Optimal Environmental Tax-Subsidy Regime in the Presence of Increasing Returns

Wenli Cheng and Dingsheng Zhang[*]

This paper develops a set of three models to study the optimal tax-subsidy regime in an economy characterised by two deviations from the perfect competition model — negative externality from pollution by the "dirty" industry, and increasing returns in the "clean" industry. Its main conclusions are: (1) the optimal single pollution tax is higher than the Pigouvian level; (2) a combination of pollution tax and quantity subsidy increases consumer welfare at a lower level of pollution tax; (3) the optimal pollution tax can be further lowered and consumer welfare further increased if the quantity subsidy is supplemented by a lump-sum subsidy.

Key Words: Optimal pollution tax; Clean subsidy; Increasing returns; Monopolistic competition.

JEL Classification Number: H23.

1. INTRODUCTION

The idea of searching for optimal taxes and subsidies to internalize negative and positive externalities is usually traced back to Pigou (1932), who contends that, "under conditions of simple competition", for every industry in which the value of the marginal social net product is different from the marginal private net product, there will be certain rates of subsidies or taxes that would have the optimal effect of increasing "the size of the national dividend" and "economic welfare" (Part II, Chapter 11, §11.).

The rate of tax or subsidy as suggested by Pigou is one that makes the net marginal private product equal to the net marginal social product. This is known as the "Pigouvian" tax or subsidy which is optimal "under conditions of simple competition". How might the optimal tax or subsidy differ from the "Pigouvian" level in situations more complex than "simple

* Cheng: Department of Economics, Monash University; Zhang: Corresponding author. EMS and IAS, Wuhan University and CEMA, Central University of Finance and Economics.

competition"? This is a question we ask in this paper. Specifically, the purpose of the paper is to study optimal tax and subsidy in an economy characterized by two deviations from the perfect competition model: the negative externality from pollution by a "dirty" industry, and increasing returns in a "clean" industry. Because of these deviations, prices are different from marginal costs and the number of firms/varieties in the "clean" industry is in general not optimal. These consequences can be addressed by an appropriate tax-subsidy regime. In this paper, we derive optimal tax-subsidy rates under three different tax-subsidy regimes: (1) a single pollution tax, (2) a combination of a pollution tax and a unit subsidy to the clean industry with increasing returns, and (3) a combination of a pollution tax, a unit subsidy and a lump sum subsidy to the clean industry with increasing returns. We investigate the extent to which the optimal tax is different from the Pigouvian tax for each case, and compare the market outcome and consumer welfare across the policy regimes.

Economists after Pigou have shown an un-waning interest in optimal taxation beyond the conditions of simple competition. Buchanan (1969) considers the case of a polluting monopolist which imposes two external diseconomies, pollution that leads to environmental damage, and output restriction that hurts consumers. He shows that a pollution tax on the monopolist's output will decrease welfare if the second diseconomy is more highly valued than the former. Later, Oates and Strassman (1984) estimate that the welfare gains from pollution control are likely to outweigh the potential losses from the various imperfections in the economy; therefore the case for a pollution tax is not seriously compromised by deviations from competitive behavior. Their conclusion has been challenged by Vetter (2009) who studies a polluting industry in which the number of firms is endogenous and suggests that the imposition of a Pigovian tax can in practice be inferior to no tax at all. Apart from the impact of imperfect competition, researchers have investigated how other deviations from the perfect competition model may affect the magnitude of the optimal pollution tax. For example, Bovenberg and Mooij (1994) and Bovenberg and Goulder (1996) show that with a pre-existing tax on labor income, an environmental levy may exacerbate the income tax distortion, hence the optimal environmental levy would typically be lower than the Pigovian tax. However, their results may be reversed under different structures of consumption and labor taxes (Fullerton, 1997, Ng, 2000).

In common with the aforementioned literature, this paper provides an example of the theory of second best in that it investigates how the optimal tax/subsidy may differ from the first best case (the Pigouvian level) in the presence of negative externality and increasing returns. The paper also has some distinct features.

Firstly, it is motivated by the observation that the case of pollution control is typically characterised by a negative externality of pollution, and increasing returns related to the discovery and adoption of clean technologies. As a result, in the absence of public policy intervention, the adoption of clean technology would be doubly underprovided by markets (Jaffe, Newell, and Stavins, 2005). Thus intuitively, a combination of a tax to reduce pollution and a subsidy to encourage the adoption of clean technology should out-perform a single pollution tax (Fischer and Newell, 2008; Heinzel and Winkler, 2011). Moreover, if the pollution tax is imposed in conjunction with a subsidy to clean technology, the optimal level of pollution tax is likely to be lower because the policy combination has the effect of reallocating resources away from the polluting industry and to the increasing return industry. This reasoning is supported by Sartzetakis and Tsigaris (2005) who show that if pollution levies are ear-marked towards subsidizing the clean technology, the tax is lower than that in the case of a single tax.

Secondly, this paper highlights the role of increasing returns in affecting the number of product varieties that is desirable to supply (Spence, 1976). Increasing returns are studied in the context of economies of specialisation (Cheng and Yang, 2004) or more commonly in monopolistic competition models pioneered by Dixit and Stiglitz (1977). The theoretical importance of increasing returns in economic analysis has been documented in Buchanan and Yoon (1994) and Arrow, Ng and Yang (1998), and some of the public policy implications have been shown by Ng and Zhang (2007). This paper extends Dixit and Stiglitz's (1977) monopolistic competition model to study the optimal environmental policy in an economy with a dirty industry that pollutes and a clean industry that exhibits increasing returns.

Thirdly, in addition to a pollution tax and a unit subsidy, this paper considers the implications of adding a lump sum subsidy to the policy mix. It shows that a policy combination of a pollution tax, a unit and a lump sum subsidy to the clean industry generates higher consumer welfare than policies without a lump sum subsidy. Since the presence of fixed costs gives rise to increasing returns and limits product varieties in the clean industry, the use of a lump sum subsidy moderates the degree of increasing returns and allows a larger number of clean product varieties in the market.

The rest of the paper is organised as follows. Section 2 presents a set of three models to determine the optimal tax/subsidy levels under three different tax-subsidy regimes: (1) a single pollution tax; (2) a pollution tax plus a unit subsidy; and (3) a pollution tax plus a unit subsidy and a lump sum subsidy. Section 3 compares the market outcome and consumer welfare across different policy regimes, and presents a numerical comparative static analysis of the equilibrium in each model. Section 4 concludes.

2. MODELS

Consider an economy with L consumers who derive utility from consuming goods X, Y and Z, and suffer from pollution. X is a good produced using a constant returns "dirty" technology such that for each unit of good X produced, a unit of harmful pollutants is emitted. Z is a good produced with a constant return "clean" technology. Y is a group of n varieties of differentiated goods, each produced by a single firm using increasing returns "clean" technologies. The representative consumer's utility function is:

$$U = x^\alpha y^\beta z^{1-\alpha-\beta} X^{-\gamma}, \quad y \equiv \left(\sum_{i=1}^{n} y_i^\rho \right)^{\frac{1}{\rho}} \tag{1}$$

where x is the quantity demanded for good X; X is the total amount of pollutants generated in the production of X; y is the quantity demanded for good Y, which is a CES function of y_i; z is the quantity demanded for good Z; α and β are preference parameters, γ is a measure of disutility from pollution.

While consumers' utility is reduced by pollution, each consumer perceives that his/her individual choice has a negligible impact on the level of pollution. Accordingly, the total amount of pollutants X is taken as given in the consumer's utility maximization problem.

The production functions for goods X and Z are:

$$X = a_x L_x, \quad Z = a_z L_z \tag{2}$$

where L_x, L_z are labor used in the production of good X and Z, respectively.

The production of each variety of good Y involves a fixed cost. For reasons of tractability, we follow Dixit and Stiglitz (1977) and assume symmetry in all varieties of good Y. This does not necessarily imply that all varieties of good Y are identical because the unit for each product variety can be defined differently, thus the symmetry assumption is less restrictive than it appears (Krugman, 1981). The production function for each variety of Y is:

$$Y_i = a_{yi} L_{yi} - f_i \tag{3}$$

where L_{yi} is labor devoted to producing variety i of good Y. The presence of a fixed cost means that the production of Y_i exhibits increasing returns. This model specification of monopolistic competition and fixed costs captures two key characteristics of the clean technology industry: the importance of fixed costs, for example, the costs of research and development and of advertising; and competition among different product varieties.

Given the consumer preferences and technologies, we now consider the market outcome of three different policy regimes in turn, and determine the optimal tax/subsidy level under each regime.

2.1. Model 1: A single pollution tax

In this model, the government imposes a single unit pollution tax on the X industry. The tax revenue is returned to consumers in a lump sum payment. This implicitly assumes that consumers have the right to a clean environment, and they sell a part of that right to the X industry in the form of pollution permits, using the government as an agent; the rate of pollution tax is the price of pollution permits.

Each consumer is endowed with one unit of labor which earns a wage. The consumer's decision problem is to choose the mix of his/her consumption basket to maximise utility function (1), subject to the following budget constraint.

$$p_x x + \sum_{i=1}^{n} p_{yi} y_i + p_z z = w + R \tag{4}$$

where p_x, p_y and p_z are the prices of goods X, Y and Z; w is the wage rate normalized to be 1; and R is a transfer payment from the government's sale of pollution permits.

All firms maximize profits subject to their technologies. Their decision problems are, respectively:

$$\text{Firms producing } X : \max \pi_x = (p_x - t)X - L_x \quad \text{subject to } X = a_x L_x \tag{5}$$
$$\text{Firms producing } Y_i : \max \pi_{yi} = p_{yi}(Y_i)Y_i - L_{yi} \quad \text{subject to } Y_i = a_{yi} L_{yi} - f_i \tag{6}$$
$$\text{Firms producing } Z : \max \pi_z = p_z Z - L_z \quad \text{subject to } Z = a_z L_z \tag{7}$$

In equilibrium all firms earn zero profits. The zero profit condition is automatically met under perfect competition and constant returns technologies. In the monopolistically competitive market for good Y varieties, the following zero profit condition is satisfied:

$$\pi_{yi} = p_{yi}(Y_i)Y_i - L_{yi} = 0 \tag{8}$$

Also, all markets clear and the total tax revenue are transferred to consumers, so we have:

Market for good X :	$X = Lx$	(9)
Markets for good Y varieties :	$Y_i = Ly_i$	(10)
Market for good Z :	$Z = Lz$	(11)
Market for labor :	$L_x + nL_y + L_z = L$	(12)
Balanced government budget :	$tX = LR$	(13)

TABLE 1.

Equilibrium Solutions

Model 1	
Prices	$w = 1$, $p_x = t + \frac{1}{a_x}$, $p_y = \frac{1}{\rho a_y}$, $p_z = \frac{1}{a_z}$
Quantities	$x = \frac{\alpha a_x}{1+(1-\alpha)a_x t}$, $y_i = \frac{1}{L}\frac{\rho f}{1-\rho}$, $z = \frac{(1-\alpha\beta)a_z(1+a_x t)}{1+(1-\alpha)a_x t}$ $n = a_y L \frac{(1-\rho)\beta}{f}\frac{(1+a_x t)}{1+(1-\alpha)a_x t}$
Utility	$U = C_1(1+(1-\alpha)a_x t)^{-\frac{(1-\rho)\beta}{\rho}-1+\gamma}(1+a_x t)^{1-\alpha+\frac{(1-\rho)\beta}{\rho}}$ where $C_1 \equiv (\alpha a_x)^{\alpha-\gamma}(\beta a_y)^{\frac{\beta}{\rho}}[(1-\alpha-\beta)a_z]^{1-\alpha\beta}L^{\frac{(1-\rho)\beta}{\rho}-\gamma}f^{-\frac{(1-\rho)\beta}{\rho}}\rho^\beta(1-\rho)^{\frac{(1-\rho)\beta}{\rho}}$
Optimal tax	$t^* = \frac{(1-\alpha)\gamma+\frac{\alpha\beta(1-\rho)}{\rho}}{a_x(1-\alpha)(\alpha-\gamma)}$
Model 2	
Prices	$w = 1$, $p_x = t + \frac{1}{a_x}$, $p_y = \frac{1}{\rho a_y}\frac{\rho\beta(1+a_x t)(1-\tau)}{\rho\beta(1+a_x t)+[(1-\alpha-\rho\beta)(1+a_x t)+\alpha]\tau}$, $p_z = \frac{1}{a_z}$
Quantities	$x = \frac{\alpha a_x(1-\tau)}{1+(1-\alpha)a_x t}$, $y_i = \frac{1}{L}\frac{\rho f}{1-\rho}\frac{\rho\beta(1+a_x t)+[(1-\alpha-\rho\beta)(1+a_x t)+\alpha]\tau}{\rho\beta(1+a_x t)(1-\tau)}$ $z = \frac{(1-\alpha-\beta)a_z(1-\tau)(1+a_x t)}{1+(1-\alpha)a_x t}$, $n = a_y L \frac{(1-\rho)\beta}{f}\frac{(1-\tau)(1+a_x t)}{1+(1-\alpha)a_x t}$
Utility	$U = C_2(1-\tau)^{\frac{(1-\rho)\beta}{\rho}+1-\beta-\gamma}(1+(1-\alpha)a_x t)^{-\frac{(1-\rho)\beta}{\rho}-1+\gamma}(1+a_x t)^{1-\alpha\beta+\frac{(1-\rho)\beta}{\rho}}\{\rho\beta(1+a_x t)+[(1-\alpha-\rho\beta)(1+a_x t)+\alpha]\tau\}^\beta$ where $C_2 \equiv (\alpha a_x)^{\alpha-\gamma}(\beta a_y)^{\frac{\beta}{\rho}}\beta^{-\beta}[(1-\alpha-\beta)a_z]^{1-\alpha-\beta}L^{\frac{(1-\rho)\beta}{\rho}-\gamma}f^{\frac{(1-\rho)\beta}{\rho}}(1-\rho)^{\frac{(1-\rho)\beta}{\rho}}$
Optimal taxes and subsidies	$t^* = \frac{(1-\alpha-\rho\beta)\gamma+\frac{\alpha\beta(1-\rho)^2}{\rho}}{a_x(1-\alpha-\rho\beta)(\alpha-\gamma)}$, $\tau^* = \frac{\beta(1-\rho)(1-\alpha-\beta)}{(1-\alpha-\rho\beta)[1-\gamma+\frac{(1-\rho)\beta}{\rho}]}$, $s^* = \frac{[\alpha+(1-\alpha)(1+a_x t^*)]\tau^*}{a_y[\rho\beta(1+a_x t^*)+(\alpha+(1-\alpha-\rho\beta)(1+a_x t^*))\tau^*]}$
Model 3	
Prices	$w = 1$, $p_x = t + \frac{1}{a_x}$, $p_y = \frac{1}{\rho a_y}\frac{\rho\beta(1+a_x t)(1-\tau)(f-g)}{B}$, $p_z = \frac{1}{a_z}$
Quantities	$x = \frac{\alpha a_x(1-\tau)}{1+(1-\alpha)a_x t}$, $y_i = \frac{\rho}{(1-\rho)L}\frac{B}{\beta(1+a_x t)(1-\tau)}$, $z = \frac{(1-\alpha-\beta)a_z(1-\tau)(1+a_x t)}{1+(1-\alpha)a_x t}$, $n = a_y L\frac{(1-\rho)\beta}{f-g}\frac{(1+a_x t)(1-\tau)}{1+(1-\alpha)a_x t}$ $B = [\rho\beta f-(\rho\beta+a_y\beta(1-\rho))g](1+a_x t)+\{[(1-\alpha-\rho\beta)(1+a_x t)+\alpha](f-g)+a_y\beta(1-\rho)g(1+a_x t)\}\tau$
Utility	$U = C_3(1-\tau)^{1-\beta-\gamma+\frac{(1-\rho)\beta}{\rho}}(f-g)^{-\frac{\beta}{\rho}}[1+(1-\alpha)a_x t]^{\gamma-1-\frac{(1-\rho)\beta}{\rho}}(1+a_x t)^{1-\alpha-\beta+\frac{(1-\rho)\beta}{\rho}}B^\beta$ where $C_3 = (\alpha a_x)^{\alpha-\gamma}(\beta a_y)^{\frac{\beta}{\rho}}\beta^{-\beta}[(1-\alpha-\beta)a_z]^{1-\alpha-\beta}L^{\frac{(1-\rho)\beta}{\rho}-\gamma}(1-\rho)^{\frac{(1-\rho)\beta}{\rho}}$
Optimal taxes and subsidies	Cannot obtain analytical solutions for t, τ, g $1-sa_y = \frac{\rho\beta(1+a_x t)(1-\tau)(f-g)}{B}$

The equilibrium prices and quantities of the economy described above can be solved and the solutions are presented in Table 1.

To obtain the optimal rate of pollution tax, we solve the government's decision problem of maximizing consumer welfare:

$$\max_{t} U = C_1 (1 + (1-\alpha)a_x t)^{\frac{(1-\rho)\beta}{\rho} - 1 + \gamma} (1 + a_x t)^{1 - \alpha + \frac{(1-\rho)\beta}{\rho}} \tag{14}$$

where $C_1 \equiv (\alpha a_x)^{\alpha - \gamma} (\beta a_y)^{\frac{\beta}{\rho}} [(1 - \alpha - \beta)a_z]^{1 - \alpha - \beta} L^{\frac{(1-\rho)\beta}{\rho} - \gamma} f^{-\frac{(1-\rho)\beta}{\rho}} \rho^{\beta} (1 - \rho)^{\frac{(1-\rho)\beta}{\rho}}$

The optimal rate of pollution tax under regime 1 is:

$$t^* = t_{pigou} + \Psi \tag{15}$$

Where $t_{pigou} = \frac{\gamma}{a_x (\alpha - \gamma)}$; $\Psi \equiv \frac{(1-\alpha)\gamma + \frac{\alpha\beta(1-\rho)}{\rho}}{a_x (1-\alpha)(\alpha - \gamma)}$.

As $\Psi > 0$, the optimal pollution tax is greater than the Pigouvian tax. Intuitively this is because the optimal pollution tax not only corrects the negative externality of pollution, but also remedies the under-provision of good Y varieties produced under increasing returns. It is to be expected that the higher the degree of increasing returns, the more the optimal tax needs to be higher than the Pigouvian tax. Equation (15) shows that this is indeed the case — the optimal tax increases with a fall in ρ. A lower ρ in this model indicates a smaller elasticity of substitution between different varieties of good Y, or a higher value for diversity. When ρ is low, proportionally more varieties of good Y are produced which means that for each variety, the fixed cost per unit of output is high; or in other words, the degree of increasing returns is high.

2.2. Model 2: A pollution tax and a unit subsidy to the clean increasing returns industry

As in model 1, the government imposes a pollution tax on the X industry and returns the revenue to consumers (or sells pollution permits to the X industry on behalf of consumers as owners of the right to a clean environment). At the same time, the government collects a lump sum tax from consumers to fund a unit subsidy to the clean increasing return industry, the Y industry. This arrangement is more flexible than the case where the government finances its subsidy through pollution tax alone because the total pollution tax in our model can be greater than, equal to or smaller than the total subsidy; or equivalently, the net tax on the consumer (i.e., transfer payment — lump sum tax) may be smaller than, equal to or smaller than zero.

The structure of this model is the same as model 1 except for the following changes. First, due to the lump sum tax on consumers, the representative

consumer's budget constraint (4) becomes:

$$p_x x + \sum_{i=1}^{n} p_{yi} y_i + p_z z = w + R - \tau \tag{16}$$

where τ is a lump sum tax.

Second, the decision problem (6) for the firm producing Y_i is now:

$$\max \pi_{yi} = (p_{yi}(Y_i) + s)y_i - L_{yi} \quad \text{subject to } Y_i = a_{yi}L_{yi} - f_i \tag{17}$$

where s is a unit subsidy.

And the zero profit condition (8) is modified to:

$$\pi_{yi} = (p_{yi}(Y_i) + s)Y_i - L_{y_i} = 0 \tag{18}$$

Finally, there is an additional requirement that the lump sum tax revenue is equal to the total amount of subsidy:

$$\tau L = nsY \tag{19}$$

The equilibrium prices and quantities for model 2 are solved in the same way as that for model 1, and the solutions are presented in Table 1. To obtain the optimal rate of pollution tax, we solve the government's decision problem of maximizing consumer welfare:

$$\max_{\tau, t} U = C_2 (1-\tau)^{\frac{(1-\rho)\beta}{\rho}+1-\beta-\gamma} (1 + (1-\alpha)a_x t)^{-\frac{(1-\rho)\beta}{\rho}-1+\gamma} \tag{20}$$

$$\times (1 + a_x t)^{1-\alpha-\beta+\frac{(1-\rho)\beta}{\rho}} \{\rho\beta(1 + a_x t) + [(1-\alpha-\rho\beta)(1 + a_x t) + \alpha]\tau\}^{\beta}$$

where $C_2 \equiv (\alpha a_x)^{\alpha-\gamma}(\beta a_y)^{\frac{\beta}{\rho}}\beta^{-\beta}[(1-\alpha-\beta)a_z]^{1-\alpha-\beta}L^{\frac{(1-\rho)\beta}{\rho}-\gamma}f^{-\frac{(1-\rho)\beta}{\rho}}(1-\rho)^{\frac{(1-\rho)\beta}{\rho}}$

The optimal pollution tax, lump sum tax and subsidy are presented in Table 1. The optimal pollution tax in model 2 is:

$$t^* = t_{pigou} + \Psi \frac{(1-\rho)(1-\alpha)}{(1-\alpha-\rho\beta)} \tag{21}$$

Clearly, the optimal pollution tax is greater than the Pigouvian tax.

Comparing equation (21) to equation (15), we also see that optimal pollution tax is lower in model 2 than in model 1 provided that consumers have preference over a third good (good Z in our model) as

$$\frac{(1-\rho)(1-\alpha)}{(1-\alpha-\rho\beta)} < 1 \text{ iff } 1-\alpha-\beta > 0 \tag{22}$$

The above result suggests that the equivalence between a pollution tax and a clean good subsidy in a two-sector model (Fuller, 1997) breaks down in a three-sector model such as ours. In a two-sector model, the pollution tax reduces the expenditure on the pollution good (X in our model), and all the reduced expenditure is diverted to buy more clean goods (Y in our model). A subsidy to the Y industry has exactly the same effect — increasing expenditure on Y which is made possible by reducing expenditure on X by the same amount. In contrast, in a three-sector model, a pollution tax still reduces the expenditure on X, but the reduced expenditure is split between buying more good Y and good Z. A subsidy to the Y industry in a sense stops some expenditure reduction from good X "leaking" to good Z. Consequently with both a pollution tax and a subsidy (model 2), the optimal pollution tax is lower than the case where there is a single pollution tax (model 1). Since consumers in reality have preferences over goods other than those targeted by government tax-subsidy policies, the 3-sector model is more relevant and the non-equivalence between a pollution tax and a subsidy to the clean sector should be noted by policy makers.

2.3. Model 3: A pollution tax, a unit subsidy and a lump sum subsidy to the clean increasing returns industry

This model is the same as model 2 except that in addition to a unit subsidy to the Y industry, the government gives a lump sum subsidy to each firm in the Y industry. Accordingly, the decision problem (18) for the firm producing Y_i changes to:

$$\max \pi_{yi} = (p_{yi}(Y_i) + s)Y_i - L_{yi} \text{ subject to } Y_i = a_{yi}L_{yi} - (f_i - g) \quad (23)$$

where g is a lump sum subsidy from the government, which has the effect of lowering the fixed component of the production costs.

We solve for the equilibrium prices and quantities and present the solutions in Table 1. To obtain the optimal pollution tax, we need solve the government's decision problem:

$$\max_{\tau,t,g} U = C_3 (1-\tau)^{1-\beta-\gamma+\frac{(1-\rho)\beta}{\rho}} (f-g)^{-\frac{\beta}{\rho}} [1 + (1-\alpha)a_x t]^{\gamma-1-\frac{(1-\rho)\beta}{\rho}}$$
$$\times (1 + a_x t)^{1-\alpha-\beta+\frac{(1-\rho)\beta}{\rho}} B^\beta \quad (24)$$

where $C_3 = (\alpha a_x)^{\alpha-\gamma}(\beta a_y)^{\frac{\beta}{\rho}}\beta^{-\beta}[(1-\alpha-\beta)a_z]^{1-\alpha-\beta}L^{\frac{(1-\rho)\beta}{\rho}-\gamma}(1-\rho)^{\frac{(1-\rho)\beta}{\rho}}$

Unfortunately we have not been able to obtain an analytical solution for the above problem. Instead we have obtained numerical solutions. These solutions can be found in Tables 2-4 which compare the 3 models for different parameter values. From Tables 2-4, we see that for all the parameter values we have chosen, the optimal pollution tax in model 3 is lower than

that in model 2, and in some cases lower than the Pigouvian tax. This is because the introduction of a lump sum subsidy to the clean increasing return industry moderates the degree of increasing returns by reducing the impact of the fixed costs on production decisions. Both subsidies attract resources to the Y industry and away from the polluting X industry, thereby reducing the need to use taxes to control pollution resulting from production in the X industry.

TABLE 2.

Comparative statics with respect to γ

$(\alpha = 0.4, \beta = 0.3, \rho = 0.8, a_x = a_y = a_z = 1, f = 1, L = 100)$

Solutions	Model 1	Model 2	Model 3
$\gamma = 0$ $t_{pigou} = 0$	$t^* = 0.1169, u^* = 0.3607,$ $y^* = 0.04, n^* = 6.2622$	$t^* = 0.0392, \tau^* = 0.0442,$ $s^* = 0.1597, u^* = 0.3617,$ $y^* = 0.0227, n^* = 5.8226$	$t^* = 0.0142, \tau^* = 0.0657,$ $g^* = 0.177, s^* = 0.1921,$ $u^* = 0.3622,$ $y^* = 0.0326, n^* = 6.8494$
$\gamma = 0.06$ $t_{pigou} = 0.1765$	$t^* = 0.3066, u^* = 0.2912,$ $y^* = 0.04, n^* = 6.6215$	$t^* = 0.1977, \tau^* = 0.0512,$ $s^* = 0.1736, u^* = 0.2930,$ $y^* = 0.0217, n^* = 6.0952$	$t^* = 0.1579, \tau^* = 0.0764,$ $g^* = 0.2141, s^* = 0.2048,$ $u^* = 0.2934,$ $y^* = 0.0316, n^* = 7.4586$
$\gamma = 0.08$ $t_{pigou} = 0.25$	$t^* = 0.3968, u^* = 0.2716$ $y^* = 0.04, n^* = 6.7693$	$t^* = 0.2908, \tau^* = 0.0504,$ $s^* = 0.1674, u^* = 0.2723,$ $y^* = 0.0222, n^* = 6.2619$	$t^* = 0.22, \tau^* = 0.0767,$ $g^* = 0.2045, s^* = 0.2045,$ $u^* = 0.2727,$ $y^* = 0.0323, n^* = 7.6844$
$\gamma = 0.1$ $t_{pigou} = 0.3333$	$t^* = 0.4978, u^* = 0.2535,$ $y^* = 0.04, n^* = 6.92$	$t^* = 0.3866, \tau^* = 0.0488,$ $s^* = 0.1596, u^* = 0.2542,$ $y^* = 0.0227, n^* = 6.4236$	$t^* = 0.3381, \tau^* = 0.0802,$ $g^* = 0.201, s^* = 0.2085,$ $u^* = 0.2545,$ $y^* = 0.0323, n^* = 7.6844$
$\gamma = 0.12$ $t_{pigou} = 0.4286$	$T^* = 0.6002, u^* = 0.2368,$ $y^* = 0.04, n^* = 7.0591$	$t^* = 0.4907, \tau^* = 0.0509,$ $s^* = 0.1624, u^* = 0.2375,$ $y^* = 0.0225, n^* = 6.5581$	$t^* = 0.4363, \tau^* = 0.0856,$ $g^* = 0.2402, s^* = 0.2086,$ $u^* = 0.2378,$ $y^* = 0.0307, n^* = 8.2192$
$\gamma = 0.14$ $t_{pigou} = 0.5385$	$t^* = 0.7131, u^* = 0.2214,$ $y^* = 0.04, n^* = 7.1986$	$t^* = 0.5963, \tau^* = 0.0494,$ $s^* = 0.1556, u^* = 0.2221,$ $y^* = 0.023, n^* = 6.7055$	$t^* = 0.5301, \tau^* = 0.0795,$ $g^* = 0.2008, s^* = 0.1981,$ $u^* = 0.2224,$ $y^* = 0.0319, n^* = 8.0227$

TABLE 3.

Comparative statics with respect to ρ

$(\alpha = 0.4, \beta = 0.3, \gamma = 0.1, a_x = a_y = a_z = 1, f = 1, L = 100); t_{pigou} = 0.3333$

solutions	Model 1	Model 2	Model 3
$\rho = 0.76$	$t^* = 0.5402, U^* = 0.2639,$ $y^* = 0.0317, n^* = 8.3749$	$t^* = 0.4176, \tau^* = 0.0571,$ $s^* = 0.1899, u^* = 0.2649,$ $y^* = 0.0182, n^* = 7.6957$	$t^* = 0.3377, \tau^* = 0.0966,$ $g^* = 0.2595, s^* = 0.2454,$ $u^* = 0.2656,$ $y^* = 0.0236, n^* = 0.7388$
$\rho = 0.78$	$t^* = 0.5185, u^* = 0.2584,$ $y^* = 0.0355, n^* = 7.6441$	$t^* = 0.4033, \tau^* = 0.054,$ $s^* = 0.1596, u^* = 0.2542,$ $y^* = 0.0201, n^* = 7.0546$	$t^* = 0.3419, \tau^* = 0.0877,$ $g^* = 0.2267, s^* = 0.2227,$ $u^* = 0.2597,$ $y^* = 0.0275, n^* = 8.6695$
$\rho = 0.8$	$t^* = 0.4978, u^* = 0.2535,$ $y^* = 0.04, n^* = 6.92$	$t^* = 0.3866, \tau^* = 0.0488,$ $s^* = 0.1596, u^* = 0.2542,$ $y^* = 0.0227, n^* = 6.4236$	$t^* = 0.3381, \tau^* = 0.0802,$ $g^* = 0.201, s^* = 0.2085,$ $u^* = 0.2545,$ $y^* = 0.0323, n^* = 7.6944$
$\rho = 0.82$	$t^* = 0.4593, u^* = 0.2492,$ $y^* = 0.0456, n^* = 6.1777$	$t^* = 0.3778, \tau^* = 0.0464,$ $s^* = 0.1499, u^* = 0.2497,$ $y^* = 0.0253, n^* = 5.7838$	$t^* = 0.3337, \tau^* = 0.0647,$ $g^* = 0.1835, s^* = 0.1693,$ $u^* = 0.25,$ $y^* = 0.0367, n^* = 6.8734$
$\rho = 0.84$	$t^* = 0.4432, u^* = 0.2453,$ $y^* = 0.0525, n^* = 5.4722$	$t^* = 0.3616, \tau^* = 0.0425,$ $s^* = 0.1361, u^* = 0.2458,$ $y^* = 0.0287, n^* = 5.1422$	$t^* = 0.3378, \tau^* = 0.0575,$ $g^* = 0.1635, s^* = 0.1529,$ $u^* = 0.246,$ $y^* = 0.0435, n^* = 6.0161$

3. MARKET OUTCOME, WELFARE IMPLICATIONS AND COMPARATIVE STATICS

From the results in Table 1, we can compare market outcome and consumer welfare under different policy regimes. Comparing model 1 (a single pollution tax) and model 2 (a pollution tax plus a unit subsidy), it is clear that the unit subsidy to the Y industry lowers the relative prices of all varieties of good Y, thereby increasing the consumption of good Y and decreasing the consumption of good X and good Z. However, the unit subsidy also reduces the number of good Y varieties (n). This is because the unit subsidy increases the difference between fixed and variable costs, which encourages the industry to take advantage of a higher degree of increasing returns by increasing the output of each variety.

Comparing model 2 and model 3 (with both a unit and a lump sum subsidy), we find that the main effect of a lump sum subsidy is that it increases the number of varieties and reduces the output of each Y variety. The reason for this is that the lump sum subsidy offsets some of the fixed

TABLE 4.

Comparative statics with respect to f

$(\alpha = 0.4, \beta = 0.3, \gamma = 0.1, \rho = 0.8, a_x = a_y = a_z = 1, L = 100)$; $t_{pigou} = 0.3333$

Solutions	Model 1	Model 2	Model 3
$f = 0.8$	$t^* = 0.4978,\ u^* = 0.2578,$ $y^* = 0.032,\ n^* = 8.65$	$t^* = 0.3863,\ \tau^* = 0.0486,$ $s^* = 0.1591,\ u^* = 0.2585,$ $y^* = 0.0182,\ n^* = 8.0306$	$t^* = 0.3295,\ \tau^* = 0.0758,$ $g^* = 0.1660,\ s^* = 0.1952,$ $u^* = 0.2588,$ $y^* = 0.0252,\ n^* = 9.7091$
$f = 0.9$	$t^* = 0.4978,\ u^* = 0.2555,$ $y^* = 0.036,\ n^* = 7.6889$	$t^* = 0.3865,\ \tau^* = 0.0487,$ $s^* = 0.1594,\ u^* = 0.2562,$ $y^* = 0.0205,\ n^* = 7.1379$	$t^* = 0.2984,\ \tau^* = 0.0802,$ $g^* = 0.1615,\ s^* = 0.2159,$ $u^* = 0.2565,$ $y^* = 0.0301,\ n^* = 8.2292$
$f = 1$	$t^* = 0.4978,\ u^* = 0.2535,$ $y^* = 0.04,\ n^* = 6.92$	$t^* = 0.3866,\ \tau^* = 0.0488,$ $s^* = 0.1596,\ u^* = 0.2542,$ $y^* = 0.0227,\ n^* = 6.4236$	$t^* = 0.3381,\ \tau^* = 0.0802,$ $g^* = 0.201,\ s^* = 0.2085,$ $u^* = 0.2545,$ $y^* = 0.0323,\ n^* = 7.6844$
$f = 1.1$	$t^* = 0.4978,\ u^* = 0.2517,$ $y^* = 0.044,\ n^* = 6.2909$	$t^* = 0.3867,\ \tau^* = 0.0489,$ $s^* = 0.1598,\ u^* = 0.2524,$ $y^* = 0.025,\ n^* = 5.8391$	$t^* = 0.3334,\ \tau^* = 0.0791,$ $g^* = 0.2585,\ s^* = 0.1969,$ $u^* = 0.2527,$ $y^* = 0.0335,\ n^* = 7.2956$
$f = 1.2$	$t^* = 0.4978,\ u^* = 0.25,$ $y^* = 0.048,\ n^* = 5.7666$	$t^* = 0.3868,\ \tau^* = 0.0489,$ $s^* = 0.16,\ u^* = 0.2507,$ $y^* = 0.0272,\ n^* = 5.3527$	$t^* = 0.3244,\ \tau^* = 0.0810,$ $g^* = 0.2890,\ s^* = 0.2013,$ $u^* = 0.2511,$ $y^* = 0.0365,\ n^* = 6.7126$

costs which lowers the degree of increasing returns, therefore more firms in the Y industry and more varieties can be sustained.

Since we have not been able to obtain an analytical solution of the optimal tax and subsidies in model 3, we cannot compare welfare of three models analytically. However, since the government has more choice variables in model 3 than they do in model 2 and model 1, it is to be expected that higher degrees of freedom should lead to higher (or least no lower) levels of consumer welfare. That is, consumer welfare in model 3 should be no lower than that in model 2, which in turn should be no lower than that in model 1. This expectation is confirmed by our numerical results presented in Tables 2-4 which show that for all parameter values we have chosen, consumer utility in model 3 is higher than that in model 2, which is in turn higher than that in model 1. These results suggest that from the perspective of consumer welfare, it may be better that the government use the combination with all three policy instruments — pollution tax, a fixed subsidy and a unit subsidy to the clean industry — instead of relying

on a pollution tax alone for pollution control and clean technology adoption, assuming the combination is feasible and does not impose much more administrative costs.

We also conduct comparative statics analysis based on numerical values to study how a change in the external harm caused by pollution (γ) and in the degree of increasing returns (influenced by ρ, f) affect the optimal levels of pollution tax and subsidies. As expected, Table 2 shows that as increases, the optimal pollution tax rises in all three models, and the optimal pollution tax is the highest in model 1, followed by that in model 2 and then that in model 3. Notably the optimal pollution tax is positive even when the external harm of pollution is zero. The reason for this apparently surprising result is that the "pollution" tax is essentially not a tax on pollution but a tax on the X industry. Since the clean industry has increasing returns, more resources should to be allocated to it than the amount determined by the market. The "pollution tax" serves to shift resources from the X industry to the clean industries.

Essentially, the degree of increasing returns measures the he extent to which the average cost of production changes in response to a change in output (Ng and Zhang, 2007). Clearly, the higher the fixed cost, the higher the degree of increasing returns. Also if consumers have a high preference for product variety (i.e., a lower elasticity of substitution between varieties, ρ), more product varieties and a smaller quantity for each variety would be produced in equilibrium, hence the degree of increasing return would be higher for each product variety. In the presence of increasing returns, there is a tendency for the market to under produce, which provides a case for subsidy. Intuitively, the higher the degree of increasing returns (i.e., a higher f and a lower ρ), the higher the optimal subsidy should be. As shown in Tables 3-4, in model 2, the optimal unit subsidy falls with an increase in ρ, and increases with an increase in f. However, in model 3 with both a lump sum subsidy and a unit subsidy, it is not clear how the optimal subsidies would react to a change in ρ and f. While both subsidies draw resources into the Y industry, they have opposite effects on the number of product varieties, and the effects seem to be nonlinear. A lump sum subsidy tends to increase product variety whereas a unit subsidy decreases it. In our numerical calculations shown in Table 3-4, both the optimal unit and lump sum subsidies fall as ρ increases. However as f changes, the optimal unit and lump sum subsidies does not seem to follow a clear pattern.

4. CONCLUSION

We have presented in this paper a set of three simple models to study the optimal tax-subsidies combination in an economy characterised by two

deviations from the perfect competition model — negative externality from pollution by the "dirty" industry, and increasing returns in the "clean" industry. Our main conclusions are threefold. First, the optimal single pollution tax is higher than the Pigouvian level. This is due to the presence of increasing returns in the non-polluting sector, thus it is the combined effect of Pigou tax and subsidy to increasing return industries shown in Ng and Zhang (2007). Secondly, a combination of pollution tax and quantity subsidy increases consumer welfare at a lower level of pollution tax. The addition of subsidy in the policy package increases welfare because the two issues of pollution and increasing returns are addressed using two separate instruments instead of one. The level of pollution tax is accordingly lower since the role of dealing with increasing returns is taken over by the subsidy. Thirdly, the optimal pollution tax can be further lowered and consumer welfare further increased if the quantity subsidy is supplemented by a lump sum subsidy. The introduction of lump sum subsidy raises welfare because in our model, increasing returns is driven by the presence of fixed cost. A lump sum subsidy lowers fixed costs, enabling more product varieties in equilibrium.

We have also shown that the optimal pollution tax increases with the marginal harm of pollution. Moreover in the model with a unit subsidy but no lump sum subsidy to the clean industry, the optimal unit subsidy to the clean industry increases with the degree of increasing returns in the clean industry. However, if both a unit subsidy and a lump sum subsidy are included, their optimal rates do not seem to follow a clear pattern due to the two subsidies' different impact on the number of product varieties.

The results of this paper are derived from models with specific functional forms. This has been necessary for tractability reasons. Once the great simplifying assumption of constant returns is dropped, one needs to rely on specific functional forms to analyze the more complex world with increasing returns (Krugman, 1983). That said, the results of this paper seem to apply in more general settings. For example, other things equal, a government with more policy levers can potentially achieve higher gains, thus one should not be surprised to observe that both lump sum and unit subsidies are given out to support clean technologies by governments. Slightly less obvious but still intuitively compelling is the result that the optimal pollution tax can be higher or lower than the Pigovian tax in the same economy depending on what other policies are in place at the same time. Thus the policy decision in relation to a pollution tax should not be made in isolation; it should take into account not only the harm caused by the pollution that the tax is designed to control, but also other policies that affect the resources drawn into/out of the relevant industries.

APPENDIX: DERIVATION OF THE PIGOUVIAN TAX

The Pigouvian tax is the optimal tax in a first best world. To derive it in our model setting, we need to remove the second best complication of increasing returns. That is, instead of having a monopolistically competitive Y industry with differentiated products, we need to have a perfectly competitive Y industry with constant return to scale technologies. This means the following changes are made to model 1. The consumer's decision problem becomes:

$$\max U = x^\alpha y^\beta z^{1-\alpha-\beta} X^{-\gamma} \quad \text{subject to } p_x x + p_y y + p_z z = w + R \quad \text{(A.1)}$$

The Y producer's decision problem is

$$\max \pi_y = p_y Y - L_y, \text{ subject to } Y = a_y L_y \quad \text{(A.2)}$$

The clearing conditions for good Y and labor change accordingly to:

Market for good Y : $\qquad\qquad\qquad\qquad Y = Ly$ (A.3)

Market for labor : $\qquad\qquad L_x + L_y + L_z = L$ (A.4)

The other equations of the model are the same as those in model 1.

Solving system of equations of this model, we get the equilibrium utility. The Pigouvian tax is determined by solving the government's decision problem:

$$\max_t U = a_x^{\alpha-\gamma} a_y^\beta a_z^{1-\alpha-\beta} \alpha^{\alpha-\gamma} \beta^\beta (1-\alpha-\beta)^{1-\alpha-\beta}$$
$$\times L^{-\gamma} [1+(1-\alpha)a_x t]^{\gamma-1} [1+a_x t]^{1-\alpha} \quad \text{(A.5)}$$

The Pigouvian tax is

$$t_{pigou} = \frac{\gamma}{a_x(\alpha-\gamma)} \quad \text{(A.6)}$$

REFERENCES

Arrow, K., Y.-K. Ng, and X. Yang (Eds.), 1998. Increasing Returns and Economic Analysis. London: Macmillan.

Buchanan, J. M. and Y. J. Yoon, 1994. The Return to increasing returns. Ann Arbor: University of Michigan Press.

Bovenberg, A. L. and L. H. Goulder, 1996. Optimal Environmental Taxation in the Presence of Other Taxes: General-Equilibrium Analyses. *American Economic Review* **86(4)**, 985-1000.

Bovenberg, A. L. and R. A. Mooij, 1994. Environmental Levies and Distortionary Taxation. *The American Economic Review* **84(4)**, 1085-1089.

Buchanan, J. M., 1969. External Diseconomies, Corrective Taxes, and Market Structure. *American Economic Review* **59(1)**, 174-177.

Cheng, W. and X. Yang, 2004. Inframarginal Analysis of Division of Labor: A Survey. *Journal of Economic Behavior and Organization* **55(2)**, 137-174.

Dixit, A. and J. Stiglitz, 1977. Monopolistic Competition and Optimum Product Diversity. *American Economic Review* **67(3)**, 297-308.

Fischer, C. and R. G. Newell, 2008. Environmental and technology policies for climate mitigation. *Journal of Environmental Economics and Management* **55(2)**, 142-162. doi: 10.1016/j.jeem.2007.11.001

Fullerton, D., 1997. Environmental Levies and Distortionary Taxation: Comment. *American Economic Review* **87(1)**, 245-251.

Heinzel, C. and R. Winkler, 2011. Distorted Time Preferences and Time-to-Build in the Transition to a Low-Carbon Energy Industry. *Environmental and Resource Economics* **49(2)**, 217-241.

Jaffe, A. B., R. G. Newell, and R. N. Stavins, 2005. A Tale of Two Market Failures: Technology and Environmental Policy. *Ecological Economics* **54(2-3)**, 164-174.

Krugman, P., 1981. Intraindustry Specialization and the Gains from Trade. *Journal of Political Economy* **89(5)**, 959-973.

Krugman, P., 1983. New Theories of Trade Among Industrial Countries. *The American Economic Review* **73(2)**, 343-347.

Ng, Y.-K. and D. Zhang, 2007. Average-Cost Pricing, Increasing Returns, and Optimal Output: Comparing Home and Market Production. *Journal of Economics (Zeitschrift fur Nationalokonomie)* **90(2)**, 167-192.

Ng, Y.-K., 2000. Efficiency, Equality, and Public Policy: With a Case for Higher Public Spending. London: Macmillan.

Oates, W. E. and D. L. Strassmann, 1984. Effluent Fees and Market Structure. *Journal of Public Economics* **24(1)**, 29-46.

Pigou, A. C., 1932. The Economics of Welfare. Library of Economics and Liberty. http://www.econlib.org/library/NPDBooks/Pigou/pgEW.html.

Sartzetakis, E. S. and P. Tsigaris, 2005. Environmental Externalities in the Presence of Network Effects: Adoption of Low Emission Technologies in the Automobile Market. *Journal of Regulatory Economics* **28(3)**, 309-326.

Spence, M., 1976. Product Selection, Fixed Costs, and Monopolistic Competition. *Review of Economic Studies* **43(2)**, 217-235.

Vetter, H., 2009. Product Variety, Scale Economies, and Environmental Taxes. *The B.E. Journal of Theoretical Economics* **9(1(Topics))**, Art. 18.