ANNALS OF ECONOMICS AND FINANCE **22-2**, 255–287 (2021)

Fiscal Policy Perceptions in a Behavioral New Keynesian Model*

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We study misperceptions of fiscal policy in a New Keynesian model based on the imperfect cognition framework of Gabaix (2020), where agents have limited ability to forecast future macroeconomic variables. We derive three main insights. First, we document the failure of Ricardian equivalence under misperception as the main transmission channel of shocks. Second, monetary policy has a weakened impact on macroeconomic variables when compared with the full information case. Third, we propose an endogenization of the cognitive parameters as an extension to Gabaix (2020). Robustness analysis reveals how households' cognitive ability plays a dominant role in affecting the size of macroeconomic fluctuations.

Key Words: Imperfect Cognition; Euler-equation; Phillips curve; Monetary Policy; Fiscal Policy; Ricardian Equivalence; Macroeconomic Fluctuations.
JEL Classification Numbers: E12, E31, E43, E52, E62, E70.

1. INTRODUCTION

In this article, we provide a simple and intuitive introduction to the derivation and the mechanics of BeNK, the **Be**havioral **N**ew **K**eynesian model based on the exposition in Gabaix (2020).We focus on the key concepts that make this modelling approach 'behavioral'. We then apply this behavioral framework to the standard New Keynesian model whereby we contrast and compare the two approaches in terms of how BeNK can change our understanding of macroeconomic policy and our interpretation of macroeconomic data. We specifically focus on the role of public debt and fiscal policies as they affect the perceptions of households.

* The views expressed in this paper are those of the authors and should not be interpreted as those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

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1529-7373/2021

We develop these ideas in the standard New Keynesian (NK) model where agents are characterized by imperfect perceptions about the future evolution of the economy's aggregates; that is, they have limited ability to forecast future variables, such as income, interest rates and inflation. We can think of this environment as one where the agents are limited - bounded - by some informational friction. In this specific case, agents are assumed to be myopic to small macroeconomic disturbances. Although agents are boundedly rational, they employ the tools, reasoning, and equilibrium concepts inherent to fully rational expectations models.

This assumption of cognitive misperception has non-trivial implications for the evolution of aggregate macro variables. We show that the extent of macroeconomic fluctuations generated by various shocks is dependent on the degree of rationality assumed for both households and firms although qualitatively the differences to the fully rational New Keynesian model are small. One key discrepancy is, however, the behavior of debt and deficits since Ricardian equivalence does no longer hold in BeNK. This channel works through a modified household Euler-equation that reflects myopia over the future evolution of fiscal variables or, in other words, fiscal misperceptions.

Based on Gabaix (2020) we present a full derivation of the model in log-linear terms. We extend the model to include three types of shocks: a technological shock, a public expenditure shock and a monetary policy shock. We also introduce explicit fiscal misperceptions on the household side. This implies that public debt affects the Euler-equation equation explicitly, which is markedly different from the standard New Keynesian model. Arguably, this introduces a new dimension for the analysis of the interaction between fiscal and monetary policy. Moreover, it presents another example of failure of Ricardian equivalence, without resorting to distortionary taxation, finite lifetimes, or other assumptions. Debt policies have real effects via real interest rate changes, which works as an additional transmission mechanism.

A second contribution is that we derive the full endogenous solution for the cognitive parameters for both households and firms. This generalization can be easily extended to more complex models by using numerical solution methods usually adopted to solve DSGE models. We simulate the model conditional to shocks and to several sets of behavioral parameters. The model is also simulated under the fully endogenized values obtained for the cognitive parameters as a function of the underlying core parameters of the model. We find that the resulting calibration is similar to one of the more extreme ones we considered with exogenous cognition parameters. It is an issue for future research to determine how plausible this parameterization is.

Our paper contributes to a recent stream of work that has challenged the assumption of fully rational agents in New Keynesian models. It is similar to the rational inattention approach pioneered by Sims (2003) where economic actors face a capacity constraint for processing information. It is also similar to the discounted-Euler equation approach where households characterized by a myopic behavior discount future consumption more heavily. With behavioral, not fully rational agents, movements in interest rates thus imply a weakened impact on today's variables. These effects are smaller the higher the degree of myopia is among households. Related work along these lines is Eusepi and Preston (2011), Massaro (2013), and García-Schmidt and Woodford (2019).

The paper is organized as follows. We lay out the building blocks of the standard New Keynesian model in section 2, which serves as the background for our introduction of behavioral aspects in section 3. We first present an overview of the behavioral approach following Gabaix (2020). We then derive the behavioral Euler-equation, the behavioral Phillips curve, and introduce fiscal perceptions. In section 4 we conduct our main qualitative analysis, where we calibrate and simulate the model for a set of shocks. We differentiate between homogenous and heterogenous perceptions and study robustness with respect to the other structural parameters of the model. Section 5 introduces our endogenization of the heretofore exogenous perception parameters. Section 6 concludes. An appendix at the end of the paper contains some of the proofs of the main results contained in the text. An additional appendix available online contains supplementary materials.[1]

2. A STANDARD NEW KEYNESIAN MODEL

We first lay out the structure of the NK model for reference purposes since the formulation of the BeNK model is based on it. Derivations are well-known and we refer to the standard literature for further details. The model contains three types of agents, a representative household, a continuum of monopolistically firms, and a set of policymakers, namely the central bank and the fiscal authority. The optimization problem of the household results in an output Euler-equation, interchangeably refereed to as the IS-curve, the firms' problem results in a Phillips-curve governing inflation dynamics, and the model is closed by specifying the behavior of the policy authorities in terms of feedback rules.

[1] The appendix can be found at: https://www.richmondfed.org/research/people/lubik

The household maximizes the intertemporal utility function:

$$U_t = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right), \tag{1}$$

subject to the budget constraint:

$$P_t C_t + P_t T_t + B_t = R_{t-1} B_{t-1} + W_t L_t + \Pi_t, \tag{2}$$

where C_t is consumption, L_t is employment for the nominal wage W_t, B_t is a risk-free nominal government bond paying a gross nominal interest rate R_t, T_t is a lump-sum tax, and the aggregate price level is P_t. The household is the residual claimant to firms' aggregate profits Π_t. $0 < \beta < 1$ is the discount factor, $\sigma > 0$ is the coefficient of relative risk aversion, and $\eta > 0$ is the labor supply elasticity. $\mathbb{E}_t$ is the standard rational expectations operator.

The first-order conditions imply a labor-leisure trade-off: $L_t^{1/\eta} = C_t^{-\sigma} W_t / P_t$ and a consumption Euler-equation: $C_t^{-\sigma} = \beta R_t \mathbb{E}_t C_{t+1}^{-\sigma} \pi_{t+1}^{-1}$. The model in levels shows all the trade-offs traditionally highlighted by the literature. In particular, from the Euler equation we again find the pivotal role played by *ex-ante* real interest, given by $R_t \mathbb{E}_t \pi_{t+1}^{-1}$ rate in deciding the intertemporal allocation of consumption between period t and $t+1$, together with the intertemporal elasticity of substitution σ. In log-linear terms, the Euler equation is:

$$\mathbb{E}_t \tilde{C}_{t+1} = \tilde{C}_t + \frac{1}{\sigma} \mathbb{E}_t (\tilde{R}_t - \tilde{\pi}_{t+1}) \tag{3}$$

where each variable with $\sim$ indicate its log-deviation from non-stochastic steady state.

On the firm side, the production technology is: $Y_t = A_t L_t^{\alpha} - \Phi$, where Y_t is output, A_t is an exogenous productivity shock, and Φ is fixed cost. Firms are monopolistically competitive and have pricing power over their final output. Final goods are imperfectly substitutable among an infinite set of different varieties indexed by $i \in [0, 1]$. Each i-th variety of final goods is produced by an i-th firm and they are aggregated via the traditional CES Dixit-Stiglitz aggregator $C_t = \left[\int_0^1 C_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$, where $\theta > 1$ is the elasticity of substitution among different final goods varieties.[2] From this we can derive a demand function for variety i, $Y_t(i) = \left[\frac{p_t(i)}{P_t} \right]^{-\theta} Y_t$, where

[2]An analogous aggregator can be defined for public expenditure G_t, which can be thought of as an aggregate term comprised of each i-th varieties, indicated by $G_t(i)$, for all i in $[0, 1]$.

demand $Y_t(i) = C_t(i) + G_t(i)$, and Y_t is aggregate demand. Similarly, $p_t(i)$ is the price ofthe i-th variety, while $P_t = \left(\int_0^1 p_{it}^{1-\theta} di\right)^{\frac{1}{1-\theta}}$ is the aggregate price index. Assuming Calvo-type price setting firms in each period can reset their prices with probability $1-\zeta$, while with probability ζ, they keep their price fixed from one period to the next.

The optimal pricing problem of a monopolistically competitive firm is to maximize profits:

$$E_t^{br} \sum_{k=0}^{\infty} (\zeta\beta)^k \lambda_{t+k} \left[P_{t,t+k} Y_{i,t+k} - MC_{t+k}(i) Y_{i,t+k}\right], \tag{4}$$

by choosing an optimal price $P_{t,t+k}$ subject to the demand function for its variety and given marginal cost $MC_{t+k}(i)$ from its production process. After lengthy derivations that are standard and well known in the literature we arrive at the New Keynesian Phillips curve:

$$\tilde{\pi}_t = \beta E_t \tilde{\pi}_{t+1} + \lambda \tilde{mc}_t, \tag{5}$$

where the coefficient λ is given by $\lambda = \frac{(1-\zeta\beta)(1-\zeta)}{\theta}$.

Monetary policy is given by an interest-rate feedback rule, where the central bank adjusts the nominal interest to changes in output and inflation relative to their target levels:

$$\frac{R_t}{R} = \left(\frac{\pi_t}{\pi}\right)^{\phi\pi} \left(\frac{Y_t}{Y}\right)^{\phi Y} \exp\left(\varepsilon_t^{MP}\right). \tag{6}$$

Similarly, fiscal policy is given by a feedback rule, where the fiscal authority adjusts the level of taxation to outstanding government debt:

$$T_t = \psi^B R_{t-1} B_{t-1}. \tag{7}$$

The model is closed by the specification of the government budget constraint:

$$B_t = R_{t-1} B_{t-1} + P_t G_t - P_t T_t,$$

where G_t is exogenous government expenditure. Combining the household budget constraint with the government budget constraint implies the standard macroeconomic equilibrium condition:

$$C_t + G_t = Y_t \tag{8}$$

Finally, the model includes three shocks: the productivity shock $A_t/A = (A_{t-1}/A)^{\rho_A} \exp(\varepsilon_t^A)$, the expenditure shock $G_t/G = (G_{t-1}/G)^{\rho_g} \exp(\varepsilon_t^g)$, and the monetary policy shock ε_t^{MP}. We assume throughout that the innovations to the shock processes i.i.d. normally distributed with zero mean and constant variances σ_x^2. As is standard in the literature, we consider a symmetric equilibrium, where all producers of differentiated products make identical pricing decisions. This results in the standard New Keynesian 3-equation system for the analysis of monetary policy and its impact on output, inflation, and the interest rate.

3. A BEHAVIORAL NEW KEYNESIAN MODEL

We now introduce behavioral aspects into the standard NK model following Gabaix (2020). The resulting BeNK model has the same structure as the standard NK counterpart. Intertemporal household decisions are governed by an Euler-equation, firms' pricing decisions are represented by a Phillips-curve and monetary and fiscal policies are implemented via feedback rules. There are three key differences to the standard environment. First, expectations are not fully rational but are behavioral in the sense that information is not fully revealed to the agents and more distant events appear fuzzier. This translates into higher cognitive discounting of forward-looking behavior and the introduction of perception parameters in the structural relationships. Second, the perception parameters change the cross-equation and cross-coefficient restrictions implied by the standard model and thereby the dynamic behavior of the model variables in a potentially non-trivial manner. Third, the structure of the equations in the BeNK model is altered to include additional variables and driving processes that do not necessarily appear in the standard NK. It is this last feature specifically that affects how the model reacts to fiscal policy.

In the following, we first introduce some general ideas about the behavioral macroeconomics approach. We then show in more detail how a behavioral Euler-equation and a behavioral Phillips-curve is derived in this context. In terms of policy analysis we then focus on introducing behavioral perceptions of fiscal policy, which we use to close the model in a general equilibrium sense.

3.1. General Approach

Behavioral macroeconomics attempts to incorporate the idea that in contrast to rational expectations (RE) environments with fully informed agents expectation formation is not necessarily based on perfect knowledge about the future evolution of all economically relevant variables. Instead, agents can have a perception of reality that is fuzzy and possibly not fully forward-looking. Under rational expectations, each individual agent has perfect perfection about the possible future evolution of all variables. However, there are likely variables over which it might be difficult to formulate expectations in a RE sense. Such behavior can be characterized in terms of discounting information available to a comparable fully rational agent. Many behavioral models share this feature, ranging from models with rationally inattentive agents to agents performing signal extraction or simple learning. Conceptually, this cognitive discounting leads to decision rules that are superficially similar to their standard RE counterparts but they correct for the behavioral aspect.

In order to make a behavioral macroeconomic model internally consistent, two key elements are needed. First, an expectation formation mechanism that is consistent with the informational structure of the model economy, while boundedly rational. More specifically, the resulting deviations from fully rational expectations are not ad hoc but are subject to internal discipline and consistency. Gabaix (2014, 2017) shows how to derive an appropriate expectations operator via the concept of sparse dynamic programming. Second, a researcher still has to make assumptions on the agents' information set, or specifically, what economically relevant information the agents perceive. Limited information equilibria can still be perfectly consistent with rational expectations, so that behavioral macroeconomics in addition assumes behavior that follows a default state. We now provide a high-level discussion of these two aspects in turn.

Consider a generic vector of variables X_{t+k}, $k \geq 0$. Let $h(X_{t+k})$ be a generic function of X_{t+k}, with $h(0) = 0$. Define $\mathbb{E}_t$ as the expectation operator of a full RE agent. Generically, we describe behavioral expectations, that is, the beliefs of a behavioral agent, by the "discounted" equation:

$$\mathbb{E}_t^{BR}\left[h(X_{t+k})\right] = m^k \mathbb{E}_t\left[h(X_{t+k})\right], \text{for all } k \geq 0, \tag{9}$$

where $\mathbb{E}_t^{BR}$ is the subjective behavioral expectations operator and BR stands for 'bounded rationality'. The parameter m captures the degree of cognitive discounting, namely the degree by which each agent perceives reality. An agent with rational expectations is characterized by the parameter value $m = 1$. When $m \in (0, 1)$, the agent has a distorted perception. The more distant events in the future are, the more fuzzy they appear to the agent. This reflects a limited ability in formulating expectations given the full set of available information. Effectively, the parameter m dampens the effect of future expectations, thereby implying a discounted Euler-equation, for instance.[3]

[3]What distinguishes this framework from generic learning environments, say, is that agents know the structure of the model, including the laws of motion of exogenous and endogenous variables and the value of parameters. Similarly to rational inattention environments, agents do not pay full attention to all available information; instead their information set is sparse and information revelation is subject to cognitive bounds. However, these frameworks typically impart persistence because of their reliance on past information, which changes the structure of the reduced form.

Similarly, we consider the following linear evolution of a state vector X_t, which typically includes the pre-determined variables and the laws of motions of exogenous shock processes:

$$X_{t+1} = G(X_t, \epsilon_{t+1}) = \Gamma X_t + \epsilon_{t+1}, \tag{10}$$

where Γ is a matrix of coefficients and $G(\cdot)$ is a generic transition function. The second equality reflects that we consider linearized versions of underlying non-linear models. The vector ϵ_t represents innovations to the dynamic system which are typically assumed to have zero mean and are *i.i.d.* The evolution of any control variables Y_t of the dynamic model, that is, the set of non-pre-determined variables can then be described by $Y_t = F(X_t, \epsilon_t)$, where F is some function. The state equation (10) encapsulates all information that is potentially available to the agent. In this specific behavioral environment, we assume that the cognitively limited agent perceives the state with a modified version of this equation, namely as a distorted law of motion for the set of variables X_t:

$$X_{t+1} = MG(X_t, \epsilon_{t+1}) = M(\Gamma X_t + \epsilon_{t+1}), \tag{11}$$

where the matrix M contains the corresponding cognitive discounting parameters. The expected value of (11) for a boundedly rational agent is thus given by:

$$\mathbb{E}_t^{BR}(X_{t+1}) = M\Gamma X_t, \tag{12}$$

which after iterating over time results in: $\mathbb{E}_t^{BR}(X_{t+k}) = M^k \Gamma X_t$. The impact of the current state on (the expectation of) future outcomes is therefore discounted by M^k relative to its full-information RE counterpart. In that sense, behavioral macroeconomics results in the idea of a "discounted Euler-equation".[4]

The second aspect of behavioral macro makes assumptions about the informational environment that agents operate in. The main assumption is that they normally spend time in a default state, where all variables are anchored to their non-stochastic steady state except for the aggregate state variables. This is a form of rational inattention (albeit not optimally derived as in Sims, 2003) that attempts to replicate actual behavior and information processing. That is, agents on average (or in normal times) follow policy functions consistent with the steady state and only track a subset of state variables that they respond to. This also implies that agents do not necessarily react to all incoming shocks directly. It is similar to assuming a turnpike approach to consumption, say, where the single agent consumes a constant fraction of income or wealth.

[4]Detailed expositions, especially the mathematical proofs, are to be found chiefly in Gabaix (2017).

In summary, the specific behavioral macroeconomics approach we implement makes behavioral assumptions about agents' policy functions, about their perception of the stochastic environment, and a model-consistent expectation formation mechanism. In what follows we embed the structure of cognitive discounting into the standard New Keynesian model.

3.2. A Behavioral Euler Equation

The Euler-equation in a New Keynesian model describes the time path of output as a function of the real rate of interest and aggregate shocks. Ultimately, it is derived from the intertemporal consumption-savings decision of a household as laid out above. The behavioral Euler-equation is no different. However, it rests on the assumption that agents only track some key variables in their decision process and thereby react to the shocks only indirectly through their effects on these variables. This puts households into their default state much of the time. In the aggregate, evolution of economy-wide variables can then be expressed in deviations from the default state. In combination with the appropriate behavioral expectation operator this leads to a discounted Euler-equation.

Specifically, we assume that households are fully informed about and observe their labor income (wage and hours worked), but that they cannot correctly perceive aggregate income or the evolution of the future interest rate. Under these assumptions, it is straightforward to show that consumption in the default state is:

$$C_t^d = \frac{r}{R} b_t + Y - T, \tag{13}$$

where C_t^d is the level of consumption in the default state and Y and T are the steady-state levels of aggregate income and taxes. We relegate the derivation of this expression to the online appendix. Constant Y and T are artifacts of the limited perception assumption, while time variation in real debt stems from the correct perception of household-labor income. Default consumption thus only moves with real debt and is somewhat akin to hand-to-mouth consumers, which have high consumption volatility since they cannot engage in smoothing to the extent that fully unconstrained or fully perceptive agents can.

The next step in deriving the behavioral consumption Euler-equation is to express the policy function for consumption in terms of the default state and expectations of future income streams. This essentially involves iterating the intertemporal consumption-saving decision forward within a sparse dynamic programming setting. Details can be found in the online appendix. The intertemporal consumption function is thus:

$$C_t = C_t^d + \mathbb{E}_t^{br} \sum_{s=0}^{\infty} \left(\frac{1}{R}\right)^{t+s} \left[\bar{b}_r \widetilde{R}^{br}(Z_{t+s}) + \frac{r}{R} \widetilde{Y}^{br}(Z_{t+s}, L_{t+s})\right], \quad (14)$$

where $\bar{b}_y = \frac{r}{R}$, $\bar{b}_r = \frac{1}{R^2}\left(\frac{r}{R} b_0 - \frac{C_0}{\sigma}\right)$. $\widetilde{Y}$, $\widetilde{R}$, indicate deviations from the steady state of each variable, while $Y^{br}(Z_t, L_t)$ is perceived individual income.[5] Aggregate consumption can therefore be decomposed in consumption in the default state and deviations from it that are driven by expected and discounted perceptions of future fluctuations in income. Behavioral consumers therefore still engage in consumption smoothing, albeit to a lesser degree.

We now introduce imperfect cognition. We assume that each individual has a distorted view about the aggregate level of income, but a correct view about their future wage income. As in Gabaix (2020), we assume that $\mathbf{Y}^{br}(Z_t) = \mathbf{Y}^{br}(L(Z_t), Z_t) = m_y Y(Z_t)$, where $m_y \in [0, 1]$ is an attention parameter that indexes the degree of misperception. In addition, we assume that aggregate labor income is incorrectly perceived by households. Specifically, we posit that household income increases with individual-level effort L_t being above the aggregate $L(Z_t)$. The perceived level of income can then be expressed as:

$$Y^{br}(Z_t, L_t) = m_y Y(Z_t) + W(Z_t)(L_t - L(Z_t)). \quad (15)$$

Similarly, we assume that the household does not perfectly perceive future interest rate paths. Consequently, there is a difference between the true interest rate $r(Z_t)$ and the perceived one $r^{br}(Z_t)$. This implies: $r^{br}(Z_t) = m_r r(Z_t)$, where $m_r \in [0, 1]$ is an attention parameter. Finally, we assume that at a steady state there are no perception differences, namely that individual and aggregate variables coincide.

We now combine these expressions and log-linearize around the steady state. This results in the following behavioral Euler-equation for output:

$$\widetilde{Y}_t = M\mathbb{E}_t \widetilde{Y}_{t+1} + \eta_R \widetilde{R}_t + \eta_G \widetilde{G}_t + \eta_b \widetilde{b}_t + \eta_T \mathbb{E}_t \widetilde{T}_{t+1}, \quad (16)$$

where all variables are expressed in log-deviations from their respective steady states. The precise coefficients are reported in the appendix. The structure of the equation is similar to the standard NK model. Current

[5]We define the set of shocks hitting the economy as Z_t. We indicate aggregate variables as $Y(Z_t)$, while variables at the level of an individual agent are simply denoted by Y_t. For instance, $L(Z_t)$ is the aggregate labor supply and $Y(Z_t) = Y(L(Z_t), Z_t)$ is the aggregate income. In case of a single agent with limited perception we write $Y(Z_t) = Y(L_t, Z_t)$.

output $\widetilde{Y}_t$ is determined by expected output $\mathbb{E}_t\widetilde{Y}_{t+1}$ and the real interest rate $\widetilde{R}_t$, and is affected by exogenous shocks hitting the economy, in this case government expenditure $\widetilde{G}_t$. There are two key differences, however. The Euler-equation is discounted in the sense that $M < \beta$ and that the responsiveness of output to the interest rate $\eta_R < \sigma$ is changed. Perhaps more importantly, output dynamics now depend on the level of debt $\widetilde{b}_t$ and expected taxes $\mathbb{E}_t\widetilde{T}_{t+1}$. Ultimately, this stems from a failure of Ricardian equivalence in this behavioral setting. Households do not fully perceive the effect that higher government purchases today have on future tax collection or that, similarly, current debt levels are backed by future tax surpluses via the intertemporal government budget constraint (assuming Ricardian fiscal policy as in Leeper, 1991). In order to close the model, we would therefore have to specify tax policy explicitly.

3.3. A Behavioral Phillips Curve

The Phillips curve (PC, henceforth) in an NK model rests on the assumption of price stickiness. This idea carries over to the behavioral Phillips curve when we introduce cognitive discounting. Specifically, we assume that the decisionmakers at firms do not pay full attention to the evolution of the future state of the economy. Much of the derivation of the behavioral version follows the standard analytics in the NK price-setting problem. We therefore relegate most steps to the online appendix and only focus on the novel aspects introduced by the behavioral environment.

Price-setting is fully under control of each firm producing variety i. The optimal choice of relative price $P_{t,t+k}$ is therefore found by maximizing:

$$\mathbb{E}_t^{BR}\sum_{k=0}^{\infty}(\zeta\beta)^k\lambda_{t+k}\left[P_{t,t+k}Y_{i,t+k} - MC_{t+k}(i)Y_{i,t+k}\right], \qquad (17)$$

subject to the demand function for $Y_{i,t+k}$ and the definition of $P_{t,t+k}$. $MC_{t+k}(i)$ is marginal cost associated with the production of the respective variety. The presence of behavioral aspects manifests itself via the use of the operator $\mathbb{E}_t^{BR}$, defined as before. The decision-relevant variables for the firm's managers are thus perceived imperfectly, and more so the further distant they are from the present.[6] At the same time, the structural relationships underlying the production and sales processes are perfectly known and understood, so that any deviation from RE stems from discounting alone.

[6] This view is supported by empirical evidence on the inflation perceptions of managers and CFOs, who should arguably be well informed of the aggregate financial environment, but seem not to be. See Coibion et al. (2018) and Coibion et al. (2020) for further discussion, analysis and evidence.

The (log-linearized) optimality condition for price-setting thus reduces to:

$$\mathbb{E}_t^{BR}\sum_{k=0}^{\infty}(\zeta\beta)^k\widetilde{P}_{t,t+k} = E_t^{BR}\sum_{k=0}^{\infty}(\zeta\beta)^k\widetilde{mc}_{t+k}. \tag{18}$$

Following the usual steps for Calvo-price setting, we can then derive the behavioral Phillips curve:

$$\widetilde{\pi}_t = \beta M^f E_t\widetilde{\pi}_{t+1} + \lambda\widetilde{mc}_t, \tag{19}$$

where the coefficients are given by: $M^f = m\left[\zeta + (1-\zeta)m_\pi\right]$ and $\lambda = \frac{(1-\zeta\beta)(1-\zeta)}{\theta}m_x$. The equation has the same structure as the standard Phillips-curve. What is different are the coefficients. The perception parameter for inflation m_π reduces the importance of expected inflation in driving current inflation (as does overall perceptions m). Similarly, output perceptions indexed by m_x reduce the PC coefficient λ relative to its RE baseline and thereby make the Phillips curve flatter. Put differently, all things being equal inflation becomes less volatile and in that sense more sticky when perceptions of the future are imperfect. Finally, we note that for $m = m_\pi = m_x = 1$, the behavioral PC reduces to the standard PC.

3.4. Introducing Fiscal Policy Perceptions

We now tackle the third component of the model, namely the specification of fiscal policy under imperfect perceptions about its future path. This is the main contribution of our paper as it extends Gabaix (2020) by allowing the interest rate on outstanding public debt to vary. Fiscal misperceptions affect household behavior since they are the only ones subject to taxation and it thereby changes the construction of the IS curve. While the functional form of the Euler Equation does not differ markedly from the generic version derived above the coefficients reflect imperfect perceptions about future fiscal policy.

We begin with a reformulation of the log-linearized version of the government budget constraint:

$$b_{t+1} = Rb_t + RR_t + g_bG_{t+1} - T_bT_{t+1}. \tag{20}$$

$R = 1 + r$ is the steady-state value of the gross real rate of return, with r the net rate; the coefficients are defined as: $g_b = G/b$, $T_b = T/b$, where we assume $b \neq 0$. We rewrite the GBC as:

$$b_{t+1} = b_t + rb_t + RR_t + g_bG_{t+1} - T_bT_{t+1} = b_t + d_{t+1}, \tag{21}$$

where we define the public deficit d_{t+1} for convenience as:

$$d_{t+1} = rb_t + RR_t + g_bG_{t+1} - T_bT_{t+1}, \tag{22}$$

namely inclusive of interest service on the outstanding debt. Using $b_{t+T} = b_t + \sum_{i=1}^{T} d_{t+i}$ we can then express the GBC in terms of future taxes and the evolution of the deficit:

$$T_{t+T+1} = \gamma_r R_{t+T} + \gamma_b b_t + \gamma_g G_{t+T+1} + \frac{1}{T_b}\left(\sum_{i=1}^{T} d_{t+i} - \gamma_d d_{t+T+1}\right), \quad (23)$$

with $\gamma_r = R/T_b$, $\gamma_b = r/T_b$, $\gamma_g = g_b/T_b = (G/b)(b/T) = G/T$, $\gamma_d = 1/T_b$. In a Ricardian economy, the behavior of the public deficit thus allows each agent to infer the future evolution of taxes.

In contrast, an agent who is limited in his perception of the future evolution of fiscal policy resorts to expecting fiscal policy to follow a *default* tax policy. We thus specify for our behavioral setting the following rule:

$$T^d_{t+1} = \gamma_b b_t. \quad (24)$$

While this rule is superficially akin to fiscal policy in the Leeper (1991) and Sims (1994) sense, namely that the government sets taxes as a function of last period's real debt, the default tax function is, in fact, a guideline for the agent in making forecasts about future level of tax pressure. Crucially, we assume that perceptions of future fiscal actions are informed only by the existing level of public debt. This misperception is evident from comparing the default tax function (24) with the actual law of motion (23).

We can now set up the fiscal perception equation as follows. The perceived law of motion for future taxes T^{br} is anchored to the level of public debt:

$$\begin{aligned} T^{br}\left(H_{t+T}, H^d_t\right) &= (1 - m_y)T(H^d_t) + m_y T(H_{t+T}) \\ &= T(H^d_t) + m_y\left[T(H_{t+T}) - T(H^d_t)\right]. \end{aligned} \quad (25)$$

Consequently, the level of future taxes perceived by the agent is:

$$\begin{aligned} &\mathbb{E}_t\left[T^{br}\left(H_{t+T}, H^d_t\right)\right] \\ &= \gamma_b b_t + m_y m^{T+1}\left(\gamma_r R_{t+T} + \gamma_r G_{t+T+1} + r\sum_{i=1}^{T} d_{t+i} - d_{t+T+1}\right) \end{aligned} \quad (26)$$

Perceptions about future taxes are anchored by the level of initial debt: the level of taxation is a distorted difference between the future level and the past behavior of fiscal deficits.

We can now use equation (26) to derive a behavioral Euler equation:

$$Y^{br}(Z_t, L_t) = m_y Y(Z_t) + \alpha(L_t - L(Z_t)) - T_y T^{br}(Z_t), \quad (27)$$

for all $t > 0$, where all variables are expressed in log-deviations from their steady state. The Euler equation with imperfect cognition about future fiscal policy is then:

$$Y_t = \mu_{b\tau} b_t - \mu_{R\tau} R_t + \mu_{G\tau} G_t + M_\tau \mathbb{E}_t Y_{t+1} - \mu_d \mathbb{E}_t d_{t+1}, \tag{28}$$

where coefficients are reported in the appendix. The full details of the derivation of equation (28) are in the online appendix.

We can also rewrite equation (28) in terms of the output gap as before:

$$X_t = -\eta_{R\tau}(i_t - \pi_{t+1} - R_t^n) + M_\tau X_{t+1}, \tag{29}$$

where the output gap is $X_t = Y_t - Y_t^n$ and the real natural rate of interest is defined as:

$$R_t^n = \eta_{GN} G_t + \eta_{AN} A_t + \frac{\mu_{b\tau}}{\mu_{R\tau}} b_t^n - \frac{\mu_d}{\mu_{R\tau}} \mathbb{E}_t d_{t+1}^n. \tag{30}$$

The coefficients are given by $\eta_{GN} = \frac{1}{\mu_{R\tau}} (\mu_{G\tau} + M_\tau \mu_g \rho_g - \mu_g)$ and $\eta_{AN} = \frac{\mu_a}{\mu_{R\tau}}(\rho_a - 1)$. We note that the natural real rate explicitly depends on fiscal variables, public debt b_t and the deficit d_{t+1}. This implies that a change in the fiscal stance also affects the level of the natural real rate.

4. MODEL ANALYSIS: FISCAL POLICY INTERACTIONS

We now study the dynamic behavior of the economy in the presence of cognitive discounting. To that end, we subject the model to three aggregate shocks: a productivity shock, a monetary policy shock, and a government expenditure shock, which are stand-ins for, respectively, supply, demand and fiscal shocks. As argued above, under cognitive discounting Ricardian equivalence need no longer apply. The question still remains, however, how quantitatively important this is. We also assess the robustness of the responses to a wide range of parameterizations.

For reference, we report the model equations below, where all variables are expressed in log-linear deviations from their non-stochastic steady state. Variable definitions are as above and the exact expressions of the coefficients are given in the appendix:

$$M\mathbb{E}_t Y_{t+1} + \mu_\tau \mathbb{E}_t T_{t+1} = Y_t - \eta_b b_t - \eta_R i_t - \eta_G G_t, \tag{31}$$

$$\beta M^f \mathbb{E}_t \pi_{t+1} = \pi_t - kY_t + k\mu_g G_t + k\mu_a A_t, \tag{32}$$

$$i_t = \phi_\pi \pi_t + \phi_y Y_t + \zeta_t^m, \tag{33}$$

$$T_t = \rho_\tau T_{t-1} + \phi_1 \frac{b_{t-1}}{\pi_t}, \tag{34}$$

$$b_{t+1} = (i_t - \mathbb{E}_t \pi_{t+1}) b_t + G_t - T_t. \tag{35}$$

The first two equations are the Euler-equation and the Phillips curve, respectively, followed by the monetary policy and fiscal policy reaction functions. The last equation is the law of motion for real debt. The model specification is completed by the monetary policy shock process: $\zeta_t^m = \rho_{mp}\zeta_{t-1}^m + \varepsilon_t^{mp}$, the productivity shock: $A_t = \rho_a A_{t-1} + \varepsilon_t^a$, and the government expenditure shock: $G_t = \rho_g G_{t-1} + \varepsilon_t^g$. The innovations ε_t to each process are assumed to be *i.i.d.* normally distributed with zero mean and constant variance.

We choose a standard benchmark parameterization and then assess its robustness. We set the discount factor β to 0.99. The share of labor in the production function α corresponds to 67 percent of total output. The Frisch elasticity of labor supply parameter η is set equal to 1. The degree of risk aversion σ is set equal to 0.5 capturing a high degree of intertemporal substitution. In the sensitivity exercise, we test the robustness of our results to different values of both η and σ. The elasticity of substitution across differentiated goods $\theta = 6$ reflects a moderate degree of monopoly power. Finally, we choose a Calvo-parameter of $\zeta = 0.75$. The response of tax revenue with respect to real debt ϕ_1 is set to 0.2, a value lying within the interval defining passive fiscal policy, as discussed by Leeper (1991). The persistence in tax revenue, identified by ρ_τ is set at 0.6. The parameter values are summarized in Table 1.

TABLE 1.

Parameter Values

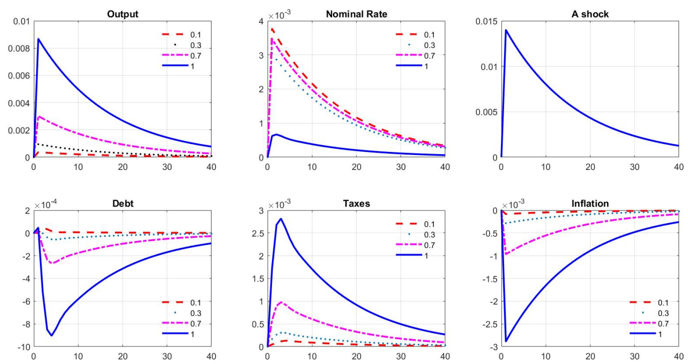
β	α	η	σ	θ	ζ	ϕ_1	ρ_τ	ρ_A	ρ_G	ρ_{mp}	ρ_τ	σ_A^2	σ_G^2	σ_{mp}^2
0.99	0.67	1	0.5	6	0.75	0.2	0.6	0.95	0.6	0.7	0.6	0.007	0.1	0.24

4.1. Homogenous Perception

We first analyze the economy's dynamics when households and firms have the same degree of behavioral inattention. We compare the various degrees of inattention in the figures below to the baseline of full attention as in the standard NK model. The solid line represents the case of behavioral parameters for both households and firms set equal to 1, indicating the full rationality case: $m = m_\pi = m_r = m_x = m_y = 1$, as well as $M = M^f = 1$. The dash-dotted line is for $m = m_\pi = m_r = m_x = m_y = 0.7$ and $M = M^f = 0.7$, the dotted line for $m = m_\pi = m_r = m_x = m_y = 0.3$ and $M = M^f = 0.3$, the dashed one for $m = m_\pi = m_r = m_x = m_y = 0.1$, $M = M^f = 0.1$. The responses to the three shocks are reported in Figures 1 - 3.

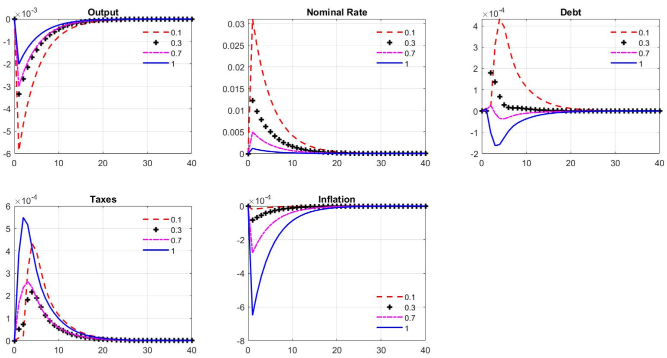
The responses to a unit productivity shock in Figure 1 reveal that the behavior for the different degrees of perception are qualitatively the same. Output, the nominal rate, taxes all increase after the shock, while inflation and debt decline. The interest rate response is driven by the policy rule, given the reaction of output. Lower inflation raises the debt burden, which triggers a positive tax response that feeds back on a lower adjustment path for debt. Under cognitive discounting the responses are much dampened, with the exception of the policy rate, the latter being driven by the relative strength (and the thus implied trade-off) of the output and inflation responses. Agents perceive the supply shock as having less of an effect on debt since the implied horizon over which they understand the intertemporal budget constraint to operate is shortened, i.e., discounted. Consequently, this triggers a weaker tax response. With very limited cognitive ability (when all behavioral parameters are at 0.1), debt can rise in response to the supply shock.

FIG. 1. Impulse Response Function, homogeneous perceptions, technological shock



In Figure 2, we report impulse responses to a contractionary monetary policy shock. The baseline response in the NK model is that output and inflation decline. Tax revenues rise to maintain intertemporal budget balance, which goes hand in hand with a decrease in debt. Under imperfect cognition the response of the former variables is qualitatively the same, while debt holdings increase. In that scenario, monetary policy is more effective in terms of a stronger output response since household perceive debt as a hedge against reduced production. As before, this does not fully wash out because of the limited perception of future tax increases. Just as before, the behavior of debt in response to shocks can serve as an indicator of the strength of cognitive discounting.

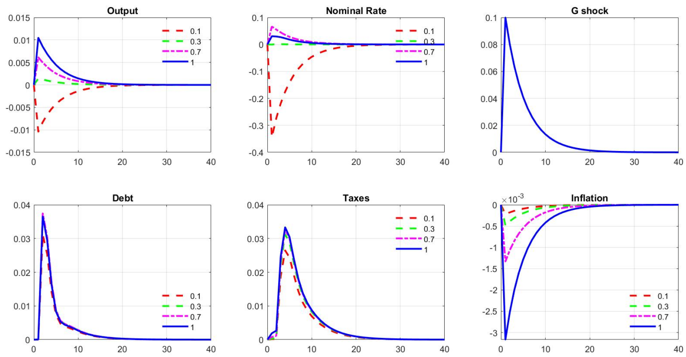
FIG. 2. Impulse Response Function, homogenous perceptions, monetary policy shock



This insight is partially confirmed by the responses to an exogenous government expenditure shock as in Figure 3. An increase of G_t in the baseline NK framework leads to an expansion of output via the usual effect on labor supply: workers anticipate higher taxes in the future because of the largely debt-financed government expansion and therefore supply more labor on account of a wealth effect. This triggers a fall in inflation and a small rise in the interest rate. Interestingly, under cognitive discounting the time paths of debt and taxes are largely identical (although the different scale between the figures can be misleading). What matters, however, is the perception of future tax increases to satisfy intertemporal budget balance. With limited perception workers feel richer than they actually are and increase labor supply by not as much. This weakened wealth effect smooths the output and inflation response and can turn the former even negative in the case of $m = 0.1$.

To summarize, the impulse response analysis demonstrates that the behavioral effects from cognitive discounting largely operate through a wealth effect; or to paraphrase, government bonds are, in fact, net wealth when agents perceive distant events less clearly than under full rational expectations. Consequently, the intertemporal budget constraint has less bite. Notably, these effects appear to make a quantitative difference when the degree of cognitive discounting is large, that is, when the perception co-

FIG. 3. Impulse Response Function, homogenous perceptions, public Expenditure shock



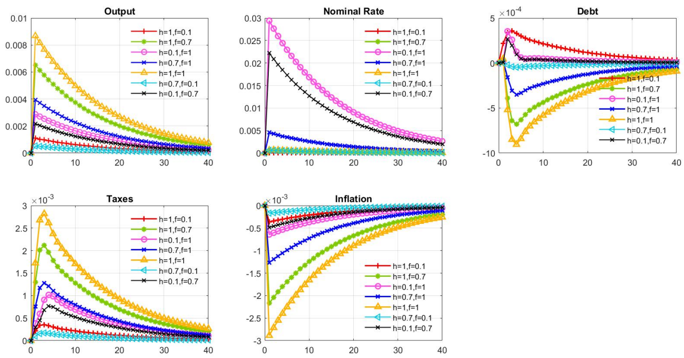
efficients m are small. In subsequent sections, we will therefore consider modifications to the basic framework to assess robustness of this conclusion.

4.2. Heterogenous Perception

Households h and firms f can harbor different degrees of cognitive ability. In the following, we deviate from the assumption of homogeneous cognitive ability in Figures 1-3 by introducing different degrees of rationality. In the following simulations we alternatively fix h and f to assume different degrees of cognitive discounting. Specifically, $h = 1$, $f = 1$ indicates the benchmark case, which is equivalent to setting the cognitive parameters to unity. For instance, when $h = 0.7$, $f = 1$, firms are fully rational but households have a limited cognitive ability: in this case, the behavioral parameters are $M^f = m_\pi = m_x = 1$, for firms and $M = m = m_r = m_y = 0.7$, for households.

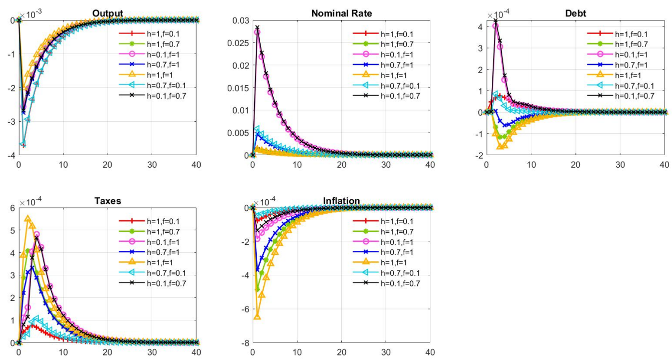
We first consider the productivity shock, reported in Figure 4, where we focus on a very limited cognition ability, equal to 0.1, assigned in turn to households and firms, an intermediate one (0.7), and a full rationality case. We find that the largest responses are associated with fully rational behavior, except for the nominal interest rate, for which the largest response occurs under limited perception $h = 0.1$, $f = 1$. The general observation from this exercise is that a lower degree of rationality implies a lower size of the responses for all variables. The discriminating factor is the behavior of debt. In the benchmark, real debt declines most in response to the technology shock. We obtain a positive debt response whenever one of the agents has very limited cognitive ability. Since households are the residual owners of firms, the wealth effect is operative through this channel, too.

FIG. 4. Heterogenous Perception, technological shock



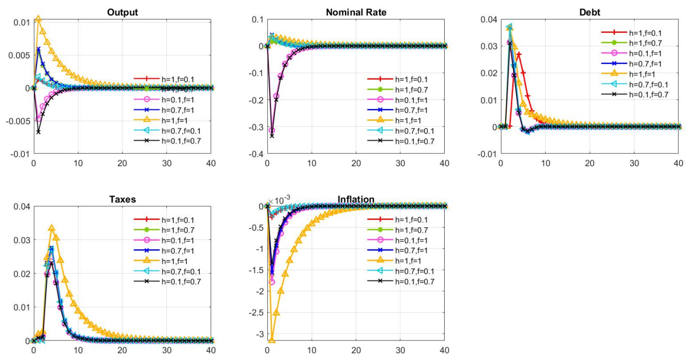
A similar reasoning applies for the case of the monetary policy shock in Figure 5. Various combinations of perception parameters show very similar qualitative responses, with the exception of real debt. In response to a policy tightening, real debt rises when one of the agents' perception is very limited, whereas in cases of smaller deviations from the fully informed benchmark debt declines. The reasoning is similar to the case of the expansionary technology shock, where limited perception implies a weakening of the wealth effect. At the same time, this aspect is more pronounced when households are more limited in perception than firms. As the upper-left hand panel of Figure 5 shows real debt increases most when $h = 0.1$, while the effect is muted under $f = 0.1$.

FIG. 5. Heterogenous Perception, monetary policy shock



Finally, we consider heterogenous perceptions in a response to an expansionary government shock in Figure 6. The differential impact of limited household perception is illustrated by the seemingly counterintuitive response of output which declines so that higher G effectively crowds out consumption. In the standard model (or with high degrees of perception) the expenditure shock is expansionary because of a wealth effect. With this not at play, households do not fully perceive that they have to work harder.

FIG. 6. Heterogenous Perception, public expenditure shock

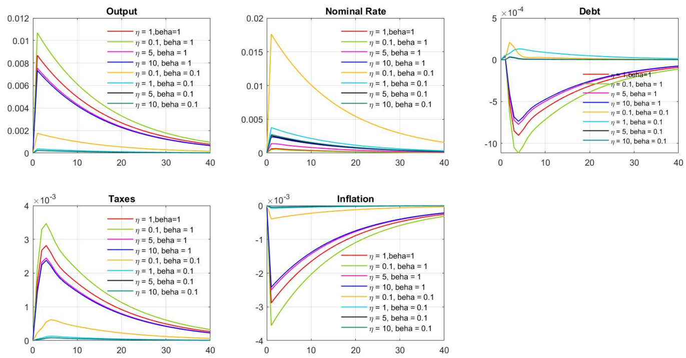


4.3. Robustness: Parameter Sensitivity

The key mechanism for the differential effects under cognitive discounting relative to the NK benchmark is the relative weakness of the wealth effect on labor supply and thus output and inflation via the NKPC. The main determinant of the responsiveness of output to changes in labor is, of course, the labor supply elasticity. We therefore assess the robustness of this mechanism to variations in this parameter. A second exercise we consider is the robustness to changes in risk aversion since this affects the intertemporal consumption trade-off and thereby the strength of intertemporal budget considerations.

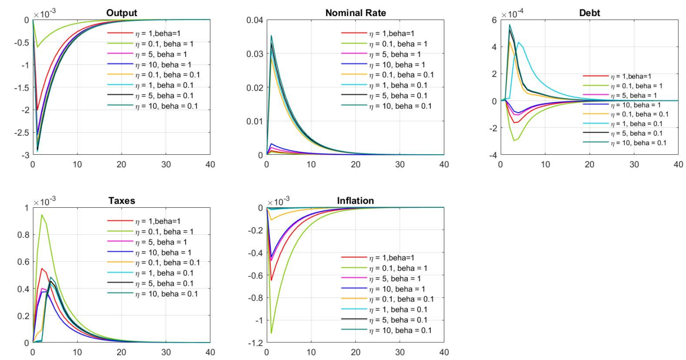
Figures 7 - 9 report impulse responses to the three previously considered shocks under various scenarios. We consider four values for the labor supply parameter η: 0.1, 1, 5, 10, where 1 is our benchmark value. Moreover, we assume homogeneous perceptions for three values: 0.1, 0.5 and 1, the latter being the fully rational case. As expected, the strength of the output response can be ranked in order of the elasticity of the labor supply as can the response of inflation and taxes under a technology shock. In terms of the overall strength of the response, behavioral parameters seem to play a more important role than η, however. When behavioral parameters are small the response of taxes does not fully cover the burden of debt servicing, caused by the weak response of inflation, which then makes public debt increase.

FIG. 7. Sensitivity with respect to η, technological shock



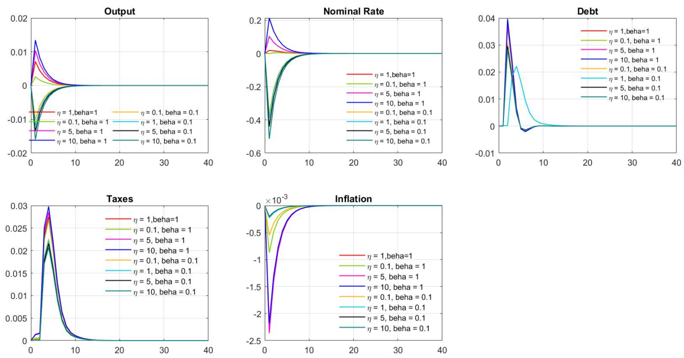
Responses to a monetary policy shock in Figure 8 can be similarly ranked. In this case, it is more noticeable that the responses cluster according to full or low perception, especially in the panels for the nominal rate and the debt responses. This follows the insight derived from above that the strength of the wealth effect driven by the degree of cognitive discounting becomes evident almost exclusively in the behavior of debt. These considerations also extend to an expenditure shock in Figure 9, albeit only as far as output and the interest rate are concerned.

FIG. 8. Sensitivity with respect to η, monetary policy shock



We now turn to assessing robustness with respect to risk aversion σ. This parameter captures the willingness of households to shift resources intertemporally and is therefore indicative of the strength of the behavioral wealth effect. We consider the following range of parameter values for σ: 0.5, 5, 0.05 and 10, which we interact with cognitive parameters for

FIG. 9. Sensitivity with respect to η, public expenditure shock



households and firms by setting m_π, m_r, m_x, m_y, and M, M^f to three different values: 0.1, 0.5 and 1. For simplicity, we assume homogeneous perceptions.

Figure 10, shows impulse responses for two values for σ, 0.5 and 5, and for three values of the behavioral parameters (0.1; 0.5; 1). The extreme values for σ are reported in Figure 11. We observe that higher values of σ reduce the size of the response of GDP and taxes after a productivity shock. When combined with small cognitive ability parameters, this leads to a strong reaction of the interest rate, with an equivalent strong response of debt and a mild reaction of taxes. More extreme values for σ are associated with weaker responses, when cognitive abilities are very small. We conclude that a technological shock is less expansionary on output, if (i) risk aversion is high; (ii) cognitive ability of both households and firms is very low.

FIG. 10. Sensitivity with respect to σ, Technology shock

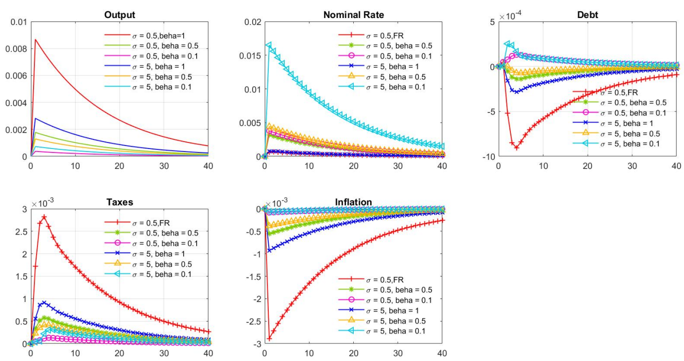
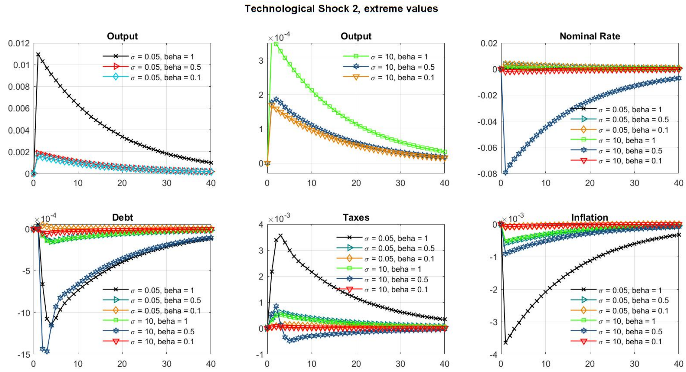
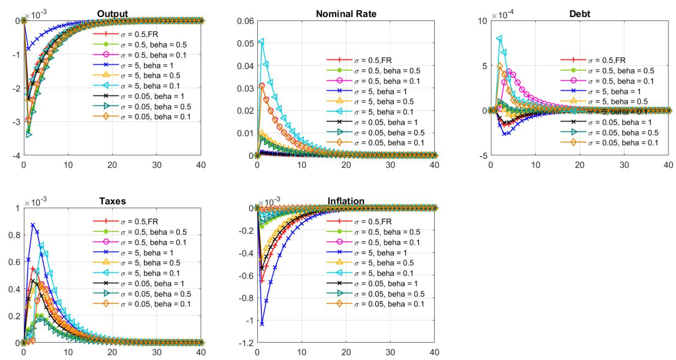


FIG. 11. Sensitivity with respect to σ, Technology shock



The responses to a monetary policy shock are reported in Figure 12. After a monetary contraction, we observe no crucial differences with respect to the sign of response of all variables: we observe a sharp and persistent reduction of both output and inflation rate. Interestingly, a stronger response can be obtained for high risk aversion and a very low behavioral parameters. Fluctuations of public debt are strongly dependent on the specific parametrization chosen: high risk aversion and low degree of cognitive ability imply that taxes do not show a strong enough reaction in light of expansion of debt. That is, the wealth effect is weaker with more behavioral agents that are more risk averse.

FIG. 12. Sensitivity with respect to σ, monetary policy shock



To summarize, we find that the key parameters in the behavioral model are those associated with the degree of perception. Household parameters are more important than those of the firm since the key transmission channel for behavioral effects is the wealth effect on account of a failure of Ricardian equivalence. In the next section we dig a little bit deeper by studying one way of micro-founding the thus far exogenous behavior parameters.

5. EXTENSION: ENDOGENIZING PERCEPTIONS

The behavioral NK model can be solved for given perception parameters, that is, the degree of misperception is treated as structural. Alternatively, it is possible to treat cognitive discounting as a decision problem, in which agents choose how much attention they would like to pay to distant future events given some constraints on cognition. Such microfoundations for cognitive discounting have been introduced by Gabaix (2014, 2017), following in the footsteps of Sims (2003). In this section we apply this approach to the NK model. We first broadly outline the approach, whereby we relegate the details to the online appendix. We then study the dynamics of the thus endogenized specification using impulse response analysis.

5.1. Outline and Summary

The microfoundations for cognitive discounting rest on the idea that agents consciously choose not to pay attention to all future eventual paths. This could either be because of limitations to information processing (e.g., Sims, 2003), lack of full knowledge of the environment (e.g., the learning approach of Evans and Honkapohja, 2001), or cognitive discounting for inate reasons. The latter can be operationalized by a two-step approach. First, decision rules are derived conditional on given perception parameters (as we have done so above), and second, based on the indirect utility and profit functions, we solve a constrained problem for the optimal level of cognitive discounting.

Specifically, following Gabaix (2014, 2017), we define a decision rule at time t as $q_t = (C_t, L_t)$ in the households' problem. We define the set of state variables $S_t = (b_t, H_t)$, where b_t is the level of public debt and H_t includes all macroeconomic variables included in the model. As in Gabaix (2020), the agent has the subjective value function:

$$\mathcal{V}_t\left(q_t, S_t, m\right) = u(q) + \beta \mathbb{E}_t \mathcal{V}_t\left(G^s\left(q_t, S_t, m, \varepsilon_t\right), m\right), \tag{36}$$

which fully characterizes his subjective view of the world captured by parameter m. $G^s\left(q_t, S_t, m, \varepsilon_t\right)$ indicates the transition function. In the first step of the endogenization problem the agent optimally chooses actions $q_t = (C_t, L_t)$, given m:

$$q(m, S_t) = \max_q \mathcal{V}_t\left(q_t, S_t, m\right). \tag{37}$$

In the second step, given the solution in (37), the agent maximizes the following indirect value function with respect to the perception parameter m:

$$\max_m \mathbb{E}_t \mathcal{V}\left(q(m, S_t), S_t, 1\right) - k_v h(m - m^d), \tag{38}$$

where m^d indicates the degree of attention associated to the default state and $(q(m, S_t), S_t, 1)$ represents the utility associated to the model with $m = 1$. Specifically, each household maximizes its utility conditional on the action $q(m, S_t)$, net of the cost of deviating attention from the default state m^d. The function $k_v h(m - m^d)$ describes the cost of implementing a degree of attention higher than that associated with the default state, that is, it can be interpreted as the 'cost of thinking'. The cost parameter $k_v \geq 0$, whereby the fully rational case corresponds to $k_v = 0$. As in Gabaix (2014), we assume the following parameterization: $h(m - m^d) = \left|m - m^d\right|$.

We implement a linear-quadratic approach via a Taylor expansion of the utility loss:

$$\max_m -\frac{1}{2}\Lambda(1-m)^2 - k_v h(m - m^d), \tag{39}$$

where $\Lambda = \lambda\sigma_S^2$, and λ is defined as:

$$\lambda = \mathbb{E}_t\left[\hat{S}_t q'^{d}_{m,s}, 0)\mathcal{V}_{qq}\left(q(m^d, 0), 0, m^d\right) q_{m,s}(m^d, 0)\hat{S}_t\right]. \tag{40}$$

$q_{m,s}$ is the second-order cross derivative and the set of variables is rescaled as follows: $\hat{S}_t = S_t - S_t^d = \sigma_S S_t^0$. The solution to this program is:

$$m = \mathcal{Q}\left(\frac{\lambda\sigma_S^2}{k_v}, m^d\right), \tag{41}$$

where:

$$\mathcal{Q}\left(\mathcal{V}, m^d\right) = \min_m \frac{1}{2}(1-m)^2\mathcal{V} + h(m, m^d) = \max_m \left(1 - \frac{1}{\mathcal{V}}, m^d\right). \tag{42}$$

Endogenizing perceptions follows a two-step procedure. First, we solve the model conditional on a specific value of the cognitive parameters. Given this solution, we then optimize with respect to perceptions. We detail all the steps needed to get the full analytical solution in the online appendix. For the numerical analysis we rely on standard method for recursive models, like Blanchard and Kahn (2000) or Sims (2001), conditional on a specific value of all cognitive parameters. The end of this process delivers endogenous perceptions for both Euler equation and the Phillips curve.

5.2. Model Dynamics

The structure of the behavioral NK model with endogenized perceptions stays the same as in our baseline. What is different is that the perception parameters are determined by deeper structural parameters so that the endogenized version imposes additional cross-coefficient restrictions. This results in specific values of these parameters, which are reported in Table 2.

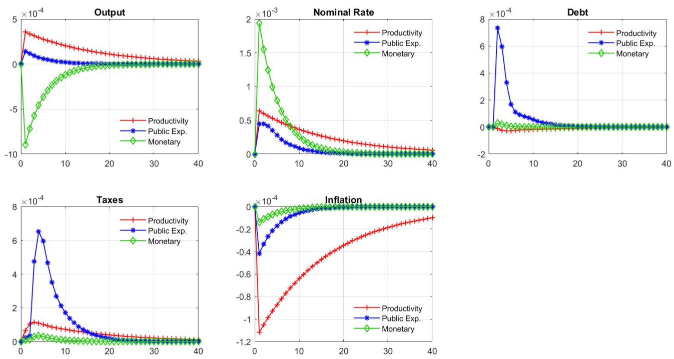
TABLE 2.

Endogenous Cognitive Parameters

M	m_y	m_r	M^f	m_x	m_{π_x}
0.328	0.88	0.66	0.67	0.22	0.2

In contrast to the various exogenous parameter cases we discussed above this results in a parameterization where the degree of household perception is relatively high, while that of the firms is comparatively low.

FIG. 13. Endogenous cognitive parameters, technological, fiscal and monetary policy shock



We report impulse responses to technology, monetary policy and government expenditure shocks for this scenario in Figure 13. As is clearly discernible from the graphs the results are in line with the results already discussed in the sensitivity analysis. Specifically, the relatively high degree of perception among the households resulting in a less discounted Euler-equation weakens the wealth effect channel that is at the core of this specific type of behavioral modelling.

6. CONCLUSION

In this paper, we revisit the behavioral New Keynesian macroeconomic framework of Gabaix (2020) and extend it to include three types of shocks: a productivity, monetary policy and government expenditure shock. By doing so we put additional focus on the perceptions of fiscal policy and the implications of the intertemporal government budget constraint. In our framework, bounded rationality implies that Ricardian equivalence does not hold. Agents do not have the ability to forecast infinitely into the future even if they are long-lived. This limited cognitive capacity makes them more short-sighted and thus incapable of rationalizing the future evolution of fiscal and monetary variables. In turn this implies that the underlying wealth channel is less operative and thereby provides a wider spectrum for dynamic behavior and monetary and fiscal policy efficacy.

We generally find that cognitive discounting implies that the responses of the model variables tend to be smoother than the full rationality benchmark. This is more pronounced as perceptions become more limited and when the limited perceptions are concentrated among households, the direct source of the wealth effect. We also propose a simple approach to endogenize the agents; cognitive parameters through a two-step procedure which links the perception coefficients to deeper structural parameters. This implies behavior that is not dramatically removed from full rationality. Nevertheless, we present in addition determinacy results for different types of interest-rate Taylor-type monetary policy rules including forward-looking inflation targeting and a rule with an inertial interest rate. We find that with boundedly rational agents Taylor-type monetary policy rules can deliver full determinacy of the equilibrium even if the interest rate response to inflation is less than one.

The paper demonstrates how the bounded rationality approach, modelled as imperfect cognition about the future evolution of the economy, can be implemented in a standard NK model. We also make tentative steps by generalizing it to more complex settings. Follow-up research includes an extension to consider investment and capital accumulation.

APPENDIX

The following sections list the analytical expressions for the coefficients in the model equations derived in the main text for reference. The detailed derivations are described in the online appendix.

A.1. DETAILS FOR THE BEHAVIORAL EULER EQUATION

The analytical expressions for the coefficients of the behavioral Euler equation derived in section 3.2 are as follows:

$$\eta_b = \frac{r}{R}\left(\frac{S_c b(1-m)}{(1-S_c)(C+\alpha_m)}\right), \qquad \eta_T = \frac{\beta m S_c G \frac{r}{R}}{(1-S_c)(C+\alpha_m)},$$

$$\eta_R = \left(\frac{S_c}{1-S_c\phi_y}\right)\left(\frac{-m_r\left(b_r\alpha\eta\frac{r}{R}\right)+mb\frac{r}{R}}{C+\alpha_m}\right), \qquad \alpha_m = \alpha\eta\sigma\left(\frac{r}{R-m}\right),$$

$$\eta_G = \frac{S_c}{1-S_c\phi_y}\left[\frac{Cg(\,-\beta m\rho_g) - \beta m\rho_g\left(\frac{r}{R}G+\alpha\eta\sigma g\right)}{S_c(C+\alpha_m)}\right], \qquad M = \frac{\beta m}{1-S_c\phi_y},$$

$$\phi_y = b_y\frac{\left(m_y+\frac{\alpha\eta\sigma}{S_c}\right)}{C+\alpha_m}.$$

with $\alpha_m > 0$ by construction since $R > m$.

A.2. DETAILS FOR BEHAVIORAL EULER EQUATION WITH FISCAL PERCEPTIONS

The coefficients of the Euler Equation in section 3.4 inclusive of fiscal perceptions are:

$$\phi_{R\tau} = \frac{m_r}{Y}\psi_y\left(-b_r+\alpha b_y\eta+b_y m_y R S_{by}\right), \qquad \psi_y = \left[1+\frac{\alpha b_y\eta\sigma}{S_c Y}\left(\frac{R}{R-m}\right)\right]^{-1},$$

$$\phi_d = \psi_y r\frac{T_y m_y}{Y}\frac{(m-1)}{(R-m)}, \qquad \phi_{b\tau} = \psi_y S_{by}\frac{r}{Y}\left(\frac{R(1-R)-m}{R(R-m)}\right),$$

$$\phi_{y\tau} = \frac{r}{R}\psi_y\frac{1}{Y}\left(m_y+\frac{\alpha\eta\sigma}{S_c}\right), \qquad \phi_{G\tau} = \left(\frac{r}{R}\right)\frac{b_y\psi_y\alpha\eta g}{S_c Y},$$

$$\phi_{G1\tau} = \psi_y\frac{m_y g}{Y}\frac{r}{R}.$$

$$\mu_{b\tau} = \frac{(1-\beta m)\,\phi_{b\tau}}{1-\phi_{y\tau}}, \qquad \mu_{R\tau} = \frac{\phi_{R\tau}}{1-\phi_{R\tau}},$$

$$\mu_{G\tau} = \frac{(g-\phi_{G\tau}-\beta m\rho_g(1+g))}{1-\phi_{R\tau}}, \qquad M_\tau = \frac{\beta m}{1-\phi_{R\tau}},$$

$$\mu_d = \frac{\beta m(\phi_d+\phi_{b\tau})}{1-\phi_{R\tau}}.$$

A.3. DETAILS FOR ENDOGENOUS PARAMETER PERCEPTIONS

The endogenization of perceptions requires a two-step procedure: in the first step, we solve the model conditional to a specific value of the cognitive parameters. In the second step, given the solution, we optimize with respect to perceptions. The rationale behind this approach is given by the need to have a set of policy functions, derived as solution of a recursive RBC-New Keynesian model, mapping each endogenous variables with the underlying shocks. These policy functions are next directly employed to recover the endogenized perceptions.

The first run solution is obtained conditional to a generic value for behavioral parameters: in this case we set all perceptions identified by various m's, to be all equal to 1, equivalent to the full cognitive ability case. This has two advantages: first of all, it reduces arbitrariness in setting up a plethora of values, without any rationale behind that. Secondly, the policy function for each endogenous variable obtained as solution, are stable. Intuitively, this assumption is equivalent to state that at each instant t, each agent conducts a mental simulation about future evolution of variable, which is assumed to be very precise: the agent is able to solve the system, at time t, with full information. However, when time goes on, agent's posterior does no longer ensure full cognitive ability and cognitive parameter (the m's) become less than one.

We could have chosen for the first run solution another value for the cognitive parameters: this would have not changed our results materially, since the resulting endogenized values emerging in the second step, would have been rescaled with respect to the case where we set all m equal to one. In this case the solution would have been strongly dependent on initial behavioral assumptions: setting behavioral parameters equal to one, is equivalent to sterilize the role of cognitive parameters in the design of the solution. Clearly, after having obtained the optimal perception parameters, the model can be solved again to get the full solution.

In the online appendix we detail all the steps needed to get the full analytical solution. Given the nature of the system, the model can be also solved numerically by using standard method for recursive models, like Blanchard and Kahn (2000) or Sims (2001), conditional to a specific value of all cognitive parameters: all numerical methods allow to get a set of policy rules whose coefficients can be fruitfully employed to get the full endogenization of cognitive parameters. The end of this process delivers endogenous perceptions for both Euler equation and Phillips curve.

A.3.1. Behavioral Euler-Equation

Endogenous cognitive parameters identifying households' perceptions for the Euler equation are:

$$m = \mathcal{Q}\left(\frac{\lambda\sigma_S^2}{k_V}, m^d\right) \tag{A.1}$$

where:

$$\lambda\sigma_S^2 = \Lambda = \lambda_u\left(c_{m,a}^2\sigma_A^2 + c_{m,g}^2\sigma_G^2\right) \tag{A.2}$$

with

$$\lambda_u \equiv -\sigma C^{-\sigma-1} + \eta^2\sigma^2 L^{\frac{1}{\eta}-1} \tag{A.3}$$

and $c_{m,a}$, $c_{m,g}$, defined as:

$$c_{m,a} = \frac{\beta\rho_a}{\left(1-\beta m^d\rho_a\right)^2}\left(\bar{b}_r m_r^d \mu_{Ra} + \bar{b}_y m_y \mu_{ya}\right) \tag{A.4}$$

$$c_{m,g} = \frac{\beta\rho_g}{\left(1-\beta m^d\rho_g\right)^2}\left(\bar{b}_r m_r^d \mu_{Rg} + \bar{b}_y m_y \mu_{yg}\right) \tag{A.5}$$

with: $\bar{b}_r = b_r/C$, $\bar{b}_y = b_y/C$, and μ_{ya}, μ_{yg}, μ_{Ra}, μ_{Rg} detailed in the online appendix.

The degree of attention for the interest rate, m_r, is:

$$m_r = \mathcal{Q}\left(\frac{\lambda\sigma_{S_r}^2}{k_V}, m^d\right) \tag{A.6}$$

with:

$$\lambda\sigma_{S_r}^2 = \Lambda_r = \lambda_u\left(c_{mr,a}^2\sigma_A^2 + c_{mr,g}^2\sigma_G^2\right) \tag{A.7}$$

with λ_u given in (A.3) and $c_{mr,a}$, $c_{mr,g}$ given by:

$$c_{mr,a} \equiv \frac{1}{1-\beta m\rho_a}\bar{b}_r\mu_{Ra} \qquad c_{mr,g} \equiv \frac{1}{1-\beta m\rho_g}\bar{b}_r\mu_{Rg} \tag{A.8}$$

The degree of attention for output m_y is:

$$m_y = \mathcal{Q}\left(\frac{\lambda\sigma_{S_y}^2}{k_V}, m^d\right) \tag{A.9}$$

with:

$$\lambda\sigma_{S_y}^2 = \lambda_u\left(c_{my,a}^2\sigma_A^2 + c_{my,g}^2\sigma_G^2\right) \tag{A.10}$$

with λ_u given in (A.3) and $c_{my,a}$, $c_{my,g}$ given by:

$$c_{my,a} \equiv \frac{\bar{b}_y \mu_{ya}}{1-\beta m \rho_a} \qquad c_{my,g} \equiv \frac{\bar{b}_y \mu_{yg}}{1-\beta m \rho_g} \tag{A.11}$$

A.3.2. Behavioral Phillips endogenous perceptions parameters

Our next step concerns the analysis of endogenous perceptions for the Phillips curve. To start with, consider the profit function, as firm's value function representation, given by $\Omega^{br}\left(\tilde{P}_{t,t+k}, Y_{t+k}\right)$:

$$\Omega^{br}\left(\tilde{P}_{t,t+k}, Y_{t+k}\right) = \mathbb{E}_t^{br} \sum_{k=0}^{\infty} (\beta\zeta)^k \omega^{br}\left(\tilde{P}_{t,t+k}, Y_{t+k}\right) \tag{A.12}$$

where $\tilde{P}_{t,t+k}$ is the log-linear version of the optimal relative price defined as $\tilde{P}_{t,t+k} = \tilde{P}_{t,t} - \sum_{i=1}^{k} \tilde{\pi}_{t+i}$, and $\omega^{br}\left(\tilde{P}_{t,t+k}, Y_{t+k}\right)$ is defined according to:

$$\omega^{br}\left(\tilde{P}_{t,t+k}, Y_{t+k}\right) = e^{(1-\theta)\tilde{P}_{t,t+k}} Y_{t+k} - MC_{t+k} e^{-\theta \tilde{P}_{t,t+k}} Y_{t+k} \tag{A.13}$$

From (A.12) and (A.13), it is possible to recover the second derivative Ω_{pp}^{br}, i.e., the second derivative of the Value Function for firms. Therefore, the endogenous cognitive parameters for firms are:

$$m^f = \mathcal{Q}\left(\frac{\lambda_f \sigma_S^2}{k_V}, m^d\right) \qquad m_x = \mathcal{Q}\left(\frac{\lambda_x \sigma_S^2}{k_V}, m^d\right) \qquad m_\pi = \mathcal{Q}\left(\frac{\lambda_\pi \sigma_S^2}{k_V}, m^d\right) \tag{A.14}$$

where:

$$\lambda_f \sigma_S^2 = p_{m,a}^2 \sigma_A^2 + p_{m,g}^2 \sigma_G^2 \tag{A.15}$$
$$\lambda_x \sigma_S^2 = p_{mx,a}^2 \sigma_A^2 + p_{mx,g}^2 \sigma_G^2 \tag{A.16}$$
$$\lambda_\pi \sigma_S^2 = p_{m\pi,a}^2 \sigma_A^2 + p_{m\pi,g}^2 \sigma_G^2 \tag{A.17}$$

where the expression of coefficients $p_{m,a}$, $p_{m,g}$, $p_{mx,a}$, $p_{mx,g}$, $p_{m\pi,a}$, $p_{m\pi,g}$, are:

$$p_{m,a} = \frac{(1-\beta\zeta)\rho_a\beta\zeta}{(1-\rho_a)}\left[\frac{a_2}{(1-\beta m\zeta)^2} - \frac{\rho_a a_2}{(1-\beta m\zeta\rho_a)^2} + \frac{m_x\mu_a^{mc}(1-\rho_a)}{(1-\beta m\zeta\rho_a)^2}\right] \tag{A.18}$$

$$p_{m,g} = \frac{(1-\beta\zeta)\rho_g\beta\zeta}{(1-\rho_g)}\left[\frac{b_2}{(1-\beta m\zeta)^2} - \frac{\rho_g b_2}{(1-\beta m\zeta\rho_g)^2} + \frac{m_x\mu_g^{mc}(1-\rho_g)}{(1-\beta m\zeta\rho_g)^2}\right] \tag{A.19}$$

$$p_{mx,a} = \frac{1-\beta}{1-\beta m\zeta\rho_a} \tag{A.20}$$

$$p_{mx,g} = \frac{1-\beta}{1-\beta m\zeta\rho_g} \tag{A.21}$$

$$p_{m\pi,a} = \frac{(1-\beta\zeta)a_2\rho_a}{(1-\rho_a)}\left[\frac{1}{1-\beta\zeta m} - \frac{1}{1-\beta\zeta m\rho_a}\right] \tag{A.22}$$

$$p_{m\pi,g} = \frac{(1-\beta\zeta)b_2\rho_g}{(1-\rho_g)}\left[\frac{1}{1-\beta\zeta m} - \frac{1}{1-\beta\zeta m\rho_g}\right] \tag{A.23}$$

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