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Factor Timing with Investor Sentiment

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This paper studies the relation between investor sentiment and factor timing portfolio within the factor zoo. We consider both the mean-variance and constant relative risk aversion (CRRA) investor utility objectives, and use a nonparametric approach to characterize the dependence of factor timing portfolio weight on investor sentiment. We focus on a sample of 55 characteristics-based factors and extract the largest sparse principal components to invest. Empirically, we find that the investor sentiment is a good guidance to factor timing from July 1965 to December 2019, and the result is also robust to out-of-sample evaluation and transaction cost.

Key Words: Factor timing; Investor sentiment; Sparse PCA; Nonparametric regression.
JEL Classification Numbers: C14, C55, G11, G12, G41.

1. INTRODUCTION

With the discovery of a large number of characteristics-based factors as the sources of risk premium in the cross-section of stocks market (Fama and French 1992,1993), the investors' investing space is expanded and now they can hold a portfolio of various factors instead of asset classes. Moreover, the performance of characteristics-based factors is linked with a wide variety of predictors, from the market fundamental (Garcia-Feijóo et al., 2015) and investor sentiment indices (Baker and Wurgler, 2006) to macroeconomic indicators (Bender et al., 2018). Taking into account the expanding of factor space and the possibility of predicting factor returns, the investor's investing problem shifts from market timing to factor timing, which combines the ideas of factor investing and market timing, and engage in allocating wealth

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in different factors such as market, size, value with changes of predictors (Shiller 1981, Fama and French 1988, Aït-Sahalia and Brandt, 2001).

In this paper, we analyze how investors could optimally time the factor zoo with investor sentiment in a non-linear fashion. Specifically, we study investors with both mean-variance and constant relative risk aversion (CRRA) preferences with different risk aversions. Technically, examining how optimal factor timing portfolio depends on predictors for a given preference is a tough work because only in the few cases where we have an explicit optimal portfolio choice formula. We hence use a non-parametric approach to characterize the problem and to avoid potential misspecification (Aït-Sahalia and Brandt, 2001). We focus on a large sample of 55 characteristics-based factors or factor zoo (Harvey et al., 2016, Hou et al., 2020), and handle the multi-dimensional challenge by using the largest sparse principal components to invest. Empirically, we find that the investor sentiment is a good guidance to factor timing from July 1965 to December 2019, and the result is also robust to out-of-sample evaluation and transaction cost.

We have three main contributions. First, our research contributes to the literature that investor sentiment can have significant effects on stock returns. For example, Baker and Wurgler (2006, 2007) and Huang et al. (2015) provide reliable indices to measure sentiment, a complex concept not easy to judge, and suggest that these sentiment indices could be statistically and economically significant predictors to predict the aggregate stock market returns. Stambaugh, Yu and Yuan (2012) explore the role of investor sentiment for the cross-sectional stock returns. But in the literature, investor sentiment is only as the predictor to these characteristics-based factors individually.

We investigate whether investor sentiment can have significant effects on the large set of factors jointly. The result of optimal timing portfolio balances the first two or even higher conditional moments of factor returns and market-wide investor sentiment indeed provides information to timing the factor zoo for investors with certain preference.

Second, our research is also related to the large set of characteristics-based factors, or "factor zoo" called by Cochrane (2011). Many studies have found a large number of characteristics-based factors that help predict the cross-sectional risk premium of stock returns. To avoid spurious overfitting and redundancy, Kozak et al. (2020) tackle "the multi-dimensional challenge" (Cochrane, 2011) of factor zoo by extracting the high-variance component of candidate factor returns through PCA and concentrating on a relatively small number of high-eigenvalue principal components (PCs) that are sufficient to explain most of the cross-sectional variation in stock returns. Based on the same assumption, Haddad et al. (2020) also use PCA to handle the dimension reduction problem and find the largest few

PCs of anomalies are strongly predictable in the optimal factor timing portfolio. But, the PCs are linear combinations of all underlying variables and usually hard to interpret. This poses a challenge for understanding the underlying economic mechanism (Rapach et al., 2019, Pelger et al., 2020, Zou et al., 2006).

To improve interpretability, we use the sparse PCA method to extract the main information of the characteristics-based 55 factors (Haddad et al., 2020) to few sparse PCs, imposing the cardinality restriction on each weight vector via the approach of expectation-maximization (EM) algorithm (Sigg and Buhmann, 2008). We restrict the number of non-zero elements on each weight vector K=11, containing 20% of the 55 factors respectively and extract the first five sparse PCs as the asset space. These sparse components jointly explain more than half of the total variance in returns and the introduction of substantive sparsity facilitates to intuitively interpret the sparse PCs. According to the components of their corresponding sparse weight vectors, we label the first five sparse PC as "trading friction", "momentum", "value", "profitability" and "growth" respectively.

Third, our research is also related to optimal portfolio design. How optimal factor timing portfolio responds to investor sentiment is not easy to address. This is true only in the few special cases where there is an explicit formula of optimal choice, such as for mean-variance utility where the optimal choice is proportional to the ratio of the conditional mean to the conditional variance of returns. Unfortunately, for most widely used utility functions, take the CRRA utility for example, optimal portfolio weights are complex functions of higher order conditional moments of returns. Moreover, there is not an explicit relationship between the broad set of anomalies and investor sentiment. Stambaugh et al. (2012) examine that the long-short strategy of each anomaly is stronger following high levels of sentiment and the relationship between anomaly and sentiment is not a simple function.

In this paper, we adopt the nonparametric approach of Brandt (1999) and Aït-Sahalia and Brandt (2001), which allows us to express optimal factor timing portfolio directly as nonlinear function of investor sentiment, avoiding specifying the conditional function and introducing the potential misspecification. The nonparametric regression method describes the flexible and smooth pattern of optimal factor timing portfolio with different levels of sentiment.

Empirically, we find that the sparse PCs of the large set characteristics-based factors do facilitate us interpret the factor timing portfolio choice, and the optimal allocation of the sparse PCs, such as "momentum", "value" and "profitability" factors, is more easily and monotonously than their corresponding factors in the Fama-French-Carhart 6 Factors. Our result also emphasizes the effect of investor sentiment on the optimal factor timing portfolio. Quantitively, the gap of certainty equivalent rate (CER) of portfolio between factor timing and static factor investing is positive, and generally broader with the number of factors invested increasing.

The out-of-sample result of factor timing with investor sentiment also performs well. For the mean-variance investor with risk aversion parameter $\gamma = 5$ and factor timing in the first five sparse PCs and market factor with BW investor sentiment index, the CER and Sharpe ratio of the annualized excess return of out-of-sample predictive optimal factor timing portfolio are 8.32% and 0.91 via a split window analysis. Considering about transaction cost, the CER and Sharpe ratio also maintain 7.12% and 0.85. The predictive estimation is still robust when including many choices, such as the investor's preferences, how investor's risk-averse is, how many factors included, replacing the investor sentiment index and the window analysis types to predict the out-of-sample performance.

The paper proceeds as follows. Section 2 lays out the conditional portfolio choice problem and discusses the use of investor sentiment index as conditional information. This section also introduces the nonparametric regression technique. Section 3 solves the investor's factor timing problem. This section first presents the data about investor sentiment and factor sets invested and then presents the main empirical results of factor timing in the cross-sectional returns of two sets of factors, Fama-French-Carhart 6 Factors and Factor Timing 55 Factors. The section also presents the out-of-sample results of predictive optimal factor timing choice in the cases with and without transaction cost. Section 4 shows the relationship between the mean-variance investor's portfolio choice of factor timing and (stochastic discount factor) SDF. The last section concludes.

2. FACTOR TIMING

In our context, factor timing is the response of the optimal allocation of factors in cross-sectional stock returns to changes in predictors. We start factor timing by defining the investor's maximization problem. In Section 2.1, we introduce the investor's portfolio problem. Section 2.2 argues why investor sentiment is good candidate for factor timing. In Section 2.3 we introduce the nonparametric regression method to solve the investor's problem.

2.1. Investor's Problem

Consider a single-period investor who maximizes the conditional expected utility of the next period's wealth W_{t+1} by trading in N risky assets and a risk-free asset, subject to the usual budget constraint:

$$\max_{\alpha_t \in \mathbb{R}} E[u(W_{t+1})|Z_t] \tag{1}$$
$$\text{subject to} W_{t+1} = W_t[R_{f,t+1} + \alpha_t(R_{t+1} - R_{f,t+1})] = W_t[R_{f,t+1} + \alpha_t R^e_{t+1}]$$

where α_t represents a vector of the proportion of wealth invested in the vector of risky assets with gross return R_{t+1}, $1 - \alpha_t$ is the proportion invested in the risk-free asset with return $R_{f,t+1}$, R^e_{t+1} is the vector of excess return on the N risky assets and W_t is the investor's wealth at time t. The function $u(\cdot)$ measures the investor's utility of next period's wealth W_{t+1}. The purpose of our study is to investigate whether state variable, investor sentiment index in this paper, represented here by Z_t, constitutes relevant information on factor timing. We denote the mapping from the state variable to the portfolio weight α_t as $\alpha_t \equiv \alpha(Z_t)$, and refer to it as the investor's portfolio allocation, choice, policy, weight, or rule, as Aït-Sahalia and Brandt (2001) do.

Furthermore, the realistic portfolio choice may be subject to the portfolio weight constraints $a \leq c(\alpha_t) \leq b$, such as borrowing or short-sale constraints, which prohibits unrealistic leveraging and short selling. However, we do not obey the constraints on portfolio weight for two reasons below. First, we consider the investor's conditional allocations of characteristics-based factors, which has already been constructed by the long-short portfolio of stocks. It is unnecessary to meet the weight constraints when the underlying assets, the long-short factor portfolios associated with characteristics, don't satisfy the constraints. Second, without the weight constraints, we can allocate the wealth in a broader asset space and research the dependence between factor timing and investor sentiment more clearly.

Obviously, different investor preferences have different relation between the portfolio choice and the conditional individual moments of the returns (Aït-Sahalia and Brandt, 2001). To see how the investor's problem conditional on the state variable with different preferences, we consider two parameterizations of the objective function $u(\cdot)$, the mean-variance and CRRA preferences, which are standard expected utility objectives.

The standard objective function of investor with mean-variance preference is:

$$u(W_{t+1}) = E[W_{t+1}] - \frac{\gamma}{2} Var[W_{t+1}] \tag{2}$$

where $\gamma \geq 0$ measures the investor's risk aversion.

The mean-variance investor's conditional problem is:

$$E[u(W_{t+1})|Z_t] = E[W_{t+1}|Z_t] - \frac{\gamma}{2} Var[W_{t+1}|Z_t] \tag{3}$$

If the assets invested includes a risk-free rate, the investor's portfolio rule on risky assets is:

$$\alpha_t = \frac{1}{\gamma W_t} \frac{E[R^e_{t+1}|Z_t]}{Var[R^e_{t+1}|Z_t]} \tag{4}$$

For the mean-variance preference, the appealing feature is that the optimal portfolio choice depends analytically only on the first two moments of returns. In other words, the mean-variance portfolio weights on risky assets are proportional to the conditional expected excess return of them and reciprocal to the conditional variance of their excess return. The vector of proportions allocated on risky assets is also the component of mean variance efficient (MVE) portfolio and reciprocal to the investor's risk aversion γ.The mean-variance investor allocates his wealth between the MVE portfolio and risk-free asset according to his risk aversion, which presents the principle of two-fund separation.

However, the simplest specification also illustrates the difficulties of the portfolio rules. The investors have to balance the first and second conditional moments of the asset returns according to the state variable and satisfy the optimal portfolio choice. Take the mean-variance investor for example, he has to estimate the ratio of the first two conditional moments by the predictor's information and there is no explicit function form of portfolio policy $\alpha(Z_t)$, which may be a highly nonlinear and even nonmonotonic function. If we simply parameterize the function form of portfolio policy, there will be the introduction of additional noise and potential misspecifications.

We also consider an investor with preference of CRRA or power utility:

$$v(W_{t+1}) = \begin{cases} \frac{W_{t+1}^{1-\gamma}}{1-\gamma}, & if\ \gamma > 1 \\ \ln W_{t+1}, & if\ \gamma = 1 \end{cases} \tag{5}$$

where γ represents the coefficient of relative risk aversion $\frac{W(\partial^2 v(W)/\partial W^2)}{\partial v(W)/\partial W}$.

The CRRA preference is the most attractive objective function largely because the portfolio weights are independent of wealth level. However, there is no closed-form solution to the CRRA investor's portfolio choice problem.

2.2. Investor Sentiment and its Information

The investor sentiment is hard to measure directly and there initially are different methods to measure it among the relevant literature. An index of buy-sell imbalance (BSI) of retail investors is built by Kumar and Lee (2006) to indicate the existence of a systematic trading activities of retail investors and reflects the investor sentiment. Da et al. (2015) construct the daily Financial and Economic Attitudes Revealed by Search (FEARS) index based on the Google search engine data as a new measure of investor sentiment and show its predictive power for short-term return reversals. Bake and Wurgler (2006) employ a number of sentiment proxies used by prior work and estimate the investor sentiment as the first PC of these proxies. The monthly investor sentiment index extracts the sentiment-related component of sentiment proxies and removes the non-sentiment-related component, and presents an excellent predictive ability of the cross-section of stock returns.

We prefer the investor sentiment index of the Baker and Wurgler (2006) (BW index hereafter) for the reasons below: first, we investigate the factor timing problem based on the representative investor of the whole market, spanning several decades of the stock market at the frequency of month. The BW index available matches the frequency of the main characteristics-based factors and the long period of market history. Second, the index is a well-established measure of investor sentiment and the predictive power of cross-sectional stock returns has been confirmed by a large amount of literature. Specifically, Stambaugh et al. (2014) refuse the possibility that investor sentiment could be a spurious regressor in different market anomalies in a vast simulation test, which strongly supports the point that the BW index possesses information of stock returns.

Brown and Cliff (2005) show the empirical evidence that there is a negative relation between sentiment and following long-term aggregate return and argue that noise traders may give rise to the strong and persistent mispricing. Consistent with noise trader models, Kumar and Lee (2006) point out that retail investors who are commonly treated as noise traders tend to systematic trading, which explains return comovements for stocks with high retail concentration, especially in the group of stocks costly to arbitrage. Moreover, Baker and Wurgler (2006) confirm the theoretical hypothesis that the wave of investor sentiment has larger effects on stock whose valuations are difficult to value and hardest to arbitrage, which supports the conclusion of Kumar and Lee (2006) and extends the investor's problem of optimal allocation into a conditional one. Stambaugh et al. (2012) underline that investor sentiment has a stronger effect on the short leg of a broad set of anomalies in cross-sectional stock returns for the reason of short-sale impediment. The empirical evidence ensures that investor sentiment contains information for future cross-sectional returns.

However, the effect of investor sentiment on volatility has not been explored as extensively as expected returns. Huang et al. (2015) display that high investor sentiment predict a high aggregate stock market volatility and thus high market risk. The empirical result disagrees with their hypothesis that the market volatility risk should explain the stock return predictability of investor sentiment and seems to support that high noise trading leads to excessive volatility. The effect of investor sentiment on volatility of characteristics-based factors has little been researched.

While the influence of sentiment on both expected returns and volatility has been studies in empirical researches, the conditional portfolio optimization for sentiment has been little investigated. Yu and Yuan (2011) demonstrate empirically that investor sentiment shifts the mean-variance trade-off, finding that the stock market's aggregate expected excess return is positively related to the conditional variance in low-sentiment periods but nearly flat in high-sentiment periods. Fu et al. (2015) extend Markowitz mean-variance theory to include sentiment and provide a theoretical framework to examine the impacts of investor sentiment on the mean-variance tradeoff, concluding that sentiment influences the mean-variance relation along the efficient frontier and a rational mean-variance investor who neglects the effect of investor sentiment may end up allocating a sub-optimal portfolio.

Summing up, investor sentiment has been shown to provide information on future return in the literature above. These studies try to exploit the information mainly by predicting either aggregate and cross-sectional returns or aggregate volatility. However, the problem of how an investor with a certain preference allocates his wealth into a large set characteristics-based factors, or factor zoo, according to the information investor sentiment provided is neglected. Our paper aims to fill this gap by investigating the investor's factor timing portfolio rules by a nonparametric regression method.

2.3. Nonparametric Technique

We solve the conditional optimal factor timing allocations by using the nonparametric estimation technique of Brandt (1999) and Aït-Sahalia and Brandt (2001). The nonparametric approach is developed from the unconditional method of moments approach and the unconditional estimate is the benchmark in our study, so we introduce the nonparametric method for conditional portfolio choice starting by the unconditional problem.

To maximize the expected utility, the first order condition of the investor with a certain preference is:

$$\alpha : E[m(\alpha)] \equiv E\left[\frac{du(W_{t+1})}{dW_{t+1}} R^e_{t+1}\right] = 0 \tag{6}$$

We abandon the subscript t of portfolio weight α to emphasize the unconditional portfolio weight. We solve the problem by applying Hansen's (1982) standard generalized method of moments (GMM) to the unconditional first order condition.

When adjusting his portfolio choice with the state variable Z_t, the investor's problem is represented in expression (1), and the first order condition for the problem is:

$$\alpha : E[m_{t+1}(\alpha_t)|Z_t] \equiv E\left[\frac{du(W_{t+1})}{dW_{t+1}}(R_{t+1} - R_{f,t+1})|Z_t\right] = 0 \qquad (7)$$

Brandt (1999) shows how to solve the optimal portfolio policy $\alpha(Z_t)$ without further assumptions about the conditional return distribution or functional form of the portfolio rule. Specifically, first define a weighting function $k(Z_t; \overline{z}, h)$ to measure how similar a sample observation Z_t is to the reference point $\overline{z}$. The weighting function gives more weight to the sample closer to $\overline{z}$ and is also called kernel function. Then, to estimate the portfolio policy in state $\overline{z}$, put the sample observations with different weights, weighting by the kernel function, and the moment conditions of the portfolio policy at the reference point $\overline{z}$ is

$$\alpha(\overline{z}) : \frac{1}{Th}\sum_{t=1}^{T} m_{t+1}(\alpha(\overline{z}))k(Z_t; \overline{z}, h)\frac{1}{\sum_{t=1}^{T} k(Z_t; \overline{z}, h)} = 0 \qquad (8)$$

In practice, we use a normal kernel function $k(Z_t; \overline{z}, h) = (2\pi)^{-1/2}\exp(-d_t^2/2)$, with $d_t = (Z_t - \overline{z})/h$, where the parameter h is the bandwidth. We apply a standard GMM to (8), and the solution $\alpha(\overline{z})$ is the optimal allocation at a given reference state $\overline{z}$.

The choice of the bandwidth h is crucial. The bandwidth h scales the difference between the sample observation Z_t and the reference state $\overline{z}$, as discussed above, and also can be interpreted as a window width that controls the number of sample points used to estimate the GMM solution in (8). On the one hand, A larger h implies averaging across more sample points and weighting downward less, thus reducing the variance but increasing the bias; On the other hand, a smaller h means sample observation points in the GMM less and weighted downward more, thus reducing the bias but increasing the variance (Ghysels et al., 2008).

The best choose of the bandwidth is the tradeoff between variance and bias. Theoretically, the best bandwidth minimizing the mean squared error (MSE) of the estimates is the form $h = \lambda\sigma(Z_t)T^{-1/(K+4)}$, where $\sigma(Z_t)$ is the unconditional standard deviation of the sample observation Z_t, T is the sample size, K is the dimension of Z, equaling to 1 in our problem, and λ is a constant, and the problem how to choose best bandwidth h turns to λ (Brandt, 1999). In practice, Brandt (1999) prefers "leave-one-out" cross-validation approach to identify the best λ. Although the optimal policy observation α_t does not exist, Brandt (1999) choose λ by minimize the sum of squared predictive errors through an estimate matrix, which may introduce additional errors. Ghysels et al. (2008) directly choose λ with three different values for daily, weekly, and monthly frequencies of samples respectively and examine that the estimates are not insensitive to reasonable variations of bandwidths by comparing portfolio choice of cross-validation bandwidths with the ones of bandwidths 25 percent smaller or 25 percent larger than the cross-validation bandwidths, though the degree of nonlinearity depends on the bandwidths.

There is no good guidance in literature for bandwidth selection with unobservable estimates, as is the case of Brandt (1999), Ghysels (2008) and our problem. We directly choose the standard deviation of the predictor as the best bandwidth to regress the nonparametric estimate and we assume it can put different sample observations with suitable weights in the nonparametric regression.

3. EMPIRICAL RESULTS

In this section we test how investors with different preferences allocate their wealth in the characteristics-based factors with changes of investor sentiment, or factor timing in the factor zoo with investor sentiment, using the nonparametric methods described in Section 2. We first describe the sentiment indices that we use to identify the investor sentiment and two sets of characteristics-based factors, Fama-French-Carhart 6 Factors and Factor Timing 55 Factors, where investors allocate their wealth, and we present the unconditional and conditional optimal factor timing allocations of investors with different preferences respectively. We also present the robust test of out-of-sample factor timing predictive estimation in Section 3.4 and show the result of predictive estimation in case with transaction costs in Section 3.5.

3.1. Sentiment Indices and Factors

We measure investor sentiment using the monthly composite sentiment series constructed by Baker and Wurgler (2006) which is available from Jeffrey Wurgler's website who provides the updated data[1]. We update the data of proxies based on the data provided on Jeffrey Wurgler's website and expand the monthly sentiment index to December 2019 by the method of Baker and Wurgler (2006) to match the time period of characteristics-based factors. The BW sentiment index spans over 54 years, from July 1965 to December 2019 and has been widely used in many studies about investor sentiment such as Baker and Wurgler (2006), Yu and Yuan (2011), Stambaugh, Yu and Yuan (2012, 2014), and others. Baker and Wurgler constructed their composite index by taking the first PC of six proxies related to investor sentiment and these proxies are closed-end fund discount, NYSE share turnover, the number and the average first-day returns on

[1] The web page is http://people.stern.nyu.edu/jwurgler/.

initial public offerings, the equity share in new issues, and the dividend premium[2].

Huang et.al (2015) argue that the first PC of the six sentiment proxies may potentially contain common approximation errors that are not relevant for forecasting returns and apply the partial least squares (PLS) approach to extract the most relevant investor sentiment component and filter out the irrelevant component in the sentiment proxies. The aligned sentiment index (PLS index hereafter) presents a better performance in predicting the aggregate stock market returns and the cross-sectional returns than its counterparts, BW index (Huang et.al, 2015). We also use the PLS index to measure the investor sentiment to test the robustness of the empirical results about factor timing.

Though the two sentiment indices, BW index and PLS index, are constructed with different methods, they are highly correlated with each other and the correlation coefficient is 0.66. The two investor sentiment indices are plotted in Figure 1 at the monthly frequency and these indices appears to capture most anecdotal accounts of fluctuations in sentiment as described by Baker and Wurgler (2006) and Huang et al. (2015).

Investor sentiment was low in the first few years of the sample after the 1961 crash of growth stocks. It reached to a peak in the 1968 and 1969 during the electronics bubble. Sentiment fell again to a trough during the 1973 to 1974 stock market crash and was at a low level in the early 1970s bear market. It picked up and rose to a peak in the period of the early 1980s biotech bubble. Sentiment dropped in the late 1980s but rose again in the early 1990s. It again reached another peak in the late 1990s bubble of technology stocks. Sentiment was low during the 2008 to 2009 subprime crisis but rises stably in 2010s, and relatively flat in the following period.

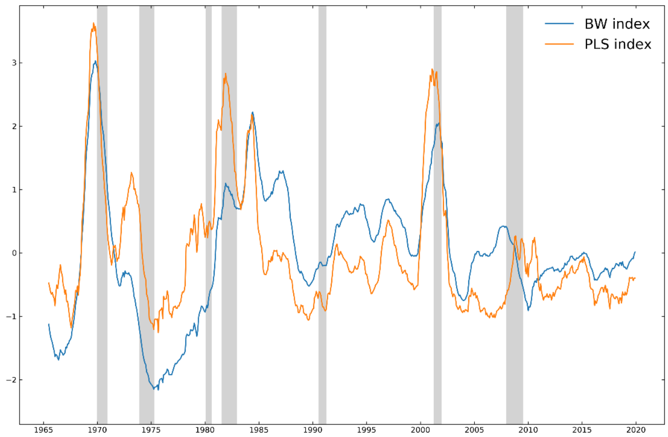
In our empirical analysis, we investigate the factor timing problem to two sets of characteristics-based factors from July 1965 to December 2019. The first set is the 5 factors from Fama and French (2015, 2016), adding the momentum factor from Carhart (1997) (henceforth Fama-French-Carhart 6 Factors or FF6F for short)[3]. The second large set is based on the 55 equity characteristics underlying common "anomalies" in the literature, constructed by Kozak et al. (2020), Giglio et al. (2020) and is applied to factor timing by Haddad et al. (2020)[4] (henceforth Factor Timing 55 Factors, or FT55F for short). The data set includes the stocks of CRSP and

[2]Unlike in Baker and Wurgler (2006), NYSE share turnover has been dropped as one of the six sentiment indicators and the sentiment index provided on Jeffrey Wurgler's website is based on five other indicators. We follow the Baker and Wurgler's change in indicators and use the five other indicators to composite different sentiment indices with the method of Baker and Wurgler (2006) and Huang et.al (2015).

[3]All Fama and French factors are downloaded from https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[4]Factor Timing 55 Factors data obtained from https://www.serhiykozak.com/data

FIG. 1. The investor sentiment indices from July 1965 to December 2019.



The blue line depicts the Baker and Wurgler (2006) investor sentiment index, BW index, which is the first principal component of six underlying proxies of investor sentiment: the closed-end fund discount, NYSE share turnover, the number and the average first-day returns on initial public offerings, the equity share in new issues, and the dividend premium. The orange line depicts Huang et al. (2014) investor sentiment index, PLS index, aligned by applying the partial least squares method to the same six proxies of investor sentiment. Both estimated investor sentiment indices are standardized to have zero mean and unit variance. The vertical bars correspond to the periods of NBER-dated recessions.

COMPUSTAT and sorts them into 10 value-weighted portfolios for each of the 55 characteristics studied in Kozak et al. (2020). Portfolios include all NYSE, AMSE, and NASDAQ firms and the breakpoint use only NYSE firms as in Fama and French (2016). We construct the long-short anomalies as spread between each anomaly's return on portfolio 10 minus the return on portfolio 1. We fill the empty values of factors with zero and use the sparse PCA method to reduce the set of factors to a few dominant easy-to-interpretation components, its largest PCs. The investment horizon is one month in all of the investor's factor timing problem.

Table 1 describes the Fama-French-Carhart 6 Factors and two investor sentiment indices, BW index and PLS index. Panel A of the Table 1 presents the univariable descriptive statistics of the annualized returns and sentiment indices. Panels B shows the autocorrelations of the factors and sentiment indices with a lag of 1, 3, 6, and 12 months. Panel C shows the pairwise correlations of the sentiment indices and their corresponding lag with the excess return of Fama-French-Carhart 6 Factors.

TABLE 1.

Description of Fama-French-Carhart 6 Factors returns and sentiment indices

Panel A: Descriptive Statistics									
	Fama-French-Carhart 6 Factors							Sentiment index	
	Mkt-RF	SMB	HML	RMW	CMA	Mom	RF	BW	PLS
Mean	0.064	0.028	0.037	0.032	0.034	0.079	0.046	0.000	0.000
Std	0.154	0.105	0.099	0.076	0.070	0.147	0.009	1.000	1.000
Skew	−0.540	0.362	0.191	−0.357	0.308	−1.293	0.606	0.224	1.585
Kurtosis	1.876	2.996	1.827	12.184	1.595	10.082	0.613	0.377	2.021
Median	0.110	0.014	0.031	0.029	0.014	0.091	0.048	−0.076	−0.303
Max	1.932	2.166	1.512	1.606	1.147	2.203	0.162	3.031	3.627
Min	−2.789	−1.783	−1.326	−2.218	−0.823	−4.127	0.000	−2.162	−1.257
Panel B: Autocorrelations									
ρ_1	0.063	0.059	0.164	0.147	0.123	0.050	0.972	0.996	0.990
ρ_3	0.016	−0.055	0.034	−0.043	0.060	0.011	0.940	0.972	0.946
ρ_6	−0.054	0.034	0.017	0.065	0.061	0.044	0.905	0.907	0.833
ρ_{12}	0.022	0.141	0.030	−0.051	−0.027	0.084	0.834	0.703	0.515
Panel C: Correlations									
Mkt-RF	1.00								
SMB	0.28	1.00							
HML	−0.26	−0.07	1.00						
RMW	−0.22	−0.35	0.08	1.00					
CMA	−0.39	−0.11	0.70	−0.02	1.00				
Mom	−0.15	−0.04	−0.19	0.10	−0.02	1.00			
RF	−0.08	−0.04	0.08	0.00	0.07	0.06	1.00		
BW	−0.06	−0.13	0.06	0.14	0.09	0.00	0.15	1.00	
PLS	−0.15	−0.05	0.16	0.12	0.17	0.02	0.44	0.66	1.00
BW_{t-1}	−0.06	−0.12	0.06	0.13	0.08	0.00	0.16	1.00	0.67
PLS_{t-1}	−0.14	−0.04	0.16	0.11	0.16	0.02	0.44	0.65	0.99

Panel A of the table shows descriptive statistics of annualized returns of Fama-French-Carhart 6 Factors. The panel also shows descriptive statistics of two indices of investor sentiment, BW index and PLS index. Panel B shows the autocorrelation of the Fama French 6 Factors and sentiment indices with lags of 1,3,6,12 months. Panel C shows correlations of the sentiment indices and their corresponding lag with the excess return of Fama-French-Carhart 6 Factors. The data is sampled monthly from July 1965 through December 2019 and there are 654 observations.

3.2. Factor Timing in Fama-French-Carhart 6 Factors

We first study the optimal factor timing portfolio of investors with different preferences in Fama-French-Carhart 6 Factors. We estimate the unconditional portfolio choice as a benchmark of factor timing and then the conditional portfolio choice, to investigate the patterns of factor timing with investor sentiment.

3.2.1. Unconditional Portfolio Choice

We present our main results of this paper beginning with the unconditional portfolio choice of investors with different utility preferences, or factor investing. The unconditional portfolio choice serves as a benchmark for following conditional portfolio choice with investor sentiment and is helpful for the factor timing portfolio choice in Section 3.2.2.

Panel A of Table 2 presents the results of unconditional portfolio choice of investors with mean-variance preference and the investors' risk aversion parameter γ is equal to 2, 5, 10, and 20. The entries in the number of factors 3 of all panels in Table 2 are for a portfolio choice between Fama-French 3 Factors (FF3F), Mkt-RF, SMB, and HML (Fama and French, 1992, 1993). The entries in the number of factors 6 of all panels in Table 2 are for a portfolio choice between Fama-French-Carhart 6 Factors, Mkt-RF, SMB, HML, RMW, and CMA, and adding the momentum factor Mom. We add the risk-free rate, RF, to the asset space of the portfolio choice problem. Specially, we take the mean of the risk-free rate as the time series values of RF. The numbers below each estimate are corresponding standard errors.

There are some interesting features of mean-variance optimal portfolio choice. Investors with mean-variance preference holds the same ratio of different factors, irrespective of the level of risk aversion. For example, when the number of factors is 3, all mean-variance investors with different risk aversions hold the same ratio of SMB to HML, equaling to 0.29. It means that mean-variance investors hold the same position of the risky factors, independent on the level of risk aversion. Take the same example of the factor number equal to 3, mean-variance investors hold the position of 15 or 52 percent SMB or HML factor to the whole risky factors. The second feature is that the risky aversion only effects how mean-variance investors allocate the optimal position between risky factors and risk-free factors. When the number of factors is 3, the mean-variance investors' optimal ratio of risk factors to risk-free rate is 5.02 for risk aversion parameter $\gamma = 2$ and 0.50 for $\gamma = 20$, which is inversely proportional to γ. Combining with the first feature, it also means the optimal position of single risky factor is inversely proportional to risk aversion. This is because the optimal ratio that mean-variance investors allocating their wealth in the risky tangency portfolio is inversely proportional to investors' risk aversion γ, and the equation (4) gives a theoretical support. These features are also coincident when the asset space of risky factors changes, for example, from FF3F to

TABLE 2.

Unconditional portfolio choice with expected utility preferences in Fama-French-Carhart 6 Factors

Panel A Mean-Variance Investors									
	Number of Factors = 3			Number of Factors = 6					
	Mkt-RF	SMB	HML	Mkt-RF	SMB	HML	RMW	CMA	Mom
$\gamma = 2$	1.642	0.763	2.611	2.957	1.791	0.674	4.527	5.817	2.218
	0.473	0.668	0.711	0.507	0.704	1.006	0.981	1.461	0.483
$\gamma = 5$	0.657	0.306	1.044	1.183	0.716	0.269	1.811	2.327	0.887
	0.189	0.267	0.285	0.203	0.282	0.402	0.392	0.584	0.193
$\gamma = 10$	0.328	0.153	0.522	0.591	0.358	0.135	0.905	1.163	0.444
	0.095	0.134	0.142	0.101	0.141	0.201	0.196	0.292	0.097
$\gamma = 20$	0.164	0.076	0.261	0.296	0.179	0.067	0.453	0.582	0.222
	0.047	0.067	0.071	0.051	0.070	0.101	0.098	0.146	0.048
Panel B Constant Relative Risk Averse Investors									
	Number of Factors = 3			Number of Factors = 6					
	Mkt-RF	SMB	HML	Mkt-RF	SMB	HML	RMW	CMA	Mom
$\gamma = 2$	1.460	0.770	2.510	2.104	1.951	0.456	4.352	4.851	1.125
	0.460	0.688	0.723	0.483	0.741	1.027	1.029	1.488	0.423
$\gamma = 5$	0.609	0.313	1.033	0.947	0.789	0.205	1.874	2.166	0.593
	0.195	0.276	0.302	0.222	0.312	0.430	0.452	0.655	0.222
$\gamma = 10$	0.308	0.157	0.520	0.489	0.395	0.107	0.954	1.113	0.318
	0.099	0.138	0.152	0.115	0.157	0.218	0.231	0.335	0.118
$\gamma = 20$	0.155	0.079	0.261	0.248	0.198	0.054	0.480	0.563	0.164
	0.050	0.069	0.076	0.058	0.079	0.109	0.116	0.169	0.060

The table shows estimates of the unconditional portfolio choice of investors with following objectives:

$$PanelA : \max_{\alpha} \left(E[W_{t+1}] - \frac{\gamma}{2} Var[W_{t+1}] \right)$$
$$PanelB : \max_{\alpha} \left(\frac{W_{t+1}^{1-\gamma}}{1-\gamma} \right)$$

where W_{t+1} is next period's wealth and α is the fractions of current wealth $W_t = 1$ invested in different factors, respectively. The entries in 3 factors are for a portfolio choice between Mkt-RF, SMB and HML (FF3F) and the entries in 6 factors are for a portfolio choice between Mkt-RF, SMB, HML, RMW, CMA and Mom (FF6F). The risk-free rate, RF, is added to the asset space of the portfolio choice problem. The numbers below each estimate are standard errors.

FF6F. The optimal allocation of mean-variance investors observes the two-fund separation rule and the factors hold the same risky position in the tangency portfolio.

Panel B of Table 2 presents the results of unconditional portfolio choice of investors with CRRA preference and the investors' relative risk aversion parameter γ is equal to 2, 5, 10, and 20. The results are different from those of mean-variance preference. Firstly, the CRRA investors hold more ratio of SMB or HML factors and less ratio of Mkt-RF factor, relative to mean-variance investors with the corresponding risk aversion parameters. These differences in the optimal allocation are mainly due to the positive skewness in SMB and HML returns and the negative skewness in Mkt-RF returns that we represent in Panel A of Table 1. Moreover, CRRA investors don't hold the same risky position of risky factors as mean-variance investors do and the two-fund separation for the mean-variance portfolio choice with a risk-free rate does not apply to investors with CRRA preference. This is theoretically because the optimal portfolio of CRRA preference is also dependent on higher order moments, which induces the differences of optimal choice between CRRA and mean-variance investors.

3.2.2. Conditional Portfolio Choice

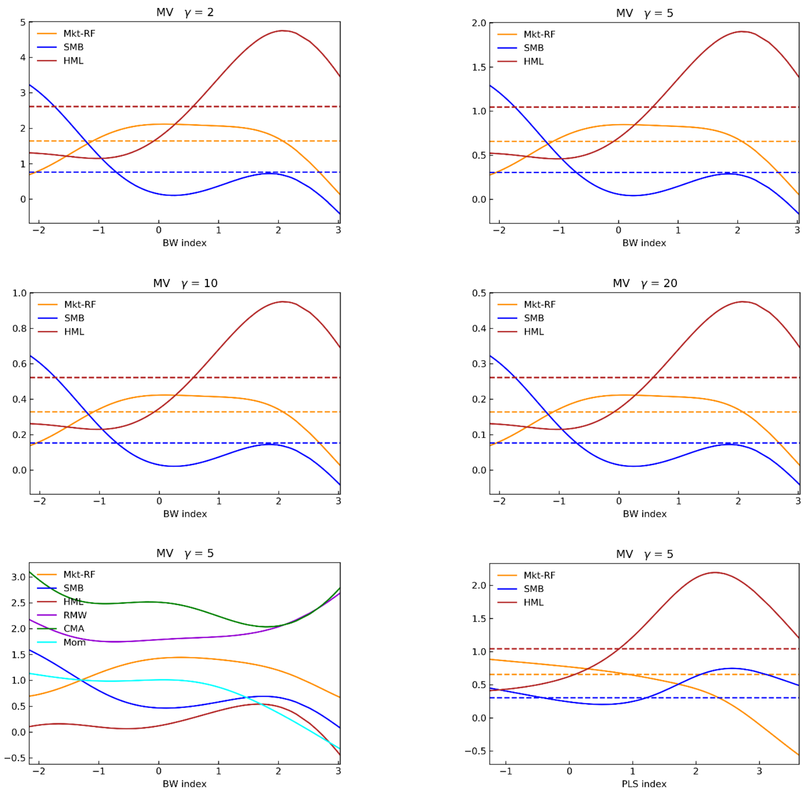
In this part, we present the results of factor timing in FF6F with investor sentiment. Figure 2 plots the conditional portfolio choice in Fama-French-Carhart 6 Factors of investors with mean-variance preference in Panel A, and CRRA preference in Panel B. The portfolio choice is conditional on an index of investor sentiment. The first four and the last plots in each panel represent the factor timing results in Fama-French 3 Factors with investor sentiment, and the fifth plot represents the results in Fama-French-Carhart 6 Factors. The bold lines in each plot show the optimal choice of risky factors as a function of investor sentiment and the dashed lines in the plots with three risky factors in each panel represent the optimal unconditional allocation, which is given in Table 2. Moreover, the investor sentiment index is BW index in the first five plots in each panel, and is PLS index in the last plot in each panel.

There are some interesting features emerging. All investors we consider engage in factor timing around the unconditional policy with investor sentiment and the allocation patterns are not monotonic with investor sentiment. Without restriction of functional form between portfolio choice of the risky factors and investor sentiment, the nonparametric estimation determines the complex relationship between them. The kernel function of the nonparametric estimation putting the sample observations with different weights, describes the conditional policy of factor timing around the unconditional policy, which puts the sample observations with equal weights.

Less risk-averse investors tend to more factor timing, and vice versa. Take the first and fourth plots in Panel A for example, the portfolio weight

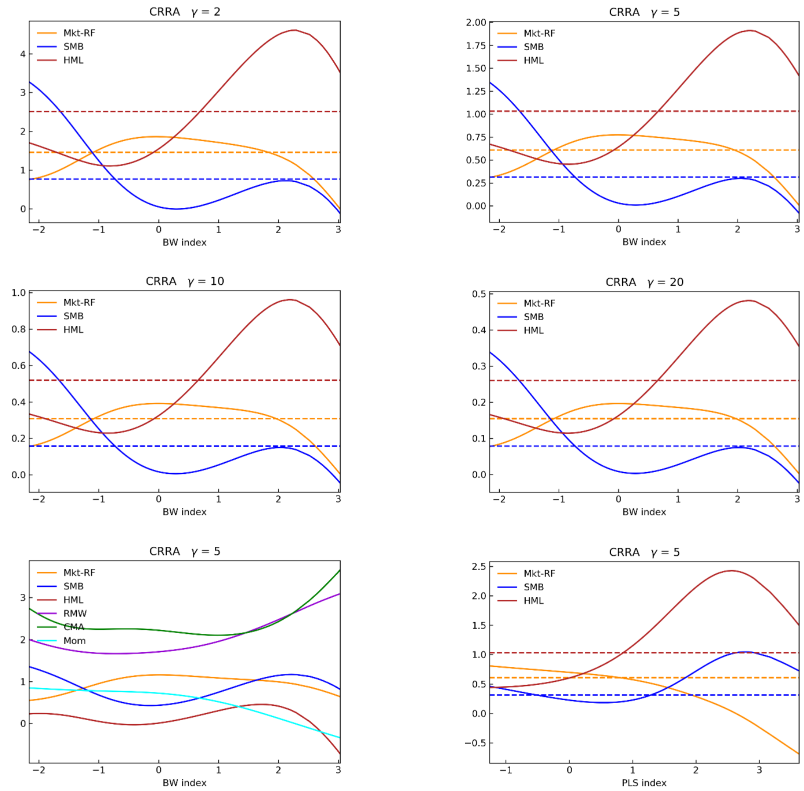
FIG. 2. Conditional portfolio choice in Fama-French-Carhart 6 Factors with different utility preferences.

Panel A Mean-Variance Investors



of HML for a mean-variance investor with $\gamma = 2$ increases from the minimal weight 1.15 to maximal weight 4.75 with changes of investor sentiment, but it only ranges from 0.11 to 0.48 for the case $\gamma = 20$. More risk-averse investors are conservative about factor timing and react less to changes of sentiment.

Mean-variance investors with different levels of risk aversion hold the same relative positions in different risky factors. The optimal choice of the Fame-French 3 Factors in the first four plots of Panel A in Figure 2 is the same without considering the weight on the vertical axis. Moreover, the optimal weights of the risky factors are inversely proportional to the level of risk aversion parameter γ with changes of investor sentiment, Panel B Constant Relative Risk Averse Investors



This figure plots the conditional portfolio choice of investors with mean-variance preference in Panel A, and CRRA preference in Panel B in Fama-French-Carhart 6 Factors. The portfolio choice is conditional on an index of investor sentiment. The first four and the last plots in each panel represent the factor timing results in Fama-French 3 Factors with investor sentiment, and the fifth plot represents the results in Fama-French-Carhart 6 Factors. The bold lines in each plot show the optimal choice to risky Fama-French factors as a function of investor sentiment. The dashed lines in the plots with three risky factors in each panel represent the optimal unconditional allocation, which is estimated in Table 2. Moreover, the investor sentiment index is BW index in the first five plots in each panel, and the one is PLS index in the last plot in each panel. The sample is from July 1965 to December 2019.

which is supported by equation (4). The relative weights of different risky factors of conditional tangency portfolio change with investor sentiment and mean-variance investors just adjust the relative weights between the "dynamical tangency portfolio" and risk-free rate with their risk aversion parameters and observe the principle of two-fund separation as in the case of unconditional portfolio.

The portfolio choices of mean-variance and CRRA investors with the same risk aversion parameter are similar according to the first four plots in the Panel A and Panel B, which shows that the higher order moments of the return distribution may not contribute to the portfolio choice much. Though the portfolio choices of CRRA investors with different coefficients of relative risk aversion γ look similar one another, the principle of two-fund separation is not applied to CRRA investors.

Comparing the fifth plot with Fama-French-Carhart 6 Factors in each panel to corresponding one with Fama-French 3 Factors, the portfolio choice of investors with given preference and risk aversion shifts with changes in the number of risky factors. The factor space expands with the increase of the risky factors where the investor can allocate their wealth, and the importance or relative position of risky factors also shifts with the wave of investor sentiment. Take mean-variance investors for example, the relative position of HML decreases and Mkt-RF increases, and the relative positions of RMW and CMA are always high with different levels of investor sentiment.

We also take the PLS index as the measure of investor sentiment and the results are presented in the last plot of each panel. The portfolio choice with PLS index is different from the one with BW index. However, the empirical features of factor timing mentioned above are consistent with the previous measure and robust with different investor sentiment measures.

Analyzing the relation between the portfolio choice of single risky factor and investor sentiment, there are obvious turns for all risky factors when the sentiment is high and BW index is over 2, though only 23 samples are in this case. The conditional means of annualized returns of Mkt-RF and SMB are -0.241 and -0.164 respectively and the standard deviations of them don't change much when the BW index is over two standard deviations, which is the reason why the portfolio choice of Mkt-RF and SMB is dropped with high investor sentiment. Though the conditional mean of annualized returns of HML is 0.031 when sentiment is over 2 and near to the one of the whole samples, 0.037, the conditional correlation of HML factor with the lag of BW index is -0.39, which means a high investor sentiment suggests a drop in the return of HML in the next period. The relation of Mom with high investor sentiment is also negative and slope of its portfolio choice is dropped. There is a soft turn in the factor timing portfolio choice when investor sentiment is lower than its standard deviation. The pattern of portfolio choice should change timely when investor sentiment is ranged out of the standard deviation, especially in the peak.

When investor sentiment is in the moderate level, ranged in its one standard deviation, there are some interesting features of factor timing portfolio choice. The most obvious pattern is the weight of HML rises with investor sentiment, which means long of the high value stocks and short of the low value stocks should be persistent with investor sentiment rise until sentiment rise to the peak. The weight of Mkt-RF in the portfolio choice is relative flat compared with HML factor. The SMB factor's weight is a U shape in the interval.

Table 3 compares the nonparametric estimates of conditional portfolio choice in Figure 2 to unconditional estimate in Table 2. This table reports the mean and standard deviation of the annualized excess returns on wealth generated by factor timing portfolio policy. It also shows the gap of the annualized CER of returns between conditional and unconditional portfolio choice. Less risk-averse investors prefer to more factor timing, as a result, the mean and standard deviation also increase with less risk-averse parameters. The expanding factor space increases the choice where investors allocate their wealth, and the gap of CER of returns widens with the number of risky factors increasing.

3.3. Factor Timing in Factor Zoo

In this part, we present the main results in our paper: the patterns of factor timing with investor sentiment in the large set of characteristics-based factors, Factor Timing 55 Factors. Table 4 shows the mean and standard deviation of annualized excess returns on the anomaly portfolios. We estimate the unconditional portfolio choice as a benchmark and then the conditional portfolio choice of factor timing, as we have done in Section 3.2. Furthermore, we first reduce the set of factors to a few dominant components by sparse PCA method before estimating the optimal choice of factor timing.

3.3.1. Sparse Principal Component Analysis

We first extract the main information of the characteristics-based Factor Timing 55 Factors to several sparse PCs by sparse PCA. Like Rapach et al. (2019), we implement sparse PCA via the method of Sigg and Buhmann (2008). Their method induces sparsity by directly imposing the restriction on the number of each weight vector through the expectation-maximization (EM) algorithm (Dempster et al., 1977) to compute sparse weight vectors

TABLE 3.

Estimates of the unconditional portfolio choice versus conditional portfolio choice in Fama-French-Carhart 6 Factors

	Unconditional		BW index			PLS index		
	Mean	Std	Mean	Std	ΔCER%	Mean	Std	ΔCER%
Panel A Mean-Variance Investors								
Number of Factors = 3								
$\gamma = 2$	0.222	0.333	0.314	0.361	7.272	0.329	0.383	7.143
$\gamma = 5$	0.089	0.133	0.126	0.144	2.909	0.132	0.153	2.855
$\gamma = 10$	0.044	0.067	0.063	0.072	1.455	0.066	0.077	1.428
$\gamma = 20$	0.022	0.033	0.031	0.036	0.727	0.033	0.038	0.714
Number of Factors = 6								
$\gamma = 2$	0.778	0.624	0.868	0.641	6.839	0.893	0.645	8.797
$\gamma = 5$	0.311	0.250	0.347	0.256	2.737	0.357	0.258	3.518
$\gamma = 10$	0.156	0.125	0.174	0.128	1.368	0.179	0.129	1.759
$\gamma = 20$	0.078	0.062	0.087	0.064	0.684	0.089	0.064	0.880
Panel B Constant Relative Risk Averse Investors								
Number of Factors = 3								
	Unconditional		BW index			PLS index		
	Mean	Std	Mean	Std	ΔCER%	Mean	Std	ΔCER%
$\gamma = 2$	0.207	0.311	0.291	0.333	7.119	0.313	0.372	7.481
$\gamma = 5$	0.086	0.129	0.121	0.138	2.908	0.131	0.155	3.074
$\gamma = 10$	0.043	0.065	0.061	0.070	1.459	0.066	0.078	1.540
$\gamma = 20$	0.022	0.033	0.030	0.035	0.730	0.033	0.039	0.770
Number of Factors = 6								
$\gamma = 2$	0.596	0.491	0.675	0.507	7.253	0.725	0.554	9.408
$\gamma = 5$	0.269	0.219	0.303	0.226	2.937	0.325	0.243	3.766
$\gamma = 10$	0.139	0.113	0.156	0.116	1.460	0.166	0.124	1.860
$\gamma = 20$	0.071	0.057	0.079	0.059	0.727	0.084	0.062	0.923

This table shows the annualized mean and standard deviation of the excess return on wealth generated by the estimates of unconditional and conditional portfolio choice of investors with mean-variance and CRRA preferences. The unconditional estimates are shown in Table 2 and the conditional estimates are based on nonparametric regressions for the individual moments, plotted in Figure 2. The table also shows the annualized CER of return required to make the investors indifferent between the conditional portfolio choice and the unconditional portfolio choice.

and corresponding sparse PCs. Sigg and Buhmann (2008) compute successive sparse weight vectors by iterative deflation, so the certain weight vector is computed after projecting the data onto the orthogonal subspace defined by the constructed sparse PCs. Specially, we adjust the positive direction of weight vector to satisfy the mean excess return of the corresponding sparse PC positive. We impose the cardinality restriction on each

TABLE 4.
The mean and standard deviation of annualized excess returns on Factor Timing 55 Factors, %

	Mean, %	Std, %		Mean, %	Std, %
Accruals	4.97	10.91	Momentum (6m)	4.30	21.94
Firm age	0.57	14.61	Momentum (12m)	15.33	24.22
Asset Turnover	4.99	13.21	Momentum-Reversals	5.17	16.77
Beta Arbitrage	1.33	21.57	Net Issuance (A)	8.67	10.12
Cash Flows/Price	4.30	15.30	Net Issuance (M)	7.06	11.19
Composite Issuance	5.97	11.44	Net Operating Assets	5.03	11.03
Debt Issuance	1.47	6.20	Price	0.75	23.50
divg	−0.48	11.44	Gross Profitability	4.83	11.84
Dividend Yield	1.86	17.99	Share Repurchases	1.52	6.01
Duration	4.62	16.97	Return on Assets (Q)	6.56	16.53
Earnings/Price	6.90	16.64	Return on Assets (A)	2.41	14.29
exchsw	2.38	14.39	Return on Book Equity (A)	7.58	16.73
F-score	0.96	5.66	Return on Book Equity (Q)	1.21	15.32
Growth in LTNOA	−0.60	10.19	Return on Market Equity	13.49	17.14
Gross Margins	0.77	11.68	Seasonality	9.24	13.76
Asset Growth	3.20	12.12	Sales Growth	−0.76	12.79
Investment Growth	4.27	9.57	Short Interest	−0.48	14.42
Industry Momentum	4.98	21.56	Share Volume	−0.28	20.71
Ind. Mom-Reversals	12.92	12.18	Size	2.59	16.58
Industry Rel. Reversals	11.37	14.25	Sales/Price	5.38	14.82
Industry Rel. Rev. (L.V.)	15.04	10.65	Short-Term Reversals	4.43	18.33
Investment/Assets	5.32	10.66	Earnings Surprises	6.12	14.15
invaci	1.25	19.68	Value-Momentum	5.62	17.47
Investment/Capital	1.12	17.30	Value-Momentum-Prof.	9.61	16.71
ipo	−0.48	16.51	Value-Profitability	8.31	13.39
Idiosyncratic Volatility	6.80	24.65	Value (A)	4.62	16.05
Leverage	2.56	16.18	Value (M)	3.71	20.59
Long Run Reversals	4.00	17.78			

The table lists all Factor Timing 55 Factors used in our analysis and shows the annualized mean return and standard deviation of the anomaly portfolios. Anomaly definitions from Kozak et al. (2020), Haddad et al. (2020), Kozak (2020), Kozak and Santosh (2019) and Giglio et al. (2020). The data set includes the stocks of CRSP and COMPUSTAT and sorts them into 10 value-weighted portfolios for each of the 55 characteristics studied in Kozak et al. (2020). The long-short anomalies are constructed as spread between each anomaly's return on portfolio 10 minus the return on portfolio 1. Excess returns on beta arbitrage portfolios are scaled by their respective betas. F-score, Debt Issuance, and Share Repurchases are binary sorts. Monthly data from July 1965 to December 2019.

weight vector $K = 11$, so that each weight vector contains 20% of the 55 factors.

Table 5 shows the percentage of variance explained by the first few dominant anomaly PCs and sparse PCs. The more PCs we use, the more cumulative variance explained, however, more difficult we analyze the results of factor timing. How many components should we use to investigate the investor's factor timing problem? Following the assumption of Haddad et al. (2020), the harmonic mean of PCs' contribution to the total variance of returns should be higher than the ratio of total R^2 to maximum squared Sharpe ratio. Using the variance explained in Table 5, we should use the first four or five sparse PCs as the asset space. The first five sparse components jointly explain more than half of the total variance in returns and the number of sparse PCs adding the aggregate market return also is equal to the dataset of Fama-French-Carhart 6 Factors, which is convenient to compare the results of the large set of factors with Fama-French-Carhart 6 Factors.

TABLE 5.

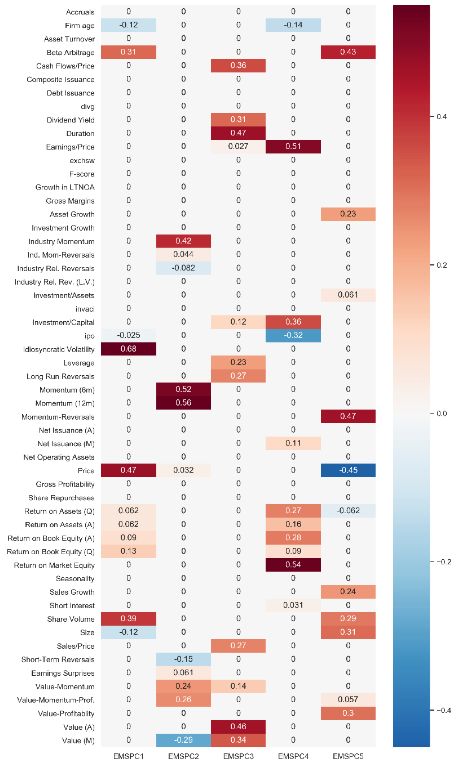
Percentage of variance explained by anomaly PCs and sparse PCs

Panel A PCA					
	PC1	PC2	PC3	PC4	PC5
%var. explained	25.2	18.4	12.3	4.6	3.4
Cumulative	25.2	43.6	55.8	60.4	63.8
Panel B Sparse PCA					
	EMSPC1	EMSPC2	EMSPC3	EMSPC4	EMSPC5
%var. explained	14.6	14.1	11.7	8.2	5.0
Cumulative	14.6	28.7	40.4	48.6	53.6

The table shows the percentage of variance explained by the largest PCs and sparse PCs of Factor Timing 55 Factors. The sparse PCs is constructed via the approach of Sigg and Buhmann (2008) through the expectation-maximization (EM) algorithm, so the sparse PCs are named after EMSPCs. The first row of each panel is the percentage of variance explained by each PC or sparse PC of the characteristics-based Factor Timing 55 Factors and the second row is the cumulative percentage of variance explained by the PCs or sparse PCs.

Figure 3 plots the weight vector of the first five sparse PCs of the Factor Timing 55 Factors via the method of Sigg and Buhmann (2008). The red color represents the positive factor weights of each sparse PC and blue represents the negative factor weights, and deeper colors mean heavier weights in weight vectors. The substantive sparsity in each weight vector facilitates us to interpret the first five sparse PCs as follows. The first column of the heat map in Figure 3 shows the first sparse PCs is predominantly a linear combination of anomalies relative to trading frictions, such as idiosyncratic volatility, price, share volume, beta arbitrage, size. We label the first sparse

FIG. 3. The factor weights of each sparse principal component of the Factor Timing 55 Factors.



This figure plots the weight vector of first five sparse PCs of the characteristics-based Factor Timing 55 Factors. The sparse PCs is constructed via the approach of Sigg and Buhmann (2008), directly imposing the sparsity restriction on the number of each weight vectors through the expectation-maximization (EM) algorithm. The direction of weight vector is adjusted to satisfy the mean excess return of the corresponding sparse PC positive. The sparse weight vectors are computed successive by iterative deflation, projecting the data onto the orthogonal subspace defined by the constructed sparse PCs and the sparse PCs are ranged by the sequence of extracting from the data. The red color represents the positive factor weights of each sparse PC and blue represents the negative factor weights, and deeper colors mean heavier weights in weight vectors. The colorbar is limited from -0.45 to 0.55.

PC as "trading friction". The second sparse PC is related to momentum and reversal and the active elements of this weight vector include most of anomalies about momentum and reversal, so we call this sparse PC as "momentum". From the third column of Figure 3, we can see the third sparse

PC is essentially a combination of value-versus-growth related anomalies, such as value (book equity to market equity), cash flow to market equity, value (book-to-market ratio using the most up-to-date prices and book equity), dividend yield (dividend scaled by price), sales-to-price, leverage (total assets over the market value of equity). We thus label the third sparse PC as "value". We refer to the fourth sparse PC as "profitability", as the major elements in its weight vector is combined with various measures of profitability, such as return on market equity, earnings to price, return on book equity, return on assets, return on assets(annual), return on book equity(annual). According to the last column of Figure 3, it's obvious that the fifth sparse PC is a linear combination of anomalies from various categories and there are same anomalies with the first sparse PC. Unlike the first sparse PC, the weight of price on the fifth sparse PC is negative, and the weights of momentum-reversal is positive, which means the portfolio can obtain long-term persistent profit. The positive weights on asset growth and sale growth also reflect the portfolio in combined with assets of growth potentiality. The fifth sparse PC, which we label as "growth", exhibits the good growth potentiality of the portfolio.

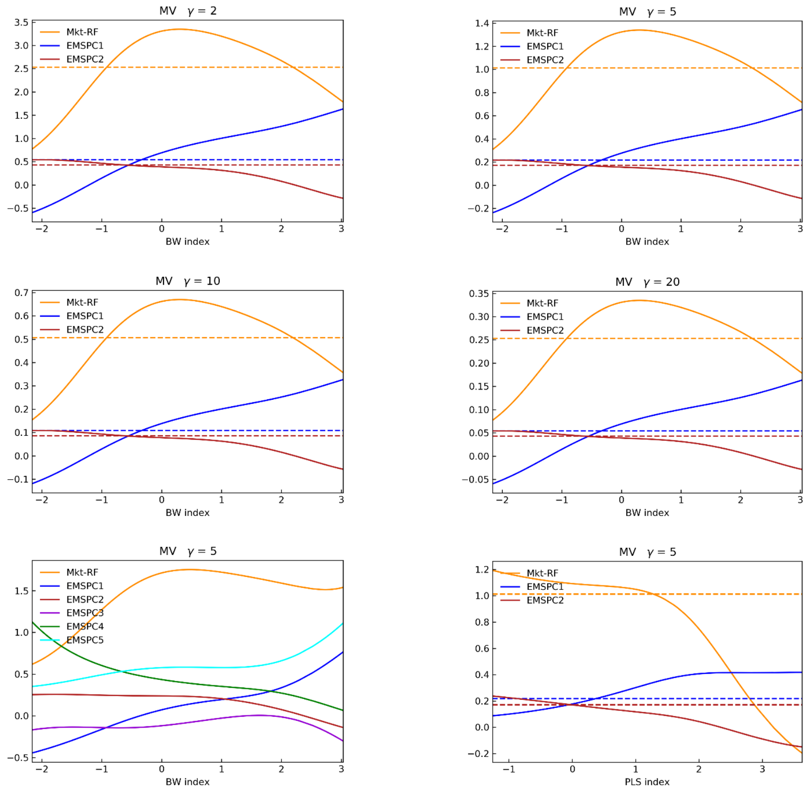
Fortunately, the improved interpretability of the sparse PCs is at relatively little cost in terms of the explanatory ability: despite the high degree of sparsity, the first five sparse PCs still explain 53.6% of the total variation in the Factor Timing 55 Factors, compared to 63.8% for the first five conventional PCs.

3.3.2. Factor Timing in Factor Timing 55 Factors

In this part, we show the results of factor timing with investor sentiment in the sparse PC space of the large set of characteristics-based Factor Timing 55 Factors. Figure 4 plots the unconditional and conditional portfolio choice in the sparse PC space of Factor Timing 55 Factors of mean-variance investors in Panel A and CRRA investors in Panel B. The plots of each panel in Figure 4 are arranged as same as corresponding panel in Figure 2, except that the asset space is the first few sparse PCs of the FT55F adding the aggregate market return. The investors also hold the same allocation patterns as we analyze in Section 3.2, for example, investors' allocation function with investor sentiment are complex, less risk-averse investors more engage in factor timing, mean-variance investors adjust the relative weights between "dynamic tangency portfolio" and the risk-free rate but the CRRA investors don't observe the principle of two-fund separation, and the relative position of portfolio choice shifts with the number of sparse PCs.

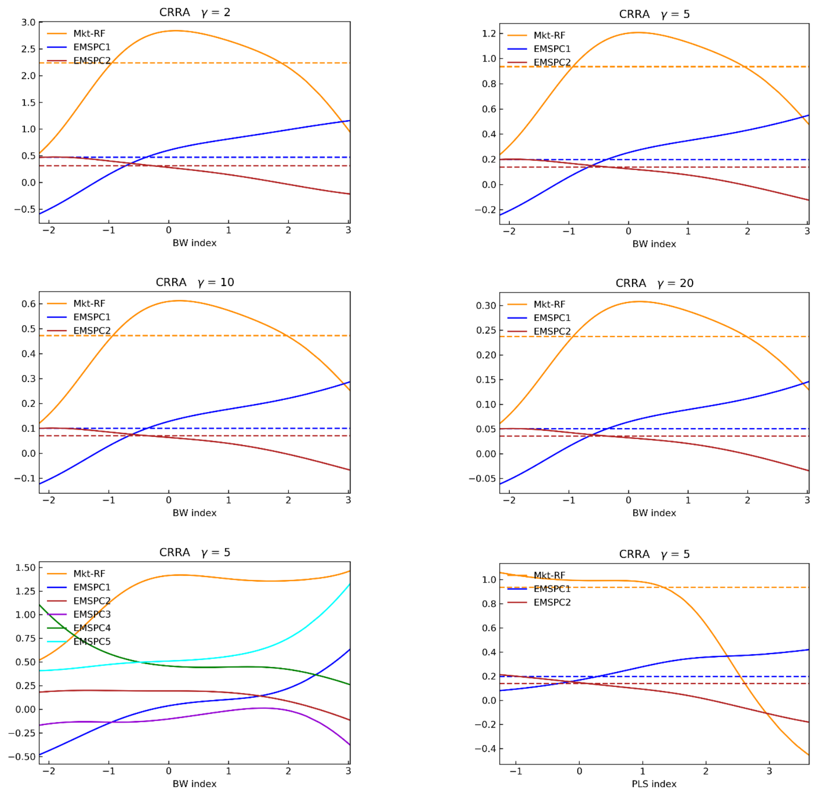
FIG. 4. Conditional portfolio choice in Factor Timing 55 Factors with different utility.

Panel A Mean-Variance Investors



For the Mkt-RF factor, the portfolio choice of it increases with the BW index when investor sentiment is low and decreases when investor sentiment is high, which is the same as the case of FF6F, but the range of Mkt-RF weight changes more when factor timing in FT55F. The portfolio weight of the first sparse PC, labeled as "trading friction", is small when investor sentiment is low, and rises a little when sentiment is high. It's mainly because when investor sentiment is low, the portfolio return of "trading friction" is low. For the second sparse PC, "momentum", the conditional weight is very close to the unconditional and the slope of it is

Panel B Constant Relative Risk Averse Investors



This figure plots the conditional portfolio choice in sparse PCs of investors with mean-variance preference in Panel A, and CRRA preference in Panel B. The portfolio choice is conditional on an index of the investor sentiment. The first four and the last plots in each panel represent the factor timing results in first two sparse PCs of Factor Timing 55 Factors with investor sentiment and the fifth plot represent the results in first five sparse PCs, adding the aggregate market factor. The bold lines in each plot show the optimal allocation to risky sparse PCs factors as a function of the investor sentiment. The dashed lines in the plots with three risky sparse PCs factors in each panel represent the unconditional portfolio choice. Moreover, the investor sentiment index in the first five plots in each panel is BW index, and the one in the last plot is PLS index. The sample is from July 1965 to December 2019.

not significantly different from zero in the case of low investor sentiment. When the sentiment is high, the optimal weight of momentum is not stable and its slope is down, where the pattern is similar to Mom factor in FF6F. The pattern of the third sparse PC, "value", is also similar to HML factor in the FF6F and there is an obvious decline for its portfolio weight when the sentiment is high, though the whole weight is more stable than HML factor. However, the fourth sparse PC, "profitability" behavior different from the RMW factor, the profitability factor in FF6F: the weight of "profitability" factor monotonously declines in the portfolio choice with the investor sentiment rising. The slope of last sparse PC, "growth", reflecting the assets growth potentiality in the portfolio, monotonously inclines with the investor sentiment, even if the investor sentiment is very high and most of the weights of other sparse PCs are dropped.

Table 6 reports the annualized CER of return required to make an investor without factor timing indifferent between with factor timing in different number of sparse PCs. Generally, the gap of CER of return between conditional and unconditional portfolio choices is positive, as shown in Table 6, no matter the investors' preferences and their risk aversion, and the result is robust with different measure of investor sentiment. Theoretically, the investors can allocate their wealth in a more expanded asset space with the sparse PC number increasing and the CER of return should also rise relative to the investors without factor timing. This pattern doesn't appear be obeyed strictly when the sparse PC number is increased, but the general trend of CER's gap broadens with the number of assets invested increasing.

3.4. Robustness

Our nonparametric method evaluates the factor timing portfolio choice is based on the entire samples. However, it is possible that the portfolio choice does not perform well on out-of-sample or fully withheld data. To test the robustness of the portfolio choice, we conduct a pure out-of-sample test. We present the out-of-sample results in Table 7 with Factor Timing 55 Factors. Our predictive estimation includes many choices such as investors' preferences, how investor risk-averse is, how many factors included, replacing the investor sentiment index and the window analysis types to predict the out-of-sample performance. In Table 7, we explore the robustness of our predictive results to changes in these specifications. Panel A and Panel B of Table 7 present the out-of-sample results of forecast portfolio choices of the investor with mean-variance and CRRA preference, respectively.

In Table 7 we report the mean, standard deviation, CER and Sharpe ratio of the annualized out-of-sample excess return of predictive portfolio choice with a wide variety of parameters and forecasting methods. Our main

TABLE 6.

Conditional portfolio choice versus unconditional portfolio choice in Factor Timing 55 Factors

Panel A Mean-Variance Investors					
Number of factors	2	3	4	5	6
BW index					
$\gamma = 2$	9.62	8.98	6.59	7.20	8.43
$\gamma = 5$	3.85	3.59	2.64	2.88	3.37
$\gamma = 10$	1.92	1.80	1.32	1.44	1.69
$\gamma = 20$	0.96	0.90	0.66	0.72	0.84
PLS index					
$\gamma = 2$	5.57	6.13	5.46	5.99	7.29
$\gamma = 5$	2.23	2.45	2.19	2.40	2.91
$\gamma = 10$	1.11	1.23	1.09	1.20	1.46
$\gamma = 20$	0.56	0.61	0.55	0.60	0.73
Panel B Constant Relative Risk Averse Investors					
BW index					
$\gamma = 2$	8.11	7.80	6.26	6.77	7.65
$\gamma = 5$	3.44	3.28	2.58	2.80	3.26
$\gamma = 10$	1.75	1.66	1.30	1.41	1.64
$\gamma = 20$	0.88	0.83	0.65	0.71	0.82
PLS index					
$\gamma = 2$	5.09	5.92	5.90	6.35	7.32
$\gamma = 5$	2.12	2.44	2.38	2.59	3.07
$\gamma = 10$	1.07	1.23	1.19	1.29	1.54
$\gamma = 20$	0.54	0.62	0.59	0.64	0.77

This tables show the annualized CER of return required to make an investor without factor timing indifferent between with factor timing. The conditional and unconditional factor portfolio choices are shown in the Figure 4. Panel A represents the CER's gap between factor timing with invest sentiment and unconditional portfolio choice of mean-variance investors and Panel B represent the CER's gap of CRRA investors. The investors with different preferences and risk aversions allocate their wealth into different asset space from the first 1 sparse PCs to the first 5 sparse PCs of the FT55F, adding Mkt-RF factor and the number of factors indicates the total factors invested in the asset space. The BW index and PLS index are used to measure the investor sentiment in each panel.

out-of-sample analysis uses a split window analysis where the predictive portfolio rule is estimated using the first 474 months' samples, from July 1965 to December 2004, according to the investor sentiment index in the reference month and uses the rest of the data to construct out-of-sample

TABLE 7.

Predictive portfolio choice with various choices

γ	Number of factors	Sentiment Index	Window analysis	Mean, %	Std., %	CER, %	Sharpe Ratio
Panel A Mean-Variance Investors							
5	6	BW	split	17.89	19.56	8.32	0.91
2	6	BW	split	44.72	48.90	20.81	0.91
10	6	BW	split	8.94	9.78	4.16	0.91
20	6	BW	split	4.47	4.89	2.08	0.91
5	2	BW	split	9.82	10.69	6.96	0.92
5	3	BW	split	9.43	15.03	3.79	0.63
5	4	BW	split	10.16	17.69	2.33	0.57
5	5	BW	split	13.10	17.46	5.47	0.75
5	6	PLS	split	18.95	21.35	7.55	0.89
5	6	BW	rolling	14.83	18.19	6.56	0.82
5	6	BW	expanding	19.85	19.81	10.04	1.00
Panel B Constant Relative Risk Averse Investors							
5	6	BW	split	15.95	17.76	7.77	0.90
2	6	BW	split	36.23	40.94	18.73	0.88
10	6	BW	split	8.18	9.08	3.91	0.90
20	6	BW	split	4.14	4.58	1.96	0.90
5	2	BW	split	9.44	10.26	6.68	0.92
5	3	BW	split	8.96	14.37	3.35	0.62
5	4	BW	split	8.75	15.78	1.98	0.55
5	5	BW	split	11.95	15.76	5.37	0.76
5	6	PLS	split	16.16	18.45	7.28	0.88
5	6	BW	rolling	14.17	16.50	6.74	0.86
5	6	BW	expanding	18.47	18.13	9.90	1.02

This table reports the summary performance of predictive portfolio choice with various choices in Factor Timing 55 Factors. Panel A shows the results of portfolio choice of mean-variance and Panel B shows the results of CRRA. The first column reports the investor's risk aversion parameters and the second one reports the number of factors invested as risky assets. The table also reports the replacement of the investor sentiment index and the methods of predictive window, and the third and fourth columns report the corresponding results. The table reports the annualized mean, standard deviation as well as Sharpe ratio of the excess return of predictive portfolio choice. The table also shows the CER of return required to make the investors indifferent between the predictive conditional portfolio choice and risk-free rate.

performance, from January 2005 to December 2019. We consider two other alternatives, rolling window and expanding window analysis. For rolling window, the window of sample used to estimate the portfolio rules rolls with the reference point and keeps the same length with split window of

474 months to the reference point. The expanding window analysis uses the samples to estimate the out-of-sample portfolio rules from the start point, July 1965, to the reference point. Moreover, we only use the first 474 months' split sample to estimate the sparse weight vectors and construct the corresponding out-of-sample sparse PCs.

The first row of each panel in Table 7 reports the results for our base specification. The next three rows we consider investors with different risk aversion parameters. Less risk-averse investors tending to more factor timing gain a higher mean of excess return of wealth next period and meanwhile a higher standard deviation. Specifically, the CER of the portfolio rule of a mean-variance investor is inversely proportional to their risk aversion parameters, however, the Sharpe ratio is same to all mean-variance investors, which is not applied to CRRA investors. The empirical results prove that out-of-sample predictive portfolio choice of mean-variance investors also observes the "dynamical two-fund separation rule" and relative weights of different risky factors of conditional tangency portfolio change with investor sentiment.

The next four rows show that our results are robust to vary the number of included factors from two to five and again obtain similar out-of-sample results. Next, we also replace the investor sentiment index, BW index, with PLS index, and the PLS index performs comparable predictive results to BW index. The last two rows show that our results are robust to use rolling window and expanding window methods to estimate the out-of-sample portfolio choice.

3.5. Transaction Costs

In this section, we examine the impact of transaction cost on the predictive portfolio choice. We take transaction costs to be constant and in cross-section at 0.5% (Brandt et al., 2009). The transaction cost of portfolio choice at time t is $c_t = z * T_t$ where z is the constant transaction cost in cross-section and T_t represents the turnover at time t. We define the portfolio choice's monthly turnover at time t as

$$T_t = \sum_i \left| w_{i,t} - \frac{w_{i,t-1} r_{i,t}}{w_{t-1} r_t} \right| \tag{9}$$

and the average monthly turnover as

$$Turnover = \frac{1}{T} \sum_{t=1}^{T} T_t \tag{10}$$

where w_t is the weight vector of the wealth invested in assets with gross return r_{t+1}, and $w_{i,t}$ is the element i in w_t and represents the optimal weight of assets i with gross return $r_{i,t+1}$. Specifically, $w_t = (1-\alpha_t, \alpha_t)$ and $r_{t+1} = (R_{f,t+1}, R_{t+1})$ corresponding to the expression (1).

Table 8 presents the results for predictive portfolio choice with constant transaction cost described above. For comparison, the corresponding annualized Sharpe ratio and CER of return required to make the investors indifferent between the predictive conditional portfolio choice and risk-free rate in the case with no transaction costs is in Table 7.

The table shows that transaction costs consume the gains from the return of portfolio choice and have a negative impact on predictive performance on CER and Sharpe ratio. Less risk-averse investors tend to more factor timing, so that their portfolio choice's CER over risk-free rate and Sharpe ratio reduce more than that of more risk-averse investors.

4. MEAN-VARIANCE EFFICIENT PORTFOLIO

For investors with mean-variance preference, they observe the two-fund separation rule and allocate their wealth between the risk-free fund and risky fund, which is also the tangency portfolio as we analyze above. The tangency portfolio is also a mean-variance efficient (MVE) portfolio and we can estimate the dynamic position of risky factor as the MVE portfolio by set the risk aversion parameter γ of mean-variance preference to 1, using the nonparametric method.

Figure 5 plots the time-series of one-year overlapping returns on the MVE portfolio. This figure plots MVE portfolio returns implied by the portfolio choice of factor timing with the mean-variance preference that is constructed using the first five sparse PCs of the Factor Timing 55 Factors and the Mkt-RF factor with BW sentiment index as the predictor.

We then estimate abnormal returns of this out-of-sample MVE portfolio with respect to five benchmarks: CAPM (Sharpe,1964, Lintner,1965), Fama-French 3 Factors (Fama and French 1992,1993), Fama-French-Carhart 4 Factors (FF3F and momentum factor, Carhart,1997), Fama-French 5 Factors (Fama and French 2015,2016), and Fama-French-Carhart 6 Factors (FF5F and momentum factor). The MVE portfolio returns are scaled to have the same standard deviation as the aggregate market in the out-of-sample period, which is equivalent to adjust the risk aversion parameter γ of mean-variance preference. Table 9 confirms that the MVE portfolio implied by factor timing performs well in the withheld data. The table reports the intercepts (alpha) from time-time regressions of the out-of-sample MVE

TABLE 8.

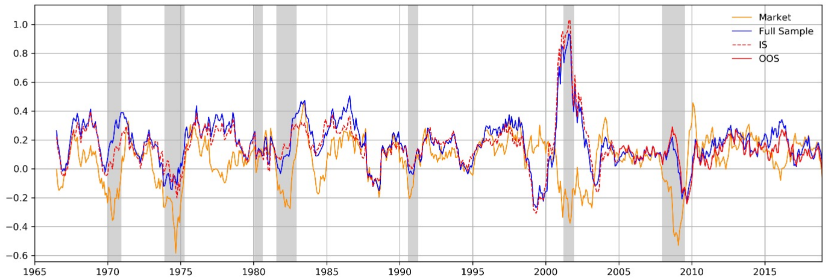
Predictive portfolio choice with transaction cost

γ	Number of factors	Sentiment Index	Window analysis	Turnover, %	Turnover/N, %	CER % $z = 0.005$	SR $z = 0.005$
Panel A Mean-Variance Investors							
5	6	BW	split	24.49	4.08	7.12	0.85
2	6	BW	split	136.63	22.77	15.48	0.81
10	6	BW	split	9.03	1.50	3.66	0.86
20	6	BW	split	4.18	0.70	1.84	0.87
5	2	BW	split	4.54	2.27	6.91	0.91
5	3	BW	split	7.40	2.47	3.57	0.61
5	4	BW	split	14.40	3.60	1.67	0.54
5	5	BW	split	18.38	3.68	4.50	0.69
5	6	PLS	split	26.84	4.47	6.22	0.82
5	6	BW	rolling	46.74	7.79	4.65	0.71
5	6	BW	expanding	30.10	5.02	8.67	0.93
Panel B Constant Relative Risk Averse Investors							
5	6	BW	split	20.93	3.49	6.65	0.84
2	6	BW	split	94.98	15.83	14.63	0.79
10	6	BW	split	8.25	1.38	3.42	0.85
20	6	BW	split	3.93	0.65	1.72	0.85
5	2	BW	split	4.33	2.17	6.62	0.91
5	3	BW	split	6.84	2.28	3.13	0.61
5	4	BW	split	12.37	3.09	1.39	0.52
5	57BW	split	15.94	3.19	4.46	0.70	
5	6	PLS	split	21.98	3.66	6.11	0.81
5	6	BW	rolling	43.09	7.18	4.93	0.75
5	6	BW	expanding	25.79	4.30	8.64	0.95

The table reports the results of the predictive portfolio choice with the constant transaction cost in Factor Timing 55 Factors. Panel A shows the results of portfolio choice of mean-variance and Panel B shows the results of CRRA. The first column reports the investor's risk aversion parameters and the second one reports the number of factors invested as risky assets. The table also reports the replacement of the investor sentiment index and the methods of predictive window, and the third and fourth columns report the corresponding results. The table reports the turnover in the factor timing and the average turnover among the factors invested. The table also reports the annualized CER of return required to make the investors indifferent between the predictive conditional portfolio choice and risk-free rate and Sharpe ratio for the predictive portfolio choice with adjustment for transaction costs. The proportional transaction cost z is 0.5%, constant across stocks and over time.

portfolio returns in the benchmarks mentioned above. The result shows that the out-of-sample MVE implied by the factor timing portfolio choice offers a large abnormal return relative to the benchmarks above. Take the Fama-French 6 Factors, the benchmark with the least alpha, for example,

FIG. 5. Time-series of returns on the MVE portfolio.



The figure plots the time-series of one-year overlapping returns on the MVE portfolio and excess returns on market. The MVE portfolio returns is implied by the portfolio choice of factor timing with the mean-variance preference that was constructed using the first five sparse PCs of the Factor Timing 55 Factors and the Mkt-RF factor with BW sentiment index as the predictor. The solid blue line shows the full sample estimates. The red dashed line shows the in-sample estimates in the pre-2005 and solid red line depicts pure OOS MVE portfolio returns in the post-2005 using the split window analysis. The orange solid line depicts the excess return on aggregate market for comparison. MVE portfolio returns are scaled to have the same standard deviation as the aggregate market. The MVE return is from July 1965 to December 2019.

we estimate an abnormal annualized return of 4.10% which is near to two standard errors from zero. The abnormal returns are even larger for other benchmarks because of the less factors included. The results in the table ensure that factor timing obtains the abnormal returns do not adequately been described by the static benchmarks and the MVE portfolio implied by factor timing portfolio choice with investor sentiment do improve the wealth returns of investor.

5. CONCLUSION

This paper documents the empirical relation between optimal factor portfolio weights and investor sentiment. We show how to use investor sentiment index to factor timing with given preferences in the asset space of FF6F and in the large set characteristics-based factors, or factor zoo. Investor sentiment is a good index to factor timing and contributes the information of market cross-sectional return, not only about first two moments, but also higher order moments. We introduce the investor sentiment into the investor's factor timing problem. The robust in- and out-of-sample empirical evidence ensures that investor sentiment do contains information

TABLE 9.

MVE portfolio's annualized OOS α in the withheld sample (2005-2019), %

	CAMP	FF3F	FF4F	FF5F	FF6F
Alpha	10.54	11.22	9.42	5.74	4.10
Std	3.72	3.62	2.88	3.19	2.30

The table shows the annualized alphas (in %) computed from the time-series regression of the out-of-sample MVE portfolio's returns relative to five benchmarks: CAPM, Fama-French 3 Factors, Fama-French-Carhart 4 Factors, Fama-French 5 Factors, and Fama-French-Carhart 6 Factors. The out-of-sample MVE in the post-2005 is estimated using the split window analysis based on the pre-2004 data sample. The out-of-sample MVE portfolio returns are normalized to have the same standard deviation as the aggregate market in the out-of-sample period.

for the future cross-sectional returns, which broadens the classical finance theory leaving no role for irrational factor.

Beyond the introduction of the investor sentiment as the guidance of factor timing, we also use the sparse PCA method to the large set characteristics-based anomalies to facilitate us interpret the factor timing portfolio choice. The sparse PCs of the Factor Timing 55 Factors is allocated more easily and monotonously in the portfolio choice compared with corresponding factors in the Fama-French-Carhart 6 Factors, such as the "momentum", "value" and "profitability" factors we label. Moreover, the empirical results also show that the portfolio choice of "trading friction" and "growth" sparse PCs we identify is also consistent with the financial intuition about investor sentiment.

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