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Heterogeneous Beliefs, Limited Participation and Flight-to-Quality*

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This paper extends the multi-asset model of Huang, Zhang and Zhu (2017) to four types of investors — insider, sophisticated, naïve, and noise investors — with different information structures and re-explores the limited participation phenomenon under correlation ambiguity. We investigate whether asset allocations depend on incomplete information under market equilibrium, namely, whether investors with less information could trade more intensively than investors with more information. As correlation coefficient increases and asset quality rises, we find that investors with less information escape from low-quality assets to high-quality assets and investors with more information escape to low-quality assets from high-quality assets. Thus, in equilibrium, all the investors exhibit a flight-to-quality trading pattern.

Key Words: Ambiguity aversion; General equilibrium; Non-participation; Asset allocation; CAPM analysis.
JEL Classification Numbers: G02, G11, D80, D81.

1. INTRODUCTION

In this paper, we explain flight-to-quality phenomenon endogenously by introducing two types of investors, one with partial information and one with no information on the correlation coefficient, to Huang, Zhang and Zhu (2017), which examines only two types of investors, one with inside information and one with partial information. Huang, Zhang and Zhu (2017) explore the limited participation phenomenon under the framework of correlation ambiguity when flight-to-quality is observed. The flight-to-

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quality phenomenon presents in information revelation when the four types of investors know more and more information. In our paper, four types of investors with heterogeneous beliefs participate in the economy. Inside investors know the true value of the correlation coefficient; sophisticated and naïve investors, however, know partial information, and sophisticated investors know more information than naïve investors; further, noise investors do not know any information. In this framework of ambiguous information, the unique equilibrium prevailing in the economy has seven alternative types depending on the quality ratio. The analysis of the equilibrium regions shows the existence of non-participation, minor participation, major participation, and full participation equilibria. In equilibrium, investors with less information might hold a larger position than investors with more information. Furthermore, correlation ambiguity brings about the flight-to-quality phenomenon endogenously. Investors with less information escape from low-quality assets to high-quality assets and investors with more information escape to low-quality assets from high-quality assets.

Flight-to-quality has long been a common phenomenon in financial markets, where market participants abruptly refuse to bear high risk and transfer their portfolios to safer assets. A large amount of the existing literature has regarded flight-to-quality (liquidity) as a main cause of market collapses, even describing it as a "financial accelerator" in times of recession (Bernanke, Gertler and Gilchrist, 1996). This is because small shocks in financial markets decrease the value of risky assets and increase volatility only slightly, whereas rising financial risks often lead to overwhelming market panic, which in turn drives market participants to escape from assets with more risks. Successively, small shocks are amplified and generating financial crises.

A related but distinct phenomenon that serves as one aspect of asset quality has been investigated in depth over recent decades: flight-to-liquidity. It is hard to distinguish flight-to-quality and flight-to-liquidity in volatile times, especially in fixed income markets. Some researchers view them as similar observations in financial markets and attempt to find the reasons for these anomalies. They conclude that market uncertainty plays a key role in the appearance of flight-to-quality. For example, Bernanke, Gertler and Gilchrist (1996) suggest that firms and households tend to be vulnerable at cyclical peaks, therefore, an adverse shock may impair their access to credit as their need for external funds rises; it is these high agency costs that push market participants to flee from low quality assets. Nevertheless, Vayanos (2004) attributes the flight-to-liquidity (quality) phenomenon to investors' increasing effective risk aversion. In his paper, a dynamic model is set up to examine how liquidity premia vary over time, and he shows that investors are more risk averse during volatile periods. Further, the pairwise correlation between assets can increase and the negative effect of volatility in downturns is strongly reflected in illiquid assets, thereby driving investors toward more liquid assets. Longstaff (2004) observes a large flight-to-liquidity premium in Treasury bond prices and shows that flight-to-quality premia are directly related to market sentiment. Comparable findings are obtained by Bethke, Gehde-Trapp and Kempf (2017), who use a sample of U.S. corporate bonds and find that bad investor sentiment leads to flight-to-quality trading behavior.

Caballero and Krishnamurthy (2008) introduce Knightian uncertainty into their theoretical model of crises based on liquidity shortages, noting that an increase in Knightian uncertainty or a decrease in aggregate liquidity can result in flight-to-quality effects. Dungey, McKenzie and Tambakis (2009) model a world in which fund managers face uncertainty about the state in the next period, finding that they tend to allocate funds to safer assets. Moreover, the flight-to-quality strategy taken by fund managers leads to asymmetric volatility responses in bond markets. Rösch and Kaserer (2014) also empirically confirm that in times of increased market uncertainty, flight-to-quality seems to take place. In their paper, they use a unique dataset on market liquidity to show the positive relationship between credit risk and liquidity risk. In times of crisis, assets with high credit quality become more liquid than low-quality assets, which corroborates the existence of the flight-to-quality phenomenon in stock markets. Guerrieri and Shimer (2014) argue that flight-to-quality occurs because of fire sales and the adverse selection problem during a depression. They develop a dynamic model of asset markets affected by adverse selection and demonstrate that in the equilibrium, the owner of high-quality assets will choose to hold out for a better selling price, whereas the shortage of buyers makes these high-quality assets less liquid. Sellers therefore may depress the value of all securities except the ones with the highest quality. Thus, the adverse selection problem in the market generates a flight-to-quality episode. Opitz and Szimayer (2018) adopt the conditional copula approach to quantify flight-to-quality and provide empirical evidence ascribing the flight-to-quality phenomenon to the Treasury bill rate. That is, a decrease in the Treasury bill rate increases the flight-to-quality risk indicator, with the decline usually related to a rise in market uncertainty.

The correlation between assets also plays an essential part in the occurrence of flight-to-quality. Some papers have implied that the rising market uncertainty in a financial crisis is usually accompanied by an increase in assets' pairwise correlations (Epstein and Halevy, 2019), which induces flight-to-quality behavior by investors with growing risk aversion (Vayanos, 2004). Cho, Choi, Kim and Kim (2016) consider the correlations between assets in global equity market conditions. They study the pairwise correlations between currency and stock returns in 9 developed and 12 emerging markets and show that currency returns in emerging markets are positively correlated with stock returns. This relationship is amplified during down markets; however, they attribute the positive correlations to the capital movements caused by flight-to-quality. Bethka, Gehde-Trapp and kempf (2017) explore the dynamics of bond correlations using a dataset of U.S. corporate bonds. They find that when investor sentiment worsens, the correlation between corporate bonds increases. Moreover, they ascribe the increasing correlation to investors' flight-to-quality decision, which is consistent with Cho, Choi, Kim and Kim (2016)'s conclusion.

Huang, Zhang and Zhu (2017) examine the limited participation phenomenon under the framework of correlation ambiguity when flight-to-quality is observed. In their paper, two types of investors with heterogeneous beliefs participate in the economy: insider investors and naïve investors. Insider investors know the true value of the correlation coefficient, whereas naïve investors do not and instead consider an interval of possible values. Naïve investors thus hold incomplete information, making them ambiguity-averse agents who have rational expectations about the marginal distributions of assets but perceive the correlation coefficient of risky assets as ambiguous. In this paper, we extend Huang, Zhang and Zhu (2017) by adding two types of investors, one with partial information and one with no information on the correlation coefficient. The investors who do not know the exact value of the correlation coefficient are called sophisticated investors and the investors who do not know any information on correlation coefficient are called noise investors herein. Therefore, we concentrate on four types of investors with heterogeneous beliefs: insider investors, sophisticated investors, naïve investors, and noise investors. In this framework of ambiguous information, the unique equilibrium prevailing in the economy has seven alternative types depending on the quality ratio. The analysis of the equilibrium regions shows the existence of non-participation, minor participation, major participation and full participation equilibria. Changes in the ambiguity of the correlation coefficient of sophisticated and naïve investors lead to the flight-to-quality phenomenon.

Similar to Huang, Zhang and Zhu (2017), a unique equilibrium prevails in an economy with seven alternative types depending on the quality ratio. When the quality ratio of the low-quality asset relative to the high-quality asset is infinitesimal, sophisticated and naïve and noise investors long buy the high-quality asset but do not trade in the low-quality asset; then, the equilibrium is called the non-participation one. When the quality ratio of the low-quality asset relative to the high-quality asset is tiny, naïve and noise investors long buy the high-quality asset but do not trade in the low-quality asset, whereas sophisticated investors participate in both assets; then, the equilibrium is called the minor participation one. When the quality ratio of the low-quality asset relative to the high-quality asset is small, only noise investors long buy the asset with high quality but do not trade in the asset with low quality, whereas sophisticated and naïve investors participate in both assets; then, the equilibrium is called the major participation one. Finally, when the quality ratio of the low-quality asset relative to the high-quality asset is medium, sophisticated and naïve and noise investors long buy both assets; then, the equilibrium is called the full participation one. Therefore, ambiguous (sophisticated and naïve and noise) investors are willing to choose risky assets with high quality, but do not participate in risky assets with low quality. The equilibrium regions provide limited participation for sophisticated and naïve and noise investors or naïve and noise investors or only noise investors for risky assets with low quality.

However, we cannot conclude that the trading position for investors with more information is greater than that for investors with less information. We show that, in equilibrium, investors with less information could trade more intensively than investors with more information. Furthermore, we find that correlation ambiguity brings about the flight-to-quality phenomenon endogenously: less-informed investors tend to escape from low-quality assets to high-quality assets and more-informed investors tend to escape from high-quality assets to low-quality assets.

The capital asset pricing model (CAPM) analysis reveals that no matter whether the economy is under a participating equilibrium (for sophisticated and naïve and noise investors) or a non-participating one (for noise investors or for naïve and noise investors or for sophisticated and naïve and noise investors), sophisticated and naïve and noise investors will favor the asset with higher quality even to an irrational degree, making its price increase and return lower compared with that standard models forecast. Therefore, no matter whether the equilibrium is a non-participating, partially participating, or fully participating one, risky assets with lower quality will generate positive excess returns, whereas assets with higher quality will generate negative excess returns.

In this paper, we investigate the implications of correlation ambiguity for investor behavior and asset prices. In our model, individuals' decision making incorporates both risk and ambiguity, and we demonstrate that limited participation arises from the rational decision by sophisticated and naïve and noise investors to avoid correlation ambiguity. In equilibrium, investors with less information might hold a larger position than investors with more information. Furthermore, correlation ambiguity brings about the flight-to-quality phenomenon endogenously.The remainder of this paper is structured as follows. In Section 2, we develop a multi-asset model of the general equilibrium including insider, sophisticated, naïve and noise investors. Section 3 describes the equilibrium regions and limited participation where equilibrium types can be shifted as the maximum correlation coefficient of sophisticated and naïve investors changes. In Section 4, we discuss the relationship between the asset allocation and information structure. Section 5 analyzes the flight-to-quality phenomenon. In Section 6, we provide the CAPM analysis. We conclude in Section 7. The proofs are provided in the appendices.

2. BASIC MODEL

We analyze an economy with three assets. One risk-free asset is money, which has a constant price of 1; the two risky financial assets are normally distributed with payoffs, $\tilde{X}_j \sim N(\mu_j, \sigma_j^2)$ for $j = 1, 2$. The correlation coefficient between the payoffs of risky assets is measured by ρ. Therefore the payoffs of risky assets follow a two-dimensional normal distribution $\tilde{X} \sim \mathbf{N}(\mu, \Sigma(\rho))$ where

$$\tilde{X} = \begin{pmatrix} \tilde{X}_1 \\ \tilde{X}_2 \end{pmatrix}, \qquad \mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \qquad \Sigma(\rho) = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}.$$

All investors have CARA utility for wealth, with the risk aversion parameter set equal to α:

$$u(w) = -e^{-\alpha w}.$$

There are four types of investors with heterogeneous beliefs in the economy: insider investors (I), sophisticated investors (S), naïve investors (N), and noise investors (U). They constitute fractions θ_I, θ_S, θ_N and θ_U in $(0, 1)$ of all investors, respectively, where $\theta_I + \theta_S + \theta_N + \theta_U = 1$. In our framework, sophisticated and naïve and noise investors are simplified collectively as **SNU investors**, sophisticated and naïve investors as **SN investors**, naïve and noise investors as **NU investors**. As insider investors know the true correlation coefficient, they sometimes can be called "informed investors" because they know complete information. Let $\hat{\rho}$ denote the true correlation coefficient. Since insider traders have rational expectations, they know the true value $\hat{\rho}$. From their viewpoint, the payoffs follow normally distributed $\tilde{X} \sim \mathbf{N}(\mu, \Sigma(\hat{\rho}))$. Inside investors are standard expected utility maximizers with rational expectations about payoff parameters. Sophisticated and naïve investors, however, differ from insider investors in that they do not know the exact value of the correlation coefficient and hence consider an interval $[\underline{\rho}_\Lambda, \overline{\rho}_\Lambda] \subset [-1, 1]$ of possible values with $-1 < \underline{\rho}_\Lambda < \overline{\rho}_\Lambda < 1$ for $\Lambda = S, N$. We assume that sophisticated investors know more information than naïve investors, meaning that sophisticated investors know more information on a narrow interval $[\underline{\rho}_S, \overline{\rho}_S]$ of possible values, whereas naïve investors know less information on a wide interval $[\underline{\rho}_N, \overline{\rho}_N]$ of possible values. Therefore, $[\underline{\rho}_S, \overline{\rho}_S] \subset [\underline{\rho}_N, \overline{\rho}_N] \subset [-1, 1]$

with $-1 < \underline{\rho}_N < \underline{\rho}_S < \overline{\rho}_S < \overline{\rho}_N < 1$ and both types of investors do not have a prior among them. Noise investors do not know the correlation coefficient at all, they sometimes can be called "uninformed investors" because they have no information. They consider the unit interval $[\underline{\rho}_U, \overline{\rho}_U] = [-1, 1]$ of values possible. For any $\rho \in [\underline{\rho}_\Lambda, \overline{\rho}_\Lambda]$ for $\Lambda = S, N, U$, SNU investors must face the possible economy $\tilde{X} \sim \mathbf{N}(\mu, \Sigma(\rho))$. Hence, they take them all into account when they make decisions. Following Gilboa and Schmeidler's (1989) axiomatic foundation for ambiguity aversion, we model SNU investors as choosing a portfolio to maximize their minimum expected utility over the set of possible distributions. To make our analysis of the equilibrium interaction among insider, sophisticated and naïve traders interesting, we assume that the true parameter value for insider traders is a convex combination of the extreme values considered possible by sophisticated traders, $\hat{\rho} \in [\underline{\rho}_S, \overline{\rho}_S] \subset [\underline{\rho}_N, \overline{\rho}_N] \subset [\underline{\rho}_U, \overline{\rho}_U] = [-1, 1]$.

The per capita endowments of the risky assets are (Z_1^0, Z_2^0). The exact distribution of endowments over investors does not affect their demand for risky assets, so we do not specify it. We denote a typical investor's wealth by w. Where no confusion would occur, we drop the investor index. The investor's budget constraint is

$$w = m + p_1 z_1 + p_2 z_2,$$

where m is the quantity of money, p_j is the price of asset j, and z_j is the quantity demanded of risky asset j. Investors are allowed to go long or short for each asset. If an investor chooses portfolio (m, z_1, z_2), her random next period wealth will be

$$\tilde{w} = m + \tilde{X}_1 z_1 + \tilde{X}_2 z_2.$$

Equivalently, we indicate the investor choice as $(w - p_1 z_1 - p_2 z_2, z_1, z_2)$, then her random next period wealth will be written as

$$\tilde{w} = w + (\tilde{X}_1 - p_1) z_1 + (\tilde{X}_2 - p_2) z_2.$$

For an **insider investor** with CARA utility of wealth and correlation coefficient parameter $\hat{\rho}$, the expected utility of this random wealth is a strictly increasing transformation of $f(z_1, z_2, \hat{\rho})$, where

$$\begin{aligned} f(z_1, z_2, \rho) &= w + (\mu_1 - p_1) z_1 + (\mu_2 - p_2) z_2 - \frac{1}{2}\alpha[\sigma_1^2 z_1^2 + 2\rho\sigma_1\sigma_2 z_1 z_2 + \sigma_2^2 z_2^2] \\ &= w + \sigma_1 R_1 z_1 + \sigma_2 R_2 z_2 - \frac{1}{2}\alpha[\sigma_1^2 z_1^2 + 2\rho\sigma_1\sigma_2 z_1 z_2 + \sigma_2^2 z_2^2], \end{aligned}$$

$R_j = \dfrac{\mu_j - p_j}{\sigma_j}$ is defined as the Sharpe ratio (Sharpe 1966), which measures the average of how much additional profit one can get by taking one more unit of risk for $j = 1, 2$ (Huang, Zhang and Zhu 2017). Then the insider investor's demand function for risky assets is given by

$$\begin{aligned} Z_I^* &= \begin{pmatrix} Z_{I1}^* \\ Z_{I2}^* \end{pmatrix} = \frac{1}{\alpha\sigma_1^2\sigma_2^2(1-\hat{\rho}^2)} \begin{pmatrix} \sigma_2^2(\mu_1 - p_1) - \hat{\rho}\sigma_1\sigma_2(\mu_2 - p_2) \\ \sigma_1^2(\mu_2 - p_2) - \hat{\rho}\sigma_1\sigma_2(\mu_1 - p_1) \end{pmatrix} \\ &= \frac{1}{\alpha(1-\hat{\rho}^2)} \begin{pmatrix} \dfrac{R_1 - \hat{\rho}R_2}{\sigma_1} \\ \dfrac{R_2 - \hat{\rho}R_1}{\sigma_2} \end{pmatrix}. \end{aligned} \tag{1}$$

An **SNU investor** evaluates the expected utility of wealth for each correlation coefficient parameter and chooses the portfolio that maximizes the minimum of these expected utilities. In effect, SNU investors try to avoid the worst-case outcomes and so choose a portfolio that explicitly limits exposure to such adverse outcomes. The expected utility of random wealth, given the correlation coefficient parameter $\rho \in [\underline{\rho}_\Lambda, \overline{\rho}_\Lambda]$ for $\Lambda = S, N, U$, is a strictly increasing transformation of $f(z_1, z_2, \rho)$. Thus, the SNU investor's decision problem can be written as a bi-level mathematical programming

$$\begin{aligned} \max_{(z_1,z_2)} \min_{\rho\in[\underline{\rho}_\Lambda,\overline{\rho}_\Lambda]} f(z_1, z_2, \rho) &= w + \sigma_1 R_1 z_1 + \sigma_2 R_2 z_2 \\ &- \frac{1}{2}\alpha[\sigma_1^2 z_1^2 + 2\rho\sigma_1\sigma_2 z_1 z_2 + \sigma_2^2 z_2^2] \end{aligned} \tag{2}$$

for $\Lambda = S, N, U$.

For **SN investors**, there are two approaches to this bi-level programming (2.2). Huang, Zhang and Zhu (2017) first solve the bi-level programming problem directly. Appendix A1 adopts Sion's Minimax Theorem (Sion, 1958) to exchange the order of bi-level optimization problem

$$\max_{(z_1,z_2)} \min_{\rho\in[\underline{\rho}_\Lambda,\overline{\rho}_\Lambda]} f(z_1, z_2, \rho) = \min_{\rho\in[\underline{\rho}_\Lambda,\overline{\rho}_\Lambda]} \max_{(z_1,z_2)} f(z_1, z_2, \rho).$$

The SN investor's demand function for risky assets is given by

$$Z_\Lambda^* = \begin{pmatrix} Z_{\Lambda 1}^* \\ Z_{\Lambda 2}^* \end{pmatrix} = \begin{cases} \dfrac{1}{\alpha(1-\underline{\rho}_\Lambda^2)} \begin{pmatrix} \dfrac{R_1 - \underline{\rho}_\Lambda R_2}{\sigma_1} \\ \dfrac{R_2 - \underline{\rho}_\Lambda R_1}{\sigma_2} \end{pmatrix}, & \text{if} \quad \begin{cases} R_1 < \underline{\rho}_\Lambda R_2 \\ R_2 > \underline{\rho}_\Lambda R_1 \end{cases} \text{or} \quad \begin{cases} R_1 > \underline{\rho}_\Lambda R_2 \\ R_2 < \underline{\rho}_\Lambda R_1 \end{cases} \\ \dfrac{1}{\alpha} \begin{pmatrix} 0 \\ \dfrac{R_2}{\sigma_2} \end{pmatrix}, & \text{if} \quad \begin{cases} \overline{\rho}_\Lambda R_2 \leqslant R_1 \leqslant \underline{\rho}_\Lambda R_2 \\ R_2 < 0 \end{cases} \text{or} \quad \begin{cases} \underline{\rho}_\Lambda R_2 \leqslant R_1 \leqslant \overline{\rho}_\Lambda R_2 \\ R_2 > 0 \end{cases} \\ \dfrac{1}{\alpha} \begin{pmatrix} \dfrac{R_1}{\sigma_1} \\ 0 \end{pmatrix}, & \text{if} \quad \begin{cases} R_1 < 0 \\ \overline{\rho}_\Lambda R_1 \leqslant R_2 \leqslant \underline{\rho}_\Lambda R_1 \end{cases} \text{or} \quad \begin{cases} R_1 > 0 \\ \underline{\rho}_\Lambda R_1 \leqslant R_2 \leqslant \overline{\rho}_\Lambda R_1 \end{cases} \\ \dfrac{1}{\alpha(1-\overline{\rho}_\Lambda^2)} \begin{pmatrix} \dfrac{R_1 - \overline{\rho}_\Lambda R_2}{\sigma_1} \\ \dfrac{R_2 - \overline{\rho}_\Lambda R_1}{\sigma_2} \end{pmatrix}, & \text{if} \quad \begin{cases} R_1 < \overline{\rho}_\Lambda R_2 \\ R_2 < \overline{\rho}_\Lambda R_1 \end{cases} \text{or} \quad \begin{cases} R_1 > \overline{\rho}_\Lambda R_2 \\ R_2 > \overline{\rho}_\Lambda R_1. \end{cases} \end{cases} \tag{3}$$

for $\Lambda = S, N$.

For **noise investors**, $[\underline{\rho}_U, \overline{\rho}_U] = [-1, 1]$. The noise investor's decision problem (2.2) can be written as a bi-level mathematical programming

$$\begin{aligned} \max_{(z_1,z_2)} \min_{\rho\in[-1,1]} f(z_1, z_2, \rho) &= w + \sigma_1 R_1 z_1 + \sigma_2 R_2 z_2 \\ &- \frac{1}{2}\alpha[\sigma_1^2 z_1^2 + 2\rho\sigma_1\sigma_2 z_1 z_2 + \sigma_2^2 z_2^2]. \end{aligned} \tag{4}$$

We examine the inner minimization problem that for any portfolio the minimum occurs at the minimum possible correlation if trading strategies for the two assets have different signs and at the maximum possible correlation if trading strategies for the two assets have the same signs. Whether the minimum occurs at the maximum or minimum mean payoff depends on the sign of $z_1 z_2$.

$$\min_{\rho\in[-1,1]} f(z_1, z_2, \rho) = \begin{cases} f(z_1, z_2, -1), & \text{if} \quad z_1 z_2 < 0 \\ f(0, z_2, \rho), & \text{if} \quad z_1 = 0 \\ f(z_1, 0, \rho), & \text{if} \quad z_2 = 0 \\ f(z_1, z_2, 1), & \text{if} \quad z_1 z_2 > 0. \end{cases} \tag{5}$$

Equation (2.5) represents a segmented curved surface. It indicates that for any portfolio the minimum occurs at the endpoints of the interval $[-1, 1]$. Consequently, what matters to an investor is not the correlation coefficient values insider the set, but rather the extreme values of the correlation coefficient. Whether the minimum occurs at -1 or 1 depends on the investor's positions on the assets. The minimum occurs at -1 if the investor goes long on one risky asset and short on the other and at 1 if the investor goes long (or short) on both risky assets.

Appendix A2 provides the noise investor's demand function for risky assets as

$$Z_U^* = \begin{pmatrix} Z_{U1}^* \\ Z_{U2}^* \end{pmatrix} = \begin{cases} \text{with } \sigma_1 Z_{U1}^* - \sigma_2 Z_{U2}^* = \dfrac{R_1}{\alpha} = -\dfrac{R_2}{\alpha}, & \text{if } \quad R_1 = -R_2 \\ \dfrac{1}{\alpha}\begin{pmatrix} 0 \\ \dfrac{R_2}{\sigma_2} \end{pmatrix}, & \text{if } \begin{cases} R_2 \leqslant R_1 \leqslant -R_2 \\ R_2 < 0 \end{cases} \text{ or } \begin{cases} -R_2 \leqslant R_1 \leqslant R_2 \\ R_2 > 0 \end{cases} \\ \dfrac{1}{\alpha}\begin{pmatrix} \dfrac{R_1}{\sigma_1} \\ 0 \end{pmatrix}, & \text{if } \begin{cases} R_1 < 0 \\ R_1 \leqslant R_2 \leqslant -R_1 \end{cases} \text{ or } \begin{cases} R_1 > 0 \\ -R_1 \leqslant R_2 \leqslant R_1 \end{cases} \\ \text{with } \sigma_1 Z_{U1}^* + \sigma_2 Z_{U2}^* = \dfrac{R_1}{\alpha} = \dfrac{R_2}{\alpha}, & \text{if } \quad R_1 = R_2. \end{cases} \tag{6}$$

Remark. In the cases $R_1 = -R_2$ and $R_1 = R_2$, there is just an equation for two unknown variables. It looks like that we do not solve the demand function. In fact, we have an additional restriction $R_1 = -R_2$ and $R_1 = R_2$, respectively, which helps us to work out general equilibrium.

Huang, Zhang and Zhu (2017) list the properties of SN investors' demand functions as follows: (1) An SN investor's demand function is continuous in price but has kinks at several prices. (2) The non-participation phenomenon for SN investors can be observed. (3) An SN investor's decision about whether to hold assets is independent of the set of correlation coefficients the investor believes possible as discrete or continuous. (4) Inside investors and SN investors trade in the same direction, except for the scenarios when non-participation occurs.

Fact 1. Insider investors and SN investors trade in the same direction, $Z_{Ij}^* Z_{\Lambda j}^* \geqslant 0$, except for the scenarios when non-participation occurs, $Z_{\Lambda j}^* = 0$, for $\Lambda = S, N$ and $j = 1, 2$. Specifically, when insider investors go long (or short) on an asset, SN investors will also go long (or short) or do not trade the same one.

(1) If $Z_{Ij}^* < 0$, then $Z_{\Lambda j}^* \leqslant 0$ for $\Lambda = S, N$ and $j = 1, 2$.
(2) If $Z_{Ij}^* = 0$, then $Z_{\Lambda j}^* = 0$ for $\Lambda = S, N$ and $j = 1, 2$.
(3) If $Z_{Ij}^* > 0$, then $Z_{\Lambda j}^* \geqslant 0$ for $\Lambda = S, N$ and $j = 1, 2$.

The proof of Fact 1 is in Huang, Zhang and Zhu (2017). We can use the same proof to check the results for sophisticated and naïve investors and for SN and noise investors, who trade in the same direction, except for the scenarios when non-participation occurs, see Appendix A3.

Fact 2. Sophisticated and naïve investors trade in the same direction, $Z_{Sj}^* Z_{Nj}^* \geqslant 0$, except for the scenarios when non-participation occurs, $Z_{Nj}^* = 0$, for $j = 1, 2$. Specifically, when sophisticated investors go long (or short) on an asset, naïve investors will also go long (or short) or do not trade the same one.

(1) If $Z_{Sj}^* < 0$, then $Z_{Nj}^* \leqslant 0$ for $j = 1, 2$.
(2) If $Z_{Sj}^* = 0$, then $Z_{Nj}^* = 0$ for $j = 1, 2$.

(3) If $Z^*_{Sj} > 0$, then $Z^*_{Nj} \geqslant 0$ for $j = 1, 2$.

Fact 3. SN and noise investors trade in the same direction, $Z^*_{\Lambda j} Z^*_{Uj} \geqslant 0$, for $\Lambda = S, N$, except for the scenarios when non-participation for noise investors occurs, $Z^*_{Uj} = 0$, for $j = 1, 2$. Specifically, when SN investors go long (or short) on an asset, noise investors will also go long (or short) or do not trade the same asset.

(1) If $Z^*_{\Lambda j} < 0$, then $Z^*_{Uj} \leqslant 0$ for $j = 1, 2$.

(2) If $Z^*_{\Lambda j} = 0$, then $Z^*_{Uj} = 0$ for $j = 1, 2$.

(3) If $Z^*_{\Lambda j} > 0$, then $Z^*_{Uj} \geqslant 0$ for $j = 1, 2$.

First, Facts $1 \sim 3$ help us eliminate impossible cases of the equilibrium. Under the assumption that both assets have positive supply, these three facts immediately rule out the equilibrium in which SNU investors short either or both of the risky assets. Thus, only seven types of equilibria are possible in our economy. Second, this result tells us that these four types of investors are simultaneously on the demand side or supply side of the risky assets. This prevents insider investors from exploiting ambiguity averse investors' lack of information or confidence when the latter wrongly short or long the assets. We can also consider this as a result of SNU investors' prudence in trading, since the investor will not long or short an asset when insider investors do otherwise; and further, the investors with less information trade cautiously because they follow the trading direction as the investors with more information.

We consider the equilibrium condition: the per capita demand for assets equals per capita supply. Equating the demands from Equations (2.1) - (2.3) - (2.6) to this supply then results in

$$\theta_I Z^*_I + \theta_S Z^*_S + \theta_N Z^*_N + \theta_U Z^*_U = Z^0 \tag{7}$$

or $\theta_I Z^*_{Ij} + \theta_S Z^*_{Sj} + \theta_N Z^*_{Nj} + \theta_U Z^*_{Uj} = Z^0_j$ for $j = 1, 2$.

To simplify notations, we define, for $-1 < \rho < \rho_S < \rho_N < 1$,

$$\begin{aligned} k(\rho, \rho_S, \rho_N) &\equiv \frac{\theta_I}{1-\rho^2}\rho + \frac{\theta_S}{1-\rho_S^2}\rho_S + \frac{\theta_N}{1-\rho_N^2}\rho_N \qquad \text{and} \\ K(\rho, \rho_S, \rho_N) &\equiv \frac{\theta_I}{1-\rho^2} + \frac{\theta_S}{1-\rho_S^2} + \frac{\theta_N}{1-\rho_N^2}, \end{aligned}$$

$$\begin{aligned}
\dot{k}(\rho,\rho_S) &\equiv k(\rho,\rho_S,0)=\frac{\theta_I}{1-\rho^2}\rho+\frac{\theta_S}{1-\rho_S^2}\rho_S \quad \text{and}\\
\dot{K}(\rho,\rho_S) &\equiv K(\rho,\rho_S,0)=\frac{\theta_I}{1-\rho^2}+\frac{\theta_S}{1-\rho_S^2}+\theta_N,\\
\ddot{k}(\rho) &\equiv k(\rho,0,0)=\frac{\theta_I}{1-\rho^2}\rho \quad \text{and}\\
\ddot{K}(\rho) &\equiv K(\rho,0,0)=\frac{\theta_I}{1-\rho^2}+\theta_S+\theta_N,
\end{aligned}$$

then

$$\begin{aligned}
|k(\rho,\rho_S,\rho_N)| &< K(\rho,\rho_S,\rho_N) \text{ and hence } \frac{k(\rho,\rho_S,\rho_N)}{K(\rho,\rho_S,\rho_N)}\in(\rho,\rho_N),\\
|\dot{k}(\rho,\rho_S)| &< \dot{K}(\rho,\rho_S)-\theta_N<\dot{K}(\rho,\rho_S) \text{ and hence } \frac{\dot{k}(\rho,\rho_S)}{\dot{K}(\rho,\rho_S)-\theta_N}\in(\rho,\rho_S),
\end{aligned}$$

and $|\ddot{k}(\rho)|<\ddot{K}(\rho)-\theta_S-\theta_N<\ddot{K}(\rho)$.

We define a measure of quality of risky asset j as the product of standard deviation and per capita endowment, $\sigma_j Z_j^0$. We denote the ratio of quality of the two risky assets as

$$E_{12}=\frac{\sigma_1 Z_1^0}{\sigma_2 Z_2^0} \qquad \text{and} \qquad E_{21}=\frac{\sigma_2 Z_2^0}{\sigma_1 Z_1^0}.$$

We define, for $-1<\hat{\rho}<\overline{\rho}_S<\overline{\rho}_N<1$,

$$\begin{aligned}
H(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) &= \frac{1-\dfrac{\theta_S}{1-\overline{\rho}_S^2}(\overline{\rho}_N-\overline{\rho}_S)(\overline{\rho}_S-\hat{\rho})}{\dfrac{\theta_I}{1-\hat{\rho}^2}(\overline{\rho}_N-\hat{\rho})+\dfrac{\theta_S}{1-\overline{\rho}_S^2}(\overline{\rho}_N-\overline{\rho}_S)}-\hat{\rho} \quad \text{and}\\
h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) &= \frac{1}{H(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)}\\
M(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) &= 1+\frac{\theta_U}{\dfrac{\theta_I}{1+\hat{\rho}}+\dfrac{\theta_S}{1+\overline{\rho}_S}+\dfrac{\theta_N}{1+\overline{\rho}_N}} \quad \text{and}\\
m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) &= \frac{1}{M(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)},
\end{aligned}$$

then $0<h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)<m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)<1<M(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)<H(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$. From Equilibria 2.2 and 2.3 or Equilibria 3.2 and 3.3 in Appendix A4, we

have

$$\begin{aligned} H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) &= \frac{\left[\dot{K}(\hat{\rho}, \overline{\rho}_S) + \theta_U\right] - \overline{\rho}_N \dot{k}(\hat{\rho}, \overline{\rho}_S)}{\overline{\rho}_N \left[\dot{K}(\hat{\rho}, \overline{\rho}_S) - \theta_N\right] - \dot{k}(\hat{\rho}, \overline{\rho}_S)} \\ &= \frac{[K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) + \theta_U] - \overline{\rho}_N k(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)}{\overline{\rho}_N K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) - k(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)}. \end{aligned}$$

From either Equilibria 2.1 and 2.2 or Equilibria 3.1 and 3.2 in Appendix A4, we have

$$\begin{aligned} H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) &= \frac{1-\hat{\rho}^2}{\theta_I(\overline{\rho}_S - \hat{\rho})} - \hat{\rho} = \frac{\left[\ddot{K}(\hat{\rho}) + \theta_U\right] - \overline{\rho}_S \ddot{k}(\hat{\rho})}{\overline{\rho}_S \left[\ddot{K}(\hat{\rho}) - \theta_S - \theta_N\right] - \ddot{k}(\hat{\rho})} \\ &= \frac{\left[\dot{K}(\hat{\rho}, \overline{\rho}_S) + \theta_U\right] - \overline{\rho}_S \dot{k}(\hat{\rho}, \overline{\rho}_S)}{\overline{\rho}_S \left[\dot{K}(\hat{\rho}, \overline{\rho}_S) - \theta_N\right] - \dot{k}(\hat{\rho}, \overline{\rho}_S)}. \end{aligned}$$

From Equilibria 2.3, 3.3 and 4 in Appendix A4, we have

$$M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) = \frac{[K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) + \theta_U] - k(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)}{K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) - k(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)}.$$

It is easy to check $1 < H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$ for $-1 < \hat{\rho} < \overline{\rho}_S < \overline{\rho}_N < 1$, then

$$\begin{aligned} 0 &< h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) < h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < 1 \\ &< M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S). \end{aligned}$$

Appendix 4 proves Theorem 1 on the existence of the general equilibrium.

THEOREM 1. *There exists a unique equilibrium in the markets. It is one of the following seven types:*

1 Non-participating in Asset 1 for SNU Investors *If the quality of asset 1 is infinitesimal relative to asset 2, $E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$, the equilibrium prices of the risky assets are given by Equations (A4.3a) and (A4.3b). In equilibrium, an insider investor holds positions of risky assets in Equations (A4.4a) and (A4.4b), and an SNU investor holds positions of risky assets in Equations (A4.5a) and (A4.5b).*

2 Non-participating in Asset 1 for NU Investors *If the quality of asset 1 is tiny relative to asset 2, $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) < E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, the* *equilibrium prices of the risky assets are given by Equations (A4.8a) and (A4.8b). In equilibrium, an insider investor holds positions of risky assets in Equations (A4.9a) and (A4.9b), a sophisticated investor holds positions of risky assets in Equations (A4.10a) and (A4.10b), and an NU investor holds positions of risky assets in Equations (A4.11a) and (A4.11b).*

3 Non-participating in Asset 1 for Noise Investors *If the quality of asset 1 is small relative to asset 2,* $h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < E_{12} \leqslant m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$, *the equilibrium prices of the risky assets are given by Equations (A4.14a) and (A4.14b). In equilibrium, an insider investor holds positions of risky assets in Equations (A4.15a) and (A4.15b), a sophisticated investor holds positions of risky assets in Equations (A4.16a) and (A4.16b), a naïve investor holds positions of risky assets in Equations (A4.17a) and (A4.17b), and a noise investor holds positions of risky assets in Equations (A4.18a) and (A4.18b).*

4 Participating in Both Assets for SNU Investors *If the quality of asset 1 is medium relative to asset 2,* $m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < E_{12} < M(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$, *the equilibrium prices of the risky assets are given by Equations (A4.38a) and (A4.38b). In equilibrium, an insider investor holds positions of risky assets in Equations (A4.39a) and (A4.39b), a sophisticated investor holds positions of risky assets in Equations (A4.40a) and (A4.40b), a naïve investor holds positions of risky assets in Equations (A4.41a) and (A4.41b), and a noise investor holds positions of risky assets in Equations (A4.42a) and (A4.42b).*

5 Non-participating in Asset 2 for Noise Investors *If the quality of asset 1 is big relative to asset 2,* $M(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) \leqslant E_{12} < H(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$, *the equilibrium prices of the risky assets are given by Equations (A4.32a) and (A4.32b). In equilibrium, an insider investor holds positions of risky assets in Equations (A4.33a) and (A4.33b), a sophisticated investor holds positions of risky assets in Equations (A4.34a) and (A4.34b), a naïve investor holds positions of risky assets in Equations (A4.35a) and (A4.35b), and a noise investor holds positions of risky assets in Equations (A4.36a) and (A4.36b).*

6 Non-participating in Asset 2 for NU Investors *If the quality of asset 1 is huge relative to asset 2,* $H(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) \leqslant E_{12} < H(\hat{\rho},\overline{\rho}_S,\overline{\rho}_S)$, *the equilibrium prices of the risky assets are given by Equations (A4.26a) and (A4.26b). In equilibrium, an insider investor holds positions of risky assets in Equations (A4.27a) and (A4.27b), a sophisticated investor holds positions of risky assets in Equations (A4.28a) and (A4.28b), and an NU investor holds positions of risky assets in Equations (A4.29a) and (A4.29b).*

7 Non-participating in Asset 2 for SNU Investors *If the quality of asset 1 is infinitude relative to asset 2,* $H(\hat{\rho},\overline{\rho}_S,\overline{\rho}_S) \leqslant E_{12}$, *the equilibrium prices of the risky assets are given by Equations (A4.21a) and (A4.21b). In equilibrium, an insider investor holds positions of risky assets in Equations*

(A4.22a) and (A4.22b), and an SNU investor holds positions of risky assets in Equations (A4.23a) and (A4.23b).

Remark 1. As shown in the previous subsection, the decision on whether to hold an asset is made by comparing the Sharpe ratios of the two assets. In equilibrium, after the Sharpe ratios are solved endogenously, comparing the Sharpe ratio is equivalent to comparing the exogenous quality of risky asset $\sigma_j Z_j^0$ for $j = 1, 2$. If the quality ratio is infinitesimal, $E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$, SNU investors make non-participation decisions on asset 1 (but insider investors long buy this asset) and all investors long buy asset 2. If the quality ratio is tiny, $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) < E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, NU investors make non-participation decisions on asset 1 (but insider and sophisticated investors long buy this asset) and all investors long buy asset 2. If the quality ratio is small, $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{12} \leqslant m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, noise investors make non-participation decisions on asset 1 (but insider and SN investors long buy this asset) and all investors long buy asset 2. If the quality ratio is medium, $m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{12} < M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, all investors long buy both assets. If the quality ratio is big, $M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \leqslant E_{12} < H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, noise investors make non-participation decisions on asset 2 (but insider and SN investors long buy this asset) and all investors long buy asset 1. If the quality ratio is huge, $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \leqslant E_{12} < H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$, NU investors make non-participation decisions on asset 2 (but insider and sophisticated investors long buy this asset) and all investors long buy asset 1. If the quality ratio is infinitude, $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) \leqslant E_{12}$, SNU investors make non-participation decisions on asset 2 (but insider investors long buy this asset) and all investors long buy asset 1. Thus the higher the quality, the more favorable the asset is.

Remark 2. According to symmetry, the conditions in Theorem 1 can be written for quality of asset 2 relative to asset 1. In **Equilibrium Type [1]**, if the quality ratio is infinitesimal, $E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$, SNU investors do not trade in asset 1. In **Equilibrium Type [7]**, if the quality ratio is infinitude, $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) \leqslant E_{12}$, SNU investors do not trade in asset 2. We call Types [1] and [7] the **"non-participation equilibrium"**. In **Equilibrium Type [2]**, if the quality ratio is tiny, $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) < E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, NU investors do not trade in asset 1. In **Equilibrium Type [6]**, if the quality ratio is huge, $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \leqslant E_{12} < H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$, NU investors do not trade in asset 2. We call Types [2] and [6] the **"minor-participation equilibrium"**. In **Equilibrium Type [3]**, if the quality ratio is small, $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{12} \leqslant m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, noise investors do not trade in asset 1. In **Equilibrium Type [5]**, if the quality ratio is big, $M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \leqslant E_{12} < H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, noise investors do not trade in asset 2. We call Types [3] and [5] the **"major-participation equilibrium"**. In **Equilibrium Type**

[4], if the quality ratio is medium, $m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{12} < M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, all investors trade in both assets. We call Type [4] the **"full-participation equilibrium"**.

Remark 3. Huang, Zhang and Zhu (2017) examine two types of investors (insider and naïve investors) and show three types of equilibria: if the quality ratio is medium, there exists one **participating equilibrium in both assets for naïve investors**; and if the quality ratio is small or big, there exists one **non-participating equilibrium in risky asset 1 or 2 for naïve Investors**, respectively. In this paper we add another two types of investors, sophisticated investors with part of information and noise investors (or uninformed investors) without information, and four types of investors (insider, sophisticated, naïve and noise investors) participate in the financial markets, then our Theorem 1 presents seven types of equilibra: if the quality ratio is medium, there exists one **participating equilibrium in both assets for SNU Investors**; if the quality ratio is small or big, there exists one **non-participating equilibrium in risky asset 1 or 2 for noise investors**, respectively; if the quality ratio is tiny or huge, there exists one **non-participating equilibrium in risky asset 1 or 2 for NU Investors**, respectively; if the quality ratio is infinitesimal or infinitude, there exists one **non-participating equilibrium in risky asset 1 or 2 for SNU Investors**, respectively. The conclusion suggests that the investors who hold more information are willing to participate in trading, while the investors who hold less information are not willing to participate in trading, which is limited participation phenomenon due to incomplete information. Our result is more complicated than the existence of general equilibrium for two types of investors found in Huang, Zhang and Zhu (2017) with three types of equilibria.

Remark 4. Both $m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ and $M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ form the thresholds that determine whether the equilibrium is a participating one or not for noise investors (given that SN investors purchase both risky assets). Both $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ and $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ form the thresholds that determine whether the equilibrium is a non-participating one or not for naïve investors (given that sophisticated investors purchase both risky assets and that noise investors do not trade the risky asset with low quality). Their extreme values $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$ and $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$ form the thresholds that determine whether the equilibrium is a non-participating one or not for sophisticated investors (given that NU investors do not trade the risky asset with low quality). These two critical values $m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ and $M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ are determined by the maximum correlation coefficients for SN investors. Function $M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ is strictly increasing in the maximum correlation coefficients $\overline{\rho}_S$ and $\overline{\rho}_N$ for SN investors, while function $m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ is strictly decreasing in the maximum correlation coefficients $\overline{\rho}_S$ and $\overline{\rho}_N$ for SN investors. These two critical values $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ and $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ are determined by the maximum correlation coefficients for SN investors. Function $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ is strictly decreasing in the maximum correlation coefficient $\overline{\rho}_N$ for naïve investors with $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) = \dfrac{1-\hat{\rho}^2}{\theta_I(\overline{\rho}_S - \hat{\rho})} - \hat{\rho}$ and $H(\hat{\rho}, \overline{\rho}_S, 1) = \dfrac{\theta_N + \theta_U}{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S}} + 1$, while function $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ is strictly increasing in the maximum correlation coefficient $\overline{\rho}_N$ for naïve investors. Furthermore, function $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$ is strictly decreasing in the maximum correlation coefficient $\overline{\rho}_S$ for sophisticated investors, while function $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$ is strictly increasing in the maximum correlation coefficient $\overline{\rho}_S$ for sophisticated investors.

From Theorem 1 we know that the minimum correlation coefficients $\underline{\rho}_S$ and $\underline{\rho}_N$ for sophisticated and naïve investors do not matter with respect to deciding which equilibrium type occurs in the economy or affecting equilibrium prices (Huang, Zhang and Zhu, 2017).

Another interesting feature of the limited participation is that non-participation on both risky assets cannot happen simultaneously under equilibrium. This can be observed directly from Theorem 1. This phenomenon is unique since it cannot be found in the models of ambiguity for expected payoffs or volatilities. Intuitively, if SNU investors decide not to trade the low-quality asset (and investors with less information also decide not to trade this asset), the individual avoids correlation ambiguity and invests rationally in the high-quality asset.

3. EQUILIBRIUM REGIONS AND LIMITED PARTICIPATION

As stated in Theorem 1, if the quality ratio is infinitesimal, $E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$, then there exists a unique non-participation equilibrium in the markets, and SNU investors will not trade risky asset 1 but they hold long positions in risky asset 2; if the quality ratio is tiny, $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) < E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, then there exists a unique minor-participation equilibrium in the markets, and NU investors will not trade risky asset 1 but they hold long positions in risky asset 2, while sophisticated investors hold long positions in both risky assets; if the quality ratio is small, $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{12} \leqslant m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, then there exists a unique major-participation equilibrium in the markets, and noise investors will not trade risky asset 1 but they hold long positions in risky asset 2, while SN investors hold long positions in both risky assets; if the quality ratio is medium, $m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{12} < M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, then there exists a unique full-participation equilibrium in the markets, and SNU investors will hold long positions in both assets; if the quality ratio is big, $M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \leqslant E_{12} < H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, then

FIG. 1. Equilibrium Regions and Limited Participation

FIG 1.1: Equilibrium Regions with Changes in Maximum Correlation Coefficient $\overline{\rho}_N$ for Naïve Investors

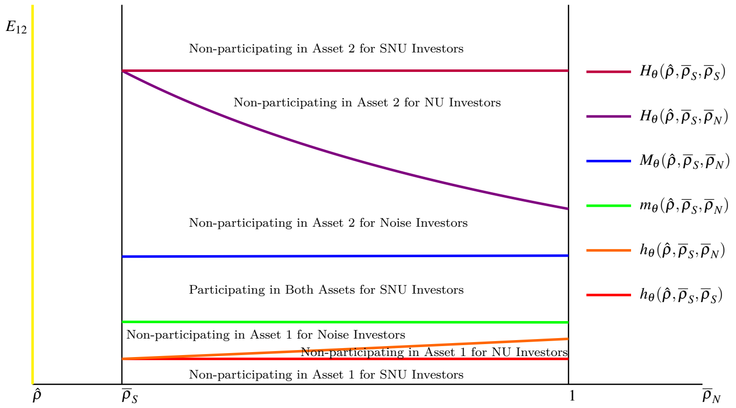
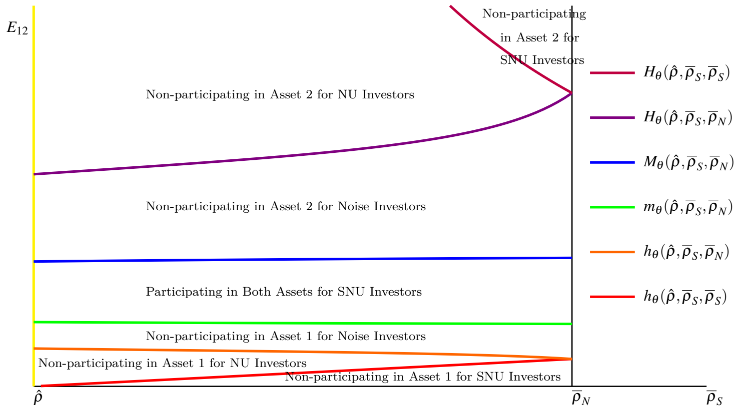


FIG 1.2: Equilibrium Regions with Changes in Maximum Correlation Coefficient $\overline{\rho}_S$ for Sophisticated Investors



there exists a unique major-participation equilibrium in the markets, noise investors will not trade risky asset 2 but they hold long positions in risky asset 1, while SN investors hold long positions in both risky assets; if the quality ratio is huge, $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \leqslant E_{12} < H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$, then there exists a unique minor-participation equilibrium in the markets, NU investors will not trade risky asset 2 but they hold long positions in risky asset 1, while sophisticated investors hold long positions in both risky assets; and if the quality ratio is infinitude, $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) \leqslant E_{12}$, then there exists a unique non-participation equilibrium in the markets, and SNU investors will not trade risky asset 2 but they hold long positions in risky asset 1. Figure 1 reports the seven equilibrium regions with changes in maximum correlation coefficient $\overline{\rho}_N$ for naïve investors in plane $\overline{\rho}_N - O - E_{12}$ for the seven cases in Theorem 1, given maximum correlation coefficient $\overline{\rho}_S$ for sophisticated investors, and the seven equilibrium regions with changes in maximum correlation coefficient $\overline{\rho}_S$ for sophisticated investors in plane $\overline{\rho}_S - O - E_{12}$ for the seven cases in Theorem 1, ceteris paribus.

3.1. Maximum Correlation Coefficient for Naïve Investors

For any given maximum correlation coefficient $\overline{\rho}_S$ for sophisticated traders, the point $(\overline{\rho}_N, E_{12})$ lies in one of seven regions shown in plane $\overline{\rho}_N - O - E_{12}$ in Figure 1.1, thus implying seven different types of equilibria. If the point $(\overline{\rho}_N, E_{12})$ is in the rectangle $(\overline{\rho}_S, 1) \times (0, h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)]$, then there exists a **Type [1] Equilibrium** — *non-participating one in which SNU traders do not trade asset 1* (but they long buy asset 2), and if the point $(\overline{\rho}_N, E_{12})$ is in the rectangle $(\overline{\rho}_S, 1) \times [H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S), \infty)$, then there exists a **Type [7] Equilibrium** — *non-participating one in which SNU traders do not trade asset 2* (but they long buy asset 1). Function $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ is strictly decreasing and convex in $\overline{\rho}_N$ with $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \geqslant H(\hat{\rho}, \overline{\rho}_S, 1) = 1 + \dfrac{\theta_N + \theta_U}{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S}}$, and function $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ is strictly increasing and concave in $\overline{\rho}_N$ with $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \leqslant h(\hat{\rho}, \overline{\rho}_S, 1) < 1$, therefore, if the point $(\overline{\rho}_N, E_{12})$ is in the curved triangle $(\overline{\rho}_S, 1) \times (h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S), h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)]$, then there exists a **Type [2] Equilibrium** — *non-participating one in which NU traders do not trade asset 1* (but they long buy asset 2 and sophisticated traders long buy both assets), and if the point $(\overline{\rho}_N, E_{12})$ is in the curved triangle $(\overline{\rho}_S, 1) \times [H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N), H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S))$, then there exists a **Type [6] Equilibrium** — *non-participating one in which NU traders do not trade asset 2* (but they long buy asset 1 and sophisticated traders long buy both assets). Function $M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ is strictly increasing and concave in $\overline{\rho}_N$ with

$$
\begin{aligned}
1 + \frac{\theta_U}{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S + \theta_N}{1+\overline{\rho}_S}} &= M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) \leqslant M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \\
&\leqslant M(\hat{\rho}, \overline{\rho}_S, 1) = 1 + \frac{\theta_U}{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{2}},
\end{aligned}
$$

and function $m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ is strictly decreasing and convex in $\overline{\rho}_N$ with $m(\hat{\rho}, \overline{\rho}_S, 1) \leqslant m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \leqslant m(\hat{\rho}, \overline{\rho}_S, 0)$, therefore, if the point $(\overline{\rho}_N, E_{12})$ is in the curved trapezoid $(\overline{\rho}_S, 1) \times (h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N), m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)]$, then there exists a **Type [3] Equilibrium** — *non-participating one in which noise traders do not trade asset 1* (but they long buy asset 2 and SN traders long buy both assets), and if the point $(\overline{\rho}_N, E_{12})$ is in the curved trapezoid $(\overline{\rho}_S, 1) \times [M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N), H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N))$, then there exists a **Type [5] Equilibrium** — *non-participating one in which noise traders do not trade asset 2* (but they long buy asset 1 and SN traders long buy both assets). Finally, if the point $(\overline{\rho}_N, E_{12})$ is in the curved trapezoid $(\overline{\rho}_S, 1) \times (m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N), M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N))$, then there exists a **Type [4] Equilibrium** — *participating one in which SNU traders trade both assets.*

Increasing maximum correlation coefficient $\overline{\rho}_N$ for naïve investors can alter the equilibrium types, ceteris paribus. To see this mathematically, note that **Equilibrium Type [4]** occurs if and only if the quality ratio is medium, $m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{12} < M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$. Function $M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ is strictly increasing and concave in $\overline{\rho}_N$ on $[\overline{\rho}_S, 1]$, while function $m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ is strictly increasing and convex in $\overline{\rho}_N$ on $[\overline{\rho}_S, 1]$. Function $M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ increases from $M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) = 1 + \dfrac{\theta_U}{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S + \theta_N}{1+\overline{\rho}_S}}$ to $M(\hat{\rho}, \overline{\rho}_S, 1) = 1 + \dfrac{\theta_U}{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{2}}$, and function $m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ decreases from $m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$ to $m(\hat{\rho}, \overline{\rho}_S, 1)$. Thus, if the quality ratio E_{12} satisfies $m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) \leqslant E_{12} \leqslant M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$, the equilibrium must be at Type [4]. We now report the results for left-hand side: $E_{12} < m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$. If $m(\hat{\rho}, \overline{\rho}_S, 1) < E_{12} < m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$, increasing maximum correlation coefficient $\overline{\rho}_N$ for naïve investors can alter the relation between E_{12} and $m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, thus switches the equilibrium from Type [3] to Type [4]. If the quality ratio E_{12} satisfies $h(\hat{\rho}, \overline{\rho}_S, 1) \leqslant E_{12} \leqslant m(\hat{\rho}, \overline{\rho}_S, 1)$, the equilibrium must be at Type [3]. Function $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ is strictly increasing and concave in $\overline{\rho}_N$ on $[\overline{\rho}_S, 1]$, and increases from $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$ to $h(\hat{\rho}, \overline{\rho}_S, 1)$. If $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) < E_{12} < h(\hat{\rho}, \overline{\rho}_S, 1)$, increasing maximum correlation coefficient $\overline{\rho}_N$ for naïve investors can alter the relation between E_{12} and $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, thus switches the equilibrium from Type [3] to Type [2]. If the quality ratio E_{12} satisfies $E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$, the equilibrium must be at Type [1]. We then report the results for right-hand side: $M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) < E_{12}$. If $M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) < E_{12} < M(\hat{\rho}, \overline{\rho}_S, 1)$, increasing maximum correlation coefficient $\overline{\rho}_N$ for naïve investors can alter the relation between E_{12} and $M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, thus switches the equilibrium from Type [5] to Type [4]. If the quality ratio E_{12} satisfies $M(\hat{\rho}, \overline{\rho}_S, 1) \leqslant E_{12} \leqslant H(\hat{\rho}, \overline{\rho}_S, 1)$, the equilibrium must be at Type [5]. Function $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ is strictly decreasing and convex in $\overline{\rho}_N$ on $[\overline{\rho}_S, 1]$, and decreases from $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) = \dfrac{1-\hat{\rho}^2}{\theta_I(\overline{\rho}_S - \hat{\rho})} - \hat{\rho}$ to $H(\hat{\rho}, \overline{\rho}_S, 1) =$

$1+\dfrac{\theta_N+\theta_U}{\dfrac{\theta_I}{1+\hat{\rho}}+\dfrac{\theta_S}{1+\overline{\rho}_S}}$. If $H(\hat{\rho},\overline{\rho}_S,1)<E_{12}<H(\hat{\rho},\overline{\rho}_S,\overline{\rho}_S)$, increasing maximum correlation coefficient $\overline{\rho}_N$ for naïve investors can alter the relation between E_{12} and $H(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$, thus switches the equilibrium from Type [5] to Type [6]. If the quality ratio E_{12} satisfies $H(\hat{\rho},\overline{\rho}_S,\overline{\rho}_S)\leqslant E_{12}$, the equilibrium must be at Type [7].

We can also understand this intuitively (Huang, Zhang and Zhu, 2017). To hold both assets, ambiguous (SNU) investors must tolerate the correlation ambiguity if the quality ratio is medium (at a small interval). When the quality ratio is small or big enough, the two assets have a significant distinction for NU investors. As the market becomes more uncertain to them and maximum correlation coefficient $\overline{\rho}_N$ for naïve investors increases, to avoid ambiguity noise investors will choose to hold only the asset with higher quality, resulting in a non-participating equilibrium in asset with lower quality for noise investors (but SN investors trade in both assets). When the quality ratio is tiny or huge enough, the two assets have a significant distinction for SN investors. As the market becomes more uncertain to them and maximum correlation coefficient $\overline{\rho}_N$ for naïve investors increases, to avoid ambiguity naïve investors will choose to hold only the asset with higher quality, resulting in a non-participating equilibrium in asset with lower quality for naïve investors (but sophisticated investors trade in both assets). Furthermore, when the quality ratio is infinitesimal or infinitude enough, the two assets also have a significant distinction for insider and sophisticated investors. As the market becomes more uncertain to them and maximum correlation coefficient $\overline{\rho}_N$ for naïve investors increases, to avoid ambiguity both types of sophisticated investors will choose to hold only the asset with higher quality, resulting in a non-participating equilibrium in asset with lower quality for sophisticated investors.

Proposition 1. When the quality ratio is in the middle interval, $m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_S)\leqslant E_{12}\leqslant M(\hat{\rho},\overline{\rho}_S,\overline{\rho}_S)$, changes in maximum correlation coefficient $\overline{\rho}_N$ for naïve investors do not alter the equilibrium type and the equilibrium can be only at Type [4] (*a participating one in both assets for SNU investors*). If E_{12} lies in the close intervals on the both sides, $m(\hat{\rho},\overline{\rho}_S,1)<E_{12}<m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_S)$ or $M(\hat{\rho},\overline{\rho}_S,\overline{\rho}_S)<E_{12}<M(\hat{\rho},\overline{\rho}_S,1)$, increasing maximum correlation coefficient $\overline{\rho}_N$ for naïve investors may shift the equilibrium type to Type [4] (*a participating one in both assets for SNU investors*) from Type [3] or [5] (*a non-participating one in asset 1 or 2 for noise investors*), respectively, where SN investors will hold both assets, while noise investors will hold the higher-quality asset and do not trade the lower-quality asset. If E_{12} lies in the near intervals on the both sides, $h(\hat{\rho},\overline{\rho}_S,1)\leqslant E_{12}\leqslant m(\hat{\rho},\overline{\rho}_S,1)$ or $M(\hat{\rho},\overline{\rho}_S,1)\leqslant E_{12}\leqslant H(\hat{\rho},\overline{\rho}_S,1)$, changes in maximum correlation coefficient $\overline{\rho}_N$ for naïve investors do not alter the

equilibrium type and the equilibrium can be only at Type [3] or [5] (*a non-participating one in asset 1 or 2 for noise investors*), respectively. If E_{12} lies in the far intervals on the both sides, $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) < E_{12} < h(\hat{\rho}, \overline{\rho}_S, 1)$ or $H(\hat{\rho}, \overline{\rho}_S, 1) < E_{12} < H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$, increasing maximum correlation coefficient $\overline{\rho}_N$ for naïve investors may shift the equilibrium type form Type [3] (*a non-participating one in asset 1 for noise investors*) to Type [2] (*a non-participating one in asset 1 for NU investors*) or from Type [5] (*a non-participating one in asset 2 for noise investors*) to Type [6] (*a non-participating one in both assets for NU investors*), respectively. If E_{12} lies in the remote intervals on the both sides, $E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$ or $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) \leqslant E_{12}$, changes in maximum correlation coefficient $\overline{\rho}_N$ for naïve investors do not alter the equilibrium type and the equilibrium will remain at Type [1] or [7] (*a non-participating one in asset 1 or 2 for SNU investors*), respectively, where ambiguous and noise investors will hold the higher-quality asset and do not trade the lower-quality asset.

3.2. Maximum Correlation Coefficient for Sophisticated Investors

For any given maximum correlation coefficient $\overline{\rho}_N$ for naïve traders, the point $(\overline{\rho}_S, E_{12})$ lies in one of seven regions shown in plane $\overline{\rho}_S - O - E_{12}$ in Figure 1.2, thus implying seven different types of equilibria. Function $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$ is strictly decreasing and convex in $\overline{\rho}_S$ and function $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$ is strictly increasing and concave in $\overline{\rho}_S$, thus, if the point $(\overline{\rho}_S, E_{12})$ is in the curved triangle $(\hat{\rho}, \overline{\rho}_N) \times (0, h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)]$, then there exists a **Type [1] Equilibrium** — *non-participating one in which SNU traders do not trade asset 1* (but they long buy asset 2), and if the point $(\overline{\rho}_S, E_{12})$ is in the curved triangle $(\hat{\rho}, \overline{\rho}_N) \times [H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S), \infty)$, then there exists a **Type [7] Equilibrium** — *non-participating one in which SNU traders do not trade asset 2* (but they long buy asset 1). Function $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ is strictly decreasing in $\overline{\rho}_S$ and function $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ is strictly increasing in $\overline{\rho}_S$, therefore, if the point $(\overline{\rho}_S, E_{12})$ is in the curved triangle $(\hat{\rho}, \overline{\rho}_N) \times (h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S), h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)]$, then there exists a **Type [2] Equilibrium** — *non-participating one in which NU traders do not trade asset 1* (but they long buy asset 2 and sophisticated traders long buy both assets), and if the point $(\overline{\rho}_S, E_{12})$ is in the curved triangle $(\hat{\rho}, \overline{\rho}_N) \times [H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N), H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S))$, then there exists a **Type [6] Equilibrium** — *non-participating one in which NU traders do not trade asset 2* (but they long buy asset 1 and sophisticated traders long buy both assets). Function $M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ is strictly increasing and concave in $\overline{\rho}_S$ with

$$1+\frac{\theta_U}{\dfrac{\theta_I+\theta_S}{1+\hat{\rho}}+\dfrac{\theta_N}{1+\overline{\rho}_N}} = M(\hat{\rho},\hat{\rho},\overline{\rho}_N) \leqslant M(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$$

$$\leqslant M(\hat{\rho},\overline{\rho}_N,\overline{\rho}_N) = 1+\frac{\theta_U}{\dfrac{\theta_I}{1+\hat{\rho}}+\dfrac{\theta_S+\theta_N}{1+\overline{\rho}_N}},$$

and function $m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$ is strictly decreasing and convex in $\overline{\rho}_S$ with $m(\hat{\rho},\overline{\rho}_N,\overline{\rho}_N) \leqslant m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) \leqslant m(\hat{\rho},\hat{\rho},\overline{\rho}_N)$, therefore, if the point $(\overline{\rho}_S, E_{12})$ is in the curved trapezoid $(\hat{\rho},\overline{\rho}_N) \times (h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N), m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)]$, then there exists a **Type [3] Equilibrium** — *non-participating one in which noise traders do not trade asset 1* (but they long buy asset 2 and SN traders long buy both assets), and if the point $(\overline{\rho}_S, E_{12})$ is in the curved trapezoid $(\hat{\rho},\overline{\rho}_N) \times [M(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N), H(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N))$, then there exists a **Type [5] Equilibrium** — *non-participating one in which noise traders do not trade asset 2* (but they long buy asset 1 and SN traders long buy both assets). Finally, if the point $(\overline{\rho}_S, E_{12})$ is in the curved trapezoid $(\hat{\rho},\overline{\rho}_N) \times (m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N), M(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N))$, then there exists a **Type [4] Equilibrium** — *participating one in which SNU traders trade both assets.*

Increasing maximum correlation coefficient $\overline{\rho}_S$ for sophisticated investors can alter the equilibrium types, ceteris paribus. To see this mathematically, note that the **Equilibrium Type [4]** occurs if and only if the quality ratio is medium, $m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < E_{12} < M(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$. Function $M(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$ is strictly increasing and concave in $\overline{\rho}_S$ on $[\hat{\rho},\overline{\rho}_N]$, while function $m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$ is strictly decreasing and convex in $\overline{\rho}_S$ on $[\hat{\rho},\overline{\rho}_N]$. $M(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$ increases from $M(\hat{\rho},\hat{\rho},\overline{\rho}_N) = 1+\dfrac{\theta_U}{\dfrac{\theta_I+\theta_S}{1+\hat{\rho}}+\dfrac{\theta_N}{1+\overline{\rho}_N}}$ to $M(\hat{\rho},\overline{\rho}_N,\overline{\rho}_N) = 1+\dfrac{\theta_U}{\dfrac{\theta_I}{1+\hat{\rho}}+\dfrac{\theta_S+\theta_N}{1+\overline{\rho}_N}}$, and $h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$ decreases from $h(\hat{\rho},\hat{\rho},\overline{\rho}_N)$ to $h(\hat{\rho},\overline{\rho}_N,\overline{\rho}_N)$. Thus, if the quality ratio E_{12} satisfies $m(\hat{\rho},\hat{\rho},\overline{\rho}_N) \leqslant E_{12} \leqslant M(\hat{\rho},\hat{\rho},\overline{\rho}_N)$, the equilibrium must be at Type [4]. We now report the results for left-hand side: $E_{12} < m(\hat{\rho},\hat{\rho},\overline{\rho}_N)$. If $m(\hat{\rho},\overline{\rho}_N,\overline{\rho}_N) < E_{12} < m(\hat{\rho},\hat{\rho},\overline{\rho}_N)$, increasing maximum correlation coefficient $\overline{\rho}_S$ for sophisticated investors can alter the relation between E_{12} and $m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$, thus switches the equilibrium from Type [3] to Type [4]. If the quality ratio E_{12} satisfies $h(\hat{\rho},\hat{\rho},\overline{\rho}_N) \leqslant E_{12} \leqslant m(\hat{\rho},\overline{\rho}_N,\overline{\rho}_N)$, the equilibrium must be at Type [3]. Function $h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$ is strictly decreasing in $\overline{\rho}_S$ on $[\hat{\rho},\overline{\rho}_N]$ and $h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$ decreases from $h(\hat{\rho},\hat{\rho},\overline{\rho}_N)$ to $h(\hat{\rho},\overline{\rho}_N,\overline{\rho}_N)$. If $h(\hat{\rho},\overline{\rho}_N,\overline{\rho}_N) < E_{12} < h(\hat{\rho},\hat{\rho},\overline{\rho}_N)$, increasing maximum correlation coefficient $\overline{\rho}_S$ for sophisticated investors can alter the relation between E_{12} and $h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$, thus switches the equilibrium from Type [2] to Type [3]. Function $h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_S)$ is strictly increasing and concave in $\overline{\rho}_S$ on $[\hat{\rho}, \overline{\rho}_N]$ and $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$ increases from $h(\hat{\rho}, \hat{\rho}, \hat{\rho}) = 0$ to $h(\hat{\rho}, \overline{\rho}_N, \overline{\rho}_N)$. If the quality ratio E_{12} satisfies $E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_N, \overline{\rho}_N)$, increasing maximum correlation coefficient $\overline{\rho}_S$ for sophisticated investors can alter the relation between E_{12} and $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, thus switches the equilibrium from Type [2] to Type [1]. We then report the results for right-hand side: $M(\hat{\rho}, \hat{\rho}, \overline{\rho}_N) < E_{12}$. If $M(\hat{\rho}, \hat{\rho}, \overline{\rho}_N) < E_{12} < M(\hat{\rho}, \overline{\rho}_N, \overline{\rho}_N)$, increasing maximum correlation coefficient $\overline{\rho}_S$ for sophisticated investors can alter the relation between E_{12} and $M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, thus switches the equilibrium from Type [5] to Type [4]. If the quality ratio E_{12} satisfies $M(\hat{\rho}, \overline{\rho}_N, \overline{\rho}_N) \leqslant E_{12} \leqslant H(\hat{\rho}, \hat{\rho}, \overline{\rho}_N)$, the equilibrium must be at Type [5]. Function $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ is strictly increasing in $\overline{\rho}_S$ on $[\hat{\rho}, \overline{\rho}_N]$, and $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ increases from $H(\hat{\rho}, \hat{\rho}, \overline{\rho}_N) = \dfrac{1-\hat{\rho}^2}{(\theta_I + \theta_S)(\overline{\rho}_N - \hat{\rho})} - \hat{\rho}$ to $H(\hat{\rho}, \overline{\rho}_N, \overline{\rho}_N) = \dfrac{1-\hat{\rho}^2}{\theta_I(\overline{\rho}_N - \hat{\rho})} - \hat{\rho}$. If $H(\hat{\rho}, \hat{\rho}, \overline{\rho}_N) < E_{12} < H(\hat{\rho}, \overline{\rho}_N, \overline{\rho}_N)$, increasing maximum correlation coefficient $\overline{\rho}_S$ for sophisticated investors can alter the relation between E_{12} and $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, thus switches the equilibrium from Type [6] to Type [5]. Function $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) = \dfrac{1-\hat{\rho}^2}{\theta_I(\overline{\rho}_S - \hat{\rho})} - \hat{\rho}$ is strictly decreasing and convex in $\overline{\rho}_S$ on $[\hat{\rho}, \overline{\rho}_N]$, and $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$ decreases from $H(\hat{\rho}, \hat{\rho}, \hat{\rho}) = \infty$ to $H(\hat{\rho}, \overline{\rho}_N, \overline{\rho}_N) = \dfrac{1-\hat{\rho}^2}{\theta_I(\overline{\rho}_N - \hat{\rho})} - \hat{\rho}$. If the quality ratio E_{12} satisfies $H(\hat{\rho}, \overline{\rho}_N, \overline{\rho}_N) \leqslant E_{12}$, increasing maximum correlation coefficient $\overline{\rho}_S$ for sophisticated investors can alter the relation between E_{12} and $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, thus switches the equilibrium from Type [6] to Type [7].

We can also understand this intuitively. To hold both assets, SN investors must tolerate the correlation ambiguity. When the quality ratio is small or large enough, the two assets have a significant distinction. As the market becomes more uncertain to them and maximum correlation coefficient $\overline{\rho}_S$ for sophisticated investors increases, to avoid ambiguity they will choose to hold only the asset with higher quality, resulting in a non-participating equilibrium in asset with lower quality for SN investors.

Proposition 2. When the quality ratio is in the middle interval, $m(\hat{\rho}, \hat{\rho}, \overline{\rho}_N) \leqslant E_{12} \leqslant M(\hat{\rho}, \hat{\rho}, \overline{\rho}_N)$, changes in maximum correlation coefficient $\overline{\rho}_S$ for sophisticated investors do not alter the equilibrium type and the equilibrium can be only at Type [4] (*a participating one in both assets for SNU investors*). If E_{12} lies in the close intervals on the both sides, $m(\hat{\rho}, \overline{\rho}_N, \overline{\rho}_N) < E_{12} < m(\hat{\rho}, \hat{\rho}, \overline{\rho}_N)$ or $M(\hat{\rho}, \hat{\rho}, \overline{\rho}_N) < E_{12} < M(\hat{\rho}, \overline{\rho}_N, \overline{\rho}_N)$, increasing maximum correlation coefficient $\overline{\rho}_S$ for sophisticated investors may shift the equilibrium type to Type [4] (*a participating one in both assets for SNU investors*) from Type [3] or [5] (*a non-participating one in asset 1 or 2 for noise investors*), respectively, where SN investors will hold both assets, while noise investors will hold the higher-quality asset and do not trade the lower-quality asset. If E_{12} lies in the near intervals on the both sides,

$h(\hat{\rho}, \hat{\rho}, \overline{\rho}_N) \leqslant E_{12} \leqslant m(\hat{\rho}, \overline{\rho}_N, \overline{\rho}_N)$ or $M(\hat{\rho}, \overline{\rho}_N, \overline{\rho}_N) \leqslant E_{12} \leqslant H(\hat{\rho}, \hat{\rho}, \overline{\rho}_N)$, changes in maximum correlation coefficient $\overline{\rho}_S$ for sophisticated investors do not alter the equilibrium type and the equilibrium can be only at Type [3] or [5] (*a non-participating one in asset 1 or 2 for noise investors*), respectively. If E_{12} lies in the far intervals on the both sides, $h(\hat{\rho}, \overline{\rho}_N, \overline{\rho}_N) < E_{12} < h(\hat{\rho}, \hat{\rho}, \overline{\rho}_N)$ or $H(\hat{\rho}, \hat{\rho}, \overline{\rho}_N) < E_{12} < H(\hat{\rho}, \overline{\rho}_N, \overline{\rho}_N)$, increasing maximum correlation coefficient $\overline{\rho}_S$ for sophisticated investors may shift the equilibrium type from Type [2] — *a non-participating one in asset 1 for NU investors* to Type [3] (*a non-participating one in asset 1 for noise investors*) or from Type [6] (*a non-participating one in both assets for NU investors*) to Type [5] (*a non-participating one in asset 2 for noise investors*), respectively. If E_{12} lies in the remote intervals on the both sides, $E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_N, \overline{\rho}_N)$ or $H(\hat{\rho}, \overline{\rho}_N, \overline{\rho}_N) \leqslant E_{12}$, increasing maximum correlation coefficient $\overline{\rho}_S$ for sophisticated investors may shift the equilibrium type from Type [2] (*a non-participating one in asset 1 for NU investors*) to Type [1] (*a non-participating one in asset 1 for SNU investors*) or from Type [6] (*a non-participating one in both assets for NU investors*) to Type [7] (*a non-participating one in asset 2 for SNU investors*), respectively.

4. ASSET ALLOCATIONS ON INFORMATION STRUCTURE

In this section, we present the relation between information structure and equilibrium positions. We compare the sizes of demand functions for four types of investors, and then compare their equilibrium positions.

4.1. Demand Functions on Information Revelation

Facts 1, 2 and 3 suggest the trading directions for different types of investors, but do not show the trading volume for investors. Sometimes, we conjecture that the demand functions for investors who hold more information are greater than that for investors who hold less information. In fact, it is an intuitive fallacy! Huang, Zhang and Zhu (2017) provide Fact 4 that sophisticated and naïve investors might hold larger positions (long or short) than the insider investors.

Fact 4. Sophisticated and naïve investors might hold larger positions (long or short) than insider investors.

We can use the same proof to check the result for sophisticated and naïve investors. Compared with sophisticated investors who avoid ambiguity and require compensation, a naïve investor also avoids ambiguity in the distribution of payoffs, thus reduces the size of the position in the risky asset. However, we use the following proposition to demonstrate that even when ambiguity aversion distorts naïve investors' behavior, they still might select a more aggressive position to hold.

Fact 5. Naïve investors might hold larger positions (long or short) than sophisticated investors.

Therefore, we cannot conclude that the more information investors hold, the larger the positions they hold in risky assets. On the other hand, since there exist two free variables in the noise investor's demand function, we can not compare the positions between noise investors and other types of investors. Noise investors might hold larger or smaller positions (long or short) than SN investors.

4.2. Equilibrium Positions on Information Structure

Theorem 1 provides the equilibrium positions for insider, sophisticated, and naïve investors, then we can compare the sizes of equilibrium positions between types of investors in Propositions 3, 4, and 5.

Appendix 5 compares equilibrium positions for insider and sophisticated investors as follows:

[1] For **Equilibrium Type [1]**, if the quality ratio is infinitesimal, $E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$, then $Z^*_{S2} > Z^*_{I2}$ for $\hat{\rho} > 0$.

[2] For **Equilibrium Type [2]**, if the quality ratio is tiny, $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) < E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, then $Z^*_{S2} > Z^*_{I2}$ for $E_{12} < \dfrac{\theta_I \overline{\rho}_S + \theta_S \hat{\rho}}{1 + (\theta_N + \theta_U)\hat{\rho}\overline{\rho}_S}$.

[3] For **Equilibrium Type [3]**, if the quality ratio is small, $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{12} \leqslant m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, then $Z^*_{S2} > Z^*_{I2}$ for $E_{12} < \dfrac{\theta_I \overline{\rho}_S + \theta_S \hat{\rho} + \theta_N \dfrac{(\hat{\rho} + \overline{\rho}_S) - \overline{\rho}_N(1 + \hat{\rho}\overline{\rho}_S)}{1 - \overline{\rho}_N^2}}{1 + \theta_N \dfrac{(\overline{\rho}_N - \hat{\rho})(\overline{\rho}_N - \overline{\rho}_S)}{1 - \overline{\rho}_N^2} + \theta_U \hat{\rho}\overline{\rho}_S}$.

[5] For **Equilibrium Type [5]**, if the quality ratio is big, $M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \leqslant E_{12} < H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, then $Z^*_{S1} > Z^*_{I1}$ for $E_{21} < \dfrac{\theta_I \overline{\rho}_S + \theta_S \hat{\rho} + \theta_N \dfrac{(\hat{\rho} + \overline{\rho}_S) - \overline{\rho}_N(1 + \hat{\rho}\overline{\rho}_S)}{1 - \overline{\rho}_N^2}}{1 + \theta_N \dfrac{(\overline{\rho}_N - \hat{\rho})(\overline{\rho}_N - \overline{\rho}_S)}{1 - \overline{\rho}_N^2} + \theta_U \hat{\rho}\overline{\rho}_S}$.

[6] For **Equilibrium Type [6]**, if the quality ratio is huge, $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \leqslant E_{12} < H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$, then $Z^*_{S1} > Z^*_{I1}$ for $E_{21} < \dfrac{\theta_I \overline{\rho}_S + \theta_S \hat{\rho}}{1 + (\theta_N + \theta_U)\hat{\rho}\overline{\rho}_S}$.

[7] For **Equilibrium Type [7]**, if the quality ratio is infinitude, $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) \leqslant E_{12}$, then $Z^*_{S1} > Z^*_{I1}$ for $\hat{\rho} > 0$.

From the analysis we obtain that the range for $Z^*_{S1} > Z^*_{I1}$ is

$$
\begin{aligned}
&\{E_{21} \leqslant h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_S) \text{ for } \hat{\rho}>0\} \cup \left\{ E_{21} < \frac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}}{1+(\theta_N+\theta_U)\hat{\rho}\overline{\rho}_S} \,\middle|\, h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_S) < E_{21} \leqslant h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) \right\} \\
&\quad \cup \left\{ E_{21} < \frac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}+\theta_N\dfrac{(\hat{\rho}+\overline{\rho}_S)-\overline{\rho}_N(1+\hat{\rho}\overline{\rho}_S)}{1-\overline{\rho}_N^2}}{1+\theta_N\dfrac{(\overline{\rho}_N-\hat{\rho})(\overline{\rho}_N-\overline{\rho}_S)}{1-\overline{\rho}_N^2}+\theta_U\hat{\rho}\overline{\rho}_S} \,\middle|\, h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < E_{21} \leqslant m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) \right\} \\
&= \begin{cases} \left\{ E_{21} < \dfrac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}}{1+(\theta_N+\theta_U)\hat{\rho}\overline{\rho}_S} \right\} \cup \left\{ h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < E_{21} < \dfrac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}+\theta_N\dfrac{(\hat{\rho}+\overline{\rho}_S)-\overline{\rho}_N(1+\hat{\rho}\overline{\rho}_S)}{1-\overline{\rho}_N^2}}{1+\theta_N\dfrac{(\overline{\rho}_N-\hat{\rho})(\overline{\rho}_N-\overline{\rho}_S)}{1-\overline{\rho}_N^2}+\theta_U\hat{\rho}\overline{\rho}_S} \right\}, & 0<\hat{\rho}<\overline{\rho}_S<\overline{\rho}_N<\dfrac{\hat{\rho}+\overline{\rho}_S}{1+\hat{\rho}\overline{\rho}_S} \\ \left\{ E_{21} < \dfrac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}}{1+(\theta_N+\theta_U)\hat{\rho}\overline{\rho}_S} \right\}, & 0<\hat{\rho}<\overline{\rho}_S<\dfrac{\hat{\rho}+\overline{\rho}_S}{1+\hat{\rho}\overline{\rho}_S}<\overline{\rho}_N \end{cases}
\end{aligned}
$$

and the range for $Z_{S2}^* > Z_{I2}^*$ is

$$
\begin{aligned}
&\{E_{12} \leqslant h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_S) \text{ for } \hat{\rho}>0\} \cup \left\{ E_{12} < \frac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}}{1+(\theta_N+\theta_U)\hat{\rho}\overline{\rho}_S} \,\middle|\, h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_S) < E_{12} \leqslant h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) \right\} \\
&\quad \cup \left\{ E_{12} < \frac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}+\theta_N\dfrac{(\hat{\rho}+\overline{\rho}_S)-\overline{\rho}_N(1+\hat{\rho}\overline{\rho}_S)}{1-\overline{\rho}_N^2}}{1+\theta_N\dfrac{(\overline{\rho}_N-\hat{\rho})(\overline{\rho}_N-\overline{\rho}_S)}{1-\overline{\rho}_N^2}+\theta_U\hat{\rho}\overline{\rho}_S} \,\middle|\, h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < E_{12} \leqslant m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) \right\} \\
&= \begin{cases} \left\{ E_{12} < \dfrac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}}{1+(\theta_N+\theta_U)\hat{\rho}\overline{\rho}_S} \right\} \cup \left\{ h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < E_{12} < \dfrac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}+\theta_N\dfrac{(\hat{\rho}+\overline{\rho}_S)-\overline{\rho}_N(1+\hat{\rho}\overline{\rho}_S)}{1-\overline{\rho}_N^2}}{1+\theta_N\dfrac{(\overline{\rho}_N-\hat{\rho})(\overline{\rho}_N-\overline{\rho}_S)}{1-\overline{\rho}_N^2}+\theta_U\hat{\rho}\overline{\rho}_S} \right\}, & 0<\hat{\rho}<\overline{\rho}_S<\overline{\rho}_N<\dfrac{\hat{\rho}+\overline{\rho}_S}{1+\hat{\rho}\overline{\rho}_S} \\ \left\{ E_{12} < \dfrac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}}{1+(\theta_N+\theta_U)\hat{\rho}\overline{\rho}_S} \right\}, & 0<\hat{\rho}<\overline{\rho}_S<\dfrac{\hat{\rho}+\overline{\rho}_S}{1+\hat{\rho}\overline{\rho}_S}<\overline{\rho}_N \end{cases}
\end{aligned}
$$

Therefore, the allocation of high-quality assets for sophisticated investors is higher than that for insider investors if the true correlation coefficient is positive.

Proposition 3. In equilibrium, sophisticated investors might hold larger positions than insider investors. Specifically, if $\hat{\rho} > 0$, then $Z_{S1}^* > Z_{I1}^*$ for

$$
E_{21} < \frac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}}{1+(\theta_N+\theta_U)\hat{\rho}\overline{\rho}_S} \quad \text{or} \quad h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < E_{21} < \frac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}+\theta_N\dfrac{(\hat{\rho}+\overline{\rho}_S)-\overline{\rho}_N(1+\hat{\rho}\overline{\rho}_S)}{1-\overline{\rho}_N^2}}{1+\theta_N\dfrac{(\overline{\rho}_N-\hat{\rho})(\overline{\rho}_N-\overline{\rho}_S)}{1-\overline{\rho}_N^2}+\theta_U\hat{\rho}\overline{\rho}_S}
$$

and $Z_{S2}^* > Z_{I2}^*$ for

$$
E_{12} < \frac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}}{1+(\theta_N+\theta_U)\hat{\rho}\overline{\rho}_S} \quad \text{or} \quad h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < E_{12} < \frac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}+\theta_N\dfrac{(\hat{\rho}+\overline{\rho}_S)-\overline{\rho}_N(1+\hat{\rho}\overline{\rho}_S)}{1-\overline{\rho}_N^2}}{1+\theta_N\dfrac{(\overline{\rho}_N-\hat{\rho})(\overline{\rho}_N-\overline{\rho}_S)}{1-\overline{\rho}_N^2}+\theta_U\hat{\rho}\overline{\rho}_S}.
$$

Appendix 5 compares equilibrium positions for insider and naïve investors as follows:

[1] + [2] For **Equilibrium Types [1] and [2]**, if the quality ratio is infinitesimal or tiny, $E_{12} \leqslant h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$, then $Z_{N2}^* > Z_{I2}^*$ for $\hat{\rho} > 0$.

FIG. 2. Comparison of Equilibrium Positions for Sophisticated and Insider Investors ($\hat{\rho} > 0$)

FIG 2.1: Comparison of Equilibrium Positions for Maximum Correlation Coefficient $\overline{\rho}_N$ for Naïve Investors

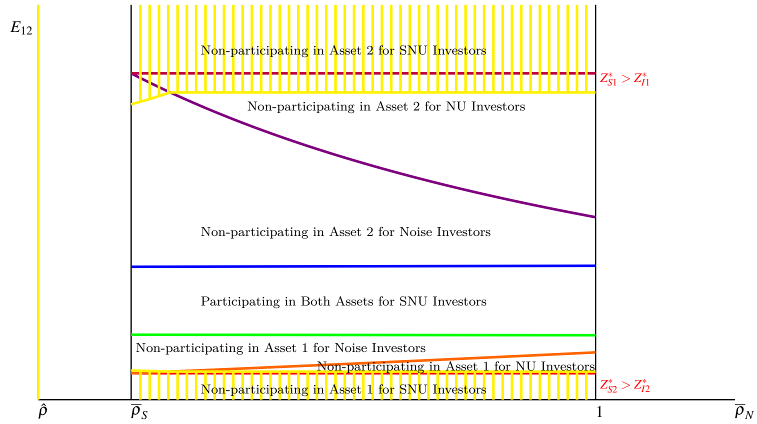
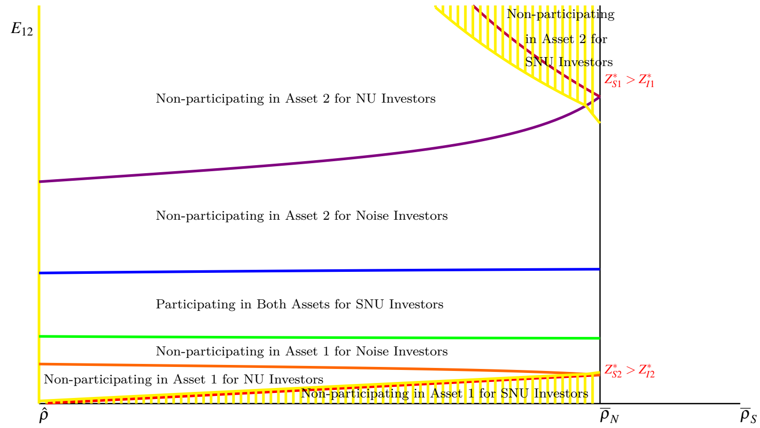


FIG 2.2: Comparison of Equilibrium Positions for Maximum Correlation Coefficient $\overline{\rho}_S$ for Sophisticated Investors



[3] For **Equilibrium Type [3]**, if the quality ratio is small, $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{12} \leqslant m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, then $Z^*_{N2} > Z^*_{I2}$ for $E_{12} < \dfrac{\theta_I \overline{\rho}_N + \theta_S \dfrac{(\hat{\rho} + \overline{\rho}_N) - \overline{\rho}_S(1 + \hat{\rho}\overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_N \hat{\rho}}{1 + \theta_S \dfrac{(\overline{\rho}_S - \hat{\rho})(\overline{\rho}_S - \overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_U \hat{\rho}\overline{\rho}_N}$.

[5] For **Equilibrium Type [5]**, if the quality ratio is big, $M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \leqslant E_{12} < H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, then $Z^*_{N1} > Z^*_{I1}$ for $E_{21} < \dfrac{\theta_I \overline{\rho}_N + \theta_S \dfrac{(\hat{\rho} + \overline{\rho}_N) - \overline{\rho}_S(1 + \hat{\rho}\overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_N \hat{\rho}}{1 + \theta_S \dfrac{(\overline{\rho}_S - \hat{\rho})(\overline{\rho}_S - \overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_U \hat{\rho}\overline{\rho}_N}$.

[6] + [7] For **Equilibrium Types [6] and [7]**, if the quality ratio is huge or infinitude, $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \leqslant E_{12}$, then $Z^*_{N1} > Z^*_{I1}$ for $\hat{\rho} > 0$.

From the analysis we obtain that the range for $Z^*_{N1} > Z^*_{I1}$ is

$$\begin{aligned}
&\{E_{21} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \text{ for } \hat{\rho} > 0\} \\
&\bigcup \left\{ E_{21} < \frac{\theta_I \overline{\rho}_N + \theta_S \dfrac{(\hat{\rho} + \overline{\rho}_N) - \overline{\rho}_S(1 + \hat{\rho}\overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_N \hat{\rho}}{1 + \theta_S \dfrac{(\overline{\rho}_S - \hat{\rho})(\overline{\rho}_S - \overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_U \hat{\rho}\overline{\rho}_N} \,\middle|\, h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{21} \leqslant m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \right\} \\
&= \left\{ E_{21} < \frac{\theta_I \overline{\rho}_N + \theta_S \dfrac{(\hat{\rho} + \overline{\rho}_N) - \overline{\rho}_S(1 + \hat{\rho}\overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_N \hat{\rho}}{1 + \theta_S \dfrac{(\overline{\rho}_S - \hat{\rho})(\overline{\rho}_S - \overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_U \hat{\rho}\overline{\rho}_N} \right\} \qquad \text{for } \hat{\rho} > 0
\end{aligned}$$

and the range for $Z^*_{N2} > Z^*_{I2}$ is

$$\begin{aligned}
&\{E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \text{ for } \hat{\rho} > 0\} \\
&\bigcup \left\{ E_{12} < \frac{\theta_I \overline{\rho}_N + \theta_S \dfrac{(\hat{\rho} + \overline{\rho}_N) - \overline{\rho}_S(1 + \hat{\rho}\overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_N \hat{\rho}}{1 + \theta_S \dfrac{(\overline{\rho}_S - \hat{\rho})(\overline{\rho}_S - \overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_U \hat{\rho}\overline{\rho}_N} \,\middle|\, h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{12} \leqslant m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \right\} \\
&= \left\{ E_{12} < \frac{\theta_I \overline{\rho}_N + \theta_S \dfrac{(\hat{\rho} + \overline{\rho}_N) - \overline{\rho}_S(1 + \hat{\rho}\overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_N \hat{\rho}}{1 + \theta_S \dfrac{(\overline{\rho}_S - \hat{\rho})(\overline{\rho}_S - \overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_U \hat{\rho}\overline{\rho}_N} \right\} \qquad \text{for } \hat{\rho} > 0.
\end{aligned}$$

Therefore, the allocation of high-quality assets for naïve investors is higher than that for insider investors if the true correlation coefficient is positive.

Proposition 4. In equilibrium, naïve investors might hold larger positions than insider investors. Specifically, if $\hat{\rho} > 0$, then $Z^*_{N1} > Z^*_{I1}$ for

$$E_{21} < \frac{\theta_I \overline{\rho}_N + \theta_S \dfrac{(\hat{\rho} + \overline{\rho}_N) - \overline{\rho}_S(1 + \hat{\rho}\overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_N \hat{\rho}}{1 + \theta_S \dfrac{(\overline{\rho}_S - \hat{\rho})(\overline{\rho}_S - \overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_U \hat{\rho}\overline{\rho}_N}$$

and $Z^*_{N2} > Z^*_{I2}$ for

$$E_{12} < \frac{\theta_I \overline{\rho}_N + \theta_S \dfrac{(\hat{\rho} + \overline{\rho}_N) - \overline{\rho}_S(1 + \hat{\rho}\overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_N \hat{\rho}}{1 + \theta_S \dfrac{(\overline{\rho}_S - \hat{\rho})(\overline{\rho}_S - \overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_U \hat{\rho}\overline{\rho}_N}.$$

FIG. 3. Comparison of Equilibrium Positions for Naïve and Insider Investors ($\hat{\rho} > 0$)

FIG 3.1: Comparison of Equilibrium Positions for Maximum Correlation Coefficient $\overline{\rho}_N$ for Naïve Investors

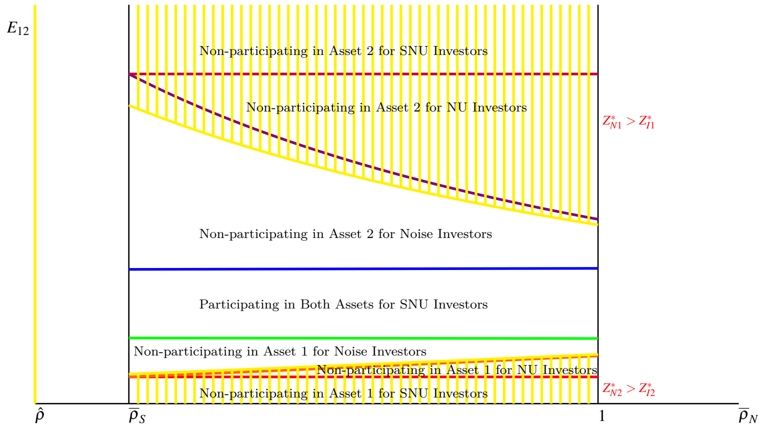
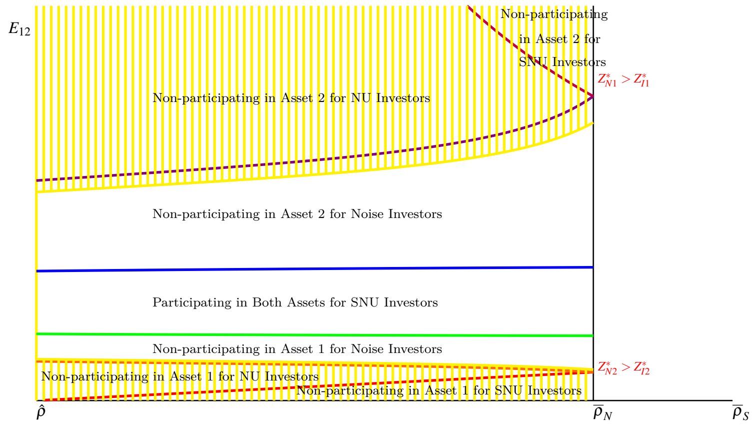


FIG 3.2: Comparison of Equilibrium Positions for Maximum Correlation Coefficient $\overline{\rho}_S$ for Sophisticated Investors



Appendix 5 compares equilibrium positions for sophisticated and naïve investors as follows:

[2] For **Equilibrium Type [2]**, if the quality ratio is tiny, $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) < E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, then $Z^*_{N2} > Z^*_{S2}$ for $\overline{\rho}_S > 0$.

[3] For **Equilibrium Type [3]**, if the quality ratio is small, $h(\hat\rho,\overline{\rho}_S,\overline{\rho}_N) < E_{12} \leqslant m(\hat\rho,\overline{\rho}_S,\overline{\rho}_N)$, then $Z^*_{N2} > Z^*_{S2}$ for $E_{12} < \dfrac{\theta_I \dfrac{(\overline{\rho}_S+\overline{\rho}_N)-\hat\rho(1+\overline{\rho}_S\overline{\rho}_N)}{1-\hat\rho^2} + \theta_S\overline{\rho}_N + \theta_N\overline{\rho}_S}{1+\theta_I\dfrac{(\hat\rho-\overline{\rho}_S)(\hat\rho-\overline{\rho}_N)}{1-\hat\rho^2} + \theta_U\overline{\rho}_S\overline{\rho}_N}$.

[5] For **Equilibrium Type [5]**, if the quality ratio is big, $M(\hat\rho,\overline{\rho}_S,\overline{\rho}_N) \leqslant E_{12} < H(\hat\rho,\overline{\rho}_S,\overline{\rho}_N)$, then $Z^*_{N1} > Z^*_{S1}$ for $E_{21} < \dfrac{\theta_I \dfrac{(\overline{\rho}_S+\overline{\rho}_N)-\hat\rho(1+\overline{\rho}_S\overline{\rho}_N)}{1-\hat\rho^2} + \theta_S\overline{\rho}_N + \theta_N\overline{\rho}_S}{1+\theta_I\dfrac{(\hat\rho-\overline{\rho}_S)(\hat\rho-\overline{\rho}_N)}{1-\hat\rho^2} + \theta_U\overline{\rho}_S\overline{\rho}_N}$.

[6] For **Equilibrium Type [6]**, if the quality ratio is huge, $H(\hat\rho,\overline{\rho}_S,\overline{\rho}_N) \leqslant E_{12} < H(\hat\rho,\overline{\rho}_S,\overline{\rho}_S)$, then $Z^*_{N1} > Z^*_{S1}$ for $\overline{\rho}_S > 0$.

From the analysis we obtain that the range for $Z^*_{N1} > Z^*_{S1}$ is

$$
\begin{aligned}
&\{h(\hat\rho,\overline{\rho}_S,\overline{\rho}_S) < E_{21} \leqslant h(\hat\rho,\overline{\rho}_S,\overline{\rho}_N) \text{ for } \overline{\rho}_S > 0\}\\
\cup\ &\left\{ E_{21} < \frac{\theta_I \dfrac{(\overline{\rho}_S+\overline{\rho}_N)-\hat\rho(1+\overline{\rho}_S\overline{\rho}_N)}{1-\hat\rho^2} + \theta_S\overline{\rho}_N + \theta_N\overline{\rho}_S}{1+\theta_I\dfrac{(\hat\rho-\overline{\rho}_S)(\hat\rho-\overline{\rho}_N)}{1-\hat\rho^2} + \theta_U\overline{\rho}_S\overline{\rho}_N} \;\middle|\; h(\hat\rho,\overline{\rho}_S,\overline{\rho}_N) < E_{21} \leqslant m(\hat\rho,\overline{\rho}_S,\overline{\rho}_N) \right\}\\
=\ &\left\{ h(\hat\rho,\overline{\rho}_S,\overline{\rho}_S) < E_{21} < \frac{\dfrac{\theta_I}{1-\hat\rho^2}[(\overline{\rho}_S+\overline{\rho}_N)-\hat\rho(1+\overline{\rho}_S\overline{\rho}_N)] + \theta_S\overline{\rho}_N + \theta_N\overline{\rho}_S}{\dfrac{\theta_I}{1-\hat\rho^2}[(1+\overline{\rho}_S\overline{\rho}_N)-\hat\rho(\overline{\rho}_S+\overline{\rho}_N)] + (\theta_S+\theta_N)} \right\} \qquad \text{for } \overline{\rho}_S > 0
\end{aligned}
$$

and the range for $Z^*_{N2} > Z^*_{S2}$ is

$$
\begin{aligned}
&\{h(\hat\rho,\overline{\rho}_S,\overline{\rho}_S) < E_{12} \leqslant h(\hat\rho,\overline{\rho}_S,\overline{\rho}_N) \text{ for } \overline{\rho}_S > 0\}\\
\cup\ &\left\{ E_{12} < \frac{\theta_I \dfrac{(\overline{\rho}_S+\overline{\rho}_N)-\hat\rho(1+\overline{\rho}_S\overline{\rho}_N)}{1-\hat\rho^2} + \theta_S\overline{\rho}_N + \theta_N\overline{\rho}_S}{1+\theta_I\dfrac{(\hat\rho-\overline{\rho}_S)(\hat\rho-\overline{\rho}_N)}{1-\hat\rho^2} + \theta_U\overline{\rho}_S\overline{\rho}_N} \;\middle|\; h(\hat\rho,\overline{\rho}_S,\overline{\rho}_N) < E_{12} \leqslant m(\hat\rho,\overline{\rho}_S,\overline{\rho}_N) \right\}\\
=\ &\left\{ h(\hat\rho,\overline{\rho}_S,\overline{\rho}_S) < E_{12} < \frac{\dfrac{\theta_I}{1-\hat\rho^2}[(\overline{\rho}_S+\overline{\rho}_N)-\hat\rho(1+\overline{\rho}_S\overline{\rho}_N)] + \theta_S\overline{\rho}_N + \theta_N\overline{\rho}_S}{\dfrac{\theta_I}{1-\hat\rho^2}[(1+\overline{\rho}_S\overline{\rho}_N)-\hat\rho(\overline{\rho}_S+\overline{\rho}_N)] + (\theta_S+\theta_N)} \right\} \qquad \text{for } \overline{\rho}_S > 0.
\end{aligned}
$$

Therefore, the allocation of high-quality assets for naïve investors is higher than that for sophisticated investors if the maximum correlation coefficient for sophisticated investors is positive.

Proposition 5. In equilibrium, naïve investors might hold larger positions than sophisticated investors. Specifically, if $\overline{\rho}_S > 0$, then $Z^*_{N1} > Z^*_{S1}$ for

$$
h(\hat\rho,\overline{\rho}_S,\overline{\rho}_S) < E_{21} < \frac{\dfrac{\theta_I}{1-\hat\rho^2}[(\overline{\rho}_S+\overline{\rho}_N)-\hat\rho(1+\overline{\rho}_S\overline{\rho}_N)] + \theta_S\overline{\rho}_N + \theta_N\overline{\rho}_S}{\dfrac{\theta_I}{1-\hat\rho^2}[(1+\overline{\rho}_S\overline{\rho}_N)-\hat\rho(\overline{\rho}_S+\overline{\rho}_N)] + (\theta_S+\theta_N)}
$$

and $Z^*_{N2} > Z^*_{S2}$ for

$$h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) < E_{12} < \frac{\dfrac{\theta_I}{1-\hat{\rho}^2}[(\overline{\rho}_S + \overline{\rho}_N) - \hat{\rho}(1 + \overline{\rho}_S\overline{\rho}_N)] + \theta_S\overline{\rho}_N + \theta_N\overline{\rho}_S}{\dfrac{\theta_I}{1-\hat{\rho}^2}[(1 + \overline{\rho}_S\overline{\rho}_N) - \hat{\rho}(\overline{\rho}_S + \overline{\rho}_N)] + (\theta_S + \theta_N)}.$$

FIG. 4. Comparison of Equilibrium Positions for Sophisticated and Insider Investors ($\overline{\rho}_S > 0$)

FIG 4.1: Comparison of Equilibrium Positions for Maximum Correlation Coefficient $\overline{\rho}_N$ for Naïve Investors

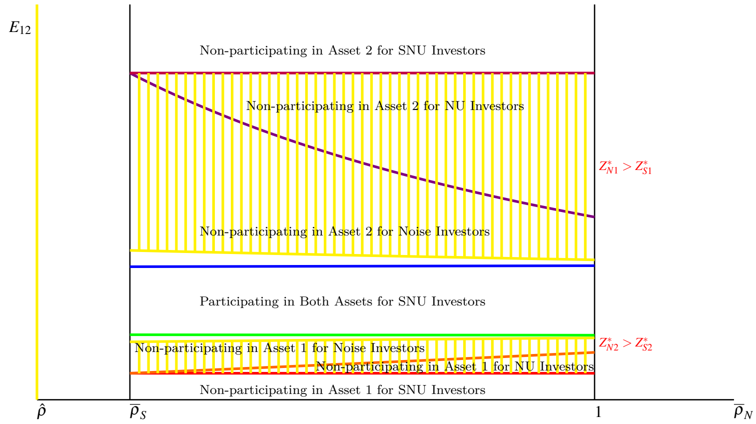
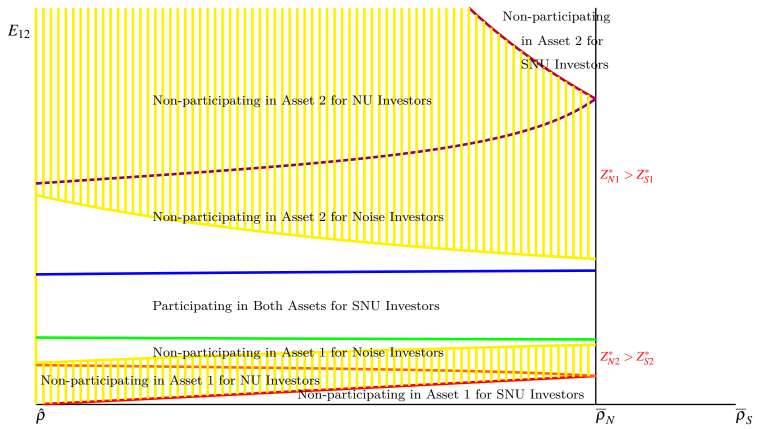


FIG 4.2: Comparison of Equilibrium Positions for Maximum Correlation Coefficient $\overline{\rho}_S$ for Sophisticated Investors



Theorem 1 also provides equilibrium positions for noise investors, then we can compare their equilibrium positions with insider and SN investors'. The results are reported in Propositions 6, 7 and 8.

Appendix 5 compares equilibrium positions for insider and noise investors as follows:

[1] + [2] + [3] For **Equilibrium Types [1] and [2] and [3]**, if the quality ratio is infinitesimal or tiny or small, $E_{12} \leqslant m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, then $Z^*_{U2} > Z^*_{I2}$ for $\hat{\rho} > 0$.

[4] For **Equilibrium Type [4]**, if the quality ratio is medium, $m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{12} < M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, then $Z^*_{U1} > Z^*_{I1}$ for $E_{21} < \dfrac{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U \hat{\rho}}{1+\hat{\rho}}}{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U}{1+\hat{\rho}}}$ and $Z^*_{U2} > Z^*_{I2}$ for $E_{12} < \dfrac{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U \hat{\rho}}{1+\hat{\rho}}}{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U}{1+\hat{\rho}}}$.

[5] + [6] + [7] For **Equilibrium Types [5] and [6] and [7]**, if the quality ratio is big or huge or infinitude, $M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \leqslant E_{12}$, then $Z^*_{U1} > Z^*_{I1}$ for $\hat{\rho} > 0$.

From the analysis we obtain that the range for $Z^*_{U1} > Z^*_{I1}$ is

$$
\begin{aligned}
&\{E_{21} \leqslant m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \text{ for } \hat{\rho} > 0\} \\
&\cup \left\{ E_{21} < \frac{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U \hat{\rho}}{1+\hat{\rho}}}{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U}{1+\hat{\rho}}} \,\middle|\, m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{21} \leqslant M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \right\} \\
&= \left\{ E_{21} < \frac{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U \hat{\rho}}{1+\hat{\rho}}}{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U}{1+\hat{\rho}}} \right\} \quad \text{for } \hat{\rho} > 0
\end{aligned}
$$

and the range for $Z^*_{U2} > Z^*_{I2}$ is

$$
\begin{aligned}
&\{E_{12} \leqslant m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \text{ for } \hat{\rho} > 0\} \\
&\cup \left\{ E_{12} < \frac{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U \hat{\rho}}{1+\hat{\rho}}}{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U}{1+\hat{\rho}}} \,\middle|\, m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{12} \leqslant M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \right\} \\
&= \left\{ E_{12} < \frac{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U \hat{\rho}}{1+\hat{\rho}}}{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U}{1+\hat{\rho}}} \right\} \quad \text{for } \hat{\rho} > 0
\end{aligned}
$$

Therefore, the allocation of high-quality assets for noise investors is higher than that for insider investors if the true correlation coefficient is positive.

Proposition 6. In equilibrium, noise investors might hold larger positions than insider investors. Specifically, if $\hat{\rho} > 0$, then $Z_{U1}^* > Z_{I1}^*$ for

$$E_{21} < \frac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U \hat{\rho}}{1+\hat{\rho}}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\hat{\rho}}}$$

and $Z_{U2}^* > Z_{I2}^*$ for

$$E_{12} < \frac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U \hat{\rho}}{1+\hat{\rho}}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\hat{\rho}}}.$$

Appendix 5 compares equilibrium positions for sophisticated and noise investors as follows:

[2] + [3] For **Equilibrium Types [2] and [3]**, if the quality ratio is tiny or small, $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) < E_{12} \leqslant m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, then $Z_{U2}^* > Z_{S2}^*$ for $\overline{\rho}_S > 0$.

[4] For **Equilibrium Type [4]**, if the quality ratio is medium, $m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{12} < M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, then $Z_{U1}^* > Z_{S1}^*$ for $E_{21} < \dfrac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U \overline{\rho}_S}{1+\overline{\rho}_S}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\overline{\rho}_S}}$ and $Z_{U2}^* > Z_{S2}^*$ for $E_{12} < \dfrac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U \overline{\rho}_S}{1+\overline{\rho}_S}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\overline{\rho}_S}}$.

[5] + [6] For **Equilibrium Types [5] and [6]**, if the quality ratio is big or huge, $M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \leqslant E_{12} < H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$, then $Z_{U1}^* > Z_{S1}^*$ for $\overline{\rho}_S > 0$.

From the analysis we obtain that the range for $Z_{U1}^* > Z_{S1}^*$ is

$$\begin{aligned} &\{h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) < E_{21} \leqslant m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \text{ for } \overline{\rho}_S > 0\} \\ \cup \ &\left\{ E_{21} < \frac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U \overline{\rho}_S}{1+\overline{\rho}_S}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\overline{\rho}_S}} \,\middle|\, m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{21} \leqslant M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \right\} \\ = \ &\left\{ h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) < E_{21} < \frac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U \overline{\rho}_S}{1+\overline{\rho}_S}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\overline{\rho}_S}} \right\} \quad \text{for } \overline{\rho}_S > 0 \end{aligned}$$

and the range for $Z_{U2}^* > Z_{S2}^*$ is

$$
\begin{aligned}
&\{h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_S) < E_{12} \leqslant m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) \text{ for } \overline{\rho}_S > 0\} \\
\cup &\left\{ E_{12} < \frac{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U\overline{\rho}_S}{1+\overline{\rho}_S}}{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U}{1+\overline{\rho}_S}} \,\middle|\, m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < E_{12} \leqslant M(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) \right\} \\
= &\left\{ h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_S) < E_{12} < \frac{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U\overline{\rho}_S}{1+\overline{\rho}_S}}{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U}{1+\overline{\rho}_S}} \right\} \quad \text{for } \overline{\rho}_S > 0
\end{aligned}
$$

Therefore, the allocation of high-quality assets for noise investors is higher than that for sophisticated investors if the maximum correlation coefficient for sophisticated investors is positive.

Proposition 7. In equilibrium, noise investors might hold larger positions than sophisticated investors. Specifically, if $\overline{\rho}_S > 0$, then $Z^*_{U1} > Z^*_{S1}$ for

$$h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_S) < E_{21} < \frac{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U\overline{\rho}_S}{1+\overline{\rho}_S}}{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U}{1+\overline{\rho}_S}}$$

and $Z^*_{U2} > Z^*_{S2}$ for

$$h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_S) < E_{12} < \frac{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U\overline{\rho}_S}{1+\overline{\rho}_S}}{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U}{1+\overline{\rho}_S}}.$$

Appendix 5 compares equilibrium positions for naïve and noise investors as follows:

[3] For **Equilibrium Type [3]**, if the quality ratio is small, $h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < E_{12} \leqslant m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$, then $Z^*_{U2} > Z^*_{N2}$ for $\overline{\rho}_N > 0$.

[4] For **Equilibrium Type [4]**, if the quality ratio is medium, $m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < E_{12} < M(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$, then $Z^*_{U1} > Z^*_{N1}$ for $E_{21} < \dfrac{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U\overline{\rho}_N}{1+\overline{\rho}_N}}{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U}{1+\overline{\rho}_N}}$ and $Z^*_{U2} > Z^*_{N2}$ for $E_{12} < \dfrac{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U\overline{\rho}_N}{1+\overline{\rho}_N}}{\dfrac{\theta_I}{1+\hat{\rho}} + \dfrac{\theta_S}{1+\overline{\rho}_S} + \dfrac{\theta_N}{1+\overline{\rho}_N} + \dfrac{\theta_U}{1+\overline{\rho}_N}}$.

[5] For **Equilibrium Type [5]**, if the quality ratio is big, $M(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) \leqslant E_{12} < H(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$, then $Z^*_{U1} > Z^*_{N1}$ for $\overline{\rho}_N > 0$.

From the analysis we obtain that the range for $Z_{U1}^* > Z_{SN}^*$ is

$$\{h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < E_{21} \leqslant m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) \text{ for } \overline{\rho}_N > 0\} \cup \left\{ E_{21} < \frac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U\overline{\rho}_N}{1+\overline{\rho}_N}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\overline{\rho}_N}} \middle| m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < E_{21} \leqslant M(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) \right\}$$

$$= \left\{ h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < E_{21} < \frac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U\overline{\rho}_N}{1+\overline{\rho}_N}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\overline{\rho}_N}} \right\} \quad \text{for } \overline{\rho}_N > 0$$

and the range for $Z_{U2}^* > Z_{N2}^*$ is

$$\{h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < E_{12} \leqslant m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) \text{ for } \overline{\rho}_N > 0\} \cup \left\{ E_{12} < \frac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U\overline{\rho}_N}{1+\overline{\rho}_N}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\overline{\rho}_N}} \middle| m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < E_{12} \leqslant M(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) \right\}$$

$$= \left\{ h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < E_{12} < \frac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U\overline{\rho}_N}{1+\overline{\rho}_N}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\overline{\rho}_N}} \right\} \quad \text{for } \overline{\rho}_N > 0$$

Therefore, the allocation of high-quality assets for noise investors is higher than that for naïve investors if the maximum correlation coefficient for naïve investors is positive.

Proposition 8. In equilibrium, noise investors might hold larger positions than naïve investors. Specifically, if $\overline{\rho}_N > 0$, then $Z_{U1}^* > Z_{N1}^*$ for

$$h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < E_{21} < \frac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U\overline{\rho}_N}{1+\overline{\rho}_N}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\overline{\rho}_N}}$$

and $Z_{U2}^* > Z_{N2}^*$ for

$$h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < E_{12} < \frac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U\overline{\rho}_N}{1+\overline{\rho}_N}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\overline{\rho}_N}}.$$

5. FLIGHT-TO-QUALITY PHENOMENON

Section 4 compares equilibrium positions between different types of investors and obtains the relation between information structure and equilibrium positions. The investors who hold more information must not have more equilibrium positions than those who hold less information. This

FIG. 5. Comparison of Equilibrium Positions for Noise and ISN Investors ($\hat{\rho} > 0$)

FIG 5.1: Comparison of Equilibrium Positions for Maximum Correlation Coefficient $\overline{\rho}_N$ for Naïve Investors

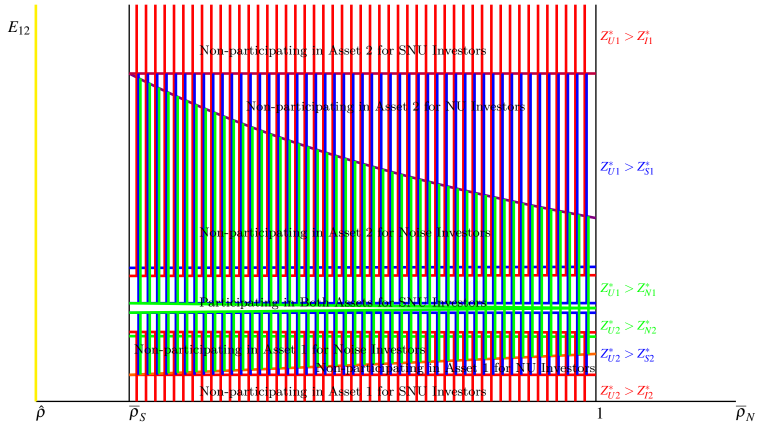


FIG 5.2: Comparison of Equilibrium Positions for Maximum Correlation Coefficient $\overline{\rho}_S$ for Sophisticated Investors

section explores the decision making of the investors with different information, the investors who hold less information escape from low-quality assets to high-quality assets. To compare the equilibrium positions, we examine each scenario in Theorem 1 in sequence.

[1] For **Type [1] Equilibrium** (*Non-participating One in Asset 1 for SNU Investors*), the quality ratio is infinitesimal, $E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$, then

$$Z^*_{U1} = Z^*_{N1} = Z^*_{S1} = 0 < Z^*_{I1}$$
$$Z^*_{U2} = Z^*_{N2} = Z^*_{S2} \lesseqqgtr Z^*_{I2} \quad \text{if and only if} \quad \hat{\rho} \lesseqqgtr 0.$$

SNU investors make non-participation decisions on asset 1, but insider investors long buy this asset; and all investors long buy asset 2: SNU investors will always take less (more) equilibrium positions than insider investors if and only if the true correlation coefficient is negative (positive). As the true correlation coefficient increases, SNU investors hold more positions on asset 2 and insider investors hold fewer positions on asset 2. Therefore, SNU investors escape from asset 1 to asset 2 and insider investors escape from asset 2 to asset 1. That is, investors with less information escape from low-quality assets to high-quality assets and investors with more information escape from high-quality assets to low-quality assets.

[2] For **Type [2] Equilibrium** (*Non-participating One in Asset 1 for NU Investors*), the quality ratio is tiny, $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) < E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, then, for asset 1,

$$Z^*_{U1} = Z^*_{N1} = 0 < Z^*_{S1} < Z^*_{I1} \quad \text{if} \quad E_{21} > \frac{\theta_I \overline{\rho}_S + \theta_S \hat{\rho} + (\theta_N + \theta_U)(\hat{\rho} + \overline{\rho}_S)}{\theta_I + \theta_S}$$
$$Z^*_{U1} = Z^*_{N1} = 0 < Z^*_{I1} < Z^*_{S1} \quad \text{if} \quad E_{21} < \frac{\theta_I \overline{\rho}_S + \theta_S \hat{\rho} + (\theta_N + \theta_U)(\hat{\rho} + \overline{\rho}_S)}{\theta_I + \theta_S}.$$

NU investors make non-participation decisions on asset 1, but IS investors long buy this asset. If $E_{21} > \frac{\theta_I \overline{\rho}_S + \theta_S \hat{\rho} + (\theta_N + \theta_U)(\hat{\rho} + \overline{\rho}_S)}{\theta_I + \theta_S}$, then $Z^*_{U1} = Z^*_{N1} = 0 < Z^*_{S1} < Z^*_{I1}$, insider investors will take a greater position on asset 1 than sophisticated investors; and if $E_{21} < \frac{\theta_I \overline{\rho}_S + \theta_S \hat{\rho} + (\theta_N + \theta_U)(\hat{\rho} + \overline{\rho}_S)}{\theta_I + \theta_S}$, then $Z^*_{U1} = Z^*_{N1} = 0 < Z^*_{I1} < Z^*_{S1}$, sophisticated investors will take a greater position on asset 1 than insider investors.

All investors long buy asset 2.

[2.1] If $\hat{\rho} < \overline{\rho}_S < 0$, then $\frac{\theta_I \overline{\rho}_S + \theta_S \hat{\rho}}{1 + (\theta_N + \theta_U)\hat{\rho}\overline{\rho}_S} < 0 < h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) < E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$ and hence $Z^*_{U2} = Z^*_{N2} < Z^*_{S2} < Z^*_{I2}$.

[2.2] If $\hat{\rho} < 0 < \overline{\rho}_S$, then $Z^*_{S2} < Z^*_{U2} = Z^*_{N2} < Z^*_{I2}$.

[2.3] If $0 < \hat{\rho} < \overline{\rho}_S$, then we have the conclusions as follows:

[2.3.1] if $E_{12} > \frac{\theta_I \overline{\rho}_S + \theta_S \hat{\rho}}{1 + (\theta_N + \theta_U)\hat{\rho}\overline{\rho}_S}$, then $Z^*_{S2} < Z^*_{I2} < Z^*_{U2} = Z^*_{N2}$;

[2.3.2] if $E_{12} < \frac{\theta_I \overline{\rho}_S + \theta_S \hat{\rho}}{1 + (\theta_N + \theta_U)\hat{\rho}\overline{\rho}_S}$, then $Z^*_{I2} < Z^*_{S2} < Z^*_{U2} = Z^*_{N2}$.

As the true correlation coefficient (and the maximum correlation coefficient for sophisticated investors) increases, NU investors hold more positions on asset 2 and insider investors hold fewer positions on asset 2. Therefore, NU investors escape from asset 1 to asset 2 and insider investors escape from asset 2 to asset 1. That is, investors with less information escape from low-quality assets to high-quality assets and investors with more information escape from high-quality assets to low-quality assets.

For the positive true correlation coefficient $0 < \hat{\rho}$, NU investors hold greater positions on asset 2 than IS investors, then NU investors escape from asset 1 to asset 2. Furthermore, if $E_{12} > \frac{\theta_I\overline{\rho}_S + \theta_S\hat{\rho}}{1+(\theta_N+\theta_U)\hat{\rho}\overline{\rho}_S}$, then $Z^*_{S2} < Z^*_{I2} < Z^*_{U2} = Z^*_{N2}$, insider investors will take a greater position on asset 2 than sophisticated investors; and if $E_{12} < \frac{\theta_I\overline{\rho}_S + \theta_S\hat{\rho}}{1+(\theta_N+\theta_U)\hat{\rho}\overline{\rho}_S}$, then $Z^*_{I2} < Z^*_{S2} < Z^*_{U2} = Z^*_{N2}$, sophisticated investors will take a greater position on asset 2 than insider investors.

[3] For **Type [3] Equilibrium** (*Non-participating One in Asset 1 for noise Investors*), the quality ratio is small, $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{12} \leqslant m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, then, for asset 1,

[3.0.1] if $$E_{21} > \frac{\theta_I\frac{(\overline{\rho}_S+\overline{\rho}_N)-\hat{\rho}(1+\overline{\rho}_S\overline{\rho}_N)}{1-\hat{\rho}^2} + \theta_S\overline{\rho}_N + \theta_N\overline{\rho}_S + \theta_U(\overline{\rho}_S+\overline{\rho}_N)}{\theta_I\frac{(1+\overline{\rho}_S\overline{\rho}_N)-\hat{\rho}(\overline{\rho}_S+\overline{\rho}_N)}{1-\hat{\rho}^2} + \theta_S + \theta_N},$$
then $Z^*_{U1} = 0 < Z^*_{N1} < Z^*_{S1} < Z^*_{I1}$;

[3.0.2] if $$\frac{\theta_I\frac{(\overline{\rho}_S+\overline{\rho}_N)-\hat{\rho}(1+\overline{\rho}_S\overline{\rho}_N)}{1-\hat{\rho}^2} + \theta_S\overline{\rho}_N + \theta_N\overline{\rho}_S + \theta_U(\overline{\rho}_S+\overline{\rho}_N)}{\theta_I\frac{(1+\overline{\rho}_S\overline{\rho}_N)-\hat{\rho}(\overline{\rho}_S+\overline{\rho}_N)}{1-\hat{\rho}^2} + \theta_S + \theta_N} > E_{21} >$$
$$\frac{\theta_I\overline{\rho}_N + \theta_S\frac{(\hat{\rho}+\overline{\rho}_N)-\overline{\rho}_S(1+\hat{\rho}\overline{\rho}_N)}{1-\overline{\rho}_S^2} + \theta_N\hat{\rho} + \theta_U(\hat{\rho}+\overline{\rho}_N)}{\theta_I + \theta_S\frac{(1+\hat{\rho}\overline{\rho}_N)-\overline{\rho}_S(\hat{\rho}+\overline{\rho}_N)}{1-\overline{\rho}_S^2} + \theta_N},$$ then $Z^*_{U1} = 0 < Z^*_{S1} < Z^*_{N1} < Z^*_{I1}$;

[3.0.3] if $$\frac{\theta_I\overline{\rho}_N + \theta_S\frac{(\hat{\rho}+\overline{\rho}_N)-\overline{\rho}_S(1+\hat{\rho}\overline{\rho}_N)}{1-\overline{\rho}_S^2} + \theta_N\hat{\rho} + \theta_U(\hat{\rho}+\overline{\rho}_N)}{\theta_I + \theta_S\frac{(1+\hat{\rho}\overline{\rho}_N)-\overline{\rho}_S(\hat{\rho}+\overline{\rho}_N)}{1-\overline{\rho}_S^2} + \theta_N} > E_{21}$$
$$> \frac{\theta_I\overline{\rho}_S + \theta_S\hat{\rho} + \theta_N\frac{(\hat{\rho}+\overline{\rho}_S)-\overline{\rho}_N(1+\hat{\rho}\overline{\rho}_S)}{1-\overline{\rho}_N^2} + \theta_U(\hat{\rho}+\overline{\rho}_S)}{\theta_I + \theta_S + \theta_N\frac{(1+\hat{\rho}\overline{\rho}_S)-\overline{\rho}_N(\hat{\rho}+\overline{\rho}_S)}{1-\overline{\rho}_N^2}},$$ then $Z^*_{U1} = 0 < Z^*_{S1} < Z^*_{I1} < Z^*_{N1}$;

[3.0.4] if $\dfrac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}+\theta_N\dfrac{(\hat{\rho}+\overline{\rho}_S)-\overline{\rho}_N(1+\hat{\rho}\overline{\rho}_S)}{1-\overline{\rho}_N^2}+\theta_U(\hat{\rho}+\overline{\rho}_S)}{\theta_I+\theta_S+\theta_N\dfrac{(1+\hat{\rho}\overline{\rho}_S)-\overline{\rho}_N(\hat{\rho}+\overline{\rho}_S)}{1-\overline{\rho}_N^2}} > E_{21}$, then $Z^*_{U1}=0<Z^*_{I1}<Z^*_{S1}<Z^*_{N1}$.

Noise investors make non-participation decisions on asset 1, but ISN investors long buy this asset. As the quality of asset 1 increases, naïve investors firstly take a larger position on it than sophisticated investors, and then even possess more than insider traders.

All investors long buy asset 2.

[3.1] If $\hat{\rho}<\overline{\rho}_S<\overline{\rho}_N<0$, then $\dfrac{\theta_I\dfrac{(\overline{\rho}_S+\overline{\rho}_N)-\hat{\rho}(1+\overline{\rho}_S\overline{\rho}_N)}{1-\hat{\rho}^2}+\theta_S\overline{\rho}_N+\theta_N\overline{\rho}_S}{\theta_I\dfrac{(\hat{\rho}-\overline{\rho}_S)(\hat{\rho}-\overline{\rho}_N)}{1-\hat{\rho}^2}+1+\theta_U\overline{\rho}_S\overline{\rho}_N} < h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)<E_{12}\leqslant m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$ and hence $Z^*_{U2}<Z^*_{N2}<Z^*_{S2}<Z^*_{I2}$.

[3.2] If $\hat{\rho}<\overline{\rho}_S<0<\overline{\rho}_N$, then $\dfrac{\theta_I\dfrac{(\overline{\rho}_S+\overline{\rho}_N)-\hat{\rho}(1+\overline{\rho}_S\overline{\rho}_N)}{1-\hat{\rho}^2}+\theta_S\overline{\rho}_N+\theta_N\overline{\rho}_S}{\theta_I\dfrac{(\hat{\rho}-\overline{\rho}_S)(\hat{\rho}-\overline{\rho}_N)}{1-\hat{\rho}^2}+1+\theta_U\overline{\rho}_S\overline{\rho}_N} < h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)<E_{12}\leqslant m(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$ and hence $Z^*_{N2}<Z^*_{U2}<Z^*_{S2}<Z^*_{I2}$.

[3.3] If $\hat{\rho}<0<\overline{\rho}_S<\overline{\rho}_N$, then $\dfrac{\theta_I\overline{\rho}_N+\theta_S\dfrac{(\hat{\rho}+\overline{\rho}_N)-\overline{\rho}_S(1+\hat{\rho}\overline{\rho}_N)}{1-\overline{\rho}_S^2}+\theta_N\hat{\rho}}{1+\theta_S\dfrac{(\overline{\rho}_S-\hat{\rho})(\overline{\rho}_S-\overline{\rho}_N)}{1-\overline{\rho}_S^2}+\theta_U\hat{\rho}\overline{\rho}_N} < h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < \dfrac{\theta_I\dfrac{(\overline{\rho}_S+\overline{\rho}_N)-\hat{\rho}(1+\overline{\rho}_S\overline{\rho}_N)}{1-\hat{\rho}^2}+\theta_S\overline{\rho}_N+\theta_N\overline{\rho}_S}{\theta_I\dfrac{(\hat{\rho}-\overline{\rho}_S)(\hat{\rho}-\overline{\rho}_N)}{1-\hat{\rho}^2}+1+\theta_U\overline{\rho}_S\overline{\rho}_N}$.

[3.3.1] If $E_{12} > \dfrac{\theta_I\dfrac{(\overline{\rho}_S+\overline{\rho}_N)-\hat{\rho}(1+\overline{\rho}_S\overline{\rho}_N)}{1-\hat{\rho}^2}+\theta_S\overline{\rho}_N+\theta_N\overline{\rho}_S}{\theta_I\dfrac{(\hat{\rho}-\overline{\rho}_S)(\hat{\rho}-\overline{\rho}_N)}{1-\hat{\rho}^2}+1+\theta_U\overline{\rho}_S\overline{\rho}_N}$, then $Z^*_{N2}<Z^*_{S2}<Z^*_{U2}<Z^*_{I2}$.

[3.3.2] If $E_{12} < \dfrac{\theta_I\dfrac{(\overline{\rho}_S+\overline{\rho}_N)-\hat{\rho}(1+\overline{\rho}_S\overline{\rho}_N)}{1-\hat{\rho}^2}+\theta_S\overline{\rho}_N+\theta_N\overline{\rho}_S}{\theta_I\dfrac{(\hat{\rho}-\overline{\rho}_S)(\hat{\rho}-\overline{\rho}_N)}{1-\hat{\rho}^2}+1+\theta_U\overline{\rho}_S\overline{\rho}_N}$, then $Z^*_{S2}<Z^*_{N2}<Z^*_{U2}<Z^*_{I2}$.

[3.4] If $0<\hat{\rho}<\overline{\rho}_S<\overline{\rho}_N$, then we have the conclusions as follows:

[3.4.1] If $0 < \hat{\rho} < \overline{\rho}_S < \overline{\rho}_N < \dfrac{\hat{\rho} + \overline{\rho}_S}{1 + \hat{\rho}\overline{\rho}_S}$, then

$$h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < \frac{\theta_I \overline{\rho}_S + \theta_S \hat{\rho} + \theta_N \dfrac{(\hat{\rho} + \overline{\rho}_S) - \overline{\rho}_N(1 + \hat{\rho}\overline{\rho}_S)}{1 - \overline{\rho}_N^2}}{1 + \theta_N \dfrac{(\overline{\rho}_N - \hat{\rho})(\overline{\rho}_N - \overline{\rho}_S)}{1 - \overline{\rho}_N^2} + \theta_U \hat{\rho}\overline{\rho}_S} < \frac{\theta_I \overline{\rho}_N + \theta_S \dfrac{(\hat{\rho} + \overline{\rho}_N) - \overline{\rho}_S(1 + \hat{\rho}\overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_N \hat{\rho}}{1 + \theta_S \dfrac{(\overline{\rho}_S - \hat{\rho})(\overline{\rho}_S - \overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_U \hat{\rho}\overline{\rho}_N}$$

$$< \frac{\theta_I \dfrac{(\overline{\rho}_S + \overline{\rho}_N) - \hat{\rho}(1 + \overline{\rho}_S\overline{\rho}_N)}{1 - \hat{\rho}^2} + \theta_S \overline{\rho}_N + \theta_N \overline{\rho}_S}{\theta_I \dfrac{(\hat{\rho} - \overline{\rho}_S)(\hat{\rho} - \overline{\rho}_N)}{1 - \hat{\rho}^2} + 1 + \theta_U \overline{\rho}_S \overline{\rho}_N}$$

and hence

[3.4.1.1] If $E_{12} > \dfrac{\theta_I \dfrac{(\overline{\rho}_S + \overline{\rho}_N) - \hat{\rho}(1 + \overline{\rho}_S\overline{\rho}_N)}{1 - \hat{\rho}^2} + \theta_S \overline{\rho}_N + \theta_N \overline{\rho}_S}{\theta_I \dfrac{(\hat{\rho} - \overline{\rho}_S)(\hat{\rho} - \overline{\rho}_N)}{1 - \hat{\rho}^2} + 1 + \theta_U \overline{\rho}_S \overline{\rho}_N}$, then $Z^*_{N2} < Z^*_{S2} < Z^*_{I2} < Z^*_{U2}$.

[3.4.1.2] If $\dfrac{\theta_I \dfrac{(\overline{\rho}_S + \overline{\rho}_N) - \hat{\rho}(1 + \overline{\rho}_S\overline{\rho}_N)}{1 - \hat{\rho}^2} + \theta_S \overline{\rho}_N + \theta_N \overline{\rho}_S}{\theta_I \dfrac{(\hat{\rho} - \overline{\rho}_S)(\hat{\rho} - \overline{\rho}_N)}{1 - \hat{\rho}^2} + 1 + \theta_U \overline{\rho}_S \overline{\rho}_N} > E_{12}$
$> \dfrac{\theta_I \overline{\rho}_N + \theta_S \dfrac{(\hat{\rho} + \overline{\rho}_N) - \overline{\rho}_S(1 + \hat{\rho}\overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_N \hat{\rho}}{1 + \theta_S \dfrac{(\overline{\rho}_S - \hat{\rho})(\overline{\rho}_S - \overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_U \hat{\rho}\overline{\rho}_N}$, then $Z^*_{S2} < Z^*_{N2} < Z^*_{I2} < Z^*_{U2}$.

[3.4.1.3] If $\dfrac{\theta_I \overline{\rho}_N + \theta_S \dfrac{(\hat{\rho} + \overline{\rho}_N) - \overline{\rho}_S(1 + \hat{\rho}\overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_N \hat{\rho}}{1 + \theta_S \dfrac{(\overline{\rho}_S - \hat{\rho})(\overline{\rho}_S - \overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_U \hat{\rho}\overline{\rho}_N} > E_{12}$
$> \dfrac{\theta_I \overline{\rho}_S + \theta_S \hat{\rho} + \theta_N \dfrac{(\hat{\rho} + \overline{\rho}_S) - \overline{\rho}_N(1 + \hat{\rho}\overline{\rho}_S)}{1 - \overline{\rho}_N^2}}{1 + \theta_N \dfrac{(\overline{\rho}_N - \hat{\rho})(\overline{\rho}_N - \overline{\rho}_S)}{1 - \overline{\rho}_N^2} + \theta_U \hat{\rho}\overline{\rho}_S}$, then $Z^*_{S2} < Z^*_{I2} < Z^*_{N2} < Z^*_{U2}$.

[3.4.1.4] If $\dfrac{\theta_I \overline{\rho}_S + \theta_S \hat{\rho} + \theta_N \dfrac{(\hat{\rho} + \overline{\rho}_S) - \overline{\rho}_N(1 + \hat{\rho}\overline{\rho}_S)}{1 - \overline{\rho}_N^2}}{1 + \theta_N \dfrac{(\overline{\rho}_N - \hat{\rho})(\overline{\rho}_N - \overline{\rho}_S)}{1 - \overline{\rho}_N^2} + \theta_U \hat{\rho}\overline{\rho}_S} > E_{12}$, then $Z^*_{I2} < Z^*_{S2} < Z^*_{N2} < Z^*_{U2}$.

[3.4.2] If $0 < \hat{\rho} < \overline{\rho}_S < \dfrac{\hat{\rho} + \overline{\rho}_S}{1 + \hat{\rho}\overline{\rho}_S} < \overline{\rho}_N$, then

$$\frac{\theta_I \overline{\rho}_S + \theta_S \hat{\rho} + \theta_N \dfrac{(\hat{\rho} + \overline{\rho}_S) - \overline{\rho}_N(1 + \hat{\rho}\overline{\rho}_S)}{1 - \overline{\rho}_N^2}}{1 + \theta_N \dfrac{(\overline{\rho}_N - \hat{\rho})(\overline{\rho}_N - \overline{\rho}_S)}{1 - \overline{\rho}_N^2} + \theta_U \hat{\rho}\overline{\rho}_S} < h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < \frac{\theta_I \overline{\rho}_N + \theta_S \dfrac{(\hat{\rho} + \overline{\rho}_N) - \overline{\rho}_S(1 + \hat{\rho}\overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_N \hat{\rho}}{1 + \theta_S \dfrac{(\overline{\rho}_S - \hat{\rho})(\overline{\rho}_S - \overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_U \hat{\rho}\overline{\rho}_N}$$
$$< \frac{\theta_I \dfrac{(\overline{\rho}_S + \overline{\rho}_N) - \hat{\rho}(1 + \overline{\rho}_S\overline{\rho}_N)}{1 - \hat{\rho}^2} + \theta_S \overline{\rho}_N + \theta_N \overline{\rho}_S}{\theta_I \dfrac{(\hat{\rho} - \overline{\rho}_S)(\hat{\rho} - \overline{\rho}_N)}{1 - \hat{\rho}^2} + 1 + \theta_U \overline{\rho}_S\overline{\rho}_N}$$

and hence

[3.4.2.1] If $E_{12} > \dfrac{\theta_I \dfrac{(\overline{\rho}_S + \overline{\rho}_N) - \hat{\rho}(1 + \overline{\rho}_S\overline{\rho}_N)}{1 - \hat{\rho}^2} + \theta_S \overline{\rho}_N + \theta_N \overline{\rho}_S}{\theta_I \dfrac{(\hat{\rho} - \overline{\rho}_S)(\hat{\rho} - \overline{\rho}_N)}{1 - \hat{\rho}^2} + 1 + \theta_U \overline{\rho}_S\overline{\rho}_N}$, then $Z_{N2}^* < Z_{S2}^* < Z_{I2}^* < Z_{U2}^*$.

[3.4.2.2] If $\dfrac{\theta_I \dfrac{(\overline{\rho}_S + \overline{\rho}_N) - \hat{\rho}(1 + \overline{\rho}_S\overline{\rho}_N)}{1 - \hat{\rho}^2} + \theta_S \overline{\rho}_N + \theta_N \overline{\rho}_S}{\theta_I \dfrac{(\hat{\rho} - \overline{\rho}_S)(\hat{\rho} - \overline{\rho}_N)}{1 - \hat{\rho}^2} + 1 + \theta_U \overline{\rho}_S\overline{\rho}_N} > E_{12}$ $> \dfrac{\theta_I \overline{\rho}_N + \theta_S \dfrac{(\hat{\rho} + \overline{\rho}_N) - \overline{\rho}_S(1 + \hat{\rho}\overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_N \hat{\rho}}{1 + \theta_S \dfrac{(\overline{\rho}_S - \hat{\rho})(\overline{\rho}_S - \overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_U \hat{\rho}\overline{\rho}_N}$, then $Z_{S2}^* < Z_{N2}^* < Z_{I2}^* < Z_{U2}^*$.

[3.4.2.3] If $\dfrac{\theta_I \overline{\rho}_N + \theta_S \dfrac{(\hat{\rho} + \overline{\rho}_N) - \overline{\rho}_S(1 + \hat{\rho}\overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_N \hat{\rho}}{1 + \theta_S \dfrac{(\overline{\rho}_S - \hat{\rho})(\overline{\rho}_S - \overline{\rho}_N)}{1 - \overline{\rho}_S^2} + \theta_U \hat{\rho}\overline{\rho}_N} > E_{12}$, then $Z_{S2}^* < Z_{I2}^* < Z_{N2}^* < Z_{U2}^*$.

As the true correlation coefficient (and the maximum correlation coefficient for sophisticated and naïve investors) increases, noise and naïve investors hold more positions on asset 2 and insider and sophisticated investors hold fewer positions on asset 2. Therefore, noise and naïve investors escape from asset 1 to asset 2 and insider and sophisticated investors escape from asset 2 to asset 1. That is, investors with less information escape from low-quality assets to high-quality assets and investors with more information escape from high-quality assets to low-quality assets.

For the positive true correlation coefficient $0 < \hat{\rho}$, noise investors hold a greater position on asset 2 than ISN investors. Furthermore, if $0 < \hat{\rho} < \overline{\rho}_S < \overline{\rho}_N < \dfrac{\hat{\rho} + \overline{\rho}_S}{1 + \hat{\rho}\overline{\rho}_S}$, then, as the quality of asset 1 increases, the investors who know less information will hold more positions on asset 2, while the investors who know more information will hold smaller position on asset 2; and if $0 < \hat{\rho} < \overline{\rho}_S < \dfrac{\hat{\rho} + \overline{\rho}_S}{1 + \hat{\rho}\overline{\rho}_S} < \overline{\rho}_N$, then, as the quality of asset 1 increases, the investors who know less information will hold more position on asset 2, while the investors who know more information will hold smaller position on asset 2, but insider investors hold more position on asset 2 than sophisticated investors.

[4] For **Type [4] Equilibrium** (*Participating One in both Assets for SNU Investors*), the quality ratio is medium, $m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{12} < M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, all investors long buy both assets.

[4.1.1] If $E_{21} > \dfrac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U \overline{\rho}_N}{1+\overline{\rho}_N}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\overline{\rho}_N}}$, then $Z^*_{U1} < Z^*_{N1} < Z^*_{S1} < Z^*_{I1}$;

[4.1.2] if $\dfrac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U \overline{\rho}_N}{1+\overline{\rho}_N}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\overline{\rho}_N}} > E_{21} > \dfrac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U \overline{\rho}_S}{1+\overline{\rho}_S}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\overline{\rho}_S}}$, then $Z^*_{N1} < Z^*_{U1} < Z^*_{S1} < Z^*_{I1}$;

[4.1.3] if $\dfrac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U \overline{\rho}_S}{1+\overline{\rho}_S}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\overline{\rho}_S}} > E_{21} > \dfrac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U \hat{\rho}}{1+\hat{\rho}}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\hat{\rho}}}$, then $Z^*_{N1} < Z^*_{S1} < Z^*_{U1} < Z^*_{I1}$;

[4.1.4] if $\dfrac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U \hat{\rho}}{1+\hat{\rho}}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\hat{\rho}}} > E_{21}$, then $Z^*_{N1} < Z^*_{S1} < Z^*_{I1} < Z^*_{U1}$.

[4.2.1] If $E_{12} > \dfrac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U \overline{\rho}_N}{1+\overline{\rho}_N}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\overline{\rho}_N}}$, then $Z^*_{U2} < Z^*_{N2} < Z^*_{S2} < Z^*_{I2}$;

[4.2.2] if $\dfrac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U \overline{\rho}_N}{1+\overline{\rho}_N}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\overline{\rho}_N}} > E_{12} > \dfrac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U \overline{\rho}_S}{1+\overline{\rho}_S}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\overline{\rho}_S}}$, then $Z^*_{N2} < Z^*_{U2} < Z^*_{S2} < Z^*_{I2}$;

[4.2.3] if $\dfrac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U \overline{\rho}_S}{1+\overline{\rho}_S}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\overline{\rho}_S}} > E_{12} > \dfrac{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U \hat{\rho}}{1+\hat{\rho}}}{\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N} + \frac{\theta_U}{1+\hat{\rho}}}$, then $Z^*_{N2} < Z^*_{S2} < Z^*_{U2} < Z^*_{I2}$;

[4.2.4] if $\dfrac{\dfrac{\theta_I}{1+\hat\rho}+\dfrac{\theta_S}{1+\overline\rho_S}+\dfrac{\theta_N}{1+\overline\rho_N}+\dfrac{\theta_U\hat\rho}{1+\hat\rho}}{\dfrac{\theta_I}{1+\hat\rho}+\dfrac{\theta_S}{1+\overline\rho_S}+\dfrac{\theta_N}{1+\overline\rho_N}+\dfrac{\theta_U}{1+\hat\rho}} > E_{12}$, then $Z^*_{N2} < Z^*_{S2} < Z^*_{I2} < Z^*_{U2}$.

Insider investors hold greater positions on both risky assets than sophisticated investors, and sophisticated investors hold greater positions on both risky assets than naïve investors. As the quality of risky asset increases, the noise investors will hold more and more equilibrium positions.

[5] For **Type [5] Equilibrium** (*Non-participating One in Asset 2 for noise Investors*), the quality ratio is big, $M(\hat\rho,\overline\rho_S,\overline\rho_N) \leqslant E_{12} < H(\hat\rho,\overline\rho_S,\overline\rho_N)$, then all investors long buy asset 1.

[5.1] If $\hat\rho < \overline\rho_S < \overline\rho_N < 0$, then $\dfrac{\theta_I\dfrac{(\overline\rho_S+\overline\rho_N)-\hat\rho(1+\overline\rho_S\overline\rho_N)}{1-\hat\rho^2}+\theta_S\overline\rho_N+\theta_N\overline\rho_S}{\theta_I\dfrac{(\hat\rho-\overline\rho_S)(\hat\rho-\overline\rho_N)}{1-\hat\rho^2}+1+\theta_U\overline\rho_S\overline\rho_N} < h(\hat\rho,\overline\rho_S,\overline\rho_N) < E_{21} \leqslant m(\hat\rho,\overline\rho_S,\overline\rho_N)$ and hence $Z^*_{U1} < Z^*_{N1} < Z^*_{S1} < Z^*_{I1}$.

[5.2] If $\hat\rho < \overline\rho_S < 0 < \overline\rho_N$, then $\dfrac{\theta_I\dfrac{(\overline\rho_S+\overline\rho_N)-\hat\rho(1+\overline\rho_S\overline\rho_N)}{1-\hat\rho^2}+\theta_S\overline\rho_N+\theta_N\overline\rho_S}{\theta_I\dfrac{(\hat\rho-\overline\rho_S)(\hat\rho-\overline\rho_N)}{1-\hat\rho^2}+1+\theta_U\overline\rho_S\overline\rho_N} < h(\hat\rho,\overline\rho_S,\overline\rho_N) < E_{21} \leqslant m(\hat\rho,\overline\rho_S,\overline\rho_N)$ and hence $Z^*_{N1} < Z^*_{U1} < Z^*_{S1} < Z^*_{I1}$.

[5.3] If $\hat\rho < 0 < \overline\rho_S < \overline\rho_N$, then $\dfrac{\theta_I\overline\rho_N+\theta_S\dfrac{(\hat\rho+\overline\rho_N)-\overline\rho_S(1+\hat\rho\overline\rho_N)}{1-\overline\rho_S^2}+\theta_N\hat\rho}{1+\theta_S\dfrac{(\overline\rho_S-\hat\rho)(\overline\rho_S-\overline\rho_N)}{1-\overline\rho_S^2}+\theta_U\hat\rho\overline\rho_N} < h(\hat\rho,\overline\rho_S,\overline\rho_N) < \dfrac{\theta_I\dfrac{(\overline\rho_S+\overline\rho_N)-\hat\rho(1+\overline\rho_S\overline\rho_N)}{1-\hat\rho^2}+\theta_S\overline\rho_N+\theta_N\overline\rho_S}{\theta_I\dfrac{(\hat\rho-\overline\rho_S)(\hat\rho-\overline\rho_N)}{1-\hat\rho^2}+1+\theta_U\overline\rho_S\overline\rho_N}$.

[5.3.1] If $E_{21} > \dfrac{\theta_I\dfrac{(\overline\rho_S+\overline\rho_N)-\hat\rho(1+\overline\rho_S\overline\rho_N)}{1-\hat\rho^2}+\theta_S\overline\rho_N+\theta_N\overline\rho_S}{\theta_I\dfrac{(\hat\rho-\overline\rho_S)(\hat\rho-\overline\rho_N)}{1-\hat\rho^2}+1+\theta_U\overline\rho_S\overline\rho_N}$, then $Z^*_{N1} < Z^*_{S1} < Z^*_{U1} < Z^*_{I1}$.

[5.3.2] If $E_{21} < \dfrac{\theta_I\dfrac{(\overline\rho_S+\overline\rho_N)-\hat\rho(1+\overline\rho_S\overline\rho_N)}{1-\hat\rho^2}+\theta_S\overline\rho_N+\theta_N\overline\rho_S}{\theta_I\dfrac{(\hat\rho-\overline\rho_S)(\hat\rho-\overline\rho_N)}{1-\hat\rho^2}+1+\theta_U\overline\rho_S\overline\rho_N}$, then $Z^*_{S1} < Z^*_{N1} < Z^*_{U1} < Z^*_{I1}$.

[5.4] If $0 < \hat\rho < \overline\rho_S < \overline\rho_N$, then we have the conclusions as follows:

[5.4.1] If $0 < \hat\rho < \overline\rho_S < \overline\rho_N < \dfrac{\hat\rho+\overline\rho_S}{1+\hat\rho\overline\rho_S}$, then

$$h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < \frac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}+\theta_N\dfrac{(\hat{\rho}+\overline{\rho}_S)-\overline{\rho}_N(1+\hat{\rho}\overline{\rho}_S)}{1-\overline{\rho}_N^2}}{1+\theta_N\dfrac{(\overline{\rho}_N-\hat{\rho})(\overline{\rho}_N-\overline{\rho}_S)}{1-\overline{\rho}_N^2}+\theta_U\hat{\rho}\overline{\rho}_S} < \frac{\theta_I\overline{\rho}_N+\theta_S\dfrac{(\hat{\rho}+\overline{\rho}_N)-\overline{\rho}_S(1+\hat{\rho}\overline{\rho}_N)}{1-\overline{\rho}_S^2}+\theta_N\hat{\rho}}{1+\theta_S\dfrac{(\overline{\rho}_S-\hat{\rho})(\overline{\rho}_S-\overline{\rho}_N)}{1-\overline{\rho}_S^2}+\theta_U\hat{\rho}\overline{\rho}_N}$$

$$< \frac{\theta_I\dfrac{(\overline{\rho}_S+\overline{\rho}_N)-\hat{\rho}(1+\overline{\rho}_S\overline{\rho}_N)}{1-\hat{\rho}^2}+\theta_S\overline{\rho}_N+\theta_N\overline{\rho}_S}{\theta_I\dfrac{(\hat{\rho}-\overline{\rho}_S)(\hat{\rho}-\overline{\rho}_N)}{1-\hat{\rho}^2}+1+\theta_U\overline{\rho}_S\overline{\rho}_N}$$

and hence

[5.4.1.1] If $E_{21} > \dfrac{\theta_I\dfrac{(\overline{\rho}_S+\overline{\rho}_N)-\hat{\rho}(1+\overline{\rho}_S\overline{\rho}_N)}{1-\hat{\rho}^2}+\theta_S\overline{\rho}_N+\theta_N\overline{\rho}_S}{\theta_I\dfrac{(\hat{\rho}-\overline{\rho}_S)(\hat{\rho}-\overline{\rho}_N)}{1-\hat{\rho}^2}+1+\theta_U\overline{\rho}_S\overline{\rho}_N}$, then $Z^*_{N1} < Z^*_{S1} < Z^*_{I1} < Z^*_{U1}$.

[5.4.1.2] If $\dfrac{\theta_I\dfrac{(\overline{\rho}_S+\overline{\rho}_N)-\hat{\rho}(1+\overline{\rho}_S\overline{\rho}_N)}{1-\hat{\rho}^2}+\theta_S\overline{\rho}_N+\theta_N\overline{\rho}_S}{\theta_I\dfrac{(\hat{\rho}-\overline{\rho}_S)(\hat{\rho}-\overline{\rho}_N)}{1-\hat{\rho}^2}+1+\theta_U\overline{\rho}_S\overline{\rho}_N} > E_{21}$ $> \dfrac{\theta_I\overline{\rho}_N+\theta_S\dfrac{(\hat{\rho}+\overline{\rho}_N)-\overline{\rho}_S(1+\hat{\rho}\overline{\rho}_N)}{1-\overline{\rho}_S^2}+\theta_N\hat{\rho}}{1+\theta_S\dfrac{(\overline{\rho}_S-\hat{\rho})(\overline{\rho}_S-\overline{\rho}_N)}{1-\overline{\rho}_S^2}+\theta_U\hat{\rho}\overline{\rho}_N}$, then $Z^*_{S1} < Z^*_{N1} < Z^*_{I1} < Z^*_{U1}$.

[5.4.1.3] If $\dfrac{\theta_I\overline{\rho}_N+\theta_S\dfrac{(\hat{\rho}+\overline{\rho}_N)-\overline{\rho}_S(1+\hat{\rho}\overline{\rho}_N)}{1-\overline{\rho}_S^2}+\theta_N\hat{\rho}}{1+\theta_S\dfrac{(\overline{\rho}_S-\hat{\rho})(\overline{\rho}_S-\overline{\rho}_N)}{1-\overline{\rho}_S^2}+\theta_U\hat{\rho}\overline{\rho}_N} > E_{21}$ $> \dfrac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}+\theta_N\dfrac{(\hat{\rho}+\overline{\rho}_S)-\overline{\rho}_N(1+\hat{\rho}\overline{\rho}_S)}{1-\overline{\rho}_N^2}}{1+\theta_N\dfrac{(\overline{\rho}_N-\hat{\rho})(\overline{\rho}_N-\overline{\rho}_S)}{1-\overline{\rho}_N^2}+\theta_U\hat{\rho}\overline{\rho}_S}$, then $Z^*_{S1} < Z^*_{I1} < Z^*_{N1} < Z^*_{U1}$.

[5.4.1.4] If $\dfrac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}+\theta_N\dfrac{(\hat{\rho}+\overline{\rho}_S)-\overline{\rho}_N(1+\hat{\rho}\overline{\rho}_S)}{1-\overline{\rho}_N^2}}{1+\theta_N\dfrac{(\overline{\rho}_N-\hat{\rho})(\overline{\rho}_N-\overline{\rho}_S)}{1-\overline{\rho}_N^2}+\theta_U\hat{\rho}\overline{\rho}_S} > E_{21}$, then $Z^*_{I1} < Z^*_{S1} < Z^*_{N1} < Z^*_{U1}$.

[5.4.2] If $0 < \hat{\rho} < \overline{\rho}_S < \dfrac{\hat{\rho}+\overline{\rho}_S}{1+\hat{\rho}\overline{\rho}_S} < \overline{\rho}_N$, then

$$\frac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}+\theta_N\dfrac{(\hat{\rho}+\overline{\rho}_S)-\overline{\rho}_N(1+\hat{\rho}\overline{\rho}_S)}{1-\overline{\rho}_N^2}}{1+\theta_N\dfrac{(\overline{\rho}_N-\hat{\rho})(\overline{\rho}_N-\overline{\rho}_S)}{1-\overline{\rho}_N^2}+\theta_U\hat{\rho}\overline{\rho}_S} < h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N) < \frac{\theta_I\overline{\rho}_N+\theta_S\dfrac{(\hat{\rho}+\overline{\rho}_N)-\overline{\rho}_S(1+\hat{\rho}\overline{\rho}_N)}{1-\overline{\rho}_S^2}+\theta_N\hat{\rho}}{1+\theta_S\dfrac{(\overline{\rho}_S-\hat{\rho})(\overline{\rho}_S-\overline{\rho}_N)}{1-\overline{\rho}_S^2}+\theta_U\hat{\rho}\overline{\rho}_N}$$

$$< \frac{\theta_I\dfrac{(\overline{\rho}_S+\overline{\rho}_N)-\hat{\rho}(1+\overline{\rho}_S\overline{\rho}_N)}{1-\hat{\rho}^2}+\theta_S\overline{\rho}_N+\theta_N\overline{\rho}_S}{\theta_I\dfrac{(\hat{\rho}-\overline{\rho}_S)(\hat{\rho}-\overline{\rho}_N)}{1-\hat{\rho}^2}+1+\theta_U\overline{\rho}_S\overline{\rho}_N}$$

and hence

[5.4.2.1] If $E_{21} > \dfrac{\theta_I \dfrac{(\overline{\rho}_S+\overline{\rho}_N)-\hat{\rho}(1+\overline{\rho}_S\overline{\rho}_N)}{1-\hat{\rho}^2}+\theta_S\overline{\rho}_N+\theta_N\overline{\rho}_S}{\theta_I \dfrac{(\hat{\rho}-\overline{\rho}_S)(\hat{\rho}-\overline{\rho}_N)}{1-\hat{\rho}^2}+1+\theta_U\overline{\rho}_S\overline{\rho}_N}$, then $Z^*_{N1} < Z^*_{S1} < Z^*_{I1} < Z^*_{U1}$.

[5.4.2.2] If $\dfrac{\theta_I \dfrac{(\overline{\rho}_S+\overline{\rho}_N)-\hat{\rho}(1+\overline{\rho}_S\overline{\rho}_N)}{1-\hat{\rho}^2}+\theta_S\overline{\rho}_N+\theta_N\overline{\rho}_S}{\theta_I \dfrac{(\hat{\rho}-\overline{\rho}_S)(\hat{\rho}-\overline{\rho}_N)}{1-\hat{\rho}^2}+1+\theta_U\overline{\rho}_S\overline{\rho}_N} > E_{21}$ $> \dfrac{\theta_I\overline{\rho}_N+\theta_S\dfrac{(\hat{\rho}+\overline{\rho}_N)-\overline{\rho}_S(1+\hat{\rho}\overline{\rho}_N)}{1-\overline{\rho}_S^2}+\theta_N\hat{\rho}}{1+\theta_S\dfrac{(\overline{\rho}_S-\hat{\rho})(\overline{\rho}_S-\overline{\rho}_N)}{1-\overline{\rho}_S^2}+\theta_U\hat{\rho}\overline{\rho}_N}$, then $Z^*_{S1} < Z^*_{N1} < Z^*_{I1} < Z^*_{U1}$.

[5.4.2.3] If $\dfrac{\theta_I\overline{\rho}_N+\theta_S\dfrac{(\hat{\rho}+\overline{\rho}_N)-\overline{\rho}_S(1+\hat{\rho}\overline{\rho}_N)}{1-\overline{\rho}_S^2}+\theta_N\hat{\rho}}{1+\theta_S\dfrac{(\overline{\rho}_S-\hat{\rho})(\overline{\rho}_S-\overline{\rho}_N)}{1-\overline{\rho}_S^2}+\theta_U\hat{\rho}\overline{\rho}_N} > E_{21}$, then $Z^*_{S1} < Z^*_{I1} < Z^*_{N1} < Z^*_{U1}$.

As the true correlation coefficient (and the maximum correlation coefficient for sophisticated and naïve investors) increases, noise and naïve investors hold more positions on asset 1 and insider and sophisticated investors hold fewer positions on asset 1. Therefore, noise and naïve investors escape from asset 2 to asset 1 and insider and sophisticated investors escape from asset 1 to asset 2. That is, investors with less information escape from low-quality assets to high-quality assets and investors with more information escape from high-quality assets to low-quality assets.

For the positive true correlation coefficient $0 < \hat{\rho}$, noise investors hold a greater position on asset 1 than ISN investors. Furthermore, if $0 < \hat{\rho} < \overline{\rho}_S < \overline{\rho}_N < \dfrac{\hat{\rho}+\overline{\rho}_S}{1+\hat{\rho}\overline{\rho}_S}$, then, as the quality of asset 1 increases, the investors who know less information will hold more positions on asset 1, while the investors who know more information will hold smaller position on asset 1; and if $0 < \hat{\rho} < \overline{\rho}_S < \dfrac{\hat{\rho}+\overline{\rho}_S}{1+\hat{\rho}\overline{\rho}_S} < \overline{\rho}_N$, then, as the quality of asset 1 increases, the investors who know less information will hold more position on asset 1, while the investors who know more information will hold less position on asset 1, but insider investors cannot be changed into the investors who holds the minimum position on asset 1.

For asset 2,

[5.0.1] if $E_{12} > \dfrac{\theta_I \dfrac{(\overline{\rho}_S+\overline{\rho}_N)-\hat{\rho}(1+\overline{\rho}_S\overline{\rho}_N)}{1-\hat{\rho}^2}+\theta_S\overline{\rho}_N+\theta_N\overline{\rho}_S+\theta_U(\overline{\rho}_S+\overline{\rho}_N)}{\theta_I\dfrac{(1+\overline{\rho}_S\overline{\rho}_N)-\hat{\rho}(\overline{\rho}_S+\overline{\rho}_N)}{1-\hat{\rho}^2}+\theta_S+\theta_N}$,

then $Z^*_{U2} = 0 < Z^*_{N2} < Z^*_{S2} < Z^*_{I2}$;

[5.0.2] if $\dfrac{\theta_I \dfrac{(\overline{\rho}_S+\overline{\rho}_N)-\hat{\rho}(1+\overline{\rho}_S\overline{\rho}_N)}{1-\hat{\rho}^2}+\theta_S\overline{\rho}_N+\theta_N\overline{\rho}_S+\theta_U(\overline{\rho}_S+\overline{\rho}_N)}{\theta_I\dfrac{(1+\overline{\rho}_S\overline{\rho}_N)-\hat{\rho}(\overline{\rho}_S+\overline{\rho}_N)}{1-\hat{\rho}^2}+\theta_S+\theta_N} > E_{12} >$ $\dfrac{\theta_I\overline{\rho}_N+\theta_S\dfrac{(\hat{\rho}+\overline{\rho}_N)-\overline{\rho}_S(1+\hat{\rho}\overline{\rho}_N)}{1-\overline{\rho}_S^2}+\theta_N\hat{\rho}+\theta_U(\hat{\rho}+\overline{\rho}_N)}{\theta_I+\theta_S\dfrac{(1+\hat{\rho}\overline{\rho}_N)-\overline{\rho}_S(\hat{\rho}+\overline{\rho}_N)}{1-\overline{\rho}_S^2}+\theta_N}$, then $Z^*_{U2}=0<Z^*_{S2}<Z^*_{N2}<Z^*_{I2}$;

[5.0.3] if $\dfrac{\theta_I\overline{\rho}_N+\theta_S\dfrac{(\hat{\rho}+\overline{\rho}_N)-\overline{\rho}_S(1+\hat{\rho}\overline{\rho}_N)}{1-\overline{\rho}_S^2}+\theta_N\hat{\rho}+\theta_U(\hat{\rho}+\overline{\rho}_N)}{\theta_I+\theta_S\dfrac{(1+\hat{\rho}\overline{\rho}_N)-\overline{\rho}_S(\hat{\rho}+\overline{\rho}_N)}{1-\overline{\rho}_S^2}+\theta_N} > E_{12}$ $> \dfrac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}+\theta_N\dfrac{(\hat{\rho}+\overline{\rho}_S)-\overline{\rho}_N(1+\hat{\rho}\overline{\rho}_S)}{1-\overline{\rho}_N^2}+\theta_U(\hat{\rho}+\overline{\rho}_S)}{\theta_I+\theta_S+\theta_N\dfrac{(1+\hat{\rho}\overline{\rho}_S)-\overline{\rho}_N(\hat{\rho}+\overline{\rho}_S)}{1-\overline{\rho}_N^2}}$, then $Z^*_{U2}=0<Z^*_{S2}<Z^*_{I2}<Z^*_{N2}$;

[5.0.4] if $\dfrac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}+\theta_N\dfrac{(\hat{\rho}+\overline{\rho}_S)-\overline{\rho}_N(1+\hat{\rho}\overline{\rho}_S)}{1-\overline{\rho}_N^2}+\theta_U(\hat{\rho}+\overline{\rho}_S)}{\theta_I+\theta_S+\theta_N\dfrac{(1+\hat{\rho}\overline{\rho}_S)-\overline{\rho}_N(\hat{\rho}+\overline{\rho}_S)}{1-\overline{\rho}_N^2}} > E_{12}$, then $Z^*_{U2}=0<Z^*_{I2}<Z^*_{S2}<Z^*_{N2}$.

Noise investors make non-participation decisions on asset 2, but ISN investors long buy this asset. As the quality of asset 2 increases, naïve investors firstly take a larger position on it than sophisticated investors, and then even possess more than insider traders.

[6] For **Type [6] Equilibrium** (*Non-participating One in Asset 2 for NU Investors*), the quality ratio is huge, $H(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)\leqslant E_{12}<H(\hat{\rho},\overline{\rho}_S,\overline{\rho}_S)$, then all investors long buy asset 1.

[6.1] If $\hat{\rho}<\overline{\rho}_S<0$, then $\dfrac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}}{1+(\theta_N+\theta_U)\hat{\rho}\overline{\rho}_S}<0<h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_S)<E_{21}\leqslant h(\hat{\rho},\overline{\rho}_S,\overline{\rho}_N)$ and hence $Z^*_{U1}=Z^*_{N1}<Z^*_{S1}<Z^*_{I1}$.

[6.2] If $\hat{\rho}<0<\overline{\rho}_S$, then $Z^*_{S1}<Z^*_{U1}=Z^*_{N1}<Z^*_{I1}$.

[6.3] If $0<\hat{\rho}<\overline{\rho}_S$, then we have the conclusions as follows:

[6.3.1] if $E_{21}>\dfrac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}}{1+(\theta_N+\theta_U)\hat{\rho}\overline{\rho}_S}$, then $Z^*_{S1}<Z^*_{I1}<Z^*_{U1}=Z^*_{N1}$.

[6.3.2] if $E_{21}<\dfrac{\theta_I\overline{\rho}_S+\theta_S\hat{\rho}}{1+(\theta_N+\theta_U)\hat{\rho}\overline{\rho}_S}$, then $Z^*_{I1}<Z^*_{S1}<Z^*_{U1}=Z^*_{N1}$.

As the true correlation coefficient (and the maximum correlation coefficient for sophisticated investors) increases, NU investors hold more positions on asset 1 and insider investors hold fewer positions on asset 1. Therefore, NU investors escape from asset 2 to asset 1 and insider investors escape from asset 1 to asset 2. That is, investors with less information escape from low-quality assets to high-quality assets and investors with more information escape from high-quality assets to low-quality assets.

For the positive true correlation coefficient $0 < \hat{\rho}$, NU investors hold greater positions on asset 2 than IS investors, then NU investors escape from asset 2 to asset 1. Furthermore, if $E_{12} > \frac{\theta_I \overline{\rho}_S + \theta_S \hat{\rho}}{1 + (\theta_N + \theta_U)\hat{\rho}\overline{\rho}_S}$, then $Z^*_{S2} < Z^*_{I2} < Z^*_{U2} = Z^*_{N2}$, insider investors will take a greater position on asset 1 than sophisticated investors; and if $E_{12} < \frac{\theta_I \overline{\rho}_S + \theta_S \hat{\rho}}{1 + (\theta_N + \theta_U)\hat{\rho}\overline{\rho}_S}$, then $Z^*_{I2} < Z^*_{S2} < Z^*_{U2} = Z^*_{N2}$, sophisticated investors will take a greater position on asset 1 than insider investors.

For asset 2,

$$
\begin{aligned}
Z^*_{U2} = Z^*_{N2} = 0 < Z^*_{S2} < Z^*_{I2} \quad \text{if} \quad E_{12} > \frac{\theta_I \overline{\rho}_S + \theta_S \hat{\rho} + (\theta_N + \theta_U)(\hat{\rho} + \overline{\rho}_S)}{\theta_I + \theta_S} \\
Z^*_{U2} = Z^*_{N2} = 0 < Z^*_{I2} < Z^*_{S2} \quad \text{if} \quad E_{12} < \frac{\theta_I \overline{\rho}_S + \theta_S \hat{\rho} + (\theta_N + \theta_U)(\hat{\rho} + \overline{\rho}_S)}{\theta_I + \theta_S}.
\end{aligned}
$$

NU investors make non-participation decisions on asset 2, but IS investors long buy this asset. If $E_{12} > \frac{\theta_I \overline{\rho}_S + \theta_S \hat{\rho} + (\theta_N + \theta_U)(\hat{\rho} + \overline{\rho}_S)}{\theta_I + \theta_S}$, then $Z^*_{U2} = Z^*_{N2} = 0 < Z^*_{S2} < Z^*_{I2}$, insider investors will take a greater position on asset 2 than sophisticated investors; and if $E_{12} < \frac{\theta_I \overline{\rho}_S + \theta_S \hat{\rho} + (\theta_N + \theta_U)(\hat{\rho} + \overline{\rho}_S)}{\theta_I + \theta_S}$, then $Z^*_{U2} = Z^*_{N2} = 0 < Z^*_{I2} < Z^*_{S2}$, sophisticated investors will take a greater position on asset 2 than insider investors.

[7] For **Type [7] Equilibrium** (*Non-participating One in Asset 2 for SNU Investors*), the quality ratio is infinitude, $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) \leqslant E_{12}$, and hence

$$
\begin{gathered}
Z^*_{U1} = Z^*_{N1} = Z^*_{S1} \lesseqgtr Z^*_{I1} \quad \text{if and only if} \quad \hat{\rho} \lesseqgtr 0 \\
Z^*_{U2} = Z^*_{N2} = Z^*_{S2} = 0 < Z^*_{I2}.
\end{gathered}
$$

All investors long buy asset 1: SNU investors will always take fewer (more) equilibrium positions than insider investors if and only if the true correlation coefficient is negative (positive); and SNU investors make non-participation decisions on asset 1, but insider investors long buy this asset. As the true correlation coefficient increases, SNU investors hold more positions on asset 1 and insider investors hold fewer positions on asset 1. Therefore, SNU investors escape from asset 2 to asset 1 and insider investors escape from asset 1 to asset 2. That is, investors with less information escape from low-quality assets to high-quality assets and investors with more information escape from high-quality assets to low-quality assets.

Summarizing the above seven settings, we obtain the following theorem on the trading pattern.

Theorem 2. In equilibrium, all investors exhibit a 'flight-to-quality' trading pattern. Less-informed investors tend to escape faster to high-quality assets.

6. IMPLICATIONS FOR ASSET PRICES — CAPM ANALYSIS

In this section, we examine the equilibrium and its implications for asset prices more in depth. We are particularly interested in the CAPM analysis. To shed light on the pricing effects of correlation ambiguity and non-participation, we now turn to the returns of risky assets and explore whether they have excess returns (alpha) that deviate from the CAPM. For asset j, the return is defined as

$$\tilde{Y}_j = \frac{\tilde{X}_j}{p_j} - 1, \qquad j = 1, 2,$$

and the return on the market (holding the entire supply of both risky assets) is

$$\tilde{Y}_M = \frac{\tilde{X}_1 Z_1^0 + \tilde{X}_2 Z_2^0}{p_1 Z_1^0 + p_2 Z_2^0} - 1.$$

Now imagine that there is a representative agent (A) in this economy with the common CARA utility function. In order for the equilibrium prices to be the same, the representative agent must hold the same beliefs on means as $\mu_j^A = \mu_j$ for $j = 1, 2$, and some special beliefs on standard deviations σ_j^A for $j = 1, 2$, and correlation coefficient ρ^A dependently upon different types of equilibria.

The representative agent has correct beliefs about mean payoffs, so she will also have correct beliefs on mean returns: $\tilde{Y}_j$ for asset $j = 1, 2$ and $\tilde{Y}_M$ for the market portfolio. Further, as assets are priced correctly from the point of view of the representative agent, the CAPM must hold from her perspective. Since we normalize the payoff of the risk-free asset to 0, the excess return of $\tilde{Y}_j$ and $\tilde{Y}_M$ is $\overline{Y}_j$ and $\overline{Y}_M$, respectively, for $j = 1, 2$.

$$\overline{Y}_j = \beta_j^A \overline{Y}_M, \qquad j = 1, 2,$$

in which β_j^A is the beta for asset j according to the representative agent and is given by

$$\beta_1^A = \frac{Cov^A(\tilde{Y}_M, \tilde{Y}_1)}{Var^A(\tilde{Y}_M)} = \frac{p_1 Z_1^0 + p_2 Z_2^0}{p_1} \frac{\left[\sigma_1^A\right]^2 Z_1^0 + \rho^A \sigma_1^A \sigma_2^A Z_2^0}{\left[\sigma_1^A\right]^2 \left[Z_1^0\right]^2 + 2\rho^A \sigma_1^A \sigma_2^A Z_1^0 Z_2^0 + \left[\sigma_2^A\right]^2 \left[Z_2^0\right]^2},$$

$$\beta_2^A = \frac{Cov^A(\tilde{Y}_M, \tilde{Y}_2)}{Var^A(\tilde{Y}_M)} = \frac{p_1 Z_1^0 + p_2 Z_2^0}{p_2} \frac{\rho^A \sigma_1^A \sigma_2^A Z_1^0 + \left[\sigma_2^A\right]^2 Z_2^0}{\left[\sigma_1^A\right]^2 \left[Z_1^0\right]^2 + 2\rho^A \sigma_1^A \sigma_2^A Z_1^0 Z_2^0 + \left[\sigma_2^A\right]^2 \left[Z_2^0\right]^2}.$$

In this expression, the covariance of the return on the market and the return on asset $j = 1, 2$, and the variance of the return on the market are calculated using the artificial beliefs of the representative agent, rather than correct beliefs.

The representative agent's beliefs about the standard deviations of returns to both assets and the correlation coefficient may not be correct, and so her beliefs about the variance of the market and the covariance of the market and returns to each asset may be incorrect. Thus, the betas calculated from the representative agent's point of view are not the betas that would be computed from actual payoff data. Now consider an outside econometrician who has the rational belief on the whole economy, that is, she knows the true value of the correlation coefficient $\hat{\rho}$. Thus from her point of view,

$$\beta_1 = \frac{Cov(\tilde{Y}_M, \tilde{Y}_1)}{Var(\tilde{Y}_M)} = \frac{p_1 Z_1^0 + p_2 Z_2^0}{p_1} \frac{\sigma_1 \left[\sigma_1 Z_1^0 + \hat{\rho} \sigma_2 Z_2^0\right]}{\sigma_1^2 \left[Z_1^0\right]^2 + 2\hat{\rho} \sigma_1 \sigma_2 Z_1^0 Z_2^0 + \sigma_2^2 \left[Z_2^0\right]^2},$$

$$\beta_2 = \frac{Cov(\tilde{Y}_M, \tilde{Y}_2)}{Var(\tilde{Y}_M)} = \frac{p_1 Z_1^0 + p_2 Z_2^0}{p_2} \frac{\sigma_2 \left[\hat{\rho} \sigma_1 Z_1^0 + \sigma_2 Z_2^0\right]}{\sigma_1^2 \left[Z_1^0\right]^2 + 2\hat{\rho} \sigma_1 \sigma_2 Z_1^0 Z_2^0 + \sigma_2^2 \left[Z_2^0\right]^2},$$

where the variances and covariances are computed using the true distribution of equilibrium returns.

Both of the actual betas differ from the representative agent's betas and thus both assets are mispriced if we consider the CAPM model in the actual economy. This mispricing can be captured in α_j, the market-adjusted returns

$$\alpha_j = \overline{Y}_j - \beta_j \overline{Y}_M = (\beta_j^A - \beta_j)\overline{Y}_M, \qquad j = 1, 2.$$

We examine the market-adjusted returns by the seven types of equilibria in Theorem 2.

Suppose that the equilibrium prevailing in the economy is **Equilibrium Type [1]** (*Non-participating Equilibrium in Asset 1 for SNU Investors*). Then from the Theorem 2, if the quality ratio is infinitesimal,

$E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$, the equilibrium prices of the risky assets are given by Equations (A4.3a) and (A4.3b).

In order for the equilibrium prices to be the same, the representative agent must hold the following beliefs

$$\sigma_1^A = \sigma_1 \sqrt{\frac{1-(1-\theta_I)\hat{\rho}^2}{\theta_I}} \quad \text{and} \quad \sigma_2^A = \sigma_2 \quad \text{and} \quad \rho^A = \hat{\rho}\sqrt{\frac{\theta_I}{1-(1-\theta_I)\hat{\rho}^2}}.$$

Then the actual parameters β_j^A for $j = 1, 2$ are calculated as

$$\beta_1^A = \frac{Cov^A(\tilde{Y}_M, \tilde{Y}_1)}{Var^A(\tilde{Y}_M)} = \frac{p_1 Z_1^0 + p_2 Z_2^0}{p_1} \frac{\sigma_1 \left[\frac{1-(1-\theta_I)\hat{\rho}^2}{\theta_I}\sigma_1 Z_1^0 + \hat{\rho}\sigma_2 Z_2^0\right]}{\frac{1-(1-\theta_I)\hat{\rho}^2}{\theta_I}\sigma_1^2 \left[Z_1^0\right]^2 + 2\hat{\rho}\sigma_1\sigma_2 Z_1^0 Z_2^0 + \sigma_2^2 \left[Z_2^0\right]^2},$$

$$\beta_2^A = \frac{Cov^A(\tilde{Y}_M, \tilde{Y}_2)}{Var^A(\tilde{Y}_M)} = \frac{p_1 Z_1^0 + p_2 Z_2^0}{p_2} \frac{\sigma_2 \left[\hat{\rho}\sigma_1 Z_1^0 + \sigma_2 Z_2^0\right]}{\frac{1-(1-\theta_I)\hat{\rho}^2}{\theta_I}\sigma_1^2 \left[Z_1^0\right]^2 + 2\hat{\rho}\sigma_1\sigma_2 Z_1^0 Z_2^0 + \sigma_2^2 \left[Z_2^0\right]^2}.$$

We can verify that $\beta_1^A > \beta_1$ and $\beta_2^A < \beta_2$, hence $\alpha_1 = (\beta_1^A - \beta_1)\overline{Y}_M > 0$ and $\alpha_2 = (\beta_2^A - \beta_2)\overline{Y}_M < 0$.

Suppose that the equilibrium prevailing in the economy is **Equilibrium Type [2]** (*Non-participating Equilibrium in Asset 1 for NU Investors*). Then from the Theorem 2, if the quality ratio is tiny, $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) < E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, the equilibrium prices of the risky assets are given by Equations (A4.8a) and (A4.8b).

In order for the equilibrium prices to be the same, the representative agent must hold the following beliefs

$$\begin{aligned}
\sigma_1^A &= \sigma_1 \sqrt{\frac{\dot{K}(\hat{\rho}, \overline{\rho}_S) + \theta_U}{\left[\dot{K}(\hat{\rho}, \overline{\rho}_S) - \theta_N\right]\left[\dot{K}(\hat{\rho}, \overline{\rho}_S) + \theta_U\right] - k^2(\hat{\rho}, \overline{\rho}_S)}} \quad \text{and} \\
\sigma_2^A &= \sigma_2 \sqrt{\frac{\dot{K}(\hat{\rho}, \overline{\rho}_S) - \theta_N}{\left[\dot{K}(\hat{\rho}, \overline{\rho}_S) - \theta_N\right]\left[\dot{K}(\hat{\rho}, \overline{\rho}_S) + \theta_U\right] - \dot{k}^2(\hat{\rho}, \overline{\rho}_S)}} \quad \text{and} \\
\rho^A &= \frac{\dot{k}(\hat{\rho}, \overline{\rho}_S)}{\sqrt{\left[\dot{K}(\hat{\rho}, \overline{\rho}_S) - \theta_N\right]\left[\dot{K}(\hat{\rho}, \overline{\rho}_S) + \theta_U\right]}}.
\end{aligned}$$

Then the actual parameters β_j^A for $j = 1, 2$ are calculated as

$$\beta_1^A = \frac{Cov^A(\tilde{Y}_M, \tilde{Y}_1)}{Var^A(\tilde{Y}_M)} = \frac{p_1 Z_1^0 + p_2 Z_2^0}{p_1} \frac{\sigma_1 \left\{ \left[\dot{K}(\hat{\rho}, \overline{\rho}_S) + \theta_U \right] \sigma_1 Z_1^0 + \dot{k}(\hat{\rho}, \overline{\rho}_S) \sigma_2 Z_2^0 \right\}}{\left[\dot{K}(\hat{\rho}, \overline{\rho}_S) + \theta_U \right] \sigma_1^2 \left[Z_1^0 \right]^2 + 2\dot{k}(\hat{\rho}, \overline{\rho}_S) \sigma_1 \sigma_2 Z_1^0 Z_2^0 + \left[\dot{K}(\hat{\rho}, \overline{\rho}_S) - \theta_N \right] \sigma_2^2 \left[Z_2^0 \right]^2},$$

$$\beta_2^A = \frac{Cov^A(\tilde{Y}_M, \tilde{Y}_2)}{Var^A(\tilde{Y}_M)} = \frac{p_1 Z_1^0 + p_2 Z_2^0}{p_2} \frac{\sigma_2 \left\{ \dot{k}(\hat{\rho}, \overline{\rho}_S) \sigma_1 Z_1^0 + \left[\dot{K}(\hat{\rho}, \overline{\rho}_S) - \theta_N \right] \sigma_2 Z_2^0 \right\}}{\left[\dot{K}(\hat{\rho}, \overline{\rho}_S) + \theta_U \right] \sigma_1^2 \left[Z_1^0 \right]^2 + 2\dot{k}(\hat{\rho}, \overline{\rho}_S) \sigma_1 \sigma_2 Z_1^0 Z_2^0 + \left[\dot{K}(\hat{\rho}, \overline{\rho}_S) - \theta_N \right] \sigma_2^2 \left[Z_2^0 \right]^2}.$$

We can verify that $\beta_1^A > \beta_1$ and $\beta_2^A < \beta_2$, hence $\alpha_1 = (\beta_1^A - \beta_1)\overline{Y}_M > 0$ and $\alpha_2 = (\beta_2^A - \beta_2)\overline{Y}_M < 0$.

Suppose that the equilibrium prevailing in the economy is **Equilibrium Type [3]** (*Non-participating Equilibrium in Asset 1 for Noise Investors*). Then from the Theorem 2, if the quality ratio is small, $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{12} \leqslant m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, the equilibrium prices of the risky assets are given by Equations (A4.14a) and (A4.14b).

In order for the equilibrium prices to be the same, the representative agent must hold the following beliefs

$$\begin{aligned}
\sigma_1^A &= \sigma_1 \sqrt{\frac{K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) + \theta_U}{K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \left[K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) + \theta_U \right] - k^2(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)}} \quad \text{and} \\
\sigma_2^A &= \sigma_2 \sqrt{\frac{K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)}{K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \left[K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) + \theta_U \right] - k^2(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)}} \quad \text{and} \\
\rho^A &= \frac{k(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)}{\sqrt{K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \left[K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) + \theta_U \right]}}.
\end{aligned}$$

Then the actual parameters β_j^A for $j = 1, 2$ are calculated as

$$\beta_1^A = \frac{Cov^A(\tilde{Y}_M, \tilde{Y}_1)}{Var^A(\tilde{Y}_M)} = \frac{p_1 Z_1^0 + p_2 Z_2^0}{p_1} \frac{\sigma_1 \left\{ \left[K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) + \theta_U \right] \sigma_1 Z_1^0 + k(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \sigma_2 Z_2^0 \right\}}{\left[K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) + \theta_U \right] \sigma_1^2 \left[Z_1^0 \right]^2 + 2k(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \sigma_1 \sigma_2 Z_1^0 Z_2^0 + K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \sigma_2^2 \left[Z_2^0 \right]^2},$$

$$\beta_2^A = \frac{Cov^A(\tilde{Y}_M, \tilde{Y}_2)}{Var^A(\tilde{Y}_M)} = \frac{p_1 Z_1^0 + p_2 Z_2^0}{p_2} \frac{\sigma_2 \left\{ k(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \sigma_1 Z_1^0 + K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \sigma_2 Z_2^0 \right\}}{\left[K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) + \theta_U \right] \sigma_1^2 \left[Z_1^0 \right]^2 + 2k(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \sigma_1 \sigma_2 Z_1^0 Z_2^0 + K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \sigma_2^2 \left[Z_2^0 \right]^2}.$$

We can verify that $\beta_1^A > \beta_1$ and $\beta_2^A < \beta_2$, hence $\alpha_1 = (\beta_1^A - \beta_1)\overline{Y}_M > 0$ and $\alpha_2 = (\beta_2^A - \beta_2)\overline{Y}_M < 0$.

Suppose that the equilibrium prevailing in the economy is **Equilibrium Type [4]** (*Participating Equilibrium in Both Assets for SNU Investors*). Then from the Theorem 2, if the quality ratio is medium, $m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{12} < M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, the equilibrium prices of the risky assets are given by Equations (A4.38a) and (A4.38b).

In order for the equilibrium prices to be the same, the representative agent must hold the following beliefs

$$\sigma_j^A = \frac{\sigma_j}{\sqrt{2\left[\frac{\theta_I}{1+\hat{\rho}} + \frac{\theta_S}{1+\overline{\rho}_S} + \frac{\theta_N}{1+\overline{\rho}_N}\right] + \theta_U}} \quad \text{for } j = 1,2 \qquad \text{and} \qquad \rho^A = 1.$$

Then the actual parameters β_j^A for $j = 1,2$ are calculated as

$$\beta_1^A = \frac{Cov^A(\tilde{Y}_M, \tilde{Y}_1)}{Var^A(\tilde{Y}_M)} = \frac{p_1 Z_1^0 + p_2 Z_2^0}{p_1} \frac{\sigma_1\left[\sigma_1 Z_1^0 + \sigma_2 Z_2^0\right]}{\sigma_1^2\left[Z_1^0\right]^2 + 2\sigma_1\sigma_2 Z_1^0 Z_2^0 + \sigma_2^2\left[Z_2^0\right]^2},$$

$$\beta_2^A = \frac{Cov^A(\tilde{Y}_M, \tilde{Y}_2)}{Var^A(\tilde{Y}_M)} = \frac{p_1 Z_1^0 + p_2 Z_2^0}{p_1} \frac{\sigma_2\left[\sigma_1 Z_1^0 + \sigma_2 Z_2^0\right]}{\sigma_1^2\left[Z_1^0\right]^2 + 2\sigma_1\sigma_2 Z_1^0 Z_2^0 + \sigma_2^2\left[Z_2^0\right]^2}.$$

We can verify that $\beta_1^A \lesseqqgtr \beta_1$ for $\sigma_2 Z_2^0 \lesseqqgtr \sigma_1 Z_1^0$ and $\beta_2^A \lesseqqgtr \beta_2$ for $\sigma_1 Z_1^0 \lesseqqgtr \sigma_2 Z_2^0$, hence $\alpha_1 = (\beta_1^A - \beta_1)\overline{Y}_M \lesseqqgtr 0$ for $\sigma_2 Z_2^0 \lesseqqgtr \sigma_1 Z_1^0$ and $\alpha_2 = (\beta_2^A - \beta_2)\overline{Y}_M \lesseqqgtr 0$ for $\sigma_1 Z_1^0 \lesseqqgtr \sigma_2 Z_2^0$.

Suppose that the equilibrium prevailing in the economy is **Equilibrium Type [5]** (*Non-participating Equilibrium in Asset 2 for Naïve Investors*). Then from the Theorem 2, if the quality ratio is big, $M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \leqslant E_{12} < H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, the equilibrium prices of the risky assets are given by Equations (A4.32a) and (A4.32b).

In order for the equilibrium prices to be the same, the representative agent must hold the following beliefs

$$\begin{aligned} \sigma_1^A &= \sigma_2\sqrt{\frac{K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)}{\left[K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) + \theta_U\right] K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) - k^2(\hat{\rho}, \overline{\rho}_S, 0)}} \quad \text{and} \\ \sigma_2^A &= \sigma_1\sqrt{\frac{K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) + \theta_U}{\left[K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) + \theta_U\right] K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) - k^2(\hat{\rho}, \overline{\rho}_S, 0)}} \quad \text{and} \\ \rho^A &= \frac{k(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)}{\sqrt{\left[K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) + \theta_U\right] K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)}}. \end{aligned}$$

Then the actual parameters β_j^A for $j = 1,2$ are calculated as

$$\beta_1^A = \frac{Cov^A(\tilde{Y}_M, \tilde{Y}_1)}{Var^A(\tilde{Y}_M)} = \frac{p_1 Z_1^0 + p_2 Z_2^0}{p_1} \frac{\sigma_1\left\{K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)\sigma_1 Z_1^0 + k(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)\sigma_2 Z_2^0\right\}}{K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)\sigma_1^2\left[Z_1^0\right]^2 + 2k(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)\sigma_1\sigma_2 Z_1^0 Z_2^0 + \left[K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) + \theta_U\right]\sigma_2^2\left[Z_2^0\right]^2},$$

$$\beta_2^A = \frac{Cov^A(\tilde{Y}_M, \tilde{Y}_2)}{Var^A(\tilde{Y}_M)} = \frac{p_1 Z_1^0 + p_2 Z_2^0}{p_2} \frac{\sigma_2\left[k(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)\sigma_1 Z_1^0 + \left[K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) + \theta_U\right]\sigma_2 Z_2^0\right]}{K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)\sigma_1^2\left[Z_1^0\right]^2 + 2k(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)\sigma_1\sigma_2 Z_1^0 Z_2^0 + \left[K(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) + \theta_U\right]\sigma_2^2\left[Z_2^0\right]^2}.$$

We can verify that $\beta_1^A < \beta_1$ and $\beta_2^A > \beta_2$, hence $\alpha_1 = (\beta_1^A - \beta_1)\overline{Y}_M < 0$ and $\alpha_2 = (\beta_2^A - \beta_2)\overline{Y}_M > 0$.

Suppose that the equilibrium prevailing in the economy is **Equilibrium Type [6]** (*Non-participating Equilibrium in Asset 2 for NU Investors*). Then from the Theorem 2, if the quality ratio is huge, $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \leqslant E_{12} < H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$, the equilibrium prices of the risky assets are given by Equations (A4.26a) and (A4.26b).

In order for the equilibrium prices to be the same, the representative agent must hold the following beliefs

$$\begin{aligned}
\sigma_1^A &= \sigma_2 \sqrt{\frac{\dot{K}(\hat{\rho}, \overline{\rho}_S) - \theta_N}{\left[\dot{K}(\hat{\rho}, \overline{\rho}_S) + \theta_U\right]\left[\dot{K}(\hat{\rho}, \overline{\rho}_S) - \theta_N\right] - k^2(\hat{\rho}, \overline{\rho}_S, 0)}} \quad \text{and} \\
\sigma_2^A &= \sigma_1 \sqrt{\frac{\dot{K}(\hat{\rho}, \overline{\rho}_S) + \theta_U}{\left[\dot{K}(\hat{\rho}, \overline{\rho}_S) + \theta_U\right]\left[\dot{K}(\hat{\rho}, \overline{\rho}_S) - \theta_N\right] - k^2(\hat{\rho}, \overline{\rho}_S, 0)}} \quad \text{and} \\
\rho^A &= \frac{\dot{k}(\hat{\rho}, \overline{\rho}_S)}{\sqrt{\left[\dot{K}(\hat{\rho}, \overline{\rho}_S) + \theta_U\right]\left[\dot{K}(\hat{\rho}, \overline{\rho}_S) - \theta_N\right]}}.
\end{aligned}$$

Then the actual parameters β_j^A for $j = 1, 2$ are calculated as

$$\begin{aligned}
\beta_1^A &= \frac{Cov^A(\tilde{Y}_M, \tilde{Y}_1)}{Var^A(\tilde{Y}_M)} = \frac{p_1 Z_1^0 + p_2 Z_2^0}{p_1} \frac{\sigma_1 \left\{\left[\dot{K}(\hat{\rho}, \overline{\rho}_S) - \theta_N\right]\sigma_1 Z_1^0 + \dot{k}(\hat{\rho}, \overline{\rho}_S)\sigma_2 Z_2^0\right\}}{\left[\dot{K}(\hat{\rho}, \overline{\rho}_S) - \theta_N\right]\sigma_1^2 \left[Z_1^0\right]^2 + 2\dot{k}(\hat{\rho}, \overline{\rho}_S)\sigma_1\sigma_2 Z_1^0 Z_2^0 + \left[\dot{K}(\hat{\rho}, \overline{\rho}_S) + \theta_U\right]\sigma_2^2\left[Z_2^0\right]^2}, \\
\beta_2^A &= \frac{Cov^A(\tilde{Y}_M, \tilde{Y}_2)}{Var^A(\tilde{Y}_M)} = \frac{p_1 Z_1^0 + p_2 Z_2^0}{p_2} \frac{\sigma_2 \left\{\dot{k}(\hat{\rho}, \overline{\rho}_S)\sigma_1 Z_1^0 + \left[\dot{K}(\hat{\rho}, \overline{\rho}_S) + \theta_U\right]\sigma_2 Z_2^0\right\}}{\left[\dot{K}(\hat{\rho}, \overline{\rho}_S) - \theta_N\right]\sigma_1^2 \left[Z_1^0\right]^2 + 2\dot{k}(\hat{\rho}, \overline{\rho}_S)\sigma_1\sigma_2 Z_1^0 Z_2^0 + \left[\dot{K}(\hat{\rho}, \overline{\rho}_S) + \theta_U\right]\sigma_2^2\left[Z_2^0\right]^2}.
\end{aligned}$$

We can verify that $\beta_1^A < \beta_1$ and $\beta_2^A > \beta_2$, hence $\alpha_1 = (\beta_1^A - \beta_1)\overline{Y}_M < 0$ and $\alpha_2 = (\beta_2^A - \beta_2)\overline{Y}_M > 0$.

Suppose that the equilibrium prevailing in the economy is **Equilibrium Type [7]** (*Non-participating Equilibrium in Asset 2 for SNU Investors*). Then from the Theorem 2, if the quality ratio is infinitude, $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) \leqslant E_{12}$, the equilibrium prices of the risky assets are given by Equations (A4.21a) and (A4.21b).

In order for the equilibrium prices to be the same, the representative agent must hold the following beliefs

$$\sigma_1^A = \sigma_1 \quad \text{and} \quad \sigma_2^A = \sigma_2 \sqrt{\frac{1 - (1 - \theta_I)\hat{\rho}^2}{\theta_I}} \quad \text{and} \quad \rho^A = \hat{\rho}\sqrt{\frac{\theta_I}{1 - (1 - \theta_I)\hat{\rho}^2}}.$$

Then the actual parameters β_j^A for $j = 1, 2$ are calculated as

$$\beta_1^A = \frac{Cov^A(\tilde{Y}_M, \tilde{Y}_1)}{Var^A(\tilde{Y}_M)} = \frac{p_1 Z_1^0 + p_2 Z_2^0}{p_1} \frac{\sigma_1 \left[\sigma_1 Z_1^0 + \hat{\rho}\sigma_2 Z_2^0\right]}{\sigma_1^2 \left[Z_1^0\right]^2 + 2\hat{\rho}\sigma_1\sigma_2 Z_1^0 Z_2^0 + \frac{1-(1-\theta_I)\hat{\rho}^2}{\theta_I}\sigma_2^2 \left[Z_2^0\right]^2},$$

$$\beta_2^A = \frac{Cov^A(\tilde{Y}_M, \tilde{Y}_2)}{Var^A(\tilde{Y}_M)} = \frac{p_1 Z_1^0 + p_2 Z_2^0}{p_2} \frac{\sigma_2 \left[\hat{\rho}\sigma_1 Z_1^0 + \frac{1-(1-\theta_I)\hat{\rho}^2}{\theta_I}\sigma_2 Z_2^0\right]}{\sigma_1^2 \left[Z_1^0\right]^2 + 2\hat{\rho}\sigma_1\sigma_2 Z_1^0 Z_2^0 + \frac{1-(1-\theta_I)\hat{\rho}^2}{\theta_I}\sigma_2^2 \left[Z_2^0\right]^2}.$$

We can verify that $\beta_1^A < \beta_1$ and $\beta_2^A > \beta_2$, hence $\alpha_1 = (\beta_1^A - \beta_1)\overline{Y}_M < 0$ and $\alpha_2 = (\beta_2^A - \beta_2)\overline{Y}_M > 0$.

Summarizing the above analysis, we list our conclusions as follows. If the quality ratio is infinitesimal, $E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$, then the non-participating equilibrium (in asset 1 for SNU investors) market-adjusted returns are $\alpha_1 > 0$ and $\alpha_2 < 0$. If the quality ratio is tiny, $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) < E_{12} \leqslant h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, then the non-participating equilibrium (in asset 1 for NU investors) market-adjusted returns are $\alpha_1 > 0$ and $\alpha_2 < 0$. If the quality ratio is small, $h(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{12} \leqslant m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, then the non-participating equilibrium (in asset 1 for noise investors) market-adjusted returns are $\alpha_1 > 0$ and $\alpha_2 < 0$. If the quality ratio is medium, $m(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) < E_{12} < M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, then the participating equilibrium (in both assets for SNU investors) market-adjusted returns are $\alpha_1 > 0$ and $\alpha_2 < 0$ for $E_{12} < 1$, while $\alpha_1 < 0$ and $\alpha_2 > 0$ for $E_{12} > 1$. If the quality ratio is big, $M(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \leqslant E_{12} < H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N)$, then the non-participating equilibrium (in asset 2 for noise investors) market-adjusted returns are $\alpha_1 < 0$ and $\alpha_2 > 0$. If the quality ratio is huge, $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_N) \leqslant E_{12} < H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S)$, then the non-participating equilibrium (in asset 2 for NU investors) market-adjusted returns are $\alpha_1 < 0$ and $\alpha_2 > 0$. If the quality ratio is infinitude, $H(\hat{\rho}, \overline{\rho}_S, \overline{\rho}_S) \leqslant E_{12}$, then the non-participating equilibrium (in asset 2 for SNU investors) market-adjusted returns are $\alpha_1 < 0$ and $\alpha_2 > 0$. In general, there is a positive excess return to hold the asset that is not held by SNU investors and a negative excess expected return to hold the other risky asset. This occurs because in order to attract the insider investors to hold the supply of the asset with ambiguous returns (according to the SNU investors), its price must be low and, thus, its returns must be high. Conversely, the ambiguity-averse investors overweight their portfolios in the non-ambiguous asset, thus increasing its price and lower its returns.

Note that under a non-participating equilibrium (type [1] or [7]), SNU investors will decide not to hold the lower-quality asset; while under a minor-participating equilibrium (type [2] or [6]), NU investors will decide not to hold the lower quality asset; and while under a major-participating equilibrium (type [3] or [5]), noise investors will decide not to hold the lowe-quality asset; and while under a full-participating equilibrium (type [4]), SNU investors hold the risky assets. Thus to summarize, under seven types of equilibria, the asset with lower-quality will generate positive excess returns, while the asset with higher quality will receive negative excess returns. That is, no matter whether the economy is under a participating equilibrium (for SNU investors) or a non-participating one (for SNU investors, for NU investors, or for noise investors), the corresponding investors will favor the asset with higher quality even to an irrational degree, making its price increase and the return lower than what standard models forecast. From this we can see that correlation ambiguity can be considered as a novel approach to complementing the study of cross-sectional performance of individual stocks.

Theorem 3. No matter whether the equilibrium is a full-participating one or major-participating one or minor-participating one or non-participating one, the risky asset with lower quality will generate positive excess returns, while the asset with higher quality will generate negative excess returns.

7. CONCLUSIONS

In this paper, we extend the multi-asset model presented by Huang, Zhang and Zhu (2017), which has two types of investors (insider and naïve investors), by adding sophisticated investors with partial information and noise investors with no information. Correlation ambiguity generates four scenarios in the demand functions for sophisticated and naïve and noise investors. The properties of these demand functions lead to the existence of a unique general equilibrium with seven different types of equilibria. Thus we obtain seven different types of equilibria: full-participating equilibrium (for sophisticated and naïve and noise investors), minor-participating equilibrium (non-participating for naïve and noise investors), major-participating equilibrium (non-participating for noise investors), and non-participating equilibrium (for sophisticated and naïve and noise investors), depending upon the quality size of risky assets. Ambiguity-averse investors (including sophisticated and naïve and noise investors) rationally choose to limit participation so that they can avoid ambiguity of correlation. Limited participation with the lower-quality asset occurs endogenously for only noise investors when the quality ratio of low-quality asset relative to high-quality asset is small, for naïve and noise investors when the quality ratio of low-quality asset relative to high-quality asset is tiny, and for sophisticated and naïve and noise investors when the quality ratio of low-quality asset relative to high-quality asset is infinitesimal.

The investors with more information are willing to participate in trading some risky assets, while the investors with less information are not willing to participate in trading, which is limited participation phenomenon due to ambiguous information. However, the demand for investors with less information might be greater than that for investors with more information. Furthermore, the investors with less information might hold more equilibrium positions than the investors with more information. The main reason is that the uncertainty on correlation comes from ambiguity (not only from pure risk). In addition, we further explore that the investors with less information escape from low-quality assets to high-quality assets and the investors with more information escape from high-quality assets to low-quality assets. Thus, in equilibrium, investors exhibit a flight-to-quality trading pattern.

The main questions in this paper can be studied further for more complicated frameworks. For example, we can examine the multi-assets model with one risk-free asset and K risky assets ($K > 2$). The general correlation matrix includes $\frac{1}{2}(K-1)K$ correlation coefficients between pairwise assets. Thus we can set out complex information structure of correlation coefficient. Sometimes we can focus on a simple model with equi-correlation matrix.

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