ANNALS OF ECONOMICS AND FINANCE **22-2**, 451–465 (2021)

Longevity, Grandparents Caring, and PAYG Pensions

Wei Gao, Chengliang Yan, and Fuyang Zhao*

This paper presents an OLG model with endogenous grandparents caring. We show that the increase of longevity has positive effects on grandparents caring, which in turn increases adults' labor supply, and reinforces the positive effect of longevity on pension benefits. Contrary to existing literature, we find that under plausible parameter values, increasing longevity may improve pension benefits. This indicates that population aging driven by increasing longevity may not be a threat to the sustainability of the PAYG pension system.

Key Words: Increasing Longevity; Grandparents Caring; Pensions; Overlapping Generations Model.
JEL Classification Numbers: D91, H55, J26.

1. INTRODUCTION

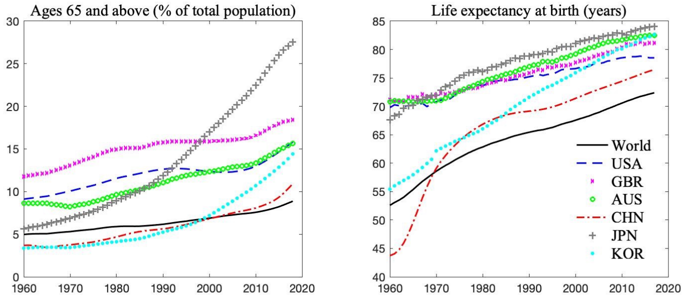
There has been an evident global trend of population aging ever since statistics began. This trend is drawing more and more attention in recent decades. Figure 1 demonstrates the relevant statistics from World Development Indicators (WDI) by the World Bank. At the global level, the proportion of people at age 65 or above has almost doubled since 1960. In developed areas, such as Europe and North America, the proportion has advanced to levels higher than 15%. In Asia, the aging process is even faster, with Japan beginning to speed up in 1990s, Korea in 2000s, and China in 2010s.

The increasing longevity is one of the main driving forces of the global aging. Also according to the statistics from WDI in Figure 1, the life expectancy at birth has risen by more than 10 years in developed areas and

* Gao: School of Economics, Central University of Finance and Economics. Email: yf0201@126.com; Yan: School of Economics, Central University of Finance and Economics. Email: yanchengliang2016@126.com; Zhao: Corresponding author. School of Economics, Central University of Finance and Economics, Beijing, 100081, China. Email: fyzhao@cufe.edu.cn. The authors are grateful to the co-editor Yulei Luo and an anonymous referee for their helpful comments and suggestions.

1529-7373/2021

FIG. 1. Population aging and life expectancy (1960-2018, source: WDI database)



more than 15 years in the whole world. This rise is even more significant in Asia, especially in China, a country with a fifth of the world's population.

Scholars are curious about the economic implications of population aging induced by increasing life expectancy. Besides the long lasting concern about the consequences on economic growth (Sala-i-Martin et al., 2004; Acemoglu and Johnson, 2007; Well, 2007; Bloom et al., 2010), more recently, the literature has also concentrated on the sustainability of the pension system (Fanti and Gori, 2008; Cipriani and Makris, 2012; Cipriani, 2014). In addition, other influencing factors, such as fertility choice (Van Groezen et al., 2003; Cremer et al., 2011) and retirement choice (Chen and Lau, 2016; Cipriani, 2018), are extensively studied in order to better comprehend this topic. In this paper, we introduce grandparent caring as another influencing factor, and investigate how it would influence the relationship between population aging and pension sustainability.

In contemporary society with a fast pace of life, parental care is not always available, especially when parents participate in the labor market. Child care provided by friends, neighbors or relatives are alternative choices, among which grandparents caring is preferred the most (Wheelock and Jones, 2002). A variety of data and studies validate the strong involvement of grandparents in child care. Based on National Survey of Families and Households (NSFH) in the U.S., Guzman (2004) reports that 38% of grandfathers and 54% of grandmothers provided different forms of child care. According to the same survey, on average, children at age 6 or below could get 23 hours of grandparents caring each week. Hank and Buber (2009) employ the 2004 Survey of Health, Aging and Retirement in Europe (SHARE2004). They find that 49% of grandfathers and 58% of grandmothers provided some kind of child care over the past 12 months, and 25% of grandfathers and 32% of grandmothers looked after their grandchildren almost weekly or more often. In Netherlands and Denmark, grandparents caring is even more prevalent: 60% of grandfathers and 65% of grandmothers provided child care over the past 12 months. Feng and

Han (2017) use the database of China Health and Retirement Longitudinal Study (CHARLS), and find that 33% of retired grandfathers and 30% of retired grandmothers looked after their grandchildren. What's more, grandparents caring is more prevalent than before. As an example, 20.5% of American children ages 0-4 with employed mothers were taken care of by their grandparents during their mothers' work hours in 2011, while the corresponding proportion was only 15.9% in 1985 (Federal Interagency Forum On Child, 2017). One important reason leading to this increasing relevance of multigenerational bonds is increasing longevity (Bengtson, 2001).

Given the above observations, we introduce grandparents caring into the model framework of Cipriani (2014). In the model, each individual lives for childhood and adult period with certainty and survives to old age with a probability. The scale of this surviving probability is an indicator of life expectancy. Adult individuals need to take a fraction of their time off from labor markets (or equivalently, a fraction of their income) to rear children. Elderly individuals care about the welfare of their adult child. What's more, they can provide free child care, which is the novel feature of our model. Taking a labor tax and a pay-as-you-go pension into account, individuals make usual intertemporal consumption and saving decisions, as well as choose the optimal level of grandparents caring to achieve a balanced tradeoff between their own utility of retirement and their descendant's welfare.

The model shows that, in the steady state, population aging due to increasing longevity does not necessarily lead to a reduction in pension payouts. In fact, the relationship between pensions and longevity is not monotonic. In our model, there are three channels that longevity affects pensions. The first is a negative effect through the old age dependency ratio (Lindh and Malmberg, 2009; Hashimoto and Tabata, 2010). The second one is a positive effect through wages (or "capital accumulation effect"). When individuals expect a longer life, they tend to save more, which results in higher capital per worker and thus higher wages (Futagami and Nakajima, 2001; Aisa and Pueyo, 2013). The third one is a positive effect through grandparents caring and labor supply. Grandparents caring provided by the elderly allow young agents to devote less time to rearing children, and more to the labor market. We find that the overall effect of the three channels induces a U-shaped relationship between longevity and pension benefits: initially decreasing with longevity and increasing after a threshold level.

We further extend our model by introducing old age individuals' bequest to their adult children. We find that grandparent caring is still increasing with longevity in the extended model. However, compared to the benchmark model, bequest from old age individuals serves as an alternative way to increase their adult children's disposable income, which partially offsets the effort of old age individuals to grandparents caring. In addition, the U-shaped relationship between longevity and pension benefits still exists with a similar threshold level of longevity as the benchmark model.

The literature using the OLG framework mainly focus on the first two effects. Assuming the capital share is consistent with empirical evidence, Fanti and Gori (2008) show that the first effect outweighs the second effect, so the overall effect of longevity on pensions is negative. After introducing endogenous fertility decisions, Cipriani (2014) shows that increasing in longevity reduces fertility rate and thus reinforces population aging, which reduces pension benefits in a further step. Studies such as Aísa et al. (2012), Cipriani (2018), Cipriani and Fioroni (2021) consider endogenous retirement decisions. They propose another channel that longevity affects pensions: increasing longevity raises old agents' labor supply, which could increase pensions. However, Aísa et al. (2012) also mention that the possibility of working at old age reduces the necessity to save during the adult period (Dedry et al., 2017), which would undermine the positive effect through capital accumulation. Besides, in Cipriani (2018), the overall effect is positive only when longevity is very low, while in Cipriani and Fioroni (2021), the overall effect is positive only when tax rate is sufficiently low. To sum up, even if adding retirement decisions, longevity's effect on pensions is ambiguous: positive only under strict conditions or still negative.

In this paper, we extend the Cipriani (2014) model by including grandparents caring. Different from the above studies, we find that, in the presence of grandparents caring, pensions decrease only when longevity is at low levels and increase when longevity is relatively large. The turning point of longevity is relatively small, implying that the latter case is more like the reality, especially at the background of long life expectancy in contemporary society. Existing literature with grandparents caring are mostly demographic, pedagogical, or sociological studies (Brandon, 2000; Arpino and Bordone, 2014). To our knowledge, this paper is one of the few studies that consider the economic implications of grandparents caring under theoretical framework. This paper contributes a new channel to understand the effect of increasing longevity on PAYG pensions.

This paper is organized as follows: model setups are described in Section 2; Section 3 explores the effect of increasing longevity on PAYG pensions; Section 4 extends the model by introducing bequest; Section 5 concludes.

2. THE MODEL

2.1. Individuals

We set up an OLG model, individuals live at most three periods: childhood, adult, and old age. In the childhood period (in period $t-1$), individuals make no economic decisions. Individuals have one unit of time in both adult period and old age period. Individuals spend time on working in labor market and rearing children in the adult period (in period t). Individuals face a probability of death at the end of the adult period, and the probability live to the old period is $\pi \in (0, 1)$. π is an indicator of longevity, and higher π implies longer life expectancy. In the old age period (in period $t+1$), individuals choose time between grandparents caring z_{t+1}, and retirement for leisure $(1 - z_{t+1})$. Individual can gain utility form consumption in the adult period c_t^y, consumption in the old age period c_{t+1}^o, as well as leisure retirement in the old age period. What's more, individuals are altruistic towards their adult children, which is defined on adult children's disposable income I_{t+1} (Lambrecht et al., 2005; Kunze, 2012). With the logarithmic utility function, individuals maximize the following utility function:

$$\max\{\ln c_t^y + \pi[\beta \ln c_{t+1}^o + \theta \ln(1 - z_{t+1}) + \gamma \ln I_{t+1}]\}, \tag{1}$$

where $\beta > 0$ is the discount factor, $\theta > 0$ denotes the degree of preference for leisure, $\gamma > 0$ denotes the degree of altruism towards adult children's disposable income.

Individuals face the following budget constraints in the adult period and the old age period respectively:

$$c_t^y + s_t = (1 - \tau)w_t[1 - (\nu n - \pi z_t/n)], \tag{2}$$

$$c_{t+1}^o = \frac{1 + r_{t+1}}{\pi} s_t + p_{t+1}, \tag{3}$$

where s_t is saving, w_t is wage rate, τ is the contribution rate of social security, ν is the time cost for rearing one child, and n is the number of children (fertility rate). And νn is the total time cost for rearing children. Assume that there are L_t adult individuals in period t, and the dynamic equation for population $L_{t+1} = nL_t$. The number of old age individuals in period t is πL_{t-1}. With total grandparents caring supply $\pi z_t L_{t-1}$, each adult can get grandparents caring $\pi z_t L_{t-1}/L_t = \pi z_t/n$. Therefore, $(\nu n - \pi z_t/n)$ is the time cost for rearing children in consideration of grandparents caring. We define $l_t = 1-(\nu n-\pi z_t/n)$ as each adult's labor supply in period t. It is obvious that, given each old age individual's grandparent caring supply z_t, higher π makes more old age individuals involved in grandparents caring, which in turn makes the adult devote more time to working in the labor market. Following Cipriani (2014, 2018), we assume that the financial market is perfect competitive, and the gross return to saving is $(1+r_{t+1})/\pi$, and p_{t+1} is pension benefits.

According to our model setup, the descendants' disposable income is:

$$I_{t+1} = (1 - \tau)w_{t+1}[1 - (\nu n - \pi z_{t+1}/n)]. \tag{4}$$

Combining Eq. (2) and Eq. (3), we could obtain individuals' lifetime budget constraint:

$$c_t^y + \frac{\pi c_{t+1}^o}{1+r_{t+1}} = (1-\tau)w_t[1-(\nu n - \pi z_t/n)] + \frac{\pi p_{t+1}}{1+r_{t+1}}, \tag{5}$$

The left hand side of Eq. (5) represents the discounted consumption expenditures in both adult period and old age period, and the right hand side of Eq. (5) is the discounted life-time income, including both labor income and pension benefits. Given the adult child's disposable income Eq. (4), and lifetime budget constraint Eq. (5), individuals choose consumption in both periods, c_t^y and c_{t+1}^o, as well as grandparents caring z_{t+1}, to maximizes the utility function (1).

We construct the following Lagrange function to solve the above optimization problem:

$$\begin{aligned} L &= \ln c_t^y + \pi[\beta \ln c_{t+1}^o + \theta \ln(1-z_{t+1}) + \gamma \ln[(1-\tau)w_{t+1}(1-(\nu n - \pi z_{t+1}/n))]] \\ &\quad + \mu_t \left\{ (1-\tau)w_t(1-(\nu n - \pi z_t/n)) + \frac{\pi p_{t+1}}{1+r_{t+1}} - c_t^y - \frac{\pi c_{t+1}^o}{1+r_{t+1}} \right\}, \end{aligned}$$

where μ_t is the Lagrange multiplier, and we obtain the following first-order conditions:

$$\frac{\partial L}{\partial c_t^y} = \frac{1}{c_t^y} - \mu_t = 0, \tag{6}$$

$$\frac{\partial L}{\partial c_{t+1}^o} = \pi\left[\frac{\beta}{c_{t+1}^o} - \frac{\mu_t}{1+r_{t+1}}\right] = 0, \tag{7}$$

$$\frac{\partial L}{\partial z_{t+1}} = \pi\left[-\frac{\theta}{1-z_{t+1}} + \frac{\gamma\pi}{(1-\nu n)n + \pi z_{t+1}}\right] = 0. \tag{8}$$

Combining Eq. (6) and Eq. (7), we obtain:

$$c_{t+1}^o = \beta(1+r_{t+1})c_t^y. \tag{9}$$

Through Eq. (8), we obtain:

$$z_{t+1} = \frac{\gamma\pi - \theta(1-\nu n)n}{\pi(\gamma+\theta)}. \tag{10}$$

2.2. Firms

Firms employ capital and labor to produce final output, and we assume the following Cobb-Douglas production function:

$$Y_t = K_t^{\phi}[(1-(\nu n - \pi z_t/n))L_t]^{1-\phi}, \tag{11}$$

where Y_t is total output, K_t is capital stock, $[1-(\nu n-\pi z_t/n)]L_t$ is total labor supply, $\phi \in (0,1)$ and $1-\phi \in (0,1)$ denotes the capital share and the labor share respectively.

Assume the final output market is competitive, and capital stock is fully depreciated after one period. Firms maximize the following profit function:

$$\max\{K_t^{\phi}[(1-(\nu n-\pi z_t/n))L_t]^{1-\phi}-(1+r_t)K_t-w_t[(1-(\nu n-\pi z_t/n))L_t]\}.$$

Denote $k_t = K_t/[(1-(\nu n-\pi z_t/n))L_t]$ and $y_t = Y_t/[(1-(\nu n-\pi z_t/n))L_t]$ as capital per worker and output per worker respectively. The production function Eq. (11) can be written in the intensive form $y_t = k_t^{\phi}$. Solving firms' optimization problem, we obtain the following first-order conditions:

$$1+r_t = \phi k_t^{\phi-1}, \tag{12}$$

$$w_t = (1-\phi)k_t^{\phi}. \tag{13}$$

2.3. Government

We assume that the government adopts the Pay-as-you-go (PAYG) social security system and balances the following budget constraint: $\pi p_{t+1}L_t = \tau w_{t+1}[1-(\nu n-\pi z_{t+1}/n)]L_{t+1}$. Using the dynamic equation for population $L_{t+1}/L_t = n$, we derive the pension benefits:

$$p_{t+1} = \frac{\tau w_{t+1}[1-(\nu n-\pi z_{t+1}/n)]n}{\pi}. \tag{14}$$

2.4. Equilibrium

The market clearing condition for capital is:

$$K_{t+1} = s_t L_t. \tag{15}$$

Dividing both sides of Eq. (15) by total labor in period $t+1$, $[1-(\nu n-\pi z_{t+1}/n)]L_{t+1}$, we derive the following dynamic equation for capital stock:

$$k_{t+1} = \frac{s_t}{[1-(\nu n-\pi z_{t+1}/n)]n}. \tag{16}$$

Substituting Eq. (9) into Eq. (5), we obtain:

$$c_t^y = \frac{1}{1+\beta\pi}\left\{(1-\tau)w_t[1-(\nu n-\pi z_t/n)] + \frac{\pi p_{t+1}}{1+r_{t+1}}\right\}. \tag{17}$$

Substituting Eq. (17) into Eq. (2), we obtain:

$$s_t = \frac{\beta\pi}{1+\beta\pi}(1-\tau)w_t[1-(\nu n-\pi z_t/n)] - \frac{\pi p_{t+1}}{(1+\beta\pi)(1+r_{t+1})}. \tag{18}$$

Substituting Eq. (10), Eq. (12-14), and Eq. (18) into Eq. (16), we derive the following dynamic equation:

$$k_{t+1} = \frac{\beta\phi(1-\tau)(1-\phi)\pi}{[(1+\beta\pi)\phi + \tau(1-\phi)]n} k_t^{\phi}. \tag{19}$$

Since the production function is neoclassical and without exogenous technology change, the steady state can be defined as $k_{t+1} = k_t = k^*$, $z_{t+1} = z_t = z^*$, $p_{t+1} = p_t = p^*$. According to Eq. (19), we can derive the unique, globally stable steady state capital k^*:

$$k^* = \left\{ \frac{\beta\phi(1-\tau)(1-\phi)\pi}{[(1+\beta\pi)\phi + \tau(1-\phi)]n} \right\}^{1/(1-\phi)}. \tag{20}$$

From Eq. (20), we can derive $dk^*/d\pi > 0$, which implies that increasing longevity has a positive effect on steady state capital stock. Intuitively, increasing longevity makes individuals save more to smooth consumption during the adult period and the old age period, which in turn increase investment and steady state capital stock.

Eq. (10) reveals that the steady state grandparents caring is:

$$z^* = \frac{\gamma\pi - \theta(1-\nu n)n}{\pi(\gamma+\theta)}. \tag{21}$$

We focus our attention on the case of grandparents caring $z^* > 0$ and adult individuals' time cost for rearing children $\nu n - \pi z^*/n > 0$. These two inequalities reveal that the parameter values should satisfy $(\theta + \gamma\nu n)n > \gamma\pi > \theta(1-\nu n)n$. From Eq. (21), it is easy to show that $dz^*/d\pi > 0$, which indicates that increase of longevity has a positive effect on grandparents caring. The intuition is that, since the old individuals are concerned with their adult children's disposable income, with higher π, the marginal benefit of grandparents caring on adult children's disposable income is higher, which makes the old age individuals more willing to devote more time to grandparents caring.

3. RESULTS

In this section, we explore how the pension benefits change with longevity. Eq. (14) implies the steady state pension benefits as the function of longevity:

$$p^* = p^*\{\pi, w(k^*(\pi)), l(\pi, z^*(\pi))\}. \tag{22}$$

From Eq. (22), increase of longevity can affect pensions through direct and indirect channels. Differentiating Eq. (22) with respect to π, we obtain:

$$\frac{dp^*}{d\pi} = \overbrace{\frac{\partial p^*}{\partial \pi}}^{-} + \underbrace{\overbrace{\frac{\partial p^*}{\partial w}}^{+} \overbrace{\frac{\partial w}{\partial k^*}}^{+} \overbrace{\frac{dk^*}{d\pi}}^{+}}_{+} + \underbrace{\overbrace{\frac{\partial p^*}{\partial l^*}}^{+} \left(\overbrace{\frac{\partial l^*}{\partial \pi}}^{+} + \overbrace{\frac{\partial l^*}{\partial z^*}}^{+} \overbrace{\frac{dz^*}{d\pi}}^{+} \right)}_{+}. \tag{23}$$

According to Eq. (23), we can decompose the effect of longevity on pension into three effects. The first part of the right-hand side of Eq. (23) represents the direct effect of longevity on pension, which indicates a higher old-age dependency ratio, and thus dilutes pension benefits per pensioner. The second part shows that longevity could increase pension through the channel of capital stock and wage rate. Intuitively, increase of longevity improves the weight of utility from consumption in the old age period, which in turn induces individuals to save more during the adult period, and thus increasing steady state capital stock and wage rate. These two effects have been stressed by previous literature, such as Fanti and Gori (2008), Cipriani (2014). Innovatively in this paper, the third part of right-hand side of Eq. (23) shows that increasing longevity can improve pensions through the channel of grandparents caring and adults' labor supply. On the one hand, an increase in longevity raise old age population, and thus brings more grandparents caring providers. On the other hand, the increase in longevity makes each old age individual devote more time to grandparents caring. More grandparents caring reduces adult individuals' time cost for rearing children, and thus have positive effect on adults' labor supply. In comparison with Fanti and Gori (2008) and Cipriani (2014), this reinforces the positive effect of increasing longevity on pension. Therefore, the overall effect of longevity on pension may not be monotonic. And we summarize the effect in the following proposition.

PROPOSITION 1. *When the capital-output elasticity, ϕ, is smaller than $1/2$, there is a U-shaped relationship between longevity and pension benefits. And the threshold value of longevity, $\bar{\pi}$ satisfies*

$$\bar{\pi} = \frac{(1-2\phi)[\phi + \tau(1-\phi)](1-\nu n)n}{\phi[\phi + \tau(1-\phi) - \beta(1-\phi)(1-\nu n)n]} \in (0,1). \tag{24}$$

Proof. Substituting Eq. (13), Eq. (20) and Eq. (21) into Eq. (14), we derive the steady state pension benefits:

$$p^* = \tau(1-\phi)\left\{\frac{\beta\phi(1-\tau)(1-\phi)\pi}{[(1+\beta\pi)\phi+\tau(1-\phi)]n}\right\}^{\phi/(1-\phi)}\left\{\frac{\gamma\pi+\gamma(1-\nu n)n}{(\gamma+\theta)n}\right\}\frac{n}{\pi}. \tag{25}$$

Differenting Eq. (25) with respect to π, we have:

$$sign\left\{\frac{dp^*}{d\pi}\right\} = sign\{(2\phi-1)[\phi+\tau(1-\phi)](1-\nu n)n+\phi[\phi+\tau(1-\phi)-\beta(1-\phi)(1-\nu n)n]\pi\}. \tag{26}$$

Empirical evidence reveals that the capital share is roughly one third (Mankiw et al., 1992). And if we consider capital K under a broader concept, the capital share parameter ϕ should be larger than one third. We concentrate our analysis on the case when the capital share $\phi < 1/2$ and obtain the proposition. ▮

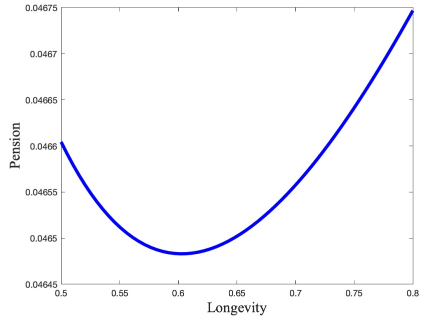
The above proposition implies that, for $\pi < \bar{\pi}$, we have $dp^*/d\pi < 0$, while for $\pi > \bar{\pi}$, we could obtain $dp^*/d\pi > 0$.

We next give a numerical example. In this example, we set the value of the discount factor β as 0.5. Assume that old age individuals equally evaluate consumption and leisure, then we have $\gamma = 0.5$. Assume that the old age individuals value less in their adult children's disposable income, so we set $\theta = 0.25$. Assume that 20% of labor income is contributed to social security, then $\tau = 0.2$. Assume that rearing one child costs young age individuals 35% of their time endowment, then $\nu = 0.35$. According to the data of the World Bank, the world-wide population growth rate is 1.21% annually since 2000. Then if setting one generation as 35 years, we have that $n = 1.0121^{35} = 1.5234$. According to the data of the Penn-World Table, for most countries including the United States and western countries, the labor share of total income has been decreasing. The lasted values of the labor share for countries listed in Figure 1 are all below 60%, so we set ϕ as 40%.

Figure 2 illustrates the impact of increasing longevity on pension benefits. We can see that there exists a U-shaped relationship between longevity and pension. When longevity is less than a threshold value, increase of longevity has a negative effect on pension. When longevity surpasses the threshold value, longevity has a positive effect on pension. From Eq. (24), we can calculate the threshold value of longevity, which is 0.6029. If we assume that the childhood period is 20 years, the adult period is 35 years, and the old age period is 35 years. The above analysis indicates that when lifetime expectancy exceeds $20 + 35 + 35 * 0.6029 = 76.1$ years old, increase of longevity can improve pension. For countries listed in Figure 1, including the United States and China, the life expectancies are all larger than 76.1.

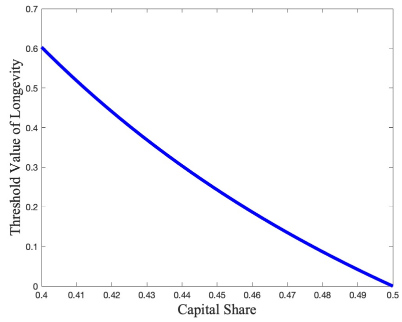
Therefore, contrary to Fanti and Gori (2008), Cipriani (2014), we find that for parameter values of the capital share consistent with empirical evidence, increase of longevity may not be a threat to sustainability of the PAYG pension.

FIG. 2. Effects of increasing longevity on pension



The critical parameter we are interested in is ϕ. And we can explore how the turning point $\bar{\pi}$ change with ϕ. Figure 3 shows that the turning point value of $\bar{\pi}$ is negatively correlated with ϕ. In recent decades, there is an evident global trend that the labor share is decreasing while the capital share is increasing. But the capital shares are still smaller than 1/2 in most countries, indicating that higher longevity is more likely to increase pension benefits.

FIG. 3. Relationship between the capital share and threshold value of longevity



4. AN EXTENSION WITH THE BEQUEST MOTIVE

In our model, the essential reason for old age individuals to provide grandparents caring is to increase their adult children's disposable income. In reality, besides grandparents caring, old age individuals can leave some heritage to their adult children. In this section, we can extend out model by introducing old age individuals' bequest to their adult children.

Assume that each old age individual chooses to leave b_t as bequest to their adult children. Each adult can obtain $\pi b_t L_{t-1}/L_t = \pi b_t/n$ from his or her parents. Therefore, individuals maximizing problem could be rewritten as

$$\max_{c_t^y, c_{t+1}^o, s_t, z_{t+1}, b_{t+1}} \{\ln c_t^y + \pi[\beta \ln c_{t+1}^o + \theta \ln(1 - z_{t+1}) + \gamma \ln I_{t+1}]\}, \tag{27}$$

$$\text{s.t.} \quad c_t^y + s_t = (1-\tau)w_t[1 - (\nu n - \pi z_t/n)] + \pi \frac{b_t}{n}, \tag{28}$$

$$c_{t+1}^o + \pi b_{t+1}/n = \frac{1 + r_{t+1}}{\pi} s_t + p_{t+1}, \tag{29}$$

$$I_{t+1} = (1-\tau)w_{t+1}(1 - [\nu n - \pi z_{t+1}/n)] + \pi \frac{b_{t+1}}{n}. \tag{30}$$

In Eq. (28) and Eq. (30), we can see that there is a new term in adults' disposable income, which is now consist of labor income and bequest from their parents. In Eq. (29), there is also a new term in old age individuals' expenditure, which is now consist of old age consumption and bequest to their children.

Solving the extended model, we can derive the steady state capital per worker k^*:

$$k^* = \left[\frac{n}{\gamma}\left(\frac{1}{\pi\phi} + \beta + \gamma\right)\right]^{1/(\phi-1)}, \tag{31}$$

as well as the steady state grandparents caring z^*:

$$z^* = 1 - \frac{1 + (1 - \nu n)n/\pi}{1 + (1-\tau)(1-\phi)(\beta+\gamma)/\theta}. \tag{32}$$

Both k^* and z^* are increasing with longevity π. However, it is hard to derive analytical solutions for the steady state bequest b^* or pension p^*. So in Figure 4, we numerically solve b^* and p^* under different longevity π.

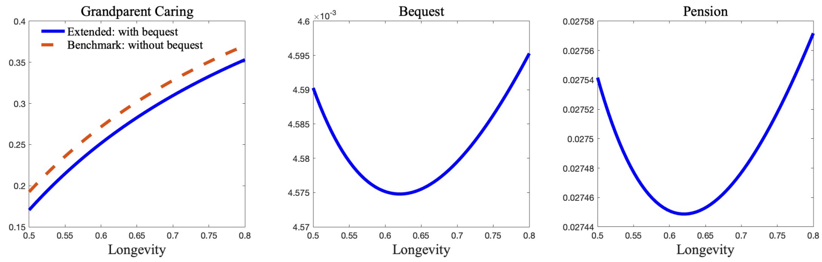
Firstly in Figure 4, as in the benchmark model, grandparent caring is increasing with longevity in the extended model. In addition, under the same longevity, the level of grandparent caring in the extended model is smaller than that in the benchmark model. The reason is that in the extended model, bequest from old age individuals serves as an alternative way to increase their adult children's disposable income, which partially substitute the effort of old age individual to grandparents caring.

In the second place, as we can see in Figure 4, there exists a U-shaped relationship between longevity and bequest, with threshold value 0.6220.

This implies that when life expectancy is smaller than 76.8, old age individuals tend to leave less bequest to their adult children under the increase of longevity, and vise-versa.

Finally, Figure 4 also demonstrates the relationship between longevity and pension. Same as in the benchmark model, the relationship is also a U-shaped one in the extended model. And the threshold value of is 0.6220, meaning that an increase of longevity has a positive effect on pension when life expectancy is larger than 76.8. This indicates that our baseline result is robust even after taking bequest into consideration.

FIG. 4. Effects of increasing longevity on grandparents caring and pension



5. CONCLUSIONS

According to the conventional view, the increase of longevity affects pension benefits through the negative effect of the larger old age dependency ratio and the positive effect of capital accumulation and wage rise. The overall effect is usually negative. In this paper, we introduce grandparents caring into the OLG model, and reinvestigate the effect of increasing longevity on PAYG pensions. Besides the two mentioned effects, longevity can improve pensions by increasing old age individuals' grandparents caring and adult individuals' labor supply. We show that under parameters consistent with empirical evidence, a higher longevity can increase pensions. And the result is robust even after taking bequest into consideration. It implies that population aging induced by increasing longevity may not be a threat to the sustainability of the PAYG social security system. The policy implication is that government should encourage retired individuals to participate in grandparents caring, so as to liberate adults' labor supply in the market.

The paper could also be extended from two dimensions: firstly, we could endogenize the fertility decision in this model. In this case, an increase of longevity has a positive effect on grandparents caring, which in turn reduces the time costs for rearing child, and may have positive effect on fertility. These can affect both the old-age dependency ratio and adult's labor supply, which are the determinants of PAYG pension benefits. Secondly, we could endogenize labor supply in the old period. In this case, an increasing longevity can affect old individuals' time allocation between grandparents caring and working in the labor market, which complicates the effects of longevity on pension benefits.

Acknowledgements: The authors acknowledge the financial support from National Social Science Foundation of China (grant number: 16ZDA005; 19ZDA069) and National Natural Science Foundation of China (grant number: 72003211).

REFERENCES

Acemoglu, Daron and Simon Johnson, 2007. Disease and Development: The Effect of Life Expectancy On Economic Growth. *Journal of Political Economy* **115(6)**, 925-985.

Aisa, Rosa and Fernando Pueyo, 2013. Population Aging, Health Care, and Growth: A Comment On the Effects of Capital Accumulation. *Journal of Population Economics* **26(4)**, 1285-1301.

Aísa, Rosa, Fernando Pueyo, and Marcos Sanso, 2012. Life Expectancy and Labor Supply of the Elderly. *Journal of Population Economics* **25(2)**, 545-568.

Arpino, Bruno and Valeria Bordone, 2014. Does Grandparenting Pay Off? The Effect of Child Care On Grandparents' Cognitive Functioning. *Journal of Marriage and Family* **76(2)**, 337-351.

Bengtson, Vern L., 2001. Beyond the Nuclear Family: The Increasing Importance of Multigenerational Bonds: The Burgess Award Lecture. *Journal of Marriage and Family* **63(1)**, 1-16.

Bloom, David E., David Canning, and Günther Fink, 2010. Implications of Population Ageing for Economic Growth. *Oxford Review of Economic Policy* **26(4)**, 583-612.

Brandon, Peter, 2000. An Analysis of Kin-Provided Child Care in the Context of Intrafamily Exchanges: Linking Components of Family Support for Parents Raising Young Children. *American Journal of Economics and Sociology* **59(2)**, 191-216.

Chen, Yanyou and Sau-Him Paul Lau, 2016. Mortality Decline, Retirement Age, and Aggregate Savings. *Macroeconomic Dynamics* **20(3)**, 715-736.

Cipriani, Giam Pietro, 2014. Population Aging and PAYG Pensions in the OLG Model. *Journal of Population Economics* **27(1)**, 251-256.

Cipriani, Giam Pietro, 2018. Aging, Retirement, and Pay-As-You-Go Pensions. *Macroeconomic Dynamics* **22(5)**, 1173-1183.

Cipriani, Giam Pietro and Miltiadis Makris, 2012. PAYG Pensions and Human Capital Accumulation: Some Unpleasant Arithmetic. *The Manchester School* **80(4)**, 429-446.

Cipriani, Giam Pietro and Tamara Fioroni, 2021. Endogenous Demographic Change, Retirement, and Social Security. *Macroeconomic Dynamics* **25(3)**, 609-631.

Cremer, Helmuth, Firouz Gahvari, and Pierre Pestieau, 2011. Fertility, Human Capital Accumulation, and the Pension System. *Journal of Public Economics* **95(11-12)**, 1272-1279.

Dedry, Antoine, Harun Onder, and Pierre Pestieau, 2017. Aging, Social Security Design, and Capital Accumulation. *The Journal of the Economics of Ageing* **9**, 145-155.

Fanti, Luciano and Luca Gori, 2008. Longevity and PAYG Pension Systems Sustainability. *Econ Bull* **10(2)**, 1-8.

Federal Interagency Forum On Child, 2017. America's Children: Key National Indicators of Well-Being 2017, Government Printing Office.

Feng, Jin and Xu Han, 2017. The Impact of Retirement Age Policy on Household Care Provision and Labor Participation in China (In Chinese). *The Journal of World Economy* **6(40)**, 145-166.

Futagami, Koichi and Tetsuya Nakajima, 2001. Population Aging and Economic Growth. *Journal of Macroeconomics* **1(23)**, 31-44.

Guzman, Lina, 2004. Grandma and Grandpa Taking Care of the Kids: Patterns of Involvement. Child Trends Research Brief. Washington DC: ERIC.

Hank, Karsten and Isabella Buber, 2009. Grandparents Caring for their Grandchildren: Findings From the 2004 Survey of Health, Ageing, and Retirement in Europe. *Journal of Family Issues* **30(1)**, 53-73.

Hashimoto, Ken-Ichi and Ken Tabata, 2010. Population Aging, Health Care, and Growth. *Journal of Population Economics* **23(2)**, 571-593.

Kunze, Lars, 2012. Funded Social Security and Economic Growth. *Economics Letters* **115(2)**, 180-183.

Lambrecht, Stéphane, Philippe Michel and Jean-Pierre Vidal, 2005. Public Pensions and Growth. *European Economic Review* **49(5)**, 1261-1281.

Lindh, Thomas and Bo Malmberg, 2009. European Union Economic Growth and the Age Structure of the Population. *Economic Change and Restructuring* **42(3)**, 159-187.

Mankiw, N. Gregory, David Romer, and David N. Weil, 1992. A Contribution to the Empirics of Economic Growth. *The Quarterly Journal of Economics* **107(2)**, 407-437.

Sala-I-Martin, Xavier, Gernot Doppelhofer, and Ronald I. Miller, 2004. Determinants of Long-Term Growth: A Bayesian Averaging of Classical Estimates (BACE) Approach. *American Economic Review*, 813-835.

Van Groezen, Bas, Theo Leers, and Lex Meijdam, 2003. Social Security and Endogenous Fertility: Pensions and Child Allowances as Siamese Twins. *Journal of Public Economics* **87(2)**, 233-251.

Well, David N., 2007. Accounting for the Effect of Health On Economic Growth. *The Quarterly Journal of Economics* **122(3)**, 1265-1306.

Wheelock, Jane and Katharine Jones, 2002. 'Grandparents are the Next Best Thing': Informal Childcare for Working Parents in Urban Britain. *Journal of Social Policy* **31(3)**, 441-463.