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Consumer Search with Price Sorting*

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This article considers costly sequential search in an online-shopping platform in which either ascending or descending price sorting can be applied prior to the search process. Our model generates price dispersion in the unique symmetric equilibrium for each type of price sorting. Moreover, we show that consumers choose ascending/descending price sorting when the proportion of high-quality firms is high/low. Finally, when search costs are small, the availability of price sorting improves consumer surplus but has no impact on industry profit.

Key Words: Consumer search; Product differentiation; Sorting; Search accuracy; Price dispersion.
JEL Classification Numbers: D43, D83, L13.

1. INTRODUCTION

Price dispersion in homogeneous product markets is a well-documented phenomenon in both offline and online markets (see Baye et al. (2006) for a survey). Ever since Stigler (1961) introduced the term "search" in his seminal article, many researchers have attempted to rationalize price dispersion using search-theoretical models, in which consumers incur a positive cost of acquiring each additional price quote.[1] These models assume that it is possible for companies to charge different prices because it is costly for consumers to find the lowest price due to incomplete information.

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[1] Examples include Reinganum (1979), MacMinn (1980), Burdett and Judd (1983), Rob (1985), Stahl (1989, 1996), and Janssen and Moraga-González (2004).

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The rapid growth of the Internet and flourishing electronic commerce have enabled consumers to use various price-searching tools to be better informed about prices. One of the most commonly observed tools in online-shopping platforms is price sorting. For example, both Amazon.com and eBay.com provide customers with the option of sorting products based on their price, in either ascending or descending order. When all the firms and products are homogenous, consumers would choose to sort prices in an ascending order and costlessly identify the product with the lowest price. This leads firms to engage in Bertrand competition, and as a result, all firms sell products at marginal cost and as a result, price dispersion should disappear.

However, in online-shopping platforms, price dispersion does exist despite consumers' ability to price sort. There is abundant empirical evidence of a substantial amount of online price dispersion. Pan et al. (2004) provide a comprehensive survey of more than 40 empirical studies on online price dispersion. With the development of information technology and easier access to online data, even more studies have emerged after Pan et al. (2004)'s survey.[2] Although these articles use different data, different measures of price dispersion, and different empirical strategies, the general finding is that online price dispersion is significant, persistent, and ubiquitous.

Why, then, do we still observe price dispersion in online markets where consumers can price sort by "low price first"? Further, why would consumers choose the option of sorting by "high price first" when they could instead use ascending price sorting? One reason could be that products within one category generally have different qualities and the cheapest results are often not what one is looking for. Further, if one chooses ascending price sorting, e-commerce platforms may display cheap and irrelevant products, an extreme case of low-quality products. For example, consider a consumer who searches for the "Garmin Forerunner 610" under the category of "Electronics" at Amazon.com. After sorting the results by "low price first," the buyer will see a variety of accessories such as USB cables, carrying cases, and other irrelevant products on the first five pages, with most of these products priced at less than $10. However, the buyer can find the right product "Garmin Forerunner 610" immediately if he or she sorts the results by "high price first." Another example is book shopping online. If consumers search for a particular book by "low price first" on Amazon, it is very likely that they will see the book summary first instead of the actual book[3].

[2]See e.g., Tang et al. (2010); Jaeger and Storchmann (2011); Sengupta and Wiggins (2014); and Overby and Forman (2015).

[3]Consumers complained that they often mistakenly bought book summaries because the covers of the summary and the original book were very similar (See: "Book Shopping

In this study, we consider a market in which products have different qualities within one category and consumers have the option of choosing how to sort prices. Products are categorized into two, high quality or low quality. Consumers attach low value to the low-quality products.

Three types of price sorting are considered in this study: random price sorting, ascending price sorting, and descending price sorting. Under random price sorting, searchers sequentially sample the products in a random order, as is commonly assumed in the literature. We use random price sorting as the benchmark model to study the impacts of the other two types of sorting. When ascending (descending) price sorting is applied, products are displayed from low price to high price (from high price to low price). Consumers sequentially sample the products in the order that they are displayed in. Following Stahl (1989), we assume that a fraction of consumers can search costlessly, whereas the remaining consumers have identical and positive search costs.

We first assume that the type of price sorting (random, ascending, or descending) is publicly known and taken as given. For each scenario, we find that there is a unique symmetric equilibrium,[4] in which high-quality firms choose the same mixed pricing strategy, while low-quality firms charge marginal cost and earn zero profit. Thus, our model generates price dispersion for high-quality firms even in the presence of price sorting. Intuitively, the existence of shoppers rules out the possibility that high-quality firms' equilibrium price distribution have atoms at prices above the marginal cost. By contrast, high-quality firms setting prices equal to the marginal cost does not constitute an equilibrium because each high-quality firm is able to earn a positive expected profit. This is because of the positive probability that each high-quality firm is the only one that offers the high-quality product. This implies that there are no pure-strategy symmetric equilibria. In the mixed pricing equilibrium, high-quality firms trade off the gains from higher prices against the benefits of being sampled first if they price low in the case of ascending price sorting, as well as against the benefits of selling to shoppers in the case of descending price sorting.

We then study how total welfare, consumer surplus, and industry profit change with the type of price sorting. This proves to be technically difficult because it involves computing and comparing price distributions. The main difficulty arises from the case of descending price sorting. Standard random searching models are simpler because consumers' searching rule

on Amazon? Don't Be Duped Into Buying a Summary," by Jeffrey A. Trachtenberg, *The Wall Street Journal*, 7 February 2019).

[4]Following Stahl (1989), this article only focuses on symmetric Nash Equilibrium. In an asymmetric equilibrium, consumers have different posterior distributions on prices based on their observations, which makes it difficult to characterize the search behavior and firms' pricing strategies.

can be characterized by a history-invariant reservation price above which consumers continue searching. Under descending price sorting, however, the reserve price depends on both the price and the number of firms that have already been sampled. Such a history-dependent property makes the computation extremely complicated and intractable. However, despite of all these difficulties, we are able to obtain clear and neat results for the case of small search costs, which is highly relevant in the era of the Internet.

In online-shopping platforms, searching usually means clicking the links in the website, and the cost of each search is believed to be quite small. Interestingly, with small search costs, both ascending and descending price sorting have no impact on industry profit. That is, the equilibrium expected profit of a high-quality firm is the same under each type of price sorting. This is because under each sorting regime, the upper bound of the support of the equilibrium price distribution always equals the monopoly price. Note that charging the monopoly price gives a high-quality firm the same expected profit, regardless of the type of sorting, because consumers will buy from this firm if and only if all other firms are offering low-quality products.[5] Because any price in the equilibrium price support gives the same expected profit, a high-quality firm should earn identical expected profits under all sorting patterns.

We also find that, compared to random price sorting, both ascending and descending price sorting improve total welfare. Intuitively, when the search cost is small, in equilibrium, consumers always purchase the high-quality product as long as there is at least one high-quality firm in the market. Thus, price sorting affects total welfare only through the total costs of search activities. The less frequently consumers search, the higher total welfare will be. Note that, both ascending and descending sorting patterns provide useful price information that prevents inefficient searches by consumers. More precisely, with small search costs, consumers under random price sorting almost always seek to continue searching because the benefit of making an additional sample is very likely to be higher than the search cost. However, under ascending price sorting, a consumer who has sampled a high-quality product will stop searching, because the next product is still high-quality but with a higher price. Similarly, under descending price sorting, the consumer never continues searching once a low-quality product in encountered. Thus, consumers search less frequently under both ascending and descending price sorting regimes.

It is thus an immediate consequence that consumer surplus will increase when price sorting is applied. The fact that consumer surplus is affected

[5]This result is true only when the search cost is small. For very large search costs, an extreme case is that the consumer will never search. Thus, a relevant firm can sell at the monopoly price as long as its product is firstly sampled by consumers, even when there are other relevant products in the market.

by both equilibrium prices and total search costs indicates that the main impact of price sorting is to reduce the total occurrence of search costs, and its influences on pricing strategies are negligible. Actually, as the search cost approaches zero, sorting only yields second-order effects on firms' equilibrium pricing strategies.

Whereas our result on the total welfare and consumer surplus would be trivial if we held the price distribution fixed, it is a significant one because we take into account the strategic pricing of firms. In particular, even when high-quality firms have incentives to raise prices to be sampled early in the case of descending price sorting, the total welfare and consumer surplus are nevertheless improved relative to random search.

Finally, we endogenize the search order by allowing consumers to choose the sorting regime. We show that the game has a generically unique equilibrium in which consumers chooses the welfare-maximizing search order. Random price sorting is never chosen in equilibrium, and consumers always take advantage of sorting options whenever they are available. This is because switching from random price sorting to either ascending or descending price sorting benefits consumers by saving their total search costs, with only negligible influence on their purchase surplus. Whether consumers choose ascending or descending sorting depends on whether the market has a high proportion of high-quality firms. When the proportion of high-quality firms is low, descending sorting is welfare optimal. The intuition is that descending price sorting helps consumers reduce their search costs because it enables them to find the high-quality products faster.

Our model is related to the branch of the search literature concerned with price dispersion. Previous studies include Rob (1985), who studied a model of sequential search with heterogeneous consumer search costs in a Nash-Stackelberg game. He assumed that each firm acts as a Stackelberg leader so that consumers are able to observe deviations by firms before they actually search. Stahl (1989, 1996) considered the same situation, but with the Nash paradigm, which assumes that consumers know only the equilibrium price distribution and observe the actual price only when a search is made. Stahl (1989) assumed that there are two types of consumers: shoppers, who have zero search costs, and searchers, who have a common positive search cost. Stahl (1996) considered a more general distribution of consumers' search costs, which is atomless except possibly for a mass of shoppers. All the above models yield equilibria in which firms take mixed pricing strategies, thus produce price dispersion. Reinganum (1979) considered a sequential search model in which firms have different production costs. In equilibrium, each firm takes pure pricing strategy based on its marginal cost, and price dispersion is thus a result of cost heterogeneities. Finally, price dispersion can be generated by other models with fixed sample size search, including MacMinn (1980), Burdett and Judd (1983), and

Janssen and Moraga-González (2004). These models, however, did not provide explanations for price dispersion when price sorting is available.

Our work is also related to the strand of literature which considers non-random sequential search. The first type of non-random search models include Arbatskaya (2007), Armstrong et al. (2009), and Zhou (2011), who assumed that the order in which consumers search the products is exogenously fixed and common knowledge. Arbatskaya (2007) considered a market with homogenous goods, where consumers search only for better prices. The model generates an equilibrium in which firms' prices decline in the order of search. Armstrong et al. (2009) and Zhou (2011), on the other hand, considered horizontal product differentiation so that consumers search both for price and product fitness.[6] In contrast to Arbatskaya (2007), both Armstrong et al. (2009) and Zhou (2011) found equilibria in which prices increase in the order of search. The reason is that later-searched firms have more monopoly power than earlier-sampled one. Our model differs from the above ones in that the actual search order in our model can be affected by the behavior of both consumers and firms.

Our model belongs to the second type of non-random search models with endogenous search order. Weitzman (1979) considered a situation in which several heterogeneous alternatives are available for search, and the optimal search policy should specify not only when to terminate search, but also in which order the searcher should continue searching. This is regarded as the work-horse model of consumer search with endogenous search order. Based on Weitzman (1979), a series of recent studies have considered price-directed consumer search. The special feature of online markets, that price information is easy to obtain, has led these articles to assume that all prices are observable before search, and that consumers endogenously choose their search order based on prices (see e.g., Armstrong and Zhou (2011); Armstrong (2017); Haan et al. (2018); Choi et al. (2018); and Ding and Zhang (2018)). In contrast to the traditional models, a surprising result of this model is that an increase in the search cost may reduce equilibrium prices. The intuition is that consumers always visit the firm with the lowest price, and are more likely to buy from the first firm as search cost increases. Consequently, firms compete fiercely to be visited first by consumers by lowering their prices. In the present study, instead of assuming that all prices are observable before search, we consider the case in which consumers use the price sorting tools provided by online platforms, but may not be fully informed of all the prices before search, which we believe is more realistic. Moreover, because consumers always start searching

[6]Armstrong et al (2009) allowed one firm to be prominent, which will be sampled first by all consumers. Other firms will be sampled in a random order once the prominent firm's offer is rejected. Zhou (2011) generalized Armstrong et al (2009) by studying the case where the order in which firms are sampled is completely given.

from the lowest price in the aforementioned models, the existing literature cannot explain why descending price sorting exists. This is because these articles focus on products with horizontal differentiation, whereas we study a vertical differentiation model. If products are horizontally differentiated, prices do not convey any information about the valuations of the products, and consumers will search from the lowest price. In contrast, if products are vertically differentiated, high prices indicate that the products are more likely to have high values, and thus consumers may choose to search from the highest price.

The rest of this article is organized as follows. Section 2 describes the market and three types of price sorting are introduced to the search model. Section 3 considers the case in which the type of price sorting is fixed and publicly known, and derives the equilibria for the three types of sorting, respectively. Section 4 reconsiders the game by assuming that the type of price sorting is optimally chosen by consumers instead of exogenously given. Section 5 presents the impacts of ascending/descending price sorting, using the case of random price sorting as the benchmark model. Section 6 discusses the extension our model to the case where each search (sampling) may reveal information on multiple products, and shows that our main results still hold. Section 7 concludes and discusses possible extensions in the future. All the technical proofs are included in the Appendix.

2. THE MODEL

Consider an oligopoly setting in which $N > 1$ firms on an e-commerce marketplace compete in selling their products to a large group of consumers. A product can be either of high quality, with valuation a_h, or of low quality, with valuation a_l, where $a_h > a_l \geq 0$. Firms are thus of two types, high-type firms and low-type firms, which produce high-quality and low-quality products, respectively.

We assume that each firm privately observes its type, which is high with probability r, and low with probability $1 - r$, with $0 < r < 1$. Firms then announce prices simultaneously based on their types. To justify our assumption about firm types, suppose that there are a continuum of firms, of which a fraction r sell high-quality products. Each time a consumer enters a keyword, the e-commerce marketplace randomly draws N firms from the firm pool and displays them to the consumer. Finally, firms' production costs are normalized to zero, regardless of their types. Our setup implies that with probability $(1 - r)^{N-1}$, a high-quality firm only competes with low-quality firms, and is therefore effectively a monopolist. Such an assumption is important to generate price dispersion in our model. In the literature, Baye and Morgan (2001) and Janssen and Rasmusen

(2002) also rely on the assumption that a firm has a positive probability of becoming a monopolist in order to create price dispersion in their papers.

The demand side of the market consists of a unit mass of consumers, each of whom wants to buy one unit among the products. The consumer's utility from purchasing a product is given by

$$u = q - p,$$

where $q \in \{a_h, a_l\}$ is the product valuation, and p is the product price. Moreover, we assume that consumers have the option to exit the market without making any purchase. In this case, both the firm and the consumer receive zero utility.

There are two types of consumers. A proportion $\mu \in (0, 1)$ of consumers are assumed to be "shoppers,"[7] who costlessly observe all firms' prices and qualities. The remaining proportion $1 - \mu$ of consumers are "searchers"[8] who initially know nothing about the products' prices and qualities, but can obtain this information through sequential searches. As is commonly assumed in the literature, the first sample is free, but each subsequent sample costs the searcher $c > 0$. After incurring the search cost c, the searcher perfectly learns both the price and quality of the next product. Recall is costless, such that the searcher can purchase any previously sampled product at no additional cost.

Prior to the search process, searchers may sort the products by their prices. Three sorting regimes are considered: random price sorting (R), ascending price sorting (A), and descending price sorting (D). Under random price sorting, searchers sample the products in a random order, as is commonly assumed in the literature. When ascending (or descending, respectively) price sorting is applied, the products are displayed, and thus sampled from low prices to high prices (or from high prices to low prices, respectively). Thus, different from previous literature, our article studies the situation where firms' prices, even though unobservable, can affect consumers' search order.

Note that we assume consumers to be able to price sort, but cannot observe prices in the main model. One may wonder whether this is a reasonable assumption, because prices are normally observable on a web page on almost all e-commerce platforms. Our justification is as follows. There are a large number of firms that sell products of the same category

[7]The existence of shoppers is to avoid the Diamond Paradox for the cases of random search and descending price sorting. However, this assumption is not necessary for ascending price sorting, in which case searchers behave exactly the same as shoppers.

[8]We do not need the assumption of searchers to guarantee positive profits for firms, since each firm has a positive probability of being a monopolist. Actually, our model generates price dispersion even when all consumers are shoppers. The existence of searchers is to compare different types of price sorting.

on e-commerce platforms. However, a web page has limited space, so only a small number of products can be displayed. For example, Expedia.com only displays 10 different flight offers on each page. It is true that prices are observable on one page, but the prices of the products displayed on the next web page cannot be seen until consumers click and turn to the next page. Our main model is a simplified version, which assumes that only one product is displayed on each web page. In Section 6, we extend our model to allow multiple products to appear on each web page and consumers observe the prices of all products on one page, and show that our key insights are robust in this extension.

3. EXOGENOUS PRICE SORTING

This section assumes that the type of price sorting is exogenously given and becomes common knowledge to all parties. Given the type of price sorting $S \in \{R, A, D\}$, the market game proceeds as follows. Nature first draws each firm's quality type. Firms privately observe their types, and simultaneously set prices according to their types and the sorting regime. The shoppers observe all the prices and qualities, and purchase the product that gives the highest surplus, provided that this is non-negative; otherwise, they leave the market without making any purchases. Searchers, on the other hand, search optimally for the best product. The equilibrium concept we use is Perfect Bayesian Equilibrium. We focus on a symmetric equilibrium in which firms of the same type take the same pricing strategy.

Let $F_h^S(\cdot)$ and $F_l^S(\cdot)$ denote the pricing strategies for high-type and low-type firms respectively, for any sorting regime $S \in \{R, A, D\}$. For each firm type $t \in \{h, l\}$, the pricing strategy $F_t^S(\cdot)$ represents a cumulative distribution function, with $\overline{p}_t^S$ and $\underline{p}_t^S$ being the supremum and infimum of the price support,[9] respectively. For any sorting regime S and firm type t, it must satisfy $0 \leq \underline{p}_t^S \leq \overline{p}_t^S \leq a_t$.

The market equilibrium under price sorting $S \in \{R, A, D\}$ thus consists of firms' price strategies $(F_h^S(\cdot), F_l^S(\cdot))$ and consumers' search policy, such that (i) given firms' price distributions, the search policy is optimal for searchers; and (ii) for each t-type firm, $t \in \{h, l\}$, there is no strictly profitable deviation from the price distribution $F_t^S(\cdot)$ given that consumers follow the search policy, and all other firms follow the pricing strategies $(F_h^S(\cdot), F_l^S(\cdot))$ accordingly.

Given firms' pricing strategies $(F_h^S(\cdot), F_l^S(\cdot))$, the infimum of the set of all possible utilities is $\underline{u}^S = \min\left\{\underline{u}_h^S, \underline{u}_l^S\right\}$, where $\underline{u}_t^S = a_t - \overline{p}_t^S$ is the lowest

[9]In general, there may be gaps in the price support. For example, the support can be $[\underline{p}_S, \alpha] \cup [\beta, \overline{p}_S]$, where α and β are such that $\underline{p}_S < \alpha < \beta < \overline{p}_S$.

(or infimum) utility of a t-quality product, $t \in \{h, l\}$.[10] Throughout this article, we impose the following assumption to rule out uninteresting cases.
Assumption 1 The search cost c is sufficiently small such that, given each type of pricing sorting, in equilibrium, the searcher never stops sampling, and makes a purchase when she samples a product that gives the lowest possible (infimum) utility $\underline{u}^S$.

Before we proceed any further, it is helpful to first take a look at firms' equilibrium profits, which is important to simplify the analysis of optimal consumer search behavior. Our first proposition gives the firms' equilibrium profits.

PROPOSITION 1. *Under Assumption 1, for each type of price sorting, the equilibrium profit is positive for each high-type firm, but zero for low-type firms.*

Suppose that firm i is a high-type firm. Then with positive probability, all the other firms are low-types, in which case firm i can charge a price slightly below $a_h - a_l$ so that all shoppers will buy from it. This guarantees a positive expected profit for firm i. Assumption 1 implies that a product that has the utility $\underline{u}^S$ should have zero demand, because neither searchers nor shoppers would buy it. Since high-type firms earn positive profit, it must be the case that low-quality products has the utility $\underline{u}^S$, which leads to zero profits for low-type firms.[11]

Proposition 1 significantly simplifies our analysis. For the rest of this article, we assume without loss of generality that $a_l = 0$, that is, consumers attach zero valuation to low-quality products. We focus on equilibria in which low-type firms always charge zero price. The low-quality products in our model can be interpreted as the cheapest results in the online-shopping platform, which are irrelevant products that consumers would never buy.

For notational simplicity, we will denote the valuation of a high-quality product by $a > 0$ instead of a_h. For any $S \in \{R, A, D\}$, the high-type firm's pricing strategy is denoted by the price distribution $F_S(\cdot)$, with $\overline{p}_S$ and $\underline{p}_S$ denoting the upper and lower bounds of the price support respectively, which satisfies that $0 \le \underline{p}_S \le \overline{p}_S \le a$.

3.1. Consumer Search Behavior

In this subsection, we analyze consumers' optimal search strategies, given that high-type firms follow the pricing strategy $F_S(\cdot)$, for each $S \in \{R, A, D\}$.

Given high-type firms' pricing strategy $F_S(p)$, a searcher would continue searching if and only if the expected benefit of doing so exceeds the total

[10] The price support can be a right-open interval, i.e., the upper bound of the price support has zero density.

[11] For high-type firms, the equilibrium price support is right-open at the upper bound.

subsequent search costs. An additional sample gives the searcher two types of benefits,[12] (i) searching for a better quality, and (ii) searching for a lower price. In order to examine both types of benefits, we impose the following assumption.

Assumption 1' The search cost c is sufficiently small such that the searcher never stops sampling and makes a purchase with any type of price sorting when she observes a product with a utility that is no higher than $a - \overline{p}_S$.

Note that $a - \overline{p}_S$ is the lowest possible utility of a high-quality product under sorting regime $S \in \{R, A, D\}$. Assumption 1' thus implies that, whereas low-quality products are never acceptable, a high-quality product does not necessarily stop searchers from continuing their search. In particular, searchers stop sampling only when the price of the high-quality product found is low enough. Note that Assumption 1' is satisfied as the search cost c approaches zero.[13] The assumption of small search cost prevails in online-shopping platforms, where a search usually means clicking a link in the website.

Random Price Sorting

Under random price sorting, due to Assumption 1', the searcher always continues sampling when observing a low-quality product. Suppose the searcher has sampled some high-quality products, of which the lowest price is $p \in [\underline{p}_R, \overline{p}_R]$. Then, the expected benefit of sampling one more product is given by

$$\phi_R(p) = r \int_{\underline{p}_R}^{p} (p - x) dF_R(x). \tag{1}$$

Let $\hat{p}_R$ denote the reservation price, which uniquely solves the equation $\phi_R(\hat{p}_R) = c$. It follows from Kohn and Shavell (1974) and Weitzman (1979) that the optimal stopping rule under random sorting is to stop sampling and make a purchase whenever the searcher observes a high-quality product with a price below the reservation price $\hat{p}_R$; and keep sampling otherwise.

Ascending Price Sorting

The optimal search policy is quite simple under ascending price sorting. Consumers never continue sampling if they observe a high-quality product because the next product would also be of high quality but with a higher price. Thus, the optimal search policy is to keep sampling until the first high-quality product appears. In other words, under ascending price sorting, both searchers and shoppers buy the high-quality product with the lowest price.

[12]Whereas both types of benefits can exist under random price sorting, only the first (or second) type exists under ascending (or descending) price sorting.

[13]Note that Assumption 1' is stronger than Assumption 1.

Descending Price Sorting

Finally, let us consider the optimal search policy for descending price sorting. Because the low-type firms charge zero, if a searcher observes a low-quality product, it means that all the subsequent products are low-types, and all the high-quality products have been sampled. Thus, the searcher should stop searching and buy the high-quality product with the lowest price. We now investigate whether a searcher should continue sampling when the currently viewed product is of high quality.

A searcher is said to be at stage n of the search process if she has sampled n high-quality products (with $N-n$ product left unsampled), for $n = 1, \ldots, N-1$. Now consider a searcher at stage n, with $p \in [\underline{p}_D, \overline{p}_D]$ being the lowest price that she has found, that is, the price of the last sampled product. According to Bayes' Rule, the expected benefit of sampling an additional product is given by

$$\phi_D(p,n) = \int_{\underline{p}_D}^{p} (p-x) d\left(\frac{1-r+rF_D(x)}{1-r+rF_D(p)}\right)^{N-n}, \text{ for } n = 1, \ldots, N-1. \quad (2)$$

Unlike the case of random price sorting, the expected benefit of an additional sample under descending price sorting, $\phi_D(p,n)$, depends not only on the current price, but also on the number of products left unsampled.

To understand expression (2), define the random variable $\hat{X}(p,n)$ as the product price of the next sample, where $\hat{X}(p,n)$ equals zero if the next product is of low quality, and $\hat{X}(p,n) \in [\underline{p}_D, p]$ if the next product is a high-type. We will solve the distribution function of $\hat{X}(p,n)$ as follows.

Firstly, define $\hat{Y}_i$ to be the random price for each firm i ex ante, where $i = 1, \ldots, N$. That is,

$$\hat{Y}_i = \begin{cases} \hat{Z}_i & \text{with probability } r \\ 0 & \text{with probability } 1-r \end{cases},$$

where $\hat{Z}_i$s are independently and identically drawn from the interval $[\underline{p}_D, \overline{p}_D]$ according to the distribution function $F_D(\cdot)$. The $\hat{Y}_i$s are also independently and identically distributed, with $\Pr\left(\hat{Y}_i \leq y\right) = 1 - r + rF_D(y)$, for any $y \in \{0\} \cup [\underline{p}_D, \overline{p}_D]$.

When the searcher has sampled n products, with the lowest price $p \in [\underline{p}_D, \overline{p}_D]$, the price of the next product, $\hat{X}(p,n)$, must be the highest among all the remaining $\hat{Y}_i$s, for $i = n+1, \ldots, N$, conditional on these prices being no higher than p. That is,

$$\hat{X}(p,n) = \max\left\{\hat{Y}_{n+1}(p), \hat{Y}_{n+2}(p), \ldots, \hat{Y}_N(p)\right\},$$

where $\hat{Y}_i(p) = \hat{Y}_i | \hat{Y}_i \leq p$.

Thus, the cumulative distribution function $G_D(x;p,n)$ associated with $\hat{X}(p,n)$ is given by

$$\begin{aligned} G_D(x;p,n) &= \Pr\left(\hat{X}(p,n) \leq x\right) \\ &= \Pr\left(\hat{Y}_i(p) \leq x, \text{ for } i = n+1, \ldots, N\right) \\ &= \left[\Pr\left(\hat{Y}_i(p) \leq x\right)\right]^{N-n} \\ &= \left[\Pr\left(\hat{Y}_i \leq x | \hat{Y}_i \leq p\right)\right]^{N-n} \\ &= \left[\frac{\Pr\left(\hat{Y}_i \leq x\right)}{\Pr\left(\hat{Y}_i \leq p\right)}\right]^{N-n} \\ &= \left(\frac{1-r+rF_D(x)}{1-r+rF_D(p)}\right)^{N-n}, \text{ for any } x \in [\underline{p}_D, p]. \end{aligned}$$

An additional sample benefits the consumer only when the next product is still of high quality, that is, when $\hat{X}(p,n) \in [\underline{p}_D, p]$. Thus, the expected benefit of observing $\hat{X}(p,n)$ is given by

$$\phi_D(p,n) = \int_{\underline{p}_D}^{p} (p-x) dG_D(x;p,n),$$

which gives expression (2).

One can prove that $\phi_D(p,n)$ increases with p as long as $(1-r+rF_D(p))^{N-n} - (1-r)^{N-n}$ is log-concave in p.[14] Define $\hat{p}_{D,n}$ as the reservation price at stage n, which solves $\phi_D(p,n) = c$. Thus, at stage n, the benefit of sampling the next product exceeds the search cost if and only if the current price p is higher than the reservation price $\hat{p}_{D,n}$ at stage n. The following lemma states that the expected benefit $\phi_D(p,n)$ decreases with n if the total number of firms is not too large.

LEMMA 1. *There exists a positive number* $\overline{N}$ *such that* $\frac{\partial \phi_D(p,n)}{\partial n} \leq 0$ *for each* $n = 1, \ldots, N-1$, *as long as* $N \leq \overline{N}$.

Recall that the reservation price $\hat{p}_{D,n}$ solves $\phi_D(p,n) = c$. Thus, the condition $\frac{\partial \phi_D(p,n)}{\partial n} \leq 0$ implies that $\hat{p}_{D,n}$ increases along the search process.

[14] We will show later that log-concavity is actually satisfied for the optimal price distribution under descending sorting.

The next lemma states that with a group of increase reservation prices, the optimal stopping rule under descending price sorting is "myopic" and is fully characterized by the reservation prices.

Proposition 2. *Suppose $N \leq \overline{N}$. The optimal search policy under descending price sorting is that, at stage n, a searcher continues sampling if and only if his current price is higher than $\hat{p}_{D,n}$, for any $n = 1, \ldots, N-1$. In other words, the searcher stops sampling and purchases from the current firm at the first stage at which the observed price falls below the corresponding reservation price.*

The intuition is as follows. At each stage, a searcher would obviously keep sampling if his current price is higher than the reservation price. However, when the current price is below the reservation price, the expected benefit of making one more sample at that stage is lower than the search cost. Moreover, because the reservation prices are higher, the expected benefits of continuing searching at future stages will never cover the subsequent search costs. Hence, the total benefits of continuing to search is strictly less than the total search costs, even when future behaviors are taken into consideration. As a result, searchers will stop searching and buy at the current price. Note that the assumption of increasing reservation prices is crucial in obtaining a "myopic" stopping rule. To be precise, it makes the next product less worth sampling as fewer products are left unsampled, so that if an additional sample is not worth it in the current stage, it will never be worth it in the future.

However, $\frac{\partial \phi_D(p,n)}{\partial n} \leq 0$ may not always hold if there are a large number of products. In this case, searchers are "safe" at the beginning of the search process in the sense that the next product is not likely to be a low-quality. At that point of the search process, continuing searching can become more beneficial because with fewer products left, the price of the next high-quality product will be lower under descending price sorting, which brings a higher expected benefit to searchers. The optimal stopping rule for this case can be complex and no longer have the "myopic" property. In particular, searchers may decide to continue searching even when the benefit of sampling the next product in the current stage is smaller than the search cost, as long as the net benefits of future searches are high. Throughout this article, we simply assume that the number of products in the market is not large, such that $\frac{\partial \phi_D(p,n)}{\partial n} \leq 0$ for any n and the reservation prices are increasing along the search process.

3.2. Market Equilibrium

We now derive the high-type firms' optimal pricing strategies for each type of price sorting. For the case of random price sorting, we use a method similar to Stahl (1989)'s. A consumer reservation price $\hat{p}_R$ is exogenously fixed, conditional on which we derive the optimal price distribution for high-type firms. The obtained price distribution will again result in a new reservation price $\hat{p}'_R$, according to discussion in the previous section. Finally, the equilibrium requires that the pre-given reservation price be consistent, that is, $\hat{p}_R = \hat{p}'_R$. This condition gives the equilibrium reservation price.

Similarly, for the case of descending price sorting, we exogenously fix a group of reservation prices: $\hat{p}_{D,1} \leq \hat{p}_{D,2} \leq \cdots \leq \hat{p}_{D,N-1}$, and solve for high-type firms' optimal price distribution. Finally, the pre-given reservation prices should be equal to those derived from the optimal price distribution, which gives the equilibrium.

Random Price Sorting

We first consider random price sorting. Following Stahl (1989), we first exogenously fix a reservation price $\hat{p}_R$, conditional on which we derive the optimal price distribution for high-type firms. The obtained price distribution in turn produces a new reservation price $\hat{p}'_R$ according to the optimal search rule. The equilibrium will be pinned down by the consistence of the pre-given reservation price, that is, $\hat{p}_R = \hat{p}'_R$.

Let $\hat{p}_R \in (0, a)$ be a pre-given reservation price.[15] The optimal search rule is thus to continue sampling if the product is a low-type, or if it is a high-type but with a price higher than $\hat{p}_R$; stop sampling and make a purchase when observing a high-quality product with a price lower than $\hat{p}_R$. Let $F_R(p; \hat{p}_R)$ be the high-type firms' optimal price distribution conditional on the above search policy. The following lemma states that the equilibrium price distribution has no mass points.

Lemma 2. *$F_R(p; \hat{p}_R)$ is atomless.*

The intuition for Lemma 2 is that under random price sorting, a high-type firm can discretely increase its demand by slightly lowering its price below the atom. On the one hand, lowering the price obviously increases shoppers' demand discretely. On the other hand, for searchers, a lower price does not change the search order, but makes the product more attractive so that both "fresh demand" and "returning demand"[16] are increased. Finally, according to Proposition 1, setting prices equal to the marginal cost

[15]Under Assumption 1', the reservation price $\hat{p}_R$ should be strictly lower than a. Note that $\hat{p}_R = a$ means the searcher would immediately accept a high-quality product with price a, which violates Assumption 1' because this high-quality product gives the lowest possible utility among all products.

[16]Similar to Zhou (2011), we refer to "fresh demand" as the demand from searchers who buy immediately after sampling the product, and "returning demand" as the demand from searchers who have sampled all the products.

does not constitute an equilibrium because each high-type firm is able to earn a positive expected profit ex ante.

Recall that $\overline{p}_R$ and $\underline{p}_R$ are the maximal and minimal elements of the price support. The following lemma gives all the possible values of $\overline{p}_R$.

LEMMA 3. *Under Assumption 1', the upper bound of the price support* $\overline{p}_R$ *equals* a.

The reason is as follows. Suppose a high-type firm sets its price $p = \overline{p}_R < \hat{p}_R$. Then, searchers who sample this firm at the first time will buy from it; and shoppers will buy from it as long as this firm is the only high-type firm in the market. It is obvious that this firm could gain a strictly higher profit by increasing its price from $\overline{p}_R$ to $\hat{p}_R$, as doing so would not change its demand, a contradiction to the optimality of the price distribution. Similarly, if $\hat{p}_R < \overline{p}_R < a$, then charging price a is strictly better than charging $\overline{p}_R$. Thus, the upper bound of the price support should be either $\hat{p}_R$ or a. Finally, Assumption 1' rules out the case of $\overline{p}_R = \hat{p}_R$, because otherwise the searcher stops sampling whenever she observes a high-quality product, regardless of its price, which violates Assumption 1'.

Now we express the demand of a representative high-quality firm, labelled as firm i, as follows

$$D(p) = \begin{cases} \mu\left[1 - rF_R(p;\hat{p}_R)\right]^{N-1} + \frac{(1-\mu)\left[1-(1-r\hat{F}_R)^N\right]}{rN\hat{F}_R} & \text{if } \underline{p}_R \le p \le \hat{p}_R \\ \left[1 - rF_R(p;\hat{p}_R)\right]^{N-1} & \text{if } \hat{p}_R < p \le a \end{cases}, \tag{3}$$

where $F_R(p;\hat{p}_R)$ is the equilibrium price distribution, and $\hat{F}_R \equiv F_R(\hat{p}_R;\hat{p}_R)$ is the probability that a high-type firm's price is below the reservation price.

To understand the demand function, note that by charging a price above $\hat{p}_R$, firm i loses all the fresh searchers who sample its product the first time. Thus, both shoppers and searchers buy from firm i if and only if it yields the highest surplus among all the products, which happens with probability $\left[1 - rF_R(p;\hat{p}_R)\right]^{N-1}$.

For $\underline{p}_R \le p \le \hat{p}_R$, $\mu\left[1 - rF_R(p;\hat{p}_R)\right]^{N-1}$ and $\frac{(1-\mu)\left[1-(1-r\hat{F}_R)^N\right]}{rN\hat{F}_R}$ respectively represents firm i's demand from shoppers and searchers. To understand the second term, note that a price below $\hat{p}_R$ allows firm i to retain all fresh searchers. Similar to Wolinsky (1986), the second term can be rewritten as $\frac{1-\mu}{N}\sum_{m=0}^{N-1}\left(1 - r\hat{F}_R\right)^m$, of which a typical component, $\frac{1-\mu}{N}(1 - r\hat{F}_R)^m$, denotes the fraction of searchers who buy from firm i after having sampled m firms.

Notice from (3) that a high-type firm's demand drops discretely as its price p exceeds $\hat{p}_R$. Intuitively, as firm i increases its price slightly above $\hat{p}_R$, it loses the whole "fresh demand" from searchers, which results in a discrete drop in the total demand. The discontinuity of the demand function results in a gap in the support of $F_R(p;\hat{p}_R)$, which starts at the reservation price $\hat{p}_R$. Note that the demand discretely drops only once and there is at most one gap, because otherwise a high-type firm can always raise its price into the gap without affecting its demand. Thus, the support of $F_R(p;\hat{p}_R)$ is $[\underline{p}_R, \hat{p}_R] \cup [p'_R, a)$ for some $p'_R \in (\hat{p}_R, a)$ which will be determined later.

It is interesting to point out that our result is quite different from Stahl (1989)'s, where the reservation price equals the upper bound of the equilibrium price, so that there is no gap in the price support. As a result, in Stahl (1989)'s model, consumers never continue searching after they observe the first sample. Search friction provides monopoly power to each firm, which is the only reason why firms can earn positive expected profits.

Note that under random price sorting, Stahl (1989)'s model is an extreme case of ours when $r = 1$. In our model, when $r < 1$, there is positive probability that a high-type firm becomes the only high-type firm in the market. This gives a new reason that supports firms with positive profits. Thus, firms in our model can earn positive expected profits even when the upper bound of the price support exceeds the reservation price, which is the only case where a gap appears.[17]

Now we derive the conditional optimal price distribution. The optimality of $F_R(p;\hat{p}_R)$ requires that a high-type firm earns the same expected profits, denoted by π_R, when charging any price in the support. In addition, charging any price outside the price support gives an expected profit no higher than π_R. According to the demand function, we have

$$\pi_R = aD(a) = a(1-r)^{N-1}.$$

Solving $\pi_R = pD(p)$, our results are summarized in the following proposition.

PROPOSITION 3. *Under Assumption 1', the symmetric equilibrium price distribution conditional on the reservation price $\hat{p}_R \in (0,a)$ is given by*

$$F_R^*(p;\hat{p}_R) = \begin{cases} \frac{1}{r}\left\{1 - \left(\frac{\pi_R}{\mu p} - \frac{(1-\mu)\left[1-(1-r\hat{F}_R)^N\right]}{\mu r N \hat{F}_R}\right)^{\frac{1}{N-1}}\right\} & \text{if } \underline{p}_R \le p \le \hat{p}_R \\ \frac{1}{r}\left[1 - \left(\frac{\pi_R}{p}\right)^{\frac{1}{N-1}}\right] & \text{if } p'_R \le p \le a \end{cases}, \tag{4}$$

[17]A gap appears only when the search cost is sufficiently small that Assumption 1' holds. When the search cost is high, the reservation price becomes so high that it is not profitable for a high-type firm to charge a price $p = a$. In this case, the reservation price becomes the upper bound of the price support so that there is no gap.

where $\hat{F}_R$ is the solution to the following equation (5), and $\underline{p}_R$ and p'_R are given by (6) and (7), respectively.

$$\hat{p}_R = \frac{\pi_R}{\mu(1-r\hat{F}_R)^{N-1} + \frac{(1-\mu)\left[1-(1-r\hat{F}_R)^N\right]}{rN\hat{F}_R}} \tag{5}$$

$$\underline{p}_R = \frac{\pi_R}{\mu + \frac{(1-\mu)\left[1-(1-r\hat{F}_R)^N\right]}{rN\hat{F}_R}} \tag{6}$$

$$p'_R = \frac{\pi_R}{(1-r\hat{F}_R)^{N-1}} \tag{7}$$

It is easy to verify that any price outside the support $[\underline{p}_R, \hat{p}_R] \cup [p'_R, a)$ produces a profit lower than π_R. Similar to Stahl (1989), our last step is to solve for a consistent reservation price, which, according to Equation (1) and the condition that $\phi_R(\hat{p}_R) = c$, is determined by the following equation

$$r\int_{\underline{p}_R}^{\hat{p}_R} (\hat{p}_R - x)dF^*_R(x;\hat{p}_R) = c. \tag{8}$$

The following proposition gives the existence and uniqueness of the consistent reservation price.

Proposition 4. *Under Assumption 1', there is a unique $\hat{p}_R$ that solves Equation (8). Moreover, the reservation price increases as the search cost becomes larger.*

As an extreme case, when search cost c approaches zero, the probability $\hat{F}_R$ converges to zero. According to (5)-(7), this implies that all $\underline{p}_R$, $\hat{p}_R$, and p'_R converge to the same value $\pi_R = a(1-r)^{N-1}$. Thus, the gap in the equilibrium price support disappears and the limit distribution becomes

$$F^*_R(p) = \frac{1}{r}[1 - \left(\frac{\pi_R}{p}\right)^{\frac{1}{N-1}}], \text{ for } p \in [\pi_R, a). \tag{9}$$

Intuitively, this price distribution is the one that would obtain if all consumers were shoppers, so that only the product with the highest surplus would be purchased.

Ascending Price Sorting

In this subsection, we will examine the symmetric equilibria under ascending price sorting. Let $F_A(p)$ be the high-type firm's optimal pricing strategy, with $\overline{p}_A$ and $\underline{p}_A$ as the highest and lowest elements of the price support, where $0 \leq \underline{p}_A \leq \overline{p}_A \leq a$.

LEMMA 4. *$F_A(p)$ is atomless.*

The intuition is the same as that for lemma 2. On the one hand, each high-type firm's expected profit is strictly positive ex-ante, which rules out the possibility of massing the price at the marginal cost; on the other hand, a price cut below any atom above the marginal cost increases the demand discretely.

LEMMA 5. *$\overline{p}_A = a$ and there is no gap in the price support.*

The reason is as follows. If the upper bound of the price support is strictly below a, then any price between $\overline{p}_A$ and a should result in the same demand as $\overline{p}_A$ does. This obviously contradicts the optimality of $F_A(p)$. Similarly, if there is a gap in the price support, then the high-type firm can always raise its price into the gap without affecting its demand, hence strictly increasing its profits.

Given the price distribution $F_A(p)$ and the consumer's search rule, both shoppers and searchers will purchase from the high-type firm that charges the lowest price. In other words, searchers behave exactly the same as shoppers. A high-type firm's total demand is thus given by

$$D(p) = [1 - rF_A(p)]^{N-1}. \tag{10}$$

Let π_A be the high-type firm's expected profit under ascending price sorting, then

$$\pi_A = aD(a) = a\,(1-r)^{N-1}.$$

The optimal price distribution $F_A^*(p)$ can thus be obtained by solving $\pi_A = pD(p)$.

PROPOSITION 5. *Under Assumption 1', the symmetric equilibrium price distribution under ascending sorting is*

$$F_A^*(p) = \frac{1}{r}[1 - \left(\frac{\pi_A}{p}\right)^{\frac{1}{N-1}}], \text{ for } p \in [\underline{p}_A, a), \tag{11}$$

where

$$\underline{p}_A = \pi_A = a\,(1-r)^{N-1} \tag{12}$$

Descending Price Sorting

We now derive the equilibrium price distribution for descending price sorting. Let $\hat{p}_D = (\hat{p}_{D,1}, \ldots, \hat{p}_{D,N-1})$ be a set of pre-given reservation prices such that $0 < \hat{p}_{D,1} \leq \hat{p}_{D,2} \leq \ldots \leq \hat{p}_{D,N-1} < a$. The conditional optimal price distribution for high-type firms is denoted by $F_D(p; \hat{p}_D)$. Let $\overline{p}_D$ and $\underline{p}_D$ be the upper and lower bounds of the price support, where $0 \leq \underline{p}_D \leq \overline{p}_D \leq a$.

In contrast to Lemma 2 and Lemma 4, under descending price sorting, we have to assume that the fraction of shoppers is sufficiently high so that $F_D(p; \hat{p}_D)$ is atomless.[18] Intuitively, charging a price slightly below an atom has two effects. On the one hand, it would reduce a high-type firm's "fresh demand" from searchers because other firms that charge the atom price would be sampled ahead of this one. On the other hand, it discretely increases the demand from shoppers. Thus, the proportions of shoppers and searchers plays an important role in determining overall change in demand. When μ is high enough, the loss of "fresh demand" is dominated by the discrete increase of the demand from shoppers. Thus, lowering the price slightly below an atom makes a profitable deviation.

Similar to Lemma 3 and Lemma 5, the following lemma characterizes the upper bound of the price support.

Lemma 6. *Under Assumption 1', the upper bound of the price support* $\overline{p}_D$ *equals* a.

For ease of notation, define

$$\hat{p}_{D,0} \equiv \underline{p}_D \text{ and } \hat{p}_{D,N} \equiv \overline{p}_D = a.$$

We now derive the high-type firm's demand function for each price interval, $p \in (\hat{p}_{D,n}, \hat{p}_{D,n+1}]$, where $n = 0, 1, \ldots, N-1$. Let $D_0(p)$ and $D_1(p)$ be the demand from shoppers and searchers, respectively. Thus, the total demand is given by

$$D(p) = \mu D_0(p) + (1-\mu)\, D_1(p), \tag{13}$$

where $D_0(p) = [1 - rF_D(p; \hat{p}_D)]^{N-1}$ as shoppers buy at price p if and only if all other products are either low-quality or have a price higher than p.

[18] As μ approaches 1, the optimal price distribution is obviously atomless because all consumers are shoppers.

To derive the demand from searchers, note that

$$\begin{aligned} D_1(p) &= \Pr\{\text{searchers buy at price } p\} \\ &= \sum_{k=0}^{N-1} \Pr\{\text{searchers buy at price } p \text{, with } k \text{ prices higher than } p\} \\ &= \sum_{k=0}^{N-1} D_1(p,k), \end{aligned} \tag{14}$$

where $D_1(p,k) = \Pr\{$searchers buy at price p, and there are k prices higher than $p\}$, for $k = 0, 1, \ldots, N-1$.

For any $p \in (\hat{p}_{D,n}, \hat{p}_{D,n+1}]$, we have

$$D_1(p,k) = \begin{cases} C(N-1,k)\,(1-r)^{N-1-k}\,[r - rF_D(p;\hat{p}_D)]^k\,, & \text{if } k < n \\ C(N-1,k)\,(1-r+rF_D(p;\hat{p}_D))^{N-1-k}\,[r - rF_D(p;\hat{p}_D)]^k\,, & \text{if } k = n \\ C(N-1,k)\,(1-r+rF_D(p;\hat{p}_D))^{N-1-k}\left[r - r\hat{F}_{D,k}\right]^k, & \text{if } k > n \end{cases},$$

where $\hat{F}_{D,k} = F_D(\hat{p}_{D,k};\hat{p}_D)$ and $C(N-1,k)$ is given by

$$C(N-1,k) = \frac{(N-1)!}{(N-1-k)!k!}, \text{ for any } k \le N-1.$$

To understand the expression of $D_1(p,k)$, note that if there are $k < n$ prices higher than p, due to the fact that $p > \hat{p}_{D,n}$, searchers will not accept price p without further searches, which means that searchers buy at price p only if all the products with prices lower than p are low-quality. Thus, in this case, $D_1(p,k)$ equals the probability that k out of $N-1$ prices are higher than p, and the remaining $N-1-k$ prices are zero.[19]

If $k = n$, then price p is the $(n+1)\,th$ highest price of all products. Because $\hat{p}_{D,n} < p \le \hat{p}_{D,n+1}$, searchers never accept any price higher than p, and buy at p immediately without further searches. Thus, $D_1(p,k)$ equals the probability that k out of $N-1$ prices are higher than p, and the remaining $N-1-k$ prices are lower than p.

Finally, if $k > n$, due to the increasing order of reservation prices, price p will be immediately accepted by searchers because $p \le \hat{p}_{D,n+1} \le \hat{p}_{D,k}$. Thus, searchers buy at p only if they do not accept any prices higher than p, that is, the previous k prices should be higher than the reservation price $\hat{p}_{D,k}$. In other words, $D_1(p,k)$ is the probability that k out of $N-1$ prices are higher than $\hat{p}_{D,k}$, and the remaining $N-1-k$ prices are lower than p.

[19]Zero price means the product is of low quality.

According to (13) and (14), the total demand for $p \in (\hat{p}_{D,n}, \hat{p}_{D,n+1}]$ is given by

$$\begin{aligned} D(p) &= \mu\left[1 - rF_D(p;\hat{p}_D)\right]^{N-1} \qquad (15) \\ &+ (1-\mu)\sum_{k=0}^{n-1} C(N-1,k)\,(1-r)^{N-1-k}\left[r - rF_D(p;\hat{p}_D)\right]^k \\ &+ (1-\mu)\,C(N-1,n)\,(1-r+rF_D(p;\hat{p}_D))^{N-1-n}\left[r - rF_D(p;\hat{p}_D)\right]^n \\ &+ (1-\mu)\sum_{k=n+1}^{N-1} C(N-1,k)\,(1-r+rF_D(p;\hat{p}_D))^{N-1-k}\left[r - r\hat{F}_{D,k}\right]^k . \end{aligned}$$

Similar to the case of random price sorting, the above demand function is discontinuous at each reservation price $\hat{p}_{D,n}$. Precisely, for each $n = 1, \ldots, N-1$, we have that

$$\begin{aligned} &\lim_{p \to \hat{p}_{D,n}-} D(p) - \lim_{p \to \hat{p}_{D,n}+} D(p) \\ =\ & (1-\mu)\,C(N-1,n-1)\left(r - r\hat{F}_{D,n}\right)^{n-1}\left[\left(1 - r + r\hat{F}_{D,n}\right)^{N-n} - (1-r)^{N-n}\right] \\ >\ & 0. \end{aligned}$$

The intuition is as follows. When a high-type firm slightly increases its price above $\hat{p}_{D,n}$, its demand from searchers drops discretely conditional on the case that this firm charges the $n-th$ highest price among all firms, because the high-type firm no longer captures all the "fresh demand" from searchers when it is $n-th$ sampled. As a result, there must be $N-1$ gaps in distribution $F_D(p;\hat{p}_D)$'s support. The price support is the union of N intervals, which has the form $\cup_{n=0}^{N-1}[p'_{D,n}, \hat{p}_{D,n+1})$, where $p'_{D,n} \in (\hat{p}_{D,n}, \hat{p}_{D,n+1})$ for any $n = 1, \ldots, N-1$ and $p'_{D,0} = \hat{p}_{D,0} = \underline{p}_D$.

It follows from the demand function (15) that $D(a) = (1-r)^{N-1}$. Thus, the high-type firm's expected profit is

$$\pi_D = aD(a) = a(1-r)^{N-1}.$$

The conditional optimal price distribution is then given in the following proposition.

Proposition 6. *Under Assumption 1', let* $F_D^*(p;\hat{p}_D)$ *be the symmetric equilibrium price distribution conditional on the set of reservation prices such that* $0 < \hat{p}_{D,1} \le \hat{p}_{D,2} \le \cdots \le \hat{p}_{D,N-1} < a$. *Then, for any* $n =$

$0, 1, \ldots, N-1$ *and* $p \in [p'_{D,n}, \hat{p}_{D,n+1}]$, $F^*_D(p; \hat{p}_D)$ *solves*

$$\frac{\pi_D}{p} = D(p),$$

where $D(p)$ *is given in (15),* $\underline{p}_D$, $\hat{F}_{D,n}$ *and* $p'_{D,n}$ *for* $n \geq 1$ *are determined by*[20]

$$\frac{\pi_D}{\underline{p}_D} = \mu + (1-\mu) \sum_{k=0}^{N-1} C(N-1,k) \, (1-r)^{N-1-k} \left[r - r\hat{F}_{D,k} \right]^k, \tag{16}$$

$$\hat{F}_{D,n} = F^*_D(\hat{p}_{D,n}; \hat{p}_D) = F^*_D(p'_{D,n}; \hat{p}_D),$$

$$\frac{\pi_D}{p'_{D,n}} = \lim_{p \to p'_{D,n}} D(p).$$

It follows from (16) that $[1 - r + rF^*_D(p; \hat{p}_D)]^{N-n} - (1-r)^{N-n}$ is log-concave in p within the price support, which verifies that the expected benefit of sampling another product at stage n, $\int_{\underline{p}_D}^{p} (p-x) d \left(\frac{1-r+rF^*_D(x;\hat{p}_D)}{1-r+rF^*_D(p;\hat{p}_D)} \right)^{N-n}$, is increasing in p. A set of consistent reservation prices $\hat{p}_D$ thus requires that

$$\int_{\underline{p}_D}^{\hat{p}_{D,n}} (\hat{p}_{D,n} - x) d \left(\frac{1 - r + rF^*_D(x; \hat{p}_D)}{1 - r + r\hat{F}_{D,n}} \right)^{N-n} = c, \text{ for all } n = 1, \ldots, N-1. \tag{17}$$

Finally, the existence and uniqueness of the set of consistent reservation prices is given in the following proposition.

PROPOSITION 7. *Under Assumption 1', there is a unique set of reservation prices* $(\hat{p}_{D,1}, \ldots, \hat{p}_{D,N-1})$ *that solves Equation (17).*

Similar to the case of random price sorting, as search cost c approaches zero, all reservation prices $\hat{p}_{D,n}$, as well as the lower bound $\underline{p}_D$, converge to $\pi_D = a(1-r)^{N-1}$. Thus, the limit price distributions under descending and random price sorting are the same, which in turn coincide with the equilibrium price distribution under ascending price sorting, given by (11) and (12).

[20] To verify the optimality of $F^*_D(p; \hat{p}_D)$, one can easily check that any price outside the support $\cup_{n=0}^{N-1} [p'_{D,n}, \hat{p}_{D,n+1})$ cannot be a profitable deviation.

4. ENDOGENIZING PRICE SORTING

In many real-world situations, consumers have the option to choose the best sorting regime. This section studies the endogenization of price sorting. The timing of the new game is as follows. Firstly, nature draws each firm's quality type. Firms then privately observe their own qualities, and simultaneously set prices based on their quality types. Consumers then behave as follows. Shoppers are always fully informed and simply purchase the product with the highest utility, as long as it is non-negative. Searchers, on the other hand, first choose the type of price sorting,[21] and then search optimally. We still aim at symmetric perfect Bayesian equilibria.

Because choosing the type of price sorting becomes part of consumers' strategy, the equilibrium in this section should consist of the high-type firm's price distribution $F(p)$, the price sorting $S \in \{R, A, D\}$, and a search policy such that (i) given the price sorting S and the search policy, the price distribution $F(p)$ is optimal for each high-type firm; (ii) given the price distribution $F(p)$ and the type of price sorting S, the search policy is optimal for searchers; and (iii) given the price distribution $F(p)$, sorting S, as well as its associated optimal search policy, is the best option among the three, in the sense that it gives searchers the highest expected surplus.

Recall that $F_S(p)$ is the equilibrium price distribution when price sorting S is exogenously fixed, for $S \in \{R, A, D\}$. The above definition implies that, when price sorting is endogenized, there are only three equilibrium candidates $(F_R(p), R), (F_A(p), A)$ and $(F_D(p), D)$.[22] Moreover, to check whether $(F_S(p), S)$ is an equilibrium, we only need to check that, given the price distribution $F_S(p)$, whether sorting S gives the highest expected consumer surplus among the three. Our result is given in the following proposition.

PROPOSITION 8. *When price sorting is endogenized, with small search cost c, (i) consumers never choose random price sorting in an equilibrium; (ii) when $r > 1/2$, the unique equilibrium is $(F_A(p), A)$, that is, ascending sorting is chosen in equilibrium; (iii) when $r < 1/2$, the unique equilibrium is $(F_D(p), D)$, that is, descending sorting is chosen in equilibrium.*

Proposition 8 (i) states that consumers always take advantage of sorting options whenever they are available. The reason is that for any given price distribution, a searcher can always get better off by switching from random sorting to either ascending sorting or descending sorting. This is

[21] Because consumers are identical ex ante, we focus on equilibria where all searchers choose the same sorting regime.

[22] We have omitted the associated search policy in the expression of an equilibrium just for simplicity of notation.

because ascending/descending price sorting provides consumers with price information during the search process, which enables consumers to reduce the number of searches and thus lower the total costs of search activities. Parts (ii) and (iii) of Proposition 8 state that ascending price sorting better saves the total consumer search costs than descending price sorting does if and only if $r > 1/2$, that is, there are more high-type firms in the market.

5. THE IMPACTS OF PRICE SORTING

In this section, we use random price sorting as the benchmark model and study the effects of ascending price sorting and descending price sorting. We explore how the availability of price sorting affects market performance and welfare.

Suppose the type of price sorting $S \in \{R, A, D\}$ is exogenously given, and denote TW_S, Π_S, and CS_S as the total welfare, industry profit, and consumer surplus, respectively. Let CS_S^0 and CS_S^1 be the surpluses for shoppers and searchers, respectively. The total consumer surplus is thus a weighed average of CS_S^0 and CS_S^1, depending on the fraction of shoppers, that is, $CS_S = \mu CS_S^0 + (1-\mu)\, CS_S^1$. Finally, let N_S be the average number of searches made by consumers.

Our first result describes the impact of price sorting on the search intensity.

Proposition 9. *When search cost c is small, compared to random price sorting, consumers search less often under both ascending and descending price sortings regimes. Specifically, we have*

$$\begin{aligned} N_A &< N_D < N_R, \text{ if } r > 1/2; \\ \text{and } N_D &< N_A < N_R, \text{ if } r < 1/2. \end{aligned}$$

Intuitively, for the extreme case in which search is costless, consumers under random price sorting will sample all the products before making a purchase. However, under ascending price sorting, consumers would stop searching when the first high-quality product is found because they know that this product has the lowest price and thus the highest purchase surplus. Similarly, under descending price sorting, the search process terminates when all the high-quality products have been sampled. Thus, both types of sorting reduce the number of searches by providing useful price information to consumers. Finally, it is obvious that search terminates earlier under ascending (descending) price sorting when firms are more (less) likely to be high types, that is, when $r > (<) 1/2$.

The following result shows that price sorting always improves the total welfare.

PROPOSITION 10. *When search cost c is small, (i) both ascending and descending price sorting regimes improve total welfare; (ii) ascending price sorting has a greater improvement on total welfare than descending price sorting if and only if $r > 1/2$. That is,*

$$\begin{aligned} TW_A &> TW_D > TW_R, \text{ if } r > 1/2; \\ \text{and } TW_D &> TW_A > TW_R, \text{ if } r < 1/2. \end{aligned}$$

Total welfare depends on the expected benefits of the trade between firms and consumers, and the total expected costs of the search activities. Note that regardless of the type of sorting, a consumer always ends up purchasing a high-quality product, as long as there is at least one high-type firm in the market. This implies that the expected benefits of the trade is always $a\left[1-(1-r)^N\right]$, regardless of the type of price sorting. Thus, the less frequently consumers search, the higher the total welfare will be. Proposition 10 thus follows immediately from Proposition 9: price sorting always provides useful price information that prevents consumers from running inefficient searches. Moreover, ascending (descending) price sorting better saves the total costs of searches when there are more high-quality (low-quality) products in the market.

The next result describes the impacts of price sorting on industry profits.

PROPOSITION 11. *Under Assumption 1', both ascending and descending sorting regimes have no impact on industry profits. That is,*

$$\Pi_A = \Pi_D = \Pi_R$$

To explain this result, note that under each type of price sorting, a high-type firm earns the same expected profit when charging any price in the support of the optimal price distribution. In addition, when charging the monopoly price, a, which equals the high-type product's quality,[23] the high-type firm earns the same expected profit regardless of the type of price sorting, because consumers will buy from this firm if and only if all other firms are of the low type. Thus, under Assumption 1', the expected profit of a high-type firm is the same for all types of price sorting. Because a low-type firm always earns zero profit, industry profits should be the same under each type of price sorting.

[23]Consider that the price approaches a from below.

Finally, the result on consumer surplus follows immediately from Propositions 10 and 11.

PROPOSITION 12. *When search cost c is small, (i) both ascending and descending price sortings boost consumer surplus; (ii) ascending price sorting has a larger effect on consumer surplus than descending price sorting if and only if $r > 1/2$. That is,*

$$\begin{aligned} CS_A &> CS_D > CS_R, \text{ if } r > 1/2; \\ \text{and } CS_D &> CS_A > CS_R, \text{ if } r < 1/2. \end{aligned}$$

Note that total consumer surplus is a weighted average of CS_S^0 and CS_S^1. Analyses in the previous section show that as search cost c approaches zero, the optimal price distributions for all types of sorting converge to the same limit. This implies that the limits of CS_S^0 are also the same for all $S \in \{R, A, D\}$. Thus, as long as c is small, the impact of price sorting on shoppers' surplus, CS_S^0, is negligible. However, the following Proposition states that the impact of price sorting on searchers' surplus, CS_S^1, is significant and the same as that on both total welfare and total consumer surplus.

PROPOSITION 13. *When search cost c is small, we have*

$$\begin{aligned} CS_A^1 &> CS_D^1 > CS_R^1, \text{ if } r > 1/2; \\ \text{and } CS_D^1 &> CS_A^1 > CS_R^1, \text{ if } r < 1/2. \end{aligned}$$

Whereas Proposition 12 is immediate due to previous results, Proposition 13 needs a detailed proof (See Appendix). Note that searchers' surplus CS_S^1 has two components: searchers' purchase surplus, which is the expected benefits of the trade between firms and searchers, and searchers' total search costs. The intuition behind our results is that when c is small, the main impact of price sorting is to reduce the total occurrence of search costs. The effects of price sorting on firms' profits and consumers' purchase surplus are negligible, because the optimal price distribution under each type of sorting has the same limit, as search cost converges to zero.

Next, we endogenize price sorting and consider how the availability of price sorting affects market performance and welfare. Due to Proposition 8, the total welfare, industry profit and consumer surplus in this case are given by TW_S, Π_S and CS_S, respectively, with $S = A$ when $r > 1/2$, and $S = D$ when $r < 1/2$. The previous analyses in this subsection imply that, for small search costs, the availability of price sorting benefits consumers but has no impact on industry profit, and thus increases total welfare. Price sorting is therefore beneficial for a platform's long-term development. The welfare improvement (which equals the consumer surplus improvement) due to price sorting is denoted by

$$\Delta TW = \begin{cases} TW_D - TW_R & \text{if } r < 1/2 \\ TW_A - TW_R & \text{if } r > 1/2 \end{cases}.$$

Were price sorting costless, a platform would undoubtedly introduce it. Introducing price sorting, however, is costly in reality. As a result, a platform would only investigate in price sorting technology if the benefits exceeded the costs. It is therefore worthwhile to investigate how the benefit of price sorting, ΔTW, changes with search frictions and other parameters of the model. We summarize the comparative statics results in the following proposition.

PROPOSITION 14. *When search cost c is small, the benefit of price sorting increases whenever (i) the search cost increases, that is,* $\frac{\partial \Delta TW}{\partial c} > 0$*; (ii) the fraction of searchers becomes larger, that is,* $\frac{\partial \Delta TW}{\partial \mu} < 0$*; (iii) the total number of firms becomes larger, that is,* $\frac{\partial \Delta TW}{\partial N} > 0$*; (iv)* $|r - \frac{1}{2}|$ *gets larger, that is,* $\frac{\partial \Delta TW}{\partial r} < 0$ *for* $r < 1/2$ *and* $\frac{\partial \Delta TW}{\partial r} > 0$ *for* $r > 1/2$.

The above results are intuitive. Price sorting benefits consumers, and thus improves the total welfare by providing consumers with useful price information that reduces the total costs of searches. Our results show that the benefits become more significant when (i) search frictions increase; (ii) the proportion of searchers increases; or (iii) the number of products increases. Finally, to understand part (iv), note that for small search cost c, compared to random search, price sorting can more effectively target the high-quality product with the lowest price, by first going through either all the low-quality products (i.e., ascending price sorting, when $r > 1/2$) or all the high-quality products (i.e., descending price sorting, when $r < 1/2$). Price sorting becomes more advantageous when the proportion of either high-quality or low-quality products is very small, that is, $|r - \frac{1}{2}|$ gets larger.

6. EXTENSION: MULTIPLE PRODUCTS IN EACH SAMPLE

In online-shopping platforms, the search results span several web pages, and more than one product is viewed in each page. For example, Amazon.com displays 24 products per page, with all the prices and product descriptions. Consumers who search for flights at Expedia.com can find 10 different flight deals in each page, which shows the origins, destinations, travel dates and prices. In this section, we show that our main results still hold even when multiple products are viewed in each sample and consumers may learn the price and product information on all products in the sample by only incurring a search cost once.

Specifically, let us reconsider the previous game by assuming that each sample consists of M products. That is, by incurring a search cost c, searchers obtain the price and quality information of M products at the same time. All other assumptions remain the same as before. We will derive the symmetric equilibrium in which low-type firms set zero price and high-type firms take the same price strategy $F_S(p; M)$,[24] when each type of price sorting $S \in \{R, A, D\}$ is exogenously given.

The method we use to solve the equilibrium is the same as before. We first show that consumers' optimal search policy are fully characterized by some reservation prices, given that all high-type firms follow the price distribution $F_S(p; M)$. The reservation prices can be calculated as functions of the price distribution. Then, we exogenously fix the reservation price(s) and solve the conditional optimal price distribution for high-type firms. Finally, in equilibrium, the pre-given reservation price(s) should be consistent with the obtained optimal price distribution. That is, the optimal reservation price(s) calculated from the obtained price distribution should be exactly equal to the pre-given reservation price(s).

Random price sorting

We first solve the symmetric equilibrium for random price sorting. Given the high-type firms' price distribution $F_R(p; M)$, and the current lowest price, p, of all sampled high-quality products, the expected benefit of sampling the next product is

$$\phi_R(p; M) = \sum_{s=1}^{M} C(M, s) r^s \left(1 - r\right)^{M-s} \int_{\underline{p}_R}^{p} (p - x) dF_R^{(s)}(x; M), \qquad (18)$$

where $F^{(n)}(p)$ is the cumulative distribution function of the lowest of n independent random variables with the same distribution $F(p)$.

To understand benefit (18), note that with probability $C(M, s) r^s \left(1 - r\right)^{M-s}$, there are s high-quality products in the next sample. Then, the lowest price of the high-quality products has distribution $F_R^{(s)}(x; M)$, given high-type firms' price distribution $F_R(p; M)$. Thus, each typical component of the right-hand side of Expression (18) is the expected incremental utility from a lower price in the next sample, conditional on s of M products in the sample being high-types.

[24] The previous game is the case in which $M = 1$.

Define the reservation price $\hat{p}_R^M$ as the solution to the equation $\phi_R(p; M) = c$. According to the previous analysis, the optimal stopping rule in this case is fully characterized by the reservation price $\hat{p}_R^M$: at any stage of the search process, consumers continue searching if and only if the current lowest price p is above the reservation price $\hat{p}_R^M$.

Next, we derive high-type firms' optimal price distribution $F_R(p; M, \hat{p}_R^M)$ for any given reservation price $\hat{p}_R^M$. A high-type firm's demand is given as follows.

$$D(p) = \begin{cases} \begin{array}{c} \mu \left[1 - rF_R(p; M, \hat{p}_R^M)\right]^{N-1} \\ +(1-\mu)\sum_{t=0}^{\frac{N}{M}-1} \frac{M}{N} \left[1 - rF_R(p; M, \hat{p}_R^M)\right]^{M-1} \left(1 - r\hat{F}_R^M\right)^{mt} \end{array} & \text{if } p < \hat{p}_R^M \\ \left[1 - rF_R(p; M, \hat{p}_R^M)\right]^{N-1} & \text{if } p > \hat{p}_R^M \end{cases},$$

where $\hat{F}_R^M = F_R(\hat{p}_R^M; M, \hat{p}_R)$ and we treat $\frac{N}{M}$ as an integer for analytical convenience.

When price p is below the reservation price, searchers buy at p if and only if they reject all the previously sampled products and price p is the lowest price in the current sample. Note that $\frac{M}{N}\left[1 - rF_R(p; M, \hat{p}_R^M)\right]^{M-1}\left(1 - r\hat{F}_R^M\right)^{mt}$ is the probability that price p is the lowest price in the $(t+1)\,th$ sample, and all the first t samples ahead of p are rejected by searchers.

According to the definition of the equilibrium, the optimal price distribution can be solved by the constant profit condition $\pi_R = pD(p)$ for any p within the price support.

Finally, the equilibrium reservation price is solved according to the consistency condition

$$\begin{aligned} \phi_R(\hat{p}_R^M; M) &= \sum_{s=1}^{M} C(M, s) r^s (1-r)^{M-s} \int_{\underline{p}_R}^{\hat{p}_R^M} (\hat{p}_R^M - x) dF_R^{(s)}(x; M, \hat{p}_R^M) \\ &= c. \end{aligned}$$

Ascending price sorting

The equilibrium under ascending price sorting is simple. As long as the search cost c is small, the optimal search rule under ascending price sorting is to stop searching only when a high-quality product is found. Thus, searchers behave like shoppers in the sense that they always purchase the product with the highest surplus. Hence, the high-type firms' optimal price distribution becomes the same as that in Proposition 3.

Descending price sorting

Next, we solve the equilibrium for descending price sorting. Suppose each high-type firm takes the price strategy $F_D(p;M)$. Consider a state in which k pages of products have been sampled, and the last product in the current page is of high quality with price p, for $k = 1, \ldots, \frac{N}{M} - 1$. We call this stage k of the search process. Next, we will calculate the expected benefit of sampling an additional page, given the current state (p, k).

Let $\sigma(t; p, k, M)$ be the probability that there are t high-quality products left, conditional on the current state (p, k). Then, for any $t = 0, 1, \ldots, N - kM$, we have that

$$\sigma(t; p, k, M) = \frac{C(N - kM, t)\,[rF_D(p;M)]^t\,(1-r)^{N-kM-t}}{[1 - r + rF_D(p;M)]^{N-kM}}.$$

Given the current state (p, k), the price of each high-quality product left unsampled is a random variable Y with distribution $J(y) = \frac{F_D(y;M)}{F_D(p;M)}$, for any $\underline{p}_D \leq y \leq p$. Let the random variable Z be the lowest price among all the high-quality products in the next page, which is the best price in the next sample. Note that $Z = 0$ when all the remaining unsampled products are low-types. When $t > 0$, we have

$$Z = \begin{cases} Y_{(1)} & \text{if } 1 \leq t \leq M, \\ Y_{(t-M+1)} & \text{if } t > M, \end{cases}$$

where $Y_{(n)}$ is the $n-th$ order statistic (or, the $n-th$ smallest order statistic) of the sample formed by t random variables, which are independently and identically distributed according to $J(y)$. In other words, if there are less than M high-quality products left, then the best price in the next page is just the lowest price of all remaining high-quality products. On the other hand, if there are more than M high-quality products left, then the best price is the $M - th$ largest price, or $(t - M + 1) - th$ smallest price, of all the t prices.

Let $f_Z(z; t, p, k, M)$ be the density of Z, conditional on the state (p, k), and that there are t high-quality products left. Then

$$f_Z(z; t, p, k, M) = \begin{cases} t\,[1 - J(z)]^{t-1}\,J'(z) & \text{if } 1 \leq t \leq M, \\ tC(t-1, M-1)\,[1 - J(z)]^{m-1}\,[J(z)]^{t-M}\,J'(z) & \text{if } t > M, \end{cases}$$

where $J(z) = \frac{F_D(z;M)}{F_D(p;M)}$.

The expected benefit of sampling the next page, given the state (p, k) is thus

$$\phi_D(p, k; M) = \sum_{t=1}^{N-kM} \sigma(t; p, k, M) \int_{\underline{p}_D}^{p} (p - z) f_Z(z; t, p, k, M)\, dz.$$

The reservation price at stage k, $\hat{p}^M_{D,k}$, is defined to be the solution to $\phi_D(p, k; M) = c$, for $k = 1, \ldots, \frac{N}{M} - 1$. The same analysis shows that as long as the reservation prices are increasing along with the search process, the optimal stopping rule in this case is to continue searching at stage k if and only if the state price p is higher than the reservation price $\hat{p}^M_{D,k}$, for all $k = 1, \ldots, \frac{N}{M} - 1$. Finally, the property of increasing reservation prices is satisfied if the total number of pages $\frac{N}{M}$ is not high.

Similar to the case of $M = 1$, we fix a group of reservation prices and solve the conditional optimal price distribution for high-type firms. Again, we first derive a high-type firm's demand, and then solve the optimal price distribution according to the constant-profit condition. Finally, the pre-given reservation prices should stand for the optimal stopping rule conditional on the obtained price distribution, which gives the equilibrium. The derivation of a high-type firm's demand is extensive and is presented in the Appendix.

Until now, we have solved the symmetric equilibrium when each type of price sorting is exogenously given. The next Proposition states that our results for welfare comparisons still hold when $M > 1$.

Proposition 15. *Suppose M products are viewed in each sample. Given the type of price sorting $S \in \{R, A, D\}$, let TW^M_S, Π^M_S, and CS^M_S be total welfare, industry profit, and consumer surplus, respectively. With small search costs, we have*

$$\Pi^M_A = \Pi^M_D = \Pi^M_R.$$

Moreover, if $r > 1/2$,

$$TW^M_A > TW^M_D > TW^M_R, \text{ and } CS^M_A > CS^M_D > CS^M_R;$$

and if $r < 1/2$,

$$TW^M_D > TW^M_A > TW^M_R, \text{ and } CS^M_D > CS^M_A > CS^M_R.$$

Finally, for the case of endogenous price sorting, the market equilibrium can be derived the same way as we did when $M = 1$. Proposition 16 states that our main result on the choice of price sorting still holds when $M > 1$.

Proposition 16. *Suppose M products are viewed in each sample. Then, with small search costs, if consumers can decide the type of price sorting, random price sorting is never selected in equilibrium. Specifically, consumers choose ascending price sorting if $r > 1/2$, and choose descending price sorting if $r < 1/2$.*

7. CONCLUSION

This article considers ascending and descending price sorting regimes in online-shopping platforms. Each type of price sorting can be applied before the sampling process. Consumers search sequentially for products with two types of qualities. We allow a fraction of consumers to have zero search costs, and all other consumers have the same positive search cost. Price dispersion exists in the unique symmetric equilibrium. We find that, when the search cost is small, using price sorting will improve both total welfare and consumer surplus, but have no impact on industry profits. Moreover, if consumers can choose the type of price sorting for their own interests, ascending price sorting (descending price sorting) will be chosen if there are more high-quality products (low-quality products) in the market.

Our analysis has been restricted to the case in which the cost of sampling a product is small. The situation may be different if the search cost becomes large. For instance, consider the comparisons of total welfare. With large search costs, searchers may accept any high-quality product, and only continue searching when low-quality products are viewed. This means the search is never done under descending price sorting. On the other hand, consumers search more often under ascending price sorting than under random price sorting, because low-quality products are always first sampled. As a result, ascending price sorting no longer saves consumers' total search costs and lowers total welfare compared to random price sorting. When the search cost becomes even higher, no search would ever take place under all types of price sorting, so that consumers always purchase the first product they sample. In this case, compared to random price sorting, total welfare is improved under descending price sorting, but decreased under ascending price sorting. Although the examples of online purchases fit the small search cost assumption, there are many other situations in which search costs are large. It will be a desirable extension to consider these situations in a future study.

Like most of the search literature, this article considers an optimal search policy in a simple form. The only choice consumers have to make is whether they want to keep searching. To be precise, when deciding to search further, consumers need not determine which product to sample next. This is because the search order is fully determined by firms' pricing strategies and the type of sorting. The derivation of this simple search policy is based on two assumptions: first, consumers observe neither price nor quality before a product is sampled; second, all products should be sampled in the same order as they are displayed according to the type of price sorting. The first assumption does not apply in situations where prices are displayed on the website after the products have been sorted. This means that consumers not only decide whether to continue searching or not, but also decide which product they want to sample based on the price information. Consumers' search behaviors in these situations are more complex, but deserves to be studied in the future.

Finally, other types of sorting options are available to consumers while shopping online. For example, on many commerce websites, consumers can sort their products by popularity or average customer rating. There is no doubt that these sorting tools also play an important role in influencing consumer decisions and product prices. The effects of these sorting regimes can be examined in the context of dynamic games that are left to future work.

APPENDIX: MISSING PROOFS

This section provides technical proofs for all the lemmas and propositions in the main text.

Proof (**Proof of Proposition 1**). Let π_h^S and π_l^S denote the equilibrium profit for high-type and low-type firms, respectively, for each sorting regime $S \in \{R, A, D\}$.

For any S, when a high-type firm i charges a price $p = \frac{a_h - a_l}{2}$, the utility of its product is $u = a_h - p = \frac{a_h + a_l}{2} > a_l$. With probability $(1-r)^{N-1}$, all the other firms are low-type firms, so that firm i is the only high-type firm in the market. Since $u > a_l$, all shoppers will buy from firm i, which makes a profit of $\mu p > 0$. This implies that firm i's expected profit is no less than $(1-r)^{N-1} \mu \frac{a_h - a_l}{2} > 0$, which implies that, for any $S \in \{R, A, D\}$, $\pi_h^S > 0$.

Recall that $\underline{u}_t^S = a_t - \overline{p}_t^S$ is the lowest (or infimum) utility of a t-quality product, $t \in \{h, l\}$, and $\underline{u}^S = \min\left\{\underline{u}_h^S, \underline{u}_l^S\right\}$ is the lowest possible utility for all products. We first prove that $\underline{u}_h^S \geq \underline{u}_l^S$ for any $S \in \{R, A, D\}$.

For random pricing sorting, suppose that $\underline{u}_h^R < \underline{u}_l^R$ otherwise. Consider a high-type firm i charging the highest price $\overline{p}_h^R$ so that the utility of its product is $\underline{u}_h^R$. In this case, $\underline{u}_h^R = \underline{u}^R$, and according to Assumption 1, searchers will keep searching when observing product i. Thus, both shoppers and searchers buy product i only when $\underline{u}_h^R$ is the highest utility of all products, which happens with zero probability because the high-type firm's equilibrium pricing strategy is atomless at the upper bound $\overline{p}_h^R$ (a mass point at $\overline{p}_h^R$ always results in profitable deviation under Assumption 1). This contradicts the fact that $\pi_h^R > 0$. As a result, it must be that $\underline{u}_h^R \geq \underline{u}_l^R$.

Applying similar arguments, we can prove that $\underline{u}_h^S \geq \underline{u}_l^S$, for all $S \in \{R, A, D\}$. Moreover, since $\underline{u}_l^S = \underline{u}^S$, according to Assumption 1, the low-type firm's equilibrium pricing strategy has no mass point at $\overline{p}_l^S$. This is because when deciding whether to purchase the low-quality product with price $\overline{p}_l^S$, all consumers behave like shoppers, regardless of the types of price sorting. Thus, a price undercut at the mass point $\overline{p}_l^S$ is always a profitable deviation.

Finally, we prove that $\pi_l^S = 0$ for any $S \in \{R, A, D\}$. Consider a low-type firm j which charges the highest price $\overline{p}_l^S$ so that the utility of its product is $\underline{u}_l^S$. Since $\underline{u}_l^S = \underline{u}^S$, according to Assumption 1, searchers will continue sampling when observing product j. Thus, both shoppers and searchers buy product j only when $\underline{u}_l^S$ is the highest utility of all products, which happens with zero probability because the low-type firm's equilibrium pricing strategy is atomless at the upper bound $\overline{p}_l^S$, for all $S \in \{R, A, D\}$. This implies that $\pi_l^S = 0$ for any $S \in \{R, A, D\}$. ▮

Proof (**Proof of Lemma 1**). Expression (2) can be rewritten as

$$\phi_D(p,n) = \int_{\underline{p}_D}^{p} \left[\left(\frac{1-r+rF_D(x)}{1-r+rF_D(p)} \right)^{N-n} - \left(\frac{1-r}{1-r+rF_D(p)} \right)^{N-n} \right] dx.$$

Define

$$h(m;x,p) = \left(\frac{1-r+rF_D(x)}{1-r+rF_D(p)} \right)^{m} - \left(\frac{1-r}{1-r+rF_D(p)} \right)^{m}.$$

Thus we have

$$\frac{\partial \phi_D(p,n)}{\partial n} = -\int_{\underline{p}_D}^{p} \frac{\partial h(m;x,p)}{\partial m}|_{m=N-n} dx.$$

Thus it is sufficient to prove that there exists a positive number $\overline{N}$ such that as long as $m < \overline{N}$, $\frac{\partial h(m;x,p)}{\partial m} > 0$ for any x and p. Note that

$$\begin{aligned} \frac{\partial h(m;x,p)}{\partial m} &= A^m \left\{ \left(1+\frac{y}{A}\right)^m \ln(A+y) - \ln A \right\} \\ &\equiv k(m;A,y), \end{aligned}$$

where $A = \frac{1-r}{1-r+rF_D(p)} \in [1-r, 1]$ and $y = \frac{rF_D(x)}{1-r+rF_D(p)} \in [0, 1-A]$. We have that

$$\begin{aligned} \frac{\partial k(m;A,y)}{\partial y} &= (A+y)^{m-1}\left[1+m\ln(A+y)\right] \\ &> (A+y)^{m-1}\left[1+m\ln A\right]. \end{aligned}$$

Define $\overline{N} = -\frac{1}{\ln(1-r)}$. Then, as long as $m < \overline{N}$, we have that $1+m\ln A > 0$, where we have used the fact that $1-r \le A \le 1$. This means that

$\frac{\partial k(m;A,y)}{\partial y} > 0$ for all $y \in [0, 1-A]$. Moreover, it is easy to verify that $k(m; A, y = 0) = 0$ and $k(m; A, y = 1-A) = -A^m \ln A > 0$. Thus, we have $\frac{\partial h(m;x,p)}{\partial m} = k(m; A, y) \geq 0$ for all m, x and p as long as $m < \overline{N}$, which completes our proof. ▌

Proof (**Proof of Proposition 2**). It is obvious that the searcher will continue searching whenever $p_{D,n} \geq \hat{p}_{D,n}$, where $p_{D,n}$ is the observed price at stage n, for any $n = 1, ..., N-1$. We now prove by induction that it is optimal for the searcher to stop searching at stage n whenever $p_{D,n} < \hat{p}_{D,n}$. Our result is obviously true when $n = N-1$. Suppose it is true for all stages $N-1, N-2, ..., N-k$, and consider stage $N-k-1$, at which $p_{D,N-k-1} < \hat{p}_{D,N-k-1}$. If the searcher keeps sampling at stage $N-k-1$, as he will stop searching in the next stage (due to the inductive hypothesis, the increasing order of reservation prices and the decreasing order of observed prices), the total expected benefits of doing so is exactly equal to $\phi_D(p_{D,N-k-1}, N-k-1)$, which is below the search cost c as $p_{D,N-k-1} < \hat{p}_{D,N-k-1}$. Thus, the searcher would be better off if he chose to stop searching at stage $N-k-1$, which completes our proof. ▌

Proof (**Proof of Lemma 2**). First of all, $F_R(p; \hat{p}_R)$ cannot have an atom at $p = 0$ because a high-type firm can guarantee itself a positive expected profit by charging a price $p \in (0, a)$. By doing so, the high-type firm yields a positive surplus, so that all the shoppers will buy from it as long as all other firms are low-type, which happens with probability $(1-r)^{N-1}$. To prove that there are no atoms at positive prices, suppose the opposite. Then, by slightly undercutting the atom, the firm can discretely increase its demand from the shoppers without losing any demand from the searchers, which obviously contradicts the optimality of $F_R(p; \hat{p}_R)$. ▌

Proof (**Proof of Lemma 3**). Suppose $\overline{p}_R < \hat{p}_R$. Then consider $p' \in (\overline{p}_R, \hat{p}_R)$. Note that p' gives the firm the same demand as $\overline{p}_R$ does, thus making a higher profit, by contradiction. On the other hand, suppose $\hat{p}_R < \overline{p}_R < a$; similar to the previous argument, any price $p' \in (\overline{p}_R, a)$ is strictly better than $\overline{p}_R$, which is also impossible. Hence, either $\overline{p}_R = \hat{p}_R$ or $\overline{p}_R = a$. Finally, $\overline{p}_R = \hat{p}_R$ violates Assumption 1' so that we have $\overline{p}_R = a$. ▌

Proof (**Proof of Proposition 3**).
Proposition 3 follows immediately from the condition $\pi_R = pD(p)$ and the demand function (3). ▌

Before giving a proof for Proposition 4, we find the following analyses very useful.

Equation (8) can be rewritten as

$$r \int_{\underline{p}_R}^{\hat{p}_R} F_R^*(x; \hat{p}_R) dx = c. \tag{A.1}$$

Expressions (4)-(7) imply that all $F_R^*(x;\hat{p}_R), \underline{p}_R, \hat{p}_R$ and p_R' can be expressed as functions of $\hat{F}_R$. Thus, the left-hand side of (A.1) depends only on $\hat{F}_R$, which in turn can be expressed as a function of c. Obviously, as $\hat{F}_R \to 0$, all $\underline{p}_R, \hat{p}_R$ and $p_R' \to \pi_R$, so that $r\int_{\underline{p}_R}^{\hat{p}_R} F_R^*(x;\hat{p}_R)dx \to 0$. Moreover, due to (4)-(7), as $\hat{F}_R \to 0$, we have that $\frac{\partial \underline{p}_R}{\partial \hat{F}_R} \to \frac{1}{2}(N-1)\pi_R r(1-\mu)$, $\frac{\partial \hat{p}_R}{\partial \hat{F}_R} \to \frac{1}{2}(N-1)\pi_R r(1+\mu)$, $\frac{\partial p_R'}{\partial \hat{F}_R} \to (N-1)\pi_R r$, and $\frac{\partial F_R^*(x;\hat{p}_R)}{\partial \hat{F}_R} \to -\frac{1-\mu}{2\mu}$ for any $x \in [\underline{p}_R, \hat{p}_R]$.

Taking a derivative of the left-hand side of (A.1) with respect to $\hat{F}_R$ gives

$$\begin{aligned}
\frac{\partial \int_{\underline{p}_R}^{\hat{p}_R} F_R^*(x;\hat{p}_R)dx}{\partial \hat{F}_R} &= \hat{F}_R \frac{\partial \hat{p}_R}{\partial \hat{F}_R} + \int_{\underline{p}_R}^{\hat{p}_R} \frac{\partial F_R^*(x;\hat{p}_R)}{\partial \hat{F}_R} dx \\
&= \hat{F}_R \left[\frac{\partial \hat{p}_R}{\partial \hat{F}_R} + \frac{\int_{\underline{p}_R}^{\hat{p}_R} \frac{\partial F_R^*(x;\hat{p}_R)}{\partial \hat{F}_R} dx}{\hat{F}_R} \right].
\end{aligned}$$

Because

$$\lim_{\hat{F}_R \to 0} \frac{\partial \hat{p}_R}{\partial \hat{F}_R} = \frac{1}{2}(N-1)\pi_R r(1+\mu)$$

and

$$\begin{aligned}
\lim_{\hat{F}_R \to 0} \frac{\int_{\underline{p}_R}^{\hat{p}_R} \frac{\partial F_R^*(x;\hat{p}_R)}{\partial \hat{F}_R} dx}{\hat{F}_R} &= \lim_{\hat{F}_R \to 0} \frac{-(1-\mu)}{2\mu} \left(\frac{\partial \hat{p}_R}{\partial \hat{F}_R} - \frac{\partial \underline{p}_R}{\partial \hat{F}_R} \right) \\
&= -\frac{1}{2}(N-1)\pi_R r(1-\mu),
\end{aligned}$$

the above expression implies that

$$\lim_{\hat{F}_R \to 0} \frac{\partial \int_{\underline{p}_R}^{\hat{p}_R} F_R^*(x;\hat{p}_R)dx}{\partial \hat{F}_R} \frac{1}{\hat{F}_R} = (N-1)\pi_R r\mu. \tag{A.2}$$

Thus, when $\hat{F}_R$ is sufficiently small, the left-hand side of (A.1) is strictly increasing in $\hat{F}_R$. This means that there is always a unique $\hat{F}_R$ which solves Equation (A.1) for small search costs.

The above useful results are summarized as follows.
Result 1: As $c \to 0$, we have
(i)

$$\begin{aligned}
\frac{\partial \underline{p}_R}{\partial \hat{F}_R} &\to \frac{1}{2}(N-1)\,\pi_R r\,(1-\mu)\,, \\
\frac{\partial \hat{p}_R}{\partial \hat{F}_R} &\to \frac{1}{2}(N-1)\,\pi_R r\,(1+\mu)\,, \\
\frac{\partial p'_R}{\partial \hat{F}_R} &\to (N-1)\,\pi_R r;
\end{aligned}$$

(ii) when $x \in [\underline{p}_R, \hat{p}_R]$, we have

$$\frac{\partial F_R^*(x;\hat{p}_R)}{\partial \hat{F}_R} \to -\frac{1-\mu}{2\mu};$$

(iii) finally, we have

$$\begin{aligned}
\frac{\partial \hat{F}_R}{\partial c}\hat{F}_R &\to \frac{1}{(N-1)\,\pi_R r^2\mu}, \\
\frac{c}{\hat{F}_R^2} &\to \frac{(N-1)\,\pi_R r^2\mu}{2}, \\
\frac{c}{\hat{F}_R} &\to 0, \\
\frac{\partial \hat{F}_R}{\partial c}c &\to 0.
\end{aligned}$$

Proof. Parts (i) and (ii) are obvious. For (iii), taking derivatives of both sides of (A.1) with respect to c gives

$$r\hat{F}_R\frac{\partial \hat{p}_R}{\partial \hat{F}_R}\frac{\partial \hat{F}_R}{\partial c} + r\frac{\partial \hat{F}_R}{\partial c}\int_{\underline{p}_R}^{\hat{p}_R}\frac{\partial F_R^*(x;\hat{p}_R)}{\partial \hat{F}_R}dx = 1,$$

or

$$r\frac{\partial \int_{\underline{p}_R}^{\hat{p}_R} F_R^*(x;\hat{p}_R)dx}{\partial \hat{F}_R}\frac{\partial \hat{F}_R}{\partial c} = 1.$$

According to (A.2), the above expression gives the first result:

$$\frac{\partial \hat{F}_R}{\partial c}\hat{F}_R \to \frac{1}{(N-1)\,\pi_R r^2\mu}.$$

For the second result, note that

$$\begin{aligned}
\lim_{c\to 0}\frac{c}{\hat{F}_R^2} &= \lim_{c\to 0}\frac{r\int_{\underline{p}_R}^{\hat{p}_R}F_R^*(x;\hat{p}_R)dx}{\hat{F}_R^2}\\
&= r\lim_{\hat{F}_R\to 0}\frac{\partial\int_{\underline{p}_R}^{\hat{p}_R}F_R^*(x;\hat{p}_R)dx/\partial\hat{F}_R}{2\hat{F}_R}\\
&= \frac{r}{2}\lim_{\hat{F}_R\to 0}\frac{\hat{F}_R\frac{\partial\hat{p}_R}{\partial\hat{F}_R}+\int_{\underline{p}_R}^{\hat{p}_R}\frac{\partial F_R^*(x;\hat{p}_R)}{\partial\hat{F}_R}dx}{\hat{F}_R}\\
&= \frac{r}{2}\lim_{\hat{F}_R\to 0}\left[\frac{\partial\hat{p}_R}{\partial\hat{F}_R}+\frac{\int_{\underline{p}_R}^{\hat{p}_R}\frac{\partial F_R^*(x;\hat{p}_R)}{\partial\hat{F}_R}dx}{\hat{F}_R}\right]\\
&= \frac{(N-1)\,\pi_R r^2\mu}{2},
\end{aligned}$$

where the second equality is due to the L'Hospital rule.

Finally, the third and fourth results of part (iii) are direct consequences of the the first two, as $\lim_{c\to 0}\hat{F}_R = 0$. ▌

Proof (**Proof of Proposition 4**). The above analysis has shown that the solution of Equation (8) always exists and is unique for small search costs. In addition, the equilibrium reservation price $\hat{p}_R$ increases with the search cost because $\frac{\partial\hat{p}_R}{\partial c} = \frac{\partial\hat{p}_R}{\partial\hat{F}_R}\frac{\partial\hat{F}_R}{\partial c} > 0$ due to Result 1. ▌

Proof (**Proof of Lemma 4**). This proof is similar to that of Lemma 2. Firstly, the equilibrium price distribution cannot have an atom at price zero because each high-type firm's expected profit is strictly positive. Secondly, to prove that there is no atom at any positive price, suppose the opposite. Then, by slightly undercutting the atom, a high-type firm not only discretely increases its demand from the shoppers, but also increases its demand from the searchers because a lower price means being sampled earlier and having a higher surplus. Thus, the high-type firm can have a strictly higher profit by doing so, which obviously contradicts the optimality of $F_A(p)$. ▌

Proof (**Proof of Lemma 5**). The proof is similar to that of Lemma 3. Suppose $\overline{p}_A < a$; then, any price $p' \in (\overline{p}_A, a)$ gives a high-type firm the same demand as $\overline{p}_A$ does, thus making a higher profit, by contradiction. To show that there is no gap in the equilibrium price support, suppose otherwise and let $(\alpha,\beta) \subset [\underline{p}_A, \overline{p}_A]$ be the largest such gap. Then charging price β yields the same demand for the firm as α does. Because $\beta > \alpha$, β will make larger profits than α, by contradiction. ▌

Proof (**Proof of Proposition 5**). Under Assumption 1', searchers never stop sampling until a high-quality product is found. Due to the increasing order of prices, each searcher ultimately purchases the high-quality product with the lowest price, and behaves exactly the same as a shopper. Thus, Expressions (11) and (12) follow immediately from the condition $\pi_A = pD(p)$ and the demand function (10). ▌

Proof (**Proof of Lemma 6**). Under Assumption 1', search takes place at each stage as long as the observed price is high enough. Thus, the upper bound of price support $\overline{p}_D$ should be above the highest reservation price $\hat{p}_{D,N-1}$. This means that charging $\overline{p}_D$ gives a high-type firm a demand $(1-r)^{N-1}$ because consumers will purchase from this firm if and only if it is the only high-type firm in the market. Suppose $\overline{p}_D < a$. Because any high-type firm can have the same demand $(1-r)^{N-1}$ by charging price a, increasing the price from $\overline{p}_D$ to a becomes a strictly profitable deviation. ▌

Proof (**Proof of Proposition 6**). Proposition 6 follows immediately from the demand function (15) and the condition that $\pi_D = pD(p)$ for all p. ▌

According to Proposition 6, the optimal price distribution function $F_D^*(p;\hat{p}_D)$ and the elements $\underline{p}_D$, $\hat{p}_{D,n}$ and $p'_{D,n}$ can be expressed as functions of $\hat{F}_{D,1}$, $\hat{F}_{D,2}$,..., $\hat{F}_{D,N-1}$, for $n = 1, ..., N-1$. More precisely, for any n and $p \in (\hat{p}_{D,n}, \hat{p}_{D,n+1}]$, $F_D(p;\hat{p}_D)$ is the solution of the following equation.

$$
\begin{aligned}
&\frac{\pi_D}{p} = \mu\left[1 - rF_D(p;\hat{p}_D)\right]^{N-1} \\
&+ (1-\mu)\sum_{k=0}^{n-1} C(N-1,k)\,(1-r)^{N-1-k}\left[r - rF_D(p;\hat{p}_D)\right]^k \\
&+ (1-\mu)\,C(N-1,n)\,(1-r+rF_D(p;\hat{p}_D))^{N-1-n}\left[r - rF_D(p;\hat{p}_D)\right]^n \\
&+ (1-\mu)\sum_{k=n+1}^{N-1} C(N-1,k)\,(1-r+rF_D(p;\hat{p}_D))^{N-1-k}\left[r - r\hat{F}_{D,k}\right]^k .
\end{aligned}
$$

Moreover, we have

$$
\frac{\pi_D}{\underline{p}_D} = \mu + (1-\mu)\sum_{k=0}^{N-1} C(N-1,k)\,(1-r)^{N-1-k}\left[r - r\hat{F}_{D,k}\right]^k ,
$$

$$\frac{\pi_D}{p'_{D,n}} = \mu\left[1 - r\hat{F}_{D,n}\right]^{N-1} + (1-\mu)\sum_{k=0}^{n-1} C(N-1,k)\,(1-r)^{N-1-k}\left[r - r\hat{F}_{D,n}\right]^k$$
$$+ (1-\mu)\,C(N-1,n)\left(1 - r + r\hat{F}_{D,n}\right)^{N-1-n}\left[r - r\hat{F}_{D,n}\right]^n$$
$$+ (1-\mu)\sum_{k=n+1}^{N-1} C(N-1,k)\left(1 - r + r\hat{F}_{D,n}\right)^{N-1-k}\left[r - r\hat{F}_{D,k}\right]^k,$$

$$\frac{\pi_D}{\hat{p}_{D,n}} = \mu\left[1 - r\hat{F}_{D,n}\right]^{N-1} + (1-\mu)\sum_{k=0}^{n-1} C(N-1,k)\,(1-r)^{N-1-k}\left[r - r\hat{F}_{D,n}\right]^k$$
$$+ (1-\mu)\,C(N-1,n)\left(1 - r + r\hat{F}_{D,n}\right)^{N-1-n}\left[r - r\hat{F}_{D,n}\right]^n$$
$$+ (1-\mu)\sum_{k=n+1}^{N-1} C(N-1,k)\left(1 - r + r\hat{F}_{D,n}\right)^{N-1-k}\left[r - r\hat{F}_{D,k}\right]^k$$
$$+ (1-\mu)\,C(N-1,n-1)\left(r - r\hat{F}_{D,n}\right)^{n-1}\left[\left(1 - r + r\hat{F}_{D,n}\right)^{N-n} - (1-r)^{N-n}\right].$$

Using the above expressions, it is not difficult to verify the following results.

Result 2: As $\hat{F}_{D,1}, \hat{F}_{D,2}, ..., \hat{F}_{D,N-1} \to 0$, we have,

(i) for any $n = 1, ..., N-1$,

$$\frac{\partial \underline{p}_D}{\partial \hat{F}_{D,n}} \to \pi_D\,(1-\mu)\,nC(N-1,n)\,(1-r)^{N-1-n}\,r^n;$$

(ii) for any $n = 1, ..., N-1$ and $m \geq n+1$,

$$\frac{\partial p'_{D,n}}{\partial \hat{F}_{D,n}} \;\to\; \pi_D r\,(N-1)\left[1 - 2\,(1-\mu)\sum_{k=n+1}^{N-1} C(N-2,k-1)\,(1-r)^{N-k-1}\,r^{k-1}\right],$$
$$\frac{\partial p'_{D,n}}{\partial \hat{F}_{D,m}} \;\to\; \frac{\partial \underline{p}_D}{\partial \hat{F}_{D,m}} \to \pi_D\,(1-\mu)\,mC(N-1,m)\,(1-r)^{N-1-m}\,r^m;$$

(iii) for any $n = 1, ..., N-1$ and $m \geq n+1$,

$$\begin{aligned}\frac{\partial \hat{p}_{D,n}}{\partial \hat{F}_{D,n}} &\to \frac{\partial p'_{D,n}}{\partial \hat{F}_{D,n}} - \frac{\partial \underline{p}_D}{\partial \hat{F}_{D,n}} \\ &\to \pi_D r\,(N-1)\left[1 - 2\,(1-\mu)\sum_{k=n+1}^{N-1} C(N-2,k-1)\,(1-r)^{N-k-1}\,r^{k-1}\right] \\ &\quad -\pi_D\,(1-\mu)\,nC(N-1,n)\,(1-r)^{N-1-n}\,r^n, \\ \frac{\partial \hat{p}_{D,n}}{\partial \hat{F}_{D,m}} &\to \frac{\partial \underline{p}_D}{\partial \hat{F}_{D,m}} \to \pi_D\,(1-\mu)\,mC(N-1,m)\,(1-r)^{N-1-m}\,r^m;\end{aligned}$$

(iv) for any $n = 1, ..., N-1$,

$$\frac{\partial \hat{p}_{D,n+1}}{\partial \hat{F}_{D,n+1}} - \frac{\partial \underline{p}_D}{\partial \hat{F}_{D,n+1}} \to \frac{\partial p'_{D,n}}{\partial \hat{F}_{D,n}}. \tag{A.3}$$

(v) for any n, any $p \in (\hat{p}_{D,n}, \hat{p}_{D,n+1}]$ and $m \geq n+1$, we have

$$\frac{\partial F_D^*(p;\hat{p}_D)}{\partial \hat{F}_{D,m}} \to -\frac{\frac{\partial \underline{p}_D}{\partial \hat{F}_{D,m}}}{\frac{\partial p'_{D,n}}{\partial \hat{F}_{D,n}}}.$$

According to (A.3), for any $p \in (\hat{p}_{D,n}, \hat{p}_{D,n+1}]$, we have

$$\frac{\partial F_D^*(p;\hat{p}_D)}{\partial \hat{F}_{D,n+1}} \to -\frac{\frac{\partial \underline{p}_D}{\partial \hat{F}_{D,n+1}}}{\frac{\partial p'_{D,n}}{\partial \hat{F}_{D,n}}} \to \frac{-\frac{\partial \underline{p}_D}{\partial \hat{F}_{D,n+1}}}{\frac{\partial \hat{p}_{D,n+1}}{\partial \hat{F}_{D,n+1}} - \frac{\partial \underline{p}_D}{\partial \hat{F}_{D,n+1}}}. \tag{A.4}$$

Note that (17) gives an equation system with $N-1$ equations and $N-1$ variables $(\hat{F}_{D,1}, \hat{F}_{D,2}, ..., \hat{F}_{D,N-1})$. For any $n = 1, ..., N-1$, define

$$\Psi_n(\hat{F}_{D,1}, \hat{F}_{D,2}, ..., \hat{F}_{D,N-1}) \equiv \int_{\underline{p}_D}^{\hat{p}_{D,n}} (\hat{p}_{D,n} - x)\,d\left(\frac{1-r+rF_D^*(x;\hat{p}_D)}{1-r+r\hat{F}_{D,n}}\right)^{N-n}.$$

Taking derivatives of both sides of (17) with respect to c gives

$$\sum_{m=1}^{N-1} \frac{\partial \Psi_n}{\partial \hat{F}_{D,m}} \frac{\partial \hat{F}_{D,m}}{\partial c} = 1 \text{ for all } n. \tag{A.5}$$

Using Result 2, it is not difficult to find that, for any $n, m = 1, ..., N-1$, as $\hat{F}_{D,1}, \hat{F}_{D,2}, ..., \hat{F}_{D,N-1} \to 0$,

$$\frac{\partial \Psi_n}{\partial \hat{F}_{D,m}} \frac{1}{\hat{F}_{D,m}} \to \begin{cases} -\frac{r(N-n)}{1-r} \frac{\partial \underline{p}_D}{\partial \hat{F}_{D,m}} & \text{if } m < n; \\ \frac{r(N-n)}{1-r} \frac{\partial \left(\hat{p}_{D,m} - \underline{p}_D\right)}{\partial \hat{F}_{D,m}} & \text{if } m = n; \\ \frac{r(N-n)}{1-r} \frac{\partial \underline{p}_D}{\partial \hat{F}_{D,m}} & \text{if } m > n. \end{cases} \tag{A.6}$$

For notational ease, we use $\hat{F}_D \to 0$ to represent $\hat{F}_{D,1}, \hat{F}_{D,2}, ..., \hat{F}_{D,N-1} \to 0$. We define

$$a_{n,m} \equiv \lim_{\hat{F}_D \to 0} \frac{\partial \Psi_n}{\partial \hat{F}_{D,m}} \frac{1}{\hat{F}_{D,m}},$$

$$b_m \equiv \lim_{\hat{F}_D \to 0} \frac{\partial \hat{F}_{D,m}}{\partial c} \hat{F}_{D,m};$$

then, (A.5) means

$$\sum_{m=1}^{N-1} a_{n,m} b_m = 1 \text{ for all } n. \tag{A.7}$$

Let matrix A be the $(N-1) \times (N-1)$ square matrix with $a_{n,m}$ as the element in the $n-th$ row and $m-th$ column. According to (A.6), it is not difficult to verify that matrix A is invertible so that there exists a unique set of $(b_1, ..., b_{N-1})$ that solves (A.7), which in turn proves that the solution of (17), $\left(\hat{F}_{D,1}, \hat{F}_{D,2}, ..., \hat{F}_{D,N-1}\right)$, uniquely exists when the search cost c is approaching zero.

Proof (**Proof of Proposition 7**). Because $(\hat{p}_{D,1}, ..., \hat{p}_{D,N-1})$ are functions of $\left(\hat{F}_{D,1}, \hat{F}_{D,2}, ..., \hat{F}_{D,N-1}\right)$, the above analysis simply shows that there exists a unique set of reservation prices $(\hat{p}_{D,1}, ..., \hat{p}_{D,N-1})$ that solves Equation (17). ▌

Proof (**Proof of Proposition 8**). The proof of this proposition follows after the proof of Proposition 14. ▌

Proof (**Proof of Proposition 9**). Under random price sorting, there is a probability $r\hat{F}_R$ that each sampled product is a Type I product, that is, the product is of high quality and with a price lower than the reservation price, in which case the searcher stops sampling and make a purchase immediately. There is probability $1 - r\hat{F}_R$ that each sampled product is a Type II product, that is, the product is either of low quality, or of high quality but with a price higher than the reservation price, in which case the searcher will continue searching.

Thus, the probability that m searches take place[1] is $\left(1-r\hat{F}_R\right)^m r\hat{F}_R$, for $m = 0, 1, ..., N-2$. This is because this happens when the first m samples are of Type II, and the $(m+1)\,th$ sample is of Type I. Finally, the probability that $N-1$ searches take place is $\left(1-r\hat{F}_R\right)^{N-1}$. This is because a searcher samples all the products if and only if all the first $N-1$ samples are of Type II.

Then the expected number of searches under random price sorting is given by

$$\begin{aligned} N_R &= \sum_{m=1}^{N-2} m\left(1-r\hat{F}_R\right)^m r\hat{F}_R + (N-1)\left(1-r\hat{F}_R\right)^{N-1} \\ &= \frac{1-\left(1-r\hat{F}_R\right)^N}{r\hat{F}_R} - 1, \end{aligned}$$

which converges to $N-1$ as $c \to 0$.

Under ascending price sorting, for small search costs, a searcher will sample all the low-quality products and purchse the high-quality product with the lowest price. So the probability that a searcher makes m samples[2] is $C(N,m)\,(1-r)^m\, r^{N-m}$ because it happens when m out of N products are of low quality, for $m = 0, 1, ..., N-2$. The probability that the searcher makes $N-1$ samples (i.e., sample all the products) is $C(N, N-1)\,(1-r)^{N-1}\, r + (1-r)^N$ because it happens when there are $N-1$ or N low-quality products. Thus, the expected number of searches under ascending price sorting is

$$\begin{aligned} N_A &= \sum_{m=1}^{N-1} mC(N,m)\,(1-r)^m\, r^{N-m} + (N-1)\,(1-r)^N \\ &= \sum_{m=1}^{N} mC(N,m)\,(1-r)^m\, r^{N-m} - (1-r)^N \\ &= N\,(1-r)\sum_{m=1}^{N} C(N-1,m-1)\,(1-r)^{m-1}\, r^{N-m} - (1-r)^N \\ &= N\,(1-r) - (1-r)^N, \end{aligned}$$

[1]We have assumed that the first sample is free and that the total number of searches we are exploring for each type of sorting is defined to be the number of samples other than the first one.

[2]The first sample is not counted.

where the third equality comes from the fact that $mC(N,m) = NC(N-1, m-1)$ and the last comes from the fact that $\sum_{m=1}^{N} C(N-1, m-1)\,(1-r)^{m-1}\,r^{N-m} = 1$.

Finally, under descending price sorting, let $p_{(n)}$ be the $n-th$ highest price out of all N prices. Given the reservation prices $0 < \hat{p}_{D,1} \leq \hat{p}_{D,2} \leq ... \leq \hat{p}_{D,N-1} < a$ and the optimal stopping rule, for any $m = 1, 2, ..., N-2$, the searcher makes m samples if and only if $p_{(m)} > \hat{p}_{D,m}$ and $p_{(m+1)} < \hat{p}_{D,m+1}$. On the other hand, $N-1$ searches take place if and only if $p_{(N-1)} > \hat{p}_{D,N-1}$. Define α_m to be the probability that m searches take place, for $m = 1, 2, ..., N-1$. Then, as the search cost $c \to 0$, $\hat{p}_{D,n} \to \underline{p}_D$ for all n, and we hence have $\alpha_m \to C(N,m)\,(1-r)^{N-m}\,r^m$ for $m = 1, 2, ..., N-2$ and $\alpha_{N-1} \to C(N, N-1)\,(1-r)\,r^{N-1} + r^N$. The expected number of searches under descending price sorting is

$$\begin{aligned}\lim_{c\to 0} N_D &= \sum_{m=1}^{N-1} mC(N,m)\,(1-r)^{N-m}\,r^m + (N-1)\,r^N \\ &= Nr - r^N,\end{aligned}$$

where the last equality follows the same way as that of the derivation of N_A.

Define $h(r) \equiv Nr - r^N$, for all $r \in [0,1]$. The above results can be summarized as

$$\lim_{c\to 0} N_R = h(1), \;\; \lim_{c\to 0} N_A = h(1-r), \text{ and } \lim_{c\to 0} N_D = h(r).$$

It is obvious that $h'(r) > 0$. This implies that

$$h(1) > h(r) \text{ and } h(1) > h(1-r)$$

and

$$h(r) > h(1-r) \text{ if and only if } r > 1/2,$$

which completes our proof. ▮

Result 3: Given any high-type firm's price distribution, when search cost c is small, compared to random price sorting, consumers search less often under both ascending and descending price sortings. Moreover, consumers search least frequently under ascending (or descending, respectively) price sorting if $r > 1/2$ (or $r < 1/2$, respectively).

Proof. The proof is the similar to that of Proposition 9. The basic logic is that, given the probability r, as the search cost c approaches zero, the expected number of searches under random price sorting is always $N-1$.

In other words, searchers will sample all the firms in the market. However, under ascending price sorting, the expected number of searches is the expected number of low-type firms in the market, $N(1-r) - (1-r)^N$. This is because searchers stop sampling only after all the low-type firms are sampled. Similarly, under descending price sorting, the expected number of searches is equal to the expected number of high-type firms in the market, $Nr - r^N$, because searchers stop as soon as they finish sampling all the high-type firms. Thus, the comparison of the total number of searches follows exactly the same way as that in Proposition 9. ∎

Proof (**Proof of Proposition 10**). When search costs are small, both shoppers and searchers always end up purchasing a high-quality product as long as there is at least one high-type firm in the market. This means that the surplus of trade under each type of price sorting is the same and equal to $a\left[1-(1-r)^N\right]$. The total welfare under price sorting $S \in \{R, A, D\}$ is given by

$$TW_S = a\left[1-(1-r)^N\right] - c(1-\mu)N_S.$$

Hence, Proposition 10 follows immediately from Proposition 9. ∎

Proof (**Proof of Proposition 11**). Proposition 11 follows from the facts that $\pi_R = \pi_A = \pi_D = a(1-r)^{N-1}$ as long as the search cost is small, and that low-type firms always earn zero profit, regardless of the types of price sorting. ∎

Proof (**Proof of Proposition 12**). Proposition 12 simply follows from Proposition 10 and 11. ∎

Proof (**Proof of Proposition 13**). We have that $CS_S = \mu CS_S^0 + (1-\mu)CS_S^1$ for all $S \in \{R, A, D\}$. According to Proposition 9, it suffices to show that

$$\lim_{c\to 0} \frac{CS_A^0 - CS_R^0}{c} = 0 \text{ and } \lim_{c\to 0} \frac{CS_D^0 - CS_R^0}{c} = 0.$$

Because CS_A^0 does not depend on c (for small c) and

$$\lim_{c\to 0} CS_R^0 = \lim_{c\to 0} CS_D^0 = CS_A^0,$$

it suffices to show that

$$\lim_{c\to 0} \frac{\partial CS_R^0}{\partial c} = 0 \text{ and } \lim_{c\to 0} \frac{\partial CS_D^0}{\partial c} = 0.$$

The shopper's surplus under random price sorting is given as follows.

$$\begin{aligned} CS_R^0 &= \sum_{m=1}^{N} C(N,m)\,(1-r)^{N-m}\,r^m \int_{\underline{p}_R}^{a} (a-x)\,dG_{R,m}(x) \\ &= \sum_{m=1}^{N} C(N,m)\,(1-r)^{N-m}\,r^m \int_{\underline{p}_R}^{a} G_{R,m}(x)dx, \end{aligned}$$

where $G_{R,m}(x) = 1 - [1 - F_R^*(x)]^m$ represents the cumulative distribution function of the smallest order statistic with m samples, as shoppers always purchase from the lowest-priced firm. To understand the above expression, note that the typical term of the right-hand side

$$C(N,m)\,(1-r)^{N-m}\,r^m \int_{\underline{p}_R}^{a} (a-x)\,dG_{R,m}(x)$$

represents the shopper's expected surplus when there are m high-type firms in the market.

Using Result 1, one can easily derive that, for any $m = 1, ..., N$, as $\hat{F}_R \to 0$,

$$\begin{aligned} \frac{\partial \int_{\underline{p}_R}^{a} G_{R,m}(x)dx}{\partial c} &= \int_{\underline{p}_R}^{a} \frac{\partial G_{R,m}(x)}{\partial c} dx \\ &= \frac{\partial \hat{F}_R}{\partial c} \int_{\underline{p}_R}^{\hat{p}_R} m\,[1 - F_R^*(x)]^{m-1}\,\frac{\partial F_R^*(x)}{\partial \hat{F}_R} dx \\ &\quad + \frac{\partial \hat{F}_R}{\partial c}\,(p_R' - \hat{p}_R)\,\frac{\partial\left[1 - \left(1 - \hat{F}_R\right)^m\right]}{\partial \hat{F}_R} \\ &\to \frac{\partial \hat{F}_R}{\partial c} \hat{F}_R \{-\frac{\hat{p}_R - \underline{p}_R}{\hat{F}_R}\,\frac{m\,(1-\mu)}{2\mu} + \frac{m\,(p_R' - \hat{p}_R)}{\hat{F}_R}\} \\ &\to 0, \end{aligned}$$

which implies that $\lim_{c\to 0} \frac{\partial CS_R^0}{\partial c} = 0$.

The shopper's surplus under descending price sorting can be derived in a similar way to that under random price sorting. We have

$$\begin{aligned} CS_D^0 &= \sum_{m=1}^{N} C(N,m)\,(1-r)^{N-m}\,r^m \int_{\underline{p}_D}^{a} (a-x)\,dG_{D,m}(x) \\ &= \sum_{m=1}^{N} C(N,m)\,(1-r)^{N-m}\,r^m \int_{\underline{p}_D}^{a} G_{D,m}(x)dx, \end{aligned}$$

where $G_{D,m}(x) = 1 - [1 - F_D^*(x)]^m$. It suffices to show that, as $c \to 0$, $\frac{\partial \int_{\underline{p}_D}^{a} G_{D,m}(x)dx}{\partial c} \to 0$ for any m.

Using Result 2, one can verify that for any $m = 1, ..., N$,

$$\begin{aligned}
\frac{\partial \int_{\underline{p}_D}^{a} G_{D,m}(x)dx}{\partial c} &= \int_{\underline{p}_D}^{a} \frac{\partial G_{D,m}(x)}{\partial c} dx \\
&= \sum_{n=1}^{N-1} \int_{p'_{D,n-1}}^{\hat{p}_{D,n}} m\,[1 - F_D^*(x)]^{m-1} \frac{\partial F_D^*(x)}{\partial c} dx \\
&\quad + \sum_{n=1}^{N-1} \frac{\partial \hat{F}_{D,n}}{\partial c} \left(p'_{D,n} - \hat{p}_{D,n}\right) m \left[1 - \hat{F}_{D,n}\right]^{m-1}.
\end{aligned}$$

As $c \to 0$, we have

$$\begin{aligned}
\frac{\partial \int_{\underline{p}_D}^{a} G_{D,m}(x)dx}{\partial c} &\to \sum_{n=1}^{N-1} \int_{p'_{D,n-1}}^{\hat{p}_{D,n}} m\,[1 - F_D^*(x)]^{m-1} \left[\sum_{m=n}^{N-1} \frac{\partial F_D^*(x)}{\partial \hat{F}_{D,m}} \frac{\partial \hat{F}_{D,m}}{\partial c}\right] dx \\
&\quad + \sum_{n=1}^{N-1} \frac{\partial \hat{F}_{D,n}}{\partial c} m \left(p'_{D,n} - \hat{p}_{D,n}\right) \\
&\to \sum_{n=1}^{N-1} \frac{m \left(\hat{p}_{D,n} - p'_{D,n-1}\right)}{\hat{F}_{D,n}} \left[\sum_{m=n}^{N-1} \frac{\partial F_D^*(p'_{D,n-1})}{\partial \hat{F}_{D,m}} \frac{\partial \hat{F}_{D,m}}{\partial c} \hat{F}_{D,n}\right] \\
&\quad + \sum_{n=1}^{N-1} \frac{\partial \hat{F}_{D,n}}{\partial c} m \left(p'_{D,n} - \hat{p}_{D,n}\right) \\
&\to \sum_{n=1}^{N-1} \frac{m \left(\hat{p}_{D,n} - p'_{D,n-1}\right)}{\hat{F}_{D,n}} \frac{\partial F_D^*(p'_{D,n-1})}{\partial \hat{F}_{D,n}} \frac{\partial \hat{F}_{D,n}}{\partial c} \hat{F}_{D,n} \\
&\quad + \sum_{n=1}^{N-1} \frac{\partial \hat{F}_{D,n}}{\partial c} \hat{F}_{D,n} \frac{m \left(p'_{D,n} - \hat{p}_{D,n}\right)}{\hat{F}_{D,n}}.
\end{aligned}$$

Thus,

$$\begin{aligned}\frac{\partial\int_{\underline{p}_D}^{a}G_{D,m}(x)dx}{\partial c} &\to \sum_{n=1}^{N-1}\left[\frac{\hat{p}_{D,n}-p'_{D,n-1}}{\hat{F}_{D,n}}\frac{\partial F_D^*(p'_{D,n-1})}{\partial\hat{F}_{D,n}}+\frac{p'_{D,n}-\hat{p}_{D,n}}{\hat{F}_{D,n}}\right]m\frac{\partial\hat{F}_{D,n}}{\partial c}\hat{F}_{D,n}\\ &\to \sum_{n=1}^{N-1}\left[\frac{\partial\left(\hat{p}_{D,n}-p'_{D,n-1}\right)}{\partial\hat{F}_{D,n}}\frac{\partial F_D^*(p'_{D,n-1})}{\partial\hat{F}_{D,n}}+\frac{\partial\left(p'_{D,n}-\hat{p}_{D,n}\right)}{\partial\hat{F}_{D,n}}\right]m\frac{\partial\hat{F}_{D,n}}{\partial c}\hat{F}_{D,n}\\ &\to \sum_{n=1}^{N-1}\left[\frac{\partial\left(\hat{p}_{D,n}-\underline{p}_D\right)}{\partial\hat{F}_{D,n}}\frac{\partial F_D^*(p'_{D,n-1})}{\partial\hat{F}_{D,n}}+\frac{\partial\underline{p}_D}{\partial\hat{F}_{D,n}}\right]m\frac{\partial\hat{F}_{D,n}}{\partial c}\hat{F}_{D,n}\\ &\to 0,\end{aligned}$$

where $p'_{D,0}\equiv\underline{p}_D$ and the last step follows from the fact that

$$\frac{\partial F_D^*(p'_{D,n-1})}{\partial\hat{F}_{D,n}}\to\frac{-\frac{\partial\underline{p}_D}{\partial\hat{F}_{D,n}}}{\frac{\partial\hat{p}_{D,n}}{\partial\hat{F}_{D,n}}-\frac{\partial\underline{p}_D}{\partial\hat{F}_{D,n}}},$$

which is due to (A.4). Thus, we have proved that $\lim_{c\to0}\frac{\partial CS_D^0}{\partial c}=0$, which completes the proof of Proposition 13. ∎

Proof (**Proof of Proposition 14**). The total welfare under price sorting $S\in\{R,A,D\}$ is

$$TW_S=a\left[1-(1-r)^N\right]-c\,(1-\mu)\,N_S.$$

Thus, the benefit of price sorting is given by

$$\Delta TW=\begin{cases}c\,(1-\mu)\,(N_R-N_D) & \text{if } r<1/2\\ c\,(1-\mu)\,(N_R-N_A) & \text{if } r>1/2\end{cases}.$$

According to the proof of Proposition 9, for each $S\in\{R,A,D\}$, N_S is continuously differentiable with respect to the parameters (c,r,N,μ) and satisfy that

$$\lim_{c\to0}N_R=N-1,$$

$$N_A=N\,(1-r)-(1-r)^N,$$

$$\lim_{c\to0}N_D=Nr-r^N.$$

For part (i), because $\lim_{c\to 0}\Delta TW = 0$, we have that

$$\lim_{c\to 0}\frac{\partial \Delta TW}{\partial c} = \lim_{c\to 0}\frac{\Delta TW}{c} = \begin{cases} \lim_{c\to 0}(1-\mu)(N_R - N_D) & \text{if } r < 1/2 \\ \lim_{c\to 0}(1-\mu)(N_R - N_A) & \text{if } r > 1/2 \end{cases}$$
$$= \begin{cases} (1-\mu)\left[N(1-r) + r^N - 1\right] & \text{if } r < 1/2 \\ (1-\mu)\left[Nr + (1-r)^N - 1\right] & \text{if } r > 1/2 \end{cases}$$

which, according to the proof of Proposition 9, is always positive for any r.

For part (ii), we have

$$\lim_{c\to 0}\frac{\partial \Delta TW}{\partial \mu}\frac{1}{c} = \begin{cases} \lim_{c\to 0}\left[-(N_R - N_D) + (1-\mu)\frac{\partial(N_R - N_D)}{\partial \mu}\right] & \text{if } r < 1/2 \\ \lim_{c\to 0}\left[-(N_R - N_A) + (1-\mu)\frac{\partial(N_R - N_A)}{\partial \mu}\right] & \text{if } r > 1/2 \end{cases}$$
$$= \begin{cases} -\left[N(1-r) + r^N - 1\right] & \text{if } r < 1/2 \\ -\left[Nr + (1-r)^N - 1\right] & \text{if } r > 1/2 \end{cases},$$

which is negative for any $r \in (0,1)$. This means that $\frac{\partial \Delta TW}{\partial \mu}$ is negative as long as the search cost is small, which proves part (ii).

Proof for part (iii) is similar to that for part (ii). It suffices to show that $\frac{\partial \Delta TW}{\partial N}$ is positive for small search costs. To this end, note that

$$\lim_{c\to 0}\frac{\partial \Delta TW}{\partial N}\frac{1}{(1-\mu)c} = \begin{cases} \lim_{c\to 0}\frac{\partial(N_R - N_D)}{\partial N} & \text{if } r < 1/2 \\ \lim_{c\to 0}\frac{\partial(N_R - N_A)}{\partial N} & \text{if } r > 1/2 \end{cases}$$
$$= \begin{cases} \frac{\partial\left[N(1-r) + r^N - 1\right]}{\partial N} & \text{if } r < 1/2 \\ \frac{\partial\left[Nr + (1-r)^N - 1\right]}{\partial N} & \text{if } r > 1/2 \end{cases}$$
$$= \begin{cases} 1 - r + r^N \ln r & \text{if } r < 1/2 \\ r + (1-r)^N \ln(1-r) & \text{if } r > 1/2 \end{cases}.$$

Thus, it suffices to show that $1 - r + r^N \ln r > 0$ for any $r \in (0,1)$ and $N > 0$. Note that $\frac{\partial\left[1 - r + r^N \ln r\right]}{\partial r} = -1 + r^{N-1} + Nr^{N-1}\ln r < 0$ so that $1 - r + r^N \ln r$ decreases with r for any $r \in (0,1)$. Moreover, when $r = 1$, we have $1 - r + r^N \ln r = 0$. This implies that $1 - r + r^N \ln r > 0$ for any $r \in (0,1)$, which proves part (iii).

For part (iv), note that

$$\lim_{c\to 0}\frac{\partial \Delta TW}{\partial r}\frac{1}{(1-\mu)\,c} = \begin{cases} \lim_{c\to 0}\frac{\partial(N_R-N_D)}{\partial r} & \text{if } r<1/2 \\ \lim_{c\to 0}\frac{\partial(N_R-N_A)}{\partial r} & \text{if } r>1/2 \end{cases}$$
$$= \begin{cases} \frac{\partial\left[N(1-r)+r^N-1\right]}{\partial r} & \text{if } r<1/2 \\ \frac{\partial\left[Nr+(1-r)^N-1\right]}{\partial r} & \text{if } r>1/2 \end{cases}$$
$$= \begin{cases} -N\left(1-r^{N-1}\right)<0 & \text{if } r<1/2 \\ N\left[1-(1-r)^{N-1}\right]>0 & \text{if } r>1/2 \end{cases}.$$

This implies that when search cost c is small, $\frac{\partial \Delta TW}{\partial r}$ is negative when $r < 1/2$, and positive when $r > 1/2$, which proves part (iv). ▮

Proof (**Proof of Proposition 8**). For part (i), given that the search cost is approaching zero, and that all high-type firms are following the optimal price distribution $F_R(p)$, searchers can always become strictly better off by switching from random price sorting to ascending price sorting. Firstly, by doing so searchers can maximize their purchase surplus because they end up paying the lowest price. Secondly, ascending price sorting saves the total search costs because the total expected number of searches is smaller under ascending price sorting than under random price sorting, as is shown in Result 3. Thus, random price sorting is never part of equilibrium.

For part (ii), the analysis is similar to that for part (i). When $r > 1/2$, according to Result 3, as the search cost approaches zero, ascending price sorting is the best sorting option for searchers for any given price distribution. This is because it not only gives the highest purchase surplus (searchers behave like shoppers), but also gives the lowest total search cost (searchers search least frequently). This proves that the only equilibrium should be $(F_A(p), A)$.

For part (iii), suppose $r < 1/2$, and the search cost approaches zero. To prove that $(F_D(p), D)$ is the unique equilibrium, we have to prove two statements: (1) given the high-type firm's price distribution $F_D(p)$, descending price sorting is better than ascending price sorting for searchers; (2) given the price distribution $F_A(p)$, descending price sorting is better than ascending price sorting.

Now we prove statement (1). Given the price distribution $F_D(p)$, let CS^1_{DD} and CS^1_{DA} be searchers' total surplus under descending price sorting and ascending price sorting, respectively; let CS^0_D be the total surplus for shoppers.[3] Let N_{DD} and N_{DA} be the expected number of searches under

[3]Shoppers do not care about the type of price sorting as they always buy from the lowest-priced high-type firm.

descending price sorting and ascending price sorting, respectively. Likewise, define CS^1_{AD}, CS^1_{AA}, CS^0_A, N_{AD}, and N_{AA} as the counterparts when the given price distribution is $F_A(p)$. To understand the notations, superscripted 1 stands for "searcher" and 0 for "shopper"; for the subscripts, the first letter $S \in \{A, D\}$ stands for the given price distribution $F_S(p)$, and the second letter $S' \in \{A, D\}$ stands for the type of price sorting chosen by searchers.

Because searchers who use descending price sorting make the same purchases as shoppers do, we have

$$\begin{aligned} CS^1_{DA} &= CS^0_D - cN_{DA} \\ CS^1_{AA} &= CS^0_A - cN_{AA}. \end{aligned}$$

Moreover, we have

$$\lim_{c \to 0} N_{DA} = \lim_{c \to 0} N_{AA} = N(1-r) - (1-r)^N,$$

which implies that

$$\lim_{c \to 0} \frac{CS^1_{DA} - CS^1_{AA}}{c} = \lim_{c \to 0} \frac{CS^0_D - CS^0_A}{c} = 0,$$

according to the proof of Proposition 13.

Thus,

$$\begin{aligned} \lim_{c \to 0} \frac{CS^1_{DD} - CS^1_{DA}}{c} &= \lim_{c \to 0} \frac{CS^1_{DD} - CS^1_{AA}}{c} \\ &> 0, \end{aligned}$$

due to Proposition 13, which proves statement (1).

Finally, we prove statement (2). We take the price distribution $F_A(p)$ as given. According to the previous analyses, the searcher's optimal stopping rule under descending price sorting can be characterized by a group of reservation prices: $\hat{p}_1 \leq \hat{p}_2 \leq ... \leq \hat{p}_{N-1}$, where $\hat{p}_n$ solves

$$\int_{\underline{p}_A}^{\hat{p}_n} (\hat{p}_n - x) d \left(\frac{1 - r + rF_A(x)}{1 - r + rF_A(\hat{p}_n)} \right)^{N-n} = c, \text{ for } n = 1, ..., N-1.$$

Especially, the reservation price at stage $N-1$, $\hat{p}_{N-1}$, satisfies that

$$\int_{\underline{p}_A}^{\hat{p}_{N-1}} (\hat{p}_{N-1} - x) d \left(\frac{1 - r + rF_A(x)}{1 - r + rF_A(\hat{p}_{N-1})} \right) = c,$$

or

$$r\int_{\underline{p}_A}^{\hat{p}_{N-1}} F_A(x)dx = c\left[1 - r + rF_A(\hat{p}_{N-1})\right]. \tag{A.8}$$

By taking the above optimal stopping rule, the searcher's expected surplus under descending price sorting is CS^1_{AD}.

Now consider the following suboptimal searching strategy under descending price sorting: at any stage, a searcher continues sampling if and only if the current price is higher than $\hat{p}_{N-1}$, which is given in (A.8). In other words, searchers behave as if the reservation prices at all stages are the same and equal to $\hat{p}_{N-1}$. Let $C\hat{S}^1_{AD}$ and $\hat{N}_{AD}$ be the searcher's expected surplus and the expected number of searches under this stopping rule, respectively. The suboptimality of the search strategy implies that $C\hat{S}^1_{AD} \leq CS^1_{AD}$. Thus, to prove statement (2), it is sufficient to show that $C\hat{S}^1_{AD} > CS^1_{AA}$ as long as the search cost is small, or

$$\lim_{c\to 0}\frac{C\hat{S}^1_{AD} - CS^1_{AA}}{c} > 0.$$

Similar to the proof of Proposition 9, one can easily see that

$$\begin{aligned}\lim_{c\to 0}\hat{N}_{AD} &= \lim_{c\to 0} N_{AD} = Nr - r^N \\ &< N(1-r) - (1-r)^N \\ &= \lim_{c\to 0} N_{AA}, \text{ when } r < 1/2.\end{aligned}$$

Under the above suboptimal stopping rule, we have that

$$\begin{aligned}C\hat{S}^1_{AD} &= \sum_{m=1}^{N} C(N,m)\,(1-r)^{N-m}\, r^m \{\int_{\hat{p}_{N-1}}^{a} (a-x)\, dG_m(x) + \\ &\quad \sum_{s=1}^{m} C(m,s)\,(1-F_A(\hat{p}_{N-1}))^{m-s}\int_{\underline{p}_A}^{\hat{p}_{N-1}} (a-x)\, dH_s(x)\} \\ &\quad -c\hat{N}_{AD},\end{aligned}$$

where $G_n(x) = 1 - [1 - F_A(x)]^n$ and $H_n(x) = [F_A(x)]^n$, representing the distribution functions for the smallest order statistic and the largest order statistic for a sample of size n, respectively.

It is easy to express CS^1_{AA} as

$$CS^1_{AA} = \sum_{m=1}^{N} C(N,m)\,(1-r)^{N-m}\, r^m \int_{\underline{p}_A}^{a} (a-x)\, dG_m(x) - cN_{AA}.$$

Thus, we have that

$$\begin{aligned}\lim_{c\to 0}\frac{C\hat{S}^1_{AD}-CS^1_{AA}}{c} &= \Delta+\lim_{c\to 0}\left(N_{AA}-\hat{N}_{AD}\right)\\ &> \Delta,\ \text{when } r<1/2,\end{aligned}$$

where

$$\begin{aligned}\Delta &= \frac{1}{c}\sum_{m=1}^{N}C(N,m)\,(1-r)^{N-m}\,r^m\{\int_{\hat{p}_{N-1}}^{a}(a-x)\,dG_m(x)\\ &\quad+\sum_{s=1}^{m}C(m,s)\,[1-F_A(\hat{p}_{N-1})]^{m-s}\int_{\underline{p}_A}^{\hat{p}_{N-1}}(a-x)\,dH_s(x)\}\\ &\quad-\frac{1}{c}\sum_{m=1}^{N}C(N,m)\,(1-r)^{N-m}\,r^m\int_{\underline{p}_A}^{a}(a-x)\,dG_m(x).\end{aligned}$$

It is then sufficient to prove that $\Delta=0$. To this end, note that

$$\begin{aligned}\Delta &= \lim_{c\to 0}\frac{1}{c}\sum_{m=1}^{N}C(N,m)\,(1-r)^{N-m}\,r^m\{-\int_{\underline{p}_A}^{\hat{p}_{N-1}}(a-x)\,dG_m(x)\\ &\quad+\sum_{s=1}^{m}C(m,s)\,(1-F_A(\hat{p}_{N-1}))^{m-s}\int_{\underline{p}_A}^{\hat{p}_{N-1}}(a-x)\,dH_s(x)\}\\ &= \lim_{c\to 0}\frac{1}{c}\sum_{m=1}^{N}C(N,m)\,(1-r)^{N-m}\,r^m\{-\int_{\underline{p}_A}^{\hat{p}_{N-1}}G_m(x)dx\\ &\quad+\sum_{s=1}^{m}C(m,s)\,(1-F_A(\hat{p}_{N-1}))^{m-s}\int_{\underline{p}_A}^{\hat{p}_{N-1}}H_s(x)dx\}\\ &= \lim_{c\to 0}\frac{1}{c}\int_{\underline{p}_A}^{\hat{p}_{N-1}}\left\{\begin{array}{c}[1-rF_A(\hat{p}_{N-1})+rF_A(x)]^N-1\\ -[1-rF_A(\hat{p}_{N-1})]^N+(1-rF_A(x))^N\end{array}\right\}dx\\ &= \lim_{c\to 0}\frac{\partial\int_{\underline{p}_A}^{\hat{p}_{N-1}}\left\{\begin{array}{c}[1-rF_A(\hat{p}_{N-1})+rF_A(x)]^N-1\\ -[1-rF_A(\hat{p}_{N-1})]^N+(1-rF_A(x))^N\end{array}\right\}dx}{\partial c}\\ &= \lim_{c\to 0}\int_{\underline{p}_A}^{\hat{p}_{N-1}}NrF'(\hat{p}_{N-1})\frac{\partial\hat{p}_{N-1}}{\partial c}\left\{\begin{array}{c}-[1-rF_A(\hat{p}_{N-1})+rF_A(x)]^{N-1}\\ +[1-rF_A(\hat{p}_{N-1})]^{N-1}\end{array}\right\}dx\\ &= -\lim_{c\to 0}NrF'(\hat{p}_{N-1})\frac{\partial\hat{p}_{N-1}}{\partial c}\int_{\underline{p}_A}^{\hat{p}_{N-1}}\left\{\begin{array}{c}[1-rF_A(\hat{p}_{N-1})+rF_A(x)]^{N-1}\\ -[1-rF_A(\hat{p}_{N-1})]^{N-1}\end{array}\right\}dx\\ &= 0,\end{aligned}$$

where the last equality comes from the following two facts

$$\lim_{c\to 0} F_A(\hat{p}_{N-1})\frac{\partial \hat{p}_{N-1}}{\partial c} = \frac{1-r}{r} \text{ (see Equation (A.8))}$$

and

$$\lim_{c\to 0} \frac{1}{F_A(\hat{p}_{N-1})}\int_{\underline{p}_A}^{\hat{p}_{N-1}} \left\{ \begin{array}{c} [1-rF_A(\hat{p}_{N-1})+rF_A(x)]^{N-1} \\ -[1-rF_A(\hat{p}_{N-1})]^{N-1} \end{array} \right\} dx = 0. \quad \text{(A.9)}$$

To see why (A.9) holds, note that as $c \to 0$, we have $\hat{p}_{N-1} \to \underline{p}_A$ and $F_A(\hat{p}_{N-1}) \to 0$, so that

$$\begin{aligned} 0 &\le \frac{1}{F_A(\hat{p}_{N-1})}\int_{\underline{p}_A}^{\hat{p}_{N-1}} \left\{ \begin{array}{c} [1-rF_A(\hat{p}_{N-1})+rF_A(x)]^{N-1} \\ -[1-rF_A(\hat{p}_{N-1})]^{N-1} \end{array} \right\} dx \\ &< \frac{1}{F_A(\hat{p}_{N-1})}\int_{\underline{p}_A}^{\hat{p}_{N-1}} \left\{ 1-[1-rF_A(\hat{p}_{N-1})]^{N-1} \right\} dx \\ &= \frac{1-[1-rF_A(\hat{p}_{N-1})]^{N-1}}{F_A(\hat{p}_{N-1})}\left(\hat{p}_{N-1}-\underline{p}_A\right) \to 0. \end{aligned}$$

▮

Now we derive a high-type firm's demand function under descending price sorting, when M products can be found in each sample. Let the pre-given reservation prices be $\hat{p}^M_{D,1} \le \hat{p}^M_{D,2} \le ... \le \hat{p}^M_{D,\frac{N}{M}-1}$ and the conditional optimal price distribution for high-type firms be $F_D(p; \hat{p}_D, M)$.

Similar to the case in which $M = 1$, define

$$\hat{p}^M_{D,0} \equiv \underline{p}^M_D \text{ and } \hat{p}^M_{D,\frac{N}{M}} \equiv \overline{p}^M_D = a.$$

We derive the high-type firm's demand function for each price interval, $p \in (\hat{p}^M_{D,n}, \hat{p}^M_{D,n+1}]$, where $n = 0, 1, ..., \frac{N}{M} - 1$. Let $D^M_0(p)$ and $D^M_1(p)$ be the demand from shoppers and searchers, respectively. Thus, the total demand is given by

$$D^M(p) = \mu D^M_0(p) + (1-\mu)\, D^M_1(p), \quad \text{(A.10)}$$

where $D_0(p) = [1 - rF_D(p; \hat{p}_D, M)]^{N-1}$ as shoppers buy at price p if and only if all other products are either low-quality or have a price higher than p.

The demand from searchers is given by

$$
\begin{aligned}
D_1^M(p) &= \Pr\{\text{searchers buy at price } p\} \\
&= \sum_{k=0}^{N-1} \Pr\{\text{searchers buy at price } p\text{, with } k \text{ prices higher than } p\} \\
&= \sum_{k=0}^{N-1} D_1^M(p,k), \qquad \text{(A.11)}
\end{aligned}
$$

where $D_1^M(p,k) = \Pr\{$searchers buy at price p, and there are k prices higher than $p\}$, for $k = 0, 1, ..., N-1$.

Denote $\hat{F}_{D,k}^M = F_D(\hat{p}_{D,k}^M; \hat{p}_D, M)$ for any $k = 0, 1, ..., \frac{N}{M} - 1$, with $\hat{F}_{D,0}^M = 0$. The searcher's demand is given as follows.

For any $p \in (\hat{p}_{D,n}^M, \hat{p}_{D,n+1}^M]$,

(i) if $k \leq M(n+1) - 2$, we have

$$D_1^M(p,k) = C(N-1,k)\,(1-r)^{N-1-k}\,[r - rF_D(p;\hat{p}_D,M)]^k\,,$$

(ii) if $k = M(n+1) - 1$, we have

$$D_1^M(p,k) = C(N-1,k)\,[1 - r + rF_D(p;\hat{p}_D,M)]^{N-1-k}\,[r - rF_D(p;\hat{p}_D,M)]^k\,,$$

(iii) if $k = M(n+s) + i$, where $s = 1, ..., \frac{N}{M} - n - 1$ and $i = 0, ..., M-2$, we have

$$
\begin{aligned}
D_1^M(p,k) &= C(N-1,i)C(N-1-i,M(n+s))\,(1-r)^{N-1-i-M(n+s)} \\
&\quad \left(r - r\hat{F}_{D,n+s}^M\right)^{M(n+s)}\,[r - rF_D(p;\hat{p}_D,M)]^i\,,
\end{aligned}
$$

(iv) if $k = M(n+s) + M - 1$, where $s = 1, ..., \frac{N}{M} - n - 1$, then

$$
\begin{aligned}
D_1^M(p,k) &= C(N-1,M-1)C(N-M,M(n+s))\,[1 - r + rF_D(p;\hat{p}_D,M)]^{N-1-k} \\
&\quad \left(r - r\hat{F}_{D,n+s}^M\right)^{M(n+s)}\,[r - rF_D(p;\hat{p}_D,M)]^{M-1}\,.
\end{aligned}
$$

To understand (i), note that if there are $k \leq M(n+1) - 2$ prices higher than p, price p will appear in one of the first n pages/samples. Because $p > \hat{p}_{D,n}^M$, searchers buy at price p only if all the products with prices lower than p are low-quality. Thus, in this case, $D_1^M(p,k)$ equals the probability that k out of $N-1$ prices are higher than p, and the rest $N-1-k$ prices are zero (i.e., they are low-quality products).

For expression (ii), when $k = M(n+1)-1$, price p appears as the last one (or the lowest one) in page $n+1$. Note that $\hat{p}^M_{D,n} < p \leq \hat{p}^M_{D,n+1}$, searchers never stop searching in the first n pages, and buy at price p immediately without further searches. Thus, $D^M_1(p,k)$ equals the probability that k out of $N-1$ prices are higher than p, and the rest $N-1-k$ prices are lower than p.

For expression (iii), if $k = M(n+s)+i$, where $s = 1, ..., \frac{N}{M} - n - 1$ and $i = 0, ..., M-2$, then price p is in the middle of page $n+s+1$ (not the last one). In this case, searchers purchase at price p if and only if two conditions are satisfied: (1) the first $M(n+s)$ prices should be no lower than the reservation price $\hat{p}^M_{D,n+s}$, so that searchers will not stop and purchase before they observe p in page $n+s+1$; (2) all the prices after p should be zero (i.e., low-quality products) because otherwise searchers will buy at a lower price in page $n+s+1$. Thus, $D^M_1(p,k)$ equals the probability that $M(n+s)$ out of $N-1$ prices are higher than $\hat{p}^M_{D,n+s}$, i prices are higher than p, and the rest $N-1-k$ prices are zero.

Finally, for expression (iv), when $k = M(n+s)+M-1$, where $s = 1, ..., \frac{N}{M} - n - 1$, price p appears as the last one in page $n+s+1$. Searchers purchase at price p if and only if they did not stop searching in the first $n+s$ pages. In other words, the first $M(n+s)$ prices should be no lower than the reservation price $\hat{p}^M_{D,n+s}$. Compared to case (iii), that searchers purchase at price p does not require that all the prices after p be zero. This is because p is the last and the lowest price in page $n+s+1$. Thus, in this case, $D^M_1(p,k)$ equals the probability that $M(n+s)$ out of $N-1$ prices are higher than $\hat{p}^M_{D,n+s}$, $M-1$ prices are higher than p, and the rest $N-1-k$ prices are lower than p.

With the expression of $D^M_1(p,k)$ for any $k = 0, ..., N-1$, the total demand function is then derived according to (A.11) and (A.10). And the conditional optimal price distribution $F_D(p; \hat{p}_D, M)$ can be solved according to the constant-profit condition $\pi_D = pD^M(p)$, for any p within the price support.

Proof (**Proof of Proposition 15**). Let N^M_S be the expected number of searches that take place under price sorting $S \in \{R, A, D\}$, when M products are found in each sample. Then it suffices to show that, as the search cost $c \to 0$,

$$N^M_A < N^M_D < N^M_R, \text{ if } r > 1/2;$$
$$\text{and } N^M_D < N^M_A < N^M_R, \text{ if } r < 1/2.$$

Following the similar arguments as those in Proposition 9, we can see that, as the search cost approaches zero, under random price sorting, searchers always sample all the products in the market; under ascending price sorting, searchers stop sampling after they have gone through all the low-quality products; under descending price sorting, searchers stop sampling only when they have gone through all the high-quality products. Thus, N_A^M and N_D^M are always smaller than N_R^M. Moreover, $N_A^M > N_D^M$ if and only if there are more low-quality products in the market, i.e., $r < 1/2$. ∎

Proof (**Proof of Proposition 16**). The logic is the same as that of Proposition 8. Random price sorting is never chosen in equilibrium because it is always dominated by ascending price sorting: compared to random price sorting, by choosing ascending price sorting, searchers can purchase high-quality products at lower prices and search less frequently. To compare ascending price sorting and descending price sorting, we can show that, given any price distribution, the difference between searchers' purchase surplus is an infinitesimal of higher order than the search cost c, as $c \to 0$. On the other hand, the total expected number of searches is smaller under ascending price sorting than under descending price sorting if and only if $r > 1/2$. This means there is always a unique equilibrium in the case of endogenous price sorting, in which consumers choose ascending price sorting if $r > 1/2$, and choose descending price sorting if $r < 1/2$. ∎

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