>9</sup>One of the major issues related to the GMM is the so-called problem of weak instruments. When weak instruments are present, it is very common that the estimated parameters vary significantly according to the vector of instruments selected. Notice that in our estimations, results are relatively stable across alternatives of instruments. Another common problem related to the GMM technique in the nonlinear setting is that estimations may vary a lot according to the starting point selected for the GMM algorithm. In the case of our estimations, the starting point of the GMM for our parameters was (1, 1, 1, 1). We tested different starting points and results were stable. For more details on the discussion of issues of the GMM technique to the Saving-Based model, see Dreyer et al. (2013).

#### 4. CALIBRATION

To investigate whether the model can resolve the equity premium paradox<sup>10</sup>, consider the following calibration. The preference parameters are set equal their average estimated values; the covariances of consumption, current and future wealth growth, and the growth of salary income with the risky return are calculated as the corresponding unconditional covariances from the data. These parameters and covariances are given in Table 4.

**TABLE 4.**

Calibration

$\beta$	.968
$\eta$	5.25
$\theta$	4.13
$\varphi$	4.56
$\frac{c}{w}$	.1513
$cov\left(\frac{c_{t+1}}{c_t}, R_{t+1}\right)$	.00045
$cov\left(\frac{w_{t+1}}{w_t}, R_{t+1}\right)$	.00636
$cov\left(\frac{w_{t+2}}{w_{t+1}}, R_{t+1}\right)$	.00047
$cov\left(\frac{\omega_{t+1}}{\omega_t}, R_{t+1}\right)$	.00075

Now reconsider the Taylor-series approximation of the premium provided by Equation (10). Using the calibrated values from Table 4, the model generates factor weights of  $\Omega_c = 9.91$ ,  $\Omega_{w1} = 11.12$ ,  $\Omega_{w2} = -6.44$ , and  $\Omega_\omega = -6.94$ . Using these weights, the model predicts a premium of  $E(R - R^f) = 9.91 * .00045 + 11.12 * .00636 - 6.44 * .0047 - 6.95 * .00075 = .066$ . Compare this to the actual, average equity premium from the data,  $E(R - R^f) = .059$ . The premium predicted by the model with leisure matches the equity premium fairly well. This seems to happen for two reasons: First,

<sup>10</sup>Part of the recent literature would advise us to test our model against first and second moments of stocks and bonds returns (Bansal and Yaron, 2004). In order to test the second moments, it is popular to use so-called variance bounds (Alvarez and Jermann, 2005). In this case, the discount factor is divided into permanent and transitory components and the implied variance bounds are analyzed. However, for the same reasons as explained by Dreyer et al. (2013) we do not make use of this technique. The reasons for not doing so are twofold: 1) Alvarez and Jermann (2005) assumed that the discount factor can be used to price at least one short-term bond, one long-term bond and one stock. Thus, an important role is played by short-term bonds. We follow the recommendation of the literature not to use short-term bonds, since these types of bonds are used neither for consumption smoothing nor for inter-temporal decisions [see more details in Dreyer et al. (2013)]. Thus, these tests are not compatible with our estimates. 2) Variance bounds tests require the stochastic discount factor to consist of a constant subjective discount factor  $\beta$  that is multiplied by a stochastic function. Our discount factor does not follow this simple multiplicative form.

note that in the absence of preferences for saving and leisure ( $\theta = \varphi = 0$ ), the factor weight on consumption growth ( $\Omega_c$ ) would only be 5.25. So, tastes for saving and leisure nearly double the effect of the covariance of consumption growth with the risky return. Second, the taste for saving ( $\theta = 4.13$ ) is so strong as to generate a very large factor weight on the growth of wealth ( $\Omega_{w1}$ ).

## 5. CONCLUSION

According to Uhlig (2007) models need to take leisure into account when pricing assets, since consumers derive utility not only from consumption, but also through leisure. In this paper we propose a version for the Saving-Based Asset Pricing (Dreyer et al. 2013) with leisure. Thus, our utility depends upon consumption, savings and non-separable leisure.

We establish three key empirical results:

- We provide evidence that the taste for leisure is significant and positive in the American economy and thus this variable should not be ignored in asset pricing models. This reinforces the importance of leisure for asset-pricing asserted by Uhlig (2007).
- We provide evidence that the taste for saving is also significant and positive in America. This is consistent with what was showed in the model without leisure in Dreyer et. al (2013).
- The comparison of the coefficient of constant relative risk aversion of 42 of the model with leisure with 10.5 from Dreyer et al. (2013) shows that the use of leisure in the model decreases the volatility of the discount factor. This calls for a higher coefficient of relative risk aversion in order to explain the equity premium.

The saving-based preference model with leisure takes us a step further along the path to explaining the equity premium puzzle, since it implies that the equity premium should depend not only on the covariance of the risky return with consumption growth and savings, but also on its covariance with salary growth. As a suggestion for future research, we believe that this version for the saving-based preference should be analyzed more deeply, perhaps combining it with models of habit formation or prospect theory.

## APPENDIX: DERIVING THE MODEL

There are two state variables for this problem, wealth and the wage rate. Let us therefore define the value function at time  $t$  as  $V(w_t, \omega_t)$ . Denote the partial derivatives of this function with respect to wealth and the wage

respectively by  $V_1$  and  $V_2$ . The Bellman equation for the problem is then:

$$V(w_t, \omega_t) = \max_{c_t, l_t, \omega_t} E \left\{ u \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \middle| \Omega_t \right\} + \beta E \{ V(w_{t+1}, \omega_{t+1}) \middle| \Omega_t \} \quad (\text{A.1})$$

Wealth accumulates according to

$$w_{t+1} = [w_t + \omega_t(1 - l_t) - c_t] R_{t+1}^w, \quad (\text{A.2})$$

where  $R_{t+1}^w = (1 - \lambda_t) R_{t+1}^f + \lambda_t R_{t+1}$ .

Now maximize Equation (A.1) with respect to  $c_t$ ,  $l_t$ , and  $\lambda_t^*$ . This yields the following first-order conditions:

$$\begin{aligned} & E \left\{ u_1 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) - E u_3 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \frac{R_{t+1}^w}{w_t} \middle| \Omega_t \right\} \\ &= \beta E \{ V_1(w_{t+1}, \omega_{t+1}) R_{t+1}^w \middle| \Omega_t \} \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} & E \left\{ u_2 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) - E u_3 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \frac{\omega_t}{w_t} R_{t+1}^w \middle| \Omega_t \right\} \\ &= \omega_t \beta E \{ V_1(w_{t+1}, \omega_{t+1}) R_{t+1}^w \middle| \Omega_t \} \end{aligned} \quad (\text{A.4})$$

$$(w_t + \omega_t l_t - c_t) E \left\{ \left[ \begin{array}{c} u_3 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \frac{1}{w_t} \\ + \beta V_1(w_{t+1}, \omega_{t+1}) \end{array} \right] (R_{t+1} - R_{t+1}^f) \middle| \Omega_t \right\} = 0 \quad (\text{A.5})$$

If  $w_t + \omega_t l_t - c_t \neq 0$  then (A.5) simplifies to

$$E \left\{ \left[ u_3 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \frac{1}{w_t} + \beta V_1(w_{t+1}, \omega_{t+1}) \right] (R_{t+1} - R_{t+1}^f) \middle| \Omega_t \right\} = 0 \quad (\text{A.6})$$

Differentiate Equation (A.1) with respect to  $w_t$  and apply the envelope theorem:

$$\begin{aligned} V_1(w_t, \omega_t) &= E \left\{ u_3 \left( c_t^*, l_t^*, \frac{w_{t+1}^*}{w_t^*} \right) \frac{w_t^* R_{t+1}^w - w_{t+1}^*}{w_t^{*2}} \middle| \Omega_t \right\} \\ &+ \beta E \{ V_1(w_{t+1}, \omega_{t+1}) R_{t+1}^w \middle| \Omega_t \} \end{aligned} \quad (\text{A.7})$$

Substituting Equation (A.7) into Equation (A.3) reveals that

$$\begin{aligned} V_1(w_t, \omega_t) &= E \left\{ u_1 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \middle| \Omega_t \right\} \\ &- E \left\{ u_3 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \frac{w_{t+1}}{w_t^2} \middle| \Omega_t \right\}. \end{aligned} \quad (\text{A.8})$$

Leading Equation (A.8) one period ahead leads to

$$\begin{aligned} V_1(w_{t+1}, \omega_{t+1}) &= E \left\{ u_1 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \middle| \Omega_{t+1} \right\} \\ &\quad - E \left\{ u_3 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \frac{w_{t+2}}{w_{t+1}^2} \middle| \Omega_{t+1} \right\}. \end{aligned} \quad (\text{A.9})$$

Combining Equations (A.9) and (A.3) yields

$$\begin{aligned} &E \left\{ u_1 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \middle| \Omega_t \right\} \\ &- E \left\{ u_3 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \frac{R_{t+1}^w}{w_t} \middle| \Omega_t \right\} \\ &= E \left\{ u_1 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) R_{t+1}^w \middle| \Omega_t \right\} \\ &- \beta E \left\{ u_3 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \frac{w_{t+2}}{w_{t+1}^2} R_{t+1}^w \middle| \Omega_t \right\}. \end{aligned} \quad (\text{A.10})$$

This is the consumption Euler equation, Equation (5) in the text.

Now substitute Equation (A.7) into Equation (A.4), which reveals that

$$\begin{aligned} V_1(w_t, \omega_t) &= E \left\{ u_2 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \frac{1}{\omega_t} \middle| \Omega_t \right\} \\ &\quad - E \left\{ u_3 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \frac{w_{t+1}}{w_t^2} \middle| \Omega_t \right\}. \end{aligned} \quad (\text{A.11})$$

Leading this one period ahead yields

$$\begin{aligned} V_1(w_{t+1}, \omega_{t+1}) &= E \left\{ u_2 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \frac{1}{\omega_{t+1}} \middle| \Omega_t \right\} \\ &\quad - E \left\{ u_3 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \frac{w_{t+2}}{w_{t+1}^2} \middle| \Omega_t \right\}. \end{aligned} \quad (\text{A.12})$$

Combining Equations (A.12) and (A.4) yields

$$\begin{aligned} &E \left\{ u_2 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \middle| \Omega_t \right\} \\ &- E \left\{ u_3 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \frac{R_{t+1}^w}{w_t} \omega_t \middle| \Omega_t \right\} \\ &= \beta E \left\{ u_2 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \frac{\omega_t}{w_{t+1}} R_{t+1}^w \middle| \Omega_t \right\} \\ &- \beta E \left\{ u_3 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \frac{w_{t+2}}{w_{t+1}} \frac{R_{t+1}^w}{w_{t+1}} \omega_t \middle| \Omega_t \right\} \end{aligned} \quad (\text{A.13})$$



This is Equation (6) in the text.

However, from equations (A.9) and (A.12) it follows that

$$\begin{aligned} & E \left\{ u_1 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \middle| \Omega_t \right\} \\ &= E \left\{ u_2 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \frac{1}{\omega_{t+1}} \middle| \Omega_t \right\}. \end{aligned} \quad (\text{A.14})$$

In other words, the wage in time  $t + 1$  equals the marginal rate of substitution. This implies that Equation (A.13) can be rewritten as

$$\begin{aligned} & E \left\{ u_2 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \middle| \Omega_t \right\} \\ &- E \left\{ u_3 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \frac{\omega_t R_{t+1}^w}{w_t} \middle| \Omega_t \right\} \\ &= \beta E \left\{ u_1 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \omega_t R_{t+1}^w \middle| \Omega_t \right\} \\ &- \beta E \left\{ u_3 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \frac{w_{t+2}}{w_{t+1}^2} \omega_t R_{t+1}^w \middle| \Omega_t \right\} = 0. \end{aligned} \quad (\text{A.15})$$

Now, combining equations (A.10) with (A.5)

$$E \left\{ (R_{t+1} - R_{t+1}^f) \left[ \begin{array}{l} \beta u_1 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \\ + u_3 \left( c_t, l_t, \frac{w_{t+1}}{w_t} \right) \frac{1}{w_t} \\ - \beta u_3 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \frac{w_{t+2}}{w_{t+1}^2} \end{array} \right] \middle| \Omega_t \right\} = 0. \quad (\text{A.16})$$

This is Equation (8) in the text.

Finally, it follows immediately from Equations (A.3) and (A.4) that

$$\frac{E \left\{ u_2 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \middle| \Omega_t \right\}}{E \left\{ u_1 \left( c_{t+1}, l_{t+1}, \frac{w_{t+2}}{w_{t+1}} \right) \middle| \Omega_t \right\}} = \omega_t. \quad (\text{A.17})$$

This is Equation (7) in the text.

If we parameterize Equations (A.10), (A.15), and (A.16) with the preferences in Equation (3), they imply:

$$E \left\{ \left( \frac{w_{t+1}}{w_t} \right)^{\theta(1-\eta)} \left[ \begin{array}{ccc} \theta \frac{c_t}{w_t} \frac{w_t}{w_{t+1}} + \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\eta} \left( \frac{\omega_t}{\omega_{t+1}} \right)^{\varphi(1-\eta)} & & \\ \left( \frac{c_{t+1}}{c_t} \right)^{\theta(1-\eta)} \left( \frac{w_{t+2}}{w_{t+1}} \right)^{\theta(1-\eta)} & & \left( 1 - \theta \frac{c_{t+1}}{w_{t+1}} \right) \end{array} \right] R_{t+1} \left| \Omega_t \right\} = 1$$

$$E \left\{ \left( \frac{w_{t+1}}{w_t} \right)^{\theta(1-\eta)} \left| \Omega_t \right\} \right. \quad (A.18)$$

$$E \left\{ \left( \frac{w_{t+1}}{w_t} \right)^{\theta(1-\eta)} \left[ \begin{array}{ccc} \theta \frac{c_t}{w_t} \frac{w_t}{w_{t+1}} + \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\eta} \left( \frac{\omega_t}{\omega_{t+1}} \right)^{\varphi(1-\eta)} & & \\ \left( \frac{c_{t+1}}{c_t} \right)^{\varphi(1-\eta)} \left( \frac{w_{t+2}}{w_{t+1}} \right)^{\theta(1-\eta)} & & \left( 1 - \theta \frac{c_{t+1}}{w_{t+1}} \right) \end{array} \right] R_{t+1}^f \left| \Omega_t \right\} = 1$$

$$E \left\{ \left( \frac{w_{t+1}}{w_t} \right)^{\theta(1-\eta)} \left| \Omega_t \right\} \right. \quad (A.19)$$

These two equations are equivalent to Equations (15) and (16) in the text, which are the focus of our empirical estimation.

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