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A Factor Model Comparison

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Recently, various models have been proposed to explain the cross section of returns in the U.S. stock markets. I present a comparison of a microcap-based factor model with other competing models. I find that the microcap factors mostly explain factors in other models, especially the models of Fama and French (2015, 2016), but not vice versa. In contrast, all-size investment and profitability factors do not perform well in explaining the microcap return spreads. In addition, I find that it is necessary to include multiple characteristics in constructing microcap factors, in order to better explain the microcap return spreads.

Key Words: Microcaps; Factor models; Investment; Profitability.
JEL Classification Numbers: G11, G12.

1. INTRODUCTION

Recently, various models have been proposed to explain the cross section of returns in the U.S. stock markets. Based on the q-theory of investment, Hou, Xie and Zhang (HXZ, 2015) develop a four-factor model, which consists of the market excess return, a size factor, an investment factor and a profitability factor. Alternatively, Fama and French (2015, 2016) develop a five-factor model which includes the three factors in their earlier model plus an alternative investment factor and an alternative profitability factor. Stambaugh and Yuan (2017) construct two factors from a set of 11 prominent characteristics (anomalies) to propose a four-factor mispricing factor model. In a recent paper, Li (2023) finds that long-short return spreads on non-microcaps are captured by those of microcaps and proposes a microcap-based factor model. In this paper, I present a detailed comparison of the microcap-based factor model with other competing models.

Barillas and Shanken (2017) show that testing assets are irrelevant when comparing nested models where all the factors are traded. Li (2018) finds that the alpha in the factor model is proportional to the residual risk of

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the traded factor. Following their insight, I perform regressions of factors in alternative models on the factors in a given model. I also perform regressions of microcap return spreads on factors in alternative models. I find that the microcap model mostly explains factors in many alternative models, especially the models of Fama and French (2015, 2016), but not vice versa. In contrast, all-size investment and profitability factors in the literature perform poorly in explaining the microcap return spreads. In addition, I find that it is necessary to include multiple characteristics in constructing microcap factors, in order to better explain microcap return spreads.

This paper is closely related to several recent studies in factor models. Fama and French (2018) consider factors that use small or big stocks versus factors that use both. Like Stambaugh and Yuan (2017), Light, Maslov and Rytchkov (2017) and Green, Hand and Zhang (2017) combine multiple characteristics or use the resulting combination portfolios in factor models. This study of the microcaps-based model with multiple characteristics reveals further insight into the relative performance of the factor models.

The reminder of the article is organized as follows. In the next section, I describe the data sources and the formations of the microcap factors and all-size factors. I then examine the results of regressions and model comparisons. The last section concludes.

2. DATA AND FACTORS

2.1. Data Description

To compare with the recent literature, especially Fama and French (2015, 2016), I use data on the value-weighted portfolios formed by Fama and French (2015, 2016) to construct factors. The data are provided by Kenneth French. The breakpoints use only NYSE stocks, but the sample is all NYSE, Amex, and NASDAQ stocks. The portfolios are formed from 5×5 quintile sorts: first on market equity (size), and then on each of the following characteristics: growth in total assets (AG), accruals (AC), net share issues (NI), operating profitability (OP), prior (2-12 month) return (PR), the variance of daily returns (Var), the variance of daily residuals (RVar) in the Fama-French (1993) three-factor model, or the book-to-market ratio (B/M). For the sake of consistency, portfolios formed from sorts with negative or zero net share issues (repurchases) are excluded. The first- and the second-pass sorts are independent for all characteristics, except that the second pass sorts on Var and RVar are conditional on size quintile. Portfolios are formed at the end of June each year, except for PR, Var and RVar, which are formed monthly. See Appendix for a detailed description of the definitions of the characteristics.

I also use data on five factors in the model of Fama and French (FF-5, 2015). The data on the five factors are also provided by Kenneth French. The five factors are the value-weighted return on a market portfolio in excess of the riskfree rate (MKT), the return on a small stock portfolio minus the return on a big stock portfolio (SMB), the return on a conservative investment (asset growth) portfolio minus the return on an aggressive investment portfolio (CMA), the return on a portfolio of stocks with robust operating profitability minus the return on a portfolio of stocks with weak operating profitability (RMW), and the return on a portfolio of stocks with high B/M ratios minus the return on a portfolio of stocks with low B/M ratios (HML). Except for MKT, Fama and French (2015) construct the factors from six value-weighted portfolios formed from 2×3 sorts on size and a characteristic such as B/M, AG or OP. All three portfolios, CMA, RMW and HML, are formed at the end of June each year.

Other data include factors in the q-factor model of HXZ (q-4, 2015): The data are provided by Lu Zhang. The size factor (ME) is the difference between the return on a small stock portfolio and the return on a big stock portfolio. The investment (I/A) factor is the difference between the return on a low I/A portfolio and the return on a high I/A portfolio. The profitability (ROE) factor is the difference between the return on a high ROE portfolio and the return on a low ROE portfolio. The three factors are constructed from value-weighted portfolios formed from $2 \times 3 \times 3$ sorts on ME, I/A, and ROE., While the I/A factor is formed at the end of June each year, the ROE factor is formed monthly.

Finally, I use data, provided by Yu Yuan, on the factors in the mispricing factor model of Stambaugh and Yuan (M-4, 2017), who use 2×3 sorts on size and another sorting variable to construct a size factor and two mispricing factors: MGMT and PERF. Unlike Fama and French (2015), who use a single characteristic, Stambaugh and Yuan (2017) use the average stock rankings with respect to a cluster of 11 characteristics as a sorting variable. MGMT is based on a cluster of six characteristics: net stock issues, composite equity issues, accruals, net operating assets, asset growth, and investment to assets. PERF is based on a cluster of five characteristics: distress, O-score, momentum, gross profitability, and return on assets. As Stambaugh and Yuan (2017) construct their size factor differently by using only stocks not used in forming MGMT and PERF, I denote their size factor as SMB'. Both MGMT and PERF are formed monthly. The sample period in this study is January 1967?December 2015 (588 months), due to the availability of data on the factors in the models of FF-5, q-4 and M-4.

2.2. Microcap and All-size Factors

This section describes how the micro-factor and all-size factors are constructed. Let $R_i(Y_j)$ denote the return on a portfolio formed from 5×5

quintile sorts on size and a characteristic, Y. Subscript i refers to a size quintile and j refers to a Y quintile, $i, j = 1, \ldots, 5$. Size and are in ascending orders. Following Fama and French (2015, 2016), size quintile 1 refers to microcaps and size quintile 5 refers to megacaps. Return spreads for each of the size quintiles are:

$$\text{(Low-high)} \quad S_i(Y) = R_i(Y_1) - R_i(Y_5) \text{ for } Y = AG, AC, NI, Var, RVar; \tag{1}$$

$$\text{(High-low)} \quad S_i(Y) = R_i(Y_5) - R_i(Y_1) \text{ for } Y = OP, PR, B/M. \tag{2}$$

The investment-related factor and a profitability-related factor are defined as follows:

$$INV_{IJ} = \begin{cases} \frac{1}{I}\sum_{i=1}^{I}(S_i(AG)), & J = 1 \\ \frac{1}{2I}\sum_{i=1}^{I}(S_i(AG) + S_i(AC)), & J = 2 \\ \frac{1}{3I}\sum_{i=1}^{I}(S_i(AG) + S_i(AC) + S_i(NI)), & J = 3 \end{cases} \tag{3}$$

$$PPR_{IJ} = \begin{cases} \frac{1}{I}\sum_{i=1}^{I}(S_i(OP)), & J = 1 \\ \frac{1}{2I}\sum_{i=1}^{I}(S_i(OP) + S_i(PR)), & J = 2 \\ \frac{1}{3I}\sum_{i=1}^{I}(S_i(OP) + S_i(PR) + S_i(Var)). & J = 3 \end{cases} \tag{4}$$

In equations (3)-(4), each factor combines return spreads from up to three characteristics and size quintiles from one to I, $I \leq 5$ including microcaps. If $I = 1$, the factors are microcap factors. If $I = 5$, the factors are all-size factors. The baseline factors in the microcap model are INV_{I3} and PPR_{I3}.

The factor, INV_{I3}, can be regarded as a broad-based investment factor, since the three characteristics in equation (3) capture information about firms' growth in total assets, changes in working capital and changes in capital expenditure through net share issues. As discussed in the section on the valuation model, the expected stock return is negatively related to accruals when operating profitability is a proxy for total earnings. I treat accruals separately from operating profitability here and use accruals to form a broad-based investment factor since accruals reflect the investment in the short-term assets like the operating working capital, which is current assets (excluding cash and marketable securities) minus current liabilities (excluding short-term debt). New share issues (NI) offer supplemental information about the change in equity that is not reflected in AG and AC. This is especially important for leveraged firms paying no or fixed dividends.

The second factor, PPR_{I3}, measures profitability, performance and risk. The profitability variable, OP, is based on annually rather than quarterly updated earnings.The variable, PR, helps to capture the momentum in firm fundamentals like earnings. As the volatility of total stock returns (expected plus unexpected) contain information about the variation of expected returns, the size- and volatility-sorted portfolios also provide insights about the variation of expected returns. Returns on the prior return- and volatility-sorted portfolios are among the most mispriced and difficult to explain, as documented by Fama and French (2016). INV_{I3} and PPR_{I3} are broader measures of investment and profitability than CMA and RMW used by Fama and French (2015) or I/A factor or ROE factor used by HXZ (2015), who construct each factor based only on one characteristic.

To construct the size factor SMB^*, I first define a size spread for each characteristic Y

$$SMB(Y) = \frac{1}{3}\sum_{j=2}^{4}(R_1(Y_j) - R_5(Y_j)), \tag{5}$$

and then average the spreads over six characteristics:

$$\begin{aligned} SMB^* \ = \ & \frac{1}{6}(SMB(AG) + SMB(AC) + SMB(NI) + SMB(OP) \\ & + SMB(PR) + SMB(Var)). \end{aligned} \tag{6}$$

The construction of the size factor here uses only microcaps and megacaps, unlike other researchers who use small and big stocks which together account for stocks in all size quintiles. The purpose is to increase the average return on the factor. Following Stambaugh and Yuan (2017), I exclude the lowest and highest characteristic quintiles, so that the size factor is neutral to extreme fluctuations in characteristics.

Following most of the literature, portfolio returns are evaluated before transaction costs, which tend to be negatively related to the firm size, but positively related to portfolio rebalancing frequencies. To mitigate the concern of transaction costs, all portfolios used as test assets or used to construct factors are value-weighted. As long as the value-weighted portfolios are traded, the costs of forming factors by equally weighting the traded portfolio return spreads should be manageable. While the profitability factor in the q-factor model of HXZ (2015) and all of the 11 anomaly variables in the mispricing factor model of Stambaugh and Yuan (2017) are formed monthly, the factors in this paper are constructed from portfolios formed annually, except for those related to momentum and volatility. As a result, the transaction costs of factor formation here should be similar to or lower than those associated with other models in the literature.

TABLE 1.

Regressions of Factors in Other Models on Microcap Factors

R_j	α_j		β_j							
	Const.		MKT		SMB^*		INV_{I3}		PPR_{I3}	
Panel A. Dependent variables are factors in Fama-French five-factor models										
CMA	−0.02	(−0.24)	−0.11	(−6.64)	0.07	(4.04)	0.63	(14.80)	0.04	(1.79)
RMW	−0.05	(−0.62)	0.01	(0.48)	−0.05	(−2.26)	−0.09	(−1.77)	0.37	(15.11)
HML	−0.14	(−1.24)	−0.08	(−3.22)	0.12	(4.13)	0.78	(11.74)	0.08	(2.29)
Panel B. Dependent variables are factors in q-factor model of Hou, Xie and Zhang										
I/A	0.09	(1.24)	−0.08	(−5.15)	0.04	(2.08)	0.53	(12.96)	0.07	(3.61)
ROE	0.19	(2.28)	0.05	(2.74)	−0.06	(−2.87)	−0.24	(−5.09)	0.50	(21.10)
Panel C. Dependent variables are factors in the model of Stambaugh and Yuan										
$MGMT$	0.19	(2.02)	−0.21	(−9.87)	−0.02	(−1.03)	0.76	(13.81)	0.13	(4.78)
$PERF$	0.34	(2.28)	−0.06	(−1.99)	0.07	(1.98)	−0.43	(−5.07)	0.59	(13.86)
$A\|\cdot\|$	0.15	(1.42)								

This table reports the estimate of $R_j = \alpha_j + \beta_j F_j + \varepsilon_j$. The microcap factor model (C-4) consists of four factors: the market excess return (MKT), the size factor (SMB^*), and the investment-related factor (INV_{I3}), and a profitability-related factor (PPR_{I3}). INV_{I3} is the average of return spreads from size quintile 1, including three characteristics: asset growth (AG), accruals (AC) and net share issues (NI). PPR_{I3} is the average of return spreads from size quintile 1, including three characteristics: operating profitability (OP), prior return (PR) and variance of daily return (Var). CMA, RMW and HML are, respectively, investment, profitability and book-to market factors in the five-factor model of Fama and French (FF-5, 2015). I/A and ROE are alternative investment and profitability factors in the q-factor model of Hou, Xie and Zhang (q-4, 2015). $MGMT$ and $PERF$ are mispricing factors in the model of Stambaugh and Yuan (M-4, 2017). In parentheses are t-statistics. $A|\cdot|$ is the average of the absolute values of alphas or t-statistics.

3. PERFORMANCE COMPARISONS

3.1. Factors

In Table 1, I present the results of regressing each factor in the FF-5, q-4 and M-4 models (except for the market and size factors) on the factors in the C-4 model with $(I, J) = (1, 3)$. For the three factors in the FF-5 model, CMA, RMW and HML, the intercepts (alphas) are −2, −5 and −14 bps, with t-statistics −0.24, −0.62 and −1.24, respectively. Hence, none of the alphas are significant at the 5 percent level, which implies that the C-4 model explains the average return on each of the three factors in the FF-5 model. In addition, for the I/A factor in the q-4 model, the estimated alpha is 9 bps, with a t-statistic of 1.24, suggesting that this factor is also explained by the C-4 model. The ROE factor in the q-4 model and two factors, MGMT and PERF in the M-4 model, however, are not fully explained by the C-4 model, as their alphas are significant at the 5 percent level. Across the three models, the average of the absolute values of alphas, $A|\cdot|$, at the bottom row of Table 1 is 15 bps, with an average of the absolute values of t-statistics of 1.42. Hence, the microcap factors explain most of the factors in the other competing models.

The estimates of factor loadings show that the investment (or management) factors, CMA, I/A and $MGMT$, have positive loadings on INV_{I3} and PPR_{I3}, but the loadings on the first are much larger and more significant. In contrast, the profitability (performance) factors, RMW, ROE and $PERF$ have positive and significant loadings on but negative loadings on INV_{I3}. The signs and magnitudes of the loadings offer additional evidence supporting the interpretation that INV_{I3} and PPR_{I3} are investment- and profitability-related factors.

TABLE 2.

Regressions of Microcap Factors on Factors in Other Models

R_j	α_j		β_j							
Panel A. Independent variables are factors in Fama-French five-factor models										
	Const.		MKT		SMB		CMA		RMW	
INV_{I3}	0.40	(7.02)	−0.02	(−1.54)	−0.05	(−2.74)	0.44	(15.08)	0.04	(1.44)
PPR_{I3}	0.85	(7.57)	−0.16	(−5.89)	−0.26	(−6.92)	0.25	(4.35)	0.82	(15.96)
Panel B. Independent variables are factors in q-factor model of Hou, Xie and Zhang										
	Const.		MKT		ME		I/A		ROE	
INV_{I3}	0.40	(6.49)	−0.04	(−3.13)	−0.03	(−1.67)	0.42	(13.14)	−0.03	(−1.32)
PPR_{I3}	0.48	(4.71)	−0.16	(−6.83)	−0.20	(−6.07)	0.37	(6.86)	0.91	(23.23)
Panel C. Independent variables are factors in the model of Stambaugh and Yuan										
	Const.		MKT		SMB'		$MGMT$		$PERF$	
INV_{I3}	0.33	(5.40)	−0.01	(−0.45)	0.02	(0.82)	0.32	(13.84)	−0.01	(−0.87)
PPR_{I3}	0.53	(4.44)	−0.05	(−1.72)	−0.28	(−7.07)	0.43	(9.46)	0.50	(16.78)
$A\|\cdot\|$	0.50	(6.18)								

See notes to Table 1 for details.

In Table 2, I report the results of regressing the factors in the microcap model, INV_{I3} or PPR_{I3} on the factors in other models, including FF-5, q-4 and M-4 models. Neither INV_{I3}, nor PPR_{I3} is explained by other models as both alphas are large and significant at the 1 percent level. In the FF-5 model, the alpha is 40 bps (t = 7.02) when is the dependent variable, and 85 bps (t = 7.57) when PPR_{I3} is the dependent variable. In the q-4 model, only the alpha for is lower than that in the FF-5 model, with a coefficient of 48 bps ($t = 4.71$). In the M-4 model, the alphas are 33 bps ($t = 5.40$) and 53 bps ($t = 4.44$). The magnitudes of the alphas and t-statistics here are greater than the corresponding values in Table 1. The average absolute alpha in panel B is 50 bps with an average absolute t-statistic of 6.18. In contrast, the average absolute alpha in Table1 is 15 bps with an average absolute t-statistic of 1.42. The differences suggest that the microcap model (C-4) mostly explains factors in other models but not vice versa.

TABLE 3.

Regressions of Microcap Return Spreads on Microcap Factors

INV_{IJ} and in C-4 are formed on microcaps only with $(I, J) =$						
Y	(1,1)		(1,2)		(1,3)	
AG	0		0.10	(2.34)	0.27	(4.04)
AC	0.09	(1.45)	−0.10	(−2.34)	0.04	(0.70)
NI	0.18	(1.92)	0.43	(3.39)	−0.31	(−3.57)
OP	0		0.20	(1.84)	−0.35	(−3.04)
PR	1.46	(6.75)	−0.20	(−1.84)	0.20	(1.24)
Var	1.48	(8.70)	1.18	(6.93)	0.15	(1.40)
$RVar$	1.38	(8.37)	1.14	(6.70)	0.10	(0.96)
B/M	0.23	(1.76)	0.66	(3.91)	−0.09	(−0.67)
$A\|\cdot\|$	0.60	(4.79)	0.43	(2.62)	0.19	(1.95)

This table reports estimates of alphas in regressions $S_1(Y) = \alpha + \beta F + \varepsilon$, (18), where Y is one of the characteristics. The factor model (C-4) consists of four factors: the market excess return (MKT), the size factor (SMB^*), and the investment-related factor (INV_{IJ}), and a profitability-related factor (PPR_{IJ}). INV_{IJ} is the average of return spreads from size quintile 1 to I ($I = 1, 5$) and including up to J ($J \leq 3$) characteristics: asset growth (AG), accruals (AC) and net share issues (NI). PPR_{IJ} is the average of return spreads from size quintile 1 to I and including up to J characteristics: operating profitability (OP), prior return (PR) and variance of daily return (Var). In parentheses are t-statistics. $A|\cdot|$ is the average of the absolute values of alphas or t-statistics.

3.2. Microcap Spreads

Next, I examine the ability of various model in explaining the microcap return spreads. I estimate the following factor model for microcap return spreads:

$$S_1(Y) = \alpha + \beta F + \varepsilon, \tag{7}$$

where Y is one of the characteristics and β is a vector of factor loadings. Table 3 reports estimates of alphas in regressions of the microcap return spreads on four factors: MKT, SMB^*, and two microcap factors INV_{I3}

and PPR_{IJ} with $(I, J) = (1,1), (1,2)$ or $(1,3)$. PPR_{IJ} is the average return spread from size quintile 1 ($I = 1$) with up to J ($J \leq 3$) characteristics: asset growth (AG), accruals (AC) and net share issues (NI). PPR_{IJ} is the average return spread from size quintile 1 with up to J characteristics: operating profitability (OP), prior return (PR) and variance of daily return (Var).

The estimated alphas and associated t-statistics in paratheses are reported in Table 3. With $J = 1$, the average absolute alpha is 60 bps, with an average $|t|$ of 4.79. With $J = 2$, the average absolute alpha is 43 bps, with an average $|t|$ of 2.62. With $J = 3$, the average absolute alpha is 19 bps, with an average $|t|$ of 1.95, which is insignificant at the 5 percent level. The results here imply that it is necessary to include multiple characteristics to form microcap factors, in order to better explain microcap return spreads.

TABLE 4.

Regressions of Microcap Return Spreads on All-Size Factors

INV_{IJ} and PPR_{IJ} in C-4 are formed on all size with $(I, J) =$						
Y	(5,1)		(5,2)		(5,3)	
AG	0.49	(6.59)	0.31	(3.86)	0.48	(5.43)
AC	0.28	(4.22)	0.14	(2.04)	0.26	(3.71)
NI	0.30	(3.23)	0.47	(3.86)	0.14	(1.34)
OP	−0.03	(−0.35)	0.26	(2.00)	−0.08	(−0.67)
PR	1.52	(7.36)	0.34	(2.88)	0.59	(3.99)
Var	1.36	(8.30)	1.24	(6.99)	0.72	(5.22)
$RVar$	1.30	(8.11)	1.22	(6.86)	0.70	(5.00)
B/M	0.36	(3.13)	0.57	(3.84)	0.21	(1.67)
$A\|\cdot\|$	0.70	(5.16)	0.57	(4.04)	0.40	(3.38)

This table reports estimates of alphas in regressions $S_1(Y) = \alpha + \beta F + \varepsilon$, where Y is one of the characteristics.

In Table 4 the microcap factors ($I = 1$) are replaced with all-size factors ($I = 5$). With all size quintiles included, $(I, J) = (5, 3)$, the average absolute alpha and an average $|t|$ are 40 bps and 3.38, respectively. Similarly, when the number of characteristics is reduced, $(I, J) = (1, 2)$, the average absolute alpha and an average $|t|$ are 43 bps and 2.62, respectively. When the number of characteristics is reduced further $(I, J) = (1, 1)$, the average absolute alpha and an average $|t|$ are 60 bps and 4.79, respectively. It is striking that in the microcap factor model with $(I, J) = (1, 3)$, only three out of eight alphas are significant with absolute t-statistics greater than 2. The inclusion of all size quintiles ($I = 5$, $J = 3$) implies that five out of eight alphas are significant at this level. The exclusion of characteristics, similarly, raises the number of significant alphas to four or more. It is also notable that the exclusion of the return spreads associated with PR and Var raises the alphas of these spreads considerably. Overall, all-size factors perform poorly compared with the microcap factors in explaining the microcap return spreads.

TABLE 5.

Regressions of Microcap Return Spreads on Factors in Other Models

Y	FF-5		q-4		M-4	
AG	0.53	(6.72)	0.56	(6.32)	0.44	(4.45)
AC	0.29	(4.30)	0.29	(4.06)	0.25	(3.35)
NI	0.36	(3.90)	0.33	(2.84)	0.30	(2.58)
OP	−0.02	(−0.24)	−0.09	(−0.76)	0.12	(0.89)
PR	1.35	(6.75)	0.69	(3.74)	0.54	(3.23)
Var	1.22	(7.23)	0.84	(4.82)	0.93	(4.72)
$RVar$	1.17	(7.08)	0.79	(4.58)	0.89	(4.56)
B/M	0.43	(4.57)	0.45	(3.21)	0.38	(2.82)
$A\|\cdot\|$	0.67	(5.10)	0.51	(3.79)	0.48	(3.33)

This table reports estimates of alphas in regressions $S_1(Y) = \alpha + \beta F + \varepsilon$, where Y is one of the characteristics. The other models are the five-factor model of Fama and French (FF-5, 2015), the q-factor model of HXZ (q-4, 2015), and the model of Stambaugh and Yuan (M-4, 2017).

Table 5 reports the estimates of regressions of microcap return spreads on factors in the FF-5, q-4 and M-4 models. In the FF-5 model, the intercepts (alphas) for the characteristics except for OP, are positive with t-statistics 3.90 or greater. The alphas range from 29-53 bps for AC, NI, B/M and AG, and 1.17-1.35 percent for $RVar$, Var, and PR. The magnitudes of the alphas reiterate the difficulty of the FF-5 model in explaining the anomalies associated with the characteristics listed here for the microcaps, as shown by Fama and French (2015, 2016).

The results are qualitatively similar in the q-4 or M-4 model. While HXZ (2015, 2016) and Stambaugh and Yuan (2017) document that the q-4 or M-4 models explain more anomalies than the FF-5 model, these models have difficulty in explaining the microcap spreads. Although alphas for PR, Var and RVar show some noticeable declines, the t-statistics associated with all alphas except for OP are 2.58 or greater. The result contrasts with the hypothesis that the asymmetry arbitrage explains the idiosyncratic volatility puzzle (Stambaugh, Yu and Yuan, 2015). The last row in each panel shows the average absolute alphas. In the FF-5 model the average is 67 bps with an average $|t|$ of 5.10. In the q-4 model, the average alpha falls to 51 bps with an average $|t|$ of 3.79. In the M-4 model, the average alpha falls further to 48 bps with an average $|t|$ of 3.33.

Finally, I compare results across Tables 3-5. All of the microcaps-based models, ($I = 1,\ J = 1, 2, 3$), have lower average absolute alphas than the FF-5 model. Two of the microcaps-based modes ($I = 1,\ J = 2, 3$) and one of the all-size models ($I = 5,\ J = 3$), produce lower average absolute alphas than the FF-5, q-4 and M-4 models. Out of the eight characteristics, the alphas for five of them (AC, PR, Var, $RVar$, and B/M) in the model with $(I, J) = (1, 3)$ are much lower than those in the FF-5, q-4 and M-4 models and insignificant. The alpha for AG, 27 bps, is about half of that in the FF-5 and q-4 model. Most strikingly, the alphas for Var and $RVar$ are 10-15 bps, remarkably lower than 1.17-1.22 percent in the FF-5 model, 79-84 bps in the q-4 model, and 89-93 bps in the M-4 model. The results imply that among all of the models here, the microcaps-based factor model with $(I, J) = (1, 3)$ does the best job in explaining the anomalies associated with microcap stocks.

4. CONCLUSIONS

I study various four-factor pricing models which consist of a market factor, a size factor, and an investment-related factor and a profitability-related factor. The last two factors are constructed from return spreads of microcaps associated with up to six characteristics, including growth in total assets, accruals, net share issues, operating profitability, prior returns, and the variance of daily returns. The model with microcaps-based factors associated with multiple characteristics outperforms the existing prominent models in a variety of ways. First, I find that the model explains the investment factor, the profitability factor and the value (book-to-market) factor in the Fama and French (2015) model plus the investment factor in the HXZ (2015) model. However, the microcap-based investment factor and the profitability factor are unexplained by all-size based factors in other prominent models. Second, the model does the best job in explaining the anomalies associated with microcaps, especially for portfolios of stocks sorted by accruals, prior returns, total volatility, residual volatility and the book-to-market ratio.

APPENDIX: DEFINITION OF CHARACTERISTICS

For portfolios formed in June of year t:

AG (investment) is the change in total assets from the fiscal year ending in year $t-2$ to the fiscal year ending in $t-1$, divided by $t-2$ total assets;

AC is the change in operating working capital per split-adjusted share from the fiscal year-end in $t-2$ to $t-1$ divided by book equity per share in $t-1$;

NI is the change in the natural log of split-adjusted shares outstanding from the fiscal year-end in $t-2$ to the fiscal year-end in $t-1$;

OP is annual revenues minus cost of goods sold, interest expense, and selling, general, and administrative expenses divided by book equity for the last fiscal year end in $t-1$; and

B/M is the book equity for the last fiscal year end in $t-1$ divided by market equity for December of $t-1$.

For portfolios for month t formed at the end of month $t-1$:

PR is prior (2-12) return;

Var is estimated using 60 days (minimum 20) of lagged returns;

$RVar$ is estimated using 60 days (minimum 20) of lagged residuals in the Fama-French (1993) three-factor model.

See the web site of Kenneth French for details.

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