ANNALS OF ECONOMICS AND FINANCE **25-2**, 649–662 (2024)

On the Range of Applicability of the First-order Approach to Agency Problems

Óscar Gutiérrez*

This note examines the range of applicability of the first-order approach to principal-agent problems under moral hazard. We show the existence of parameter configurations where the solution given by Holmstrom (1979)-Jewitt (1988) stops working to provide the second-best contract. The problem arises when the agent's reservation utility is below a certain threshold, which is associated with the fact that the contract lacks a fixed component.

Key Words: Moral Hazard; Principal-Agent Model; Second-Best Contract; First-Order Approach; Reservation Utility.
JEL Classification Numbers: D2, D8, G34, J33, M4.

1. INTRODUCTION

The principal-agent model under moral hazard describes a stylized situation where a principal hires an agent to perform a task on her behalf, but the input provided by the agent is private information that is unverifiable for the principal. By means of an incentives contract dependent on output —i.e., a sharing rule— the principal induces the agent to perform the optimal level of effort according to her interests. The principal-agent model has attracted much theoretical research attention, and has been successfully used to justify a wide array of incentives practices observed in managerial practice; see Ross (1973), Mirrlees (1976), Holmstrom (1979), Grossman and Hart (1983), Rogerson (1985), Jewitt (1988), Basu et al. (1985), Raju and Srinivasan (1994), Hemmer et al. (1999), Lambert (2001), Gutiérrez and Salas (2008), et cetera.

A problem concerning the principal-agent model lies in the extreme difficulty of obtaining solutions. The Mirrlees-Holmstrom formulation of the model greatly simplifies the problem (see Holmstrom (1979)). In the Mirrlees-Holmstrom formulation, the solution only requires the application

* Universitat Autònoma de Barcelona. Email: oscar.gutierrez@uab.es. The author did not receive support from any organization for the submitted work.

1529-7373/2024

of pointwise optimization to the Lagrangian (see Holmstrom (1979)), and the solutions are readily obtained by invoking the so-called first-order approach (FOA henceforth). The approach consists of assuming that the agent's decision problem, characterized by an incentive compatibility constraint, has an interior solution, which allows us to replace the original constraint by a relaxed one. The use of the FOA is not absent of technical problems, but given the great simplification that it entails, meritorious efforts have been devoted to provide sufficient conditions justifying its validity; see Rogerson (1985), Jewitt (1988), Sinclair-Desgagné (1994), Conlon (2009), Moroni and Swinkels (2014), Chade and Swinkles (2020), et cetera.

This note explores the range of applicability of the solution provided by Holmstrom (1979) and Jewitt (1988) to principal-agent setups with continuous output.[1] By means of standard agency models, we characterize the region where the Holmstrom-Jewitt framework cannot provide an admissible solution. The problem arises when the reservation utility falls below a certain threshold. Intuitively, the problem can be described as follows: as the reservation utility decreases, it also decreases the output-independent part of the fee (or "fixed salary"). However, under the FOA the fixed salary cannot be negative (see Section 3). This key fact limits the parameter space where the FOA works (Section 4). For some utility functions, the region where the FOA holds is explicitly given in terms of a simple expression for the lower bound of the reservation utility. The limitations described here may affect more sophisticated principal-agent models since the FOA technique is also used in dynamic agency setups (Spear and Srivastava (1987)).

2. THE FIRST-ORDER APPROACH

This section reviews the Mirrlees-Holmstrom formulation of the principal-agent model. A risk-neutral principal (she) hires a risk-averse and effort-averse agent (he) to perform an unobservable task on her behalf that influences an ex post signal X which we identify with output. The input provided by the agent, a, has an opportunity cost $C(a)$, with $C'(a) > 0$ and $C''(a) > 0$; this input (say action or effort) is private information for the agent (hidden action); principal and agent observe the output x (strictly speaking X represents the random output and x an ex-post output realization, but both terms will be used indistinctly if no confusion is possible). The principal offers the agent an output-dependent compensation $s(x)$ for the costly input in order to (partially) solve the moral hazard problem due to the unobservable action. The total (or net) utility of the

[1]The advantages of Jewitt's (1988) approach over Rogerson's (1985) are exposed in Conlon (2009).

agent is additively separable: $u(w,a) := U(w) - C(a)$, where w represents the payment from the principal (which in our case corresponds to $s(x)$) and $U(\cdot)$ is an strictly increasing and strictly concave N-M utility function (the agent is risk-averse): $U'(\cdot) > 0$, $U''(\cdot) < 0$. If output is assumed to be continuous, the optimization program for the principal is stated as follows:

$$\begin{aligned} &\max_{s(x),a} \int [x - s(x)] f(x,a) dx \\ &s.t. \int U(s(x)) f(x,a) dx - C(a) \geq R \\ &a \in \arg\max_{a'} \int U(s(x)) f(x,a') dx - C(a'), \end{aligned} \tag{P1}$$

where R represents the agent's reservation utility (or outside option), and $f(x,a)$ denotes the output density function for a given effort a. The two constraints in (P1) are respectively known as the participation constraint (PC) and the incentive compatibility constraint (ICC).

The optimization program (P1) is usually difficult to solve, and here is where the FOA plays its role. According to the FOA, the original constraint (ICC) $a \in \arg\max_{a'} \int U(s(x)) f(x,a') dx - C(a')$ is replaced by the relaxed constraint (ICC'):

$$\int U[s(x)] f_a(x,a) dx - C'(a) = 0 \tag{ICC'}$$

where $f_a \equiv \partial f / \partial a$. When the first-order approach can be invoked, see Jewitt (1988) for sufficient conditions,[2] the optimal sharing rule $s(x)$ satisfies the two following conditions:

$$\frac{1}{U'(s(x))} = \lambda + \mu \frac{f_a(x,a)}{f(x,a)} \tag{1a}$$

$$\int [x - s(x)] f_a(x,a) dx + \mu \left[\int U(s(x)) f_{aa}(x,a) dx - C''(a) \right] = 0, \tag{1b}$$

where λ and μ respectively denote the Lagrange multipliers associated to (PC) and (ICC'). The second-best optimal contract is derived from Eq. (1a) as a function of λ, μ and a^*, which are obtained from (PC) and (ICC')

[2]In particular, Jewitt shows that if the density function can be expressed as $f(x,a) = \theta(x)\varphi(a)e^{\alpha(a)B(x)}$, with $B(x)$ and f_a/f concave functions of output; expected output concave in effort; and $U(U^{-1}(1/z))$ concave for $z > 0$, then the FOA holds (see Jewitt's Cor. 1; a more relaxed set of conditions can be found in his Theorem 1).

together with Eq. (1b). Next we explain, by means of a typical principal-agent model, the reason why these conditions may not properly characterize the correct solution to (P1).

3. ILLUSTRATING THE PROBLEM

Consider next the example given by Holmstrom (1979, p. 79): a machine repairman with utility function $u(s,a) = 2\sqrt{s} - a^2$ receives a salary contingent on the lapse of time that the repaired machine keeps working. In the example, the machine will keep working for a random time that hinges on the agent's effort, following an exponential distribution with density $f(x) = (1/a)e^{-x/a}$ (so $E(X) = a$). The agent's reservation utlity is not given; Holmstrom fixes instead the Lagrange multiplier associated to constraint (PC), $\lambda = 0.5$, which is equivalent to assuming a reservation utility of $R = 0.75$. In this model the Lagrange multipliers are given by $\lambda = (R + a^2)/2$ and $\mu = a^3$. The second-best contract, calculated using Eq. (1a), Eq. (1b), (PC) and (ICC'), is $s^*(x) = (0.25 + 0.5x)^2$. This contract induces a second-best effort $a^* = 0.5$, and the expected profit is $E[x - s^*(x)] = 0.188$.

If we consider instead $R = 0.25$, the second-best contract obtained using the expressions for λ and μ given above is $s(x) = (-0.030 + 0.556x)^2$. However, for relatively low outputs this contract does not fulfill condition $\lambda + \mu f_a/f \geq 0$. This makes it non-admissible as second-best contract since it implies that the agent's marginal income utility $U'(s(x))$ is negative for low outputs; see Eq. (1a). In fact, the second-best contract must ensure that condition $\lambda + \mu f_a/f \geq 0$ holds for the entire range of output values. Although Jewitt et al. (2008) rule out the possibility of unbounded likelihood ratios in order to avoid the existence of Mirrlees forcing contracts, the inexistence of multipliers is a more fundamental reason to rule them out.

Then, for some parameter configurations, constraint $\lambda + \mu f_a/f \geq 0$ must be taken into account in the computation of the second-best contract; in particular, when the reservation utility is relatively low. For such parameter configurations, expressions $\lambda = (R + a^2)/2$ and $\mu = a^3$ are no longer valid, and the integrations involved in their calculation should be constrained to values compatible with condition $\lambda + \mu f_a/f \geq 0$. In particular, the expression $4a^3 + 2\lambda a - 1 = 0$ derived in Holmstrom (op. cit., p. 79) from his Eq. (8) (our Eq. (1b)) does not hold if the reservation utility is below 0.71. Calculating numerically the correct values of the multipliers (i.e., by imposing $\lambda + \mu f_a/f \geq 0 \;\; \forall x$), the second-best contract turns out to be $s(x) = \max[0; (-0.033 + 0.559x)^2]$. It is important to note that condition $\lambda + \mu f_a/f \geq 0$ applies to all output distributions (take into account that multipliers λ and μ are necessarily positive, and the likelihood ratio f_a/f is necessarily negative for low outputs since $\int f_a dx = 0$).

4. ON THE RANGE OF APPLICABILITY OF THE FOA

4.1. Main result

The caveat described in Section 3 is not serious since the FOA provides the correct solution, a kinked contract, by simply imposing $\lambda + \mu f_a/f \geq 0$ to the entire output range. However, for alternative specifications of the principal-agent model, this condition can invalidate the FOA in some regions of the parameter space. For the sake of illustration, consider that the agent's utility is given by $u(s,a) = \ln(s) - (k/2)a^2$ with $k > 0$, and the output distribution follows a gamma distribution of parameters p and a/p, with $p > 0$ and $a > 0$. This specification ensures $E(X) = a$, as in the previous example, but the results do not depend on this choice.[3] The corresponding density function is $f(x) = \frac{1}{(a/p)^p \Gamma(p)} x^{p-1} e^{-(p/a)x}$, so the likelihood ratio is $f_a/f = (p/a^2)(x-a)$. For this mathematical specification the second-best contract is expressible as $s(x) = \alpha + \beta x$ with α and β real numbers. However, and contrarily to the model of Section 3, we must rule out kinked contracts of the form $s(x) = (\alpha + \beta x)_+ \equiv \max[0; \alpha + \beta x]$ with $\alpha < 0$; in the example of Section 3, a null salary $s(x) = (\alpha + \beta x)_+ = 0$ for low outputs was admissible because the corresponding contribution to $Eu[s(x,a)]$ was 0, and consequently the agent's expected utility remained finite. The reason to rule out kinked contracts lies on the fact that the income utility $\ln(\cdot)$ is unbounded from below at $0+$, which is inherited by the agent's expected utility: if the range of output values for which $s(x) = 0$ (corresponding to the outcomes for which $\lambda + \mu f_a/f < 0$) has positive Lebesgue measure, then $Eu[s(x), a]$ will diverge to $-\infty$. Thus, condition $\lambda + \mu f_a/f \equiv \alpha + \beta x \geq 0$ (which in turn implies $\alpha \geq 0$) constrains the choice contract set, and the FOA only holds in a region of the parameter space. The problem described tends to arise when the reservation utility R is relatively low.

The threshold value below which the FOA fails is calculated as follows: for many typical parameter configurations (see below), there exists a reservation utility for which the second-best contract adopts the form $s(x) = \beta x$, i.e., the agent's payment is proportional to output.[4] For

[3] The gamma distribution encompasses the exponential as a particular case, and it is appealing for the analysis of incentives practices; see Basu et al. (1985), Raju and Srinivasan (1996), Hemmer et al. (1999).

[4] Decreasing R implies a decrease of component α in the contract. This regularity is checked in all the simulations carried out in the present study, but its general fulfilment is not crucial. For us, it suffices the existence of situations where the FOA fails when the reservation utility falls below a certain threshold.

this contract the agent's optimal choice on effort is simply $a^* = \sqrt{1/k}$, which leads to $\beta = 1/(1+2/p) \equiv \beta_1$; see Appendix 1. Assume that the principal optimally issues a contract for which the participation constraint is binding: $EU[s(x)] - C(a) = R$. This condition leads to $\beta = (p/a^*)\exp[-\Gamma'(p)/\Gamma(p) + 1/2 + R] \equiv \beta_2$, with $\Gamma(\cdot)$ the gamma function (which generalizes the factorial); see Appendix 1. By imposing $\beta_1 = \beta_2$ we obtain $\hat{R} \equiv \ln\left(\frac{\sqrt{1/k}}{p+2}\right) + \Gamma'(p)/\Gamma(p) - 1/2$, a threshold value of the reservation utility below which $\alpha < 0$. For the reasons given above, a contract with $\alpha < 0$ is not compatible with the FOA. Then, the FOA can only hold if $R \geq \hat{R}$. The problem described for the logarithmic utility also arises for isoelastic (or CRRA) utility functions with coefficient of relative risk aversion below one.[5] The result is summarized in Theorem 1.

THEOREM 1. *Assume that output follows a gamma distribution of parameters p and a/p, and the agent's total utility is given by $u(s,a) = U(s) - (k/2)a^2$, where $U(s) = \ln(s)$ or $U(s) = (1/\gamma)s^\gamma$ with $\gamma < 1$. The relative risk aversion (RRA) of $U(\cdot)$ is ≥ 1. Assume also that (P1) can be solved so that the participation constraint is binding.*

If the reservation utility is below a certain threshold $\hat{R}$, then the Holmstrom-Jewitt set of sufficient conditions fails to provide the solution to (P1). In particular, for $U(s) = \ln(s)$, $\hat{R} = \ln\left(\frac{\sqrt{1/k}}{p+2}\right) + \Gamma'(p)/\Gamma(p) - 1/2$, with $\Gamma(\cdot)$ the gamma function.

Proof. See the explanation above and Appendix 1. ∎

Remark: If the RRA of $U(\cdot)$ is < 1, there exists a certain threshold $\hat{R}$ below which the FOA solution is a kinked contract (as in the example of Section 3). In particular, for $U(s) = 2\sqrt{s}$, $\hat{R} = (k/2)\left(\frac{2}{k^2(1+2/p)}\right)^{2/3}$; see Appendix 2.

4.2. Discussion

(1) Theorem 1 sets bounds on the range of applicability of the FOA when the reservation utility is relatively low. By characterizing the threshold $\hat{R}$ of the reservation utility under which the FOA holds we constrain the parameter space, which splits into a region where the FOA holds and another one where it does not. These regions are fully delimited in terms of the threshold of the reservation utility, which represents a novel insight in the

[5]It is remarkable that the agency specification with $u(s,a) = \ln(s) - (k/2)a^2$, $X \sim$ *Gamma*, and the reservation utility equal to threshold $\hat{R}$, is completely solvable in closed-form, something unusual in principal-agent models.

agency literature. A graphical image can be useful to grasp the intuition: by diminishing sufficiently the reservation utility, the second-best solution approaches a contract that lacks a fixed part (i.e., it entirely depends on output), which is in turn linked to the necessary condition $\lambda + \mu f_a/f \geq 0$. Below the threshold, the FOA does not hold. Hence, the FOA can fail because an implicit restriction of economic nature constrains the mathematical agency problem for some parameter configurations. Given that the fixed salary α helps attain a higher reservation utility — which reflects the agent's bargaining power —, the failure of the FOA will be typically associated to situations where the labor market for executives is relatively competitive.

(2) In those regions where $\lambda + \mu f_a/f = \alpha + \beta x < 0$, imposing $\alpha \geq 0$ does not solve the problem. First, although a contract of the form $s(x) = \beta x$ fulfils all the Holmstrom-Jewitt requirements, a kinked contract $s(x) = \max[\delta; \alpha + \beta x]$ with $\alpha < 0$ and δ positive but negligible — which represents a slight variation of the former contract — makes the principal better off (than issuing a contract of the form $s(x) = \beta x$). Second, the principal can always improve the former contract $s(x) = \max[\delta; \alpha + \beta x]$ by making δ slightly lower (but positive). In sum, for reservation utilities below $\hat{R}$ the FOA is unable to provide the second-best contract by no means.

(3) Frequently, observed incentives are justified in efficiency terms by comparison with second-best contracts derived from the FOA; Basu et al. (1985), Raju and Srinivasan (1994), Hemmer et al. (1999), Gutiérrez and Salas (2008), et cetera.[6] The fact that the FOA stops working in some regions of the parameter space may suggest the optimality of contracts well different from those obtained from the FOA (for example discontinuous contracts; see e.g. Weinschenk (2010)).

(4) Rogerson (1985) provides another set of sufficient conditions validating the FOA. In his framework, output is a discrete variable. The continuous version of Rogerson's setup is neither absent of problems. To see it, consider that the output distribution is given by $F(x; a) = x^a$, with $x > 0$, which is the continuous version of the example provided in Rogerson (1985). We can check that the likelihood ratio is logarithmic ($f_a/f = 1/a + \ln(x)$); given that Rogerson (1985) demands an unbounded agent's utility for the FOA to hold, a (Mirrlees) forcing contract can be constructed and the FOA is not applicable.

4.3. Example

[6]This represents the optimal contracting approach, a normative approach that follows Jensen and Meckling (1976). According to the managerial power approach, the contract design is part of the agency problem, so actual incentives are explained taking into account managerial rent-seeking practices; see Bebchuk and Fried (2003).

Take $u(s,a) = \ln(s) - (k/2)a^2$; output follows a gamma distribution of parameters p and a/p. For the parameter configuration $p = 3, k = 0.02$ and $R = 1$, the second-best contract (calculated using Eqs. (1a) and (1b) and conditions (PC) and (ICC')) is $s^*(x) = \alpha+\beta x = 0.50+0.65x$, which induces an effort $a^* = 6.56$; the principal's expeced profit is $E[x - s^*(x)] = 1.79$; the agent's expected utility is $R = 1$. If $R = \hat{R} \cong 0.77$, the FOA leads to $s^*(x) = 0.60x$, with $a^* = \sqrt{1/k} = 7.07$; the principal's expeced profit is $E[x - s^*(x)] = 2.83$. For reservation utilities below $\hat{R}$, the FOA does not work.

Consider now $u(s,a) = 2\sqrt{s} - (k/2)a^2$. For the parameter configuration $p = 3, k = 0.5$ and $R = 1$, the second-best contract is $s^* = (\alpha + \beta x)^2 = (0.179+0.4x)^2$, which induces an effort $a = 1.60$. If we lower the reservation utility to $\hat{R} = (k/2)\left(\frac{2}{k^2(1+2/p)}\right)^{2/3} = 0.711$, then $\alpha = 0$ and the second-best contract is $s(x) = \beta^2x^2 = (0.421x)^2$, which induces an effort of $a = 1.68$. For $R < \hat{R} = 0.711$, the second-best contract is a kinked one.

4.4. Other Utilities

For utility functions in the CARA class, the problem is similar to the logarithmic case. Consider that the income utility of the agent is $U(s,a) = -\exp(-rs)$ with $r > 0$, i.e., CARA utility with risk-aversion coefficient r. It must be noted that the CARA utility is bounded-from-below if output X is restricted to be positive, but condition $\lambda + \mu f_a/f \geq 0$ must hold as well. If output follows a gamma distribution, the second-best contract is $s^* = (1/r)\ln(\lambda + \mu f_a/f) = (1/r)\ln(\alpha + \beta x)$ with $\alpha \geq 0,\ \beta > 0$. The expected income utility of the agent is then:

$$E[U(s,a)] = -\int \exp[-r(1/r)\ln(\alpha+\beta x)]f(x,a)dx = -\int \frac{1}{(\alpha+\beta x)} f(x,a)dx.$$

Then expression $\lambda+\mu f_a/f \equiv \alpha+\beta x$ must be strictly positive (in all intervals with positive measure) in order to ensure that the last integrand does not explode near 0. As in the logarithmic case, this limits the application of the FOA. In Appendix 3 we derive the value of the reservation utility threshold.

5. SUMMARY AND CONCLUDING REMARKS

In principal-agent models under moral hazard it is typically assumed that the agent chooses the action that maximizes his expected utility. The incentive compatibility constraint characterizes the utility-maximizing behavior of the agent, giving the optimal response to the contract issued by the principal. When the first-order approach to principal-agent problems holds, the agent's problem has an interior solution and the incentive compatibility constraint can be substituted by a relaxed constrained. The shortcut is extremely useful in agency modelling as it provides an easy way to find second-best incentive contracts, thus reducing a hard problem to a much more manageable one. This makes the question of validating the FOA to principal-agent problems a cornerstone of the theoretical incentives literature.

This note explores the range of applicability of the first-order approach to agency formulated by Holmstrom (1979) and Jewitt (1988). We show the existence of bounds in the set of parameter configurations for which the FOA works. For standard agency models, the validity of the FOA requires a non-negative fixed salary in order to fulfill the basic requirement of the salary having a positive marginal utility. The fact that the fixed salary α must be non-negative sets a threshold for the reservation utility below which the FOA fails. For some typical agency setups this threshold can be given in closed-form, which gives a simple characterization of the region where the FOA holds. Given that the problem described arises when the agent's reservation utility is relatively low, we point out that the FOA tends to fail in situations where managers lack market power. Although some previous contributions remark the role of the reservation utility in the validity of the FOA (Gutiérrez (2012), Moroni and Swinkles (2014)), none of them relates it to the fact that the contract lacks a fixed, non-contingent on output, component. Also, we make explicit the way in which a low reservation utility makes the FOA fail, giving the threshold in closed-form for some utility functions.

The appearance of limits in the range of applicability of the FOA may suggest a rationale for incentives practices alternative to those the FOA can justify. This question is left for further research.

APPENDIX 1

Logarithmic income utility

If the agent's utility is $u(s,a) = \ln(s) - ka^2/2$ and output follows a gamma distribution (in our case of parameters p and a/p), the second-best contract is $s(x) = \alpha + \beta x$ with α and β real numbers. There exist a threshold reservation utility $\hat{R}$ such that $s(x) = \beta x$. Then, $\alpha < 0$ for reservation utilities below $\hat{R}$ (which must be ruled out as it implies $\alpha + \beta x \equiv \lambda + \mu f_a/f < 0$ for relatively low outputs).

Let us calculate the threshold $\hat{R}$. For the second-best contract $s(x) = \beta x$, condition (ICC') leads to

$$\int U(s(x)) f_a(x,a)dx - C'(a) = \beta(\partial/\partial a) \int \ln(x) f(x,a)dx - ka = 0$$

(the mathematical specification allows us to differentiate through the integral). The integral term can be written as $\ln(\beta) + E[\ln(X)]$; the term

$E[\ln(X)]$ is equal to $\ln(a)$ plus other terms that only depend on parameter p (in particular $E[\ln(X)] = \ln(a) - \ln(p) + \Gamma'(p)/\Gamma(p)$, with $\Gamma(\cdot)$ the gamma function, a generalization of the factorial function). Thus, the derivative of $E[\ln(X)]$ with respect to a is simply $(\partial/\partial a)E[\ln(X)] = \partial(\ln(a))/\partial a = 1/a$, and the optimal effort for the agent reduces to $a^* = \sqrt{1/k}$, valid whenever the second-best contract adopts the form $s^*(x) = \beta x$. On the other hand, Eq. (1b) can be written as:

$$\begin{aligned}
0 &= \int [x - s(x)] f_a(x,a)dx + \mu \left[\int U(s(x)) f_{aa}(x,a)dx - C''(a) \right] \\
&= (\partial/\partial a) \int [x - s(x)] f(x,a)dx + \mu \left[(\partial^2/\partial a^2) \int U(s(x)) f(x,a)dx - C''(a) \right] \\
&\overset{s(x)=\beta x}{=\!=} (\partial/\partial a) \int (x - \beta x) f(x,a)dx + \mu \left[(\partial^2/\partial a^2) \int U(\beta x) f(x,a)dx - C''(a) \right] \\
&= (\partial/\partial a) E[(1-\beta)x] + \mu \left\{ (\partial^2/\partial a^2) \left[\ln\beta + E(\ln(X)) - (k/2)a^2 \right] \right\} \\
&= 1 - \beta + \mu \left(-\frac{1}{a^2} - k \right) \overset{a^*=\sqrt{1/k}}{=\!=} 1 - \beta - 2k\mu = 0 \\
&\Rightarrow \mu = (1-\beta)/2k.
\end{aligned}$$

Finally, making equal $\mu = (1-\beta)/2k$ and $\mu = \beta a^2/p$ (derived from the identity $\lambda + \mu f_a/f \equiv \alpha + \beta x$), we obtain $\beta = (1+2/p)^{-1}$. On the other hand, if the principal sets β to ensure that the agent's expected utility exactly reaches the reservation utility R, then $\beta = (p/a)\exp[-\Gamma'(p)/\Gamma(p) + 1/2 + R]$, using the expression for $E[\ln(X)]$ given above. Hence, for reservation utilities below $\hat{R} \equiv \ln\left(\frac{\sqrt{1/k}}{p+2}\right) + \Gamma'(p)/\Gamma(p) - 1/2$, the FOA does not provide the second-best contract. ▌

APPENDIX 2

Square root utility

If the agent's utility is $u(s,a) = 2\sqrt{s} - (k/2)a^2$, (ICC') leads to:

$$\begin{aligned}
&\int U(s(x)) f_a(x,a)dx - C'(a) = 2\beta(\partial/\partial a) \int x f(x,a)dx - C'(a) \\
&= 2\beta(\partial/\partial a)E(x) - C'(a) = 2\beta(\partial/\partial a)a - ka = 2\beta - ka = 0.
\end{aligned}$$

The agent chooses his effort according to the best-response function $a = 2\beta/k$. If $s(x) = \beta^2 x^2$, the expected utility of the agent is:

$$\begin{aligned} E[u(s,a)] =& E[2\sqrt{s(x)}] - C(a) \\ =& 2\beta \int x f(x,a) dx - (k/2)a^2 = 2\beta a - (k/2)a^2 \\ =& 4\beta^2/k - (k/2)4\beta^2/k^2 \\ =& 2\beta^2/k = ka^2/2 = C(a). \end{aligned}$$

Following similar steps as above, Eq. (1b) leads to:

$$\begin{aligned} 0 =& \int [x - s(x)] f_a(x,a) dx + \mu \left[\int U(s(x)) f_{aa}(x,a) dx - C''(a) \right] \\ =& (\partial/\partial a) \int [x - s(x)] f(x,a) dx + \mu \left[(\partial^2/\partial a^2) \int U(s(x)) f(x,a) dx - C''(a) \right] \\ \overset{s(x)=\beta^2x^2}{=\!=}& (\partial/\partial a) \int (x - \beta^2 x^2) f(x,a) dx + \mu \left[(\partial^2/\partial a^2) \int 2\beta x f(x,a) dx - C''(a) \right] \\ =& (\partial/\partial a)[a - \beta^2 a^2 (1 + 1/p)] + \mu \left\{ (\partial^2/\partial a^2)(2\beta a) - [(k/2)a^2]'' \right\} \\ =& 1 - 2\beta^2 a (1 + 1/p) + \mu(-k). \end{aligned}$$

Given that $a = 2\beta/k$ and $\mu = ka^3/2p$ (calculated from (ICC')), Eq. (1b) leads to $a^* = \left(\frac{2}{k^2(1+2/p)}\right)^{1/3}$. Imposing that β is chosen so as to ensure that the expected utility equals the reservation utility, and recalling that $E[u(s,a)] = C(a)$, the threshold $\hat{R}$ turns out to be $\hat{R} = E[u(s(x))] = C(a) = ka^2/2 = (k/2)\left(\frac{2}{k^2(1+2/p)}\right)^{2/3}$. ∎

APPENDIX 3

CARA utility

For $U(s,a) = -\exp(-rs)$ with $r > 0$, and output density $f(x) = \frac{1}{(a/p)^p \Gamma(p)} x^{p-1} e^{-(p/a)x}$, the second-best contract is $s^*(x) = (1/r)\ln(\lambda + \mu f_a/f) = (1/r)\ln(\alpha + \beta x)$ with $\alpha > 0$, $\beta > 0$. For contracts of the form

$s(x) = (1/r)\ln(\beta x)$, the agent's expected utility is:

$$\begin{aligned} E[u(s,a)] &= -\int \exp[-r(1/r)\ln(\alpha+\beta x)]f(x,a)dx - ka^2/2 \\ &= -\int \frac{1}{\alpha+\beta x} f(x,a)dx - ka^2/2 \\ &\overset{\alpha=0}{=} -\frac{1}{\beta}\int x^{-1} f(x,a)dx - ka^2/2 \\ &= -\frac{1}{\beta}\frac{p}{(p-1)a} - ka^2/2. \end{aligned}$$

For the last equality, we use the first negative moment of the gamma distribution; see Cressie et al. (1981). The derivative of the last expression is $\frac{1}{\beta}\frac{p}{(p-1)a^2} - ka$, and making it equal to zero (optimality condition for the agent's problem, ICC') we obtain the agent best response function: $a^* = \left(\frac{1}{\beta}\frac{p}{(p-1)k}\right)^{1/3}$. Taking this into account and $\mu = a^2\beta/p$ we use Eq. (1b) to obtain β:

$$\begin{aligned} 0 =& \int [x - s(x)] f_a(x,a)dx + \mu\left[\int U(s(x)) f_{aa}(x,a)dx - C''(a)\right] \\ =& \frac{\partial}{\partial a}\int [x-s(x)]f(x,a)dx + \mu\frac{\partial^2}{\partial a^2}\left[\int U(s(x))f(x,a)dx - C(a)\right] \\ =& \frac{\partial}{\partial a}[a - (1/r)\ln(\beta a)] + \mu\frac{\partial^2}{\partial a^2}\left[-\frac{p}{\beta(p-1)a} - ka^2/2\right] \\ & \text{(we have ignored terms that vanish after differentiation)} \\ =& \left(1 - \frac{1}{ra}\right) + \mu\left(-\frac{2}{\beta}\frac{p}{p-1}\frac{1}{a^3} - k\right) \\ =& \left(1 - \frac{1}{ra}\right) + \mu\left[-\frac{2}{\beta}\frac{p}{p-1}\left(\frac{1}{\frac{p}{\beta k(p-1)}}\right) - k\right] \\ =& \left(1 - \frac{1}{ra}\right) - \mu 3k = \left(1 - \frac{1}{ra}\right) - \frac{a^2\beta}{p}3k = 0 \\ \Rightarrow& \beta = \frac{p}{(p-1)k} \Big/ \left(\frac{1}{r} + \frac{3}{p-1}\right)^3, \end{aligned}$$

and the threshold value is $\hat{R} = -(3k/2)\left(\frac{1}{r} + \frac{3}{p-1}\right)^2$. ∎

REFERENCES

Basu, Amiya K., Rajiv Lal, Venkatachary Srinivasan, and Richard Staelin, 1985. Salesforce compensation plans: An agency theoretic perspective. *Marketing Science* **4(4)**, 267-291.

Bebchuk, Lucian, and Jesse Fried, 2003. Executive compensation as an agency problem. *Journal of Economic Perspectives* **17(3)**, 71-92.

Chade, Hector, and Jeroen Swinkels, 2020. The no-upward-crossing condition, comparative statics, and the moral hazard problem. *Theoretical Economics* **15(2)**, 445-476.

Conlon, John R., 2009. Two new conditions supporting the first-order approach to multisignal principal-agent problems. *Econometrica* **77(1)**, 249-278.

Cressie, Noel, Anne S. Davis, J. Leroy Folks, and J. Leroy Folks, 1981. The moment-generating function and negative integer moments. *The American Statistician* **35(3)**, 148-150.

Grossman, Sanford and Oliver Hart, 1983. An Analysis of the Principal-Agent Problem. *Econometrica* **51(1)**, 7-45.

Gutiérrez, Óscar, 2012. On the Consistency of the First-Order-Approach to Principal-Agent Problems. *Theoretical Economics Letters* **2(2)**, 157-161.

Gutiérrez, Óscar, and Vicente Salas-Fumás, 2008. Performance standards and optimal incentives. *Journal of Accounting and Economics* **45(1)**, 139-152.

Hemmer, Thomas, Oliver Kim, and Robert E. Verrecchia, 1999. Introducing convexity into optimal compensation contracts. *Journal of Accounting and Economics* **28(3)**, 307-327.

Holmstrom, Bengt, 1979. Moral hazard and observability. *Bell Journal of Economics* **10(1)**, 74-91.

Jensen, Michael, and William H. Meckling, 1976. Theory of the firm: Managerial behavior, agency costs and ownership structure. *Journal of Financial Economics* **3(4)**, 305-360.

Jewitt, Ian, 1988. Justifying the First-Order Approach to Principal-Agent Problems. *Econometrica* **56(5)**, 1177-1190.

Jewitt, Ian, Ohad Kadan, and Jeroen Swinkels, 2008. Moral hazard with bounded payments. *Journal of Economic Theory* **143(1)**, 59-82.

Lambert, Richard A., 2001. Contracting Theory and Accounting. *Journal of Accounting and Economics* **32(1)**, 3-87.

Mirrlees, James, 1976. The optimal structure of incentives and authority within an organization. *Bell Journal of Economics* **7(1)**, 105-131.

Moroni, Sofia, and Jeroen Swinkels, 2014. Existence and non-existence in the moral hazard problem. *Journal of Economic Theory* **150**, 668-682.

Raju, Jagmohan S., and Venkatachary Srinivasan, 1996. Quota-based compensation plans for multiterritory heterogeneous salesforces. *Management Science* **42(10)**, 1454-1462.

Rogerson, William P., 1985. The first-order approach to principal-agent problems. *Econometrica* **53(6)**, 1357-1367.

Sinclair-Desgagné, Bernard, 1994. The first-order approach to multi-signal principal-agent problems. *Econometrica* **62(2)**, 459-465.

Spear, Stephen E., and Sanjay Srivastava, 1987. On repeated moral hazard with discounting. *The Review of Economic Studies* **54(4)**, 599-617.

Weinschenk, Philipp, 2010. Moral hazard and ambiguity, Preprints of the Max Planck Institute for Research on Collective Goods, No. 2010.39, Max Planck Institute for Research on Collective Goods, Bonn.