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Debt Taxes During Crises, a Blessing in Disguise?

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Models with an occasionally binding credit constraint are used to study financial crises. We examine the welfare effects of implementing a policy designed for a specific type of constraint when the economy is facing a different one. To this purpose we analyze the implementation of ex ante (macroprudential) versus ex post debt taxes across four possible constraint scenarios (depending on whether creditors assess current or future and total or disposable income of debtors). Our main conclusion is that a debt tax applied only during potentially constrained periods (ex post) is a better policy in three of the four possible cases.

Key Words: Macroprudential policies; Ex post policies; Debt tax; Financial constraint; Financial crisis; Sudden stops.
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1. INTRODUCTION

A strand of economic literature has analyzed financial crises in the context of open economies that are occasionally credit constrained. Recent studies (Vargas & Parra-Polania, 2021; Ottonello et al., 2022) highlight the importance, in this analysis, of the information creditors use to assess the borrowing capacity of potential debtors, namely, the specific form of the credit constraint. Do creditors evaluate borrowing capacity based on current or future income? Moreover, do they focus on pre-tax income (total income) or post-tax income (disposable income)?

The relevance of the characteristics of the financial constraint raises several questions. A central one concerns the welfare implications of imple-

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menting a policy designed for a particular type of credit constraint in an economy actually facing a different one. For instance, what happens if a macroprudential debt tax—recommended in the literature as welfare enhancing under a current-income borrowing constraint—is applied in an economy that faces a future-income constraint? This paper addresses such questions by examining the welfare effects of implementing one of two policy interventions: an ex ante (macroprudential) debt tax or an ex post debt tax. These policies are evaluated across four potential credit constraint scenarios, derived by combining current/future income with total/disposable income.

Since there are many possible intervention policies that could be considered, we limit the analysis to these two because they are shown to equalize the decentralized equilibrium to the one obtained by a benevolent social planner operating under discretion and, following common practice in the literature, with restricted planning abilities (i.e., subject to both the same financial constraint and the same pricing rule as private agents. See, e.g., Lorenzoni, 2008; Bianchi, 2011; Benigno et al., 2016). We do not consider the social planner's problem under commitment.

We find that imposing a debt tax during periods when the economy is potentially constrained[1] (referred to as an ex post debt tax) is more beneficial for welfare than a macroprudential one (i.e., a debt tax that applies during normal periods only) if the economy faces a disposable-income constraint (whether based on current or future income) or a future-income constraint (whether based on total or disposable income). On the one hand, a macroprudential debt tax improves welfare under a current-income constraint but reduces welfare under a future-income constraint. Under a current-income constraint, a macroprudential debt tax reduces debt variability, thereby mitigating the adverse effects of binding constraints. Instead, under a future-income collateral constraint borrowing decisions are already constrained efficient, therefore implementing a macroprudential policy distorts those decisions, reducing welfare. On the other hand, an ex post debt tax (returned to households via lump-sum subsidies) increases disposable income but does not affect total income, and hence has a welfare-improving impact when the economy faces a disposable-income constraint and has no effect under a total-income constraint.

To give more context to our findings let us briefly recapitulate some results obtained in the related literature. Previous studies establish that in an economy under a current-total-income credit constraint, private agents internalize the social cost of their borrowing decisions if a macroprudential debt tax is used (e.g., Bianchi, 2011; Korinek, 2011). This result also

[1] We refer to "potentially" constrained periods since the implementation of the policy may render the borrowing constraint non-binding in some periods that would otherwise be binding.

holds for the case of an economy under a current-disposable-income credit constraint (Vargas and Parra-Polania, 2021). In economies under future-total-income constraint, private agents make constrained-efficient decisions (Ottonello et al., 2022) and hence no policy intervention is required to internalize their social cost (eventhough their shadow value of borrowing is different from the social planner one). In the present paper we demonstrate that in this case (under a future-total-income constraint), there is an ex post debt tax that entirely equalizes private agents' equilibrium with that of the social planner: that is, not only their borrowing decisions but also their shadow values of borrowing are made equal. This result is a theoretical curiosity with no policy-relevant effects (since in the absence of such an intervention borrowing decisions were already constrained efficient). In contrast, when facing a future-disposable-income constraint an ex post debt tax increases debt capacity as it affects future disposable income positively: it is expected to be collected in every potentially constrained period and to be returned to households via lump-sum subsidies.[2]

We use a standard small open economy model (proposed by Mendoza, 2002) for which we only change the collateral constraint form to consider the four possible abovementioned cases. We theoretically derive, for a general model with an implicit utility function, the mentioned results by imposing state-contingent debt taxes on debt acquired in the current period, either as a macroprudential policy, or as an ex post debt tax. To illustrate our theoretical results, we assume a specific and standard utility funcion (CRRA), simulate the different scenarios using standard parameter values, and calculate the welfare effects of each.

The remainder of this paper is organized as follows. Section 2 presents the theoretical framework and derives the core results. Section 3 presents and discusses numerical examples. Section 4 concludes.

2. THEORETICAL FRAMEWORK

We adopt a canonical small open economy model with tradable and nontradable goods and borrowing subject to collateral constraint.

The economy is populated by a continuum of households of size one. A representative household seeks to maximize its lifetime expected utility function expressed by

$$U = E_1 \left[\sum_{t=1}^{\infty} \beta^t u\left(C_t\right) \right], \tag{1}$$

[2]This requires lenders to expect the ex post debt tax to apply during future crises. We assume this expectation holds, as this is a time-consistent policy, i.e., the policymaker has no incentive to refrain from implementing it in subsequent crises (see Section 2.2 for details).

where $E[\cdot]$ is the expectations operator, $u(\cdot)$ is the well-behaved period utility function, β is the discount factor and C_t is the consumption index which aggregates tradable (T) and nontradable (N) goods:

$$C_t = C\left(C_t^T, C_t^N\right). \tag{2}$$

Every period, this household receives a stochastic (and exogenous) bundle of tradable and nontradable goods, Y_t^T and Y_t^N, and has access to international credit markets through one-period loans B_{t+1} at an interest rate r ($R \equiv 1 + r$). The budget constraint, expressed in units of tradable goods, is

$$C_t^T + P_t^N C_t^N + RB_t = Y_t^T + P_t^N Y_t^N + B_{t+1}, \tag{3}$$

where P_t^N is the price of nontradables; the price of tradable goods operates as the numeraire.

The household faces a collateral constraint; that is, it can borrow B_{t+1} up to a fraction κ of its income, such that $B_{t+1} \leq \kappa \times (\text{INCOME})$. We consider four types of collateral constraints depending on the income used for assessing borrowing capacity. These four types arise from the combination of two different features: a) whether the relevant income is the current one or the next-period income; and b) whether the relevant income is the total or the disposable one. This way, the four possible collateral constraints are:

1. Current-total-income collateral constraint:

$$B_{t+1} \leq \kappa\left(Y_t^T + P_t^N Y_t^N\right). \tag{4}$$

2. Current-disposable-income collateral constraint:

$$B_{t+1} \leq \kappa\left(Y_t^T + P_t^N Y_t^N - T_t\right), \tag{5}$$

i.e., borrowers' debt capacity is evaluated considering income after deducting taxes ($T_t > 0$) or adding subsidies ($T_t < 0$). T is taken as exogenous by the household.

3. Next-period-total-income collateral constraint:[3]

$$B_{t+1} \leq \kappa E_t \left[Y_{t+1}^T + Y_{t+1}^N P_{t+1}^N \right]. \tag{6}$$

4. Next-period-disposable-income collateral constraint:

$$B_{t+1} \leq \kappa E_t \left[Y_{t+1}^T + Y_{t+1}^N P_{t+1}^N - T_{t+1} \right]. \tag{7}$$

As mentioned in the introduction, the specific form of the collateral constraint is decisive for the analysis of financial crises. An economy with a collateral constraint determined by current income exhibits a pecuniary externality and overborrowing. Instead, in an economy with a collateral constraint determined by future income, the decentralized (DC) equilibrium is constrained efficient, and therefore there is no need for policy intervention to equalize it to the allocations obtained by a benevolent social planner (SP) under discretion.

Following common practice in the literature, we assume that private agents and the policymaker observe the current values of all variables in the model and know the probability distribution of the exogenous income variables (Y_t^T and Y_t^N). Regarding the collateral constraint, we assume that private agents know the specific form that they are facing while the policymaker does not know it. For the sake of simplicity, we do not assume that the policymaker assigns probabilities to the possible collateral constraint forms. Instead, we analyze the consequences of the policymaker assuming each form at a time, as if certain that it is the collateral private agents are facing.

2.1. Solution to the household's maximization problem

We denote by μ_t and λ_t the Lagrange multipliers associated to the budget and credit constraints, respectively. Non-tradable consumption is determined by the market-clearing condition of that market, i.e., $C_t^N = Y_t^N$. The other first-order conditions for maximization in this decentralized (DC) economy make up the following equation system:

$$\mu_t = R\beta E_t \mu_{t+1} + \lambda_t, \tag{8}$$

[3]For this and the next case we consider the expected value operator ($E[\cdot]$) as in Devereux et al. (2019). In contrast, Ottonello et al. (2022) use the minimum operator ($\min[\cdot]$) but, as they remark, their results also hold for the expected-value case. In terms of the environment to support the microfoundation of these constraint forms, the minimum operator ensures debt repayment in every possible state; however, as shown by Ottonello et al. (2022), the economy does not feature sudden stops and hence they need to add other type of shocks (either stochastic volatility or shocks to the share of income pledge as collateral). No additional shocks are needed in the expected-value case for the economy to feature sudden stops.

$$u_{T,t}\left(C_t^T\right) = \mu_t, \tag{9}$$

$$P_t^N = \frac{u_{N,t}\left(C_t^T\right)}{u_{T,t}\left(C_t^T\right)}, \tag{10}$$

$$C_t^T + RB_t = Y_t^T + B_{t+1}, \tag{11}$$

$$\lambda_t\left[\kappa \times (\text{INCOME}) - B_{t+1}\right] = 0, \tag{12}$$

which includes the market-clearing condition for tradables, i.e., Equation (11), and where $u_{T,t}\left(C_t^T\right) \equiv u'\left(C_t\right)\left(\partial C_t/\partial C_t^T\right)$ and $u_{N,t}\left(C_t^T\right) \equiv u'\left(C_t\right)\left(\partial C_t/\partial C_t^N\right)$. This five-equation system provides a solution for C_t^T, μ_t, λ_t, B_{t+1} and P_t^N for given values of $\left\{B_t,\, Y_t^T,\, Y_t^N\right\}$ and the (consistent) expected values of future variables.

If the economy is financially unconstrained in period t, then $B_{t+1} \leq \kappa \times (\text{INCOME})$ and hence $\lambda_t = 0$, from Equation (12). Given this (and the expected value of μ_{t+1}), μ_t is determined by Equation (8), in turn C_t^T is determined by Equation (9), and then P_t^N and B_{t+1} are determined by Equations (10) and (11), respectively.

If, instead, the economy is constrained, then $\lambda_t \geq 0$ and Equation (12) turns into $B_{t+1} = \kappa \times (\text{INCOME})$. The solution of the system depends then on whether the relevant income in the collateral constraint is the current or the future one. If it is the current one, then C_t^T, B_{t+1} and P_t^N are determined by the sub-system of Equations (10)-(12). Next, Equation (9) determines μ_t, and finally Equation (8) determines λ_t (given the expected value of μ_{t+1}). If instead the relevant income in the collateral constraint is the future one, B_{t+1} is determined by Equation (12) (given the expected value of either total or disposable next-period income), then C_t^T is determined by Equation (11), P_t^N in turn is determined by Equation (10), subsequently Equation (9) determines μ_t, and finally Equation (8) determines λ_t (given the expected value of μ_{t+1}).

2.2. Policy interventions and welfare effects

This class of models, with occasionally binding collateral constraints that are themselves a function of aggregate endogenous variables of the economy, are common in the study of sudden stops. With occasionally binding constraints, individual private agents may not internalize the effect of their decisions on the aggregate borrowing limit, leading to pecuniary externalities and inefficient borrowing. In this environment, a social planner (SP) who faces the same borrowing constraint as the private agents (i.e., a constrained SP) but takes into account the consequences of her choices on the market value of collateral would face a lower probability of being financially constrained relative to a DC economy.

These models have been extensively used for the analysis of optimal policy interventions that would reduce the probability and severity of sudden stops, as well as their welfare implications. The interventions are focused in two general sets of state-contingent policies: (a) macroprudential or ex ante policies (i.e., policies implemented in good times to mitigate the frequency and severity of financial crises in the future)[4], and (b) ex post policies aimed at dealing with the financial crises once it is in motion (i.e., policies implemented in potentially[5] constrained periods)[6]. Within these two general sets (ex ante vs. ex post), there are also many possible policies depending on whether they are subsidies or taxes, whether they are levied on nontradable or tradable consumption, or on their corresponding prices, or on debt. Regarding optimal interventions, they can be derived as optimal under discretion or under commitment.

Since there are many possible intervention policies that could be considered, in the present paper we limit the analysis to those two that equalize the DC equilibrium to the one obtained by a benevolent SP under discretion: an ex ante debt tax, that equalizes DC and SP equilibria in economies under current-income credit constraints (e.g., Bianchi, 2011; Korinek, 2011; Vargas and Parra-Polania, 2021), and an ex post debt tax, that equalizes DC and SP equilibria in economies under future-income credit constraints (as shown in Proposition 3 below).[7]

Specifically, the interventions we analyze are state-contingent debt taxes (τ_t) which are issued on debt acquired in period t (i.e., B_{t+1}). As a macroprudential policy, the debt tax would be positive during unconstrained times and nil during credit-constrained periods. As an ex post debt tax, it would be positive only during potentially constrained periods and nil during the unconstrained ones. In both cases, the corresponding tax is returned to the household in the same period through a lump-sum subsidy

$$T_t = -\tau_t B_{t+1} < 0.$$

[4]e.g., Bianchi, 2011 ; Bianchi and Mendoza, 2018; Jeanne and Korinek, 2019.

[5]By using the expression "potentially constrained periods", we want to emphazise that the ex post debt tax applies only in those periods when, were the tax not issued, the economy would be financially constrained. As a result of the tax, during those periods the economy might still be financially constrained but with lower severity, or it might not be financially constrained altogether.

[6]e.g., Benigno et al. 2016; Bianchi 2016; Jeanne and Korinek 2020

[7]While we consider four possible scenarios for the credit constraint, we solve only two distinct SP problems: one based on current income and the other on future income. In both cases, total income is used because the SP problem does not involve taxes or subsidies, and therefore, there is no distinction between pre- and post-tax/subsidy income. In the standard Ramsey framework for optimal taxation, taxes and subsidies serve as instruments for the policymaker in a decentralized economy to internalize the social cost of private decisions. Consequently, they are not part of the SP problem itself.

Either as an ex post or an ex ante policy, the debt tax τ_t changes Equation (8) in the system of first-order conditions to

$$(1-\tau_t)\,\mu_t = R\beta E_t \mu_{t+1} + \lambda_t. \tag{8.1}$$

Equations (9)-(11) remain unchanged.[8]

In what follows, we describe the theoretical implications of implementing these debt-tax policies in each one of the models determined by the collateral-constraint types described in Equations (4) to (7).

Current-total-income collateral and macroprudential debt tax

In an economy described by Equations (1) - (3) and a collateral constraint determined by current-total income, as in Equation (4), a macroprudential debt tax implements the SP solution in the DC economy (demonstrated in, e.g., Bianchi, 2011; Korinek, 2011). That is, a macroprudential debt tax reduces the probability of being financially constrained, implements constrained-efficient allocations, and thus increases social welfare.

Current-total-income collateral and ex post debt tax

PROPOSITION 1. *In the economy described by Equations (1) - (3) and the collateral constraint that depends on current-total income, as in Equation (4), an ex post debt tax leaves unchanged the equilibrium allocation. Therefore, social welfare is not affected by levying such a tax.*

Proof. During constrained periods, an ex post debt tax τ_t is nonzero but it only affects the determination of λ_t (as explained in subsection 2.1), and C_t^T, B_{t+1} and P_t^N are determined independently of Equation (8.1). During unconstrained periods, an ex post debt tax τ_t is nil. Therefore, the final and unique effect of τ_t on the equilibrium is rescaling λ_t during constrained periods. ∎

The current-total-income collateral constraint is the most commonly used financial constraint in the related literature. As a conclusion from the above results, when the collateral constraint is determined by current-total income, the policy intervention that implements the SP solution and increases social welfare is a macroprudential debt tax. Instead, as demonstrated in Proposition 1, an ex post debt tax causes no effect on social welfare and in that sense it is equivalent to implementing no policy.

[8] Notice that as the debt tax is returned to the household as a lump-sum transfer in the same period, they cancel each out in the budget constraint.

Current-disposable-income collateral and macroprudential debt tax

Vargas and Parra-Polania (2021, Proposition 4) prove that the SP solution can be implemented in a DC economy described by Equations (1) - (3) and facing a current-disposable-income constraint (Equation (5)) by means of the same macroprudential debt tax that implements such a solution in that DC economy but instead facing a current-total-income constraint. Consequently, imposing the macroprudential tax is welfare improving in this case.

Current-disposable-income collateral and ex post debt tax

PROPOSITION 2. *In an economy described by Equations (1) - (3) and with collateral constraint that depends on current-disposable income, as in Equation (5), an ex post debt tax increases borrowing capacity during potentially-constrained periods, and hence welfare is improved. Furthermore, the higher the debt tax, the higher the borrowing capacity, and the higher the welfare.*

Proof. The first statement of the proposition is straightforward and follows from the corresponding collateral constraint, Equation (5): during potentially-constrained periods (i.e. when the economy would be constrained in the absence of any policy intervention), the debt tax is collected ($\tau_t > 0$) and it is returned to households via a lump-sum transfer $-T_t = \tau_t B_{t+1}$, increasing borrowing capacity.

Furthermore, as explained above in this subsection, an ex post debt tax transforms Equation (8) into Equation (8.1) during potentially-constrained periods and, as explained in subsection 2.1, this only affects λ. Therefore the only relevant change for the determination of the equilibrium allocation that results from implementing such a tax is the partial (or even total) relaxation of the financial constraint, thereby making feasible allocations that yield higher levels of utility.

The last statement of the proposition can be derived from Equation (5) as well. When an ex-post debt tax is levied, this collateral constraint can be rewritten as follows

$$B_{t+1} \leq [\kappa / (1 - \kappa\tau_t)] \left(Y_t^T + P_t^N Y_t^N\right).$$

The right hand side of this equation (i.e. the borrowing capacity) is increasing in τ_t. ∎

Although borrowing capacity cannot be infinitely increased (if $\tau_t \longrightarrow 1$ borrowing capacity tends to $[\kappa / (1 - \kappa)] \left(Y_t^T + P_t^N Y_t^N\right)$), for some parameter combinations the impact might be sufficiently high such that the collateral constraint is not binding in any period, as we illustrate in our numerical example below (see Section 3). However, as remarked by Vargas and Parra-Polania (2021) regarding this type of results, implementation issues should be considered as it seems unfeasible to use debt of a specific period to increase the borrowing capacity on which that same debt depends, particularly considering that such a capacity is assessed before the loan is disbursed.

In conclusion, when the collateral constraint is determined by current-disposable income, both policies (a macroprudential and an ex post debt tax) increase social welfare. The macroprudential debt tax implements the SP allocation and an ex post debt tax has the potential to make the economy financially unconstrained. Although both interventions increase social welfare, the macroprudential debt tax results in a specific welfare gain, i.e., the one derived from implementing the SP allocation in the decentralized economy. Instead, the welfare gain from an ex post debt tax arises from increasing repayment capacity (mitigating the negative impact of the financial constraint) and, therefore, varies with the tax level imposed. This positive welfare effect is increasing in the level of the debt tax. Consequently, in many instances the policymaker could raise the tax level to obtain a welfare gain greater than the one obtained with the macroprudential policy. In fact, as we illustrate in Section 3 for a specific but standard set of parameter values, it is possible for the policymaker to make the economy financially unconstrained, through a sufficiently high ex-post debt tax.

Future-total-income collateral and ex post debt tax

Ottonello et al. (2022) show that the DC equilibrium in an economy under a future-income collateral constraint (as in Equation (6)) is already constrained efficient; that is, there is no difference between the SP equilibrium allocation and that of the DC economy (these equilibria only differ in their shadow values of borrowing λ_t^{SP} vs λ_t). Therefore, implementing an ex-post policy in this case does not affect the DC allocation. It only rescales λ in the same way that such a policy does in the economy with a collateral constraint that depends on current-total income.

For the subsequent discussion it is useful to show that there is a policy intervention that equalizes all values of both equilibria (DC and SP), i.e., including the Lagrange multipliers λ_t^{SP} and λ_t. To this purpose let us start by recalling that since each household has an insignificant impact on the market, it takes prices as given. Instead a SP, subject to the same financial constraint, internalizes the effect of borrowing and consumption decisions on prices. Following the constrained-efficiency criterion (i.e., as mentioned in the introduction, we assume the SP is constrained by the same pricing rule of the DC equilibrium), it can be verified that the first-order conditions for the SP equilibrium are equal to those for the DC equilibrium, Equations (6) and (10)-(11), except for the case of Equation (8) that turns into

$$\mu_t^{SP} = R\beta E_t \mu_{t+1}^{SP} + \lambda_t^{SP}\left(1 + E_t \psi_{t+1}^{SP}\right), \tag{8.2}$$

where $E_t\psi_{t+1}^{SP} \equiv -\kappa E_t\left[Y_{t+1}^N\left(\partial P_{t+1}^{N,SP}/\partial C_{t+1}^{T,SP}\right)\left(\partial C_{t+1}^{T,SP}/\partial B_{t+1}^{SP}\right)\right]$. Remember that during unconstrained periods $\lambda = 0$, and hence these equations, (8) and (8.2), only differ during constrained periods.

PROPOSITION 3. *For the DC economy described by Equations (1) - (3) and under the future-total-income financial constraint (6), there exist an ex post debt tax rate that implements the SP equilibrium. This tax does not alter the welfare level.*

Proof. In this economy, private agents make constrained-efficient decisions; then, the DC and SP equilibria are equal except for their shadow value of borrowing (Ottonello et al., 2022). To implement the SP equilibria it will be enough to equalize Equations (8.1) and (8.2) during constrained periods. It can be easily verified that the following debt tax fulfills that purpose:

$$\tau_t = \frac{\lambda_t E_t \psi_{t+1}}{\mu_t},$$

where we have taken into account that, as a result of this tax, both equilibria are exactly the same, including $\psi_{t+1} = \psi_{t+1}^{SP}$. Notice that during normal times (i.e., $\lambda_t = 0$) the tax is nil. ∎

The implementation of this debt tax that equalizes the SP and the DC equilibria is a theoretical curiosity with no policy relevance since the SP and DC allocations are already equal in the absence of any intervention, and the only difference between both equilibria is the fact that the shadow value of borrowing for the SP is a rescaled version of that for the DC economy.

Future-total-income collateral and macroprudential debt tax

As mentioned in the previous case, decisions in a DC economy facing a future-income constraint are already constrained efficient, and therefore no intervention is required to equalize SP and DC allocations.

PROPOSITION 4. *Implementing the macroprudential debt tax in a DC economy described by Equations (1) - (3) and under future-total-income collateral constraint, Equation (6), reduces welfare.*

Proof. As explained above in this subsection, the macroprudential debt tax transforms Equation (8) into Equation (8.1) during unconstrained periods. As explained in subsection 2.1, this change affects the determination of C_t^T, B_{t+1} and P_t^N equilibrium values.

Furthermore, as the relevant income for the collateral constraint is the total rather than the disposable one, there is no effect on borrowing capacity associated to lump-sum transfers. Therefore the implementation of the macroprudential tax does not relax the financial constraint but distorts the equilibrium allocation that was, in the absence of the tax, constrained efficient (as demonstrated by Ottonello et al., 2022) thereby reducing welfare. ▮

In conclusion, when the collateral constraint is determined by future-total income, the DC equilibrium is constrained efficient and hence equilibrium allocations are equal to those of the SP equilibrium without the need of any intervention policy. On the one hand, implementing a macroprudential debt tax reduces social welfare since then private agents' decisions would deviate from the constrained efficient ones. On the other, an ex post debt tax does not affect consumption and future debt decisions and in that sense it is equivalent to implementing no policy (as in the current total income case). Such a tax is a theoretical curiosity that equalizes the shadow value of borrowing of the DC equilibrium to that of the SP, with no effect on allocations.

Future-disposable-income collateral and ex post debt tax

PROPOSITION 5. *In an economy described by Equations (1) - (3) and under a collateral constraint determined by future-disposable income, as defined in Equation (7), an ex-post debt tax increases the borrowing capacity of the economy during potentially-constrained periods (as long as in those periods there is a positive probability that the economy will remain constrained in the next period). As a result, welfare improves.*

Proof. It follows from Equation (7) and the fact that the relevant income for determining current borrowing capacity is the one expected for the next period. To the extent that there is a positive probability that the credit constraint will bind in the next period, there is also a positive expected value of tax collection ($E_t\tau_{t+1} > 0$) that will be returned to households via a lump-sum transfer $-E_t T_{t+1} = E_t \tau_{t+1} B_{t+2} > 0$, increasing current borrowing capacity.[9]

Furthermore, as explained above (in this subsection) an ex post debt tax transforms Equation (8) into Equation (8.1) during potentially-constrained periods and, as explained in subsection 2.1, this only affects λ. Therefore, the only relevant change for the determination of the equilibrium allocation that results from implementing such a tax is the partial relaxation of the financial constraint, thereby making feasible allocations that yield higher levels of utility. ▮

In general, the expected effect on welfare would be small in this scenario (as illustrated below in the numerical example) since the impact on borrowing capacity depends not on a sure-to-occur transfer, like in the current-disposable-income case, but on the expectation of a transfer (i.e., it will occur with some probability). Furthermore, since decisions in the future-income models are constrained efficient in the absence of interventions, it is in general expected that the space for welfare improvement be smaller than the one in the current-income models.

Future-disposable-income collateral and macroprudential debt tax

Equivalent to the scenario with future-total-income collateral constraint, the DC equilibrium under the future-disposable income collateral is constrained efficient. Therefore, implementing the macroprudential debt tax has a negative effect on social welfare since it distorts decisions that are, in the absence of the intervention, constrained efficient. However, because in this scenario the relevant income for the collateral constraint is the future-disposable one, the macroprudential debt tax also has a positive effect since the borrowing capacity of the economy is increased due to the probable next period transfer (which will occur if the economy is unconstrained in the next period and hence the macroprudential tax applies). The final effect on welfare is in principle ambiguous and will depend on the parameter values. In particular, it is negative for our numerical example (see next section) which employs very standard values.

Notice that this policy is time inconsistent because of the timing of the effects: the expectation of the tax applying in the future yields a positive effect today but, when the time comes to activate it, the policymaker has incentives not to do so since there will be a negative impact on welfare (as a consequence of distorted decisions). Consequently, the activation in the future of this macroprudential tax would be noncredible.

[9]This requires that if the credit constraint binds again in $t+1$ the debt tax will apply. Notice that the policymaker will have no incentives to not activate the tax since it causes no harm to the economy in that period (i.e., it is not a time-inconsistent policy).

In conclusion, if the collateral constraint is determined by future disposable income, an ex post debt tax is expected to have a positive welfare effect, while a macroprudential policy would be time inconsistent and thus not credible (assuming full credibility, its final effect would be negative for standard parameter values).

TABLE 1.

Welfare effects of debt tax policies for each collateral constraint type

	Macroprudential debt tax	**Ex-post debt tax**
Current total income	Implements SP allocation. Increases welfare.	No effect on allocations. No effect on welfare.
Current disposable income	Implements SP allocation. Increases welfare. Same effect as with current-total-income.	Positive effect on borrowing capacity. Effect increasing on debt tax level. Increases welfare.
Future total income	It distorts agents' decisions during unconstrained periods, reducing welfare.	No effect on allocations. Therefore, no effect on welfare.
Future disposable income	Effect is ambiguous. For standard parameter values, there is reduction in welfare. Time-inconsistent policy	Positive effect on borrowing capaci ty Increases welfare.

Table 1 summarizes the derived social welfare results of applying either a macroprudential debt tax or an ex post debt tax under each of the four types of collateral constraints considered. The macroprudential debt tax increases welfare in economies with a collateral constraint that depends on current income, either total or disposable. Instead, in economies under constraints that depend on future income, the macroprudential debt tax distorts decisions that are already constrained efficient, reducing welfare. With regard to an ex post debt tax, this policy has no effect on welfare when the collateral constraint depends on total income; however, it increases welfare, by increasing borrowing capacity, when the collateral constraint depends on disposable income.

Another relevant consideration for policy makers would be the size of the welfare effects obtained from implementing one or the other policy in each context. In the next section, we present numerical results based on simulations for the scenarios considered under standard parameter values.

3. NUMERICAL EXAMPLES

For the simulations, we solve the model using a global nonlinear method similar to the one described by Bianchi (2011). It is based on a basic iteration algorithm that takes into account the existence of the credit constraint and its occasional activation. Initial values are assumed for the endogenous variables and for the relevant expectations of future variables (according to each case). An initial solution of the equation system is obtained for each state - i.e., for given values of $\{B_t, Y_t^T, Y_t^N\}$- (as described in subsection 2.1). The consistency of this solution (expectations and the binding/nonbinding condition of the constraint in each state) is verified and (according to a tolerance level) it is determined if a new iteration is required until there is convergence.

TABLE 2.

Parameter values for each type of collateral constraint

	Parameter	**Value**	
Interest rate	R	1.04	
Coefficient of relative risk aversion	σ	2.00	
Intertemporal elasticity of substitution	ξ	0.83	
Average value of tradable endowment process	y^T	1.00	
Autocorrelation of tradable endowment	ρ^T	0.50	
Standard deviation shocks to tradable endowment	σ_T	0.04	
Weight of tradables in CES	ω	0.31	
		Current Income	**Future Income**
Subjective discount factor	β	0.91	0.93
Credit constraint parameter	κ	0.32	0.29

We suppose that the household's utility function is of a CRRA form, the total consumption aggregator is a CES function between tradable and nontradable consumption. For the models with the financial constraint expressed in terms of current income, we set the same parameter values assigned by Bianchi (2011), with the non-tradable endowment Y_t^N normalized to one and the tradable endowment Y_t^T following a log AR(1) process (See Table 2). Under the current-total-income financial constraint these parameter values imply a frequency of crisis equal to 6.2%.[10] For the models with the constraint determined by future income, we use the

[10]As is standard in the previous literature, a crisis period is defined by the presence of two events: a) the collateral constraint is binding, and b) the current account value is at least one standard deviation above its steady state average.

same parameter set except for two that we recalibrate to match the same frequency of crisis: the discount factor β, and the coefficient in the credit constraint κ. The resulting values are equal to those used by Ottonello et al. (2022) in their model with future income and financial shocks, i.e., $\beta = 0.93$ and $\kappa = 0.29$.

The following analysis is based on the results for the stochastic steady states of the corresponding models, i.e., using the ergodic distribution of $\left\{B, Y^T\right\}$, obtained from 100-thousand-draw simulations. Specifically, we compute the welfare gain (loss) of implementing a specific policy following the consumption-compensating variation, that is to say, as the percentage reduction (increase) in consumption -across all periods and states- that would make the consumer indifferent between the equilibrium with the policy implemented and the one with no intervention. As in the theoretical section, in all cases we solve the model under discretion, and hence do not consider solutions under commitment.

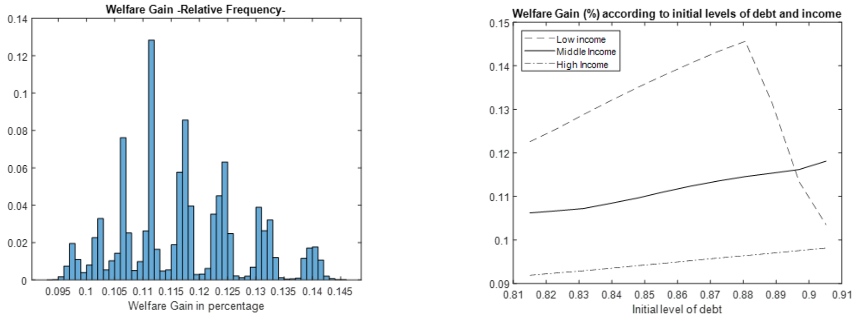
Results for collateral constraints with current income

For the current-income collateral economy, as stated in the abovementioned theoretical results, implementing the macroprudential debt tax generates welfare gains. Furthermore, those gains are equal for both the total and the disposable-income financial constraints. The mean of the welfare gain is 0.12% of consumption and the standard deviation 0.01%.[11] The left panel in Figure 1 shows the (ergodic) distribution of the welfare gain, which takes values between 0.09% and 0.15%, and the right panel illustrates how the welfare gain varies according to the initial level of debt and for three different initial levels of income: low (dashed line), medium (solid line), and high (dash-dotted line) which are, respectively, the lowest, the average, and the highest levels of tradable endowment Y_t^T in the distribution considered. The right panel shows that the welfare gain from implementing a macroprudential debt tax is, in general, increasing in the initial level of debt; however it becomes decreasing for combinations of low initial levels of income and high initial levels of debt. For those combinations, the crisis probability is high; therefore, the macroprudential debt tax has a low probability of mitigating or preventing the upcoming crisis and hence the welfare gain is lower (of course, it still has a mitigation effect on more distant potential crises).

For the case of an ex-post debt tax implemented in an economy under current-total-income financial constraint, as we demonstrated in Proposi-

[11] Figure A1, in the appendix, displays the tax value varying (between 0% and 14.8%) across levels of income and debt. They correspond to the optimal values of the macroprudential debt tax, i.e., the ones that equalize the DC to the SP equilibrium (under discretion) for the current-income collateral economy with the parameter values of Table 2.

FIG. 1. Welfare Gain: macroprudential tax in the current-income case



tion 1, there is no effect on the equilibrium allocation, and thus there is no welfare change.

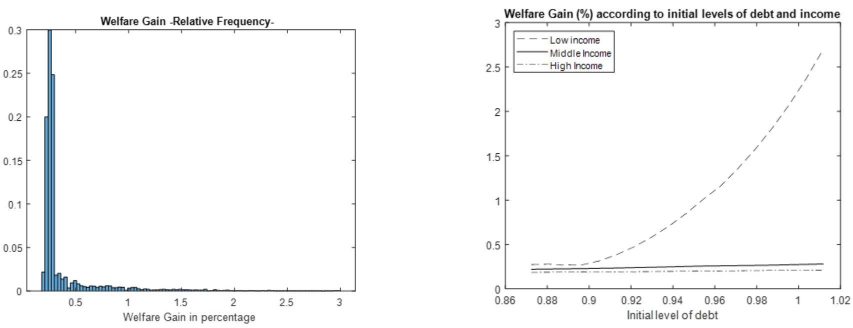
Instead, as proved in Propositon 2, issuing an ex post debt tax when current-disposable income is the relevant one in the collateral constraint results in a welfare gain which, in addition, increases with the level of the debt tax. It is even possible to avoid being financially constrained. For the parameters considered, we find that an ex post debt tax that lies between 1% and 44.5% (conditional on the circumstances: increasing in the level of debt and decreasing in the level of income), which applies only when the financial constraint would bind (in the absence of the tax), it is possible to reach a nonbinding collateral constraint equilibrium.[12] The mean of the welfare gain is 0.38% and the standard deviation 0.34%. The left panel of Figure 2 shows the (ergodic) distribution of the welfare gain, which takes values between 0.19% and 3.0%, and the right panel illustrates how the welfare gain varies according to the level of initial debt and for low (dashed line), medium (solid line) and high (dash-dotted line) initial levels of income. The welfare gain from an ex post debt tax in this economy is increasing in the initial level of debt. Since in this case the debt tax prevents being financially constrained, the greatest impact on welfare is reached for combinations of low initial levels of income and high initial levels of debt.

Results for collateral constraint that depends on future income

[12]Figure A2, in the appendix, displays the tax value varying across levels of income and debt. In this case we have made a search for values of the ex post debt tax that are sufficiently high (see Proposition 2) to reach a nonbinding collateral constraint equilibrium in a current-disposable-income collateral economy with the parameter values of Table 2.

For the future-income collateral constraint economy, we know from the result found by Ottonello et al. (2022) that there is no difference, in terms of welfare, between the DC equilibrium and the one obtained by the SP (under the same constraint). In the case of the future-total-income financial constraint, the ex-post debt tax has no effect on welfare (Proposition 3) and the macroprudential debt tax only distorts constrained-efficient decisions, thereby deteriorating welfare (Proposition 4). By implementing the same values of the macroprudential tax from the current-income case, we obtain a reduction in social well-being.[13] The mean of the welfare loss is 0.016% and the standard deviation 0.002%. The left panel of Figure 3 shows the (ergodic) distribution of the welfare loss, which takes values between 0.01% and 0.02%, and the right panel illustrates how the welfare loss from a macroprudential debt tax varies according to the initial level of debt and for low (dashed line), medium (solid line) and high (dash-dotted line) initial levels of income. The welfare loss is increasing in the initial level of debt and decreasing in the initial level of income.

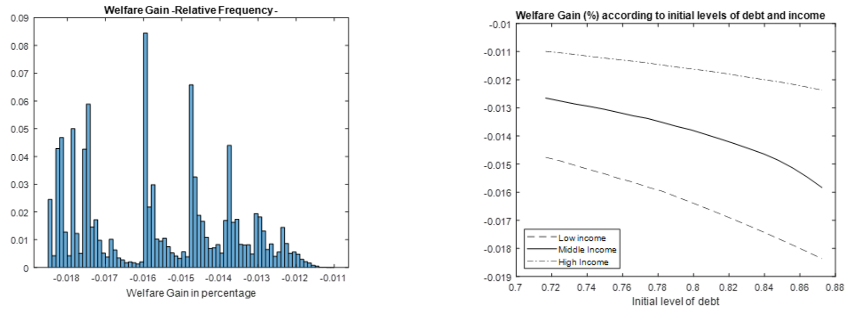
FIG. 2. Welfare Gain: ex post tax in the current-disposable-income case



Results are somewhat similar when levying a macroprudential debt tax under the future-disposable-income financial constraint. The mean of the welfare loss is 0.012% and the standard deviation 0.002%. However, as mentioned above, this scenario presents an offsetting effect coming from the fact that debt capacity is increased due to the expected lump-sum transfers (which reimburse future debt tax payments) and the welfare loss is therefore lower than in the total-income case. The benefit from increasing debt capacity is greater in cases with low initial levels of income and high initial levels of debt such that for those states the welfare loss becomes

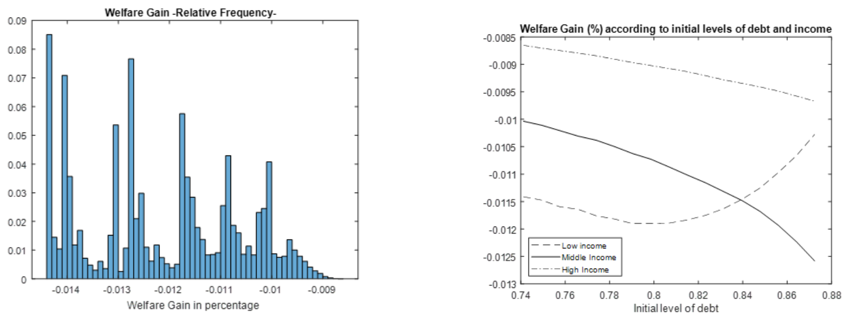
[13]We assume that the policymaker applies the optimal macroprudential debt taxes from an economy under current-income collateral constraint in this economy that actually faces a future-total-income collateral constraint.

FIG. 3. Welfare Gain: macroprudential tax in the future-total-income case



decreasing in the initial level of debt -at least for the particular calibration considered here- (see Figure 4).

FIG. 4. Welfare Gain: macroprudential tax in the future-disposable-income case



These results have been obtained for illustration purposes under the assumption that the future activation of the macroprudential tax is fully credible; however, it must be remembered that, as explained in the previous section, this scenario corresponds to a time-inconsistent policy.

Finally, let us consider the case of an ex-post debt tax implemented in an economy under a future-disposable-income financial constraint. As proved in Proposition 5, such a policy causes no distortion to constrained-efficient decisions (unlike the macroprudential policy) and increases debt capacity due to the expected transfers, improving welfare. For the particular calibration considered here, those welfare gains are relatively small[14]: the

[14]We consider an ex post tax level between 0.06% and 30.7% -increasing in the level of debt and decreasing in the level of income-. With such a tax the frequency of crisis

mean of the welfare gain is 0.013% and the standard deviation is 0.002% (see Figure 5).

FIG. 5. Welfare Gain: ex post tax in the future-disposable-income case

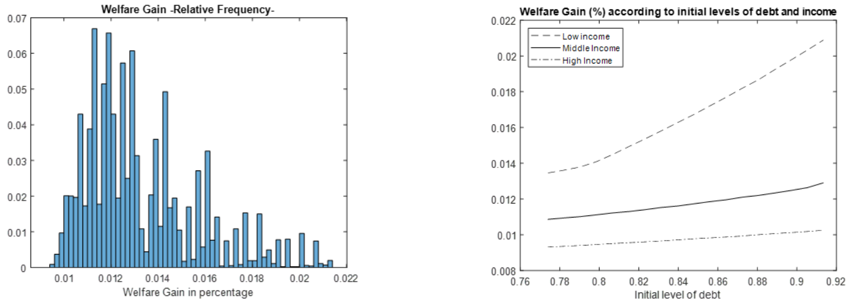


Table 3 summarizes all the welfare results from the simulations carried out in this section.

TABLE 3.

Welfare gain (consumption-compensating variation in %) by collateral constraint and debt tax policy

	Welfare gain	**Macroprudential debt tax**	**Ex post debt tax**
Current total income	Average	0.12	0
	Standard deviation	0.01	
Current disposable income	Average	0.12	0.38
	Standard deviation	0.01	0.34
Future total income	Average	−0.016	0
	Standard deviation	0.002	
Future disposable income	Average	−0.012	0.013
	Standard deviation	0.002	0.002

4. CONCLUSION

The specific form of the collateral constraint is not innocuous for the analysis of financial crises. As previous literature has shown, an economy with a collateral constraint determined by current income exhibits a pecuniary externality and overborrowing. Instead, in an economy with a collateral constraint determined by future income, the DC equilibrium is

reduces to 2.2%. Figure A3, in the appendix, displays the tax value varying across levels of income and debt.

constrained-efficient, and therefore there is no need for policy intervention to equalize the DC and SP optimal allocations.

In this paper, we study the welfare consequences of implementing a policy designed for a particular type of credit constraint in an economy that is actually facing a different one (e.g., implementing a macroprudential tax — suggested by the related literature as the convenient one for a current-income borrowing constraint — in an economy actually facing a future-income constraint). Particularly, we analyze the welfare effects of implementing either of two policies: an ex ante (or macroprudential) debt tax or an ex post debt tax, across the four possible collateral constraint scenarios (derived by combining current/future with total/disposable income).

We find that imposing an ex post debt tax is a more favorable intervention policy — with regard to welfare — than a macroprudential debt tax in three of the four scenarios considered. The macroprudential debt tax is welfare improving when the economy faces a current-income credit constraint but welfare reducing with a future-income one (whether total or disposable). Under a current-income collateral constraint, a macroprudential debt tax reduces debt variability which helps to mitigate the negative effects when the credit constraint binds. In contrast, with a future-income collateral constraint, borrowing decisions are already constrained-efficient, and the macroprudential policy reduces welfare by distorting these decisions. Instead, an ex post debt tax (returned to households via a lump-sum subsidy) increases disposable income but does not affect total income, and hence it has a welfare-improving impact when the economy faces a disposable-income credit constraint and no effect when it is a total-income one.

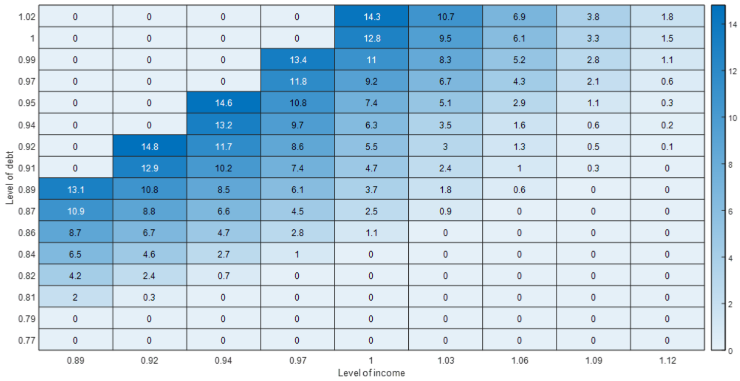
For the scenarios considered, the macroprudential debt tax reduces welfare when applied in an economy with a collateral constraint determined by future income. To avoid unintended welfare reductions, the most favorable policy is an ex post debt tax. At worst, it does not affect welfare, and at best, it can increase welfare when the collateral constraint depends on disposable income.

APPENDIX

Figure A1 displays the optimal values of the macroprudential debt tax (i.e., the ones that equalize the DC to the SP equilibrium under discretion) varying across levels of income and debt for the current-income collateral economy with the parameter values of Table 2.

Figure A2 presents an example of ex post debt tax values, varying across levels of income and debt, that are sufficiently high (see Proposition 2) to reach a nonbinding collateral constraint equilibrium in a current-

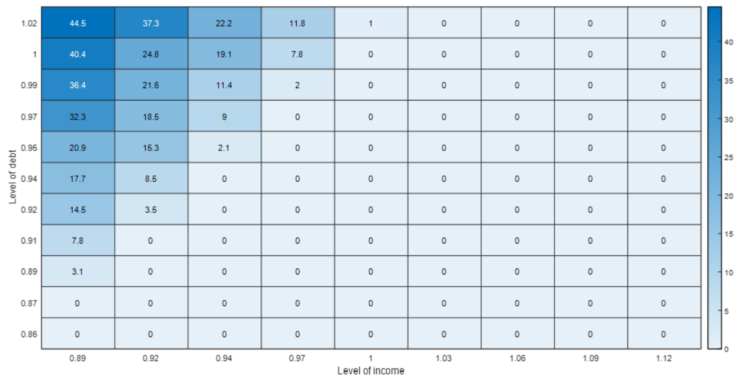
Figure A1. Macroprudential tax values



Darker blue indicates a higher level of tax (the value in percentage is inside the cell). The macroprudential debt tax is by definition nil for constrained periods (i.e., for combinations of high debt an low income)

disposable-income collateral economy with the parameter values of Table 2.

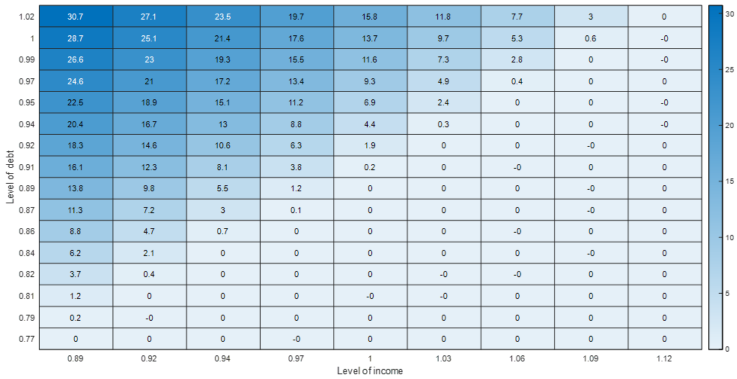
Figure A2. Ex post tax values for the current-disposable-income case



Darker blue indicates a higher level of tax (the value in percentage is inside the cell) The debt tax is nonzero for potentially constrained periods only.

Figure A3 presents an example of ex post debt tax values, varying across levels of income and debt, to illustrate that in an economy with a future-disposable-income collateral constraint -and under standard parameter values (see Table 2)- implementing such a tax yields welfare gains, albeit very modest ones. With these tax values, the frequency of crisis drops significantly, from 6.2% to 2.2%; however, the average welfare gain is small, just 0.013% (see Table 3).

Figure A3. Ex post tax values for the future-disposable-income case



Darker blue indicates a higher level of tax (the value in percentage is inside the cell)
The debt tax is nonzero for potentially constrained periods only.

REFERENCES

Benigno, Gianluca, Huigang Chen, Christopher Otrok, Alessandro Rebucci, and Eric R. Young, 2016. Optimal capital controls and real exchange rate policies: A pecuniary externality perspective. *Journal of Monetary Economics* **84**, 147-165.

Bianchi, Javier, 2011. Overborrowing and systemic externalities in the business cycle. *American Economic Review* **101**, 3400-3426.

Bianchi, Javier, 2016. Efficient bailouts? *American Economic Review* **106**, 3607-3659.

Bianchi, Javier, and Enrique Mendoza, 2018. Optimal time-consistent macroprudential policy. *Journal of Political Economy* **126**, 588-634.

Devereux, Michael B., Eric R. Young, and Changhua Yu, 2019. Capital controls and monetary policy in sudden-stop economies. *Journal of Monetary Economics* **103**, 52-74.

Jeanne, Olivier, and Anton Korinek, 2019. Managing credit booms and busts: A Pigouvian taxation approach. *Journal of Monetary Economics* **107**, 2-17.

Jeanne, Olivier, and Anton Korinek, 2020. Macroprudential regulation versus mopping up after the crash. *Review of Economic Studies* **87**, 1470-1497.

Korinek, Anton, 2011. The new economics of prudential capital controls: A research agenda. *IMF Economic Review* **59**, 523–561.

Lorenzoni, G., 2008. Inefficient credit booms. *Review of Economic Studies* **75**, 809-833.

Mendoza, Enrique, 2002. Credit, prices, and crashes: Business cycles with a sudden stop. In Preventing currency crises in emerging markets. Edited by S. Edwards and J. A. Frankel . Chicago, IL: University of Chicago Press and National Bureau of Economic Research.

Ottonello, Pablo, Diego J. Perez, and Paolo Varraso, 2022. Are collateral-constraint models ready for macroprudential policy design? *Journal of International Economics* **139**.

Vargas, Carmiña O., and Julian A. Parra-Polania, 2021. Analysis of macro-prudential and ex post financial crisis interventions: Relevance of the fiscal-policy setup. *International Journal of Finance & Economics* **26**, 3759-3769.