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Revisiting Optimal Capital Taxation with the Spirit of Capitalism*

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This article investigates optimal capital taxation within a two-sector growth model in which agents embody the spirit of capitalism. We demonstrate that the limiting capital income tax rate is generally not zero, with its sign determined by the utility specifications rather than the production technology. All three possible signs are illustrated using widely adopted utility functions. Additionally, the analysis reveals that the limiting consumption tax rate is indeterminate in the long run, while the optimal long-run taxes on labor income and expenditures for generating effective labor are both zero.

Key Words: The spirit of capitalism; Capital income taxation; Two-sector models.

JEL Classification Numbers: H21, E62.

1. INTRODUCTION

One of the most striking results in dynamic optimal tax theory is the Chamley (1986)-Judd (1985) zero capital income tax theorem. Although developed in somewhat different settings, both studies reach the same conclusion: capital should not be taxed in the steady state. The economic intuition is that capital income taxation distorts savings incentives, reduces capital accumulation, and ultimately hinders economic growth. To

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avoid these adverse effects, capital should remain untaxed in the long run. These seminal works have inspired a large body of research in dynamic public finance. While some studies confirm the theorem, others challenge it by introducing alternative settings.

In this paper, we introduce the spirit of capitalism -also referred to as wealth effects or status preferences - into dynamic optimal tax theory to reexamine optimal capital taxation.[1] Our motivation of incorporating the spirit of capitalism into optimal tax theory is fourfold. First, taxing capital income is, in effect, taxing the wealthy. Individuals with a strong spirit of capitalism are likely to be among the wealthiest members of society, and the spirit-of-capitalism channel may yield new insights for optimal tax theory. Second, prior studies have used the spirit-of-capitalism framework to modify the modified golden rule result in optimal growth theory (Kurz, 1968; Zou, 1994, 1995). A novel savings motive arising from the spirit of capitalism can lead to capital overaccumulation[2], which in turn has implications for the taxation of capital income. Third, the spirit-of-capitalism approach has been widely applied to address puzzles in economics and finance, suggesting it may also have valuable implications for optimal capital taxation.[3] Finally, we seek to examine the conjecture of Jones et al. (1997), which posit that "if the capital stock enters the objective function of the pseudo planner's problem, then the planner will tax capital income in the limit" (pp. 105-106). To this end, we embed the spirit of capitalism into a two-sector growth model and analyze how it shapes optimal capital taxation.

By incorporating the spirit of capitalism into the two-sector growth model with physical and human capital developed by Jones et al. (1997), we reexamine the Chamley-Judd zero capital income taxation theorem. We show that the limiting capital income tax rate is generally not zero; in the

[1]The modeling strategy of including capital or wealth directly in the utility function is known as the spirit of capitalism (Bakshi and Chen, 1996; Boileau and Braeu, 2007; Doepke and Zilibotti, 2008; Luo et al., 2009a, 2009b; Karnizova, 2010; Smith, 2001; Zou, 1994, 1995; Li et al., 2020; He et al., 2022; Shi, et al., 2025), Social status/norms (Cole et al., 1992; Luo and Young, 2009), or wealth effects (Kurz, 1968).

[2]The standard Ramsey model implies that the net marginal product of per capita capital equals the time preference rate, $f'(k^{mg}) = \rho$, known as the modified golden-rule level of physical capital. In contrast, Kurz (1968) and Zou (1994) show that the spirit of capitalism reduces the marginal product of capital, $f'(k^*) = \rho - U_k/U_c < \rho = f'(k^{mg})$, thereby increasing the steady-state capital stock, $k^* > k^{mg}$. Zou (1995) further develops this approach to explain the savings behavior of the very healthy.

[3]Applications include the Equity Premium Puzzle (Bakshi and Chen, 1996; Smith, 2001; Boileau and Braeu, 2007), savings and wealth accumulation (Cole et al., 1992; Zou, 1995), innovation and growth (He et al., 2022; Shi et al., 2025), consumption (Luo et al., 2009b); occupational choice (Doepke and Zilibotti, 2008), wealth distribution (Luo and Young, 2009), business cycle (Boileau and Braeu, 2007; Karnizova, 2010), and cross-country growth differences (Kurz, 1968; Zou, 1994).

long run, it may be positive, negative, or zero. The sign of the limiting capital tax rate depends on the specification of the utility function rather than on the production technology. If consumers derive utility from both the spirit of capitalism and consumption, the zero capital income taxation theorem no longer holds. Similar results for one-sector growth models are found in Li et al. (2020) and Bu and Wang (2025). Intuitively, the spirit of capitalism introduces a non-pecuniary return on capital accumulation in the consumption Euler equation, altering the effect of taxing capital. On one hand, taxing capital discourages MPK-driven capital accumulation, which-in standard Ramsey settings-reduces steady-state capital. On the other hand, a lower steady-state capital increases the marginal utility of captial in the non-pecuniary term, thereby encouraging spirit-of-capitalism-driven capital accumulation. The net effect is ambiguous, making it impossible to determine the sign of the limiting capital income tax rate in general. We present specific, widely used utility functions that yield each of the three possibilities. Moreover, we show that although the optimal capital income tax is not zero in general, the limiting labor income tax rate is always zero in the long run. Our findings reject Jones et al. (1997)'s conjecture while reinforcing their main conclusion that physical capital, as a stock, is not inherently special, and that zero-taxation results depend on model specifications. Furthermore, we demonstrate that the limiting consumption tax rate is also indeterminate (it may be positive, zero, or negative), while the limiting tax rate on expenditures for generating effective labor is always zero.

Literature review. This research is closely related to four strands of the existing literature. First, a large body of work finds various forms of zero-capital-tax results. For exogenous-growth models, see, for example, Chamley (1980, 1986), Judd (1985, 1999), Atkeson et al. (1999), and Chari and Kehoe (1999). For endogenous-growth models, see Lucas (1990) and Jones et al. (1993, 1997). For results under uncertainty, see Zhu (1992), Judd (1993), and Chari et al. (1994). For results with heterogeneous agents, see Werning (2007) and Greulich et al. (2016).

Second, other studies shows that positive or negative long-run capital taxes can be optimal. For results on capital taxation in OLG models, see, for example, Erosa and Gervais (2002). For models that assign social weights to future periods or generations, see Farhi and Werning (2010, 2013). For results with limited commitment, see Chari and Kehoe (1990), Stokey (1991), and Farhi et al. (2012). For models with incomplete markets and idiosyncratic risk, see Aiyagari (1995) and Conesa et al. (2009). For heterogeneous-agent and incomplet-information models, see Golosov et al. (2003) and Golosov et al. (2006).

Third, our paper contributes to a large literature examining the implications of the spirit of capitalism or the quest for status on various economic outcomes, including: economic growth (Zou, 1994; Futagami and Shibata, 1998; Smith, 1999; Corneo and Jeanne, 2001; He et al., 2022; Shi et al., 2025); savings (Cole et al., 1992; Zou, 1995; Carroll, 2000; Luo et al., 2009a); consumption (Luo et al., 2009b); asset pricing (Bakshi and Chen, 1996; Smith, 2001; Gong and Zou, 2002); wealth distribution (Luo and Young, 2009; Corneo and Jeanne, 2001); business cycles (Karnizova, 2010; Michaillat and Saez, 2015); Money (Gong and Zou, 2001; Michaillat and Saez, 2021); taxation (Saez and Stantcheva, 2018); comparison with recursive utility (Alaoui and Sandroni, 2018); patent protection (Pan et al., 2018); expansion of variety (Hof and Prettner, 2019); and industrialization (Chu and Wang, 2020).

Fourth, our paper also contributes the literature on how the spirit of capitalism (or wealth in utility function) affects optimal taxation. In models with wealth in utlity function, Saez and Stantcheva (2018) derive optimal linear and nonlinear tax formulas expressed in terms of empirical elasticities and social preferences, and address several important policy questions. Li et al. (2020) examine how the spirit of capitalism influences optimal capital taxes and show that the optimal rate is generally not zero-it may be positive, negative, or zero. In the special case of an additively separable utility function with respect to consumption, leisure, and capital, the sign of the limiting capital tax depends on the comparison between the capital elasticity of the marginal utility of capital and the consumption elasticity of the marginal utility of consumption. Both of these studies employ one-sector exogenous growth models. In constrast, our paper investigates how the spirit of capitalism affects optimal capital taxes in a two-sector endogenous growth model. In this way, our work complements the existing literature on the topic.

The remainder of the paper is organized as follows. Section 2 presents the model setup. Section 3 applies the primal approach to solve the Ramsey problem. Section 4 derives the optimal long-run taxes. Section 5 offers concluding remarks. The mathematical appendix is provided in Section 6.

2. THE MODEL

2.1. Model setup

Consider a production economy without uncertainty. An infinitely lived representative household derives utility from streams of consumption, leisure, and physical capital $\{c_t, l_t, k_t\}_{t=0}^{\infty}$, according to

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t, k_t), \tag{1}$$

where $\beta \in (0,1)$ is the time discount factor; $c_t \geq 0$, $l_t \geq 0$, and $k_t \geq 0$ denote, respectively, consumption, leisure, and the stock of physical capital at time t. The utility function satisfies $u_i > 0$, $u_{ii} < 0$, and $u_{ij} \geq 0$ for $i, j \in \{c, l, k\}$ with $i \neq j$.[4] The household's utility depends directly on the stock of physical capital, with $u_k > 0$ and $u_{kk} < 0$. This specification-capital in the utility function-captures the consumer's preference for the spirit of capitalism. We justify this assumption in details below.[5]

The household is endowed with one unit of time per period, which it allocates among leisure l_t, time spent accumulating human capital n_{ht}, and time spent producing effective labor n_{mt}. The time constraint is given by:

$$l_t + n_{ht} + n_{mt} = 1. \tag{2}$$

The household accumulates human capital according to

$$h_{t+1} = (1 - \delta_h) h_t + H(x_{ht}, h_t, n_{ht}), (\mu_t) \tag{3}$$

where $\delta_h \in (0,1)$ is the depreciation rate, and $H(\cdot)$ represents the human capital production function, which uses market goods x_{ht}, existing human capital h_t, and raw labor time n_{ht}. The shadow value of human capital is denoted by μ_t. Human capital is used to produce efficiency units of labor e_t (i.e., skill-adjusted labor supply):

$$e_t = M(x_{mt}, h_t, n_{mt}),$$

where x_{mt} and n_{mt} are market goods and raw labor allocated to labor-efficiency production. Both H and M exhibit constant returns to scale in (x_{jt}, h_t) for $j = \{h, m\}$, are twice continuously differentiable, and have strictly positive but diminishing marginal products in all inputs.

The single good is produced using a standard production function $F(k_t, e_t)$, where k_t denotes physical capital and e_t denotes effective labor. The function F is strictly increasing and strictly concave in each input, and it exhibits constant returns to scale. By Euler's theorem for the homogeneous functions, applied to H, M, and F, we obtain

$$\begin{aligned}
H(x_{ht}, h_t, n_{ht}) &= H_x(x_{ht}, h_t, n_{ht}) x_{ht} + H_h(x_{ht}, h_t, n_{ht}) h_t, && (4)\\
M(x_{mt}, h_t, n_{mt}) &= M_x(x_{mt}, h_t, n_{mt}) x_{mt} + M_h(x_{mt}, h_t, n_{mt}) h_t, && (5)\\
F(k_t, e_t) &= F_k(k_t, e_t) k_t + F_e(k_t, e_t) e_t.
\end{aligned}$$

[4] $u_{ii} < 0$ indicates that diminishing marginal utility in each commodity, while $u_{ij} > 0$ $(i \neq j)$ indicates that the marginal utility of one commodity increases with the consumption of another.

[5] Kurz (1968) refers to this as the "wealth effect"; Zou (1994) calls it "the spirit of capitalism"; Luo and Young (2009) describe it as the demand for social status. Mathematically, these formulations are equivalent.

Output can be consumed by households, used by the government, allocated to human capital accumulation, devoted to the production of effective labor, or invested to augment the capital stock. The resource constraint of the economy[6] is given by

$$c_t + g_t + x_{ht} + x_{mt} + k_{t+1} = F(k_t, e_t) + (1-\delta)k_t, \tag{6}$$

where $\delta \in (0,1)$ is the depreciation rate of physical capital, and $\{g\}_{t=0}^{\infty}$ denotes an exogenous sequence of government purchases.

Government. The government finances its stream of purchases $\{g_t\}_{t=0}^{\infty}$ by levying taxes on capital income, labor income, consumption and expenditures on producing effective labor, at tax rate τ_t^k, τ_t^n, τ_t^c, and τ_t^m, respectively. It can also trade one-period bonds, whose sequential trading suffices to implement any desired intertemporal trade in a world without uncertainty. Let b_t denote the government's indebtedness to the private sector, denominated in time t-goods, maturing at the beginning of period t. The government's budget constraint is

$$g_t = \tau_t^k r_t k_t + \tau_t^n w_t n_t + \tau_t^c c_t + \tau_t^m x_{mt} + \frac{b_{t+1}}{R_t} - b_t, \tag{7}$$

where r_t and w_t are the rental rate of capital and the wage rate for labor, respectively, denominated in units of time t goods, and R_t is the gross rate of return on one-period bonds held from t to $t+1$.[7] Interest earnings on bonds are assumed to be tax-exempt; this assumption is innocuous for bond exchanges between the government and the private sector. We assume that the government can commit fully and credibly to future tax rates, thereby avoiding the time-consistency problem raised in Kydland and Prescott (1977) and Barro and Gordon (1983).[8]

Households. A representative household chooses $\{c_t, l_t, x_{ht}, x_{mt}, n_{ht}, n_{mt}, e_t, k_{t+1}, b_{t+1}\}_{t=0}^{\infty}$ to maximize expression (1), subject to the sequence of budget constraints

$$\begin{aligned} &(1+\tau_t^c)\,c_t + (1+\tau_t^m)\,x_{mt} + x_{ht} + k_{t+1} + \frac{b_{t+1}}{R_t} \\ = \;& (1-\tau_t^k) r_t k_t + (1-\tau_t^n) w_t e_t + (1-\delta_k) k_t + b_t, \end{aligned} \tag{8}$$

[6]The resource constraint can be derived by combining the household flow budget constraint, the government budget constraint, and the marginal productivity conditions.

[7]One-period government bonds cannot be accumulated like the private capital; hence, they are not included in the representative household's utility function.

[8]Xie (1997) raises the time-inconsistency problem in the context of the Ramsey taxation problem.

together with the human capital accumulation equation (3) and the time-allocation constraint (2), for $t \geq 0$, given k_0, b_0, and h_0. Here, b_t denotes the real value of one-period government bond holdings maturing at the beginning of period t, denominated in units of time t consumption.

Forming the Lagrangian with multipliers λ_t and μ_t associated with constraints (8) and (3), respectively, we derive the household's optimality conditions (see Appendix A)[9]:

$$\frac{(1-\tau_t^n)\, w_t M_n(t)}{(1+\tau_t^c)} = \frac{u_l(t)}{u_c(t)} = \frac{H_n(t)}{(1+\tau_t^c)\, H_x(t)}, \tag{9}$$

$$(1-\tau_t^n)\, w_t M_x(t) = 1+\tau_t^m, \tag{10}$$

$$\frac{u_c(t)}{(1+\tau_t^c)} = \beta\left\{u_k(t+1) + \frac{u_c(t+1)}{\left(1+\tau_{t+1}^c\right)}\left[\left(1-\tau_{t+1}^k\right) r_{t+1} + 1 - \delta\right]\right\}, \tag{11}$$

$$\frac{u_c(t)}{(1+\tau_t^c)\, H_x(t)} = \beta\frac{u_c(t+1)}{\left(1+\tau_{t+1}^c\right)}\left[\left(1-\tau_{t+1}^n\right) w_{t+1} M_h(t+1) + \frac{H_h(t+1) + 1 - \delta_h}{H_x(t+1)}\right], \tag{12}$$

$$\frac{u_c(t)}{(1+\tau_t^c)} = \beta R_t \frac{u_c(t+1)}{\left(1+\tau_{t+1}^c\right)}, \tag{13}$$

$$R_t = \frac{(1-\tau_{t+1}^k) r_{t+1} + 1 - \delta}{\left(1 - \frac{\beta u_k(t+1)(1+\tau_t^c)}{u_c(t)}\right)}. \tag{14}$$

Optimality requires that the marginal contribution of the last unit of final goods be equal across consumption and human capital accumulation, and that the marginal contribution of the last unit of time be equal across utility from leisure, human capital production, and effective labor production, as shown in Equation (9). Equation (10) states that the (net) marginal product value of the market good-i.e., the marginal product of effective labor with respect to the market good, multiplied by its after-tax wage, $(1-\tau_t^n)\, w_t M_x(t)$-is equal to its marginal cost, given by the after-tax price $(1+\tau_t^m)$. Equations (11) and (13) are consumption Euler equations, in which a new positive term $\beta u_k(t+1)$ represents an additional savings channel arising from the spirit of capitalism.[10] Equation (14) is the modi-

[9]Let $u_c(t)$, $u_l(t)$, and $u_k(t)$ denote the time-t derivatives of $u(c_t, l_t, k_t)$ with respect to consumption, leisure, and physical capital, respectively.

[10]This new savings motive is clearer in the steady state, no-tax version of Equation (11): $F_k = 1/\beta - 1 + \delta_k - u_k/u_c$. Relative to the standard model without capital in the utility function, the marginal product of capital F_k is lower by a positive term $u_k/u_c > 0$.

fied no-arbitrage condition for portfolio choices, where a new positive term $\beta\frac{u_k(t+1)}{u_c(t)}$ appears in the denominator.

Firms. In each period, taking (r_t, w_t) as given, the representative firm rents capital and labor from households to solve the profit-maximizing problem:

$$\max_{\{k_t,e_t\}} F(k_t, e_t) - r_t k_t - w_t e_t.$$

The first-order conditions for this problem are

$$r_t = F_k(k_t, e_t), w_t = F_n(k_t, e_t). \tag{15}$$

In words, each input is employed until the marginal product of the last unit equals its rental price. Under constant returns to scale, this implies the standard result that pure profits are zero.

2.2. Wealth in utility function as a representation of the spirit of capitalism

In this subsection, we argue that including capital or wealth in the preference specification reflects the spirit of capitalism. To support this view, we cite and discuss the statements of three influential thinkers: Max Weber, Karl Marx, and Joseph Schumpeter.

First, Weber (1958) contends that the essence of the spirit of capitalism lies in the continual accumulation of wealth for its own sake, rather than solely for the material rewards it can provide. In "The Proestant Ethic and the Spirit of Capitalism", Weber writes:

"In fact, the summum bonum of his ethic, the earning of more and more money, combined with the strict avoidance of all sponteneous enjoyment of life, is above all completely devoid of any eudaemonistic, not to say hedonistic, admixture. It is thought of so purely as an end in itself, that from the point of view of the happiness of, or utility to, the single individual, it appears entirely transcendental and absolutely irrational. Man is dominated by the making of money, by acquisition as the ultimate purpose of his life. Economic acquisition is no longer subordinated to man as the means for the satisfaction of his material needs. This reversal of what we should call the natural relationship, so irrational from a naive point of view, is evidently a leading principle of capitalism as it is foreign to all people not under capitalistic influence." (p.53)

Second, Karl Marx (1977) argues that saving and capital accumulation constitute the essence of the capitalist system. He write:

"Accumulate, accumulate! That is Moses and the prophets! Industry furnishes the material which saving accumulates! Therefore save, save, i.e., reconvert the greatest possible portion of surplus-value into capital! Accumulation for the sake of accumulation, production for the sake of produc-

tion: this is the formula in which classcial economists expressed the historical mission of the bourgeoisie in the period of its domination." (Capital, Vol. 1, p. 742)

Thus, Marx believes that capitalists derive utility from savings and wealth accumulation.

Third, Schumpeter (1934) expresses a similar viewpoint regarding entrepreneurs' psychology and objectives:

"Then there is the will to conquer, the impulse to fight, to prove oneself superior to others, to succeed for the sake, not of the fruits of success, but of success itself, from this aspect, economic action becomes akin to sport.... The financial result is a secondary consideration, or, all all events, mainly values as an index of success and as a symptom of victory, the displaying of which very often is more important as a motive of large expenditure than the wish for the consumers' goods themselves." (p.93)

Furthermore, Schumpeter rejects the traditional hedonistic assumption that entrepreneurs' utility derive solely from consumption. Instead, he emphasizes the "psychology of entrepreneurs": entrepreneurs are strongly motivated by the "dream and the will to found a private kingdom, usually, though not necessarily, also a dynasty."

Most researchers interpret wealth in the utlity function as embodying the spirit of capitalism (Zou, 1994, 1995; Bakshi and Chen, 1996; Smith, 1999, 2001; Boileau and Braeu, 2007; Luo et al., 2009a, 2009b; Karnizova, 2010; Li et al., 2020; Shi et al., 2025). Others describe this mathematical structure as reflecting wealth effects (Kurz, 1968), social status (Futagami and Shibata, 1998; Corneo and Jeanne, 2001; Gong and Zou, 2001, 2002; Luo and Young, 2009; Pan et al., 2018; Chu et al., 2020), or social norms (Cole et al., 1992). Moreover, Saez et al. (2018) discuss three possbile microfoundations for wealth in the utility: the bequest motive, entrepreneurship, and service flows from wealth.

3. THE PRIMAL APPROACH TO THE RAMSEY PROBLEM

We examine the optimal taxes in the long run by employing the primal approach developed by Atkinson and Stiglitz (1980) and Lucas and Stokey (1983). To this end, we present the following useful definitions.

Definition 1 A competitive equilibrium consists of a price system $\{r_t, w_t, R_t\}$, *an allocation* $\{c_t, l_t, x_{ht}, x_{mt}, n_{ht}, n_{mt}, k_{t+1}, b_{t+1}\}_{t=0}^{\infty}$, *and government policies* $\left\{g_t, \tau_t^k, \tau_t^n, \tau_t^c, B_{t+1}\right\}_{t=0}^{\infty}$, *such that:*

(a) Given the price system and government policies, the allocation solves both the firm's and household's problems with $b_t = B_t$ *for all* $t \geq 0$*;*

(b) Given the allocation and price system, the government policies satisfy the sequence of government budget constraints (7) for all $t \geq 0$*;*

(c) The time allocation constraint (2) and the resource constraint (6) hold for all $t \geq 0$.

There exist multiple competitive equilibria indexed by different government policies, which motivates the Ramsey problem.

Definition 2 Given initial conditions k_0, b_0, h_0, and τ_0^k, the Ramsey problem is to choose a competitive equilibrium that maximizes the objective function (1).

Substituting the flow budget constraint repeatedly, applying the household's optimality conditions, and utilizing the linear homogeneity of $H(\cdot)$ and $M(\cdot)$, along with imposing the following two transversality conditions:

$$\lim_{T\to\infty} q_T^0 k_{T+1} = 0, \lim_{T\to\infty} q_T^0 \frac{b_{T+1}}{R_T} = 0,$$

we derive the household's present-value budget constraint of the household[11]:

$$\begin{aligned} &\sum_{t=0}^{\infty}\left[q_t^0\left(1+\tau_t^c\right)c_t + q_{t+1}^0 \frac{u_k\left(t+1\right)\left(1+\tau_{t+1}^c\right)}{u_c\left(t+1\right)} k_{t+1}\right] \\ = &\left(\begin{array}{c} \left[(1-\tau_0^n)w_0 M_h\left(0\right) + \frac{H_h(0)+1-\delta_h}{H_x(0)}\right] h_0 \\ +\left[\left(1-\tau_0^k\right) r_0 + 1 - \delta\right] k_0 + b_0 \end{array}\right). \end{aligned} \tag{16}$$

Using the definition of the Arrow-Debreu price,

$$q_t^0 \equiv \sum_{i=0}^{t-1} R_i^{-1}, \tag{17}$$

for $t \geq 1$, with the numeraire $q_0^0 = 1$, and equation (13), we have

$$q_t^0 = \frac{\beta^t u_c\left(t\right)\left(1+\tau_t^c\right)}{u_c\left(0\right)\left(1+\tau_0^c\right)}. \tag{18}$$

Substituting (18) into (16) yields the implementability condition:

$$\sum_{t=0}^{\infty} \beta^t \left[u_c\left(t\right) c_t + \beta u_k\left(t+1\right) k_{t+1}\right] = A, \tag{19}$$

where

$$A \equiv \frac{u_c\left(0\right)}{1+\tau_0^c}\left\{\left[\frac{H_h\left(0\right)+1-\delta_h}{H_x\left(0\right)} + \left(1-\tau_0^n\right) F_e\left(0\right) M_h\left(0\right)\right] h_0 + \left[\left(1-\tau_0^k\right) r_0 + 1 - \delta\right] k_0 + b_0\right\}.$$

[11]The derivation of the present-value budget constraint is provided in Appendix B.

Thus, the Ramsey problem is to maximize expression (1), subject to (19) and the resource constraint (6).

We proceed by assuming that government expenditures are sufficiently small so that the problem's constraint set is convex, allowing us to apply Lagrangian methods. Specifically, let ϕ denote the Lagrangian multiplier associated with (19), and define

$$\begin{aligned} & U\left(c_t, n_{ht}, n_{mt}, k_t, c_{t+1}, n_{ht+1}, n_{mt+1}, k_{t+1}, \phi\right) \\ = \;& u\left(c_t, 1-n_{ht}-n_{mt}, k_t\right)+\phi\left[u_c\left(t\right) c_t+\beta u_k\left(t+1\right) k_{t+1}\right]. \end{aligned}$$

We can then construct the Lagrangian

$$\mathcal{L}=\sum_{t=0}^{\infty} \beta^t\left\{\begin{array}{c} U\left(c_t, n_{ht}, n_{mt}, k_t, c_{t+1}, n_{ht+1}, n_{mt+1}, k_{t+1}, \phi\right)+ \\ \theta_t\left[F\left(k_t, M\left(x_{mt}, h_t, n_{mt}\right)\right)+\left(1-\delta_k\right) k_t-c_t-g_t-k_{t+1}-x_{mt}-x_{ht}\right] \\ v_t\left[\left(1-\delta_h\right) h_t+H\left(x_{ht}, h_t, n_{ht}\right)-h_{t+1}\right] \end{array}\right\}-\phi A,$$

where $\{\theta_t\}_{t=0}^{\infty}$ is a sequence of Lagrange multipliers for the resource constraint (6).

The first-order necessary conditions for an interior solution are

$$c_t: \beta\left[U_1\left(t\right)-\theta_t\right]+U_5\left(t-1\right)=0, \tag{20}$$

$$n_{ht}: \beta\left[U_2\left(t\right)+v_t H_n\left(t\right)\right]+U_6\left(t-1\right)=0, \tag{21}$$

$$n_{mt}: \beta\left[U_3\left(t\right)+\theta_t F_e\left(t\right) M_n\left(t\right)\right]+U_7\left(t-1\right)=0, \tag{22}$$

$$x_{ht}: \theta_t=v_t H_x\left(t\right), \tag{23}$$

$$x_{mt}: F_e\left(t\right) M_x\left(t\right)=1, \tag{24}$$

$$k_{t+1}: U_8\left(t\right)-\theta_t=\beta\left\{U_4\left(t+1\right)+\theta_{t+1}\left[F_k\left(t+1\right)+1-\delta\right]\right\}, \tag{25}$$

$$h_{t+1}: v_t=\beta\left\{\theta_{t+1} F_e\left(t+1\right) M_h\left(t+1\right)+v_{t+1}\left[H_h\left(t+1\right)+1-\delta_h\right]\right\}, \tag{26}$$

where[12]

$$U_1\left(t\right)=u_c\left(t\right)+\phi\left[u_{cc}\left(t\right) c_t+u_c\left(t\right)\right],$$

$$U_2\left(t\right)=U_3\left(t\right)=-u_l\left(t\right)-\phi u_{cl}\left(t\right) c_t,$$

$$U_4\left(t+1\right)=u_k\left(t+1\right)+\phi u_{ck}\left(t+1\right) c_{t+1},$$

[12]Note that $U_i\left(t\right), i \in \{1,2,3,4,5,6,7,8,9\}$ stands for the partial derivative of the multivariate function $U\left(\cdot\right)$ with respect to its ith argument, where all of the arguments take values at time t. For example, $U_5\left(t-1\right)$ stands for the partial derivative of $U\left(\cdot\right)$ with respect to c_{t+1}, where all the arguments take values at time $t-1$.

$$U_5\left(t-1\right)=\phi\beta u_{kc}\left(t\right)k_t,$$

$$U_6\left(t-1\right)=U_7\left(t-1\right)=-\phi\beta u_{kl}\left(t\right)k_t,$$

$$U_8\left(t\right)=\phi\beta\left[u_{kk}\left(t+1\right)k_{t+1}+u_k\left(t+1\right)\right].$$

4. OPTIMAL TAXATION

In this section, we examine the limiting optimal taxes on capital income, labor income, consumption and expenditures for generating effective labor force, respectively. Consider the special case where there exists a $T \geq 0$ such that $g_t = g$ for all $t \geq T$. We assume that a solution to the Ramsey problem exists and converges to a time-invariant allocation, so that consumption c, labor l, capital k, and human capital h remain constant after some time.[13]

4.1. Capital income tax $\left(\tau^k\right)$

In the steady state, equation (25) becomes

$$\beta\left[\frac{u_k+\phi\left(u_{kk}k+u_k+u_{ck}c\right)}{\theta}+\left(F_k+1-\delta\right)\right]=1. \tag{27}$$

Dividing both sides of (11) by $u_c\left(t\right)/\left(1+\tau_t^c\right)$ and substituting the first equality of equation (9), we obtain

$$\beta\left\{\frac{u_k}{u_l}\frac{H_n}{H_x}+\left[\left(1-\tau^k\right)F_k+1-\delta\right]\right\}=1. \tag{28}$$

Combining equations (27) and (28), we get

$$\tau^k=\frac{1}{F_k}\left[\frac{u_k}{u_l}\frac{H_n}{H_x}-\frac{u_k+\phi\left(u_{kk}k+u_k+u_{ck}c\right)}{\theta}\right]. \tag{29}$$

From the steady-state versions of equations (21) and (22), we have

$$\frac{\theta}{v}=\frac{H_n}{F_eM_n}. \tag{30}$$

Combining (23) and (30) gives

$$\frac{H_n}{H_x}=F_eM_n. \tag{31}$$

[13]Unlike the standard Ramsey model, models incorporating the spirit of capitalism (or wealth effects) may exhibit multiple equilibria, as examined by Kurz (1968). Here, we focus on the taxation problem and assume the existence of a steady state.

Solving the steady-state version of equation (22) for

$$\theta = \frac{[u_l + \phi (u_{kl} k + u_{cl} c)]}{F_e M_n}, \tag{32}$$

and substituting (32) along with (31) into (29), we derive the expression for the limiting capital income tax as follows:

$$\tau^k = \frac{F_e M_n \phi}{F_k u_l} \frac{u_k (u_{cl} c + u_{kl} k) - u_l (u_{kk} k + u_k + u_{ck} c)}{u_l + \phi (u_{cl} c + u_{kl} k)}. \tag{33}$$

where

$$\Theta \equiv u_k (u_{cl} c + u_{kl} k) - u_l (u_{kk} k + u_k + u_{ck} c).$$

The Lagrange multiplier ϕ measures the utility cost of raising government revenues through distorting taxes and is nonnegative. It is straightforward to see that the sign of the limiting capital income tax depends on the sign of the numerator inside the square brackets, denoted by Θ, which can be positive, negative, or zero. Accordingly, we have the following:

Proposition 1 Suppose the economy converges to an interior steady state in the two-sector model incorporating the spirit of capitalism. The long-run capital income tax is positive, zero, or negative, if and only if Θ is larger than, equal to, or less than zero, respectively. Namely,

$$\tau^k \begin{cases} > \\ = \\ < \end{cases} 0 \Leftrightarrow \Theta \begin{cases} > \\ = \\ < \end{cases} 0,$$

where

$$\Theta \equiv u_k (u_{cl} c + u_{kl} k) - u_l (u_{kk} k + u_k + u_{ck} c).$$

Proposition 1 shows that the limiting capital income tax is generally nonzero, since the term Θ is typically not equal to zero. Notably, the sign of the optimal capital tax rate depends solely on the specification of the utility function, rather than on the production technology. In particular, when consumers derive utility from both the spirit of capitalism and consumption, the zero capital income taxation theorem no longer holds. Our finding of a nonzero capital income tax partially support the conjecture of Jones et al. (1997), who argue that "if the capital stock enters the objective function of the pseudo planner's problem, then the planner will tax capital income in the limit".

Intuitively, the ambiguous effects of the spirit of capitalism arise from the non-pecuniary return on capital induced by the spirit of capitalism, given by $\frac{u_k}{u_c/(1+\tau^c)}$, in the asset-pricing equation (i.e., rearranged consumption Euler equation (11)). Specifically,

$$1 = \underbrace{\beta \frac{u_c(t+1)/\left(1+\tau_{t+1}^c\right)}{u_c(t)/\left(1+\tau_t^c\right)}}_{\text{SDF}} \left\{ \underbrace{\frac{u_k(t+1)}{u_c(t+1)/\left(1+\tau_{t+1}^c\right)}}_{\text{non-pecuniary return}} + \underbrace{\left[\left(1-\tau_{t+1}^k\right)F_k + 1 - \delta\right]}_{\text{pecuniary return}} \right\}. \tag{34}$$

Taxing capital discourages MPK-driven capital accumulation, which in standard Ramsey frameworks leads to a lower steady-state capital stock. However, a lower steady-state capital increases the numerator of the non-pecuniary component due to $u_{kk} < 0$, thereby encouraging capital accumulation driven by the spirit of capitalism. Because these two opposing effects interact, it is difficult to determine which dominates, and thus the sign of the limiting capital income tax cannot be determined in general. Similar results have been derived by Li et al. (2020) and Bu and Wang (2025) within a one-sector neoclassical growth model.

Corollary 1 (Jones et al. 1997) If agents have no preference for the spirit of capitalism (i.e., $u_k = 0$), the limiting capital income tax is zero (i.e., $\tau^k = 0$).

Proof If $u_k = 0$, then $\Theta = 0$, implying $\tau^k = 0$. $\square$

Corollary 1 shows that if there is no preference for the spirit of capitalism, our model reduces to that developed by Jones et al. (1997).[14] In this case, the limiting capital income tax is zero. Furthermore, If we assume $H(x_{ht}, h_t, n_{ht}) \equiv 0$, $M(x_{mt}, h_t, n_{mt}) = n_{mt}$, and $h_0 = 0$, the model further reduces to the one-sector Cass-Koopmans framework examined by Judd (1985) and Chamley (1986), in which the limiting capital income tax rate is also zero.

Next, we provide several examples illustrating all three possible cases.

Example 1 If the utility function takes the form

$$u(c,l,k) = \frac{\left(c^{\alpha} l^{1-\alpha}\right)^{1-\sigma} - 1}{1-\sigma} + \ln k,\ \alpha \in (0,1), \sigma \in [0,1),$$

then

$$\Theta = \alpha(1-\alpha)(1-\sigma)\alpha^{\alpha(1-\sigma)} l^{(1-\sigma)(1-\alpha)-1} k^{-1} > 0,$$

[14] Jones et al. (1997) establish that both capital and labor income taxes are zero in the steady state, if the following three conditions hold: (i) there are no profits from accumulating either capital stock, (ii) the tax code is sufficiently rich, and (iii) relative prices play no role in reducing the value of fixed sources of income.

which implies that the limiting capital tax rate is positive, i.e.,

$$\tau^k > 0.$$

When the elasticity of intertemporal substitution ($EIS = 1/\sigma$) for consumption and leisure is high (meaning households strongly prefer to substitute future consumption for current consumption and thus save more), the government imposes a tax on physical capital to curb the household's savings motive driven by the combination of a high EIS and the desire for the spirit of capitalism.

Example 2 If the utility function takes the form

$$u\left(c, l, k\right) = \frac{\left(c^{\alpha} l^{1-\alpha}\right)^{1-\sigma} - 1}{1-\sigma} + \ln k, \ \alpha \in \left(0, 1\right), \sigma \in \left(1, +\infty\right),$$

then

$$\Theta = \alpha\left(1-\alpha\right)\left(1-\sigma\right)\alpha^{\alpha(1-\sigma)} l^{(1-\sigma)(1-\alpha)-1} k^{-1} < 0.$$

Thus, the limiting capital tax rate is negative, i.e.,

$$\tau^k < 0.$$

When the elasticity of intertemporal substitution ($EIS = 1/\sigma$) for consumption and leisure is relatively low (meaning households save less and their savings are insufficient to support their increasing desire or utility for capital), the government should subsidize capital income to encourage household saving. On the other hand, if the utility function is:

$$u\left(c, l, k\right) = \frac{c^{1-\sigma} - 1}{1-\sigma} + v\left(l\right) + k^{\gamma},$$

where $\gamma \in (0, 1)$, $\sigma \in [0, +\infty)$, and $v' > 0$, $v'' < 0$, then the limiting tax rate is also negative, namely,

$$\tau^k = \frac{F_e M_n \phi}{F_k u_l}\left[\frac{-v'\left(l\right)\gamma^2 k^{\gamma-1}}{u_l + \phi\left(u_{cl} c + u_{kl} k\right)}\right] < 0.$$

Example 3 If the utility function is additively separable with respect to (c, l, k) and the component for physical capital is logarithmic, namely,

$$u\left(c, l, k\right) = u\left(c\right) + v\left(l\right) + \gamma \ln k,$$

where $u'\left(c\right) > 0$, $u''\left(c\right) < 0$, $v'\left(l\right) > 0$, $v''\left(l\right) < 0$, and $\gamma > 0$, then

$$\Theta = 0,$$

which implies that the limiting capital tax rate is zero, i.e.,

$$\tau^k = 0.$$

In this case, the negative effect of the elasticity of intertemporal substitution (EIS) on savings and capital accumulation exactly offsets the positive effect of the desire for the spirit of capitalism.

4.2. Labor income tax (τ^n)

In the steady state, the second equality of the first-order condition (9) for the representative consumer gives

$$\frac{H_n}{H_x} = (1 - \tau^n) F_e M_n. \tag{35}$$

If follows immediately from equations (31) and (35) that

$$\tau^n = 0, \tag{36}$$

which implies that the limiting tax rate on labor income is zero. Thus, we have:

Proposition 2 In the two-sector model with the spirit of capitalism, the limiting labor income tax is zero, i.e.,, $\tau^n = 0$.

Propositions 1 and 2 show that although the optimal capital income tax maybe nonzero in the long run, the limiting labor income tax rate is zero. This zero labor income tax result contrasts with Li et a. (2020) and Bu and Wang (2025), where the limiting labor income tax rate may be positive, negative, or zero.

4.3. Taxes on expenditures (τ^m, τ^c)

Since the limiting labor income tax is zero (i.e., $\tau^n = 0$) in the steady state, both equations (10) and (24) imply that

$$\tau^m = 0, \tag{37}$$

which means that the limiting tax on the expenditures for generating effective labor is zero.

Proposition 3 In the two-sector model with the spirit of capitalism, the limiting tax on expenditures for generating effective labor is zero, i.e., $\tau^m = 0$.

Propositions 2 and 3 show that the preference for the spirit of capitalism does not alter Jones et al. (1997)'s results of a zero limiting tax on labor income and expenditures for generating effective labor in the long run.

To derive the optimal consumption rax, we use equation (9) together with the zero limiting labor income tax ($\tau^n = 0$) to obtain

$$1+\tau^c = \frac{u_c}{u_l} F_e M_n.$$

From the steady-state versions of equations (20) and (22), we have

$$F_e M_n = \frac{u_l + \phi\left(u_{cl}c + u_{kl}k\right)}{u_c + \phi\left(u_{cc}c + u_c + u_{kc}k\right)}.$$

Combining these, we get

$$1+\tau^c = \frac{u_c}{u_l} \frac{u_l + \phi\left(u_{cl}c + u_{kl}k\right)}{u_c + \phi\left(u_{cc}c + u_c + u_{kc}k\right)}, \tag{38}$$

which shows that the limiting consumption tax is generally not zero.

Thus, we have

Proposition 4 In the two-sector model with the spirit of capitalism, the limiting consumption tax can be positive, zero, or negative, if and only if Ω *is larger than, equal to, or less than zero, respectively. Namely,*

$$\tau^c \begin{cases} > \\ = \\ < \end{cases} 0 \Leftrightarrow \Omega \begin{cases} > \\ = \\ < \end{cases} 0, \tag{39}$$

where

$$\Omega \equiv u_c\left(u_{cl}c + u_{kl}k\right) - u_l\left(u_{cc}c + u_c + u_{kc}k\right).$$

Proof Using equation (38), we solve for the limiting consumption tax rate as follows:

$$\begin{aligned} \tau^c &= \frac{u_c\left[u_l + \phi\left(u_{cl}c + u_{kl}k\right)\right] - u_l\left[u_c + \phi\left(u_{cc}c + u_c + u_{kc}k\right)\right]}{u_c + \phi\left(u_{cc}c + u_c + u_{kc}k\right)} \\ &= \phi\frac{u_c\left[\left(u_{cl}c + u_{kl}k\right)\right] - u_l\left[\left(u_{cc}c + u_c + u_{kc}k\right)\right]}{u_c + \phi\left(u_{cc}c + u_c + u_{kc}k\right)} \\ &\equiv \phi\frac{\Omega}{u_c + \phi\left(u_{cc}c + u_c + u_{kc}k\right)}, \end{aligned}$$

which establishes the results listed in (39). □

Proposition 4 shows that the limiting consumption tax is generally not zero in the long run. In particular, if the utility function takes one of the following forms:

$$u(c,l,k)=\frac{c^{1-\sigma}}{1-\sigma}v(l)+w(k)\,,\text{ or } u(c,l,k)=\ln c+v(l)+w(k)\,,$$

then $\Omega=0$, which implies that the limiting consumption tax rate is zero, i.e.,

$$\tau^c=0.$$

5. CONCLUSION

This article reexamines the Chamley-Judd zero capital income tax theorem within a two-sector growth model. In this framework, there are two types of capitals-physical and human-and agents embody the spirit of capitalism. We demonstrate that the limiting capital income tax rate is generally not zero in the long run; it may be positive, negative, or zero, depending on the utility specifications rather than the production technology. To illustrate this, we provide three widely used utility functions, each corresponding to one of these possibilities. Additionally, we find that the limiting consumption tax rate is indeterminate (positive, negative, or zero) in the long run, while the limiting taxes on labor income and expenditures for generating effective labor converge to zero. Our theoretical results partially confirm the conjecture of Jones et al. (1997) and extend the main findings of Li et al. (2020) from a one-sector to a two-sector growth model.

APPENDIX

A.1. APPENDIX A: DERIVE THE FIRST ORDER CONDITIONS

In this appendix, we solve the first order conditions of the consumer's maximization problem. Construct the following Lagrangian:

$$\mathcal{L}_0=\sum_{t=0}^{\infty}\beta^t\left\{\begin{array}{c}u(c_t,l_t,k_t)+\mu_t\left[(1-\delta_h)\,h_t+H\left(x_{ht},h_t,n_{ht}\right)-h_{t+1}\right]+\\ \lambda_t\left[\begin{array}{c}(1-\tau_t^k)r_tk_t+(1-\tau_t^n)w_te_t+(1-\delta_k)k_t+b_t\\ -\left(1+\tau_t^c\right)c_t-\left(1+\tau_t^m\right)x_{mt}-x_{ht}-k_{t+1}-\frac{b_{t+1}}{R_t}\end{array}\right]\end{array}\right\}.$$

The first order necessary conditions with respect to c_t, n_{ht}, n_{mt}, x_{mt}, x_{ht}, k_{t+1}, b_{t+1}, and h_{t+1} are

$$c_t:u_c(c_t,l_t,k_t)=\lambda_t\left(1+\tau_t^c\right),\tag{A.1}$$

$$n_{ht} : u_l(c_t, l_t, k_t) = \mu_t H_{n_h}(x_{ht}, h_t, n_{ht}), \tag{A.2}$$

$$n_{mt} : u_l(c_t, l_t, k_t) = \lambda_t(1 - \tau_t^n) w_t M_{n_m}(x_{mt}, h_t, n_{mt}), \tag{A.3}$$

$$x_{mt} : (1 - \tau_t^n) w_t M_{x_m}(x_{mt}, h_t, n_{mt}) = (1 + \tau_t^m), \tag{A.4}$$

$$x_{ht} : \lambda_t = \mu_t H_{x_h}(x_{ht}, h_t, n_{ht}), \tag{A.5}$$

$$k_{t+1} : \lambda_t = \beta \left\{ u_k(c_{t+1}, l_{t+1}, k_{t+1}) + \lambda_{t+1} \left[(1 - \tau_{t+1}^k) r_{t+1} + (1 - \delta_k) \right] \right\}, \tag{A.6}$$

$$b_{t+1} : \lambda_t = \beta R_t \lambda_{t+1}, \tag{A.7}$$

$$h_{t+1} : \mu_t = \beta \left\{ \begin{array}{c} \lambda_{t+1}(1 - \tau_{t+1}^n) w_{t+1} M_h(x_{mt+1}, h_{t+1}, n_{mt+1}) + \\ \mu_{t+1} \left[(1 - \delta_h) + H_h(x_{ht+1}, h_{t+1}, n_{ht+1}) \right] \end{array} \right\}, \tag{A.8}$$

respectively.

Substuting (A.1) into (A.3) gives rise to

$$\frac{(1 - \tau_t^n) w_t M_n(t)}{(1 + \tau_t^c)} = \frac{u_l(t)}{u_c(t)}, \tag{A.9}$$

while plugging (A.1) and (A.2) in (A.5) leads to

$$\frac{u_l(t)}{u_c(t)} = \frac{H_n(t)}{(1 + \tau_t^c) H_x(t)}. \tag{A.10}$$

Combining (A.9) and (A.10) yields us (9). Putting (A.1) in (A.6) yields us the consumption Euler eqution (i.e., (11)):

$$\frac{u_c(t)}{(1 + \tau_t^c)} = \beta \left\{ u_k(t+1) + \frac{u_c(t+1)}{\left(1 + \tau_{t+1}^c\right)} \left[\left(1 - \tau_{t+1}^k\right) r_{t+1} + 1 - \delta \right] \right\}. \tag{A.11}$$

Plugging (A.1) and (A.5) in (A.8) gives us the intertemporal optimality condition (i.e., (12)):

$$\frac{u_c(t)}{(1 + \tau_t^c) H_x(t)} = \beta \frac{u_c(t+1)}{\left(1 + \tau_{t+1}^c\right)} \left[\left(1 - \tau_{t+1}^n\right) w_{t+1} M_h(t+1) + \frac{H_h(t+1) + 1 - \delta_h}{H_x(t+1)} \right]. \tag{A.12}$$

Putting (A.1) in (A.7) leads to (i.e., (13))

$$\frac{u_c(t)}{(1 + \tau_t^c)} = \beta R_t \frac{u_c(t+1)}{\left(1 + \tau_{t+1}^c\right)}. \tag{A.13}$$

Combining (A.1), (A.6), and (A.7) leads to the no-arbitrage condition (i.e., (14))

$$R_t = \frac{(1-\tau_{t+1}^k)r_{t+1} + 1 - \delta}{\left(1 - \frac{\beta u_k(t+1)(1+\tau_t^c)}{u_c(t)}\right)}. \tag{A.14}$$

A.2. APPENDIX B: DERIVE THE PRESENT-VALUE BUDGET CONSTRAINT

We derive the present-value budget constraint in this appendix. Firstly, repeated substitution of the flow budget constraint gives rise to

$$b_0 = q_T^0 \frac{b_{T+1}}{R_T} + \sum_{t=0}^{T} q_t^0 \left[(1+\tau_t^c) c_t + (1+\tau_t^m) x_{mt} + x_{ht} + x_t - (1-\tau_t^n) w_t e_t \right]. \tag{A.15}$$

Note that

$$\begin{aligned}
& \sum_{t=0}^{T} q_t^0 x_t \\
\equiv\ & \sum_{t=0}^{T} q_t^0 \left[k_{t+1} - \left(\left(1-\tau_t^k\right) r_t + 1 - \delta \right) k_t \right] \\
=\ & \sum_{t=0}^{T} q_t^0 k_{t+1} - \sum_{t=1}^{T} q_t^0 \left[\left(1-\tau_t^k\right) r_t + 1 - \delta \right] k_t - \left(\left(1-\tau_0^k\right) r_0 + 1 - \delta \right) k_0 \\
=\ & q_T^0 k_{T+1} + \sum_{t=0}^{T-1} \left[q_t^0 - q_{t+1}^0 \left(\left(1-\tau_{t+1}^k\right) r_{t+1} + 1 - \delta \right) \right] k_{t+1} - \left(\left(1-\tau_0^k\right) r_0 + 1 - \delta \right) k_0 \\
=\ & q_T^0 k_{T+1} + \sum_{t=0}^{T-1} q_{t+1}^0 \frac{u_k(t+1)\left(1-\tau_{t+1}^c\right)}{u_c(t+1)} k_{t+1} - \left(\left(1-\tau_0^k\right) r_0 + 1 - \delta \right) k_0.
\end{aligned} \tag{A.16}$$

The fourth equality comes from the fact

$$q_t^0 - q_{t+1}^0 \left(\left(1-\tau_{t+1}^k\right) r_{t+1} + 1 - \delta \right) = q_{t+1}^0 \frac{u_k(t+1)\left(1-\tau_{t+1}^c\right)}{u_c(t+1)},$$

which can be verified by using the equality $q_t^0 = q_{t+1}^0 R_t$, the first-order condition of the household with respect to real bond holdings and the no-arbitrage condition on portfolio choices. Substituting (A.16) into (A.15), taking limits on both sides with repect to $T \to +\infty$, and imposing two transversality conditions

$$\lim_{T\to\infty} q_T^0 \frac{b_{T+1}}{R_T} = 0, \quad \lim_{T\to\infty} q_T^0 k_{T+1} = 0,$$

we obtain

$$\sum_{t=0}^{\infty}\left\{\begin{array}{c} q_t^0\left(1+\tau_t^c\right)c_t+ \\ q_{t+1}^0\frac{u_k(t+1)\left(1+\tau_{t+1}^c\right)}{u_c(t+1)}k_{t+1}\end{array}\right\}=\left\{\begin{array}{c}\underbrace{\sum_{t=0}^{\infty}q_t^0\left[(1-\tau_t^n)w_te_t-\left(1+\tau_t^m\right)x_{mt}-x_{ht}\right]}_{\equiv\Xi} \\ +\left[\left(1-\tau_0^k\right)r_0+1-\delta\right]k_0+b_0\end{array}\right\}. \tag{A.17}$$

Secondly, we prove that

$$\Xi=\left[(1-\tau_0^n)w_0M_h\left(0\right)+\frac{H_h\left(0\right)+1-\delta_h}{H_x\left(0\right)}\right]h_0. \tag{A.18}$$

Using the accumulation equation of human capital and the linear homogeneity of H, we solve for

$$x_{ht}=\frac{h_{t+1}-\left(H_h\left(t\right)+1-\delta_h\right)h_t}{H_x\left(t\right)}. \tag{A.19}$$

Substituting $(1-\tau_t^n)w_tM_x\left(t\right)=1+\tau_t^m$ and (A.19) into Ξ and using the homogeneity of M, we have

$$\begin{aligned}
\Xi &\equiv \sum_{t=0}^{\infty}q_t^0\left[(1-\tau_t^n)w_te_t-\left(1+\tau_t^m\right)x_{mt}-x_{ht}\right] \\
&= -\sum_{t=0}^{\infty}q_t^0\frac{h_{t+1}}{H_x\left(t\right)}+\sum_{t=0}^{\infty}q_t^0\left[(1-\tau_t^n)w_tM_{ht}+\frac{H_h\left(t\right)+1-\delta_h}{H_x\left(t\right)}\right]h_t \\
&= -\sum_{t=0}^{\infty}q_t^0\frac{h_{t+1}}{H_x\left(t\right)}+\sum_{t=0}^{\infty}q_{t+1}^0\left[\begin{array}{c}(1-\tau_{t+1}^n)w_{t+1}M_{ht+1} \\ +\frac{H_h(t+1)+1-\delta_h}{H_x(t+1)}\end{array}\right]h_{t+1} \\
&+ \left[(1-\tau_0^n)w_0M_h\left(0\right)+\frac{H_h\left(0\right)+1-\delta_h}{H_x\left(0\right)}\right]h_0 \\
&= -\sum_{t=0}^{\infty}q_t^0\underbrace{\left\{\frac{1}{H_x\left(t\right)}-\frac{1}{R_t}\left[\begin{array}{c}(1-\tau_{t+1}^n)w_{t+1}M_{ht+1} \\ +\frac{H_h(t+1)+1-\delta_h}{H_x(t+1)}\end{array}\right]\right\}}_{\equiv\Psi}h_{t+1} \\
&+ \left[(1-\tau_0^n)w_0M_h\left(0\right)+\frac{H_h\left(0\right)+1-\delta_h}{H_x\left(0\right)}\right]h_0 \\
&= \left[(1-\tau_0^n)w_0M_h\left(0\right)+\frac{H_h\left(0\right)+1-\delta_h}{H_x\left(0\right)}\right]h_0.
\end{aligned}$$

Note that

$$\Psi \equiv \frac{1}{H_x(t)} - \frac{1}{R_t}\left[(1-\tau^n_{t+1})w_{t+1}M_{ht+1} + \frac{H_h(t+1)+1-\delta_h}{H_x(t+1)}\right] = 0,$$

which can be proved by using the optimality conditions of human capital accumulation and bond holdings.

Finally, plugging (A.18) in (A.17) gives us the present-value budget constraint in the main text. □

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